

複素信号 (Complex Signal)

Complex signal processing is NOT complex.

複素信号処理は複雑ではない。

(Prof. Ken Martin)

Complex signal



Real signal

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Explicit Transfer Function of RC Polyphase Filter for Wireless Transceiver Analog Front-End

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- Roles of RC Polyphase Filter
- Transfer Function Derivation of RC Polyphase Filter
 - 1st-order filter
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- Summary

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Research Goal

- To establish systematic design and analysis methods of RC polyphase filters.
- As its first step, to derive explicit transfer functions of the 1st-, 2nd- and 3rd-order RC polyphase filters.

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Roles of RC Polyphase Filter in Wireless Transceiver

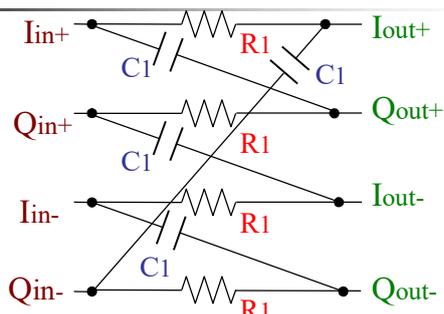
Features of RC Polyphase Filter

- Its input and output are complex signal.
- **Passive** RC analog filter
- One of key components in wireless transceiver analog front-end
 - **I, Q signal generation**
 - **Image rejection**
- Its explicit transfer function has not been derived yet.

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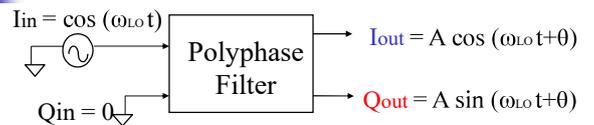
First-order RC Polyphase Filter



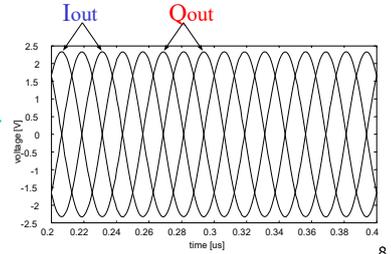
Differential Complex Input: $V_{in} = I_{in} + j Q_{in}$

Differential Complex Output: $V_{out} = I_{out} + j Q_{out}$ 7

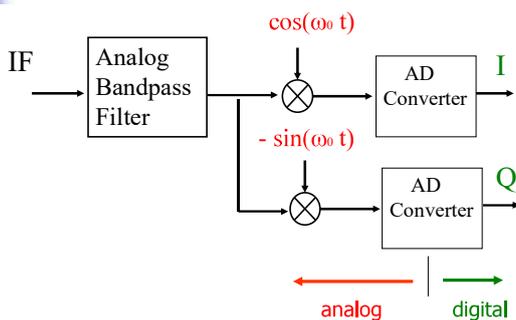
I, Q signal generation from single sinusoidal input



$$\omega_{LO} = \frac{1}{R_1 C_1}$$



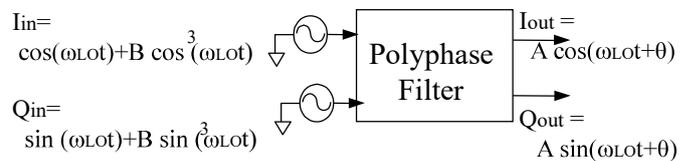
Cosine, Sine Signals in Receiver



They are used for down conversion

Pure I, Q signal generation

3rd-order harmonics rejection



With 3rd-order harmonics.

Without 3rd-order harmonics.

Simulation of 3rd-order harmonics rejection

$$I_{in}(t) = \cos(\omega_{LO} t) + a \cos^3(\omega_{LO} t)$$

$$Q_{in}(t) = \sin(\omega_{LO} t) + a \sin^3(\omega_{LO} t)$$

$$3\omega_{LO} = \frac{1}{R_1 C_1}$$

$$I_{out}(t) = A \cos(\omega_{LO} t + \theta)$$

$$Q_{out}(t) = A \sin(\omega_{LO} t + \theta)$$

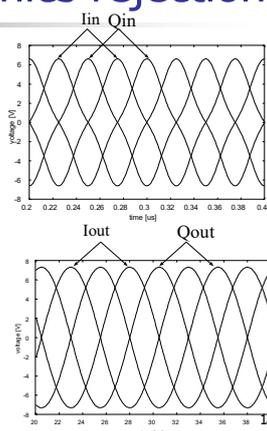
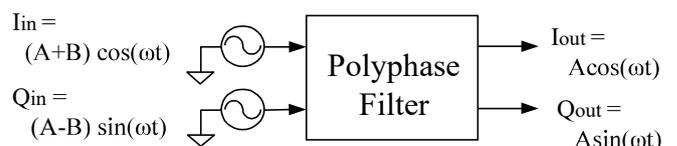


Image Rejection Filter

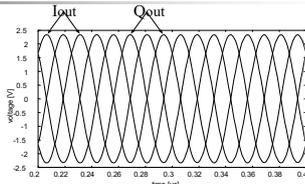


$$Ae^{j\omega t} + Be^{-j\omega t}$$

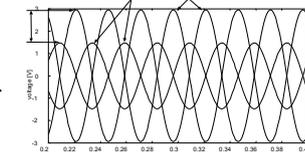
$$Ae^{j\omega t}$$

Problem when $\omega_{LO} \neq 1/R_1C_1$

$$\omega_{LO} = \frac{1}{R_1C_1}$$



$$\omega_{LO} = \frac{2}{R_1C_1}$$



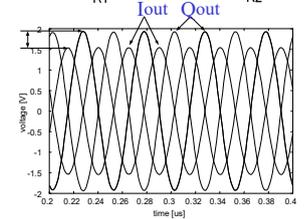
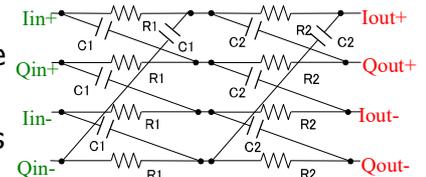
Amplitudes of I,Q signals differ significantly.

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2nd-order RC Polyphase Filter

The problem of large difference between Iout, Qout amplitudes can be alleviated

$$\omega_{LO} = \frac{2}{R_1C_1}$$

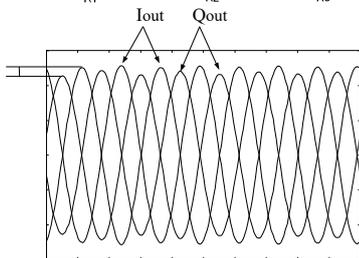


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3rd-order RC Polyphase Filter

The amplitude difference problem is further alleviated.

$$\omega_{LO} = \frac{2}{R_1C_1}$$



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Transfer Function Derivation of RC Polyphase Filter

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Transfer Function of RC Polyphase Filter

Complex Signal Theory

- Complex input
- Complex output

$$V_{in}(j\omega) = I_{in} + j \cdot Q_{in}$$

$$V_{out}(j\omega) = I_{out} + j \cdot Q_{out}$$

- Complex Transfer Function

$$G(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$$

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Transfer Function of 1st-order RC Polyphase Filter

Differential signal

$$I_{in}(t) = I_{in+}(t) - I_{in-}(t)$$

$$Q_{in}(t) = Q_{in+}(t) - Q_{in-}(t)$$

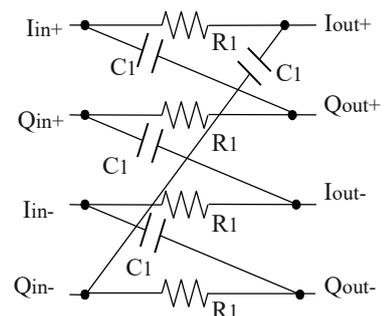
$$I_{out}(t) = I_{out+}(t) - I_{out-}(t)$$

$$Q_{out}(t) = Q_{out+}(t) - Q_{out-}(t)$$

Complex signal

$$V_{in}(t) = I_{in}(t) + jQ_{in}(t)$$

$$V_{out}(t) = I_{out}(t) + jQ_{out}(t)$$



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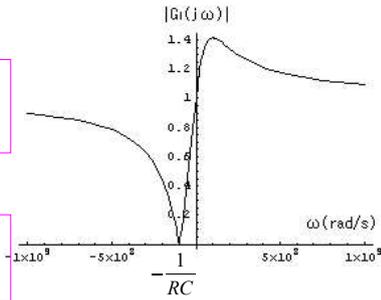
Transfer Function of 1st-order RC Polyphase Filter

•Transfer Function

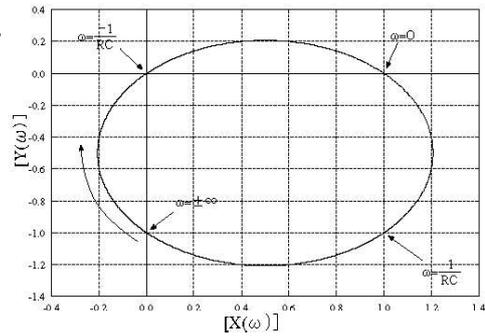
$$G_1(j\omega) = \frac{1 + \omega RC}{1 + j\omega RC}$$

•Gain

$$|G_1(j\omega)| = \frac{|1 + \omega RC|}{\sqrt{1 + (\omega RC)^2}}$$



Nyquist Chart of G1(jω)



$$G_1(j\omega) = X(\omega) + j Y(\omega)$$

Symmetric with respect to a line of $Y = -X$.

Explanation of

I, Q signal generation by G1(jω)

$$Q_{in}(t) \equiv 0, I_{in}(t) = \cos(\omega t),$$

$$V_{in}(t) = \cos(\omega t) = \frac{1}{2}[e^{j\omega t} + e^{-j\omega t}]$$

$$|G_1(j\omega)|_{\omega=\frac{1}{RC}} = 0, |G_1(j\omega)|_{\omega=\frac{1}{RC}} = \sqrt{2}, \angle G_1(j\omega) = -\frac{\pi}{4}$$

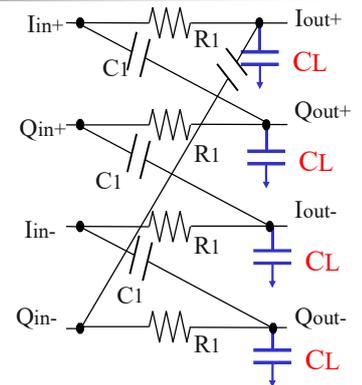
$$V_{out}(t) = \frac{1}{2} [|G_1(j\omega)| e^{j(\omega t + \angle G_1(j\omega))} + |G_1(-j\omega)| e^{j(-\omega t + \angle G_1(-j\omega))}]$$

$$= \frac{\sqrt{2}}{2} \cos(\omega t - \frac{\pi}{4}) + j \frac{\sqrt{2}}{2} \sin(\omega t - \frac{\pi}{4})$$

Output Load (CL) Effects

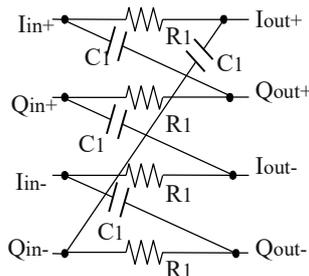
$$G_1(j\omega) =$$

$$\frac{1 + \omega R_1 C_1}{1 + j\omega R_1 (C_1 + C_L)}$$



Input Impedance

Input Impedance $Z_{in} =$
 $\frac{\text{Complex Input Voltage}}{\text{Complex Input Current}}$



$$Z_{in} = \frac{1 + j\omega R_1 C_1}{2 j\omega C_1 [1 + j(1 + \omega R_1 C_1)]}$$

Component Mismatch Effects

Mismatches δX among R's, C's

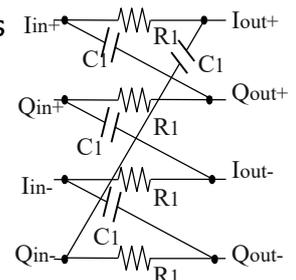
Image signal $\overline{V_{out}}$ is caused.

$$V_{out} = G_1 \cdot V_{in} + E_1 \cdot \delta X \cdot \overline{V_{in}}$$

$$\overline{V_{out}} = \overline{G_1} \cdot \overline{V_{in}} + \overline{E_1} \cdot \delta X \cdot V_{in}$$

where $E_1 := \frac{(1 + j)R_1 C_1 \omega}{2(1 + jR_1 C_1 \omega)^2}$

Image transfer function



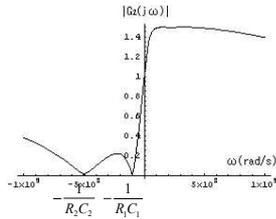
Transfer Function of 2nd-order RC Polyphase Filter

Transfer Function

$$G_2(j\omega) = \frac{(1 + \omega R_1 C_1)(1 + \omega R_2 C_2)}{1 - \omega^2 R_1 C_1 R_2 C_2 + j\omega(C_1 R_1 + C_2 R_2 + 2R_1 C_2)}$$

Derivation is very complicated, so we used "Mathematica."

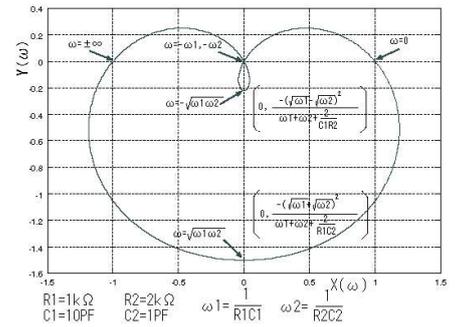
Gain $|G_2(j\omega)|$ characteristics



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Nyquist Chart of $G_2(j\omega)$

Symmetric with respect to a line of $X = 0$.



$$G_2(j\omega) = X(\omega) + j Y(\omega)$$

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Features of 2nd-order RC Polyphase Filter

$$|G_2(j\omega)| \neq |G_2(-j\omega)|, \quad |G_2(j\omega_1)| = |G_2(j\omega_2)|,$$

$$|G_2(-j\omega_1)| = |G_2(-j\omega_2)| = 0,$$

$$\lim_{\omega \rightarrow \pm\infty} |G_2(j\omega)| = 1, \quad |G_2(j0)| = 1,$$

$$\left[\frac{\partial |G_2(j\omega)|}{\partial \omega} \right]_{\omega = \sqrt{\omega_1 \omega_2}} = 0, \quad \frac{|G_2(j\sqrt{\omega_1 \omega_2})|}{|G_2(-j\sqrt{\omega_1 \omega_2})|} = \frac{(\sqrt{\omega_1} + \sqrt{\omega_2})^2}{(\sqrt{\omega_1} - \sqrt{\omega_2})^2}$$

$$\text{For arbitrary } a, \quad |G_1(ja\sqrt{\omega_1 \omega_2})| = |G_1(j\sqrt{\omega_1 \omega_2} / a)|_{27}$$

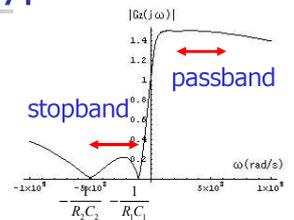
Flat passband design of 2nd-order RC Polyphase Filter

Passband: $\omega_1 \sim \omega_2$

Stopband: $-\omega_1 \sim -\omega_2$

where $\omega_1 := 1/R_1 C_1$,

$\omega_2 := 1/R_2 C_2$



● To make gain in passband flat,
 $|G_2(j\omega_1)| = |G_2(j\omega_2)| = |G_2(j\sqrt{\omega_1 \omega_2})|.$

● Image Rejection Ratio = $\left[\frac{\sqrt{\omega_2} + \sqrt{\omega_1}}{\sqrt{\omega_2} - \sqrt{\omega_1}} \right]^2$

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Explanation why a 2nd-order filter reduces I, Q amplitude difference.

Input signal

$$Q_{in}(t) \equiv 0, I_{in}(t) = \cos(\omega t) = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t})$$

Output of a 1st-order filter

$$I_{out1}(t) + jQ_{out1}(t) = \frac{1}{2}[|G_1(j\omega)|e^{j\omega t + \theta_1} + |G_1(-j\omega)|e^{-j\omega t - \theta_1}]$$

$$= |G_1(j\omega)|\left(1 + \frac{|G_1(-j\omega)|}{|G_1(j\omega)|}\right)\cos(\omega t + \theta_1) + j|G_1(j\omega)|\left(1 - \frac{|G_1(-j\omega)|}{|G_1(j\omega)|}\right)\sin(\omega t + \theta_1)$$

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Explanation why a 2nd-order filter reduces I, Q amplitude difference.

Output of a 2nd-order filter

$$I_{out2}(t) + jQ_{out2}(t) = \frac{1}{2}[|G_2(j\omega)|e^{j\omega t + \theta_2} + |G_2(-j\omega)|e^{-j\omega t - \theta_2}]$$

$$= |G_2(j\omega)|\left(1 + \frac{|G_2(-j\omega)|}{|G_2(j\omega)|}\right)\cos(\omega t + \theta_2) + j|G_2(j\omega)|\left(1 - \frac{|G_2(-j\omega)|}{|G_2(j\omega)|}\right)\sin(\omega t + \theta_2)$$

● According to the transfer functions,

$$\frac{|G_1(-j\omega)|}{|G_1(j\omega)|} \gg \frac{|G_2(-j\omega)|}{|G_2(j\omega)|} \quad \text{at } \omega \approx \omega_{LO}$$

then, the amplitude difference is reduced.

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Transfer Function of 3rd-order RC Polyphase Filter

$$G_3(j\omega) = \frac{N_3(j\omega)}{D_{3R}(\omega) + jD_{3I}(\omega)}$$

where

$$N_3(j\omega) = (1 + \omega R_1 C_1)(1 + \omega R_2 C_2)(1 + \omega R_3 C_3)$$

$$D_{3R}(\omega) =$$

$$1 - \omega^2 [R_1 C_1 R_2 C_2 + R_2 C_2 R_3 C_3 + R_1 C_1 R_3 C_3 + 2R_1 C_3 (R_2 C_2 + R_2 C_1 + R_3 C_2)]$$

$$D_{3I}(\omega) =$$

$$\omega [R_1 C_1 + R_2 C_2 + R_3 C_3 + 2(R_1 C_2 + R_2 C_3 + R_1 C_3)] - \omega^3 R_1 C_1 R_2 C_2 R_3 C_3$$

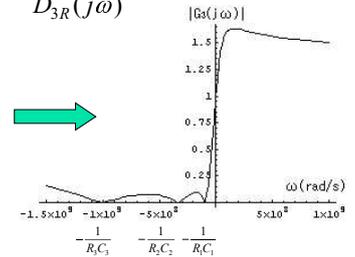
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Gain, Phase of 3rd-order RC Polyphase Filter

Gain: $|G_3(j\omega)| = \frac{|N_3(j\omega)|}{\sqrt{D_{3R}(\omega)^2 + D_{3I}(\omega)^2}}$

Phase: $\tan(\angle G_3(j\omega)) = -\frac{D_{3I}(\omega)}{D_{3R}(\omega)}$

Gain characteristics

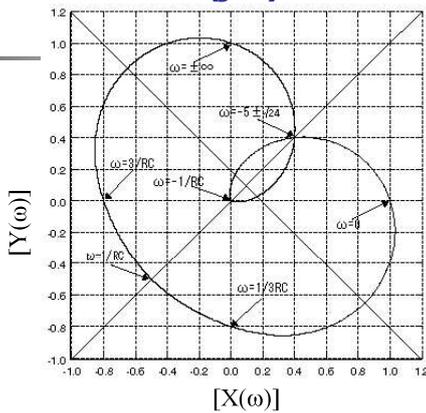


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Nyquist Chart of G₃(jω)

In case
 $R_1 C_1 = R_2 C_2$
 $= R_3 C_3$

Symmetric
 with respect to
 a line of $Y = X$.



$$G_3(j\omega) = X(\omega) + j Y(\omega)$$

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Summary

- Explicit transfer functions of 1st-, 2nd- and 3rd-order RC polyphase filters.
 - ➡ Systematic design and analysis are possible.
- On-going projects
 - Derivation of higher-order filter ones.
 - Nonideality effects in higher-order filters.
 - Systematic design method using Nyquist chart.

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