Explicit Transfer Function of RC Polyphase Filter for Wireless Transceiver Analog Front-End

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Contents

Research Goal

- To establish systematic design and analysis methods of RC polyphase filters.

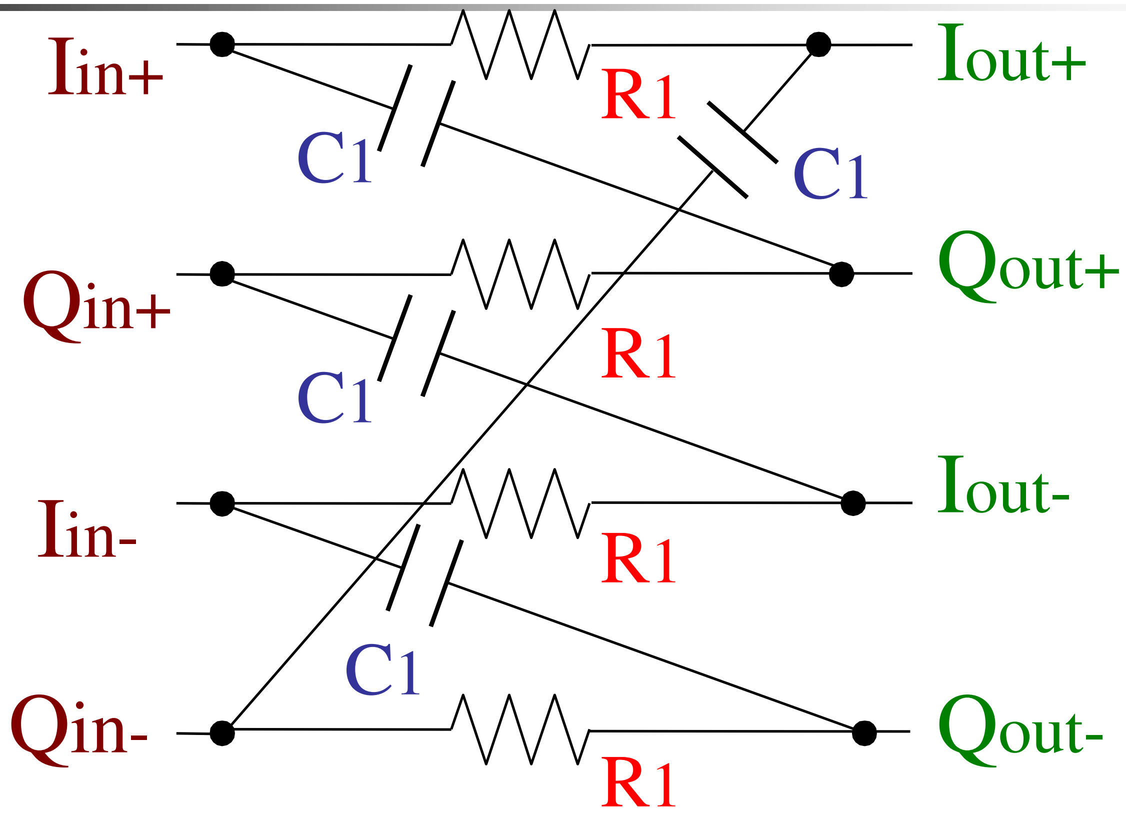
- As its first step,
  to derive explicit transfer functions of
  the 1st-, 2nd- and 3rd-order
  RC polyphase filters.

Roles of RC Polyphase Filter in Wireless Transceiver

Features of RC Polyphase Filter

- Its input and output are complex signal.
- Passive RC analog filter
- One of key components in wireless transceiver analog front-end
  - I, Q signal generation
  - Image rejection
- Its explicit transfer function has not been derived yet.

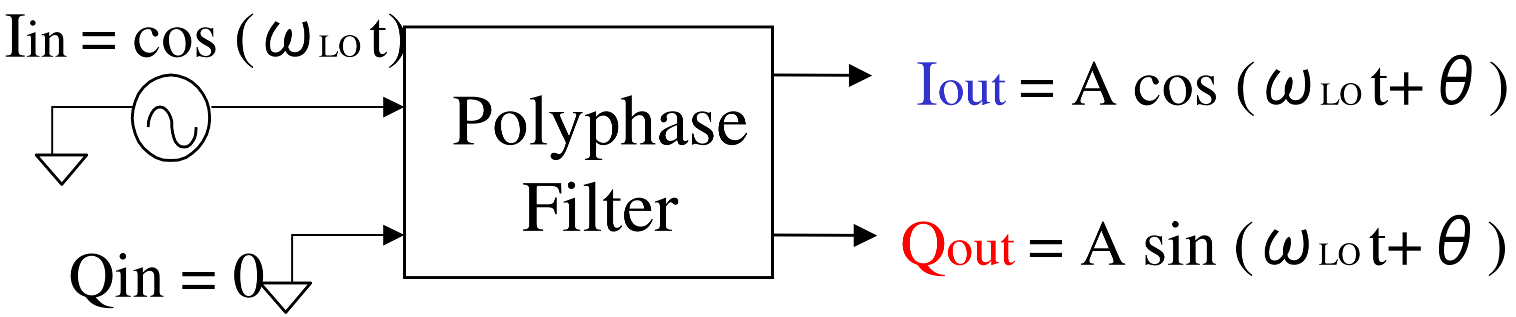
First-order RC Polyphase Filter



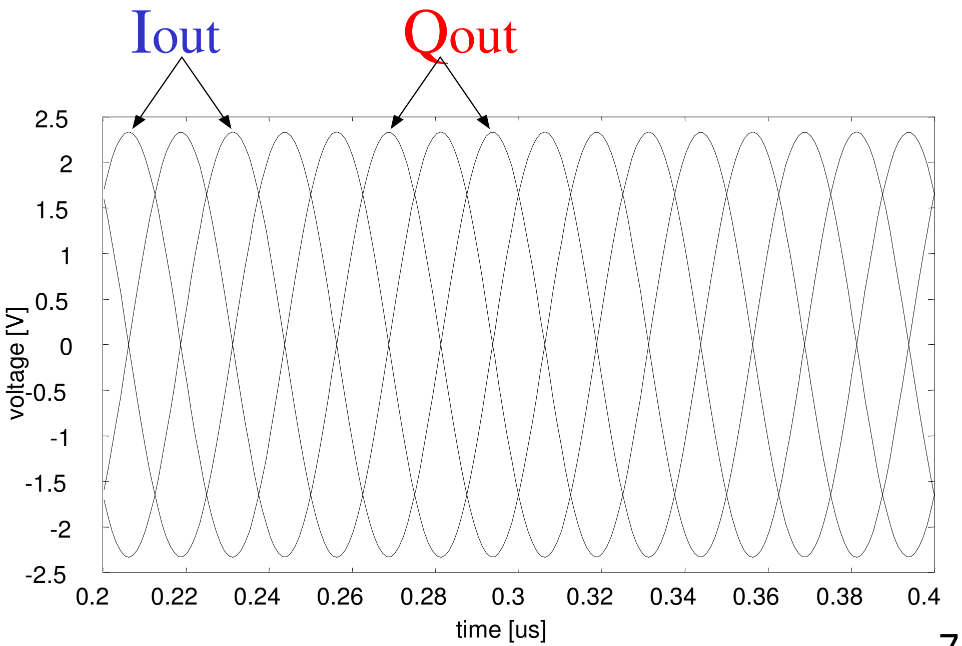
Differential Complex Input: $V_{in} = I_{in} + j\,Q_{in}$

Differential Complex Output: $V_{out} = I_{out} + j\,Q_{out}$

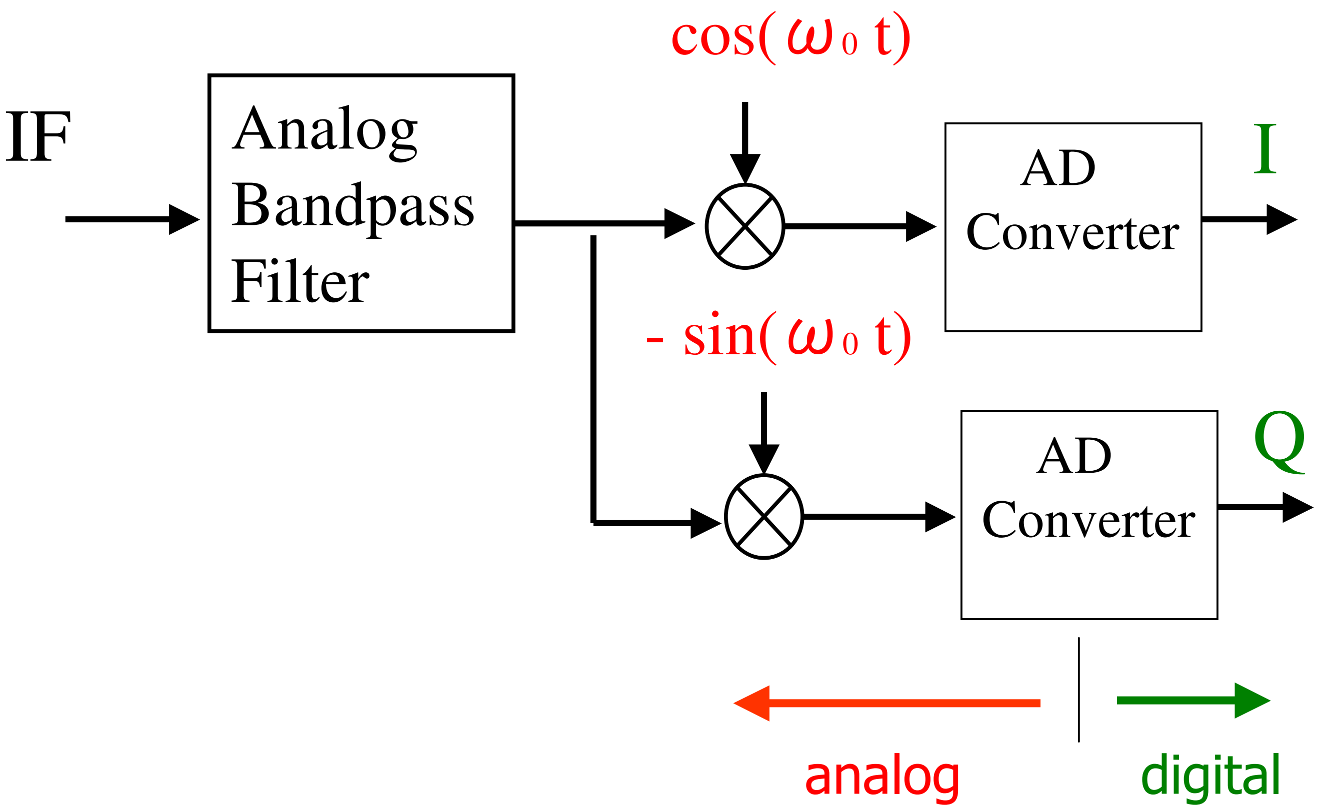
I, Q signal generation from single sinusoidal input



$$\omega_{LO} = \frac{1}{R_1 C_1}$$



Cosine, Sine Signals in Receiver



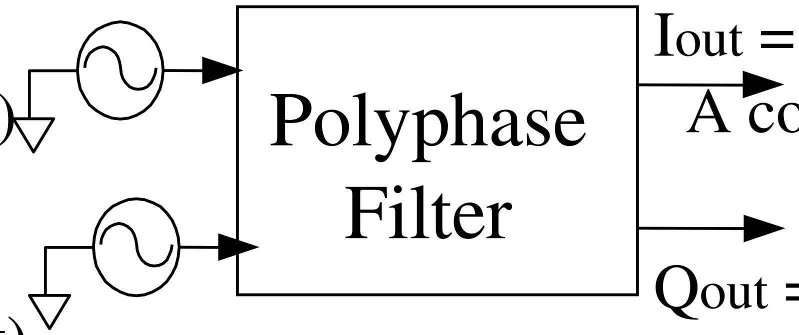
They are used for down conversion

Pure I, Q signal generation

3rd-order harmonics rejection

Iin= $\cos(\omega_{LO}t)+B\cos^3(\omega_{LO}t)$

Qin= $\sin(\omega_{LO}t)+B\sin^3(\omega_{LO}t)$



Iout = $A\cos(\omega_{LO}t+\theta)$

Qout = $A\sin(\omega_{LO}t+\theta)$

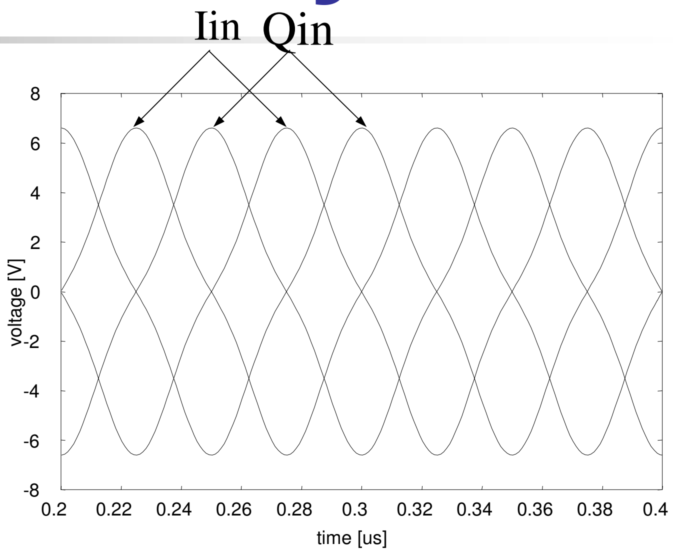
With
3rd-order harmonics.

Without
3rd-order harmonics.

Simulation of 3rd-order harmonics rejection

$$I_{in}(t) = \cos(\omega_{LO}t) + a\cos^3(\omega_{LO}t)$$

$$Q_{in}(t) = \sin(\omega_{LO}t) + a\sin^3(\omega_{LO}t)$$



$$3\omega_{LO} = \frac{1}{R_1C_1}$$

$$I_{out}(t) = A\cos(\omega_{LO}t + \theta)$$

$$Q_{out}(t) = A\sin(\omega_{LO}t + \theta)$$

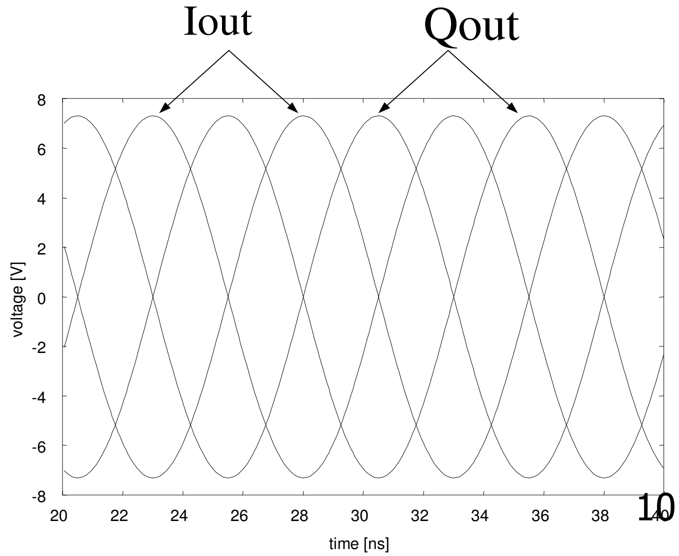
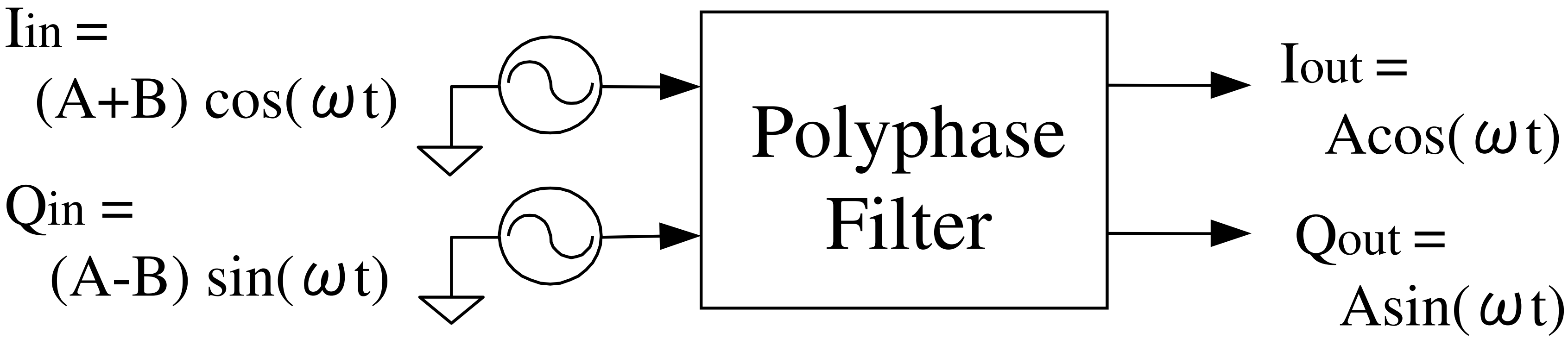


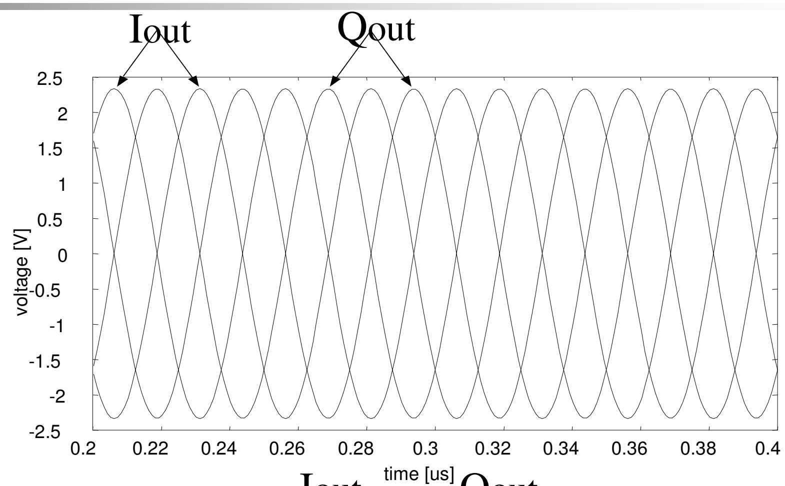
Image Rejection Filter



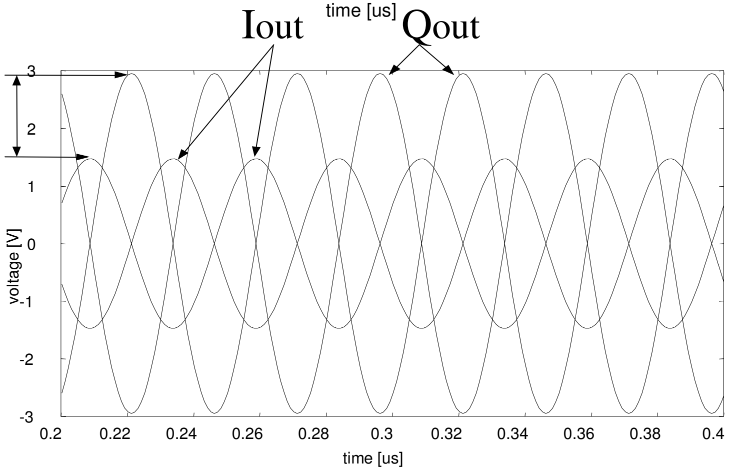
$$Ae^{j\omega t} + Be^{-j\omega t} \quad \Rightarrow \quad Ae^{j\omega t}$$

Problem when $\omega_{LO} \neq 1/R_1C_1$

$$\omega_{LO} = \frac{1}{R_1 C_1}$$



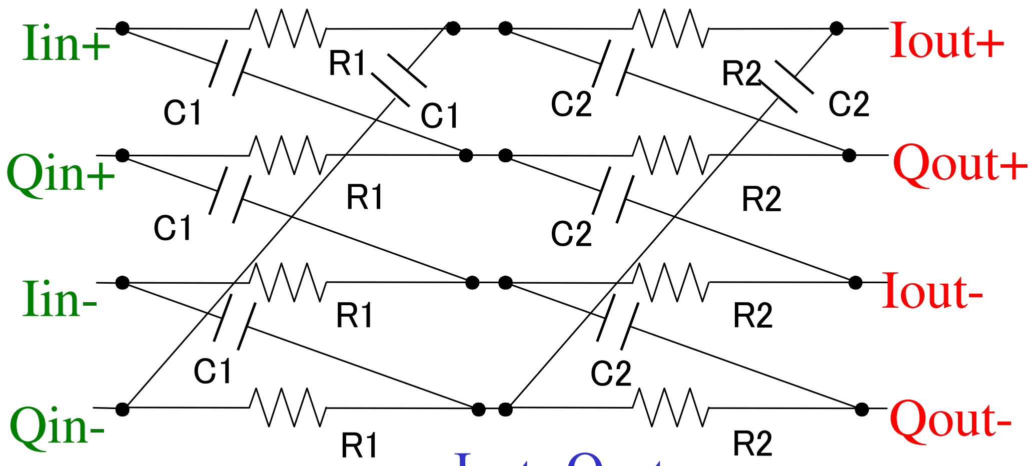
$$\omega_{LO} = \frac{2}{R_1 C_1}$$



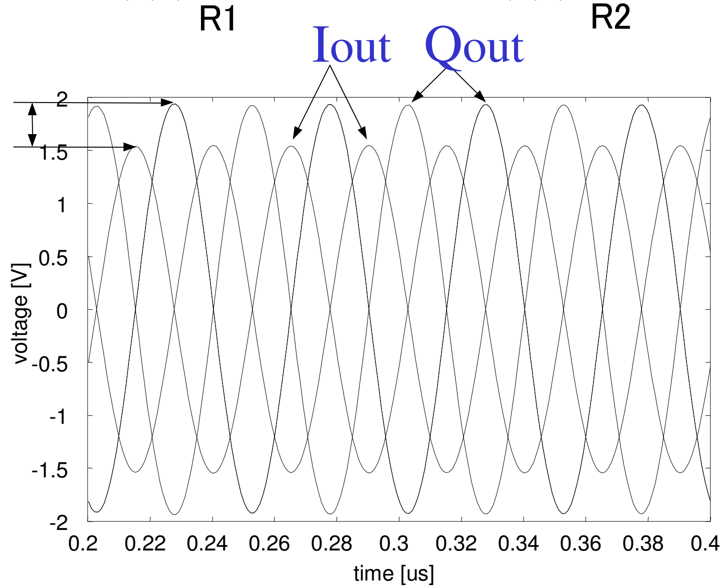
Amplitudes of I,Q signals differ significantly.

2nd-order RC Polyphase Filter

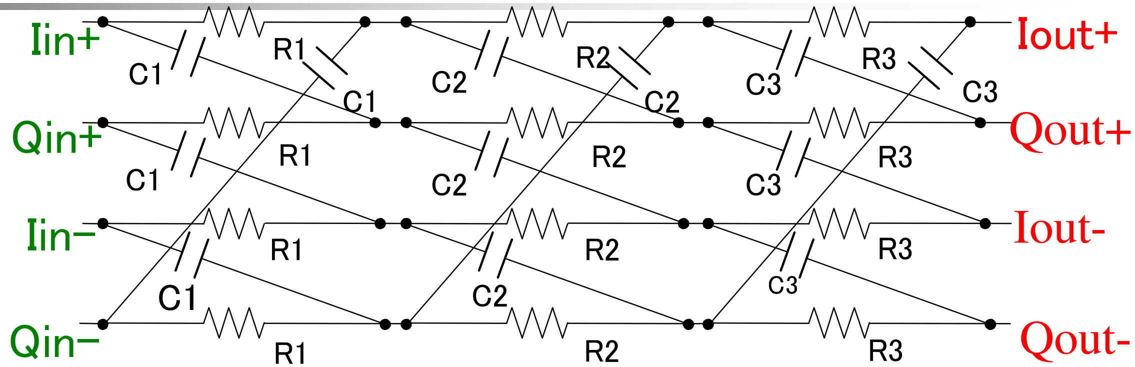
The problem of large difference between Iout, Qout amplitudes can be alleviated



$$\omega_{LO} = \frac{2}{R_1 C_1}$$

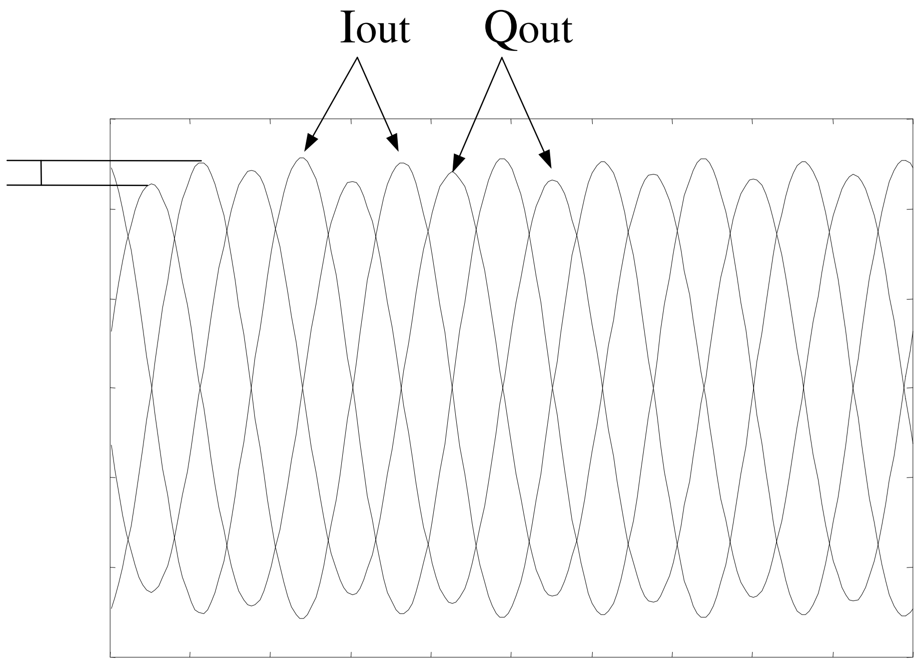


3rd-order RC Polyphase Filter



The amplitude difference problem is further alleviated.

$$\omega_{LO} = \frac{2}{R_1 C_1}$$



Transfer Function Derivation of RC Polyphase Filter

Transfer Function of RC Polyphase Filter

- Complex Signal Theory

- Complex input
- Complex output

$$V_{in}(j\omega) = I_{in} + j \cdot Q_{in}$$

$$V_{out}(j\omega) = I_{out} + j \cdot Q_{out}$$

- Complex Transfer Function

$$G(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$$

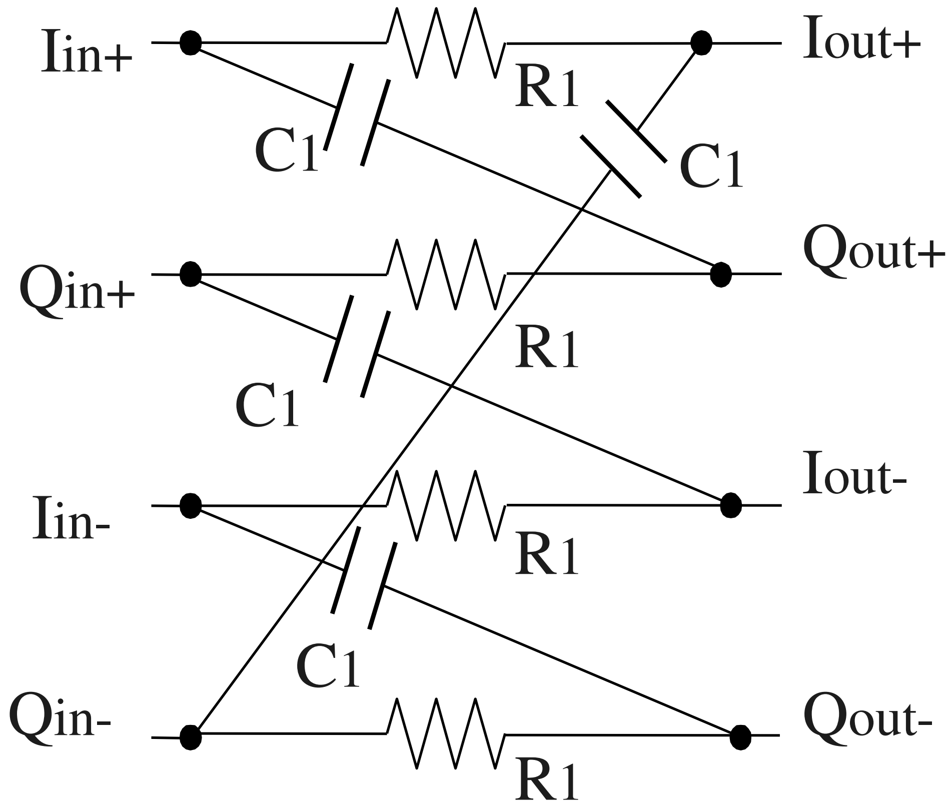
Transfer Function of 1st-order RC Polyphase Filter

Differential signal

$$I_{in}(t) = I_{in+}(t) - I_{in-}(t)$$
$$Q_{in}(t) = Q_{in+}(t) - Q_{in-}(t)$$
$$I_{out}(t) = I_{out+}(t) - I_{out-}(t)$$
$$Q_{out}(t) = Q_{out+}(t) - Q_{out-}(t)$$

Complex signal

$$V_{in}(t) = I_{in}(t) + jQ_{in}(t)$$
$$V_{out}(t) = I_{out}(t) + jQ_{out}(t)$$



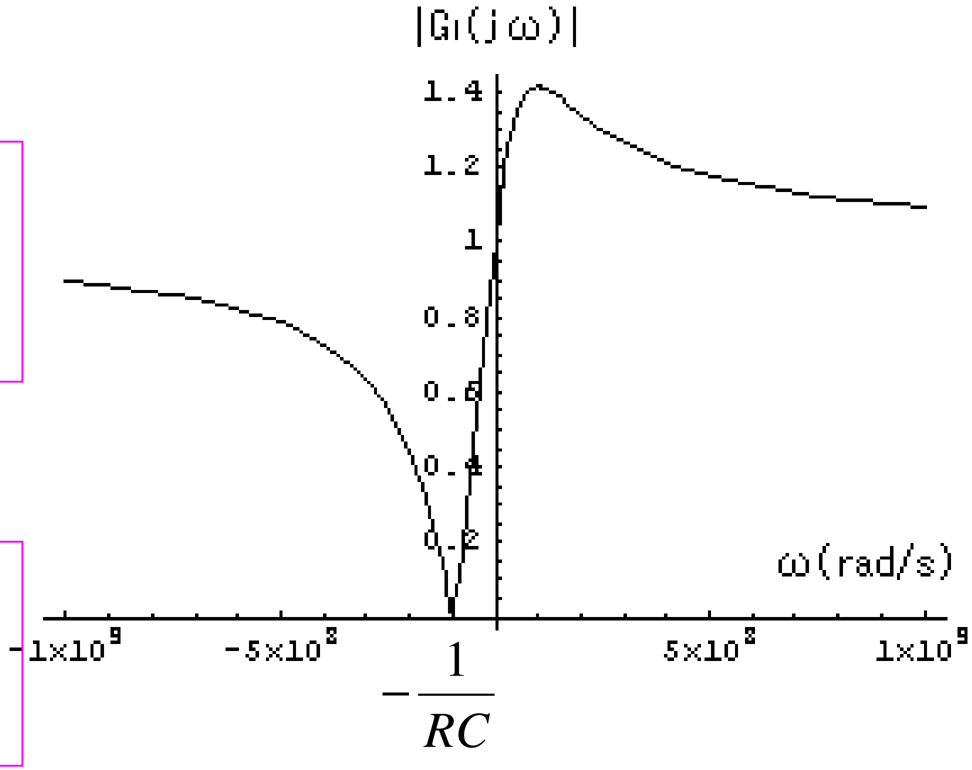
Transfer Function of 1st-order RC Polyphase Filter

- Transfer Function

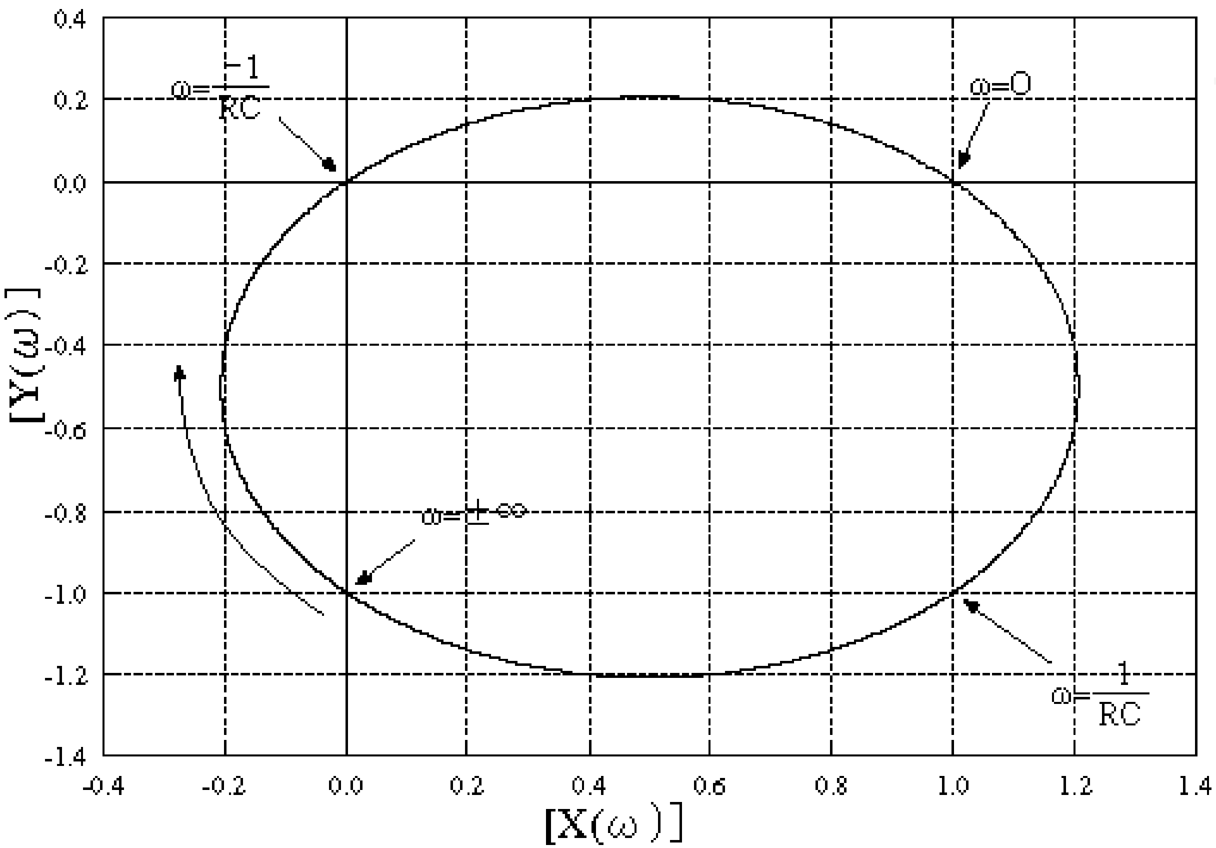
$$G_1(j\omega) = \frac{1+\omega RC}{1+j.\omega RC}$$

- Gain

$$\left|G_1(j\omega)\right| = \frac{\left|1+\omega RC\right|}{\sqrt{1+(\omega RC)^2}}$$



Nyquist Chart of G1(jw)



$G_1(j\,\omega) = X(\omega) + j\,Y(\omega)$

Symmetric with respect to a line of Y = -X.

Explanation of I, Q signal generation by G1(jω)

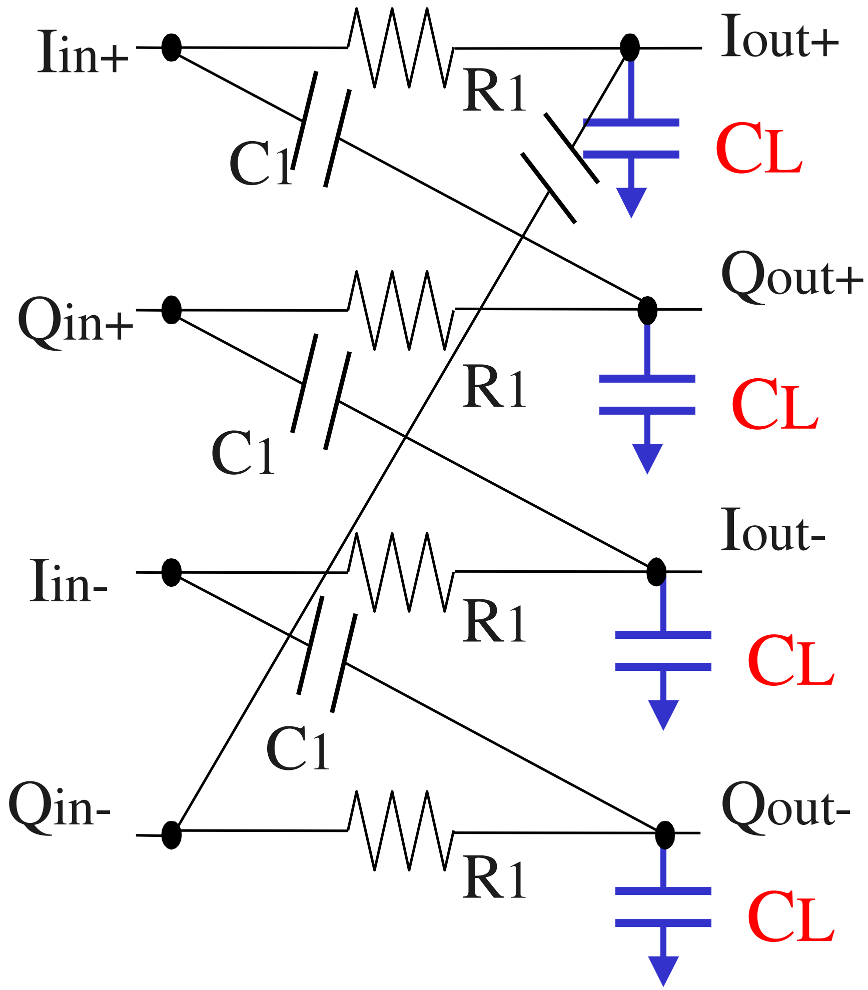
$$Q_{in}(t) \equiv 0, I_{in}(t) = \cos(\omega t),$$
$$V_{in}(t) = \cos(\omega t) = \frac{1}{2}[e^{j\omega t} + e^{-j\omega t}]$$

$$\left|G_1(j\omega)\right|_{\omega=-\frac{1}{RC}} = 0, \left|G_1(j\omega)\right|_{\omega=\frac{1}{RC}} = \sqrt{2}, \angle G_1(j\omega) = -\frac{\pi}{4}$$

$$V_{out}(t) = \frac{1}{2}[\left|G_1(j\omega)\right| e^{j(\omega t + \angle G_1(j\omega))} + \left|G_1(-j\omega)\right| e^{j(-\omega t + \angle G_1(-j\omega))}]$$
$$= \frac{\sqrt{2}}{2}\cos(\omega t - \frac{\pi}{4}) + j\frac{\sqrt{2}}{2}\sin(\omega t - \frac{\pi}{4})$$

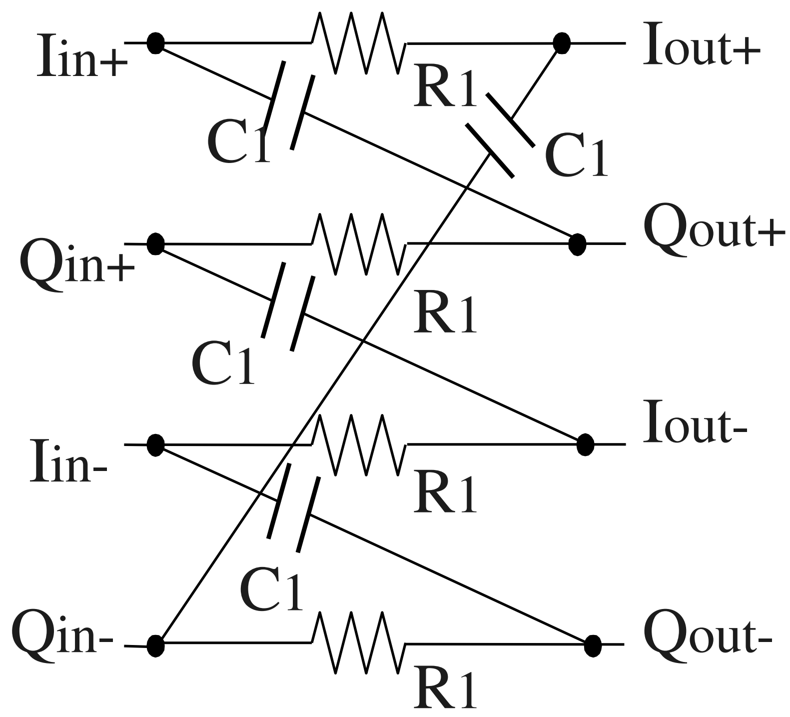
Output Load (C_L) Effects

$$G_1(j\omega) = \frac{1+\omega R_1 C_1}{1+j\omega R_1(C_1+C_L)}$$



Input Impedance

$$\text{Input Impedance } Z_{in} = \frac{\text{Complex Input Voltage}}{\text{Complex Input Current}}$$



$$Z_{in} = \frac{1+j\omega R_1 C_1}{2\,j\omega C_1\,[1+j(1+\omega R_1 C_1)]}$$

Component Mismatch Effects

Mismatches δX among R's, C's

Image signal $\overline{Vout}$ is caused.

$$\mathrm{V}out = \mathrm{G}_1 \cdot Vin + E_1 \cdot \delta X \cdot \overline{Vin}$$

$$\overline{\mathrm{V}out} = \overline{\mathrm{G}_1} \cdot \overline{Vin} + \overline{E_1} \cdot \delta X \cdot Vin$$

where $$E_1 := \frac{(1+j)R_1C_1\omega}{2(1+jR_1C_1\omega)^2}$$

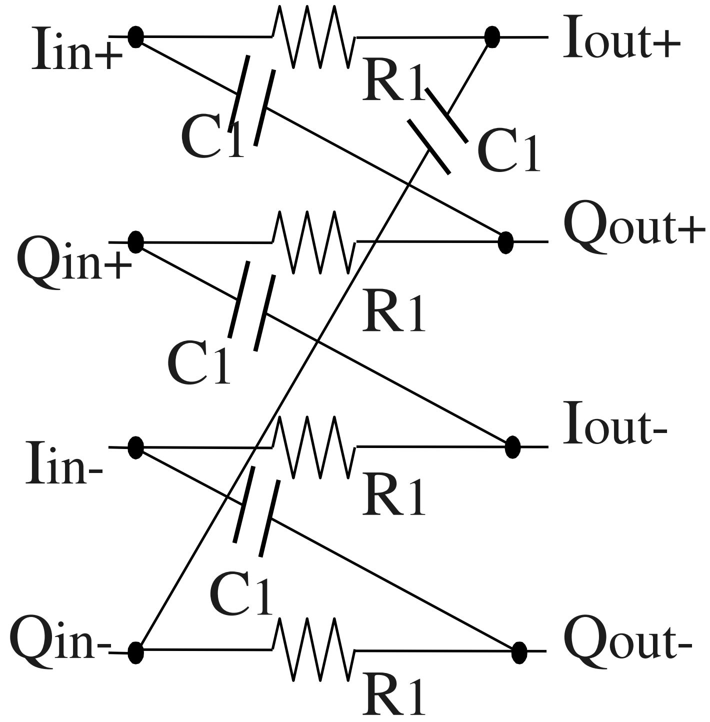


Image transfer function

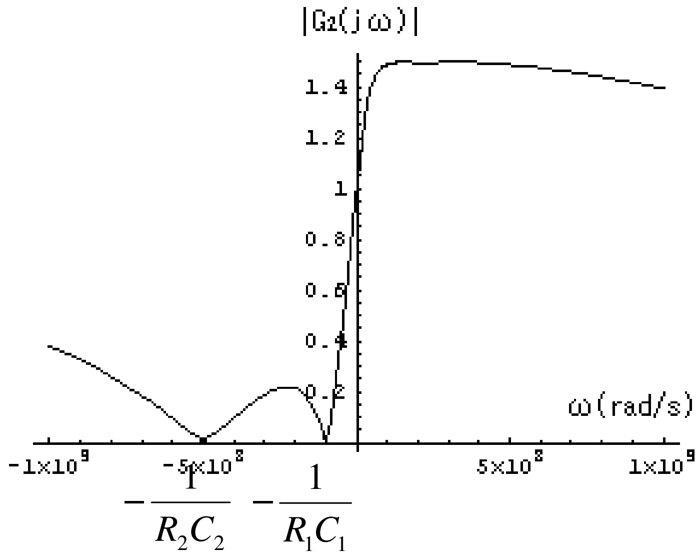
Transfer Function of 2nd-order RC Polyphase Filter

Transfer Function

$$G_2(j\omega) = \frac{(1+\omega R_1 C_1)(1+\omega R_2 C_2)}{1-\omega^2 R_1 C_1 R_2 C_2 + j\omega(C_1 R_1 + C_2 R_2 + 2R_1 C_2)}$$

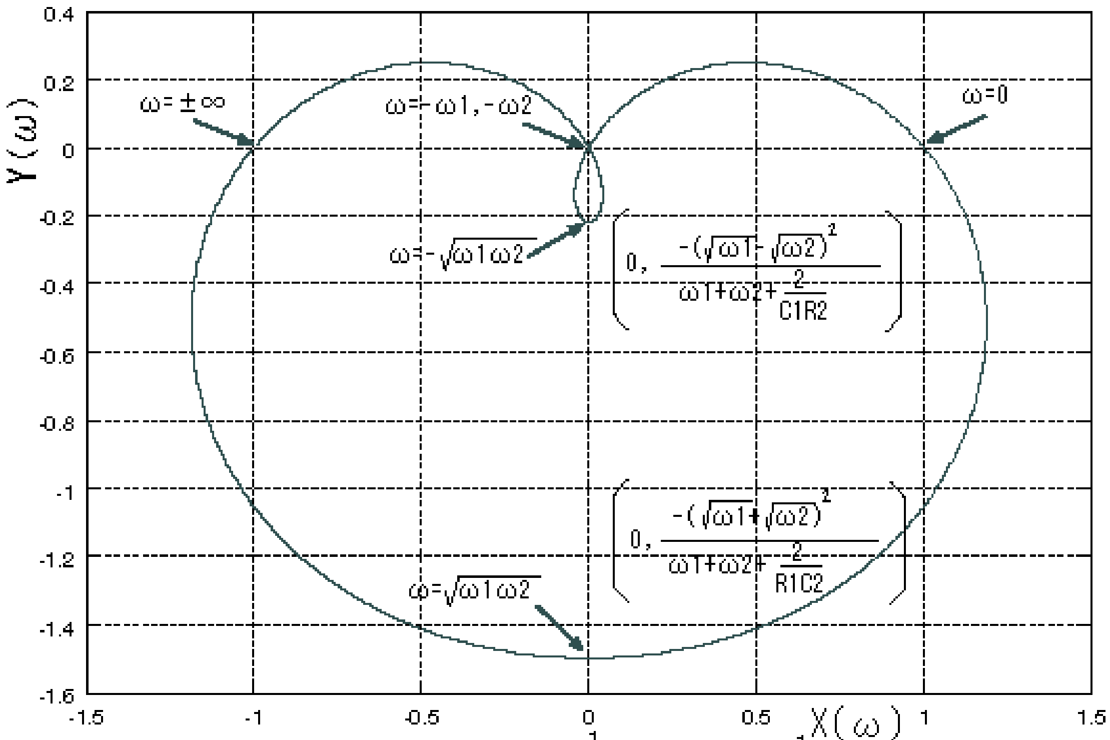
Derivation is very complicated, so we used "Mathematica."

Gain $|G_2(j\omega)|$ characteristics



Nyquist Chart of G2(jω)

Symmetric with respect to a line of X = 0.



$\omega 1 = \frac{1}{R1C1}$ $\quad \omega 2 = \frac{1}{R2C2}$

$G2(j\omega) = X(\omega) + jY(\omega)$

Features of 2nd-order RC Polyphase Filter

$$\left|G_2(j\omega)\right| \neq \left|G_2(-j\omega)\right|, \quad \left|G_2(j\omega_1)\right| = \left|G_2(j\omega_2)\right|,$$

$$\left|G_2(-j\omega_1)\right| = \left|G_2(-j\omega_2)\right| = 0,$$

$$\lim_{\omega\to\pm\infty}\left|G_2(j\omega)\right| = 1, \qquad \left|G_2(j0)\right| = 1,$$

$$\left[\frac{\partial\left|G_2(j\omega)\right|}{\partial\omega}\right]_{\omega=\sqrt{\omega_1\omega_2}} = 0, \quad \frac{|G_2(j\sqrt{\omega_1\omega_2})|}{|G_2(-j\sqrt{\omega_1\omega_2})|} = \frac{(\sqrt{\omega_1}+\sqrt{\omega_2})^2}{(\sqrt{\omega_1}-\sqrt{\omega_2})^2}$$

For arbitrary *a*, $\left|G_1(ja\sqrt{\omega_1\omega_2})\right| = \left|G_1(j\sqrt{\omega_1\omega_2}/a)\right|.$

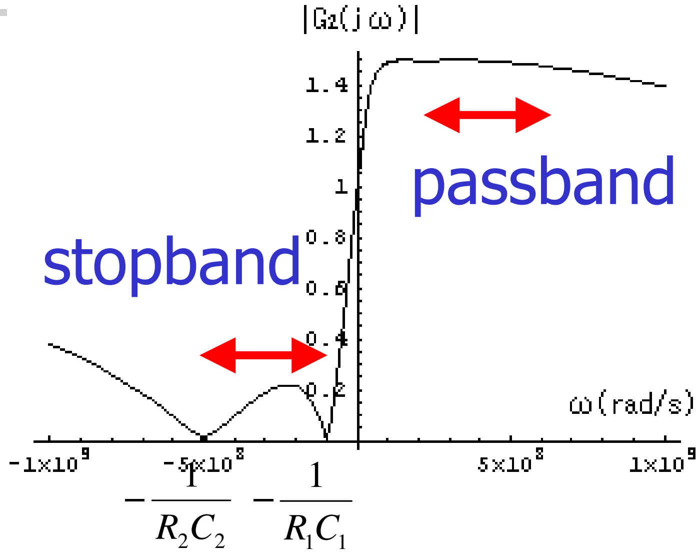
Flat passband design of 2^{nd}-order RC Polyphase Filter

Passband: $\omega_1 \sim \omega_2$

Stopband: $-\omega_1 \sim -\omega_2$

where $\omega_1 := 1/R_1C_1$,

$\omega_2 := 1/R_2C_2$



- To make gain in passband flat,
  $|G_2(j\omega_1)| = |G_2(j\omega_2)| = |G_2(j\sqrt{\omega_1\omega_2})|$.

- Image Rejection Ratio = $\left(\frac{\sqrt{\omega_2}+\sqrt{\omega_1}}{\sqrt{\omega_2}-\sqrt{\omega_1}}\right)^2$

Explanation why a 2nd-order filter reduces I, Q amplitude difference.

Input signal

$$Q_{in}(t) \equiv 0, I_{in}(t) = \cos(\omega t) = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t})$$

Output of a 1st-order filter

$$I_{out1}(t) + jQ_{out1}(t) = \frac{1}{2}[|G_1(j\omega)|e^{j\omega t+\theta 1} + |G_1(-j\omega)|e^{-j\omega t-\theta 1}]$$

$$= |G_1(j\omega)|(1+\frac{|G_1(-j\omega)|}{|G_1(j\omega)|})\cos(\omega t+\theta_1) + j|G_1(j\omega)|(1-\frac{|G_1(-j\omega)|}{|G_1(j\omega)|})\sin(\omega t+\theta_1)$$

Explanation why a 2nd-order filter reduces I, Q amplitude difference.

Output of a 2nd-order filter

$$I_{out2}(t) + jQ_{out2}(t) = \frac{1}{2}[\left|G_2(j\omega)\right| e^{j\omega t+\theta 2} + \left|G^2(-j\omega)\right| e^{-j\omega t-\theta 2}]$$

$$= \left|G_2(j\omega)\right|(1+\frac{\left|G_2(-j\omega)\right|}{\left|G_2(j\omega)\right|})\cos(\omega t+\theta_2) + j\left|G_2(j\omega)\right|(1-\frac{\left|G_2(-j\omega)\right|}{\left|G_2(j\omega)\right|})\sin(\omega t+\theta_1)$$

- According to the transfer functions,

$$\frac{\left|G_1(-j\omega)\right|}{\left|G_1(j\omega)\right|} >> \frac{\left|G_2(-j\omega)\right|}{\left|G_2(j\omega)\right|} \quad \text{at } \omega \approx \omega_{LO}$$

then, the amplitude difference is reduced.

Transfer Function of 3rd-order RC Polyphase Filter

$$G_3(j\omega) = \frac{N_3(j\omega)}{D_{3R}(\omega) + jD_{3I}(\omega)}$$

where

$$N_3(j\omega) = (1+\omega R_1 C_1)(1+\omega R_2 C_2)(1+\omega R_3 C_3)$$

$$D_{3R}(\omega) =$$

$$1-\omega^2[R_1C_1R_2C_2 + R_2C_2R_3C_3 + R_1C_1R_3C_3 + 2R_1C_3(R_2C_2 + R_2C_1 + R_3C_2)]$$

$$D_{3I}(\omega) =$$

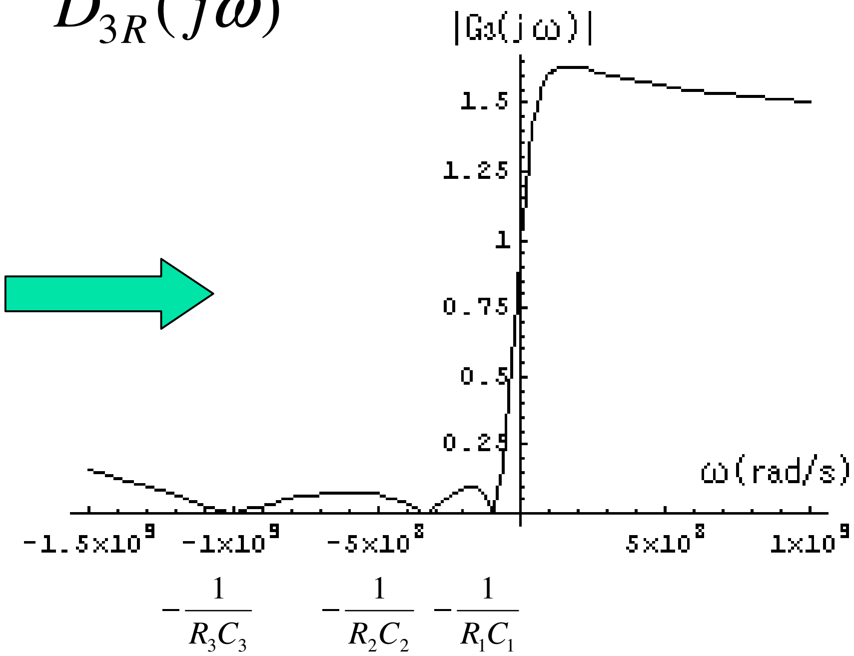
$$\omega[R_1C_1 + R_2C_2 + R_3C_3 + 2(R_1C_2 + R_2C_3 + R_1C_3)] - \omega^3 R_1C_1R_2C_2R_3C_3$$

Gain, Phase of 3rd-order RC Polyphase Filter

Gain: $$|G_3(j\omega)| = \frac{|N_3(j\omega)|}{\sqrt{D_{3R}(j\omega)^2 + D_{3I}(j\omega)^2}}$$

Phase: $$\tan(\angle G_3(j\omega)) = -\frac{D_{3I}(j\omega)}{D_{3R}(j\omega)}$$

Gain characteristics



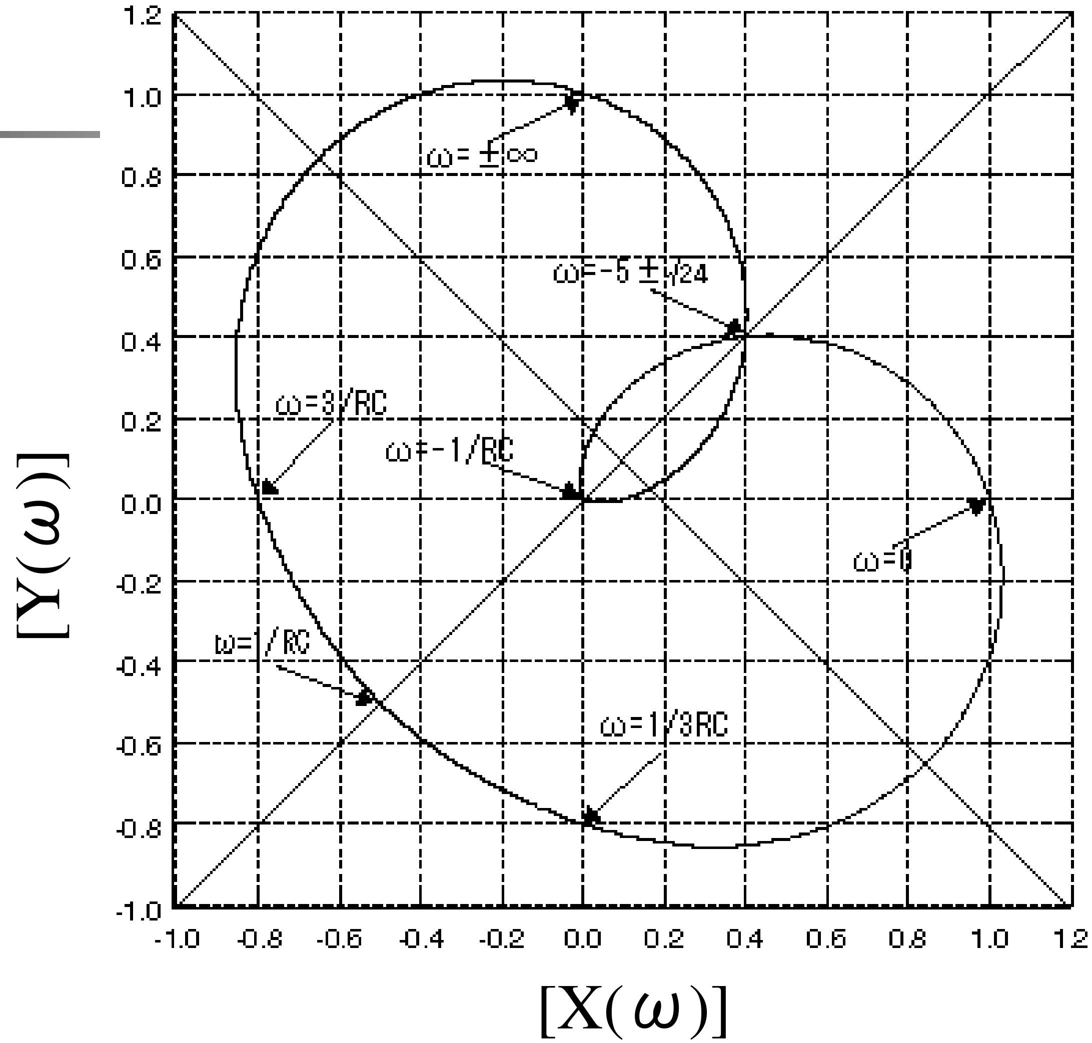
Nyquist Chart of $G_3(j\omega)$

In case

$R_1C_1=R_2C_2$

$=R_3C_3$

Symmetric with respect to a line of Y= X.



$G_3(j\,\omega) = X(\omega) + j\,Y(\omega)$

Summary

- Explicit transfer functions of
  1st-, 2nd- and 3rd-order RC polyphase filters.

  ⇨ Systematic design and analysis are possible.

- On-going projects
  - Derivation of higher-order filter ones.
  - Nonideality effects in higher-order filters.
  - Systematic design method using Nyquist chart.