Oct. 18, Thursday 10:30 ~ 11:00
Session 6B-1
Mixed Signal Designs and ATE

Highly Efficient Waveform Acquisition Condition in Equivalent-Time Sampling System

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Outline

- Motivation
- Equivalent-Time Sampling
- Golden Ratio
- Proposed Golden Ratio Sampling
- Simulation
- Conclusion

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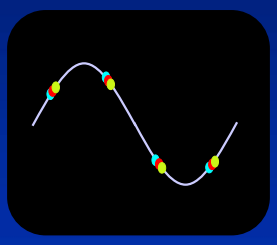
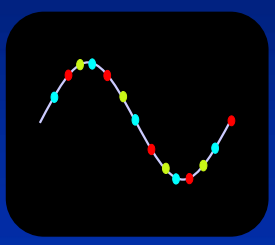
Motivation

in Equivalent-Time Sampling



Bad



Good

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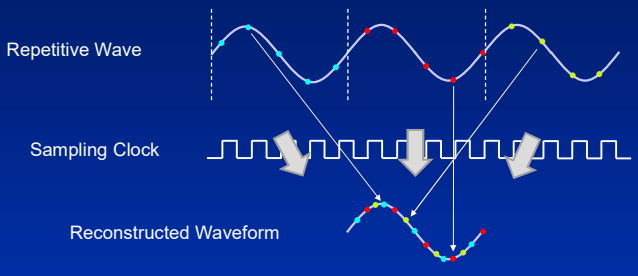
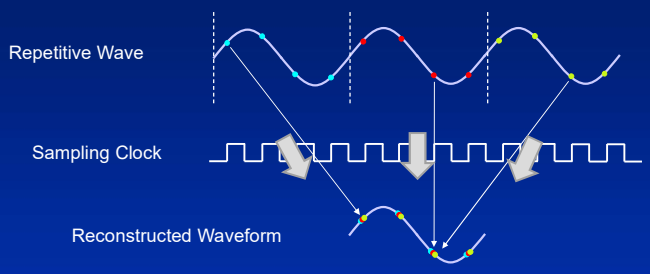
Equivalent-Time Sampling



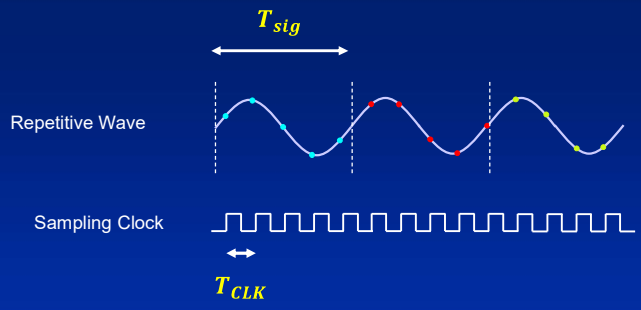
Higher time resolution than sampling clock period

Waveform Missing



Toothless waveform is appeared

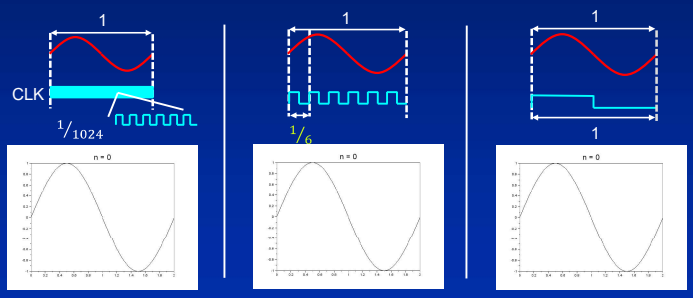
Condition



$$T_{CLK} = \boxed{?} \times T_{sig}$$

Waveform Missing Conditions

$f_{CLK} \gg f_{sin}$ $\quad f_{CLK} \approx \frac{1}{\alpha} f_{sin} \left(\alpha = 1, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \cdots, \frac{1}{6}, \cdots\right)$ $\quad f_{CLK} \approx f_{sin}$

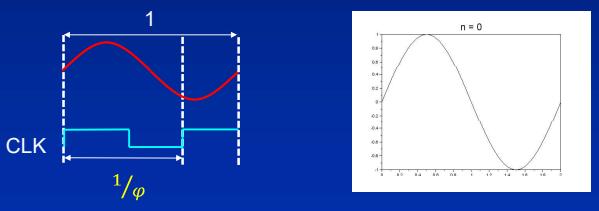


Sampling points move little ➡ Requires long time

Proposed Optimal Condition

$$f_{CLK} = \varphi \times f_{sig}$$

φ : Golden ratio (= 1.6180339887…)



Sampling points disperse uniformly through measurement

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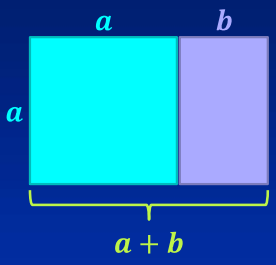
Golden Ratio

$$\varphi \equiv \frac{a+b}{a} = \frac{a}{b}$$



$$\varphi = \frac{1+\sqrt{5}}{2} = 1.6180339887\cdots$$

Fibonacci Number

$F_0 = 0$

$F_1 = 1$

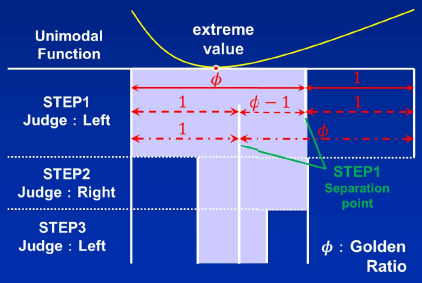
$F_{n+2} = F_n + F_{n+1}$

$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \cdots$

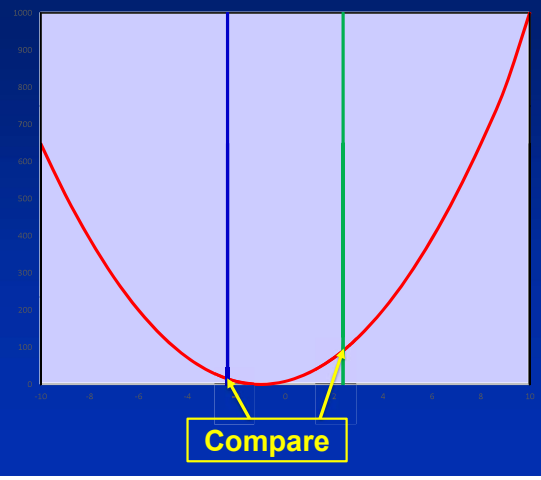
$$\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = 1.6180339887\cdots = \varphi$$

Golden Section Search

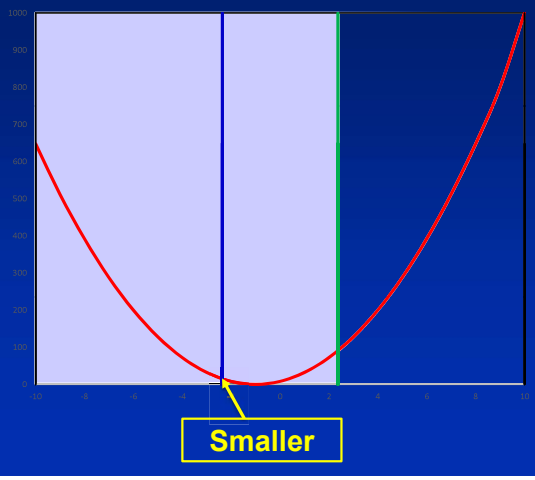
- Finds the extreme value of a unimodal function by narrowing the range
- Distances of separation points are golden ratio



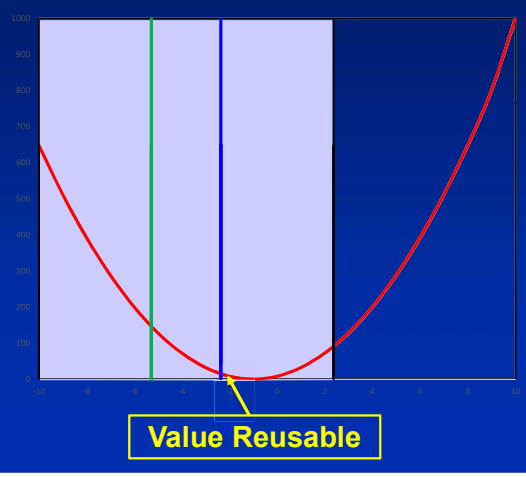
Golden Section Search (1/5)



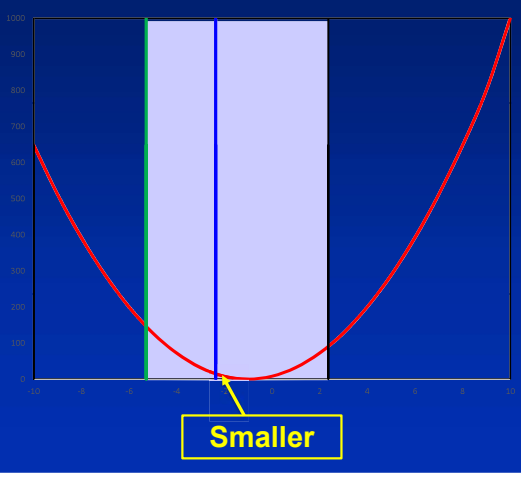
Golden Section Search (2/5)



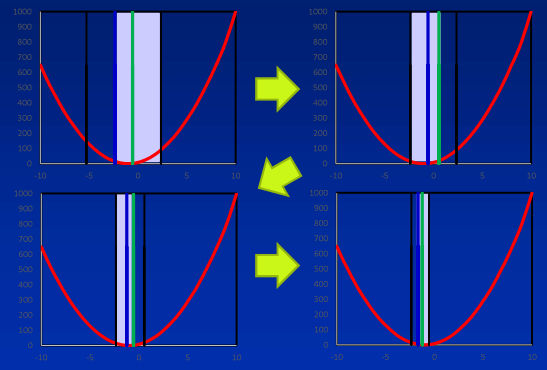
Golden Section Search (3/5)



Golden Section Search (4/5)



Golden Section Search (5/5)



Range is narrowed x 1/Φ in every steps

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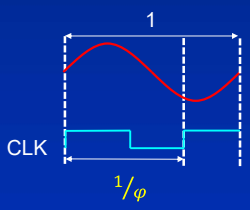
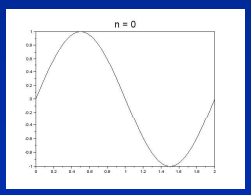
Proposed Optimal Condition

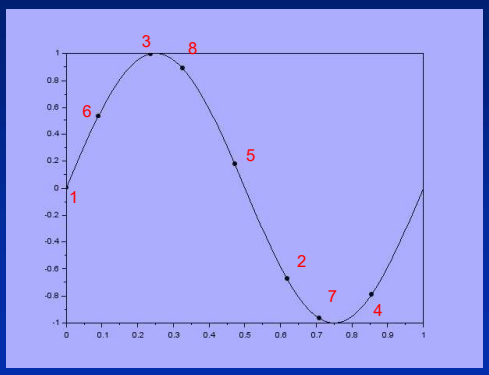
$$f_{CLK} = \varphi \times f_{sig}$$

φ : **Golden ratio** (= 1.6180339887…)

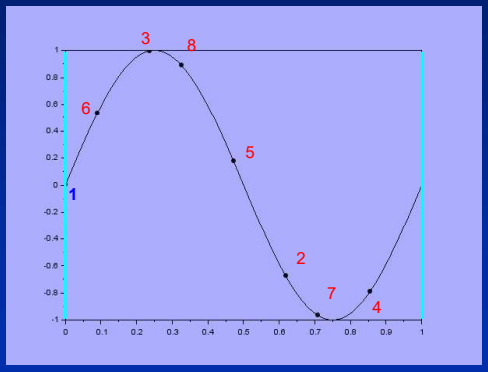




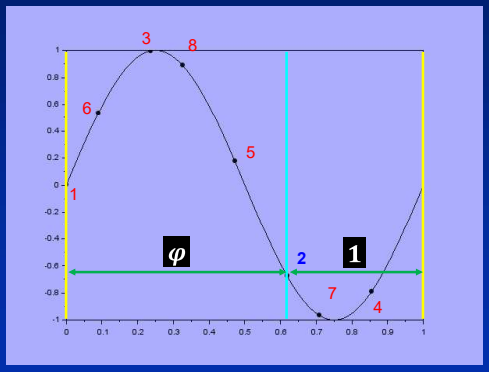
Golden Ratio Sampling (8pt.)



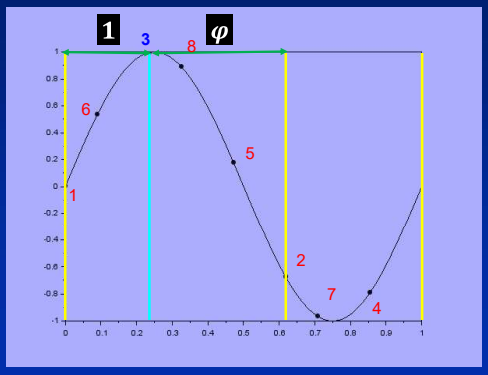
Golden Ratio Sampling (1/8)



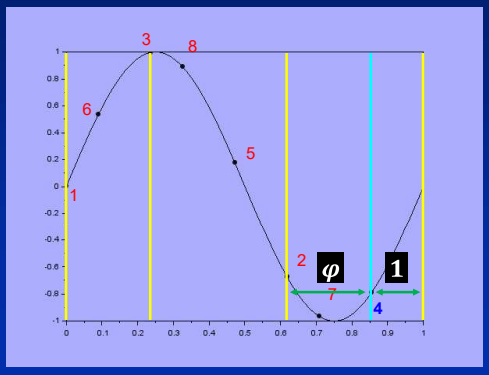
Golden Ratio Sampling (2/8)



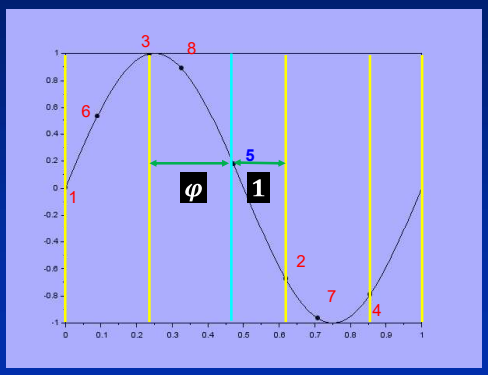
Golden Ratio Sampling (3/8)



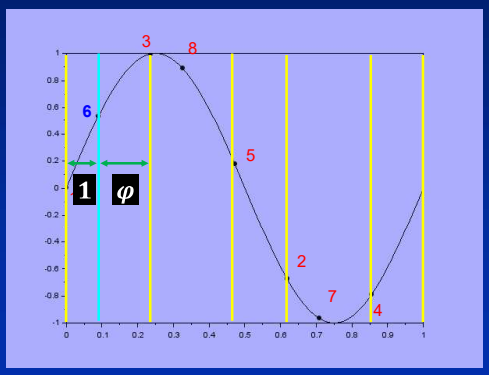
Golden Ratio Sampling (4/8)



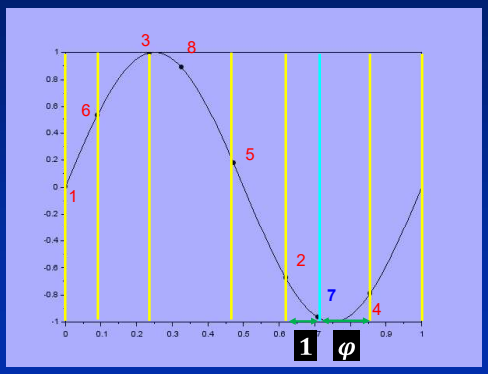
Golden Ratio Sampling (5/8)



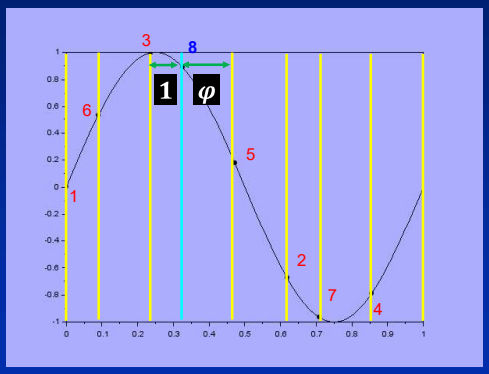
Golden Ratio Sampling (6/8)



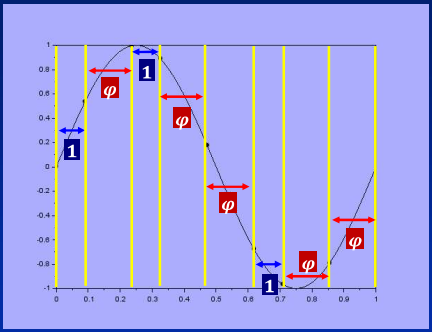
Golden Ratio Sampling (7/8)



Golden Ratio Sampling (8/8)



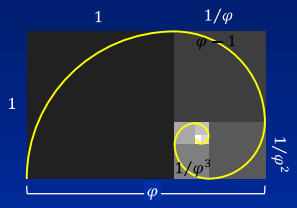
Distance



All sections are divided by golden ratio

➡ **Longer and shorter range does not exist**

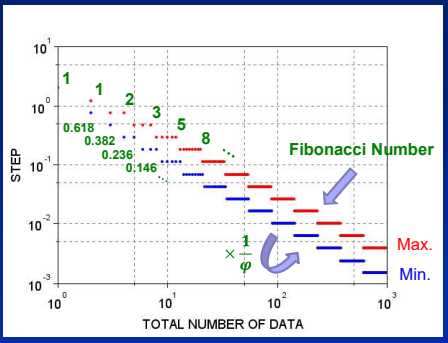
Max. & Min. Distance



Max. / Min. distances = φ or φ^2 const.

➡ Sampling points disperse uniformly through measurement

Time Resolution



Max. & **min.** distances decreases **x 1/Φ** every **Fibonacci numbers**

➡ Time resolution improves about **1 / Total Number of data**

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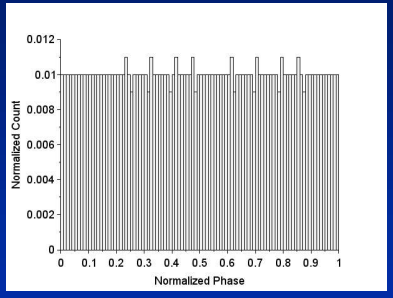
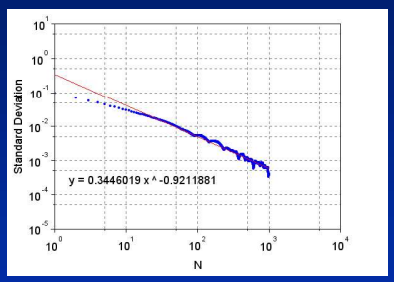
Histogram (1,000 pt.)



Uniform Distribution

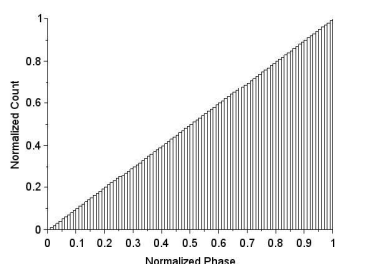
DNL Transition



DNL standard deviation from the approximate curve is **SMALL**

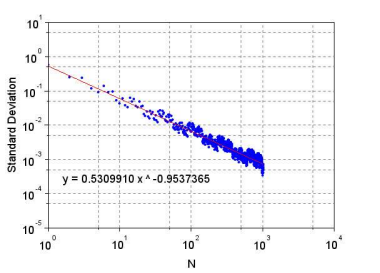
➡ Sampling phases are NOT appeared in the same bin successively

Accumulated Histogram (1,000 pt.)



Uniform Distribution

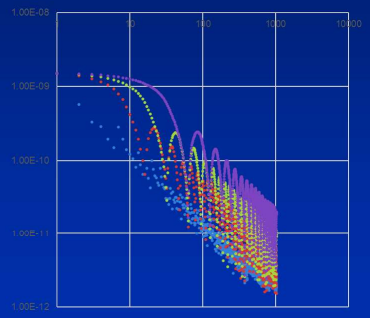
INL Transition



INL standard deviation from the approximate curve is **SMALL**

➡ Sampling phases disperse **uniformly through measurement**

INL Transition (Waveform Missing)

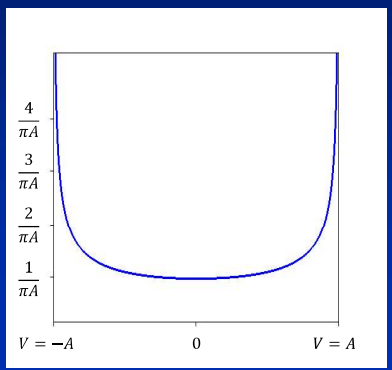


Application

- Wideband Waveform Sampling Systems
- ADC Testing with Histogram Method
- Time-to-Digital Converter Calibration
- Integral-type Time-to-Digital Converter

Probability Density Function

Sinusoidal Wave



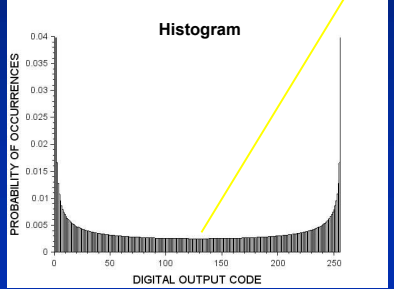
$$p(V) = \frac{1}{\pi\sqrt{A^2 - V^2}}$$

A: *Amplitude of Sinusoidal Wave*

V: *Voltage*

Ideal Probability of Each Bin

$$P(n) = \frac{1}{\pi}\left[sin^{-1}\left(\frac{B(n-2^{N-1})}{A2^N}\right) - sin^{-1}\left(\frac{B(n-1-2^{N-1})}{A2^N}\right)\right]$$

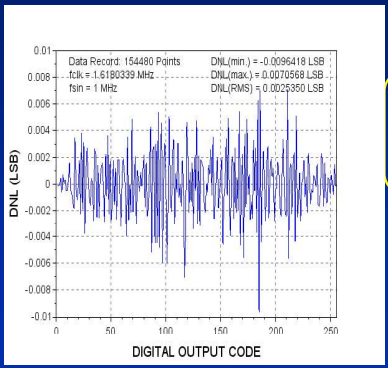


B: *Full Scale Range* (V)

N: *Resolution of ADC* (bit)

n: *Code Bin Number* (n^{th})

Differential Non-Linearity

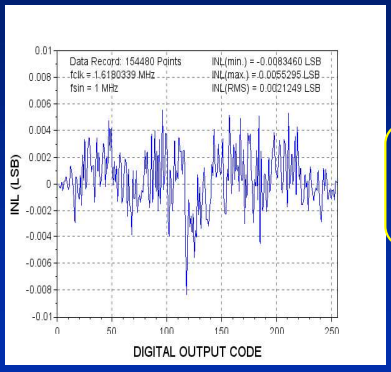


$$DNL_n(LSB) = \frac{AP_n}{IP_n} - 1$$

AP: *Measured Histgram*

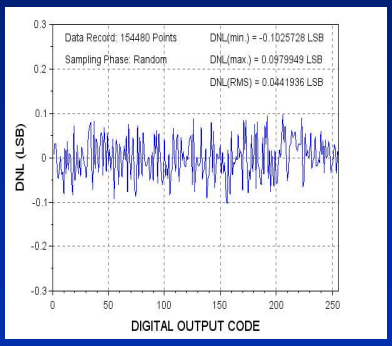
IP: *Ideal Histgram*

Integral Non-Linearity



$$INL_n(LSB) = \sum_{i=1}^{n} DNL_i$$

Required Number of Samples



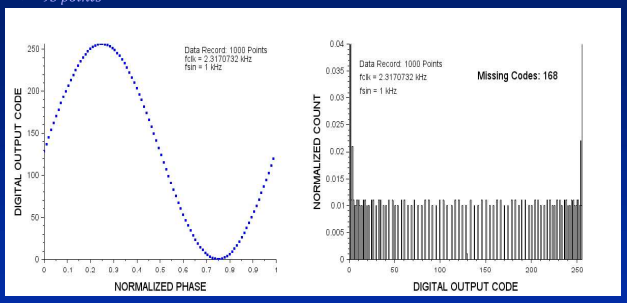
$$N_R = \frac{\pi \times 2^{N-1} \times Z_{\alpha/2}^2}{\beta^2}$$

$Z_{\alpha/2}$: *Z value*

β: *DNL error*

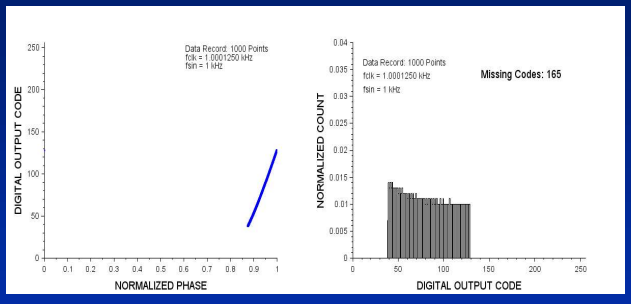
Waveform Missing (Case: 1)

$\alpha = \frac{41}{95} \rightarrow 95\ points$



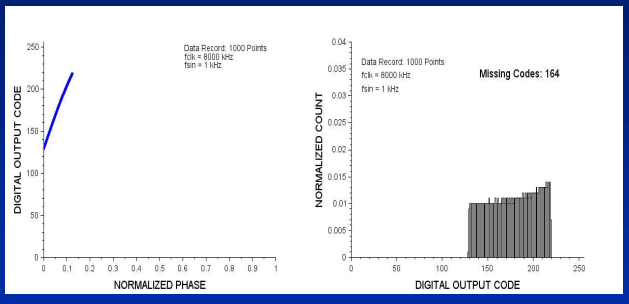
when $f_{CLK} = \frac{1}{\alpha} f_{sig}$ $\left(\alpha = 1, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \cdots\right)$

Waveform Missing (Case: 2)



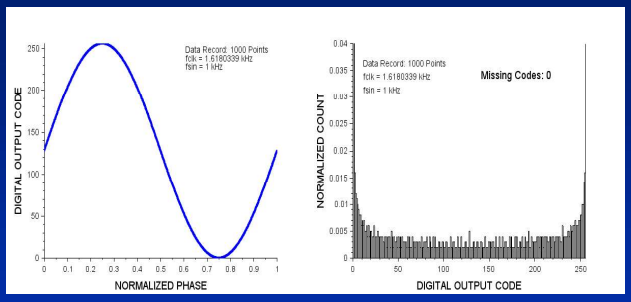
when $f_{CLK} \approx f_{sig}$

Waveform Missing (Case: 3)



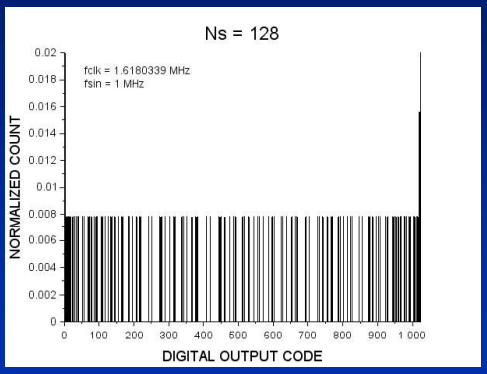
when $f_{CLK} \gg f_{sig}$

Golden Ratio Sampling



when $f_{CLK} = \varphi \times f_{sin}$

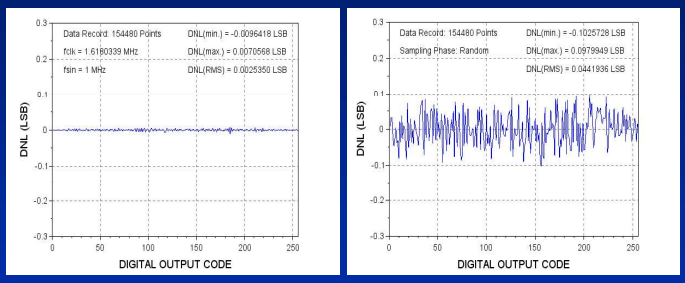
Histogram (Golden Ratio Sampling)



N_s: *Number of Samples*

Golden-ratio vs. Random Sampling

Golden-ratio Sampling | **Sampling at rand()**



$0.1\ LSB\ DNL\ error$
$95\%\ confidence\ level$
$8\ bit\ ADC$

$$N_R = \frac{3.14 \times 2^7 \times (1.96)^2}{(0.1)^2} = 154480\ samples$$

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Conclusion

Our proposed golden ratio sampling rate

In waveform equivalent-time sampling system

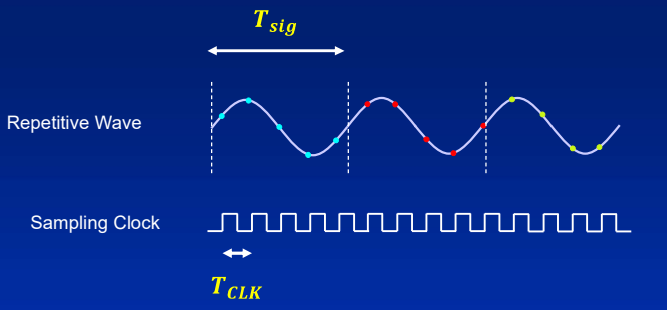
Sampling clock frequency x Input signal period

= golden ratio

- Can avoid waveform missing.
- Sampling points are dispersed uniformly through the measurement.

Appendix

Condition



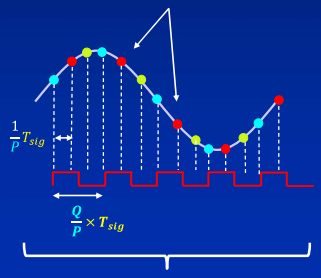
$$T_{CLK} = \frac{Q}{P} \times T_{sig}$$

Fixed Number of Data

P, Q: integers and relatively prime

$$T_{CLK} = \frac{Q}{P} \times T_{sig}$$

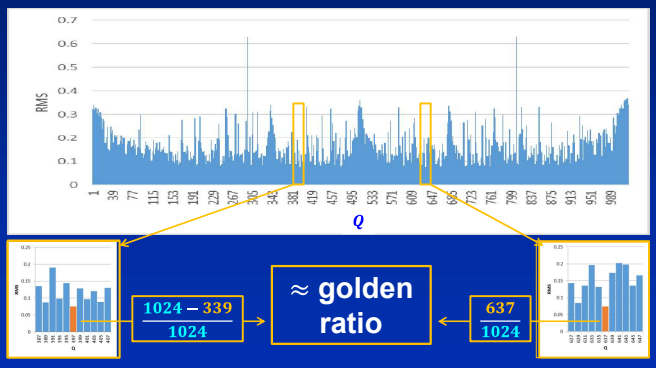
Q: determines phase distance for each sampling



P: Maximum number of total measurable sampling points

Distances of INL standard deviation from the approximate curve (RMS)

$$T_{CLK} = \frac{Q}{1024} \times T_{sig}$$

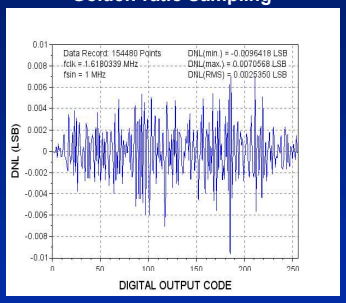


$\frac{1024 - 339}{1024}$ → ≈ golden ratio ← $\frac{637}{1024}$

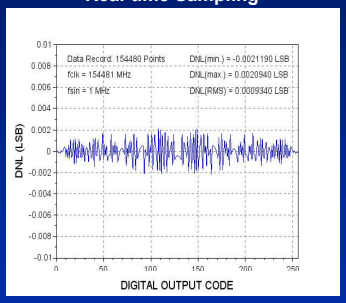
min.

Golden-ratio vs. Real-time Sampling

Golden-ratio Sampling



Real-time Sampling



$0.1\ LSB\ DNL\ error$
$95\%\ confidence\ level$
$8\ bit\ ADC$

$$N_R = \frac{3.14 \times 2^7 \times (1.96)^2}{(0.1)^2} = 154480\ samples$$