Complex Bandpass $\Delta \Sigma AD$ Modulator with Noise-coupled Architecture

Hao San

Department of Electronic Engineering Graduate School of Engineering, Gunma University Tenjin-cho 1-5-1, Kiryu, Gunma, 376-8515 Japan Telephone: +81 (277) 30–1789 Fax: +81 (277) 30–1707 Email: san@el.gunma-u.ac.jp

Abstract—Complex bandpass $\Delta \Sigma AD$ modulators can provide superior performance to a pair of real bandpass $\Delta \Sigma AD$ modulators of the same order. They process just input I and Q signals, not image signals, and AD conversion can be realized with low power dissipation, so that they are desirable for such low-IF receiver applications. This paper proposes a new architecture for complex bandpass $\Delta\Sigma AD$ modulators with noise-coupled topology, which effectively raises the order of the complex modulator and achieves higher SQNDR (Signal to Quantization Noise and Distortion Ratio) with low power dissipation. By providing the cross-coupled quantization noise injection to internal I and Q Paths, two quantizers' noise coupling can be realized in complex form, which enhances the order of noise shaping in complex domain, and provides a higher-order NTF using a lower-order loop filter in complex $\Delta \Sigma AD$ modulator. Proposed higher-order modulator can be realized just by adding some passive capacitors and switches, the additional integrator circuit composed of operation amplifier is not necessary, and the performance of the complex modulator can be effectively raised without more power dissipation. We have performed simulation with MATLAB to verify the effectiveness of the proposed architecture. The simulation results show that the proposed architecture can achieve the realization of higher-order enhancement, and improve the SQNDR of a complex bandpass $\Delta \Sigma AD$ modulator.

I. INTRODUCTION

Recently, the research for complex bandpass $\Delta\Sigma ADCs$ has become popular for their applications to RF receivers in wireless communication systems. Shifting the ADC towards the antenna side in receiver architecture relaxes the requirements placed on analog circuits at the expense of more complicated digital circuit, and allows more digital integration of analog function on a single chip, and as such results in a cheaper system with a higher level of integration. However, ADCs with high linearity, large dynamic range, bandwidth and strong image rejection capabilities are required, and a complex bandpass $\Delta \Sigma ADC$ is one of their candidates. In the RF receiver of communication systems such as cellular phones and wireless LANs, low-IF receiver architecture is frequently used so that more receiver functions, such as multi-standard and automatic gain control, can be moved to the digital part to provide more programmability. In conventional low-IF receiver architectures, two real (one input and one output) $\Delta\Sigma$ AD modulators are used for in-phase (I) and quadrature Haruo Kobayashi

Department of Electronic Engineering Graduate School of Engineering, Gunma University Tenjin-cho 1-5-1, Kiryu, Gunma, 376-8515 Japan Telephone: +81 (277) 30–1788 Fax: +81 (277) 30–1707 Email: k_haruo@el.gunma-u.ac.jp

(Q) paths. Its disadvantage is that not only input signals but also image signals are converted by ADCs. On the other hand, a complex bandpass $\Delta\Sigma AD$ modulator can provide superior performance to a pair of real bandpass $\Delta\Sigma AD$ modulators of the same order. It processes just input I and Q signals, not image signals, and AD conversion can be realized with low power dissipation. Thus, they are desirable for such low-IF receiver applications[1]-[6].

In a $\Delta\Sigma$ AD modulator, oversampling and noise-shaping techniques are used to achieve high accuracy. In order to realize higher SQNDR, higher oversampling ratio (OSR) is needed which demands higher sampling rate, and/or a highorder filter inside a modulator (and a high-order digital filter following the $\Delta\Sigma$ AD modulator) is required, which need more hardware. However, either of above techniques for higher SNQNDR will cause more power dissipation for the modulator. The best solution to the problems is at system level. By applying a complex noise-coupled structure to the front-end of internal ADCs, the order of the complex modulator will be effectively raised. The complex noise-coupled structure can be realized just by adding some passive capacitors and switches, the additional active circuits are not necessary. Therefore, it can achieve higher SQNDR with low power dissipation.

II. Complex Bandpass $\Delta\Sigma AD$ Modulator

A complex bandpass $\Delta\Sigma$ AD modulator gains its advantage by implementing the poles and zeros of its loop filter without conjugates, which are leaked in the image band for a complex single-side band signal. Fig.1 shows the signal-flow-graphics (SFG) of complex bandpass $\Delta\Sigma$ AD modulator[2][3], and Fig.2 shows its simplified structure, which is composed of a complex bandpass filter, two internal quantizers(ADCs) and two DACs. When input signal X(z), output signal Y(z) and quantizer noise $E_q(z)$ are given by complex form,

$$\begin{array}{lcl} X(z) &=& I_{in} + jQ_{in} \\ Y(z) &=& I_{out} + jQ_{out} \\ E(z) &=& E_I + jE_Q \end{array}$$

then, the transfer function of input and output of this complex modulator can be expressed as:

978-1-4244-2167-1/08/\$25.00 ©2008 IEEE



Fig. 1. SFG of complex bandpass $\Delta\Sigma AD$ modulator.



Fig. 2. Complex bandpass $\Delta\Sigma AD$ modulator structure.

$$I_{out} + jQ_{out} = (I_{in} + jQ_{in}) + \frac{1}{1 + H(z)}(E_I + jE_Q) \quad (1)$$

Here, H(z) is a complex filter transfer function, then we have signal transfer function STF(z) and noise transfer function NTF(z) as followings:

$$STF(z) = 1 \tag{2}$$

$$NTF(z) = \frac{1}{1+H(z)} \tag{3}$$

From Eq.(1), we see that complex bandpass $\Delta \Sigma AD$ modulator with two inputs and outputs of I and Q signal paths, two analog input signals are modulated in complex form, and gets two digital output signals. Quantization noise of two ADCs $E(z) = E_I + jE_Q$ is noise shaped in complex form according to NTF(z) (Eq.3) of modulator. Complex bandpass filter in the modulator has asymmetrical frequency characteristics to the axis of $\omega = 0$, which is different from a real bandpass filter. It has opposite frequency characteristics for $\omega > 0$ and $\omega < 0$, one side is signal-band (passband), the other side is image-band (attenuation band). Therefore, a complex bandpass $\Delta\Sigma AD$ modulator performs AD conversion effectively only for the positive frequency of I, Q input signals in a low-IF receiver, and hence it can be realized with lower power dissipation than a pair of real bandpass $\Delta\Sigma AD$ modulators which perform AD conversion for the negative frequency (image signal) as well as the positive frequency.

III. NOISE-COUPLED $\Delta\Sigma AD$ Modulator

Fig.3 shows the structure of a noise-coupled lowpass $\Delta\Sigma AD$ modulator[7], which is a full feedforward $\Delta\Sigma AD$



Fig. 3. Noise-coupled lowpass $\Delta\Sigma AD$ modulator.

modulator with an additional error-feedback structure of quantization noise. Notice the error-feedback structure surrounded by dotted line, we see that:

$$A(z) = Y(z) - E(z)$$

$$B(z) = E(z)$$

which means that the quantization noise E(z) is obtained by subtracting the internal ADC's input from the DAC output; after though a filter z^{-1} , a delayed replica of E(z) is fed back to the input node of ADC again[8]. While the noise transfer function of $\Delta\Sigma$ AD modulator without additional error-feedback structure is NTF(z), the transfer function of input and output of noise-coupled $\Delta\Sigma$ AD modulator shown in Fig.3 can be written as following:

$$Y(z) = X(z) + NTF'(z)E(z)$$

$$NTF'(z) = NTF(z)(1 - z^{-1})$$
(4)

From Eq.(4), we see that by providing an additional noisecoupled structure with the error-feedback topology, the NTF'(z) of $\Delta\Sigma AD$ modulator increments the NTF(z) by an extra $(1 - z^{-1})$ factor, the order of the modulator is increased by one, which is equivalent to obtaining more noise shaping in low frequency signal band.

In a noise-coupled $\Delta\Sigma AD$ modulator, the injection method of the quantization noise to modulator is similar to the cascade (or MASH) scheme, which provides a higher-order noise shaping using a lower-order loop filter. However, there is no mismatch error of the noise leakage at all. Furthermore, while multi-bit quantizer is used for the modulator, the quantization noise can be assumed under busy signal conditions. Then the injected noise also acts as merely as a dither signal, which reducing tones and harmonic spurs. Thus, the noise coupling method can raise the order of a noise transfer function, at same time, and the stability condition of the original modulator is preserved [9].

IV. PROPOSED COMPLEX BANDPASS $\Delta\Sigma$ AD MODULATOR WITH NOISE-COUPLED ARCHITECTURE

In this work, we propose a complex bandpass $\Delta\Sigma AD$ modulator with complex noise coupling, which extends the lowpass noise-coupled $\Delta\Sigma AD$ modulator to complex domain.

Fig.4 shows the SFG of proposed complex bandpass noisecoupled $\Delta\Sigma AD$ modulator. Compare with the SFG of lowpass $\Delta\Sigma AD$ modulator shown in Fig.3, we see that proposed complex modulator with two input and output signals in



Fig. 4. SFG of noise-coupled complex bandpass $\Delta\Sigma AD$ modulator.



Fig. 5. Noise-coupled complex bandpass $\Delta\Sigma AD$ modulator structure.

complex domain is almost the same to the lowpass one, except the filter at error-feedback structure is jz^{-1} . According to Eq.(4), if we can realize the jz^{-1} in the modulator, then z^{-1} factor in this equation can be replaced by jz^{-1} , so that we will get the noise transfer function by extra $(1-jz^{-1})$ factor, which has a complex notch at z=j.

Fig.5 shows the realization structure of proposed modulator with complex noise coupling. Similar to a lowpass noisecoupled modulator, proposed modulator is a conventional feedforward complex $\Delta\Sigma AD$ modulator with an additional errorfeedback structure in complex domain. Notice the complex error-feedback structure of I and Q paths surrounded by dotted line, we see that:

$$I_a = I_{out} - E_I, \quad Q_a = Q_{out} - E_Q$$
$$I_b = E_I, \qquad Q_b = E_Q.$$

Above equations mean that the quantization noise E_I and E_Q of two ADCs are obtained by subtracting the internal ADCs' input from the DACs output, respectively; after though the filter z^{-1} , delayed replica of the quantization noise E_I and E_Q are different from self-coupled lowpass modulator, they are cross-coupled to the input node of ADCQ and ADCI, but not ADCI and ADCQ, respectively. The proposed cross-coupled error-feedback structure is equivalent to the realization of j factor to the complex signals (with 90° phase-shifted), then we get following:

$$I_b + jQ_b = (-Ia + I_{out}) + j(-Qa + Q_{out})$$
$$= E_I + jE_Q$$

While the noise transfer function of original complex bandpass $\Delta \Sigma AD$ modulator without additional error-feedback

structure is NTF(z), the transfer function of input and output of complex noise-coupled $\Delta\Sigma AD$ modulator shown in Fig.5 can be written as following:

$$Y(z) = STF(z) \cdot X(z) + NTF'(z) \cdot E_q(z)$$

$$NTF'(z) = NTF(z) \cdot (1 - jz^{-1})$$
(5)

Compare the Eq.(5) to Eq.(4), we know the following:

- For a lowpass noise-coupled ΔΣAD modulator shown in Fig.3, delayed quantization noise is self-coupled to the input node of internal ADC, the NTF'(z) of ΔΣAD modulator increments the NTF(z) by an extra (1 − z⁻¹) factor, the order of noise shaping is increased by one for low frequency signal band.
- For proposed complex bandpass noise-coupled $\Delta\Sigma AD$ modulator shown in Fig.4, two delayed quantization noise of ADCI and ADCQ are cross-coupled not self-coupled to the different input node of ADCQ and ADCI with different polarities. Therefore, the NTF'(z) of proposed $\Delta\Sigma AD$ modulator increments the NTF(z) by an extra $(1-jz^{-1})$ factor, the order of noise shaping is increased by one for IF frequency signal band in complex domain.

According to above discussion, we know that the complex noise-coupled structure can be realized simply by cross coupling the two quantization noise. In the circuit implementation, the proposed structure can be realized just by adding some passive capacitors and switches, the additional complex integrator circuit is not necessary, and the performance of the complex modulator can be effectively raised without more power dissipation.

Same as lowpass noise-coupled modulator, multibit ADC/DACs are required for complex bandpass noise-coupled modulator, so that the additional noise coupling have not any damage to the stability of modulator. On the other hand, multibit DACs cannot be made perfectly linear and their nonlinearity in the feedback paths are equivalent to errors added directly to the input signals; hence, they may degrade the SQNDR of the $\Delta\Sigma$ AD modulator. However, complex dataweighted averaging (DWA) algorithm can be provided for the modulator to suppress nonlinearity effects of multibit DACs in a complex form [10].

V. SIMULATION RESULTS

We have conducted MATLAB simulations to evaluate the effectiveness of the proposed complex bandpass $\Delta\Sigma$ AD architecture with noise coupling. We made the comparison of behavioral models which are shown in Fig.2 and Fig.5. In the behavioral model of Fig.2, a second-order complex bandpass $\Delta\Sigma$ AD modulator with 3-bit internal ADC/DACs is used [5], and in the behavioral model of Fig.5, we just add the complex noise coupling structure to Fig.2.

Fig.6 shows compared simulation results of output power spectrum between behavioral model of Fig.3 and Fig.4. Around intermediate frequency (IF) input signal band of Fin=Fs/4 (F_s is sampling frequency of $\Delta\Sigma$ AD modulator),



Fig. 6. Comparison of power spectrum $(F_{in} = F_s/4)$.

the signal power of proposed modulator is the same as conventional one, but for the proposed modulator, the noise floor is lower than conventional architecture, which means that the noise power can be suppressed well in proposed modulator.

Fig.7 shows compared simulation results of SQNDR vs OSR which are calculated from above of their output power spectrum between behavioral model of Fig.3 and Fig.4. For the conventional complex bandpass $\Delta\Sigma AD$ modulator shown in Fig.2, the SQNDR increases by 15dB/Oct while OSR is increased, which shows 2nd-order characteristics of $\Delta\Sigma AD$ modulator. On the other hand, for the proposed complex bandpass $\Delta\Sigma AD$ modulator with noise coupled architecture shown in Fig.5, the SQNDR increases by 21dB/Oct while OSR is increased, which shows 3rd-order characteristics of $\Delta\Sigma AD$ modulator. It suggests that the proposed modulator realizes high order of noise shaping by complex noise-coupled architecture, it can effectively raise the order of the modulator, and suppress the noise power of interest band. Cross-coupled noise injection provides an efficient way to realize higher-order complex bandpass $\Delta\Sigma AD$ modulators. The SONDR of the proposed complex bandpass $\Delta\Sigma AD$ modulator can be higher than conventional one.

VI. CONCLUSION

We have proposed a new complex bandpass $\Delta\Sigma AD$ modulator with noise-coupled architecture. By providing the crosscoupled quantization noise injection between internal I and Q paths, complex noise coupling of two quantization noises can be realized, which effectively enhances the order of the complex modulator and achieves higher-order noise shaping. Proposed complex noise coupling structure can be realized just by adding some passive capacitors and switches. As a result, the proposed complex modulator provides a higherorder NTF using a lower-order loop filter, the additional integrator circuit which consists of an operational amplifier is not necessary, and the performance of the complex modulator can be effectively raised without more power dissipation. The



Fig. 7. Simulation results comparison of SQNDR-OSR.

MATLAB simulation results with behavioral model show that the proposed architecture can effectively raise the order of the modulator, and improve the SQNDR of a complex bandpass $\Delta\Sigma$ AD modulator.

ACKNOWLEDGMENT

The authors would like to thank Japan Science and Technology Agency (JST) which supports this research. A part of this work was performed at Gunma University Advanced Technology Research Center (ATEC) and Incubation Center.

REFERENCES

- J. Crols, M. Steyeart, "Low-IF Topologies for High-Performance Analog Front Ends of Fully Integrated Receivers," IEEE Trans. Circuits Syst. II, vol.45, no.3, pp.269-282, March 1998.
- [2] S. A. Jantzi, K. W. Martin, A. S. Sedra, "Quadrature bandpass ΣΔ modulator for digital radio," IEEE J. Solid-State Circuits, vol.32, no.12, pp.1935-1949, Dec. 1997.
- [3] K. W. Martin, "Complex Signal Processing is Not Complex," IEEE Trans. on Circuits Syst. I, vol.51, no.9, pp.1823-1836, Sep. 2004.
- [4] L. Breems, R. Rutten, R. Veldhoven and G. Weide, "A 56mW Continuous-Time Quadrature Cascaded ΣΔ Modulator With 77dB DR in a Near Zero-IF 20MHz Band," IEEE J. Solid-State Circuits, vol.42, no.12, pp.2696-2705, Dec. 2007
- [5] H. San, Y. Jingu, H. Wada, H. Hagiwara, A. Hayakawa, H. Kobayashi, T. Matsuura, K. Yahagi, J. Kudoh, H. Nakane, M. Hotta, T. Tsukada, K. Mashiko, and A. Wada, "A Second-Order Multi-bit Complex Bandpass ΔΣAD Modulator With I, Q Dynamic Matching and DWA algorithm," IEICE Trans. on Electronics, vol.E90-C, no.6, pp.1181-1188, June 2007.
- [6] H. San, Y. Jingu, H. Wada, H. Hagiwara, A. Hayasaka, J. Kudoh, K. Yahagi, T. Matsuura, H. Nakane, H. Kobayashi, M. Hotta, T. Tsukada, K.Mashiko, A.Wada, "A Multibit Complex Bandpass ΔΣAD Modulator with I, Q Dynamic Matching and DWA Algorithm," in Proc. IEEE Asian Solid-State Circuits Conf.(ASSCC), Nov. 2006, pp.55-58.
- [7] K. Lee, M. Bonu and G.C. Temes, "Noise-coupled ΔΣ ADCs," Electron. Lett., vol.42, no.24, pp.1381-1382, Nov. 2006.
- [8] R. Schreier and G.C. Temes, "Understanding Deta-Sigma Data Converters," IEEE Press, 2004.
- [9] K. Lee, J. Chae, M. Aniya, K. Hamashita, K. Takasuka, S. Takeuchi and G.C. Temes, "A Noise-Coupled Time-Interleaved ΔΣ ADC with 4.2MHz BW, -98dB THD, and 79dB SNDR," in IEEE ISSCC Dig. of Tech. Papers, Feb. 2008, pp.494-495.
- [10] H. San, H. Kobayashi, S. Kawakami, N. Kuroiwa, "A Noise-Shaping Algorithmof Multi-bit DAC Nonlinearities in Complex Bandpass ΔΣAD Modulators," IEICE Trans. on Fundamentals, vol.E87-A, no.4, pp.792-800, April 2004.