Digitally-Assisted Compensation for Timing Skew in ATE Systems

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H. Miyajima  H. Kobayashi

Advantest Corporation
Gunma University
Contents

• Research Goal
• Conventional Linear Phase Digital Filter Condition
• New Linear Phase Digital Filter Condition
  – Time-Shift, Impulse Response of Ideal Filter
  – New Linear Phase Digital Filter
• MATLAB Simulation
• Design Considerations
  – Window
  – Gain Adjustment
• Application
• Conclusion
Research Goal

Timing skew is a major problem in ATE systems

Digital compensation for timing skew
  ⇒ Linear phase is important

Conventional linear-phase digital filter ⇒ coarse timing adjustment

Proposed linear-phase digital filter ⇒ fine timing adjustment
Features of Proposed Digital Filter

- Fine time resolution
- Linear phase
- Applicable to bandpass signal
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Linear Phase FIR Filter Impulse Response

(1) Case 1
odd # of taps • even symmetry

(2) Case 2
even # of taps • even symmetry

(3) Case 3
odd # of taps • odd symmetry

(4) Case 4
even # of taps • odd symmetry
Frequency Characteristics

<table>
<thead>
<tr>
<th>$h(nT')$</th>
<th>$H(e^{j\omega T'})$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case 1</strong></td>
<td>$e^{-j\omega(N-1)T_s/2} \sum_{k=0}^{(N-1)/2} a_k \cos[\omega kT_s]$</td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td>$e^{-j\omega(N-1)T_s/2} \sum_{k=1}^{N/2} b_k \cos[\omega(k - 1/2)T_s]$</td>
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<td><strong>Case 3</strong></td>
<td>$e^{-j(\omega(N-1)T_s/2-\pi/2)} \sum_{k=0}^{(N-1)/2} a_k \sin[\omega kT_s]$</td>
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Phase: proportional to $\omega$ (linear phase)

Time resolution of group delay: $T_s/2$
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Ideal LPF

Frequency Characteristics

\[ |H(j\omega)| \]

Impulse Response

\[ h(t) = \frac{1}{T_s} \text{sinc} \left( \frac{\pi t}{T_s} \right) \]

\[ \omega_s = \frac{2\pi}{T_s} : \text{Sampling Frequency} \]
Discrete-Time Representation of Ideal LPF

\[ \sum_k H(j(\omega - k \cdot \omega_s)) \]

\[ h(t) = \sum_k \text{sinc} \left( \pi \frac{k \cdot T_s}{T_s} \right) \delta(t - k \cdot T_s) \]
Impulse Response Time-Shift

\[ \angle G(j\omega) = -\omega \Delta t \]

No change of Gain

\[ \Delta t \text{ time-shift of impulse response} \]
Time-Shift and Filter Coefficients

FIR filter

\[ h(t) = \sum_{k} \text{sinc} \left( \pi \frac{k \cdot T_s}{T_s} \right) \delta(t - k \cdot T_s) \]

IIR Filter

\[ h(t) = \sum_{k} \text{sinc} \left( \pi \frac{k \cdot T_s - \Delta t}{T_s} \right) \delta(t - k \cdot T_s) \]

Ideal Delay-Filter

\[ \Delta t = \frac{T_s}{T_s} \]
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• Conclusion
2-Tap Filter: Model

\[ |H(j\omega)| \]
\[ \angle H(j\omega) \]

\[ \pi / T_s \]
\[ -j\omega T_s / 2 \]
2-Tap Filter: Delay Model

\[ |G(j\omega)| \]

\[ \angle G(j\omega) \]

- \( j(\omega T_s / 2 + \omega \tau) \)

FIR

\[ a_0 \quad a_1 \]

\[ \frac{t}{T_s} \]

IIR

\[ a'_0 \quad a'_1 \]

\[ \frac{t}{T_s} \]

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2-Tap Filter: Delay Model

\[ G(j\omega) = a_0 + a_1 j(\omega T_s/2 + \omega \tau) \]

\[ |G(j\omega)| \]

\[ \angle G(j\omega) \]

IIR

FIR

\[ t/T_s \]

\[ a_0, a_1 \]

\[ a_0', a_1' \]
Proposed Delay Digital Filter

(a) FIR Filter

(b) Ideal Delay Filter

(c) Delay Digital Filter
# Frequency Characteristics of Proposed Delay Digital Filter

<table>
<thead>
<tr>
<th>Case</th>
<th>$g(nT)$</th>
<th>$G(e^{j\omega T})$</th>
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<tr>
<td>Case 1</td>
<td>$e^{-j\omega(N-1)T_s/2 + \omega \tau}$ $\sum_{k=0}^{(N-1)/2} a_k \cos[\omega kT_s]$</td>
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<td>$e^{-j\omega(N-1)T_s/2 + \omega \tau}$ $\sum_{k=1}^{N/2} b_k \cos[\omega(k-1/2)T_s]$</td>
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<td></td>
</tr>
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<td>Case 4</td>
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<td></td>
</tr>
</tbody>
</table>

- **Phase**: proportional to $\omega$ (linear phase)
- **Group delay time resolution $\tau$**: Arbitrary small
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Comparison of 2-Tap Filter Impulse Responses

2-Tap FIR Filter

2-tap FIR coefficients zero

Proposed Delay Filter (0.3 samples delay)

Impulse response changes.

Non-zero
Comparison of 2-Tap Filter Frequency Characteristics

No change of gain

Original filter and proposed delay filter

Phase slope changes

Proposed delay filter

Normalized Frequency (Fs=1.0)

Gain [dB]

Phase [radian]
Finite Tap Truncation of Proposed Delay Filter

61-Tap Cosine Roll-off Filter

Delay Filter (0.3 samples delay)

- Rectangular window
- Hann window

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Effects of Window

Gibbs oscillation of group delay

Frequency characteristics of delay filter with 61-tap truncation
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How to Apply Window

Centered at origin

Centered at impulse response center

Window center shifted by $\Delta t$
Frequency Characteristics of Delay Filter after Applying Window

- **Delay**: 0.3 samples
- **Filter Tap**: 100 taps
- **Window**: Han
- **Pass band**: $(0.1 \sim 0.4) \cdot F_s$
- **FFT points**: 1024 points
Group Delay Characteristics of Delay Filter after Applying Window

- **Window centered at origin**
- **Window centered at impulse response**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Delay</td>
<td>0.3 samples</td>
</tr>
<tr>
<td>Filter Tap</td>
<td>100 taps</td>
</tr>
<tr>
<td>Window</td>
<td>Han</td>
</tr>
<tr>
<td>Pass band</td>
<td>$(0.1 \sim 0.4) \cdot F_s$</td>
</tr>
<tr>
<td>FFT points</td>
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Frequency Characteristics of Delay Filter after Applying Window

Window centered at origin

Window centered at impulse response

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<td>Pass band</td>
<td>(0.05 - 0.3) \cdot Fs</td>
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Group Delay Characteristics of Delay Filter after Applying Window

- **Window centered at origin**
- **Window centered at impulse response**

Applying window centered at impulse response

**Constant group delay over entire passband**

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  – **Gain Adjustment**
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Proposed Filter DC Gain Adjustment

Digital filter DC gain: \[ \sum a_n \]

DC gain adjustment due to finite tap truncation is required

\[ \sum_{n=0}^{N} a'_n = \text{DC gain of original FIR filter} \]
Frequency Characteristics of Proposed Delay Filter

**Without DC gain adjustment**

**With DC gain adjustment**

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<tr>
<td>Cut-off Freq.</td>
<td>0.4 • Fs</td>
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Gain Characteristics of Proposed Delay Filter

With DC gain adjustment
Without DC gain adjustment

- Delay: 0.1 samples
- Filter Tap: 101 taps
- Window: Han
- Cutoff Freq.: 0.4 \times Fs
- FFT points: 1024 points
- Delay: 0.3 samples
- Filter Tap: 101 taps
- Window: Han
- Cutoff Freq.: 0.4 \times Fs
- FFT points: 1024 points
Gain Characteristics of Proposed Delay Filter

- **Original FIR filter**
- **With DC gain adjustment**
- **Without DC gain adjustment**

### Delay Filter Parameters

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Gain Characteristics of Proposed Delay Filter

- Original FIR filter
- With DC gain adjustment
- Without DC gain adjustment

DC gain adjustment

Delay filter gain

Original FIR filter gain

Normalized Frequency

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I/Q Delay Mismatch in Quadrature Modulator

I(t) = cos (2πf₀t)
Q(t) = sin(2πf₀t)

SSB signal input

DAC:
Digital-to-analog converter

SSB:
Single side band

I(t) + jQ(t)

Delay in analog domain

Image rejection ratio

DAC:
Digital-to-analog converter

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I/Q Delay Mismatch Compensation in Quadrature Modulator

\[ I(t) = \cos(2\pi f_0 t) \]
\[ Q(t) = \sin(2\pi f_0 t) \]

SSB signal

Digital timing compensation \( \tau \)

Delay in analog domain

DAC:

Digital-to-analog converter

SSB: single side band

DAC: digital-to-analog converter
Matlab Simulation Results

(a) Ideal case

(b) Timing skew case

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<td>Filter tap #</td>
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Matlab Simulation Results

(c) Compensation using delay filter
Without adjustment of window, gain

(d) Compensation using delay filter
With adjustment of window, gain

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Interleaved ADC System

- M channel ADCs  ➡️  M-times sampling rate

![Diagram of an interleaved ADC system](image-url)
Timing Skew in Interleaved ADC System

ADC: analog-to-digital converter

ADC1

ADC2

MUX

Digital output

Dout

Analog Input

$\text{f}_{\text{in}}$

$\text{CLK1}$

$\text{CLK2}$

$\text{CLK1}$

$\text{CLK2}$

$\text{Ts}$

$\tau$

$0 \rightarrow \text{f}_{\text{in}} \rightarrow \text{F}_s \rightarrow \text{f}$

$0 \rightarrow \text{f}_{\text{in}} \rightarrow (\text{F}_s - \text{f}_{\text{in}}) \rightarrow \text{f}$

$\text{F}_s = \frac{1}{\text{T}_s}$
Timing Skew Compensation in Interleaved ADC System

ADC: analog-to-digital converter

ADC1

ADC2

CLK1

CLK2

MUX

Digital output

Dout

Analog input

f_{in}

Clock skew effect compensation

CLK 1

CLK 2

f_{in}

f_{in}

F_{s}

F_{s} - f_{in}F_{s}

F_{s} = 1/T_{s}

Kobayashi. Lab @ Gunma_University
Matlab Simulation Results

(a) Ideal case

(b) Timing skew case

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Matlab Simulation Results

(c) Compensation using delay filter
   Without adjustment of window, gain

(d) Compensation using delay filter
   With adjustment of window, gain

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<thead>
<tr>
<th>Signal</th>
<th>Spurious</th>
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</thead>
<tbody>
<tr>
<td>[Normalized frequency]</td>
<td>[Magnitude [dB]]</td>
</tr>
<tr>
<td>0.0</td>
<td>-120</td>
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</tbody>
</table>

- Delay: 0.3 samples
- Filter tap: 61 taps
- Window: Han
- FFT points: 1024 points
Conclusion

- Linear phase digital filter with fine time resolution of group delay
- Design consideration
  - How to apply window
  - DC gain adjustment
- Application Examples
  - I/Q delay mismatch compensation in quadrature modulator
  - Timing skew compensation in interleaved ADC system

Future work
- Implementation issues
  - Finite word length, finite tap effects
  - LSI implementation