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Digitally-Assisted Compensation for Timing Skew in ATE Systems

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Contents

Research Goal

Timing skew is a major problem in ATE systems

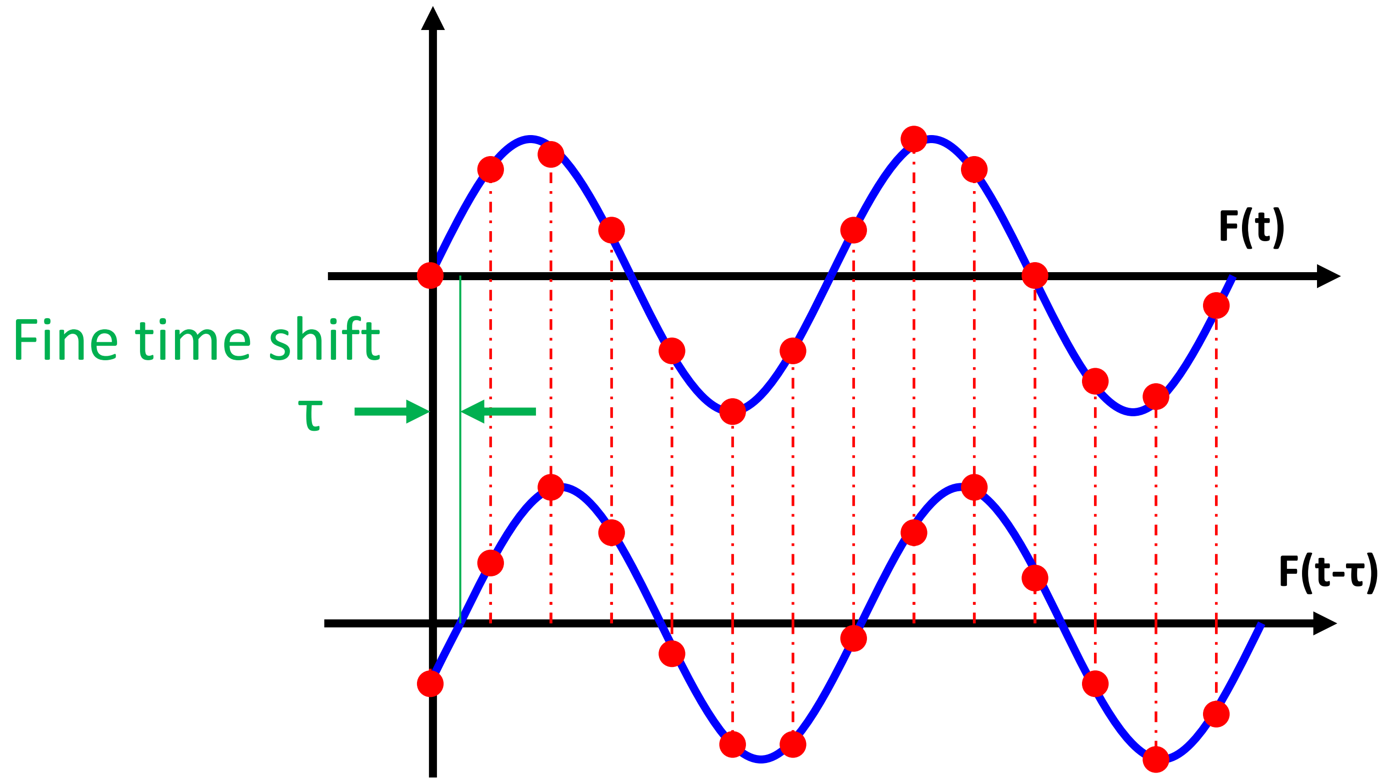
Digital compensation for timing skew

⇒ Linear phase is important

Conventional linear-phase digital filter ⇒ coarse timing adjustment

Proposed linear-phase digital filter ⇒ fine timing adjustment

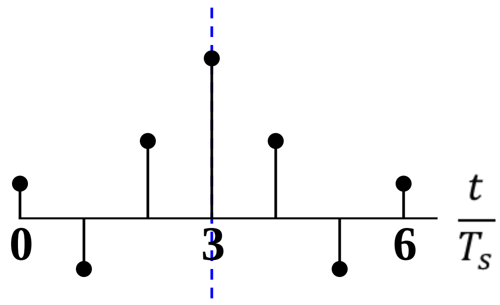
Features of Proposed Digital Filter



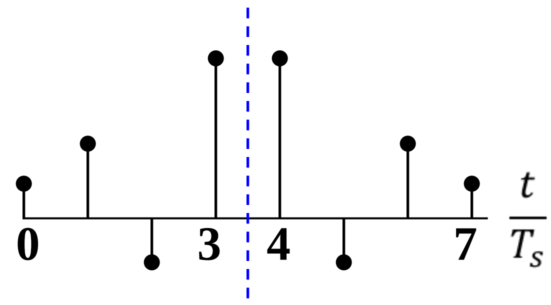
- Fine time resolution
- Linear phase (to preserve waveform in time domain)
- Applicable to bandpass signal (as well as lowpass signal)

Contents

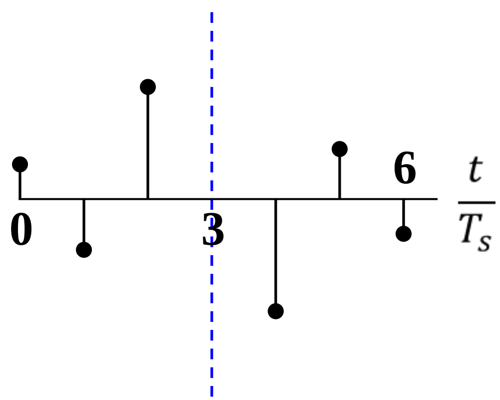
Linear Phase FIR Filter Impulse Response



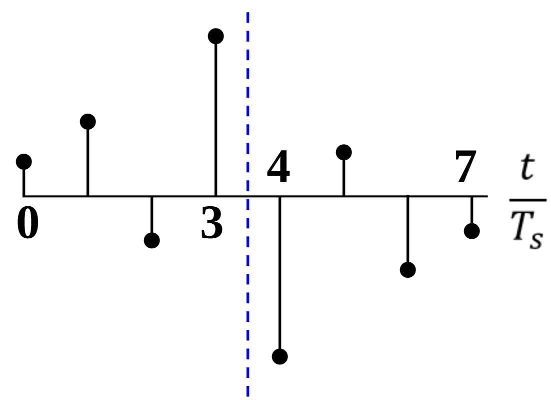
(1)Case 1
odd # of taps・even symmetry



(2) Case 2
even # of taps・even symmetry



(3) Case 3
odd # of taps・odd symmetry



(4) Case 4
even # of taps・odd symmetry

Frequency Characteristics

$h(nT)$	$H\left(e^{j\omega T}\right)$
Case 1	$e^{-j\omega(N-1)T_s/2} \sum_{k=0}^{(N-1)/2} a_k \cos[\omega k T_s]$
Case 2	$e^{-j\omega(N-1)T_s/2} \sum_{k=1}^{N/2} b_k \cos[\omega(k-1/2)T_s]$
Case 3	$e^{-j(\omega(N-1)T_s/2-\pi/2)} \sum_{k=0}^{(N-1)/2} a_k \sin[\omega k T_s]$
Case 4	$e^{-j(\omega(N-1)T_s/2-\pi/2)} \sum_{k=1}^{N/2} b_k \sin[\omega(k-1/2)T_s]$

Phase : proportional to ω (linear phase)

Time resolution of group delay Ts/2

Contents

- Research Purpose
- Conventional Linear Phase Digital Filter Condition
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  - <u>Time-Shift, Impulse Response of Ideal Filter</u>
  - New Linear Phase Digital Filter
- MATLAB Simulation
- Design Consideration
  - Window
  - Gain Adjustment
- Application
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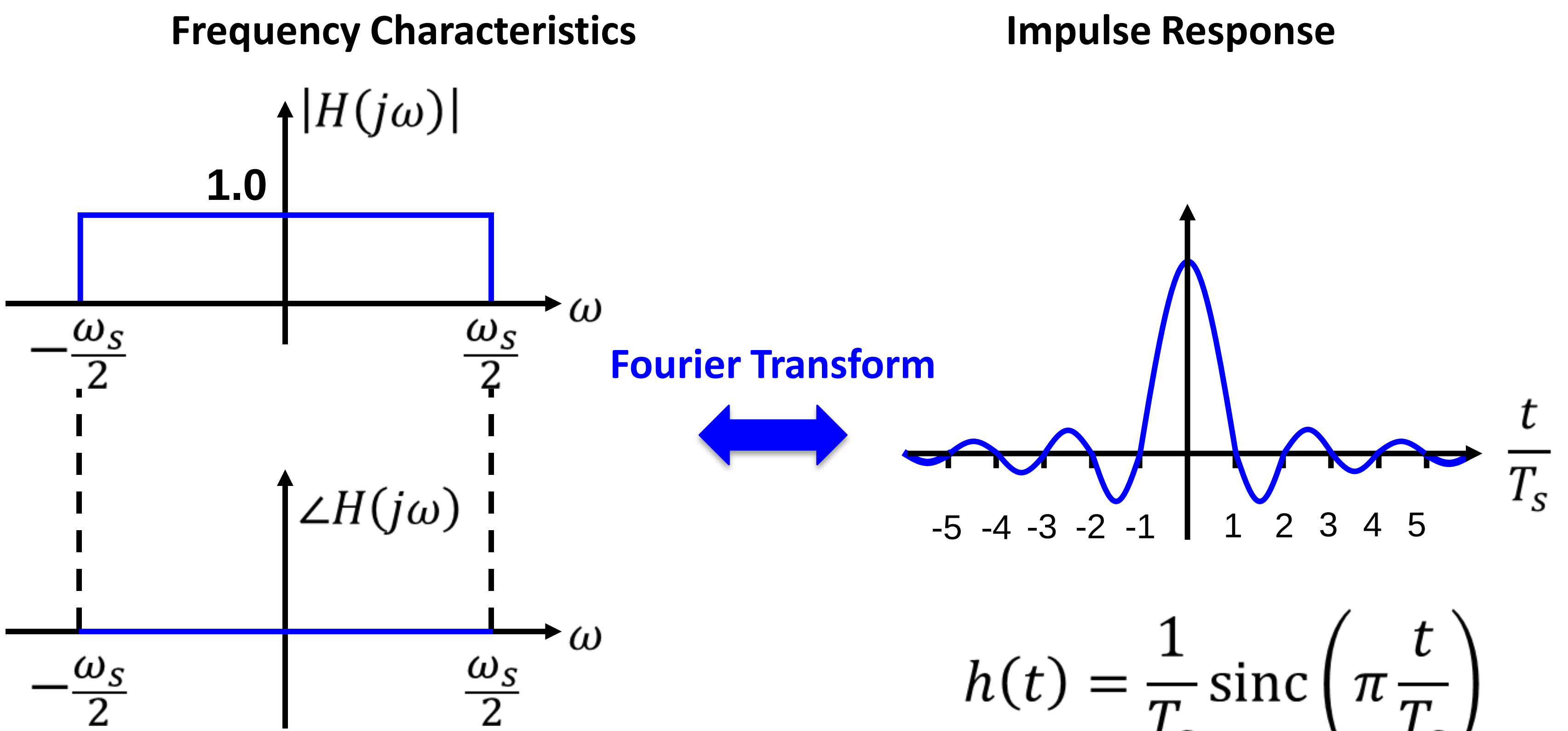
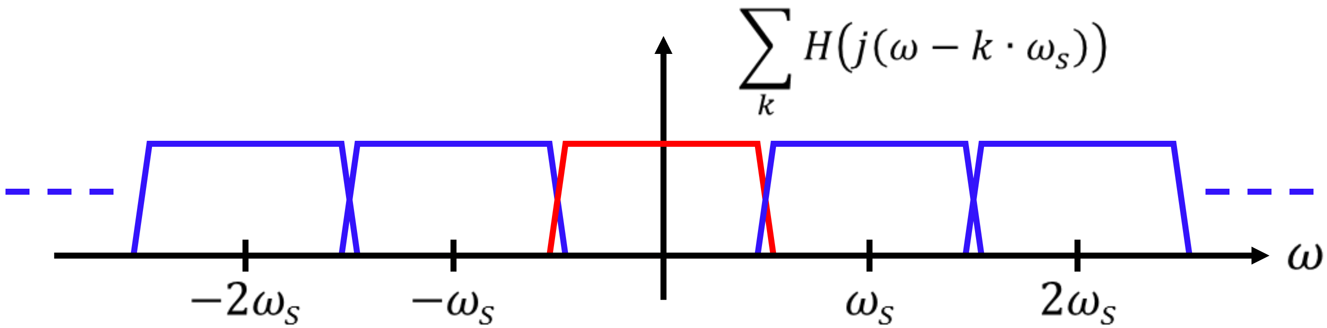
Ideal LPF



$$h(t) = \frac{1}{T_s}\,\mathrm{sinc}\left(\pi \frac{t}{T_s}\right)$$

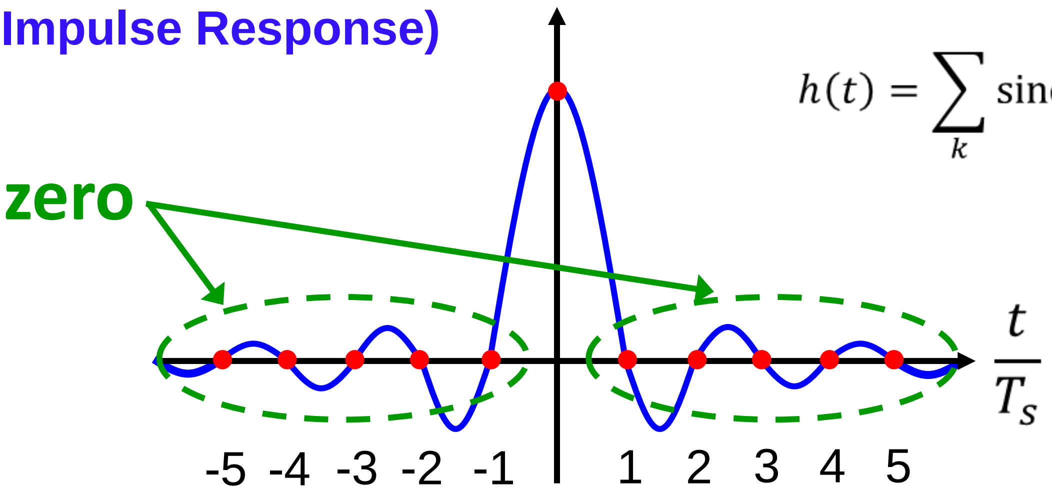
$\omega_s = \frac{2\pi}{T_s}$ **: Sampling Frequency**

Discrete-Time Representation of Ideal LPF



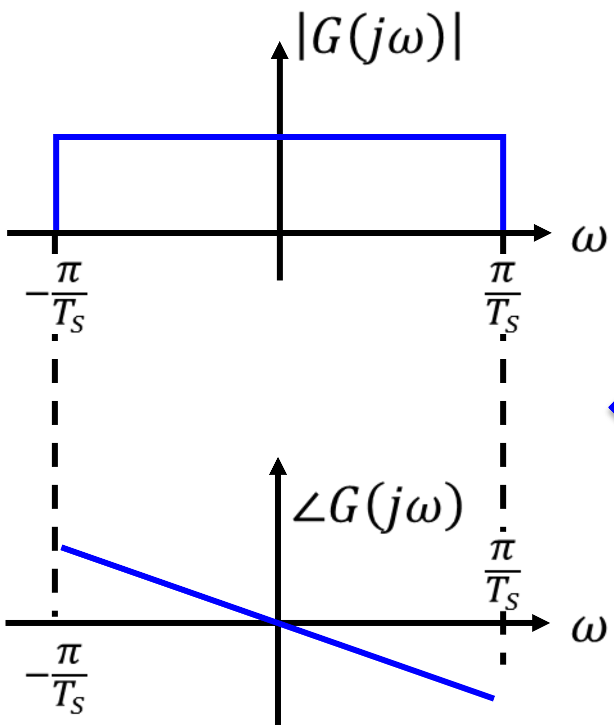
Fourier Transform

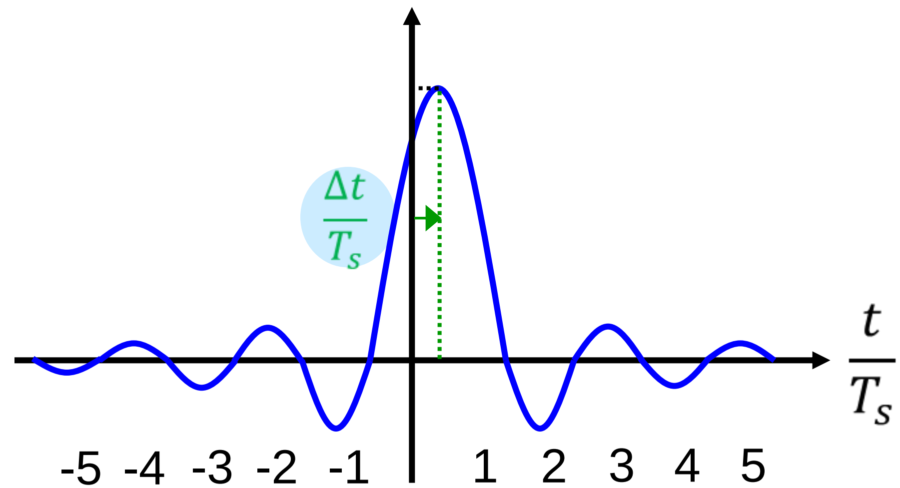
FIR (Finite Impulse Response)



$$h(t) = \sum_k \mathrm{sinc}\left(\pi \frac{k \cdot T_s}{T_s}\right) \delta(t - k \cdot T_s)$$

Impulse Response Time-Shift





$\angle G(j\omega) = -\omega\Delta t$

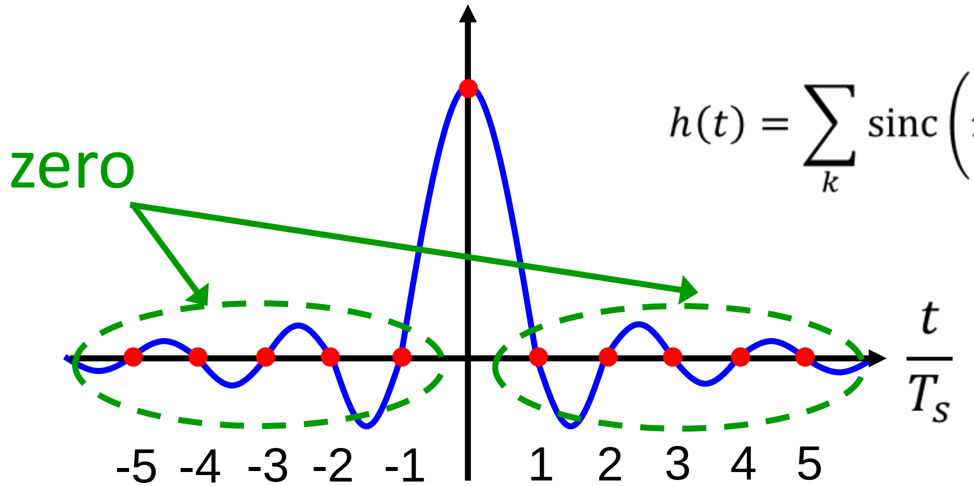
Δt time-shift of impulse response

No change of Gain

Time-Shift and Filter Coefficients

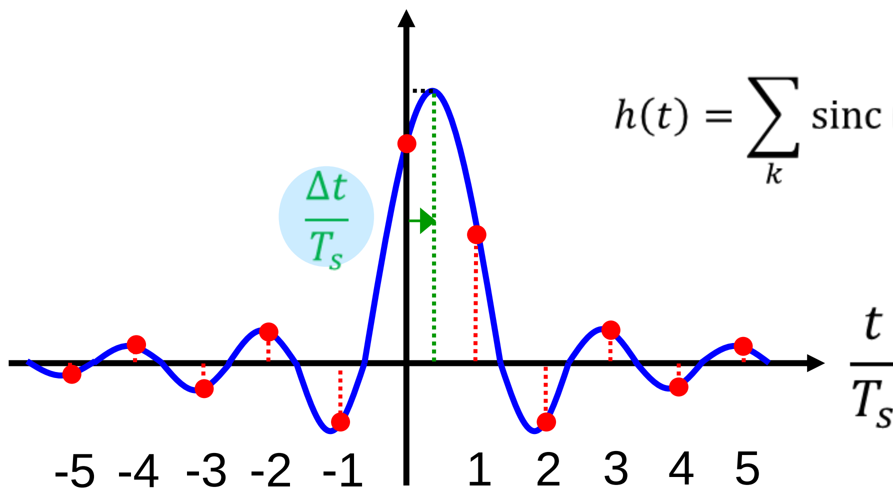
FIR filter

Time Shift



$$h(t) = \sum_k \mathrm{sinc}\left(\pi \frac{k \cdot T_s}{T_s}\right) \delta(t - k \cdot T_s)$$

IIR Filter

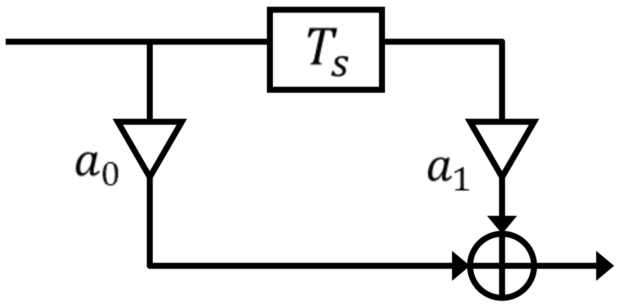


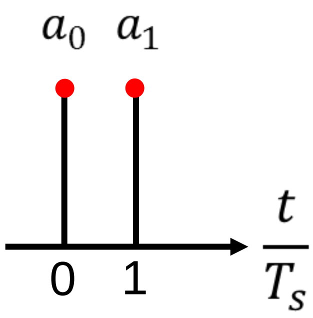
$$h(t) = \sum_k \mathrm{sinc}\left(\pi \frac{k \cdot T_s - \Delta t}{T_s}\right) \delta(t - k \cdot T_s)$$

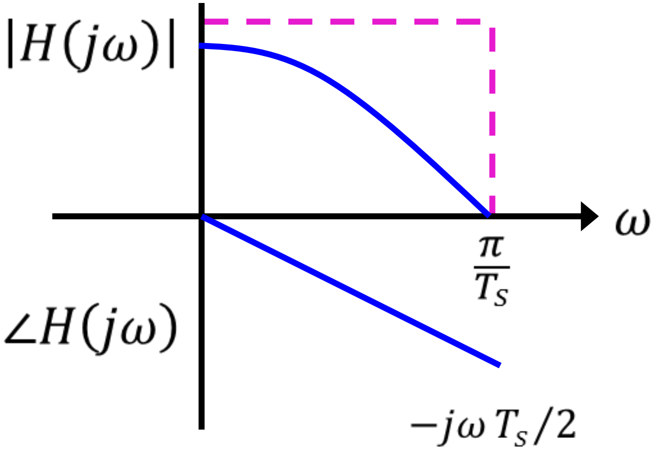
Ideal Delay-Filter

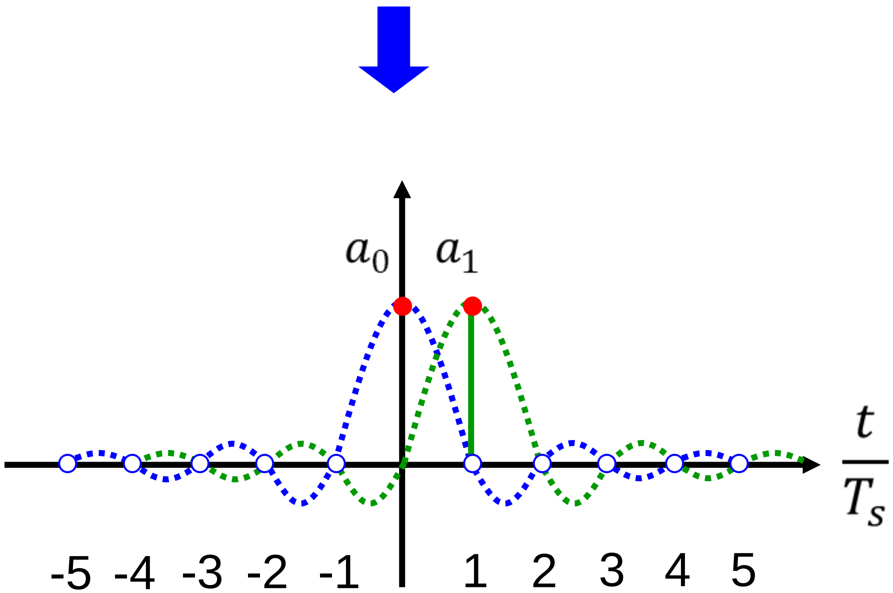
Contents

2-Tap Filter: Model

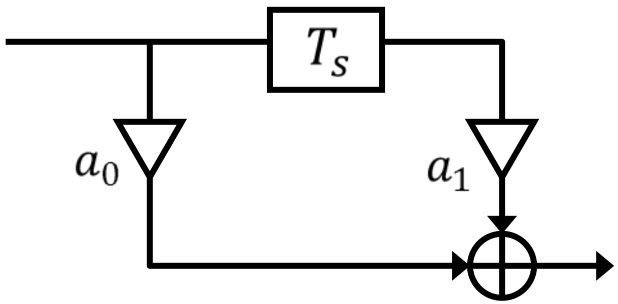


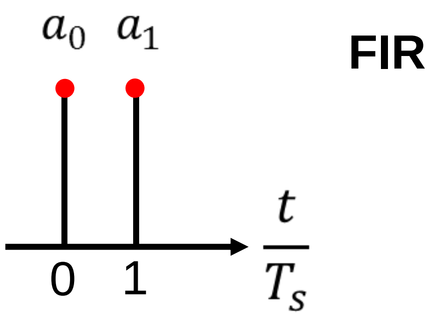


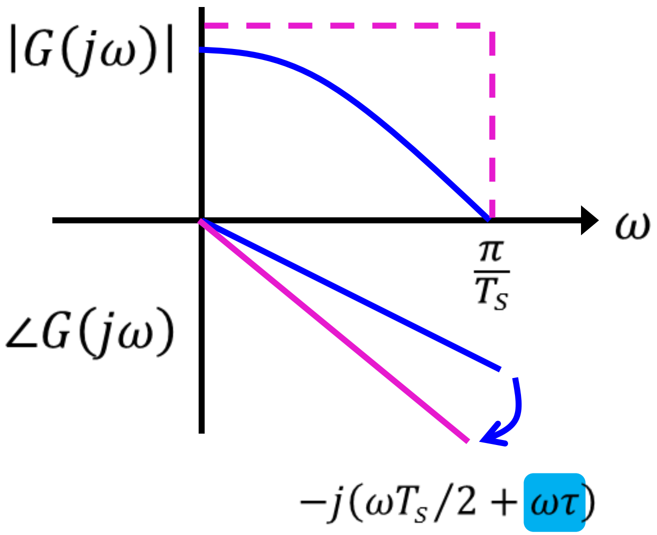


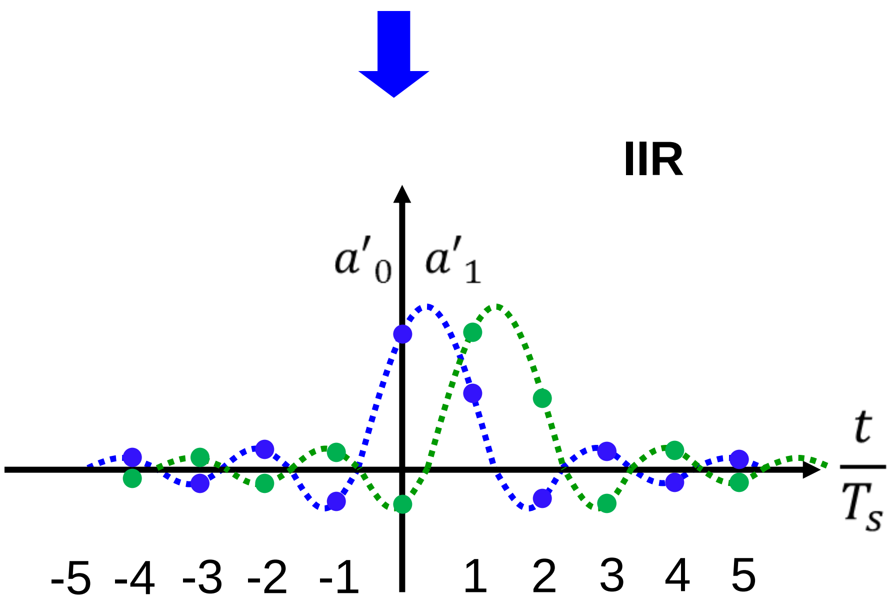


2-Tap Filter: Delay Model

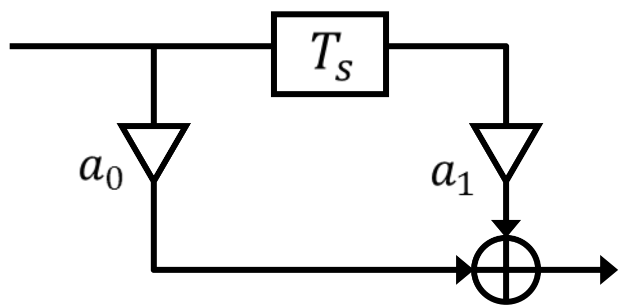


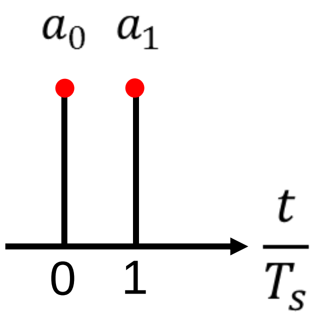




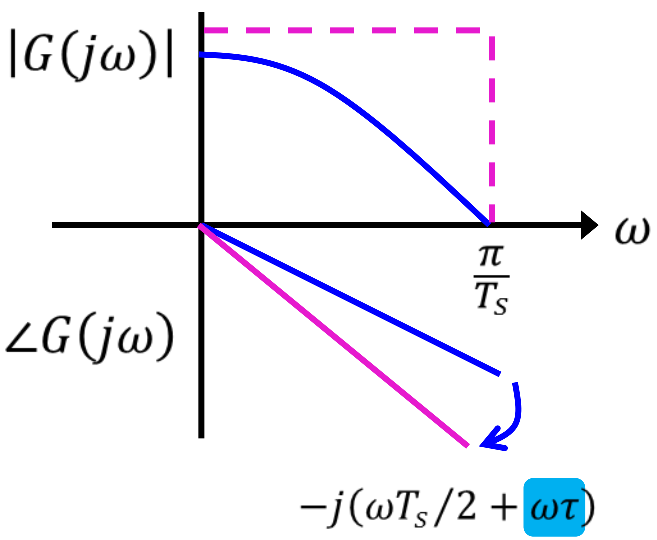


2-Tap Filter: Delay Model

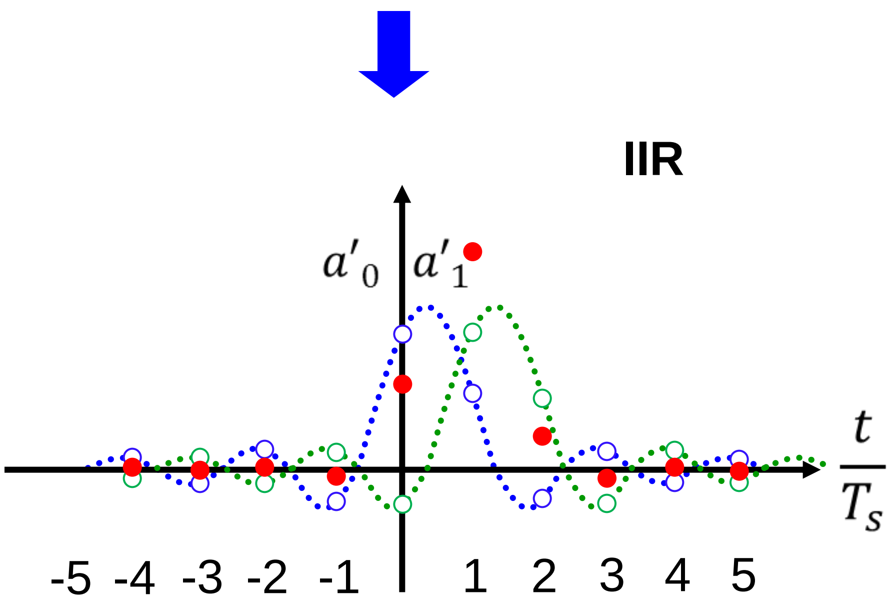




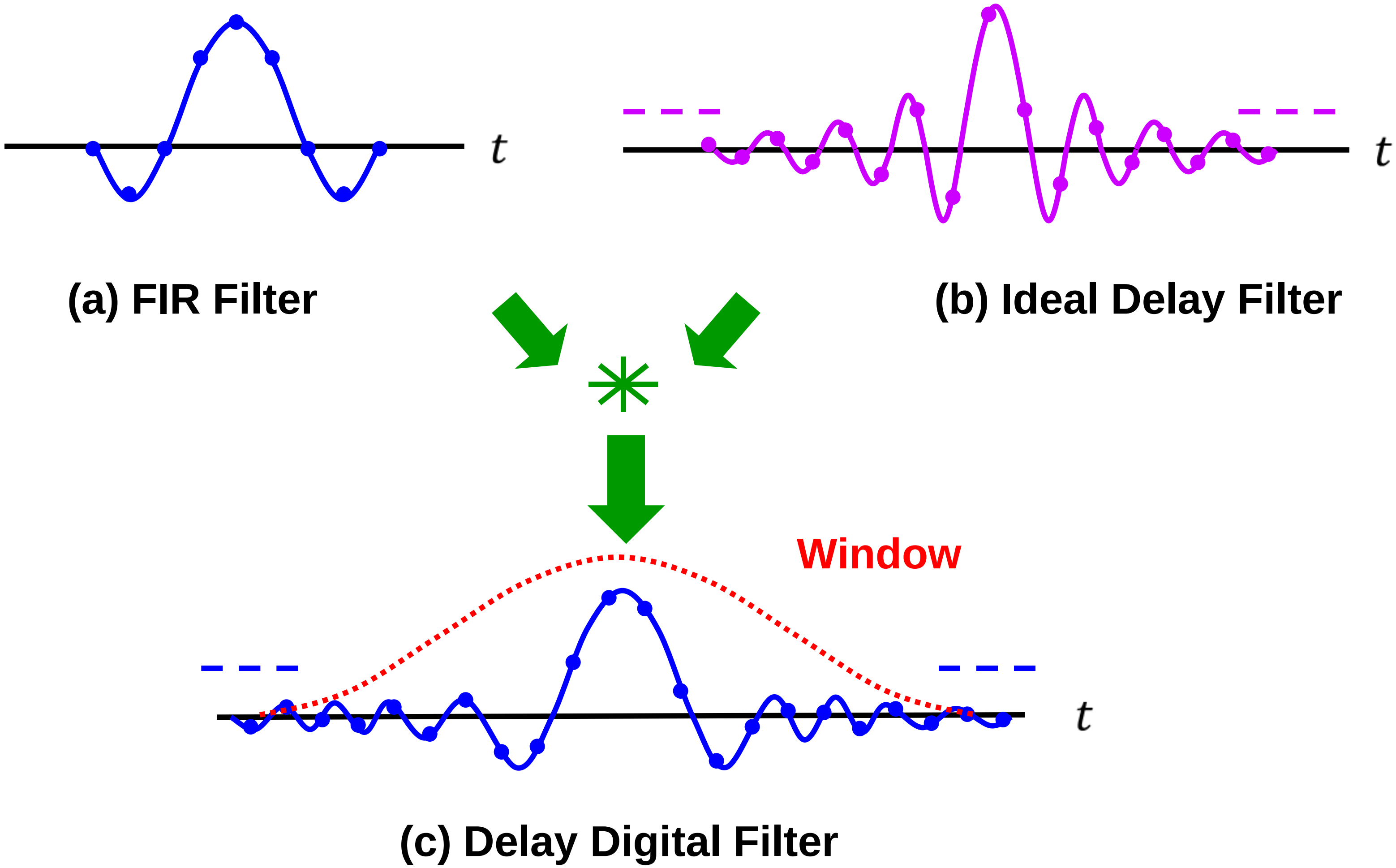
FIR



IIR



Proposed Delay Digital Filter



Frequency Characteristics of Proposed Delay Digital Filter

$g(nT)$	$G(e^{j\omega T})$
Case 1	$e^{-j\omega(N-1)T_s/2+\omega\tau}\sum_{k=0}^{(N-1)/2} a_k\cos[\omega k T_s]$
Case 2	$e^{-j\omega(N-1)T_s/2+\omega\tau}\sum_{k=1}^{N/2} b_k\cos[\omega(k-1/2)T_s]$
Case 3	$e^{-j(\omega(N-1)T_s/2-\pi/2+\omega\tau)}\sum_{k=0}^{(N-1)/2} a_k\sin[\omega k T_s]$
Case 4	$e^{-j(\omega(N-1)T_s/2-\pi/2+\omega\tau)}\sum_{k=1}^{N/2} b_k\sin[\omega(k-1/2)T_s]$

Phase : proportional to ω（linear phase）

Group delay time resolution τ : Arbitrary small

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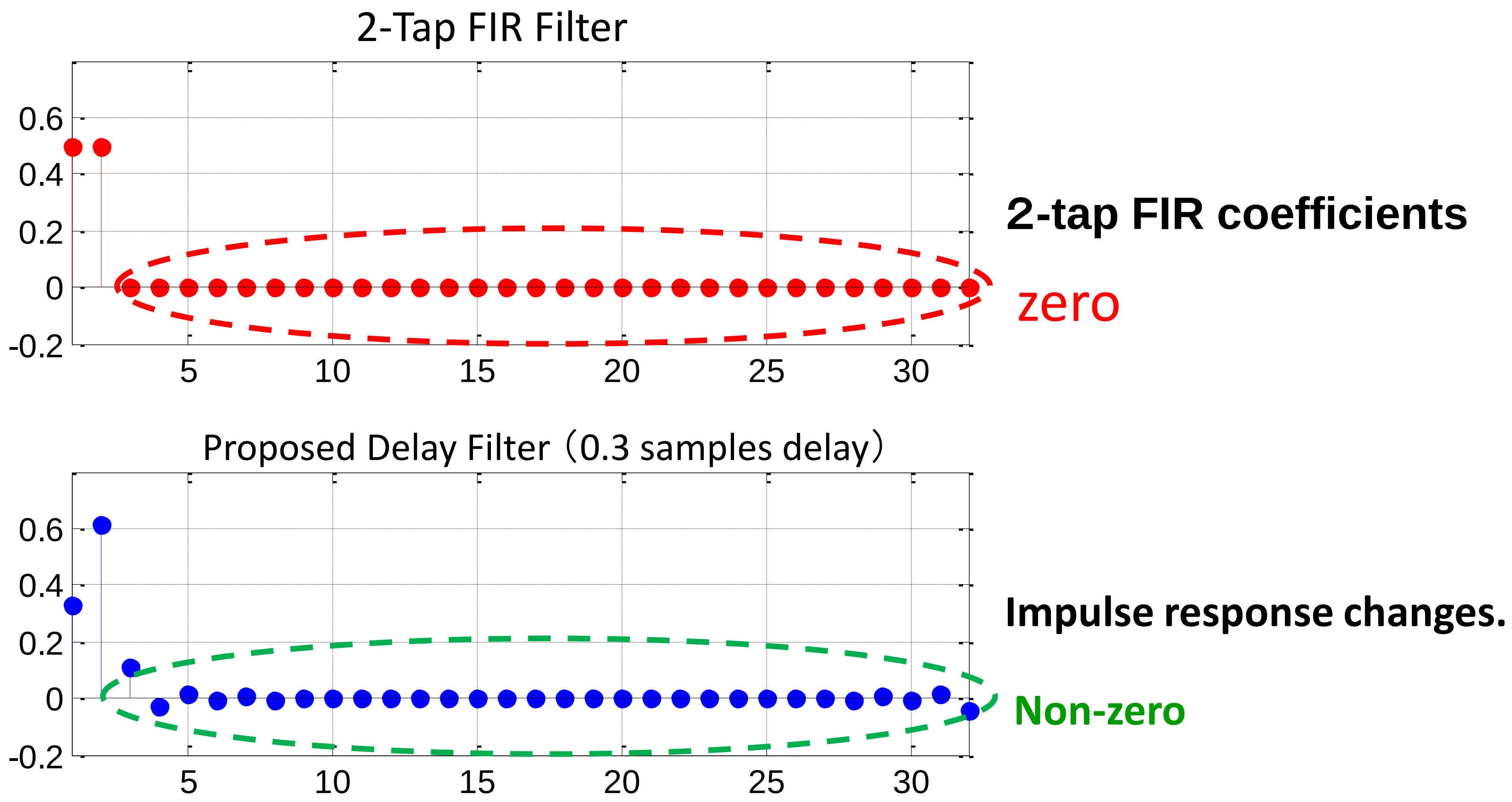
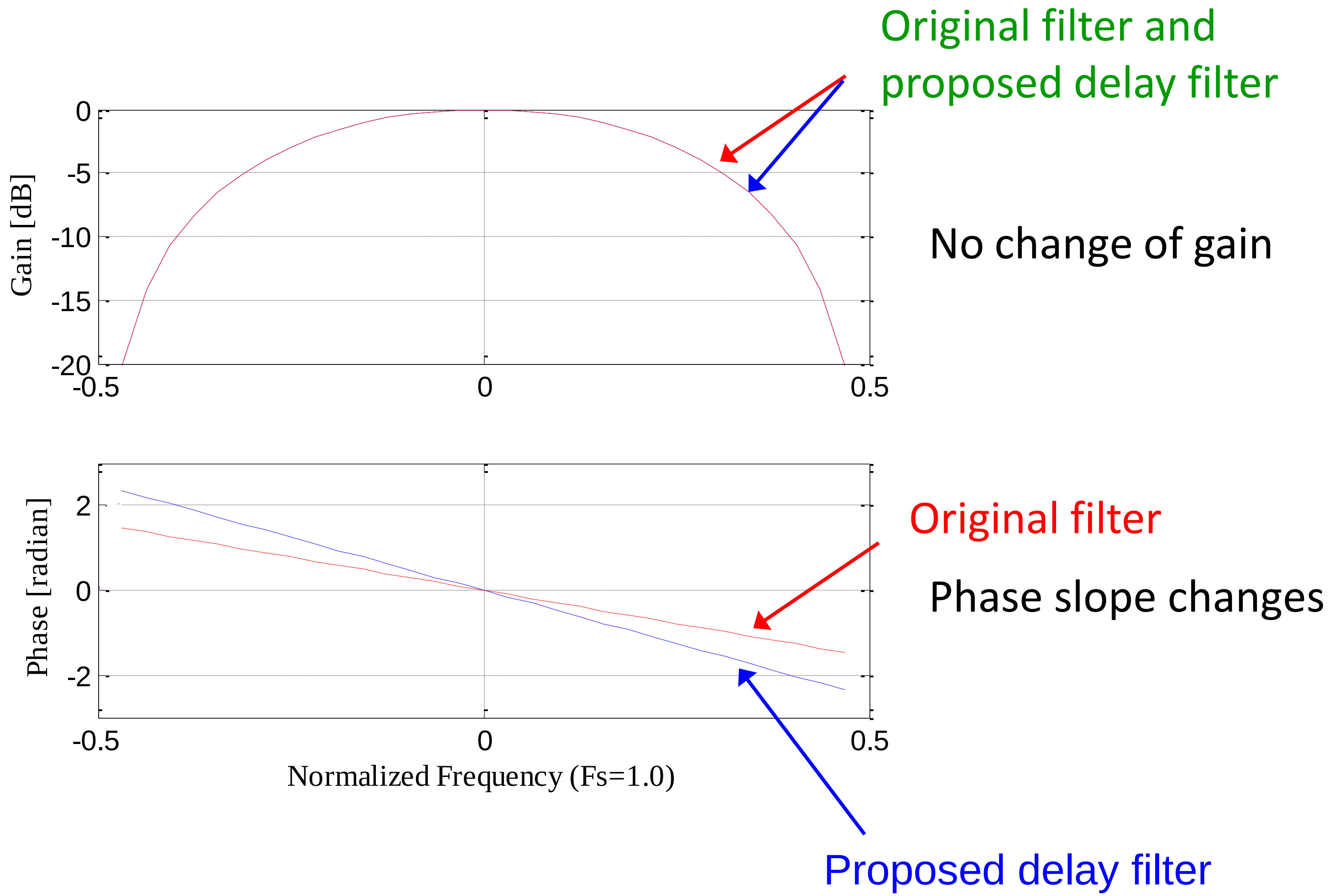
Comparison of 2-Tap Filter Impulse Responses

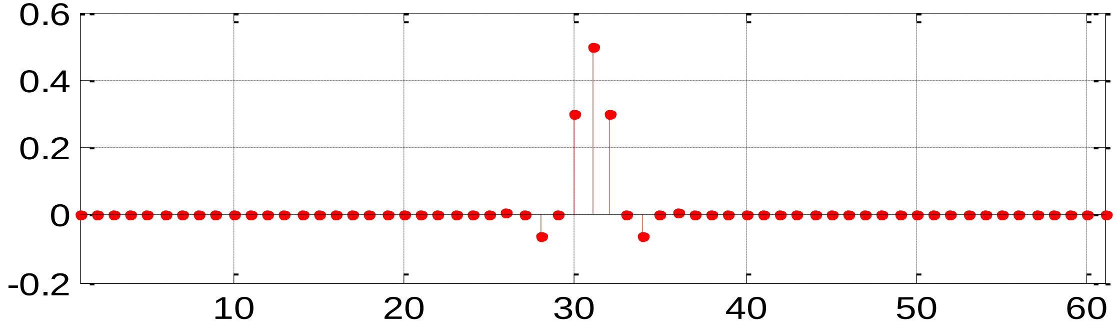


Comparison of 2-Tap Filter Frequency Characteristics

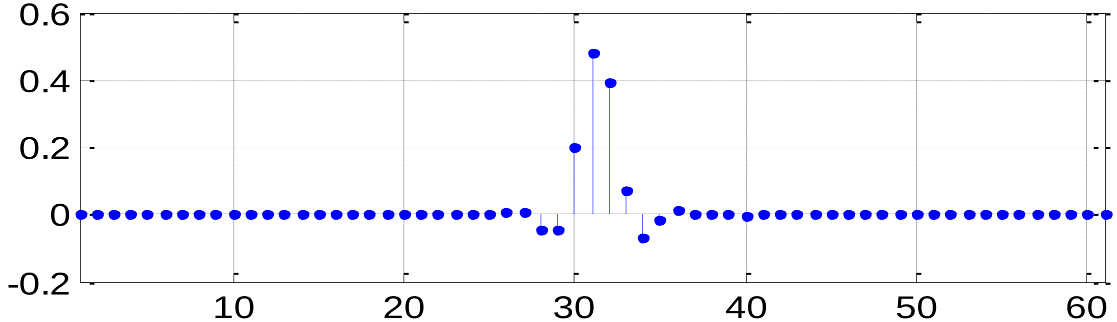


Finite Tap Truncation of Proposed Delay Filter

61-Tap Cosine Roll-off Filter

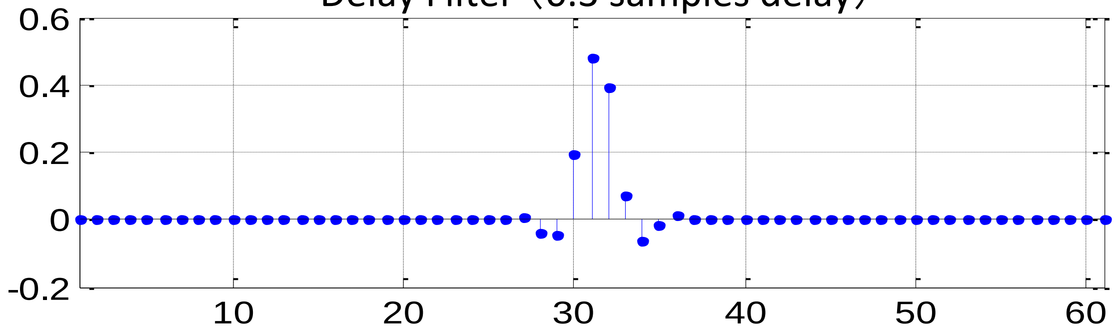


Delay Filter（0.3 samples delay）



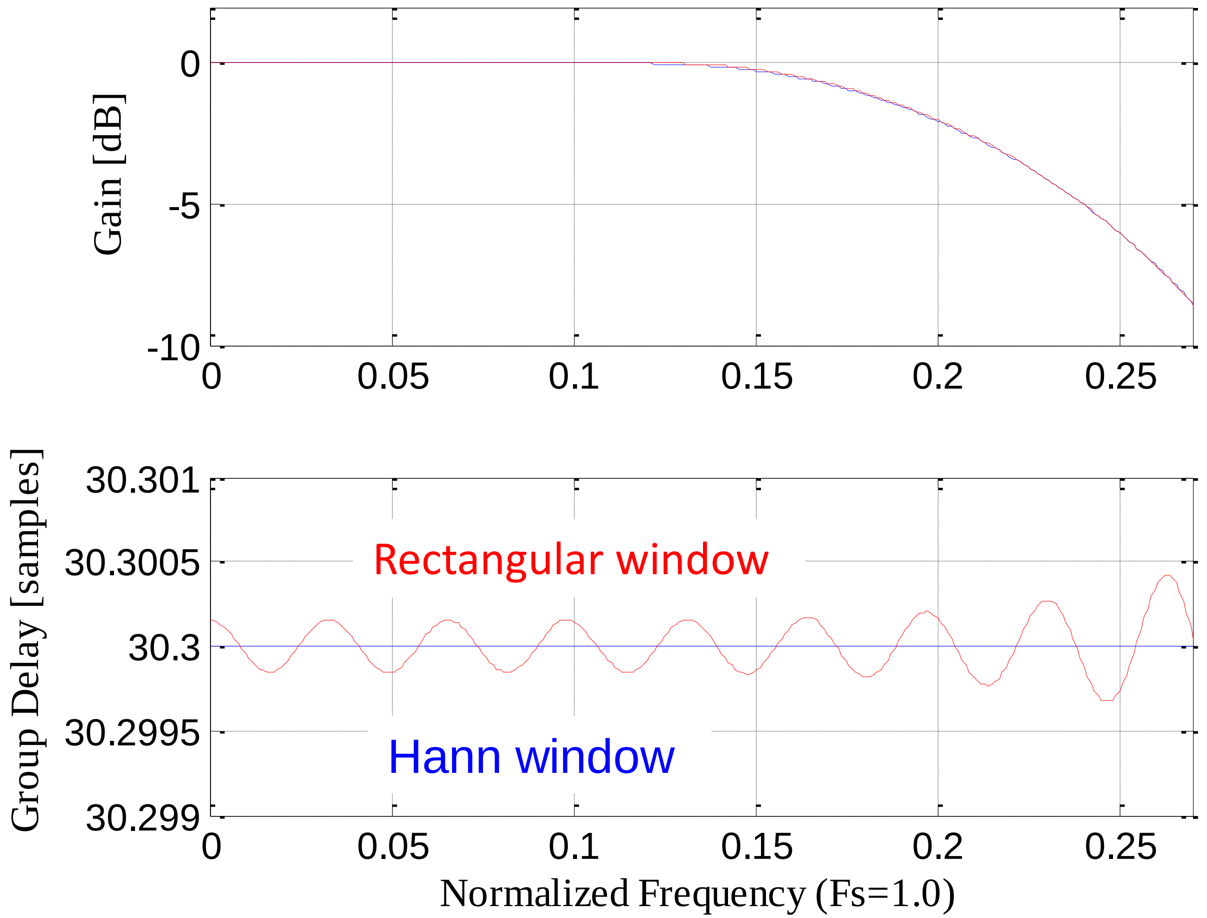
Rectangular window

Delay Filter（0.3 samples delay）



Hann window

Effects of Window



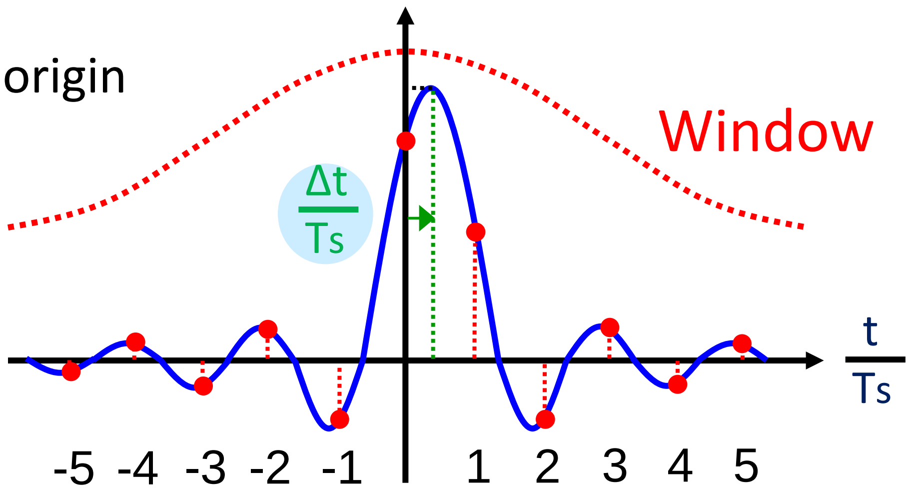
Gibbs oscillation of group delay

Frequency characteristics of delay filter with 61-tap truncation

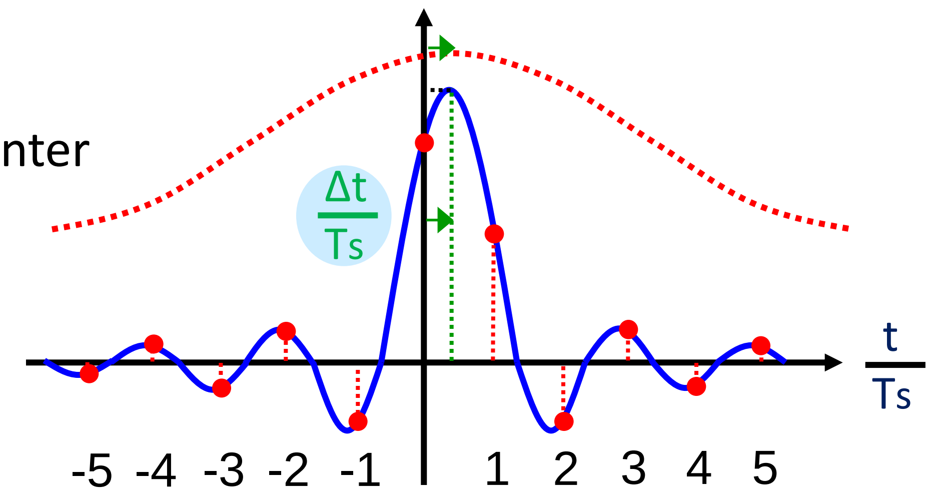
Contents

How to Apply Window

Centered at origin

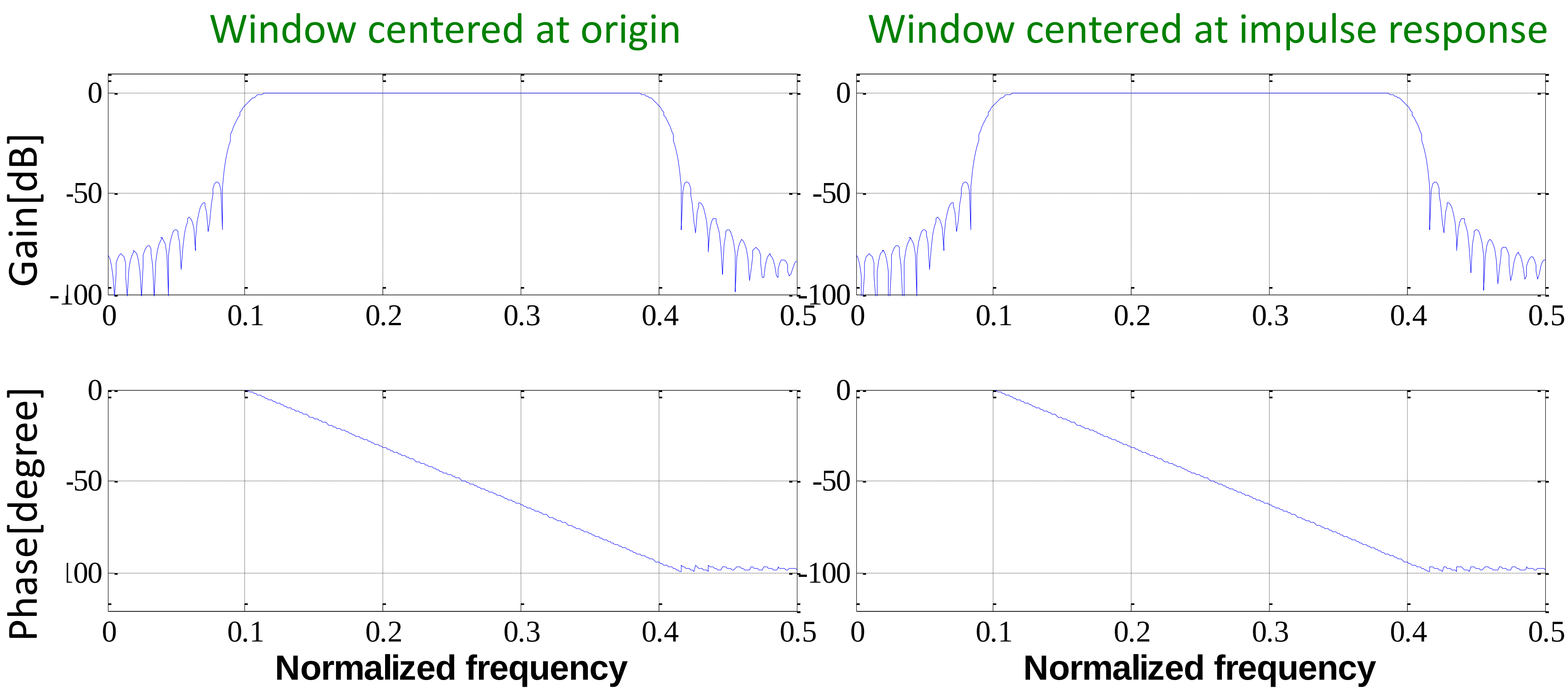


Centered at
impulse response center



Window center shifted by Δt

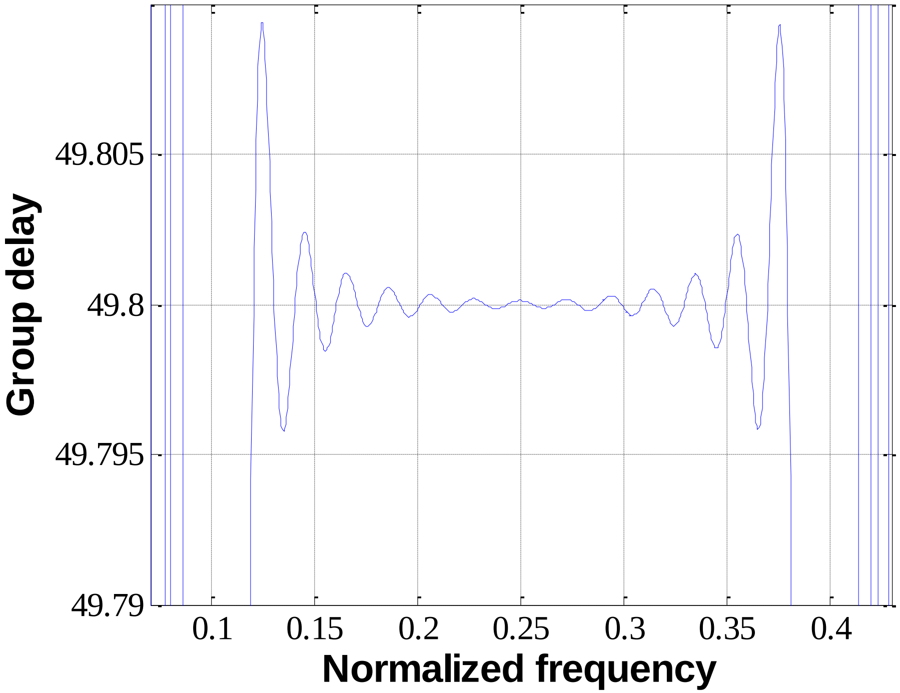
Frequency Characteristics of Delay Filter after Applying Window



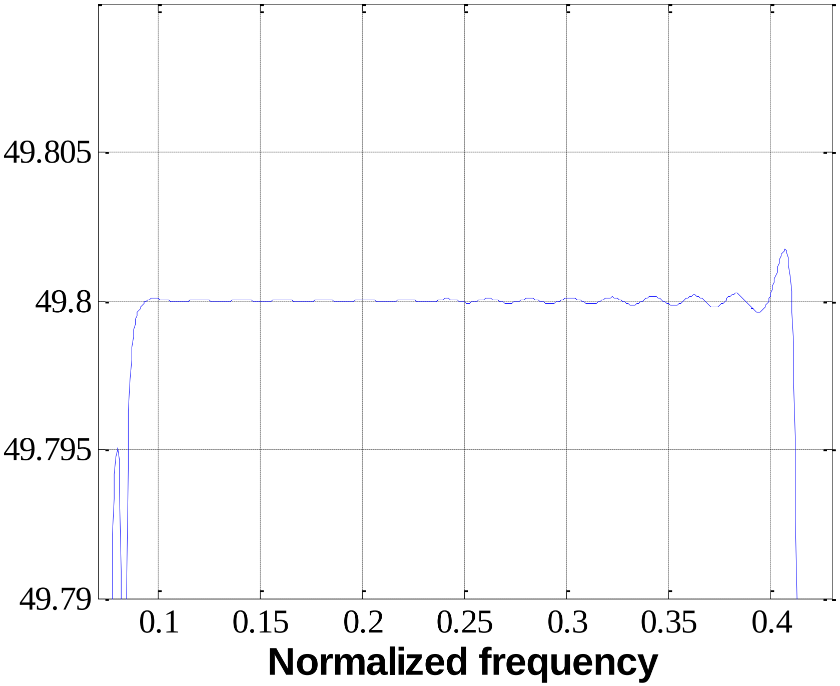
Delay	0.3 samples
Filter Tap	100 taps
Window	Han
Pass band	(0.1～0.4)･Fs
FFT points	1024 points

Group Delay Characteristics of Delay Filter after Applying Window

Window centered at origin

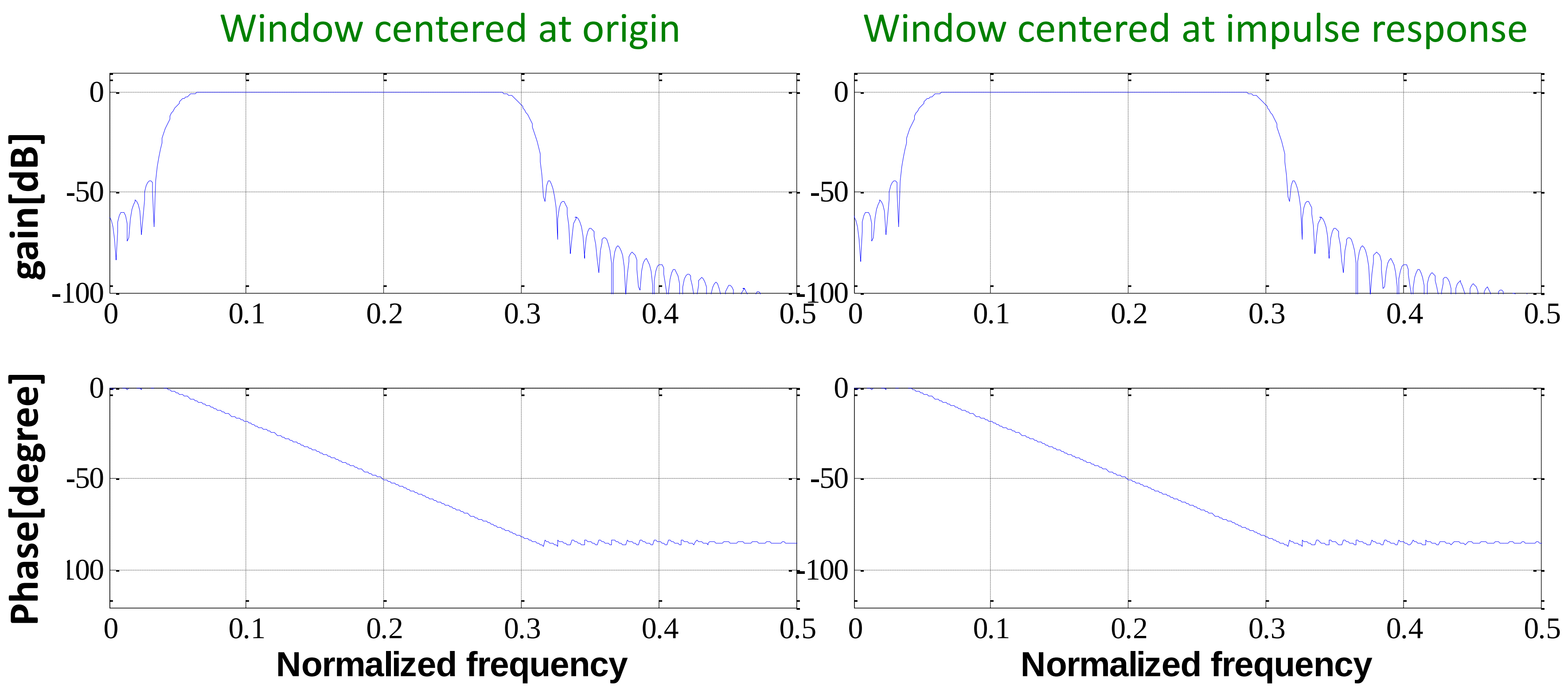


Window centered at impulse response



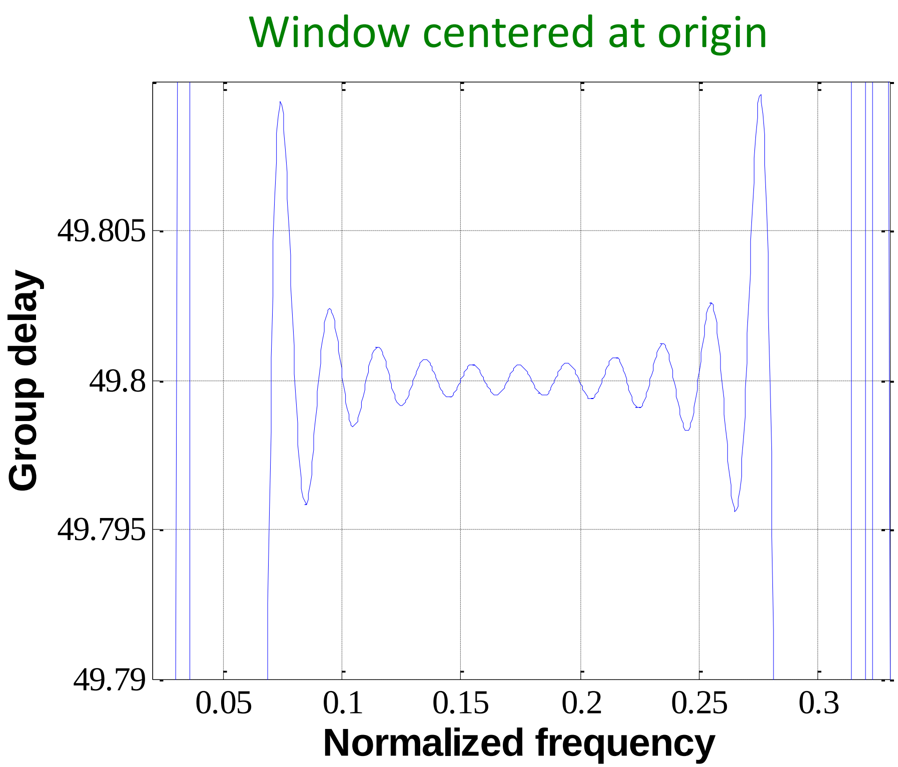
Delay	0.3 samples
Filter Tap	100 taps
Window	Han
Pass band	(0.1～0.4)・Fs
FFT points	1024 points

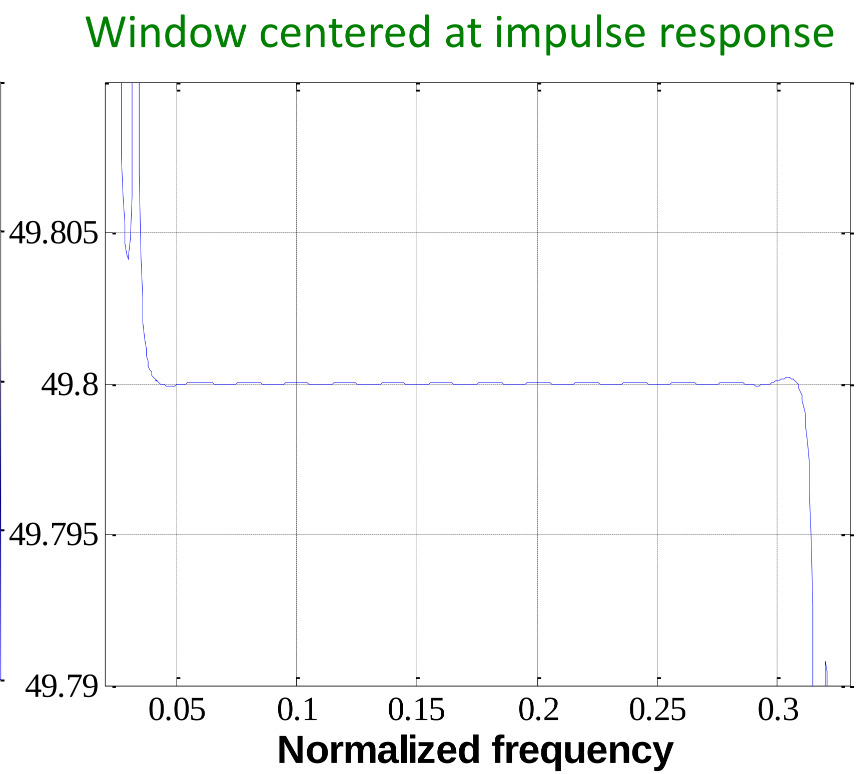
Frequency Characteristics of Delay Filter after Applying Window



Delay	0.3 samples
Filter Tap	100 taps
Window	Han
Pass band	(0.05～0.3)・Fs
FFT points	1024 points

Group Delay Characteristics of Delay Filter after Applying Window

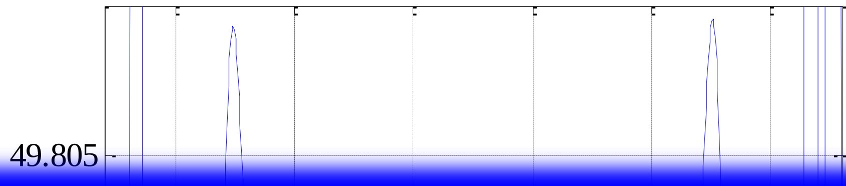




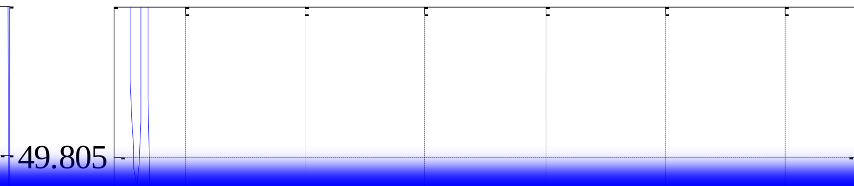
Delay	0.3 samples
Filter Tap	100 taps
Window	Han
Pass band	(0.05～0.3)･Fs
FFT points	1024 points

Group Delay Characteristics of Delay Filter after Applying Window

Window centered at origin



Window centered at impulse response



Applying window centered at impulse response

⬇

Constant group delay over entire passband

Delay	0.3 samples
Filter Tap	100 taps
Window	Han
Pass band	(0.05～0.3)･Fs
FFT points	1024 points

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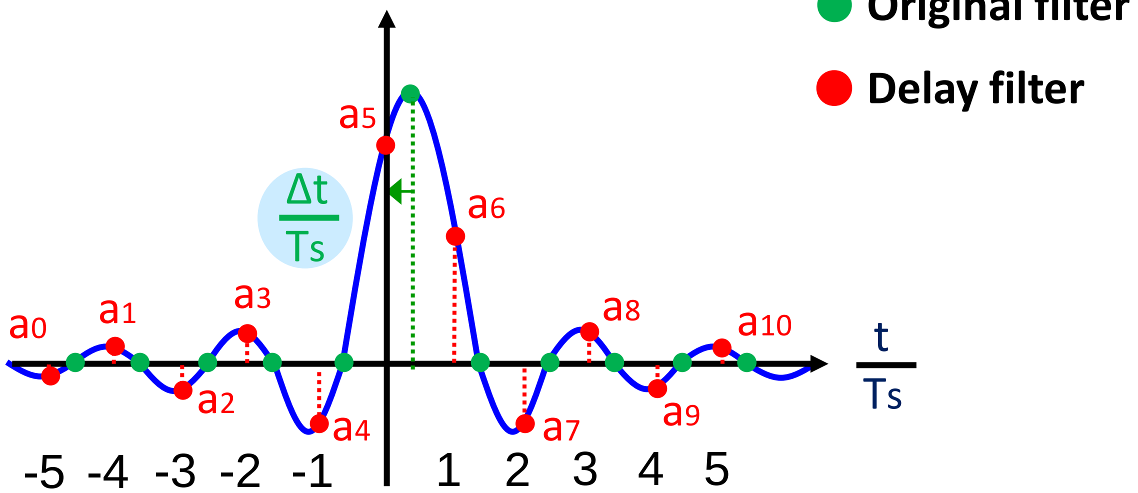
Proposed Filter DC Gain Adjustment

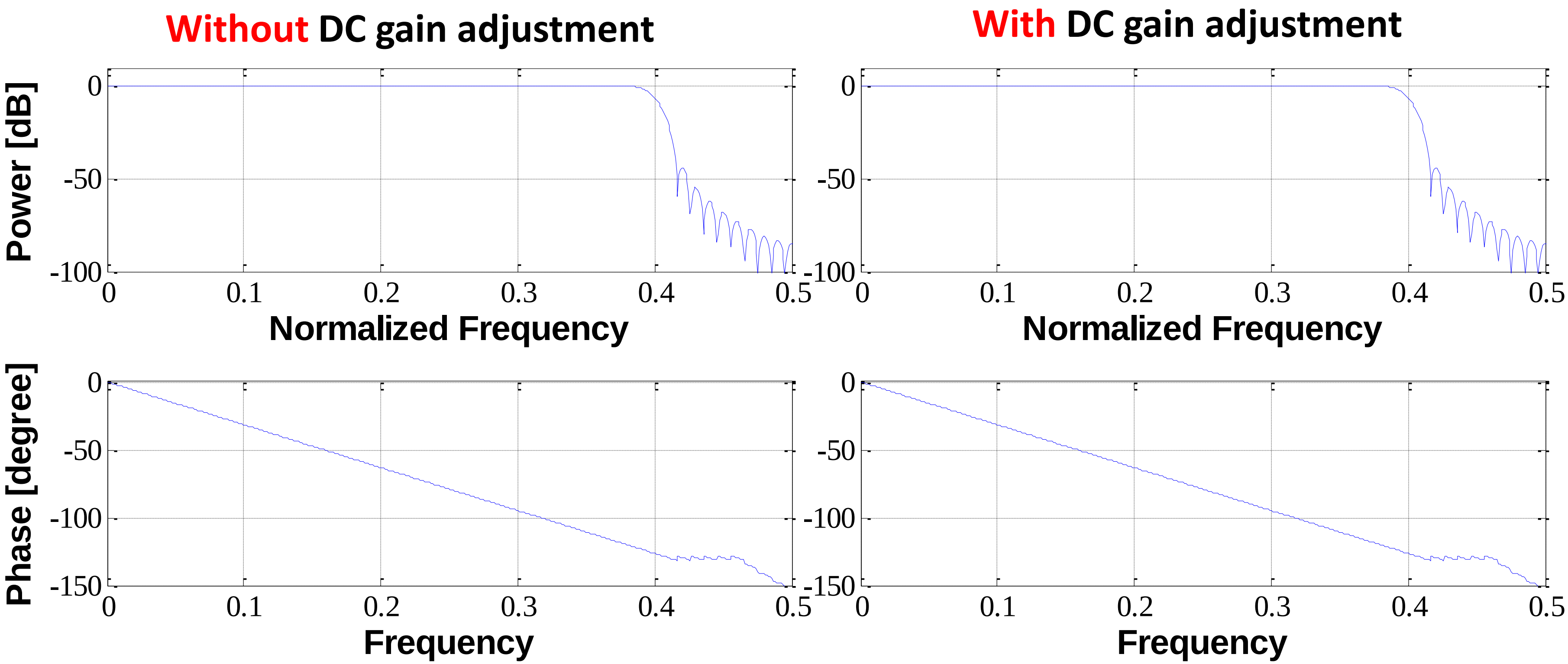
Digital filter DC gain : $\sum a_n$



DC gain adjustment due to finite tap truncation is required

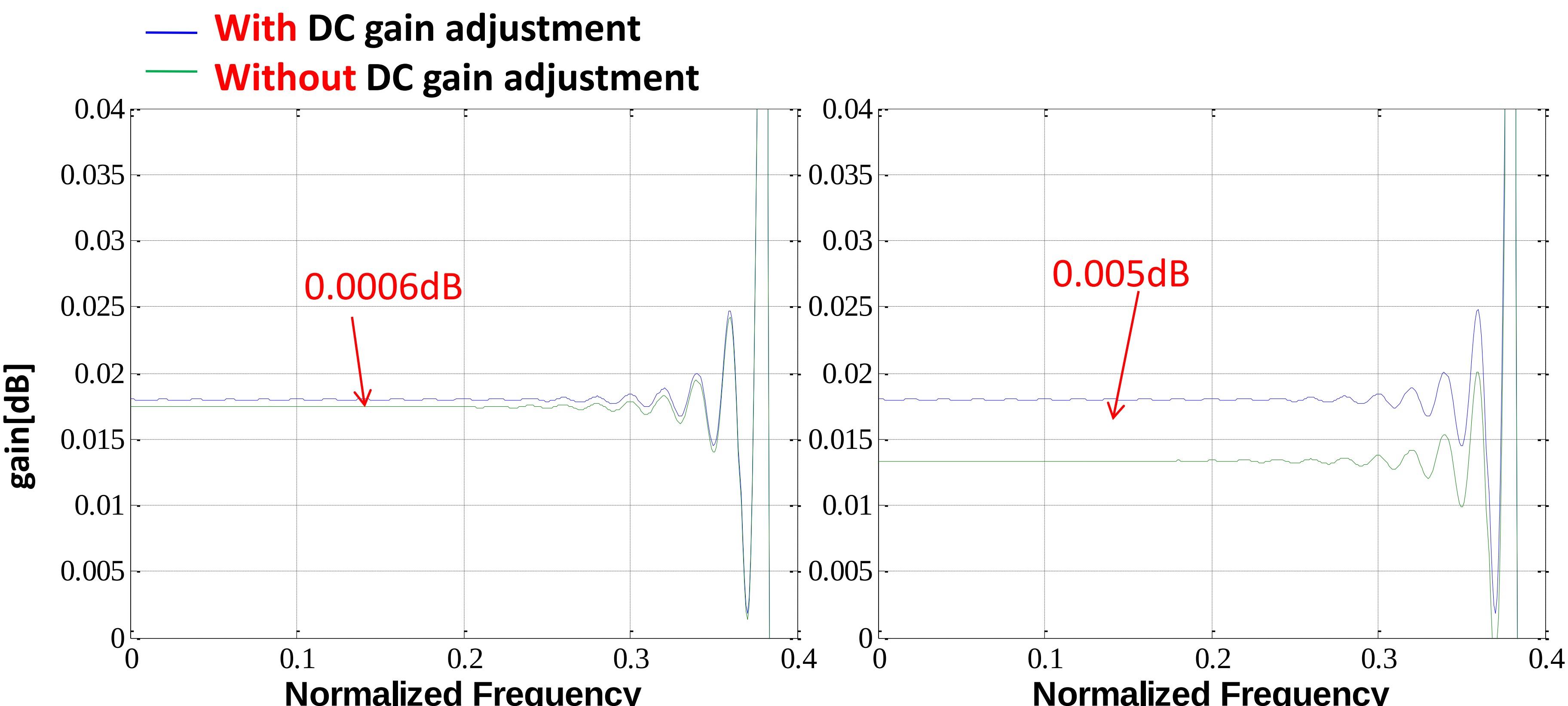
$$\sum_{n=0}^{N} a'_n = \text{DC gain of original FIR filter}$$

Frequency Characteristics of Proposed Delay Filter



Delay	0.3 samples
Filter Tap	101 taps
Window	Han
Cut-off Freq.	0.4・Fs
FFTpoints	1024 points

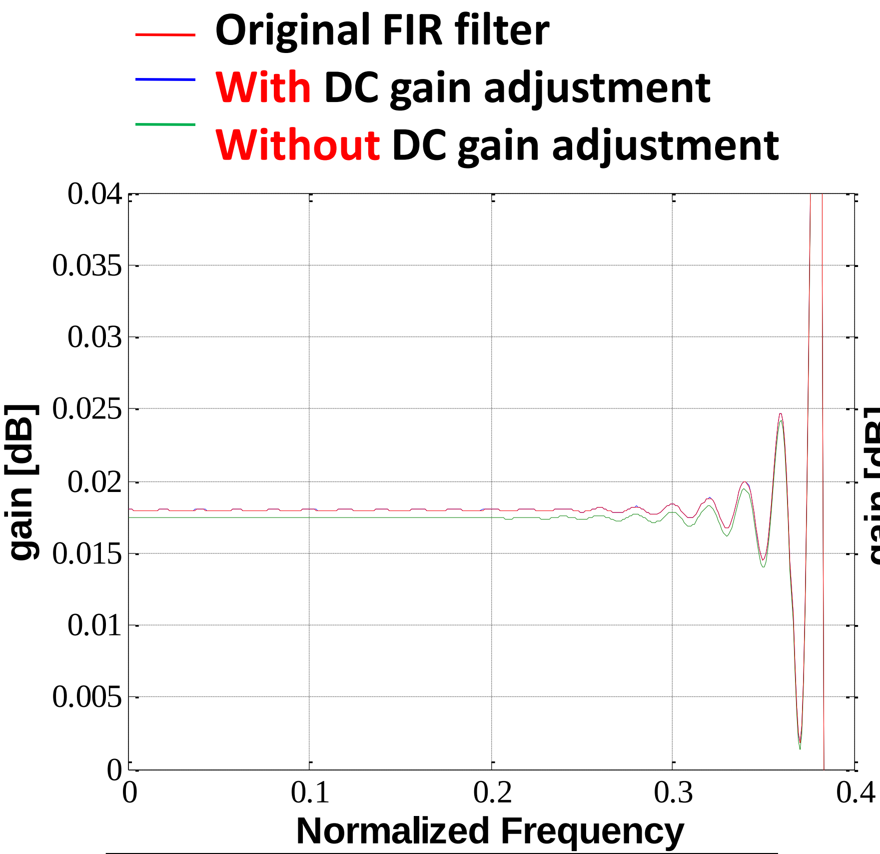
Gain Characteristics of Proposed Delay Filter



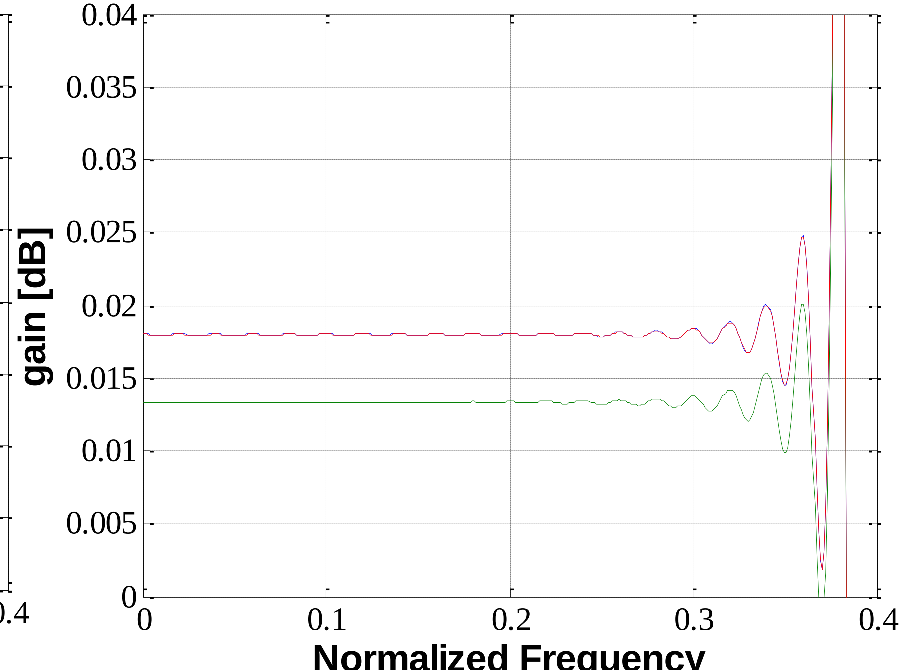
Delay	0.1samples
Filter Tap	101 taps
Window	Han
Cutoff Freq.	0.4・Fs
FFTpoints	1024 points

Delay	0.3 samples
Filter Tap	101 taps
Window	Han
Cutoff Freq.	0.4・Fs
FFT points	1024 points

Gain Characteristics of Proposed Delay Filter

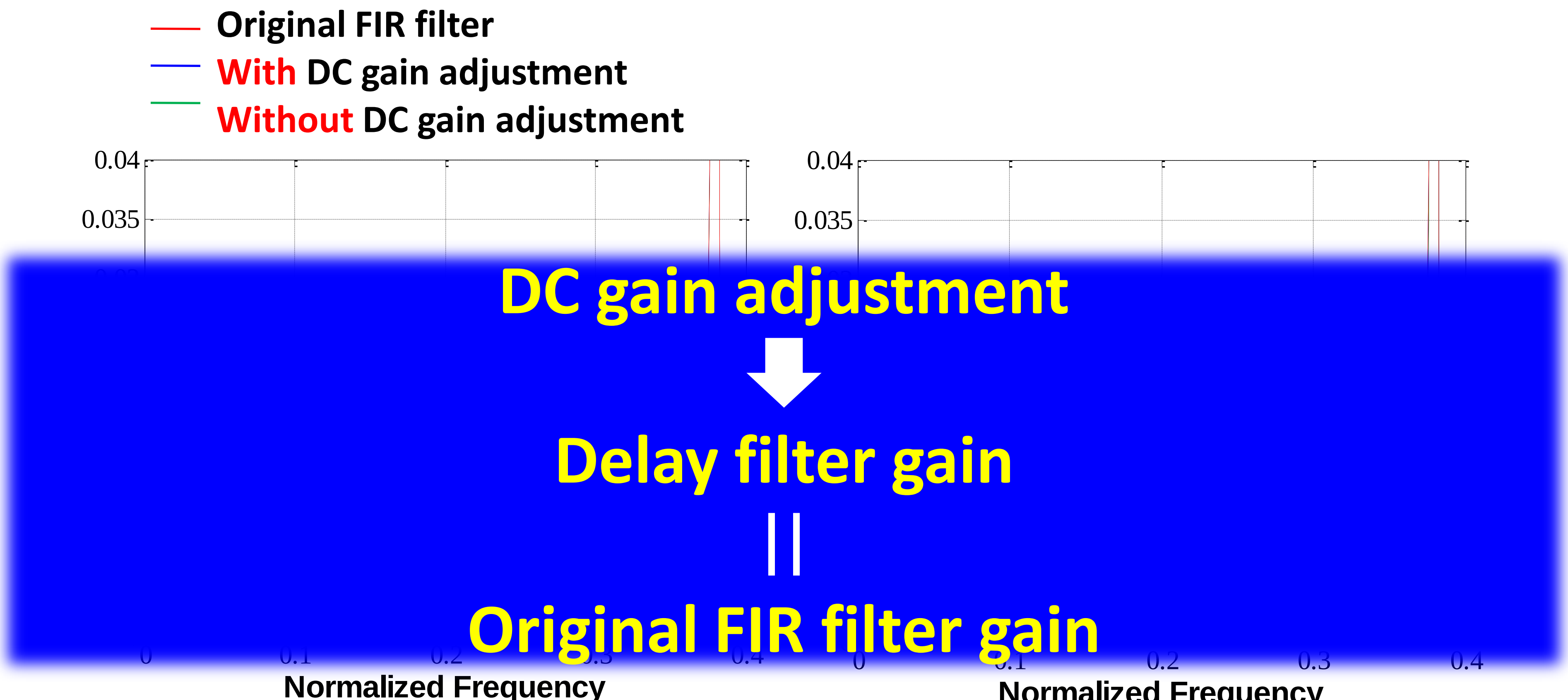


Delay	0.1samples
Filter Tap	101 taps
Window	Han
Cutoff Freq.	0.4・Fs
FFTpoints	1024 points



Delay	0.3 samples
Filter Tap	101 taps
Window	Han
Cutoff Freq.	0.4・Fs
FFT points	1024 points

Gain Characteristics of Proposed Delay Filter



Delay	0.1samples
Filter Tap	101 taps
Window	Han
Cutoff Freq.	0.4･Fs
FFTpoints	1024 points

Delay	0.3 samples
Filter Tap	101 taps
Window	Han
Cutoff Freq.	0.4･Fs
FFT points	1024 points

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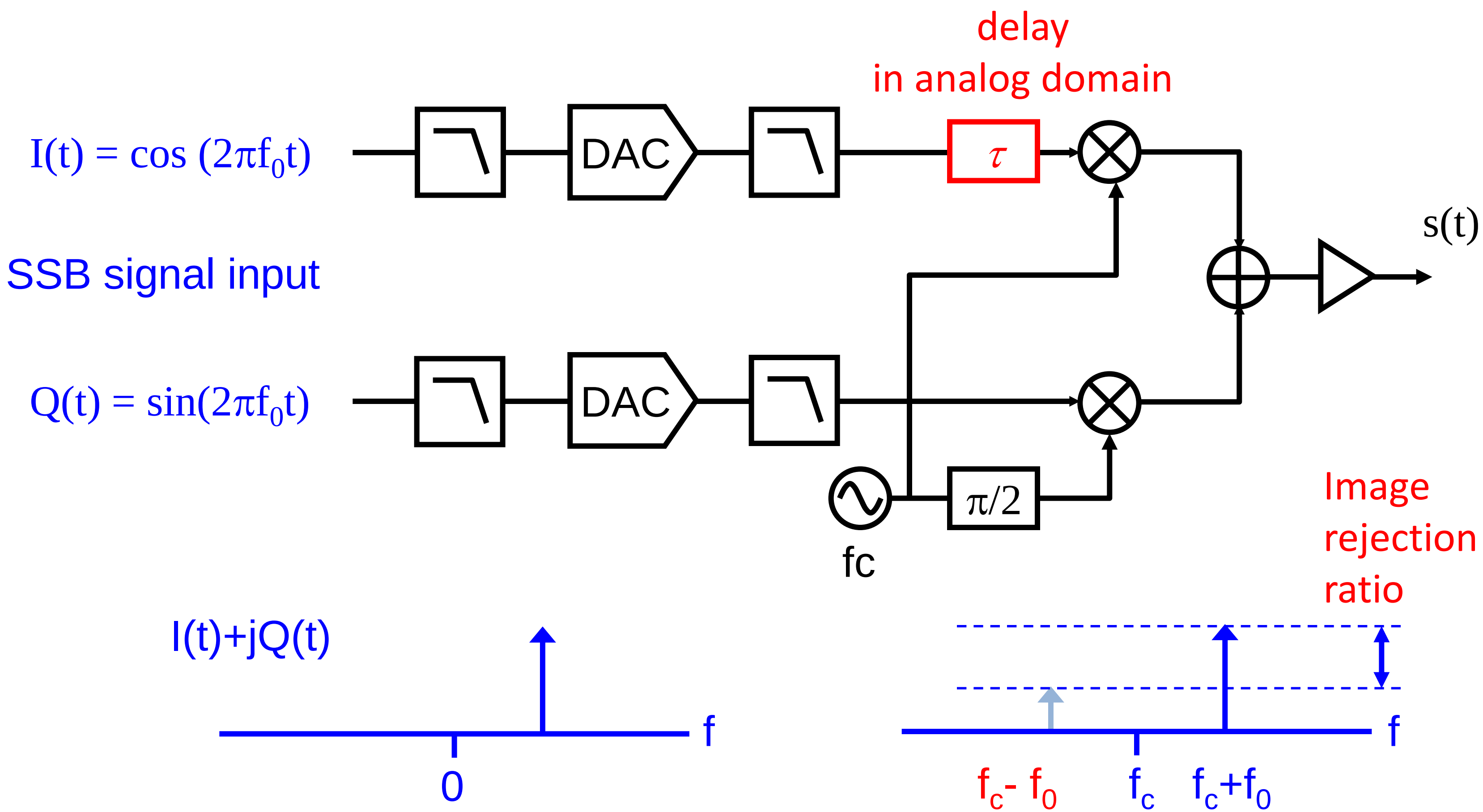
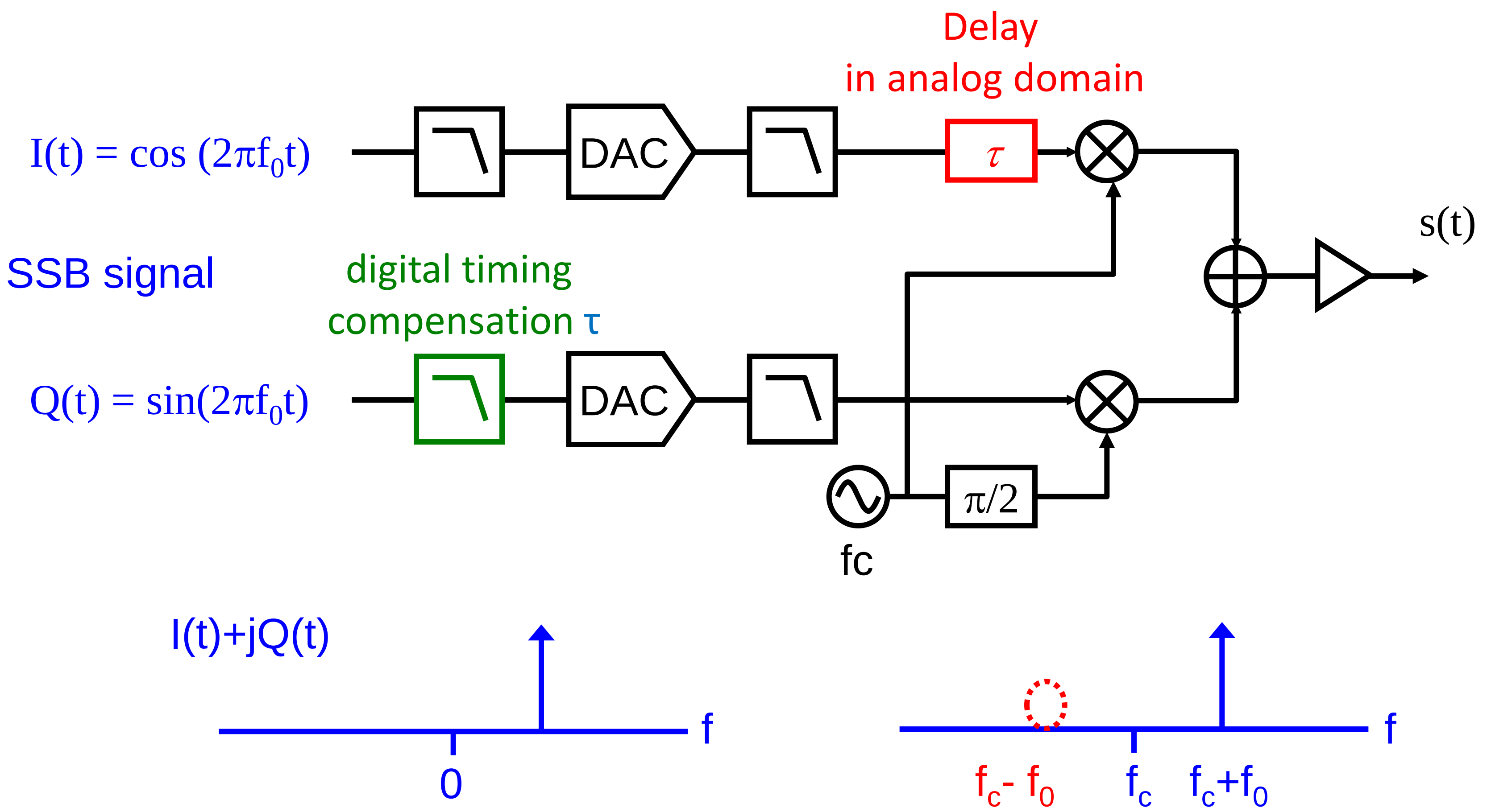
I/Q Delay Mismatch in Quadrature Modulator



SSB : single side band

DAC : digital-to-analog converter

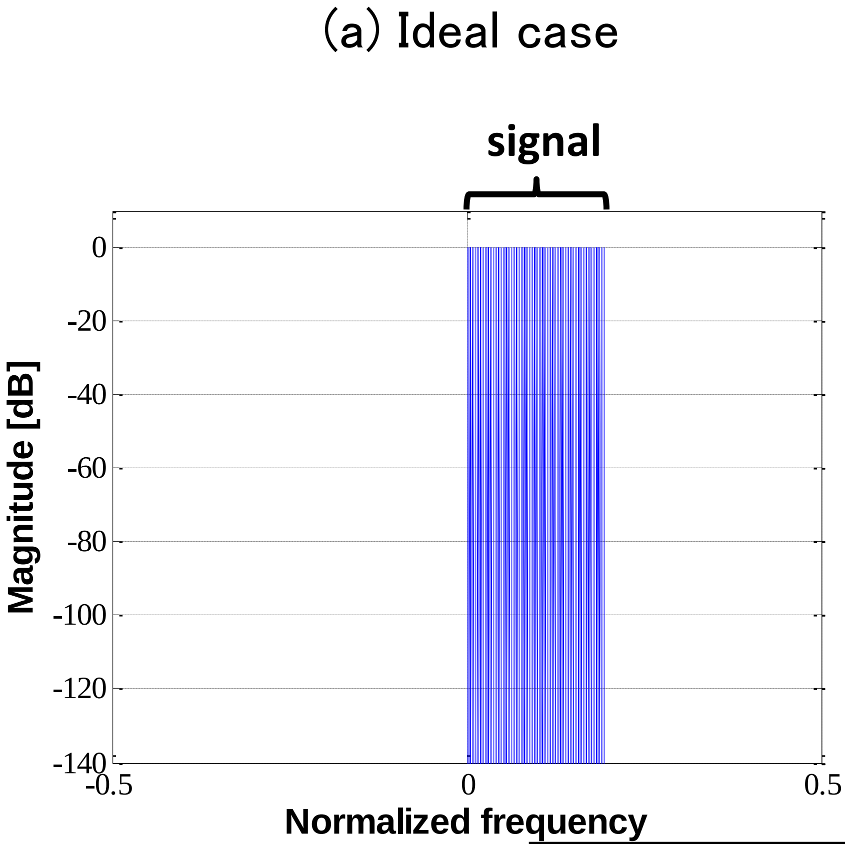
I/Q Delay Mismatch Compensation in Quadrature Modulator

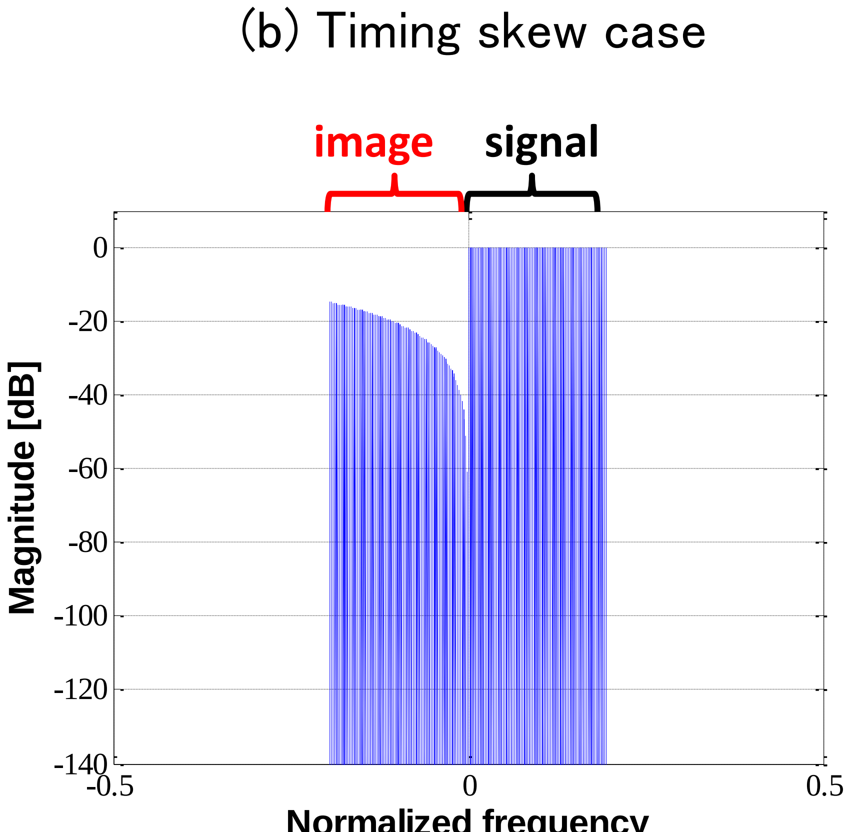


SSB : single side band

DAC : digital-to-analog converter

Matlab Simulation Results



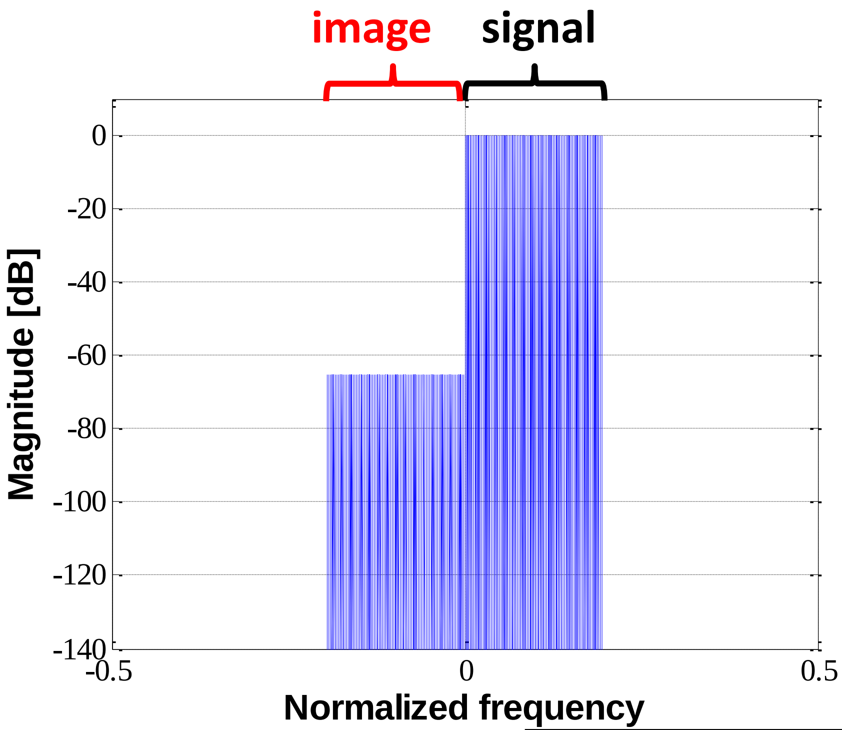


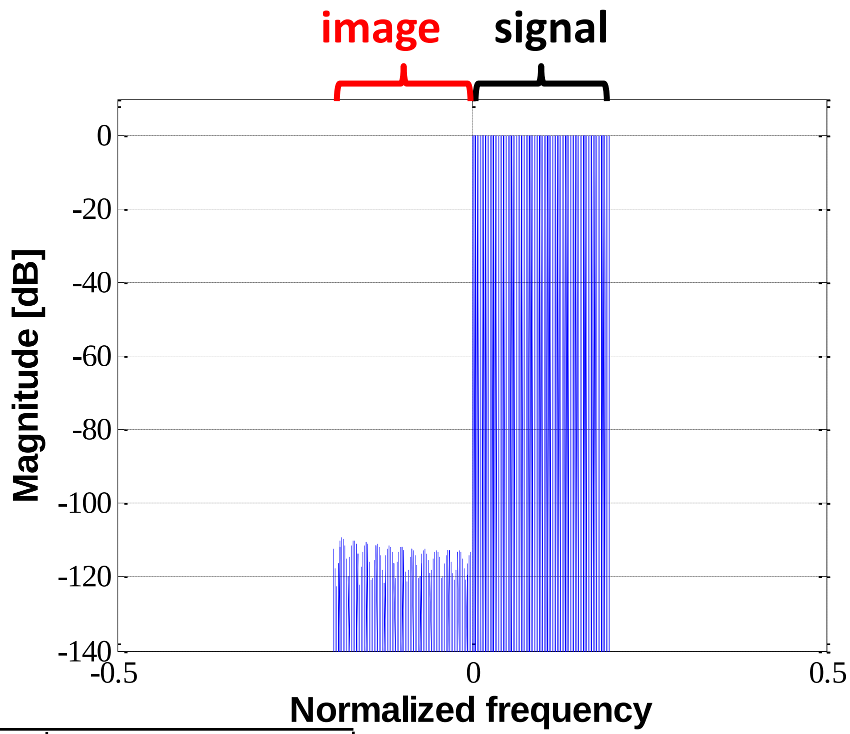
Delay	0.3 samples
Filter tap #	61 taps
Window	Han
FFT points	1024 points

Matlab Simulation Results

(c) Compensation using delay filter
Without adjustment of window, gain

(d) Compensation using delay filter
With adjustment of window, gain

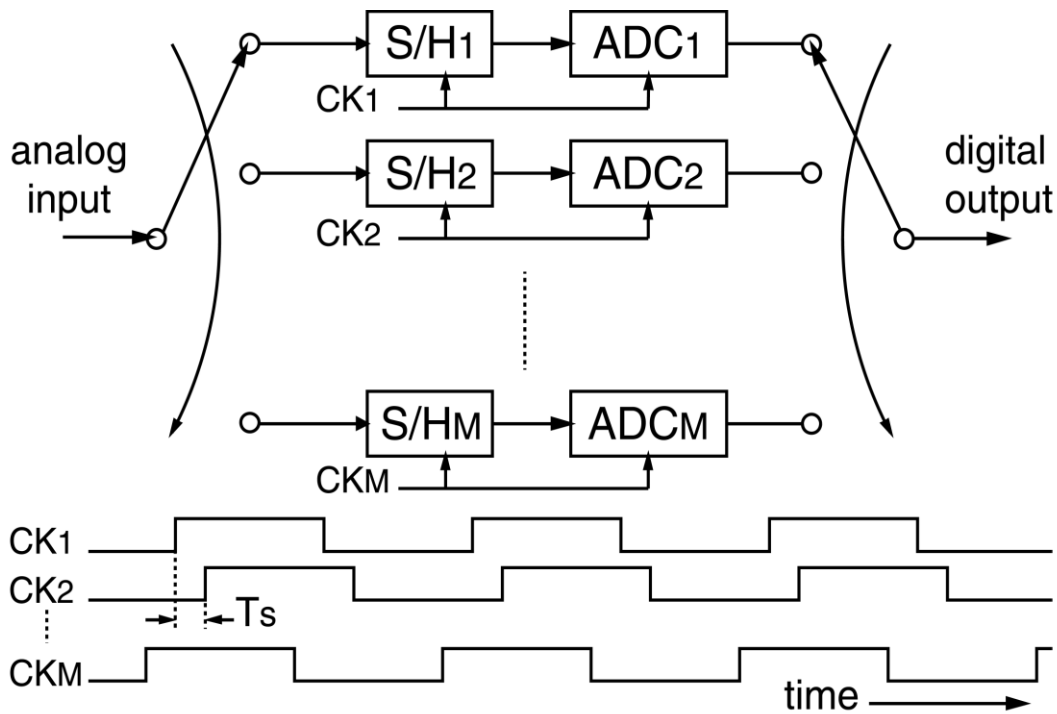




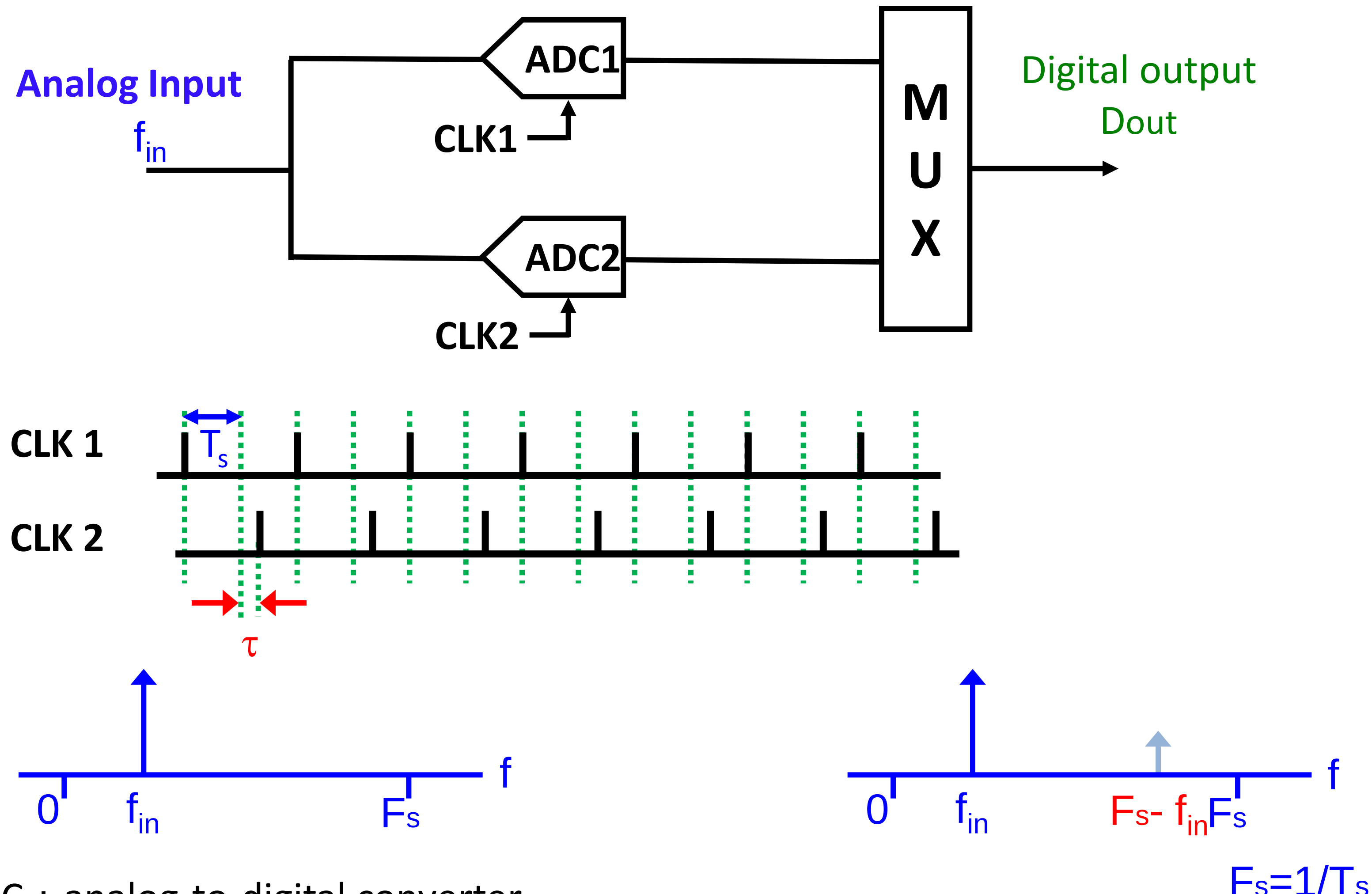
Delay	0.3 samples
Filter tap	61 taps
Window	Han
FFT points	1024 points

Interleaved ADC System

- M channel ADCs ➡ M-times sampling rate

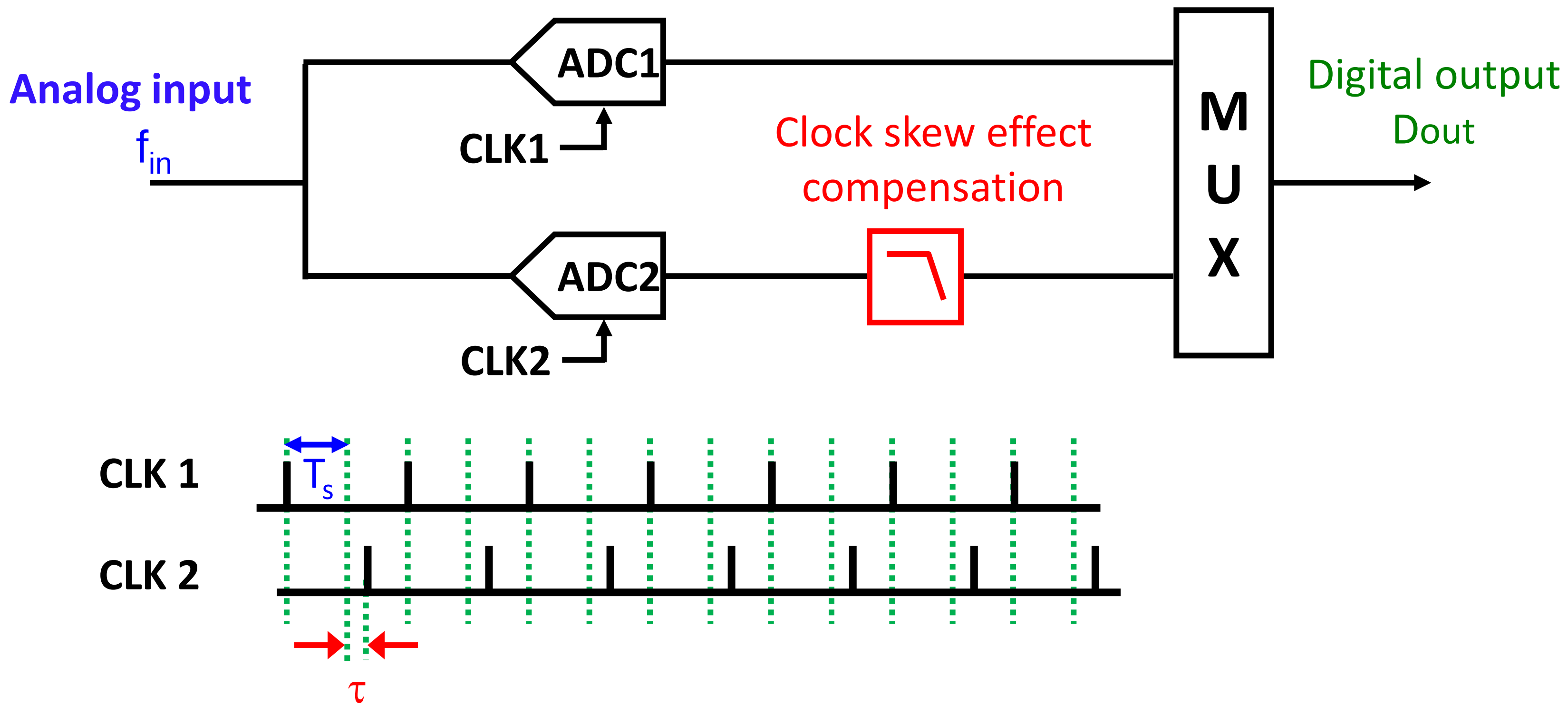


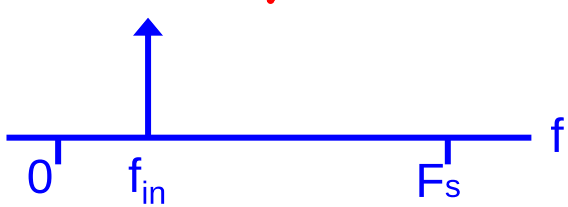
Timing Skew in Interleaved ADC System

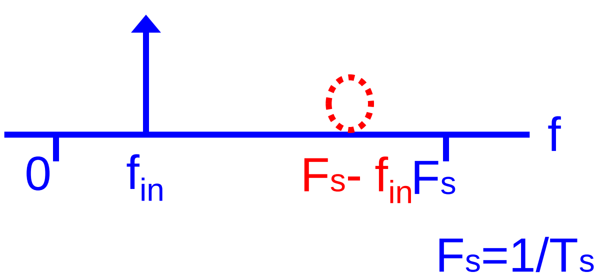


ADC：analog-to-digital converter

Timing Skew Compensation in Interleaved ADC System

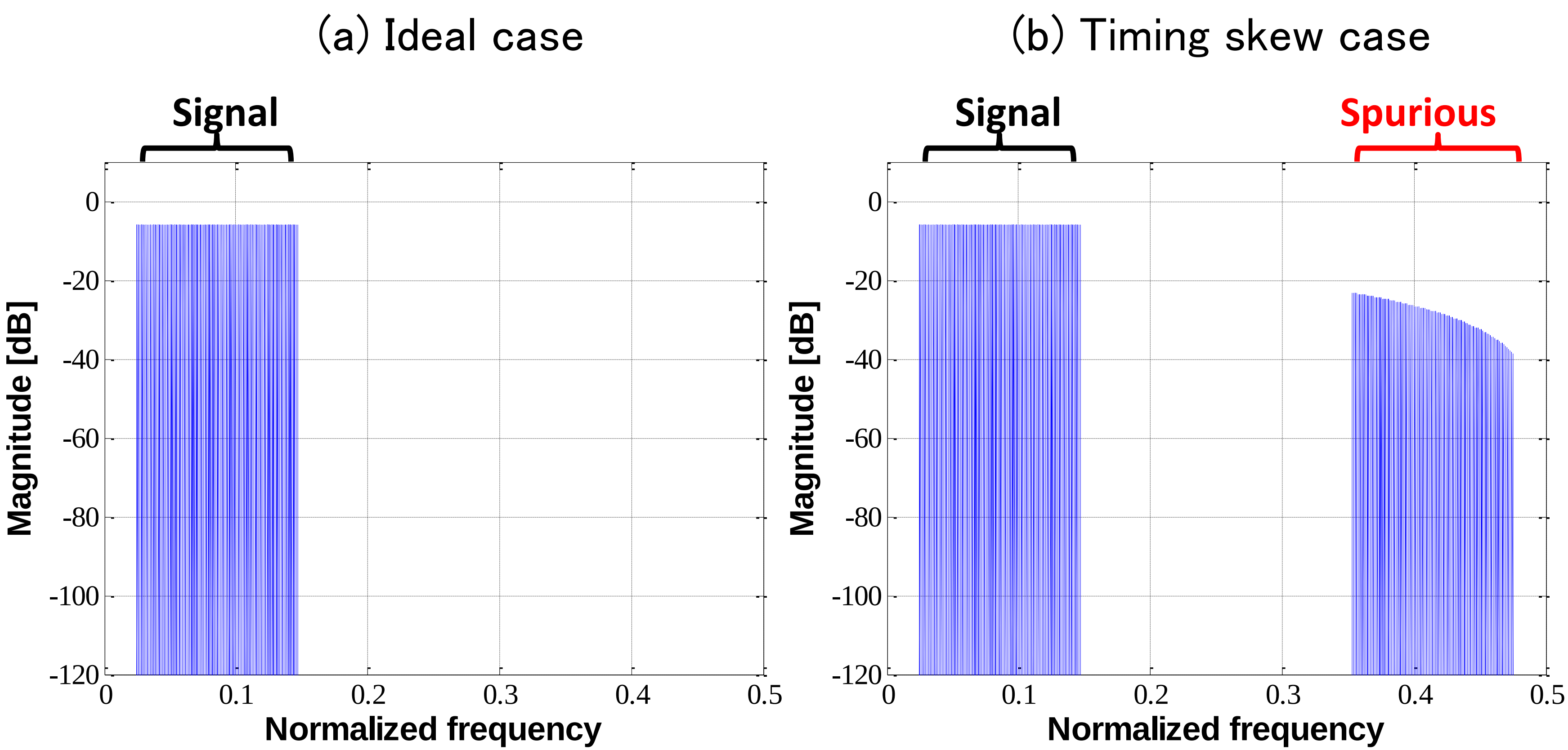






ADC：analog-to-digital converter

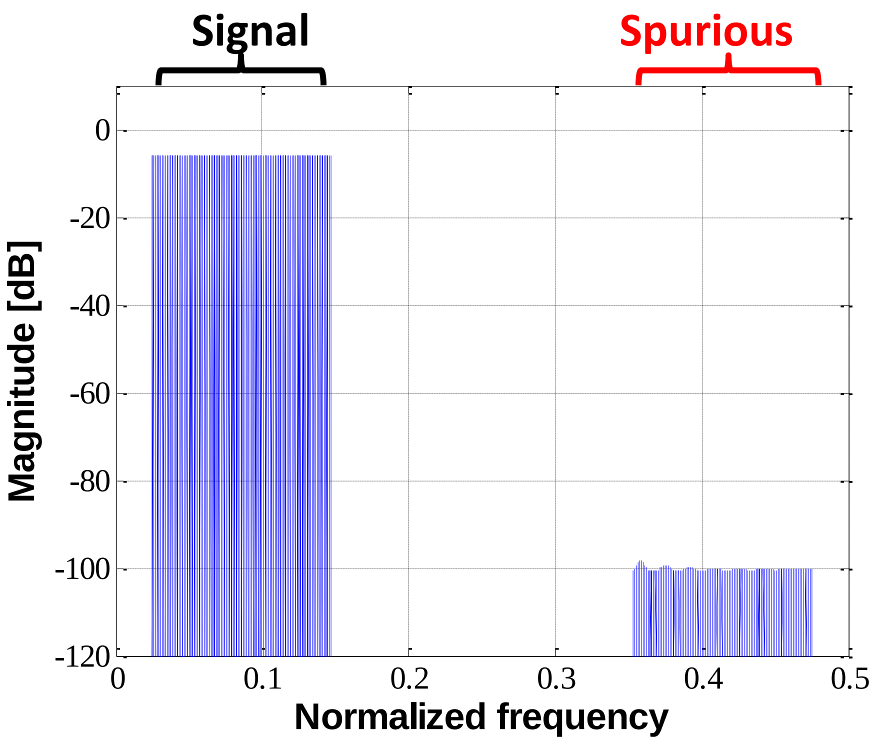
Matlab Simulation Results



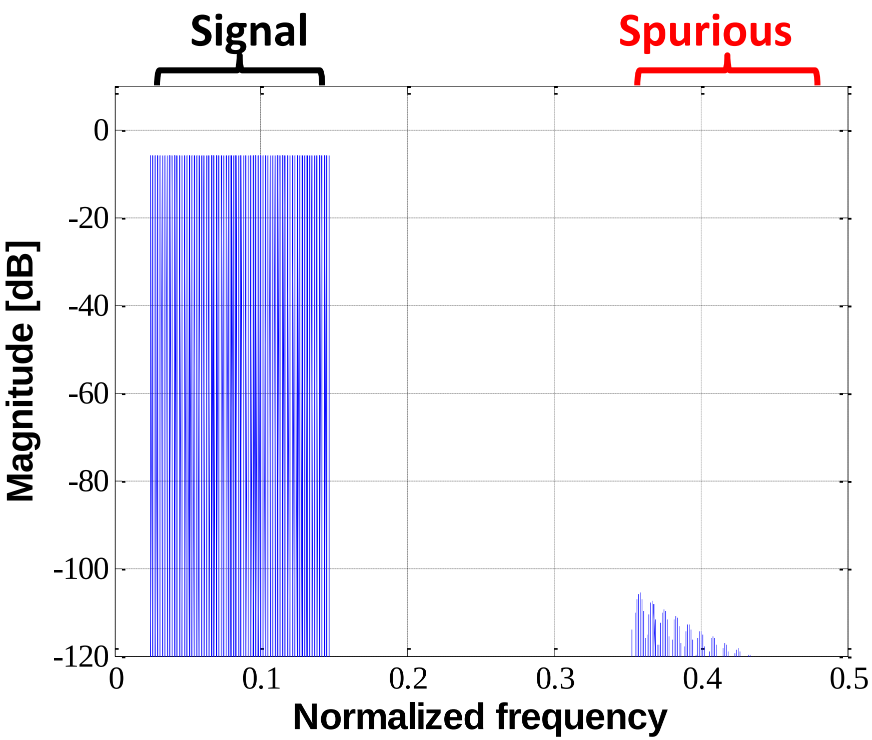
Delay	0.3 samples
Filter tap	61 taps
Window	Han
FFTpoints	1024 points

Matlab Simulation Results

(c) Compensation using delay filter
Without adjustment of window, gain



(d) Compensation using delay filter
With adjustment of window, gain



Delay	0.3 samples
Filter tap	61 taps
Window	Han
FFTpoints	1024 points

Conclusion

- Linear phase digital filter
  with fine time resolution of group delay
- Design consideration
  - How to apply window
  - DC gain adjustment
- Application Examples
  - I/Q delay mismatch compensation
    in quadrature modulator
  - Timing skew compensation in interleaved ADC system

On-going work

- Implementation issues
  – Finite word length, finite tap effects
  – LSI implementation

Digitally-Assisted Compensation Technique for Timing Skew in ATE Systems

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Abstract

This paper describes timing skew adjustment techniques in ATE systems (such as for timing skew compensation in an interleaved ADC system and an SSB signal generation system) using a digital filter with novel linear phase condition proposed in our ITC2010 paper. A conventional linear phase digital filter is an FIR filter with coefficients of odd- or even –symmetry and whose group delay NTs/2 where N is the number of the FIR filter taps and Ts is the sampling period; its group delay time resolution is Ts/2. We have generalized the linear phase condition, and with our novel linear phase condition, the group delay time resolution can be arbitrary small, and the coefficients are not necessarily odd- or even-symmetric. In this paper we discuss several practical issues for applying our digital filter to timing skew compensation in ATE systems, such as truncation of the infinite number of taps, techniques of using window and DC gain adjustment. We also compare our digital filter with the fractional delay digital filter.

Keywords: Digital Filter, Linear Phase, Digitally-Assisted Analog Technology, Timing Skew, ATE, Fractional Delay Digital Filter

1. Introduction

In this paper we describe a digital filter with novel linear phase condition and show that its delay time resolution is arbitrary fine (i.e., its group delay can be set with arbitrary small time resolution), and its practical issues for timing skew adjustment applications in ATE systems. In section 2, conventional linear phase condition for digital filter is explained, and in section 3, our novel linear phase condition is explained based on our ITC2010 paper [1]. In section 4, we investigate realization consideration for our digital filter, and in section 5, comparison with fractional delay filter is shown. Section 6 concludes the paper.

2. Conventional Linear Phase Condition

Linear phase characteristics are important for the digital filter to preserve the signal waveform in time domain. It is well-known in [2-4] that the FIR digital filter with odd or even symmetry coefficients has linear phase characteristics and it is unconditionally stable. The IIR digital filter with odd or even symmetry of both its denominator and numerator has also linear characteristics but it is unstable. Hence in almost all cases, the FIR digital filter with odd or even symmetry coefficients is used where the linear phase is required, and in such cases its group delay is (N/2)Ts where N is the number of the FIR filter taps and T_s is the sampling period; in other words the time resolution of the group delay is $T_s/2$, and this cannot be used for fine timing skew adjustment in ATE systems.

3. Novel Linear Phase Condition

In this section, we describe - based on our ITC2010 paper [1] - the extended linear phase characteristics conditions for the digital filter which has not necessarily odd or even symmetry coefficients, and its time resolution of the group delay is arbitrary small.

First we discuss without consideration of causality, for simplicity. Let us consider the following analog filter (Fig.1):

$$v_{out} = \begin{cases} a_0 v_{in}(t) & \text{in case} -\pi/T_s < \omega < \pi/\mathrm{T_s} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Then its impulse response h(t) is given as follows:

$$h(t) = a_0 T_s \operatorname{sinc}(\pi t/T_s). \tag{2}$$

We consider the case that the input $v_{in}(t)$ is band-limited to $-\pi/T_s < \omega < \pi/\mathrm{T_s}$. We sample the above impulse response with a period Ts, and use the following transformation to obtain the digital filter which corresponds to the analog filter in (1):

$$\begin{aligned} &T_s \to 1 \\ &v_{out}(nT_s) \to y(n) \\ &v_{in}(nT_s) \to x(n). \end{aligned} \tag{3}$$

Then we have the following digital filter:

$$y(n) = a_0 x(n) \tag{4}$$

This is because

$$\operatorname{sinc}(\pi n) = \begin{cases} 1 & \text{in case } n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

This digital filter has obviously linear phase characteristics (or rigorously speaking zero phase characteristics).

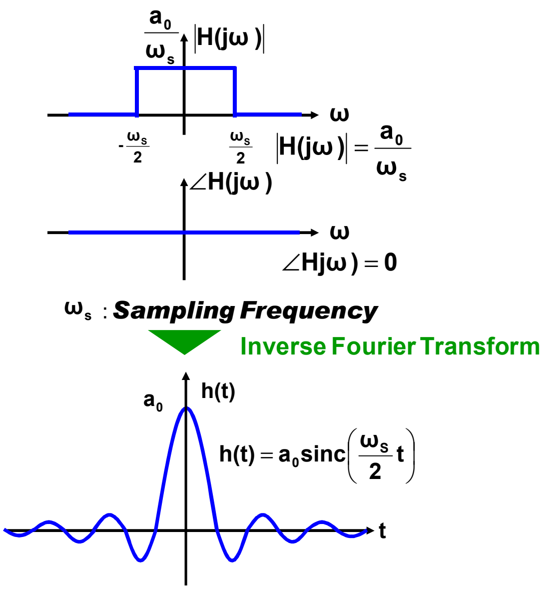

Fig. 1. An ideal analog low pass filter. Gain, phase characteristics, and impulse response.

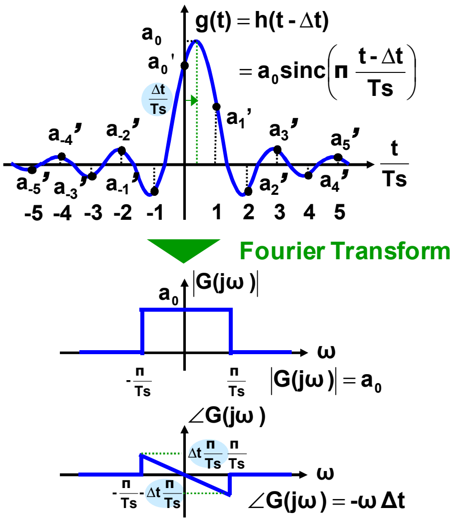

Fig. 2. Sampling timing shift can maintain the linear phase characteristics. Impulse response, and gain, phase characteristics.

Now let us consider to sample $h(t)$ at $t = nT_s + \tau$ (Fig.2), where $0 < \tau < T_s$, and use (3). characteristics). Then we have the following digital filter which corresponds to the analog filter in (1):

$$y(n) = \sum_{k=-\infty}^{\infty} a'_k x(n-k). \tag{6}$$

Here

$$a'_k = \mathrm{sinc}(\pi(k + \tau/T_s)). \tag{7}$$

In general a'_k is not necessarily zero and a'_k is not necessarily equal to a'_{-k} or $-a'_{-k}$.

Proposition 1 : The digital filter given by (6), (7) has the linear filter characteristics, and its group delay is τ.

Next we discuss in case of Fig.3, and consider the following analog filter:

$$v_{out} = \begin{cases} a_0 v_{in}(t) + a_1 v_{in}(t - T_s) \\ \quad \text{in case} - \pi/T_s < \omega < \pi/T \\ 0 \quad \text{otherwise.} \end{cases} \tag{8}$$

Note that its impulse response is given as follows:

$$h(t) = a_0 T_s \mathrm{sinc}(\pi t/T_s) + a_1 T_s \mathrm{sinc}(\pi(t/T_s - 1)). \tag{9}$$

We assume that the input $v_{in}(t)$ is band-limited to $-\pi/T_s < \omega < \pi/T_s$. Similarly we sample this filter with $t = nT_s$, and we have the following digital filter using (3):

$$y(n) = a_0 x(n) + a_1 x(n-1). \tag{10}$$

Next we sample (9) with $t = nT_s + \tau$, and we have the following digital filter:

$$y(n) = \sum_{k=-\infty}^{\infty} a'_k x(n-k). \tag{11}$$

Here

$$a'_k = a_0 \mathrm{sinc}(\pi(k + \tau/T_s)) + a_1 \mathrm{sinc}(\pi((k-1) + \tau/T_s)). \tag{12}$$

Proposition 2 : The digital filter given by (11), (12) with $a_0 = a_1$ or $a_0 = -a_1$ has the linear phase characteristics and its group delay is $T_s/2 + \tau$. Also the digital filter of (11) has the same gain characteristics as (10). The same argument holds for an N-tap FIR filter.

Proposition 3 : Let us consider an N-tap FIR digital filter with coefficients a_k of odd or even symmetry.

$$y(n) = \sum_{k=0}^{N-1} a_k x(n-k) \tag{13}$$

Then the following digital filter has the linear characteristics with group delay $(N/2)T_s + \tau$.

$$y(n) = \sum_{k=-\infty}^{\infty} a'_k x(n-k) \tag{14}$$

Here

$$a'_k = \sum_{l=0}^{N-1} a_l \mathrm{sinc}(\pi((k-l) + \tau/T_s)). \tag{15}$$

Table 1 shows the frequency characteristics of digital filters with our proposed linear phase condition.

TABLE I Frequency characteristics of the proposed linear phase digital filter

case	N	$h(n)$	$H(e^{j\omega})$
1	odd	even symmetry	$e^{-j\omega(\frac{N-1}{2} + \frac{\tau}{T_s})} \sum_{k=0}^{\frac{N-1}{2}} a_k \cos(k\omega)$
2	even	even symmetry	$e^{-j\omega(\frac{N-1}{2} + \frac{\tau}{T_s})} \sum_{k=0}^{\frac{N}{2}} a_k \cos((k\text{-}\frac{1}{2})\omega)$
3	odd	odd symmetry	$e^{-j\omega(\frac{N-1}{2} + \frac{\tau}{T_s})} \sum_{k=0}^{\frac{N-1}{2}} a_k \sin(k\omega)$
4	even	odd symmetry	$e^{-j\omega(\frac{N-1}{2} + \frac{\tau}{T_s})} \sum_{k=0}^{\frac{N}{2}} a_k \sin((k\text{-}\frac{1}{2})\omega)$

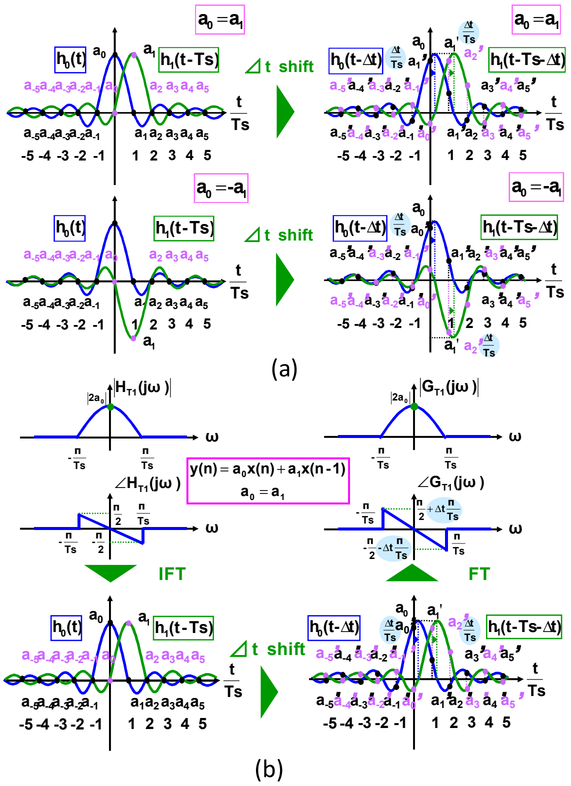

Fig. 3. 2-tap FIR filter without and with sampling timing shift. (a) Impulse response. (b) Gain and phase responses.

Now we will provide the proof for our proposed linear phase digital filter in general case: let us consider an N-tap FIR filter with conventional linear phase condition, and we have the impulse response $\tilde{h}(t)$ with continuous time and its Fourier transform $\tilde{H}(f)$:

$$\tilde{h}(t)=\sum_{n=0}^{N-1} h(nT_s)\delta(t-nT_s). \qquad (16)$$

$$\tilde{H}(f)=H(f)\star\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta(f-\frac{k}{T_s}). \qquad (17)$$

Here $\star$ indicates convolution, $\delta(\cdot)$ denotes a delta function, and

$$H(f)=|H(f)|e^{-j2\pi f\frac{N-1}{2}T_s} \quad (-\frac{1}{2T_s}\le f\le\frac{1}{2T_s}). \qquad (18)$$

When we add a delay τ to the impulse response $h(t)$ and we have its frequency characteristics as follows:

$$H'(f)=|H(f)|e^{-j2\pi f\frac{N-1}{2}T_s}\cdot e^{-j2\pi f\tau}. \qquad (19)$$

We see that the phase characteristics of $H'(f)$ is linear with respect to f. $H'(f)$ can be interpreted as the convolution between $H(f)$ and $S(f)$, where $S(f)$ is the ideal filter with a delay τ:

$$S(f)=e^{-j2\pi f\frac{N-1}{2}\tau} \quad (-\frac{1}{2T_s}\le f\le\frac{1}{2T_s}). \qquad (20)$$

Thus after the sampling operation in time domain, the ideal filter $S(f)$ in Eq.(20) leads to the following $\tilde{S}(f)$:

$$\tilde{S}(f)=S(f)\star\sum_{k=-\infty}^{\infty}\delta(f-\frac{k}{T_s})=\sum_{k=-\infty}^{\infty}S(f-\frac{k}{T_s}). \qquad (21)$$

Next we will consider the effect of the delay τ to the impulse response. The inverse Fourier transform of $\tilde{S}(f)$ is given as follows:

$$\begin{aligned}\tilde{s}(t)&=\mathrm{sinc}(\frac{\pi(t-\tau)}{T_s})\cdot\sum_{n=-\infty}^{\infty}\delta(t-nT_s)\\&=\sum_{n=-\infty}^{\infty}\mathrm{sinc}(\frac{\pi(t-\tau)}{T_s})\delta(t-nT_s).\end{aligned} \qquad (22)$$

We see from (22) that $\tilde{s}(t)$ is asymmetric with respect to $t=0$, and we have the following impulse response:

$$\begin{aligned}&\tilde{h}(t)\star\tilde{s}(t)\\&=\sum_{n=0}^{N-1}h(nT_s)\delta(t-nT_s)\star\sum_{n=-\infty}^{\infty}\mathrm{sinc}(\frac{\pi(kT_s-\tau)}{T_s})\cdot\delta(t-nT_s)\\&=\sum_{n=-\infty}^{\infty}\sum_{n=0}^{N-1}h(nT_s)\mathrm{sinc}(\frac{\pi(kT_s-\tau)}{T_s})\cdot\delta(t-(n-k)T_s).\end{aligned} \qquad (23)$$

Thus the impulse response of time delay τ with continuous time has finite values for $t\to\pm\infty$ due to the *sinc* function effects.

4. Realization Consideration

We sample the input signal with the sampling period Ts and then we consider the band-limited case to $-\pi/T_s<\omega<\pi/T_s$, in order to avoid the aliasing effects. In such case $h(t)$ does not converge to zero as t becomes plus/minus infinity. So the digital filter with our novel linear phase condition has to have the infinite number of taps and this cannot be realized. (Note that in case of $\tau=0$, $h(nT_s)$ can be zero as n becomes large which corresponds to the conventional linear phase FIR digital filter case.) So we consider to truncate the terms for large number of $|k|$ in (15) applying a window function and we approximate the digital filter of (14), (15) with the finite number of taps. We also consider here the DC gain adjustment.

4.1 Approximation with Finite Number of Taps

The ideal digital filter with our proposed linear phase condition needs infinite number of taps. However it is cannot be realized, and hence we have to approximate it as the filter with finite number of taps. We consider here the effects of the truncation to the finite number of taps. We observe from our simulation results so-called Gibbs oscillation at the edges of pass-band of the gain characteristics and also phase characteristics (Fig.4) [2], [3]; Gibbs oscillation for phase characteristics is not observed in many cases, and we have found that this Gibbs oscillation for phase characteristics is due to the asymmetry of the impulse response $h(n)$ with respect to $n=0$.

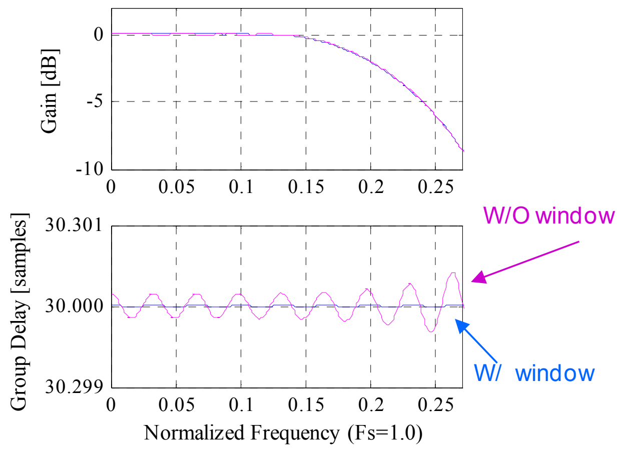

Fig. 4. Gain and phase characteristics of the proposed digital filter (with time shift τ of 0.3Ts) after truncation to finite number (N=61) of filter taps with and without applying Hann window.

4.2 Applying Window Function

Next we investigate to use window functions when we approximate the ideal filter using the one with the finite number of taps. When we use a window function, the Gibbs oscillations for gain and phase are suppressed. Fig.5 shows our simulation result with time-shift τ of 0.3Ts and applying Hann window. We have also found that this Gibbs oscillation for phase can be further suppressed if we use a window function with the time-shift τ, as shown in Fig.5 where we choose the time shift τ of 0.5Ts (which affects phase characteristics significantly) and we use a Hann window.

There can be two methods for applying a window: one is to use the window with symmetry to the Y-axis (Fig.5 (a)) and the other is to use the window with the symmetry to the center of the impulse response (Fig.5 (b)). We have performed simulation and found that the one in Fig.5 (b) is better. The Gibbs oscillation of the group delay is suppressed when the window of time-shift is used for the LPF (Fig.6).

Our proposed linear phase filter is also applicable to a bandpass filter and Fig.7 shows the group delay of the bandpass filter with the bandwidth of 0.1 fs – 0.4fs. We see that the group delay is almost constant in the wider range when the window of time-shift is used (Fig.7 (b)).

4.3. DC Gain Adjustment

The DC gain of our digital filter can be changed by truncation to the finite number of the taps after windowing, and we have to adjust it for the practical use. The DC gain adjustment technique can be described as follows:

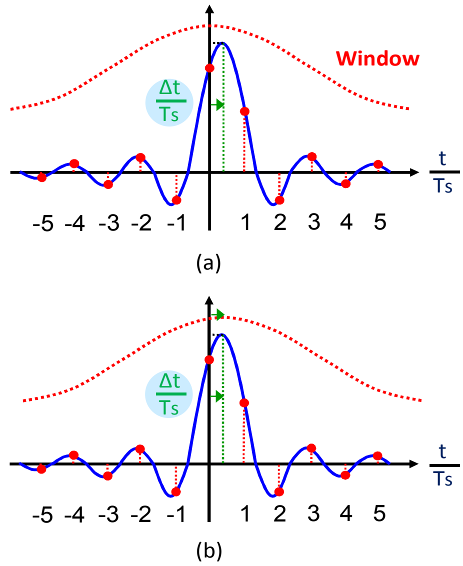

Fig.5. (a) Window with symmetry to the Y-axis (window is not time-shifted). (b) Window with symmetry to the center of the impulse response (window is time-shifted).

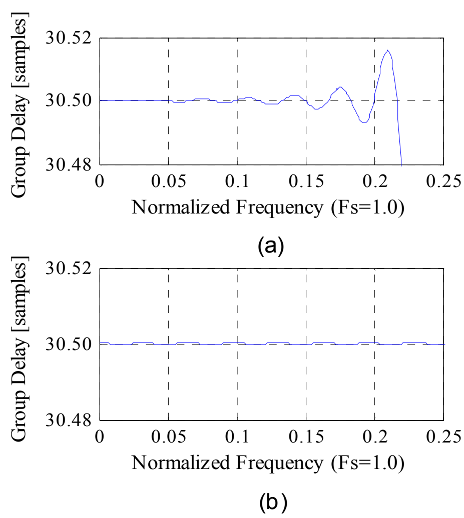

Fig.6. Group delay characteristics of the proposed digital filter (with time shift τ of $0.5T_s$) after truncation to finite number (N = 61) of filter taps. (a) With applying Hann window of no time-shift. (b) With applying Hann window time-shifted by $\tau = 0.5T_s$

Our digital filter without DC gain adjustment

$$g(n) = h(n) \text{ where } n = 0, \pm 1, \pm 2, \pm 3, \ldots \pm N$$

Our digital filter with DC gain adjustment

$$g'(n) = (G_{ideal} / G_{fnt})\, h(n)$$

$$\text{where } n = 0, \pm 1, \pm 2, \pm 3, \ldots \pm N$$

Here DC gain of the ideal filter is given by

$$G_{ideal} = \sum_{n=-\infty}^{\infty} h(n)$$

Also DC gain of the filter after truncation of the finite number (2N+1) taps is given by

$$\mathrm{Gfnt} = \sum_{n=-N}^{N} h'(n)$$

We have performed simulation to demonstrate the effectiveness of the window with time-shift and DC gain adjustment in the single-side band (SSB) signal generation system in Fig,8. We assume that there is timing skew τ in I-path and we use our timing skew compensation digital filter in Q-path. Fig. 11 (a) shows the power spectrum of the output s(t) without timing skew, and Fig.9 (b) shows the one with timing skew τ where spurious components are observed.

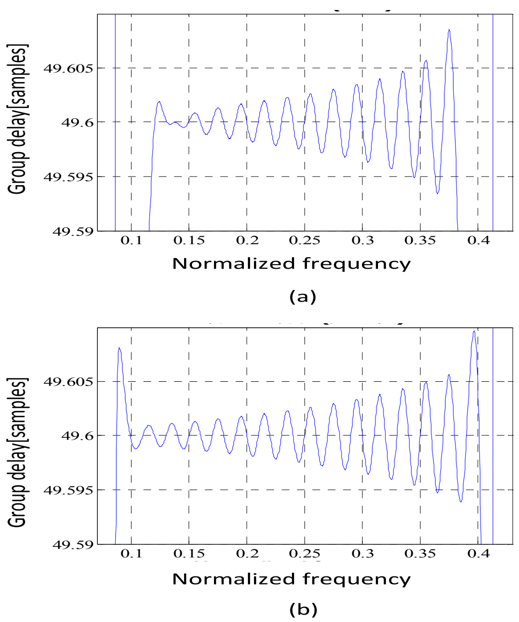

Fig.7.Group delay of bandpass filter with the bandwidth of 0.1fs – 0.4fs. (a) With applying Hann window of no-time shift. (b) With applying Hann window of time-shift.

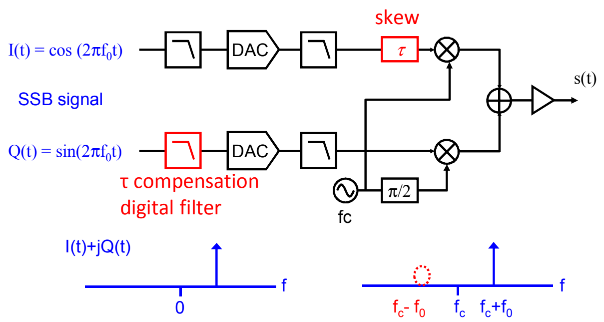

Fig. 8. SSB signal generation system with timing skew τ in I-path and the timing skew compensation digital filter in Q-path.

Fig.10 shows the simulation result using timing skew compensation with our proposed digital filter. Fig.10 (a) is the case that the window of no-time shift are used and DC gain adjustment is not used while Fig.10 (b) is the case that the window of time-shift and DC gain adjustment are used. We see that spurious components

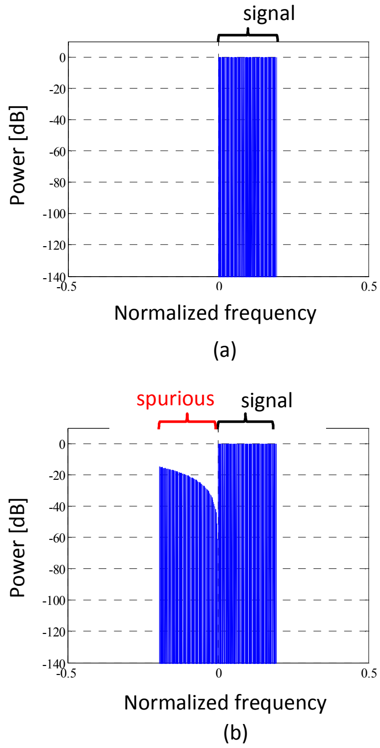

Fig.9. Simulation results of output power spectrum of the SSB signal generation system in Fig.8. (a) Without timing skew. (b) With timing skew τ.

are further suppressed when the time-shifted window and DC gain adjustment are used.

5. Comparison with Fractional Delay Filter

We would like to call the reader's attention that another digital filter with fine time resolution, so-called a fractional delay digital filter has been proposed [5-8], which mainly focuses on the waveform interpolation and reconstruction. However our proposed technique can incorporate filtering characteristics (such as a cosine roll-off filter, a Gaussian filter) as well as fine timing skew adjustment with the clear design method as described above; this is very useful in some electronic manufacturing equipment applications [1]. Furthermore, since our proposed filter is easy to design, we can obtain their coefficient values with small amount of calculation which is desirable for many applications, especially ATE systems where real-time timing calibration is required.

We have performed Matlab simulation and found that our proposed filter can apply for the signal up to the frequency close to the Nyquist rate (in other words, the bandwidth of our proposed filter is close to the Nyquist rate while that of the fractional delay filter is not); this is another advantage of our proposed filter (Fig.11).

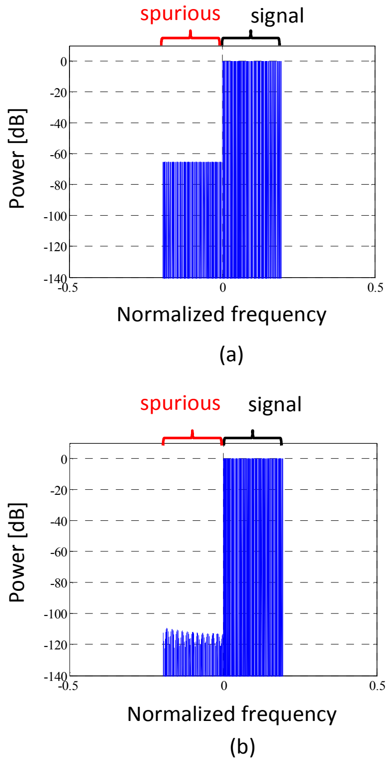

Fig.10. Simulation results of timing skew compensation with our proposed digital filter. (a) With the window of no time-shit and without DC gain adjustment. (b) With the window of time-shift and with DC gain adjustment.

6. Conclusion

We have described the digital filter with novel linear phase characteristics and the time resolution of its group delay is arbitrary small. We have investigated the truncation effects to the finite number of its filter taps, techniques of using window and DC gain adjustment as well as comparison with fractional delay filter. We believe that our proposed digital filter is opening a new research area for digital filters with linear phase and fine resolution of group delay, as well as its applications.

7. Acknowledgement

We would like to thank Zachary Nosker for his help in assembling this paper.

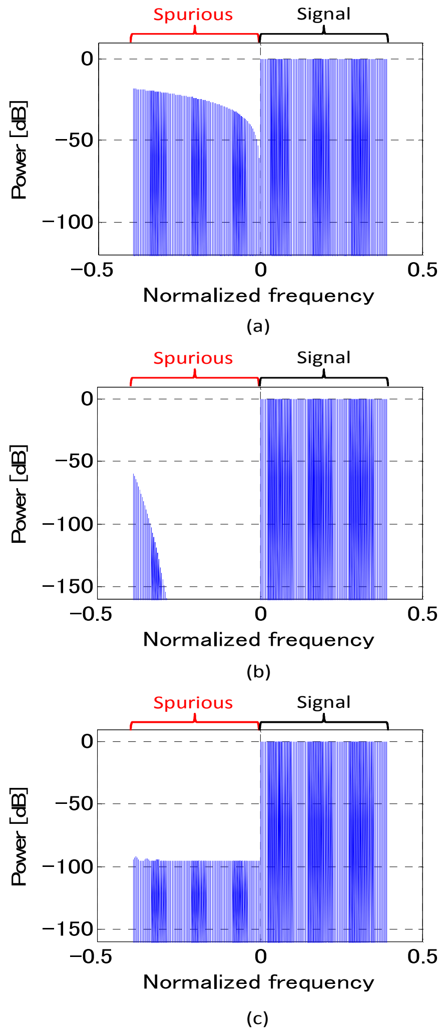

Fig.11: SSB signal power spectrum Matlab simulation results. (a) Without compensation. (b) With compensation using a 301-tap fractional delay filter and a (symmetric) blackman window. (c) With compensation using our proposed 301-tap digital filter and the same blackman window.

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Timing Skew Compensation Technique Using Digital Filter with Novel Linear Phase Condition

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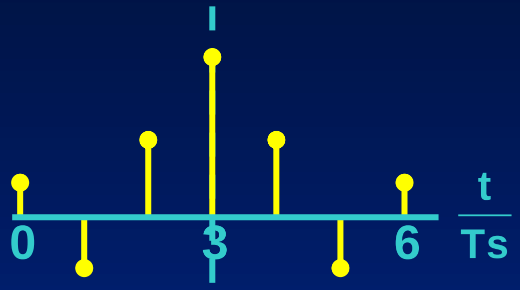
Purpose

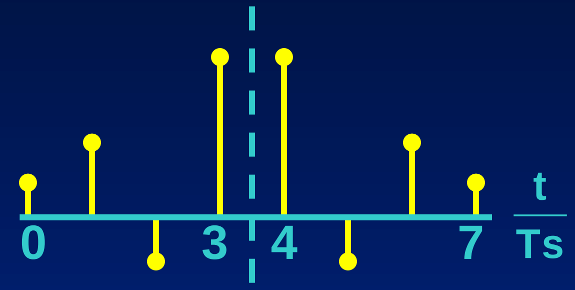
- Fine skew adjustment using a digital filter while maintaining a linear phase condition in ATE
  - Timing accuracy is important to ATE
  - Various digital filters are used for testing analog LSIs
  - Linear phase condition is required of the digital filter to preserve the analog waveform

Outline

- Conventional linear phase FIR filter
- Time-shifted ideal filter
- Construction of linear phase filter
- Application examples
- Conclusion

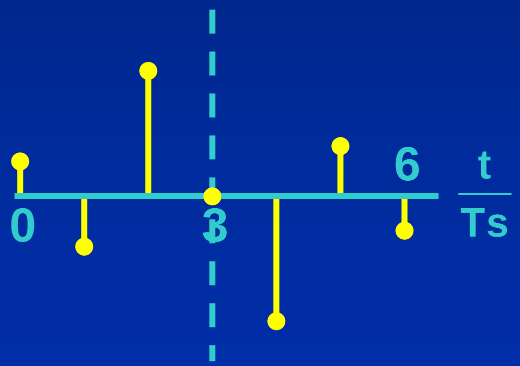
4 Types of Generalized Linear-Phase FIR Systems

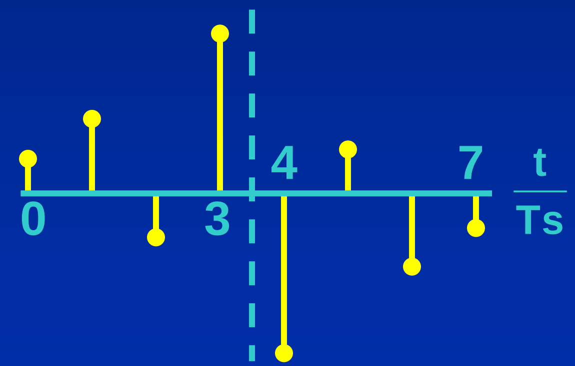




(1)Type I
symmetric
even-order

(2)Type II
symmetric
odd-order





(3)Type III
antisymmetric
even-order

(4)Type IV
antisymmetric
odd-order

Frequency Characteristics of 4 Types

$h(nT)$	$H(e^{j\omega T})$
Type I	$e^{-j\omega(N-1)T_s/2}\sum_{k=0}^{(N-1)/2} a_k \cos[\omega k T_s]$
Type II	$e^{-j\omega(N-1)T_s/2}\sum_{k=1}^{N/2} b_k \cos[\omega (k-1/2) T_s]$
Type III	$e^{-j(\omega(N-1)T_s/2-\pi/2)}\sum_{k=0}^{(N-1)/2} a_k \sin[\omega k T_s]$
Type IV	$e^{-j(\omega(N-1)T_s/2-\pi/2)}\sum_{k=1}^{N/2} b_k \sin[\omega (k-1/2) T_s]$

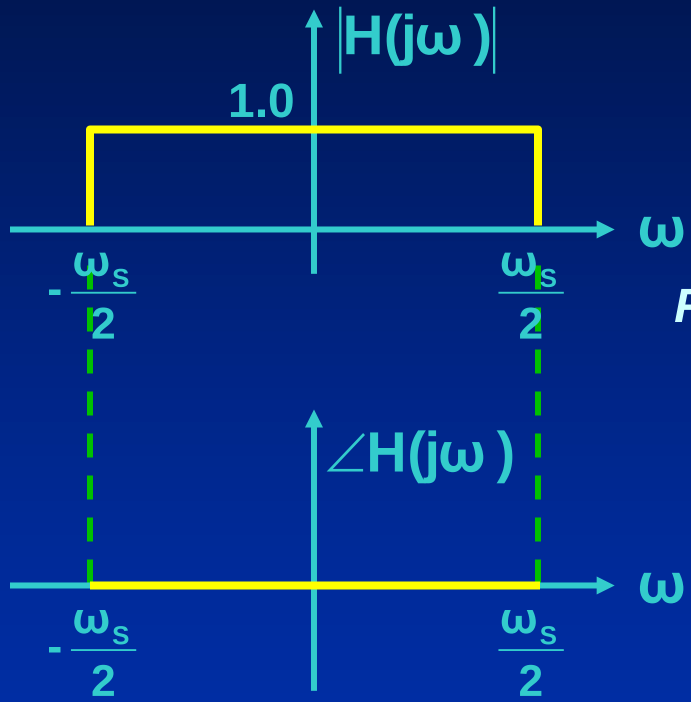
Phase : 1st order function of frequency
Delay : depends on number of Taps

Outline

- Conventional linear phase FIR filter
- Time-shifted ideal filter
- Construction of linear phase filter
- Application examples
- Conclusion

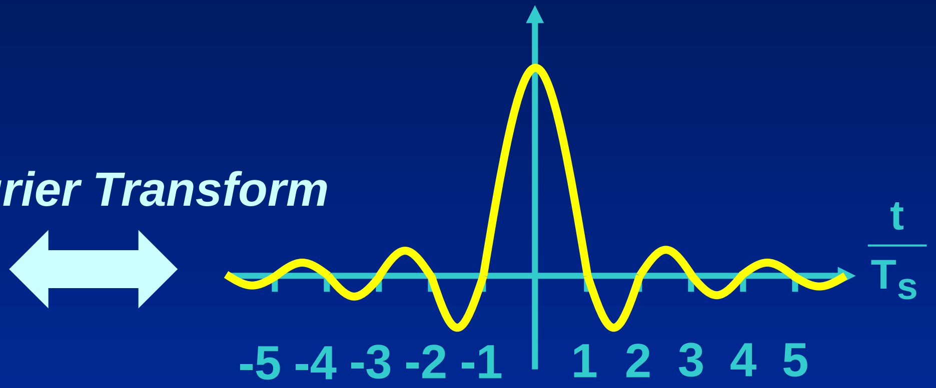
Ideal Filter Response

Frequency Response



Impulse Response

Fourier Transform

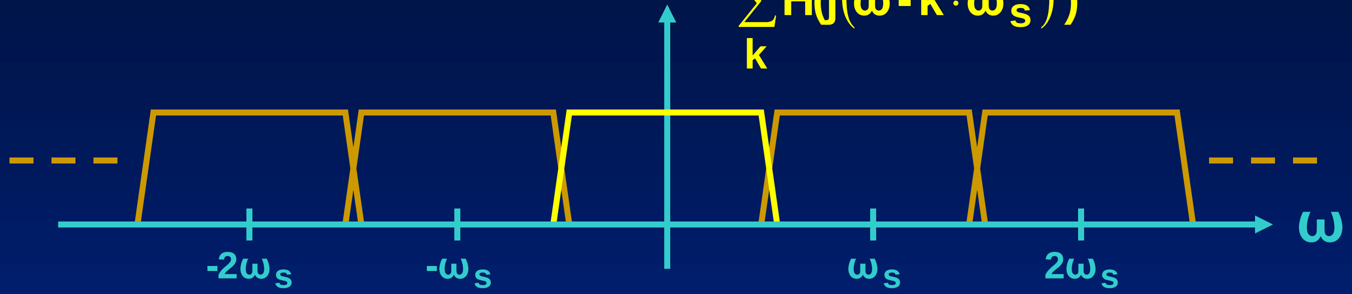


$$h(t) = \frac{1}{T_s} \cdot \mathrm{sinc}\left(\pi \frac{t}{T_s}\right)$$

$\omega_s = \frac{2\pi}{T_s}$ **: Sampling Rate**

Discrete-Time Expression

$$\sum_k H\left(j\left(\omega - k\cdot\omega_s\right)\right)$$

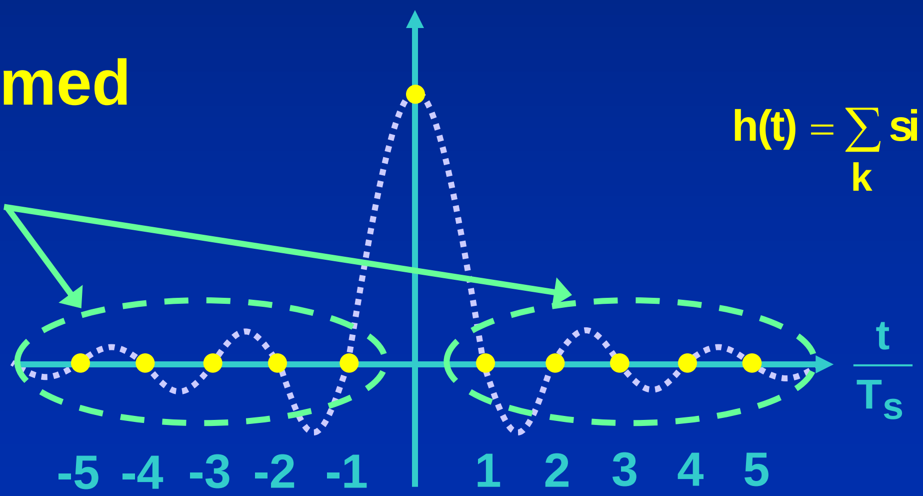


Fourier Transform

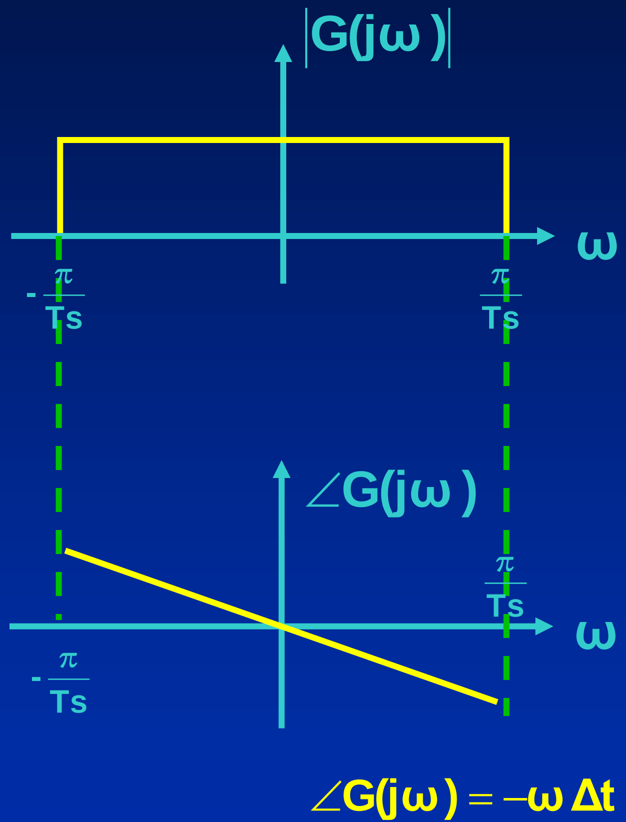
FIR is formed

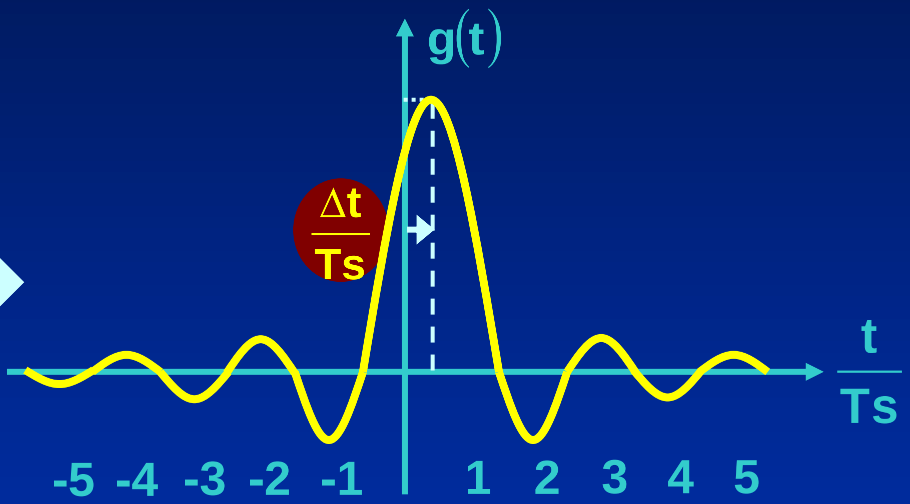
All zero

$$h(t) = \sum_k \mathrm{sinc}\left(\pi\frac{k\cdot T_s}{T_s}\right)\delta\left(t - k\cdot T_s\right)$$



Time Shifted Impulse Response





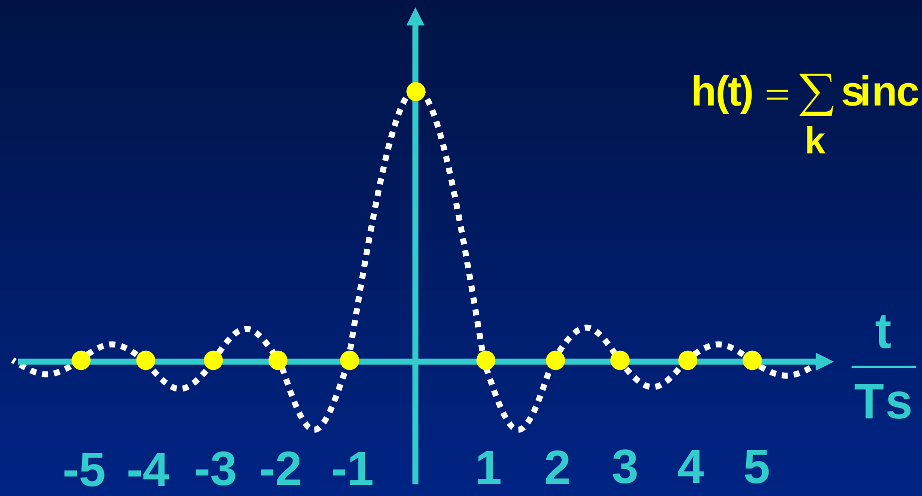
Only phase changed

Impulse response shifted Δt

Influence to Coefficients by Time Shift

FIR

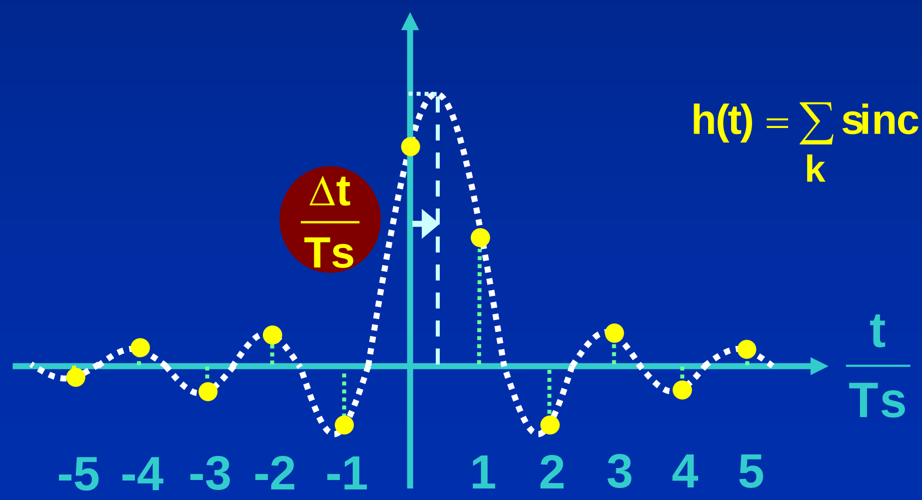
$$h(t) = \sum_k \mathrm{sinc}\left(\pi \frac{k \cdot T_S}{T_S}\right) \delta\left(t - k \cdot T_S\right)$$



Time shift

IIR

$$h(t) = \sum_k \mathrm{sinc}\left(\pi \frac{k \cdot T_S - \Delta t}{T_S}\right) \delta\left(t - k \cdot T_S\right)$$



Outline

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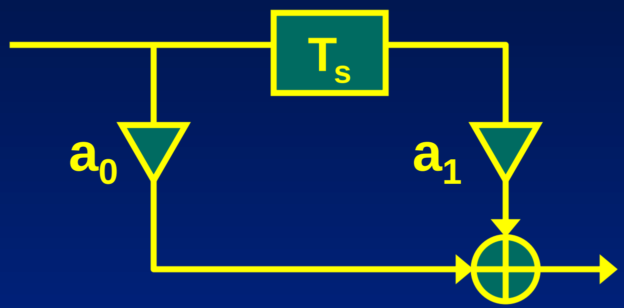
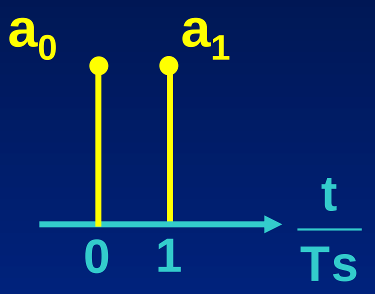
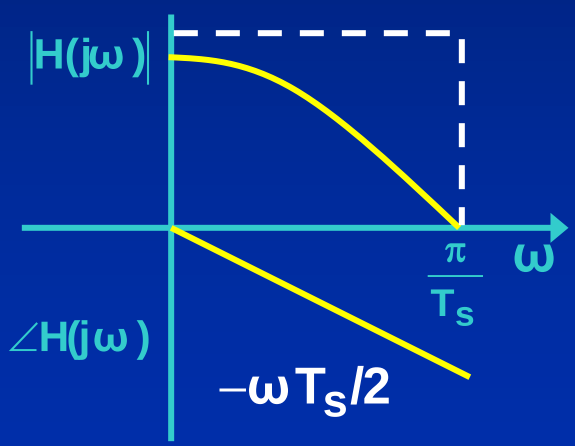
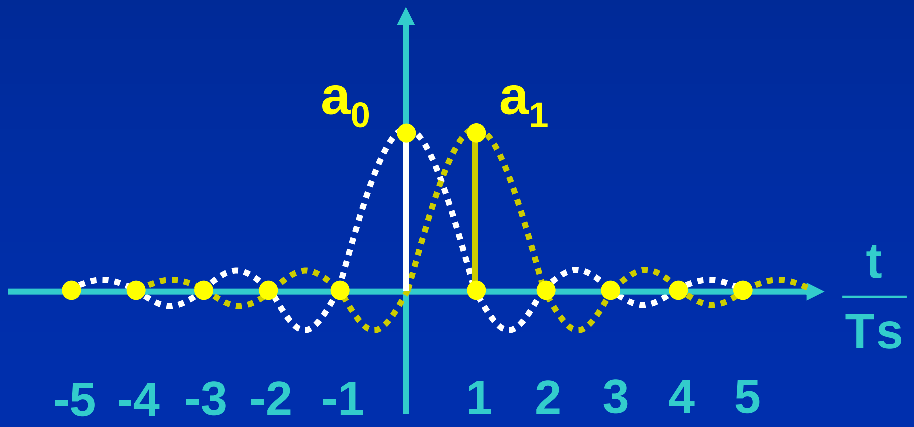
2 Tap FIR Model

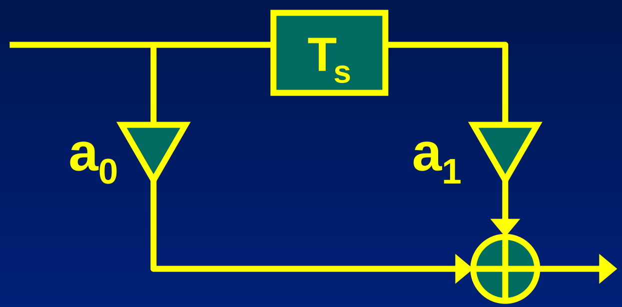


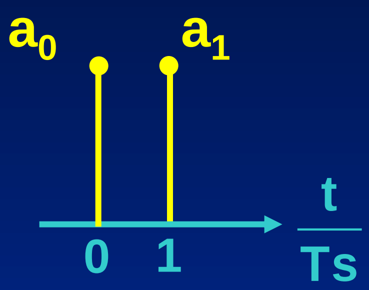


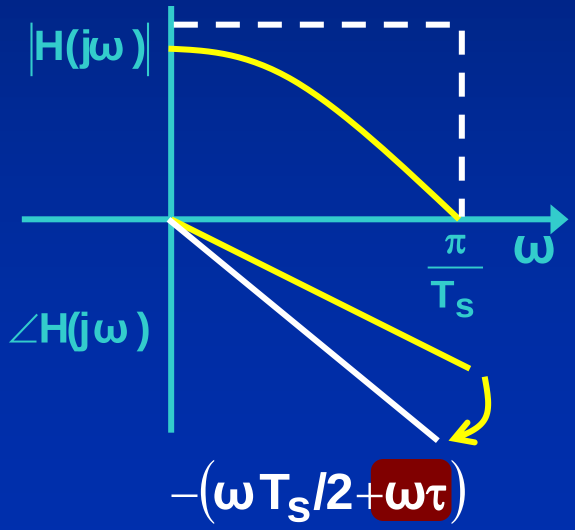


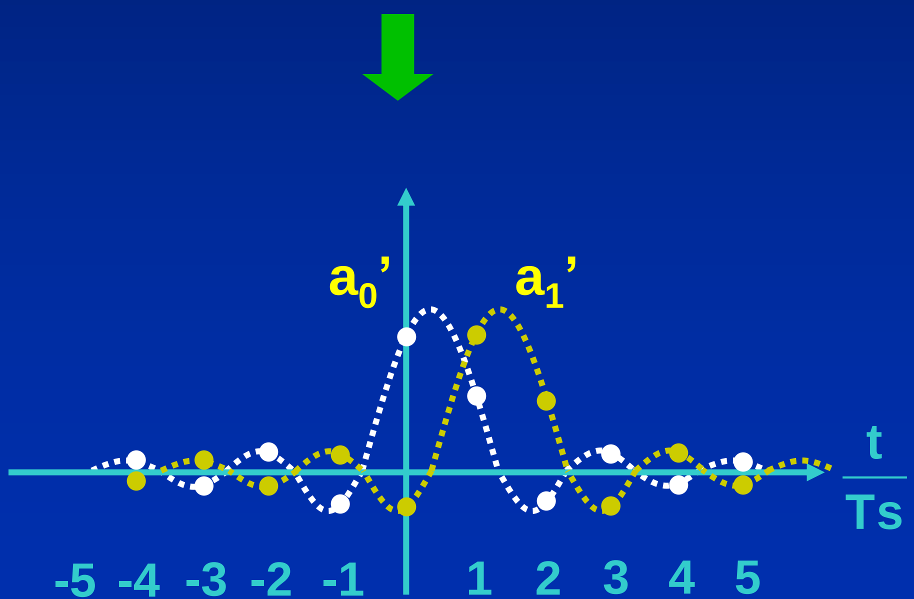


2 Tap Delayed FIR Model

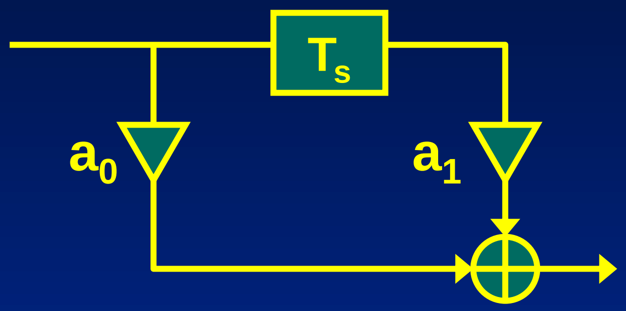


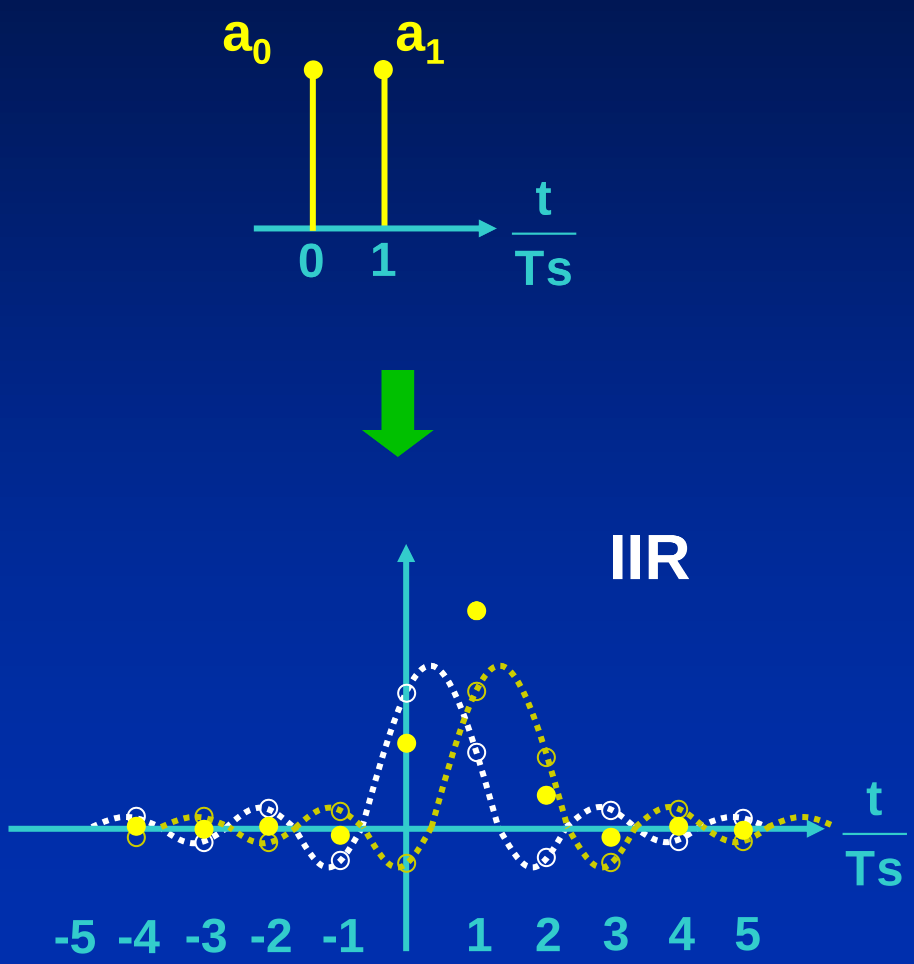


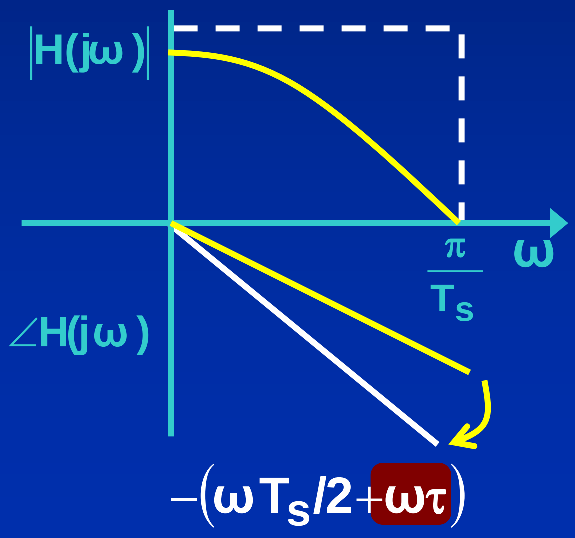




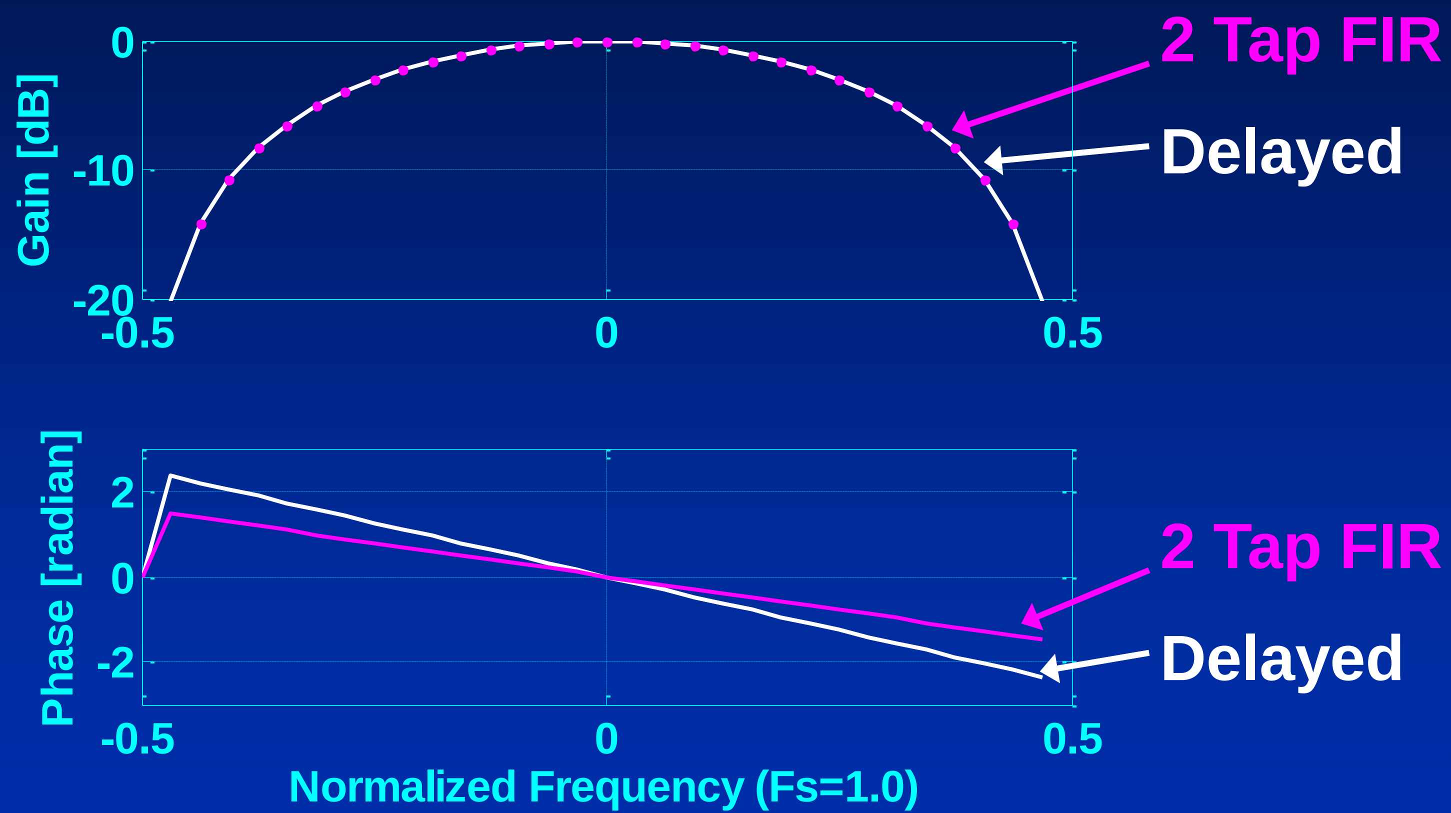
2 Tap Delayed FIR Model







Comparison of Freq. Characteristic



Only slope of phase characteristic is changed

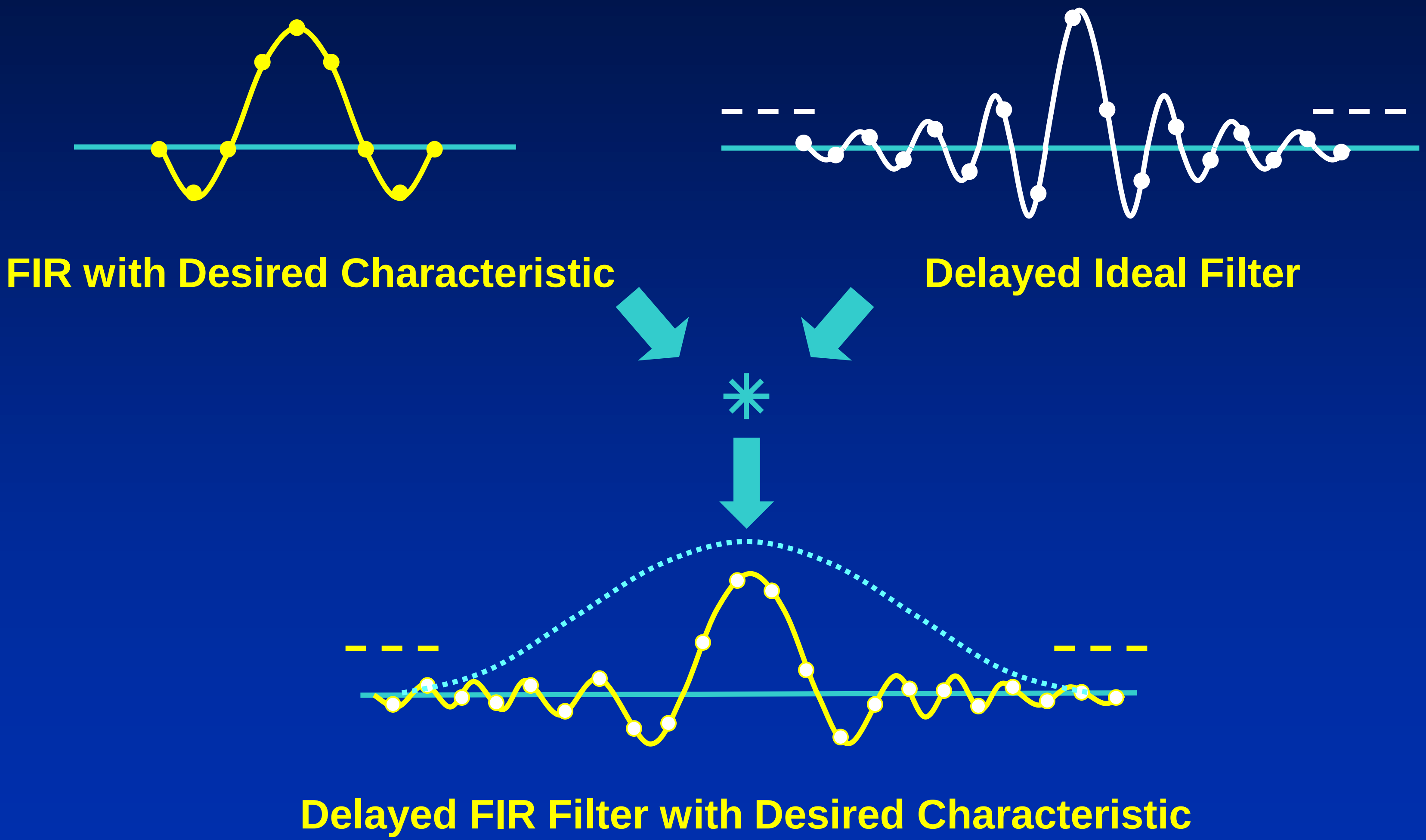
Frequency Characteristic of Proposed Filter

$g(nT)$	$G(e^{j\omega T})$
Type I	$e^{-j\left(\omega(N-1)T_S/2+\omega\tau\right)}\sum_{k=0}^{(N-1)/2} a_k \cos[\omega k T_S]$
Type II	$e^{-j\left(\omega(N-1)T_S/2+\omega\tau\right)}\sum_{k=1}^{N/2} b_k \cos[\omega(k-1/2)T_S]$
Type III	$e^{-j(\omega(N-1)T_S/2-\pi/2+\omega\tau)}\sum_{k=0}^{(N-1)/2} a_k \sin[\omega k T_S]$
Type IV	$e^{-j(\omega(N-1)T_S/2-\pi/2+\omega\tau)}\sum_{k=1}^{N/2} b_k \sin[\omega(k-1/2)T_S]$

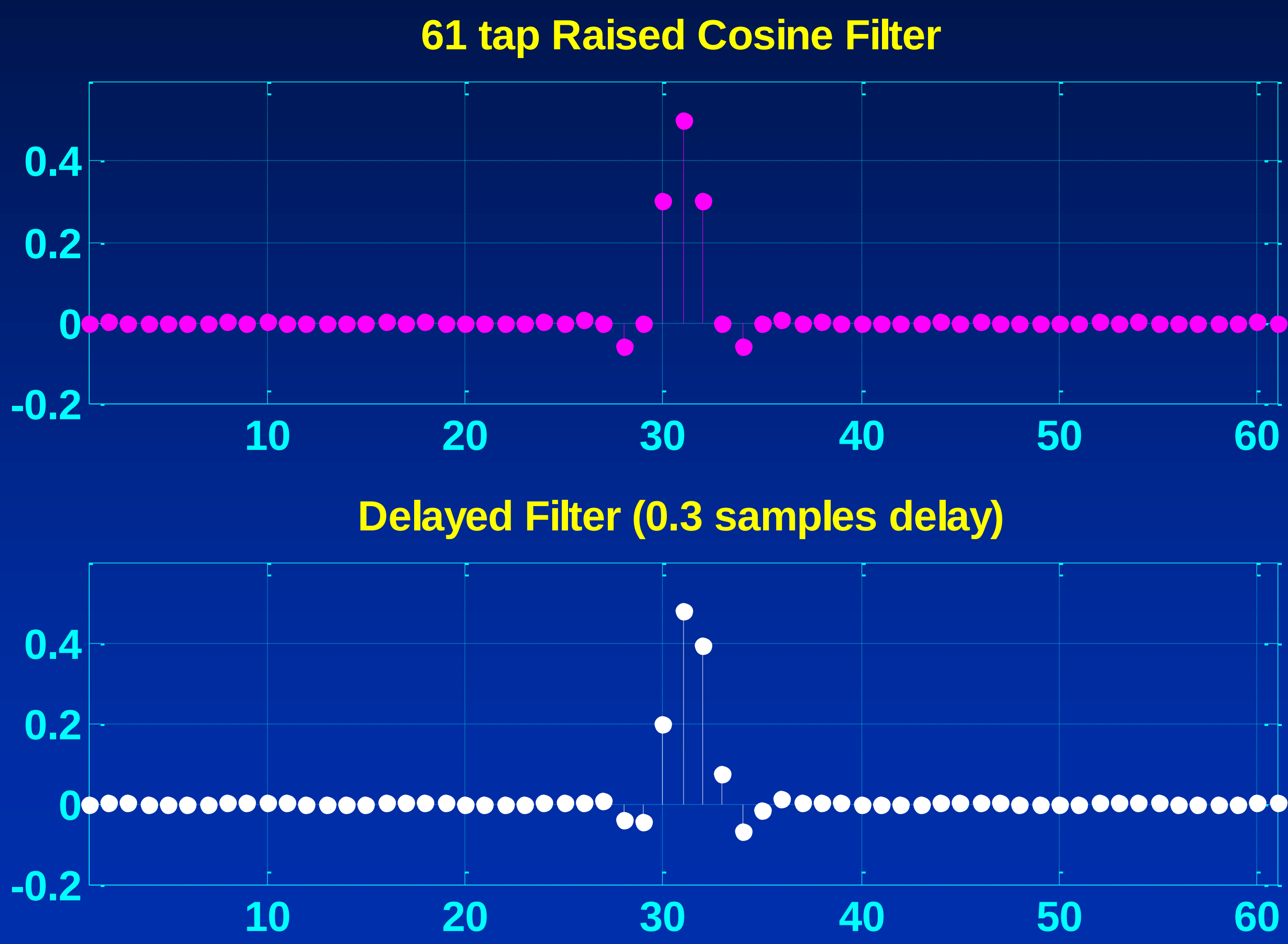
Phase : 1st order function of frequency

Delay : controllable with τ

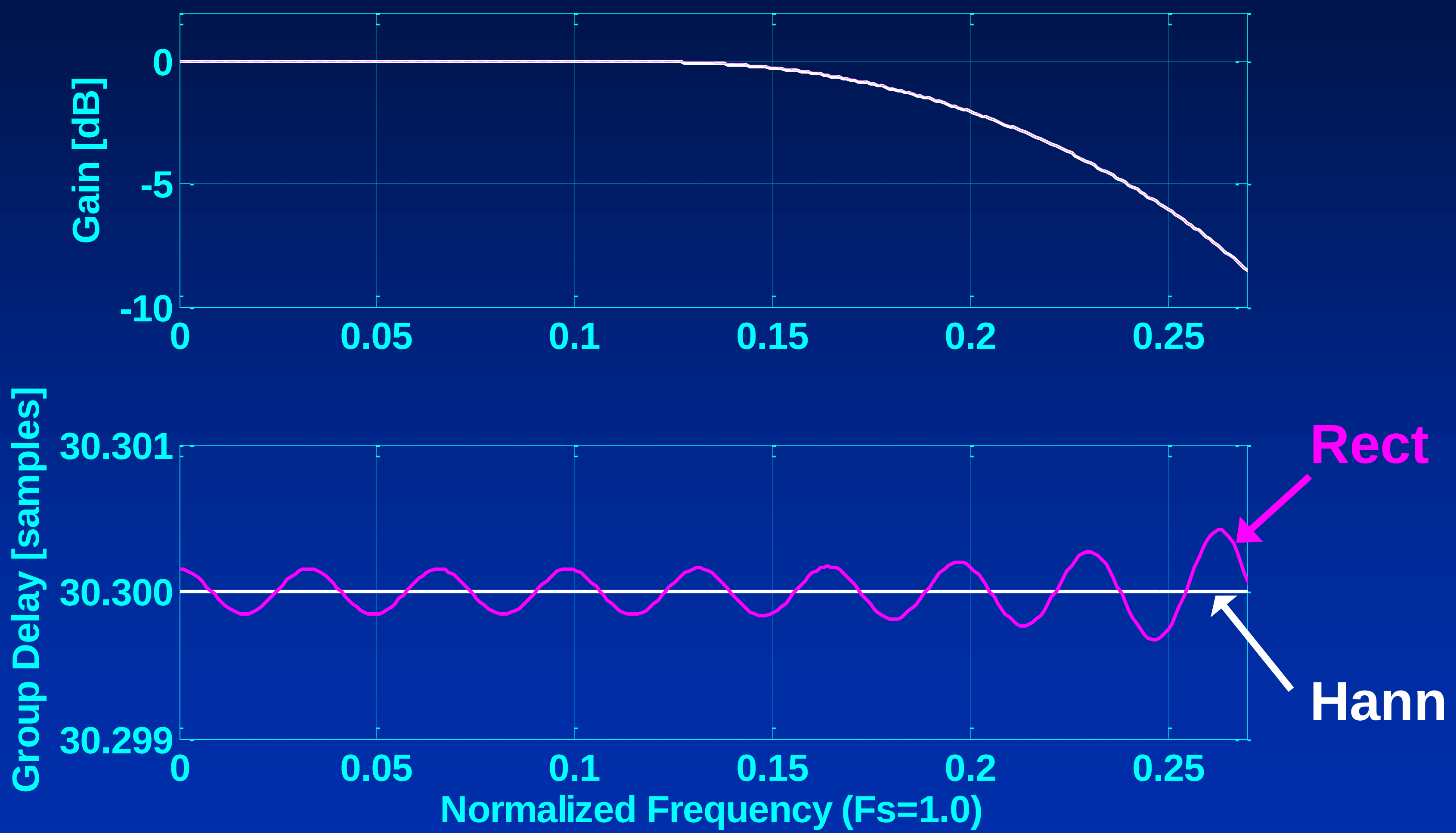
Proposed Design Technique



Example of Raised Cosine Filter



Effect of Window Function



Window function can reduce Gibbs phenomenon

Novel Linear Phase Condition of D.F.

- Original FIR filter has complete linear phase
- Original FIR filter is band-limited
- Bandwidth of signal is below Nyquist rate

⇩

Fine delay can be controlled using Ideal filter

- Delayed filter has infinite impulse response
- Window function can construct FIR effectively

Outline

- Conventional linear phase FIR filter
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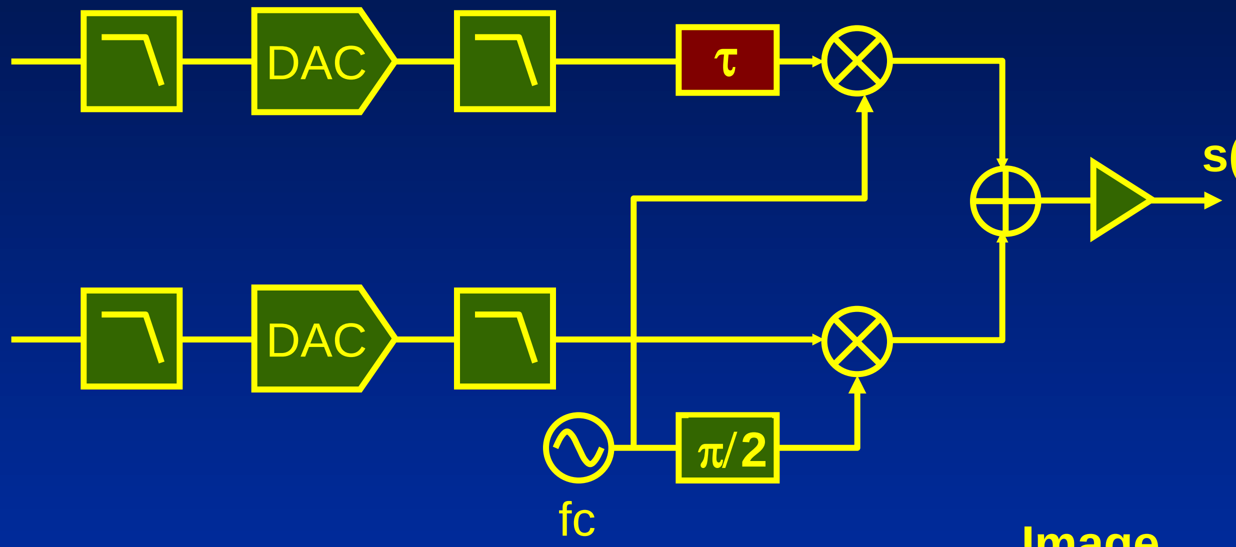
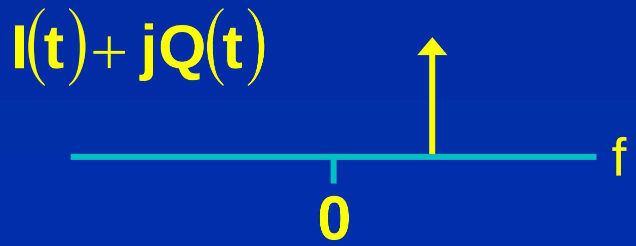
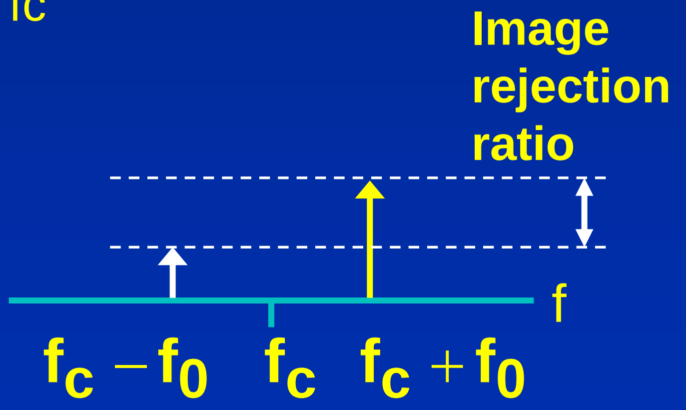
Application to Quadrature Modulator

$$I(t) = \cos(2\pi f_0 t)$$

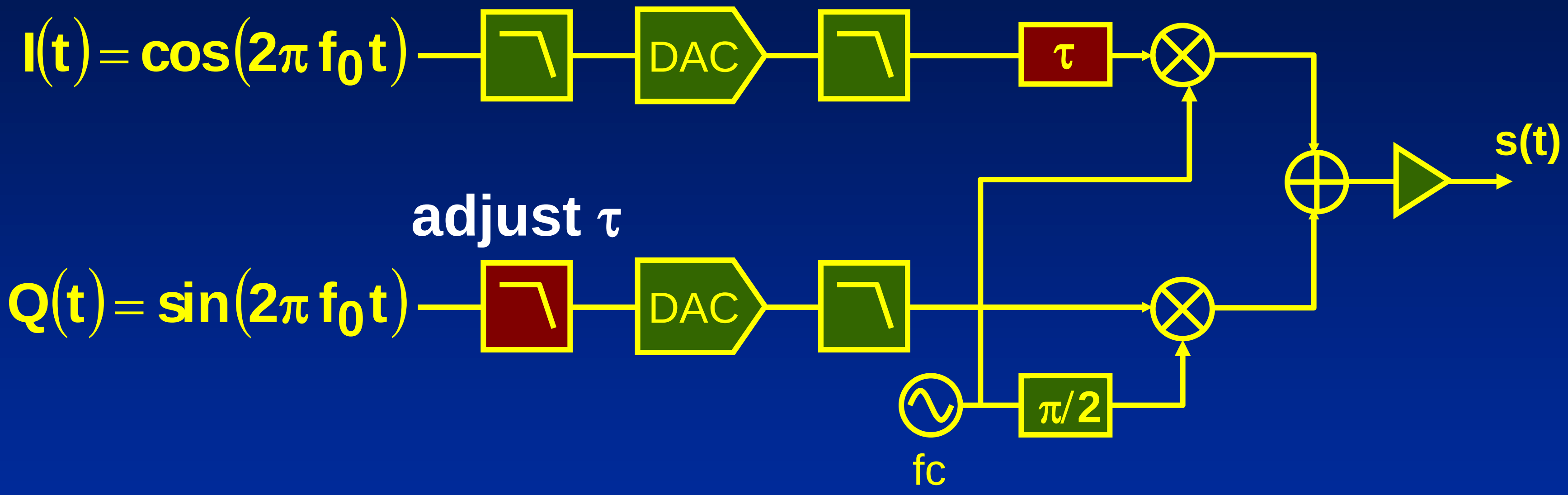
$$Q(t) = \sin(2\pi f_0 t)$$

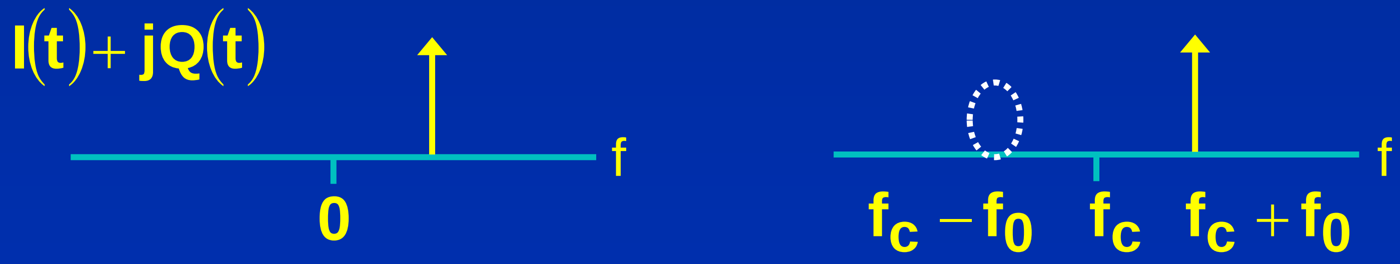




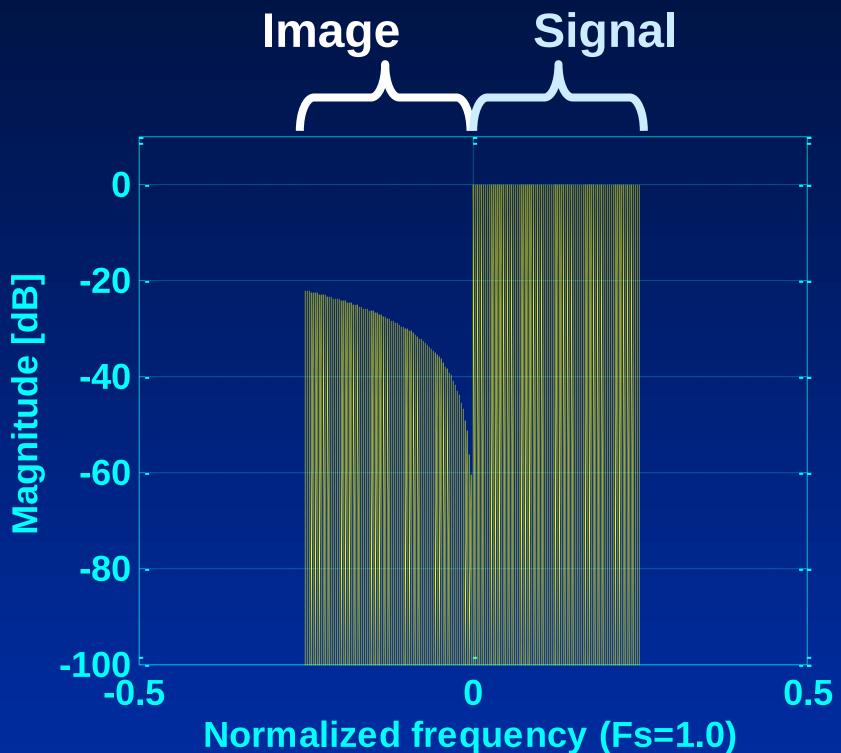


Adjustment of I/Q Skew

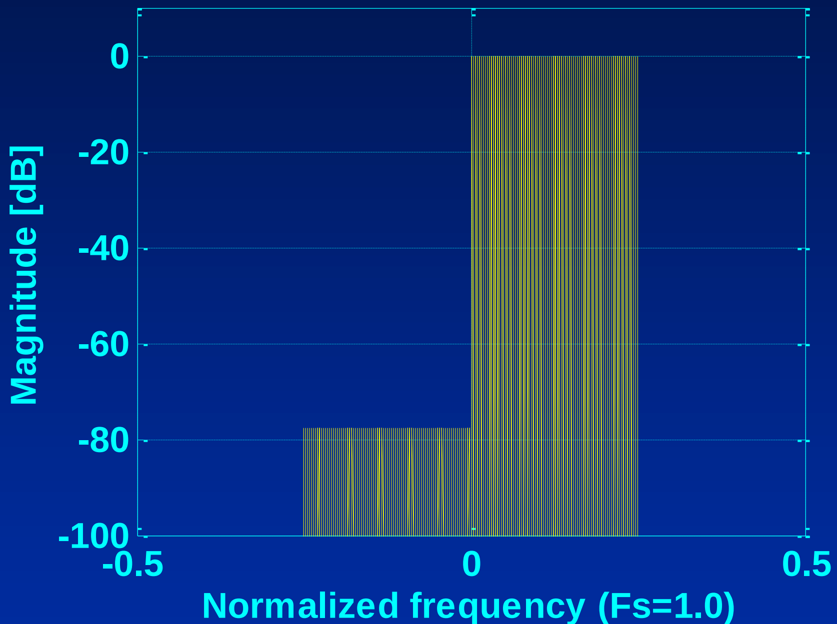




Simulation Results



(a) SSB signal with I/Q skew

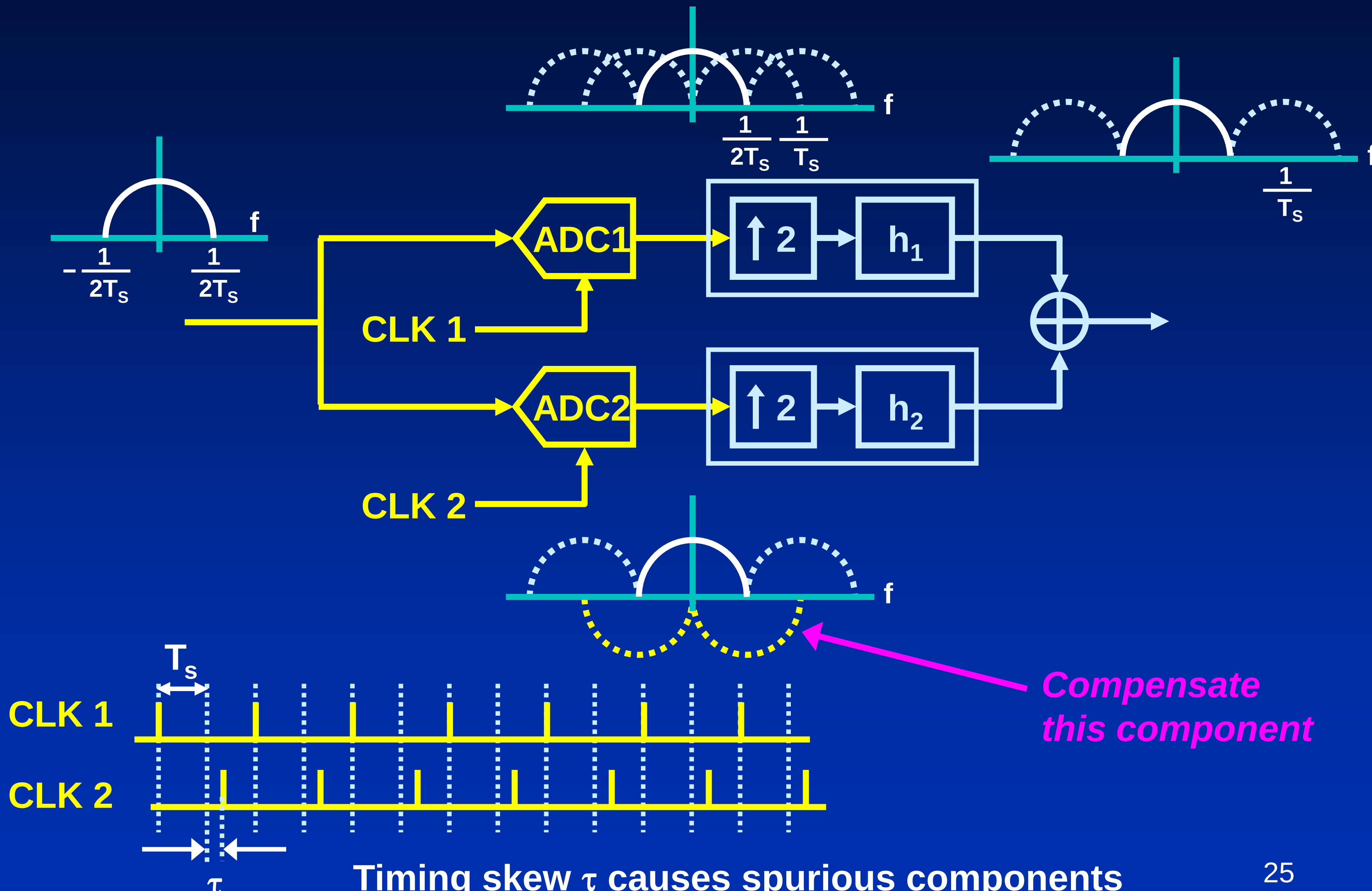


(b) SSB signal with compensation

Delay Compensation Filter

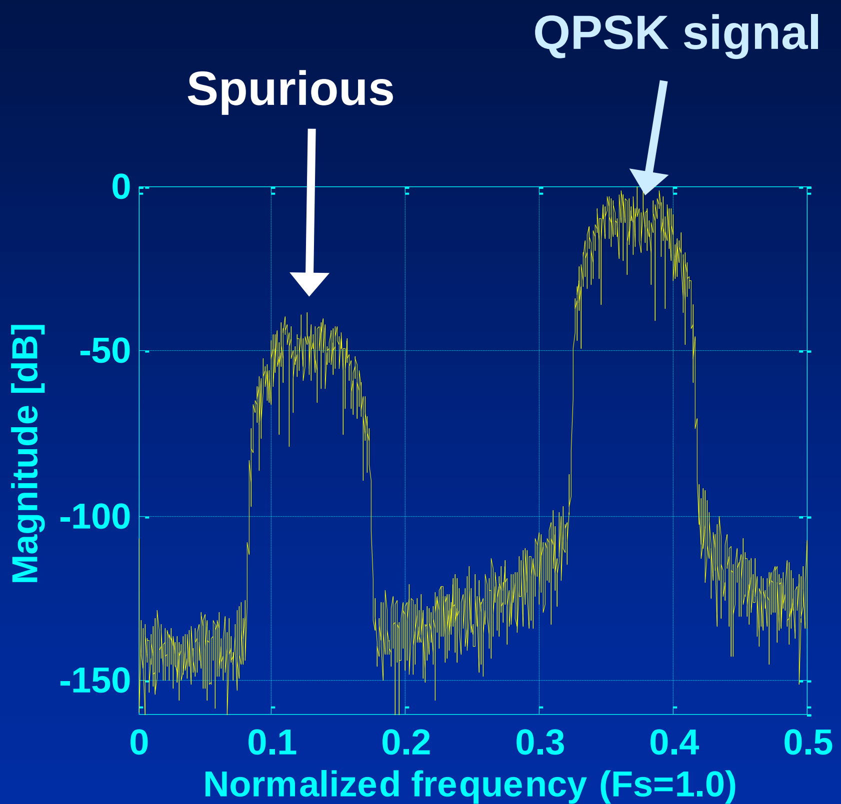
delay	0.1 sampling points
Taps	61 Taps
Window	Hann

Application to Time-Interleaved ADCs

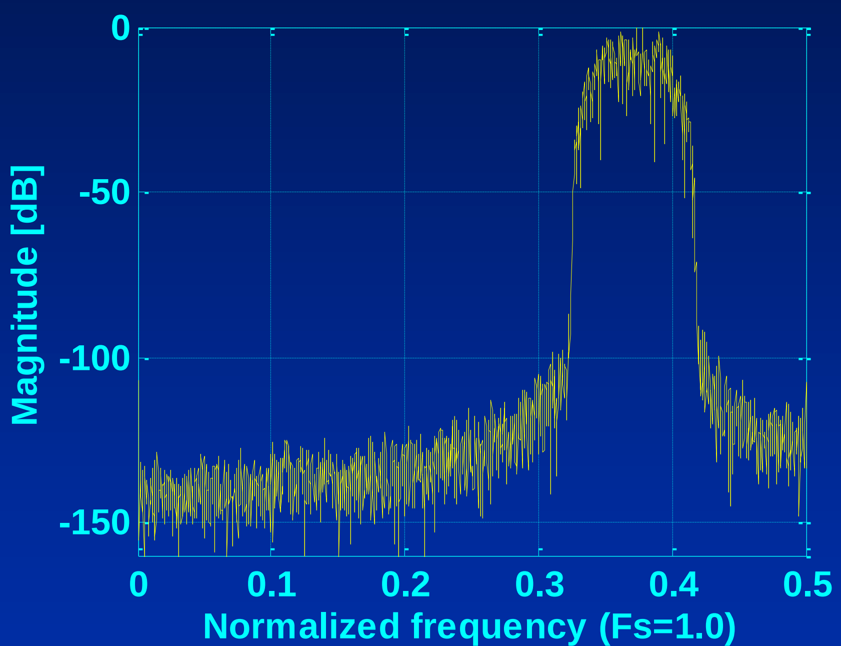


Timing skew τ causes spurious components

Simulation Results



(a) 2ch interleaved ADC with 0.01 samples skew



(b) Compensate the skew using 91 taps delay filter

Conclusion

- Fine delay controllable digital filter which maintains desired characteristics is proposed
- It is applicable not only to Low Pass Filters but also to Band Pass Filters
- It can compensate the timing skew of analog modules in ATE

Timing Skew Compensation Technique Using Digital Filter with Novel Linear Phase Condition

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Abstract

This paper describes the timing skew compensation technique using the digital filter with our novel linear phase condition. First we describe the digital filter which can set its group delay with the arbitrary fine time resolution while it maintains the linear phase characteristics; the conventional linear phase digital filter can set its group delay with the time resolution of a half of the sampling period. We will provide its structure and operation, theoretical analysis as well as simulation verification. Next we will describe the application of our proposed digital filter to compensate for timing skew in the following cases:
(1) Sampling timing skew among channels in the time-interleaved ADC system.
(2) I, Q-path timing skew in the single-side band (SSB) signal generator.
We show its effectiveness with simulation.

Keywords: Digital Filter, Linear Phase, Digitally-Assisted Analog Technology, Timing Skew, Digital Error Correction, ATE

1. Introduction

Fine timing skew adjustments are frequently used in Automatic Test Equipment (ATE) systems, where linear phase characteristics are desired in many cases to preserve signal waveforms in time domain. Digital techniques are preferred for the timing skew compensation because they are stable, reliable and easy to implement compared to analog techniques. However, the conventional digital filter with linear phase cannot be applied to the fine timing skew adjustment because its delay time resolution is limited.

In this paper we propose a digital filter with novel linear phase condition and show that its delay time resolution is arbitrary fine (i.e., its group delay can be set with arbitrary small time resolution). Ideally, our proposed linear phase digital filter has infinite number of taps which cannot be realized. Hence we approximate it with the finite number of taps. We observe Gibbs oscillations [1], [2] for phase as well as gain characteristics when we approximate it directly without applying a window function. However using proper window functions can eliminate these oscillations and their gain and phase characteristics are close to the ones with the ideal digital filter.

We also show the application of our proposed digital filter to compensate for timing skew in ATE systems in the following cases:

(1) Sampling timing skew among channels in the time-interleaved ADC system.

(2) I, Q-path timing skew in the single-side band (SSB) signal generator.

2. Conventional Linear Phase Condition

Linear phase characteristics are important for the digital filter to preserve the signal waveform in time domain. It is well-known in [1] that the FIR digital filter with odd or even symmetry coefficients has linear phase characteristics and it is unconditionally stable. The IIR digital filter with odd or even symmetry of both its denominator and nominator has also linear characteristics but it is unstable. Hence in almost all cases, the FIR digital filter with odd or even symmetry coefficients is used where the linear phase is required, and in such cases its group delay is (N/2)Ts where N is the number of the FIR filter taps and T_s is the sampling period; in other words the time resolution of the group delay is $T_s/2$, and this cannot be used for fine timing skew adjustment in ATE systems.

3. Novel Linear Phase Condition

In this section, we show the extended linear phase characteristics conditions for the digital filter which has not necessarily odd or even symmetry coefficients, and its time resolution of the group delay is arbitrary small.

First we discuss without consideration of causality, for simplicity. Let us consider the following analog filter (Fig.1):

$$v_{out} = \begin{cases} a_0 v_{in}(t) & \text{in case} -\pi/T_s < \omega < \pi/\mathrm{T_s} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Then its impulse response h(t) is given as follows:

$$h(t) = a_0 T_s \,\mathrm{sinc}(\pi t/T_s). \tag{2}$$

We consider the case that the input $v_{in}(t)$ is band-limited to $-\pi/T_s < \omega < \pi/\mathrm{T_s}$. We sample the above impulse response with a period Ts, and use the following transformation to obtain the digital filter which corresponds to the analog filter in (1):

$$\begin{aligned} &T_s \to 1 \\ &v_{out}(nT_s) \to y(n) \\ &v_{in}(nT_s) \to x(n). \end{aligned} \tag{3}$$

Then we have the following digital filter:

$$y(n) = a_0 x(n) \tag{4}$$

This is because

$$\mathrm{sinc}(\pi n) = \begin{cases} 1 & \text{in case } n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

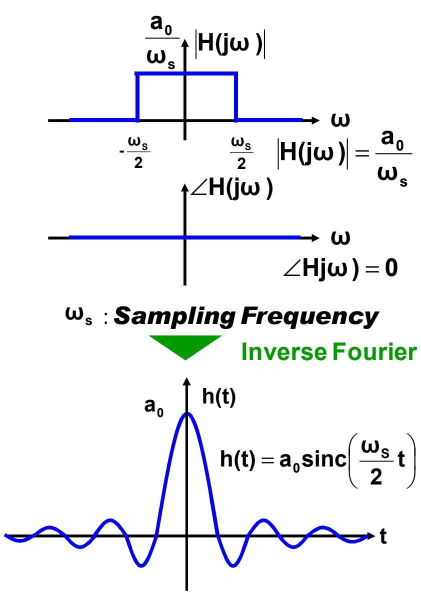

Fig. 1. An ideal analog low pass filter. Gain, phase characteristics, and impulse response.

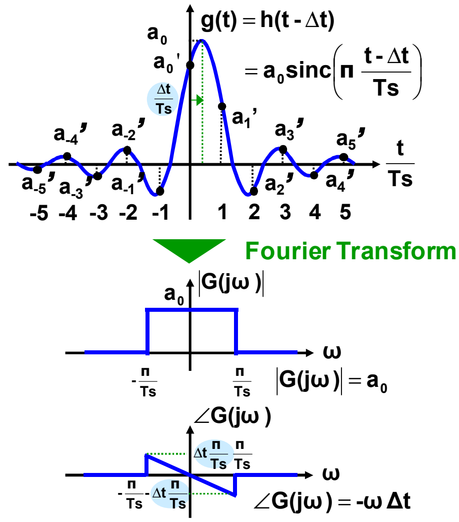

Fig. 2. Sampling timing shift can maintain the linear phase characteristics. Impulse response, and gain, phase characteristics.

This digital filter has obviously linear phase characteristics (or rigorously speaking zero phase characteristics). Now let us consider to sample $h(t)$ at $t = nT_s + \tau$ (Fig.2), where $0 < \tau < T_s$, and use (3). Then we have the following digital filter which corresponds to the analog filter in (1):

$$y(n) = \sum_{k=-\infty}^{\infty} a'_k x(n-k). \tag{6}$$

Here

$$a'_k = \mathrm{sinc}(\pi(k + \tau/T_s)). \tag{7}$$

In general a'_k is not necessarily zero and a'_k is not necessarily equal to a'_{-k} or $-a'_{-k}$.

Proposition 1 : The digital filter given by (6), (7) has the linear filter characteristics, and its group delay is τ.

Proof : The inverse Fourier transform of (6) is given by

$$Y(j\omega) = H(j\omega) X(j\omega). \tag{8}$$

It follows from (7) that in case of $\tau = 0$,

$$H(j\omega)\big|_{\tau=0} = a_0. \tag{9}$$

Then we have the following for a given τ $(0 < \tau < T_s)$:

$$H(j\omega) = a_0 e^{-j\omega\tau}. \tag{10}$$

Thus the digital filter given by (6), (7) has the linear filter characteristics, and its group delay is τ (Fig.2). (Q. E. D.)

Next we discuss in case of Fig.3, and consider the following analog filter:

$$v_{out} = \begin{cases} a_0 v_{in}(t) + a_1 v_{in}(t - T_s) \\ \quad \text{in case} - \pi/T_s < \omega < \pi/T \\ 0 \quad \text{otherwise.} \end{cases} \tag{11}$$

Note that its impulse response is given as follows:

$$h(t) = a_0 T_s \mathrm{sinc}(\pi t/T_s) + a_1 T_s \mathrm{sinc}(\pi(t/T_s - 1)). \tag{12}$$

We assume that the input $v_{in}(t)$ is band-limited to $-\pi/T_s < \omega < \pi/T_s$. Similarly we sample this filter with $t = nT_s$, and we have the following digital filter using (3):

$$y(n) = a_0 x(n) + a_1 x(n-1). \tag{13}$$

Next we sample (12) with $t = nT_s + \tau$, and we have the following digital filter:

$$y(n) = \sum_{k=-\infty}^{\infty} a'_k x(n-k). \tag{14}$$

Here

$$a'_k = a_0 \mathrm{sinc}(\pi(k + \tau/T_s)) + a_1 \mathrm{sinc}(\pi((k-1) + \tau/T_s)). \tag{15}$$

Proposition 2 : The digital filter given by (14), (15) with $a_0 = a_1$ or $a_0 = -a_1$ has the linear phase characteristics and its group delay is $T_s/2 + \tau$. Also the digital filter of (14) has the same gain characteristics as (13).

Proof : We consider the case of $a_0 = a_1$. The inverse Fourier transform of (14) is given by

$$Y(j\omega) = H(j\omega) X(j\omega). \tag{16}$$

It follows from (15) that in case of $\tau = 0$,

$$H(j\omega)\big|_{\tau=0} = 2a_0 \cos(\omega T_s) e^{-j\omega T_s/2} \tag{17}$$

Then we have the following for a given τ $(0 < \tau < T_s)$:

$$H(j\omega) = 2a_0 \cos(\omega T_s) e^{-j\omega(T_s/2+\tau)}. \tag{18}$$

Thus the digital filter given by (14), (15) has the linear filter characteristics, and its group delay is $(T_s/2 + \tau)$.

Also it follows from (17), (18) that

$$\left|H(j\omega)\right|_{\tau=0} = \left|H(j\omega)\right|_{0<\tau<T_s} = \left|2a_0 \cos(\omega T_s)\right|. \tag{19}$$

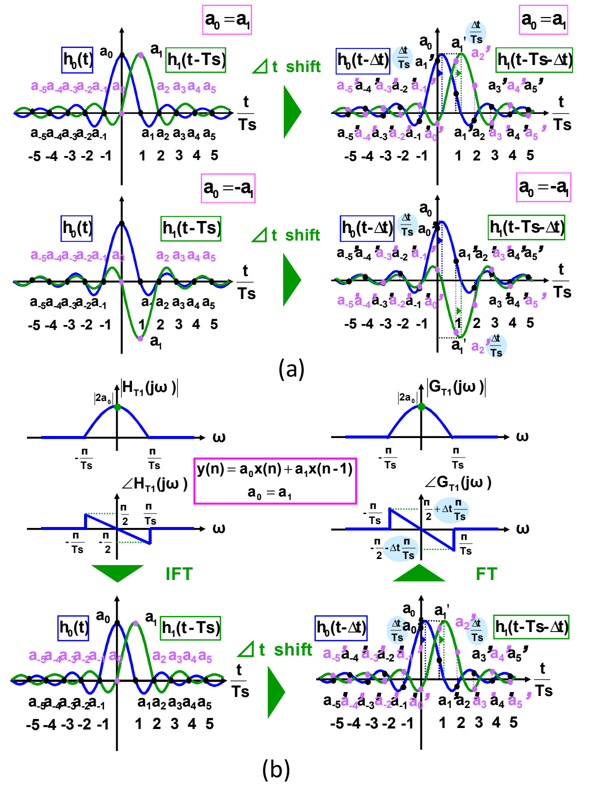

Fig. 3. 2-tap FIR filter without and with sampling timing shift. (a) Impulse response. (b) Gain and phase responses.

Then the digital filter of (14) has the same gain characteristics as (13).

Similar argument is valid in case of $a_0 = -a_1$. (Q. E. D.)

The same argument holds for the 3-tap FIR filter case (Fig.4), and also in general for an N-tap FIR filter as Propositions 1, 2.

Proposition 3 : Let us consider an N-tap FIR digital filter with coefficients a_k of odd or even symmetry.

$$y(n) = \sum_{k=0}^{N-1} a_k x(n-k) \tag{20}$$

Then the following digital filter has the linear characteristics with group delay $(N/2)T_s + \tau$.

$$y(n) = \sum_{k=-\infty}^{\infty} a'_k x(n-k) \tag{21}$$

Here

$$a'_k = \sum_{l=0}^{N-1} a_l \mathrm{sinc}(\pi((k-l) + \tau/T_s)). \tag{22}$$

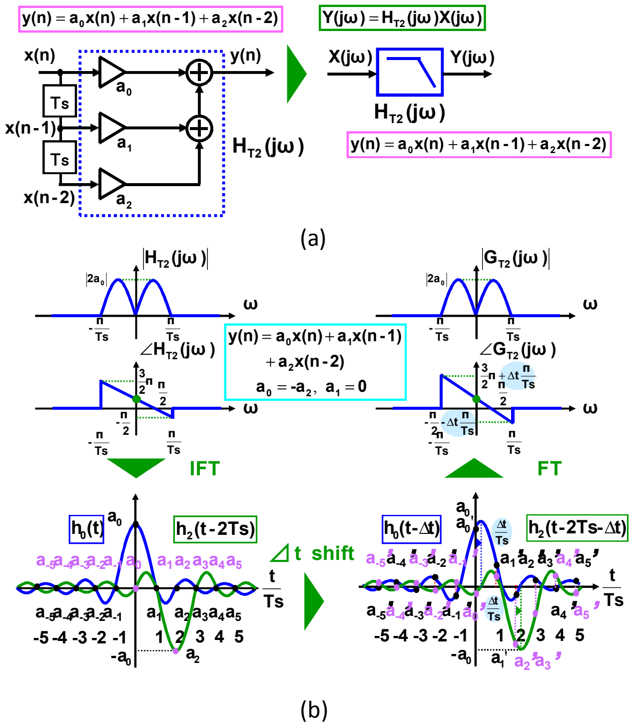

Fig. 4. 3-tap FIR filter without and with sampling timing shift. (a) Without sampling time shift. (b) Gain, phase and impulse responses with sampling timing shift.

Proposition 3 can be proved similarly in Proposition 1, 2 cases.

We have performed MATLAB simulation and checked that the above digital filter with time shift τ have linear phase characteristics and have the same gain characteristics as the one without time shift for N=1, 2, and 3.

Table 1 shows the frequency characteristics of digital filters with the conventional linear phase conditions, and Table 2 shows the ones with our proposed linear phase conditions derived from the corresponding conventional linear phase digital filter.

TABLE I

FREQUENCY CHARACTERISTICS WITH CONVENTIONAL LINEAR PHASE CONDITIONS

case	N	$h(n)$	$H(e^{j\omega})$
1	odd	even symmetry	$e^{-j\omega\frac{N-1}{2}}\sum_{k=0}^{\frac{N-1}{2}} a_k\cos(k\omega)$
2	even	even symmetry	$e^{-j\omega\frac{N-1}{2}}\sum_{k=0}^{\frac{N}{2}} a_k\cos((k-\frac{1}{2})\omega)$
3	odd	odd symmetry	$e^{-j\omega\frac{N-1}{2}}\sum_{k=0}^{\frac{N-1}{2}} a_k\sin(k\omega)$
4	even	odd symmetry	$e^{-j\omega\frac{N-1}{2}}\sum_{k=0}^{\frac{N}{2}} a_k\sin((k-\frac{1}{2})\omega)$

TABLE II

FREQUENCY CHARACTERISTICS WITH PROPOSED LINEAR PHASE CONDITIONS

case	N	$h(n)$	$H(e^{j\omega})$
1	odd	even symmetry	$e^{-j\omega(\frac{N-1}{2}+\frac{\tau}{T_s})}\sum_{k=0}^{\frac{N-1}{2}} a_k\cos(k\omega)$
2	even	even symmetry	$e^{-j\omega(\frac{N-1}{2}+\frac{\tau}{T_s})}\sum_{k=0}^{\frac{N}{2}} a_k\cos((k-\frac{1}{2})\omega)$
3	odd	odd symmetry	$e^{-j\omega(\frac{N-1}{2}+\frac{\tau}{T_s})}\sum_{k=0}^{\frac{N-1}{2}} a_k\sin(k\omega)$
4	even	odd symmetry	$e^{-j\omega(\frac{N-1}{2}+\frac{\tau}{T_s})}\sum_{k=0}^{\frac{N}{2}} a_k\sin((k-\frac{1}{2})\omega)$

Now we will provide the proof for our proposed linear phase digital filter in general case:

Let us consider an N-tap FIR filter with conventional linear phase condition, and we have the impulse response $\tilde{h}(t)$ with continuous time and its Fourier transform $\tilde{H}(f)$:

$$\tilde{h}(t) = \sum_{n=0}^{N-1} h(nT_s)\delta(t - nT_s). \qquad (23)$$

$$\tilde{H}(f) = H(f) \star \frac{1}{T}\sum_{k=-\infty}^{\infty}\delta(f - \frac{k}{T_s}). \qquad (24)$$

Here $\star$ indicates convolution, $\delta(\cdot)$ denotes a delta function, and

$$H(f) = |H(f)|e^{-j2\pi f\frac{N-1}{2}T_s} \quad (-\frac{1}{2T_s} \le f \le \frac{1}{2T_s}). \quad (25)$$

When we add a delay τ to the impulse response $h(t)$ and we have its frequency characteristics as follows:

$$H'(f) = |H(f)|e^{-j2\pi f\frac{N-1}{2}T_s} \cdot e^{-j2\pi f\tau}. \qquad (26)$$

We see that the phase characteristics of $H'(f)$ is linear with respect to f. $H'(f)$ can be interpreted as the convolution between $H(f)$ and $S(f)$, where $S(f)$ is the ideal filter with a delay τ:

$$S(f) = e^{-j2\pi f\frac{N-1}{2}\tau} \quad (-\frac{1}{2T_s} \le f \le \frac{1}{2T_s}). \qquad (27)$$

Thus the ideal filter $\tilde{S}(f)$ for (24) is given by

$$\tilde{S}(f) = S(f) \star \sum_{k=-\infty}^{\infty}\delta(f - \frac{k}{T_s}) = \sum_{k=-\infty}^{\infty} S(f - \frac{k}{T_s}). \qquad (28)$$

Next we will consider the effect of the delay τ to the impulse response. The inverse Fourier transform of $\tilde{S}(f)$ is given as follows:

$$\tilde{s}(t) = \text{sinc}(\frac{\pi(t-\tau)}{T_s}) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$
$$= \sum_{n=-\infty}^{\infty} \text{sinc}(\frac{\pi(t-\tau)}{T_s}) \delta(t - nT_s). \quad (29)$$

We see from (29) that $\tilde{s}(t)$ is asymmetric with respect to $t = 0$, and we have the following impulse response:

$$\tilde{h}(t) \star \tilde{s}(t)$$
$$= \sum_{n=0}^{N-1} h(nT_s)\delta(t - nT_s) \star \sum_{n=-\infty}^{\infty} \text{sinc}(\frac{\pi(kT_s - \tau)}{T_s}) \cdot \delta(t - nT_s)$$
$$= \sum_{n=-\infty}^{\infty} \sum_{n=0}^{N-1} h(nT_s) \text{sinc}(\frac{\pi(kT_s - \tau)}{T_s}) \cdot \delta(t - (n-k)T_s). \quad (30)$$

Thus the impulse response of time delay τ with continuous time has finite values for $t \to \pm\infty$ due to the *sinc* function effects.

4. Realization Consideration

Here we sample the input signal with the sampling period Ts and then we consider the band-limited case to $-\pi/T_s < \omega < \pi/T_s$, in order to avoid the aliasing effects. In such case $h(t)$ does not converge to zero as t becomes plus/minus infinity. So the digital filter with our novel linear phase condition has to have the infinite number of taps and this cannot be realized. (Note that in case of $\tau = 0$, $h(nT_s)$ can be zero as n becomes large which corresponds to the conventional linear phase FIR digital filter case.) So we consider to truncate the terms for large number of $|k|$ in (22) applying a window function and we approximate the digital filter of (21), (22) with the finite number of taps.

4.1 Approximation with Finite Number of Taps

The ideal digital filter with our proposed linear phase condition needs infinite number of taps. However it is cannot be realized, and hence we have to approximate it as the filter with finite number of taps. We consider here the effects of the truncation to the finite number of taps. We observe from our simulation results so-called Gibbs oscillation at the edges of pass-band of the gain characteristics and also phase characteristics (Fig.5) [1], [2]; Gibbs oscillation for phase characteristics is not observed in many cases, and we have found that this Gibbs oscillation for phase characteristics is due to the asymmetry of the impulse response $h(n)$ with respect to $n = 0$.

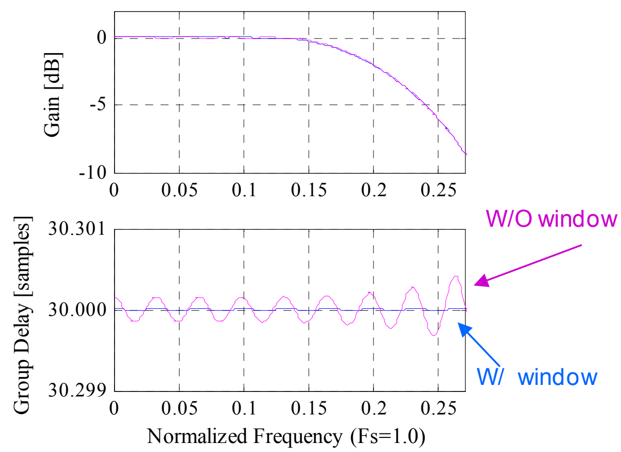

Fig. 5. Gain and phase characteristics of the proposed digital filter (with time shift τ of 0.3Ts) after truncation to finite number (N=61) of filter taps with and without applying Hann window.

4.2 Applying Window Function

Next we investigate to use window functions when we approximate the ideal filter using the one with the finite number of taps. When we use a window function, the Gibbs oscillations for gain and phase are suppressed. Fig.5 shows our simulation result with time-shift τ of 0.3Ts and applying Hann window. We have also found that this Gibbs oscillation for phase can be further suppressed if we use a window function with the time-shift τ, as shown in Fig.6 where we choose the time shift τ of 0.5Ts (which affects phase characteristics significantly) and we use a Hann window.

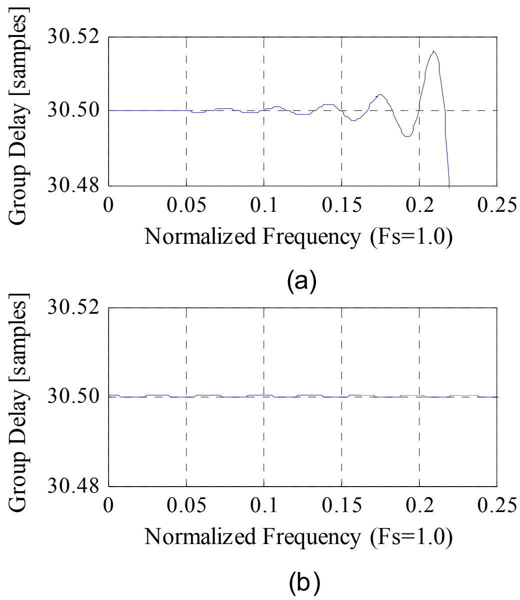

Fig. 6. Phase characteristics of the proposed digital filter (with time shift τ of $0.5T_s$) after truncation to finite number (N = 61) of filter taps. (a) With applying Hann window of no time-shift. (b) With applying Hann window time-shifted by $\tau = 0.5T_s$

5. Digital Filter Application for Timing Skew Compensation

5.1 Interleaved ADC System

A time-interleaved ADC system is an effective way to implement a high-sampling-rate ADC with relatively slow circuits (Fig.7) [4], [5], and is widely used in ATE systems. In the ADC system, several channel ADCs operate at interleaved sampling times as if they were effectively a single ADC operating at a much higher sampling rate. However, mismatches among channel ADCs - such as offset, gain and bandwidth mismatches as well as timing skew of the clocks distributed to the channels - degrade SNDR and SFDR of the ADC system as a whole.

Here we consider the timing skew problem in the interleave ADC system. Suppose that the clocks *CK1, CK2,* … *CKM* have skews $dt_1, dt_2, \cdots dt_M$ (Fig.7) [4-7]. If the input signal Vin(t) is sampled at time t+dt instead of time t, we have the sampling time error e(t) :

$$e(t) = Vin(t+dt) - Vin(t)$$

which can be approximated by

$$e(t) \fallingdotseq [dVin(t))/dt]\, dt$$

This skew causes so-called pattern noise in the ADC system, and in the time domain the largest error occurs when the input signal has the largest slew rate. The timing skew effect in the time-interleaved ADC system is serious for high frequency analog signal measurements, because its slew rate (dVin(t)/dt) becomes high.

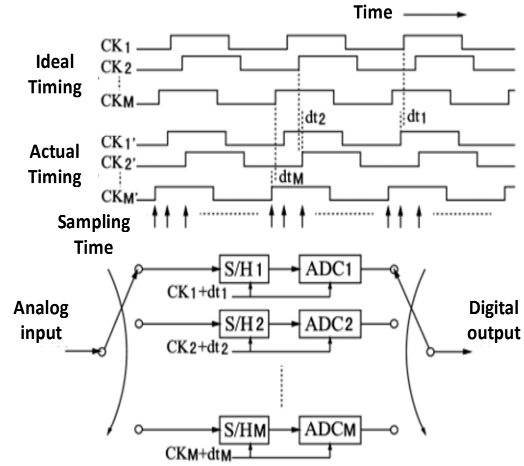

Fig. 7. Interleaved ADC system and timing skew.

Proposed Timing Skew Compensation Method 1 :

We propose to compensate for the timing skew effects using our linear phase digital filter *directly* as shown in Fig.8 (a) in the two-channel case. We have performed MATLAB simulation and obtained the result in Fig.8 (b); we see that the spurious tone is suppressed by our proposed digital filter. However this method is only applicable for the input frequency from 0 to fs/2 where fs is the channel ADC sampling frequency; this method is NOT applicable for the input frequency from fs/2 to Fs/2 where Fs is the sampling frequency of the whole interleaved ADC system and Fs=2fs in the two-channel case.

Proposed Timing Skew Compensation Method 2 :

Next we will describe a more sophisticated timing skew compensation method which also uses our proposed linear phase digital filter and is applicable for the input frequency from DC to the whole interleaved ADC sampling frequency Fs/2.

The timing skew effect in the time-interleaved ADC system is serious for high frequency analog signal measurements [4]. We present here its frequency domain compensation method based on [6], [7]. We design and apply a digital filter for the timing skew compensation so that spurious due to the timing skew is cancelled. Its principle is as follows: Let us consider the two channel case for simplicity. The output spectrum for channel 1 and 2 without mismatches are given as follows:

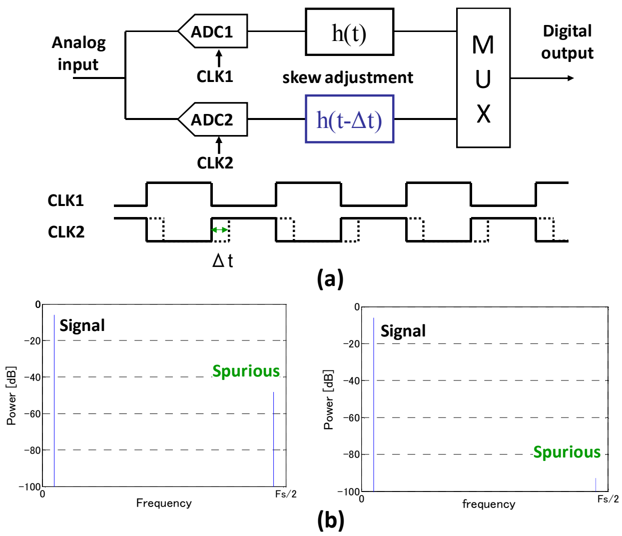

Fig. 8. Proposed timing skew compensation method 1. (a) Timing skew effect compensation in the 2-channel interleave ADC system with our novel linear phase digital filters. (b) Simulation results of the timing skew compensation method 1 with our digital filter.

$$X_1(f)=\frac{1}{2T_s}\sum_{k=-\infty}^{\infty}X\left(f-\frac{k}{2T_s}\right) \tag{31}$$

$$X_2(f)=\frac{1}{2T_s}\sum_{k=-\infty}^{\infty}X\left(f-\frac{k}{2T_s}\right)e^{-j\pi k}. \tag{32}$$

Then we have the output spectrum of the interleaved ADC system:

$$\begin{aligned}X_{1,2}(f)&=X_1(f)+X_2(f)\\&=\frac{1}{2T_s}\sum_{k=-\infty}^{\infty}X\left(f-\frac{k}{2T_s}\right)\left(1+e^{-j\pi k}\right)\end{aligned} \tag{33}$$

Here

$$1+e^{-j\pi k}=\begin{cases}2, & k: even\\ 0, & k: odd\end{cases} \tag{34}$$

and we have the rewritten output power spectrum of (33).

$$X_{1,2}(f)=\frac{1}{T_s}\sum_{k=-\infty}^{\infty}X\left(f-\frac{k}{2T_s}\right). \tag{35}$$

Now we assume here that the timing skew between channel 1 and 2 is τ, and we have the power spectrum of the channel ADCs and also the whole interleaved ADC system:

$$X_1(f)=\frac{1}{2T_s}\sum_{k=-\infty}^{\infty}X\left(f-\frac{k}{2T_s}\right) \tag{36}$$

$$X_2(f)=\frac{1}{2T_s}\sum_{k=-\infty}^{\infty}X\left(f-\frac{k}{2T_s}\right)e^{-j2\pi\tau(f-k/(2T_s))}e^{-j\pi k} \tag{37}$$

$$X_{1,2}(f)=\frac{1}{2T_s}\sum_{k=-\infty}^{\infty}X\left(f-\frac{k}{2T_s}\right)\left(1+e^{-j2\pi\tau(f-k/(2T_s))}e^{-j\pi k}\right). \tag{38}$$

For 2-channel case, we have to consider only in k=0, 1, 2 cases because the signal band is from DC to 2 (fs/2), and we have the following:

$$X_1(f)=\frac{1}{2T_s}\sum_{k=0}^{2}X\left(f-\frac{k}{2T_s}\right)$$

$$X_2(f)=\frac{1}{2T_s}\sum_{k=0}^{2}X\left(f-\frac{k}{2T_s}\right)e^{-j2\pi\tau(f-k/(2T_s))}e^{-j\pi k}.$$

$$\begin{aligned}X_1(f)&=\frac{1}{2T_s}\sum_{k=0}^{2}X\left(f-\frac{k}{2T_s}\right)\\&=\frac{1}{2T_s}\left[X(f)+X\left(f-\frac{1}{2T_s}\right)+\left(f-\frac{1}{T_s}\right)\right]\end{aligned}$$

$$\begin{aligned}X_2(f)&=\frac{1}{2T_s}\sum_{k=0}^{2}X\left(f-\frac{k}{2T_s}\right)e^{-j2\pi\tau(f-k/(2T_s))}e^{-j\pi k}\\&=\frac{1}{2T_s}\Big[X(f)e^{-j2\pi\tau f}\\&\quad+X\left(f-\frac{k}{2T_s}\right)e^{-j2\pi\tau(f-1/(2T_s))}e^{-j\pi}\\&\quad+X\left(f-\frac{k}{2T_s}\right)e^{-j2\pi\tau(f-1/(T_s))}e^{-2j\pi}\Big]\end{aligned} \tag{39}$$

We see from (39) that we can cancel the spurious component for k=1 by multiplying

$$H_2(f)=e^{j2\pi\tau(f-1/(2T_s))}. \tag{40}$$

Then we compensate for the timing skew effect using the following filters:

$$\begin{aligned}H_1(f)&=e^{-j2\pi\xi}\\H_2(f)&=e^{-j2\pi\xi}e^{j2\pi\tau(f-1/(2T_s))}.\end{aligned} \tag{41}$$

This compensation method can be realized with two ways:

(1) Frequency domain approach: We perform FFT to each channel ADC output signal and apply the above filter $H_1(f)$, $H_2(f)$ respectively.

(2) Time domain approach: We apply the digital filter $h_1(n)$, $h_2(n)$ for each channel ADC output which implements $H_1(f)$ and $H_2(f)$ respectively.

We have investigated their compensation accuracy and calculation complexity. These methods are effective over the input frequency range from DC up to M・(fs/2), where M is the number of channels and fs is the channel ADC sampling frequency; such performance was very difficult to realize with conventional methods.

We have performed simulation by applying our methods to a two-channel time-interleaved ADC system with timing skew and validated their effectiveness. Figure 9 shows $h_1(n)$, $h_2(n)$ filter characteristics used for the timing skew compensation method 2, and we see that in both cases, their group delays are constant (phases are linear) with respect to the input frequency. Figure 10 shows the simulation results and we see that spurious signals are suppressed with our proposed method 2.

5.2 SSB Signal Generation

An ATE for communication IC testing incorporates SSB signal generation function (Fig.11) [8], [9], and we consider here to generate a SSB signal with a 2-channel arbitrary waveform generator. When the timing skew between I and Q-path exists, the negative frequency component is not zero. We propose to use our linear phase digital filter to compensate for the timing skew (Fig.12). Fig.13 shows our MATLAB simulation results, and we see that the negative frequency components due to the timing skew are suppressed.

We close this section by remarking that another digital timing skew compensation technique with fine time resolution, so-called a fractional delay digital filter [10], [11], [12], [13], [14] has been proposed, which mainly focuses on the waveform interpolation and reconstruction. However our proposed technique can incorporate filtering characteristics (such as a cosine roll-off filter) as well as fine timing skew adjustment with the clear design method as described above; this is very useful in LSI testing technology applications. Furthermore, since our proposed filter is easy to design, we can obtain their coefficient values with small amount of calculation which is desirable for ATE and LSI testing technologies where real-time timing calibration is required.

6. Conclusion

We have proposed the digital filter with novel linear phase characteristics and the time resolution of its group delay is arbitrary small. Also we have shown its application for timing skew compensation in interleaved ADC systems and SSB signal generation systems. We have performed simulation to validate these results. We believe that our proposed technique will open a new research area for digital filters with linear phase and fine resolution of group delay, and give a significant impact on its application in electronic systems as digital compensation technique for timing skew and frequency characteristics, which is reliable and stable as well as suitable for fine CMOS implementation.

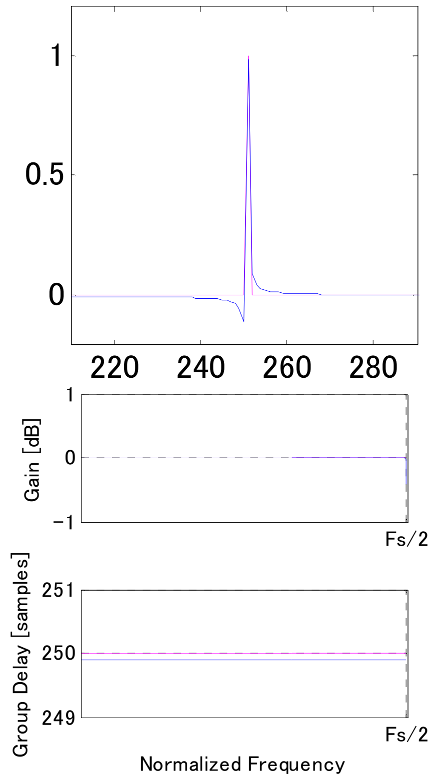

Fig. 9. $h_1(n)$ (shown in purple) and $h_2(n)$ (shown in blue) filter characteristics used for the timing skew compensation method 2. (Top) Impulse response. (Middle) Gain characteristics. (Bottom) Group delay characteristics. For $h_1(n)$ Impulse response is symmetric but for $h_2(n)$ it is asymmetric.

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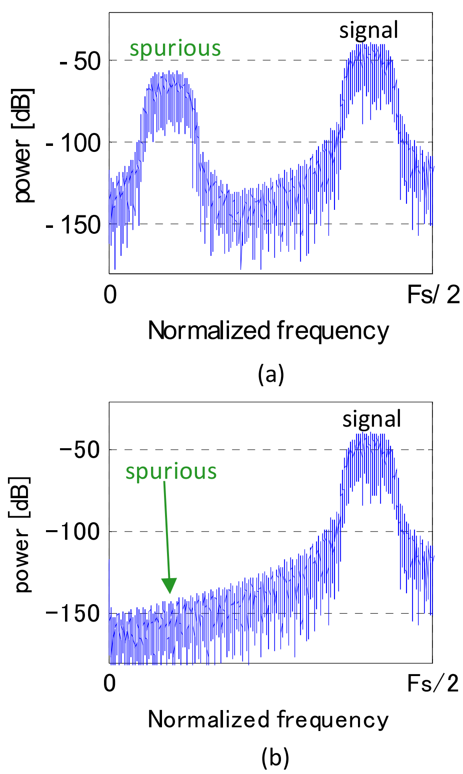

Fig. 10. Simulation results of the timing skew compensation method 2 for the QPSK input (whose signal band is within Fs/4 – Fs/2) with applying the Blackman window function for the digital filter tap truncation. (a) Without compensation. (b) With compensation.

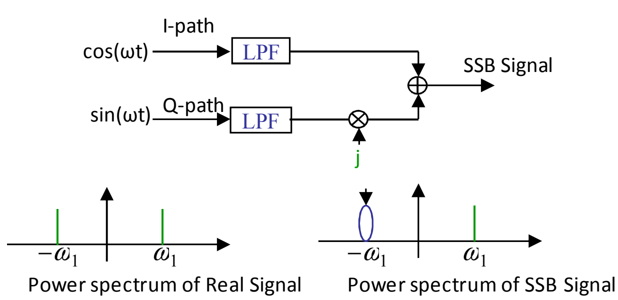

Fig. 11. Single-side band (SSB) signal.

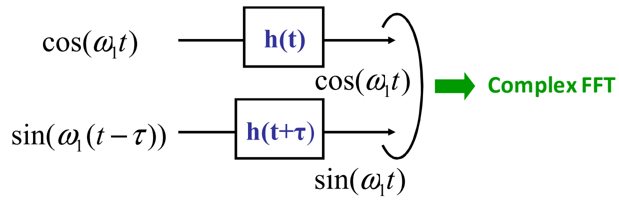

Fig. 12. Timing skew compensation for the SSB signal with our linear phase digital filter.

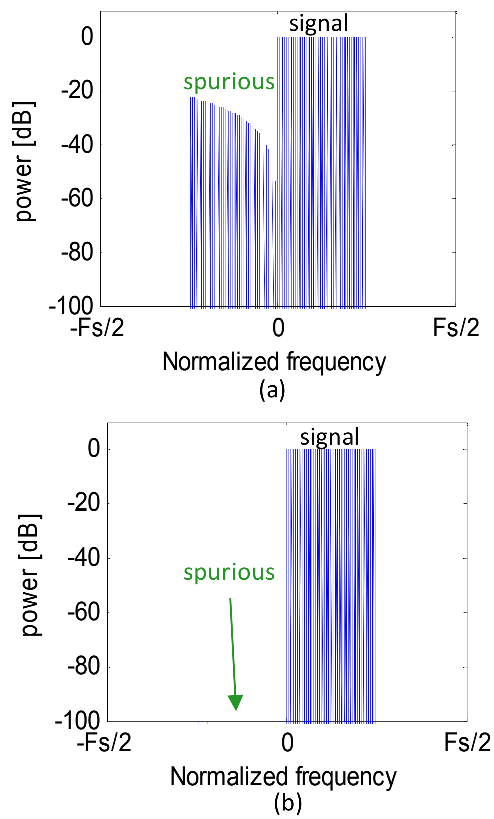

Fig. 13. Simulation results of timing skew compensation for the SSB signal with our novel linear phase digital filter for wideband input signal. (a) Without compensation. (b) With compensation using our proposed linear phase digital filter.