Timing Skew Compensation Technique Using Digital Filter with Novel Linear Phase Condition

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Abstract

This paper describes the timing skew compensation technique using the digital filter with our novel linear phase condition. First we describe the digital filter which can set its group delay with the arbitrary fine time resolution while it maintains the linear phase characteristics; the conventional linear phase digital filter can set its group delay with the time resolution of a half of the sampling period. We will provide its structure and operation, theoretical analysis as well as simulation verification. Next we will describe the application of our proposed digital filter to compensate for timing skew in the following cases:

(1) Sampling timing skew among channels in the timeinterleaved ADC system.

(2) I, Q-path timing skew in the single-side band (SSB) signal generator.

We show its effectiveness with simulation.

Keywords: Digital Filter, Linear Phase, Digitally-Assisted Analog Technology, Timing Skew, Digital Error Correction, ATE

1. Introduction

Fine timing skew adjustments are frequently used in Automatic Test Equipment (ATE) systems, where linear phase characteristics are desired in many cases to preserve signal waveforms in time domain. Digital techniques are preferred for the timing skew compensation because they are stable, reliable and easy to implement compared to analog techniques. However, the conventional digital filter with linear phase cannot be applied to the fine timing skew adjustment because its delay time resolution is limited.

In this paper we propose a digital filter with novel linear phase condition and show that its delay time resolution is arbitrary fine (i.e., its group delay can be set with arbitrary small time resolution). Ideally, our proposed linear phase digital filter has infinite number of taps which cannot be realized. Hence we approximate it with the finite number of taps. We observe Gibbs oscillations [1], [2] for phase as well as gain characteristics when we approximate it directly without applying a window function. However using proper window functions can eliminate these oscillations and their gain and phase characteristics are close to the ones with the ideal digital filter.

We also show the application of our proposed digital filter to compensate for timing skew in ATE systems in the following cases:

(1) Sampling timing skew among channels in the time-interleaved ADC system.

(2) I, Q-path timing skew in the single-side band (SSB) signal generator.

2. Conventional Linear Phase Condition

Linear phase characteristics are important for the digital filter to preserve the signal waveform in time domain. It is well-known in [1] that the FIR digital filter with odd or even symmetry coefficients has linear phase characteristics and it is unconditionally stable. The IIR digital filter with odd or even symmetry of both its denominator and nominator has also linear characteristics but it is unstable. Hence in almost all cases, the FIR digital filter with odd or even symmetry coefficients is used where the linear phase is required, and in such cases its group delay is (N/2)Ts where N is the number of the FIR filter taps and T_s is the sampling period; in other words the time resolution of the group delay is $T_s/2$, and this cannot be used for fine timing skew adjustment in ATE systems.

3. Novel Linear Phase Condition

In this section, we show the extended linear phase characteristics conditions for the digital filter which has not necessarily odd or even symmetry coefficients, and its time resolution of the group delay is arbitrary small.

First we discuss without consideration of causality, for simplicity. Let us consider the following analog filter (Fig.1):

$$v_{out} = \begin{cases} a_0 v_{in}(t) & \text{in case} - \pi/T_s < \omega < \pi/T_s \\ 0 & \text{otherwise.} \end{cases}$$
(1)

Then its impulse response h(t) is given as follows:

$$h(t) = a_0 T_s \operatorname{sinc}(\pi t / T_s).$$
⁽²⁾

We consider the case that the input $v_{in}(t)$ is band-limited to $-\pi/T_s < \omega < \pi/T_s$. We sample the above impulse response with a period Ts, and use the following transformation to obtain the digital filter which corresponds to the analog filter in (1):

$$T_{s} \rightarrow 1$$

$$v_{out}(nT_{s}) \rightarrow y(n) \qquad (3)$$

$$v_{in}(nT_{s}) \rightarrow x(n).$$

Then we have the following digital filter:

$$y(n) = a_0 x(n) \tag{4}$$

This is because

$$\operatorname{sinc}(\pi n) = \begin{cases} 1 & \text{in case } n = 0\\ 0 & \text{otherwise} \end{cases}$$
(5)





Fig. 1. An ideal analog low pass filter. Gain, phase characteristics, and impulse response.



Fig. 2. Sampling timing shift can maintain the linear phase characteristics. Impulse response, and gain, phase characteristics.

This digital filter has obviously linear phase characteristics (or rigorously speaking zero phase characteristics). Now let us consider to sample h(t) at $t = nT_s + \tau$ (Fig.2), where $0 < \tau < T_s$, and use (3). Then we have the following digital filter which corresponds to the analog filter in (1):

$$y(n) = \sum_{k=-\infty}^{\infty} a'_k x(n-k).$$
(6)

Here

$$a'_{k} = \operatorname{sinc}(\pi(k + \tau/T_{s})).$$
⁽⁷⁾

In general a'_k is not necessarily zero and a'_k is not necessarily equal to a'_{-k} or $-a'_{-k}$.

Proposition 1 : The digital filter given by (6), (7) has the linear filter characteristics, and its group delay is τ .

Proof : The inverse Fourier transform of (6) is given by

$$Y(j\omega) = H(j\omega)X(j\omega).$$
(8)

It follows from (7) that in case of $\tau = 0$,

$$H(j\omega)|_{\tau=0} = a_0. \tag{9}$$

Then we have the following for a given τ ($0 < \tau < T_s$):

$$H(j\omega) = a_0 e^{-j\omega\tau}.$$
 (10)

Thus the digital filter given by (6), (7) has the linear filter characteristics, and its group delay is τ (Fig.2). (Q. E. D.)

Next we discuss in case of Fig.3, and consider the following analog filter:

$$v_{out} = \begin{cases} a_0 v_{in}(t) + a_1 v_{in}(t - T_s) \\ \text{in case} - \pi/T_s < \omega < \pi/T \\ 0 \text{ otherwise.} \end{cases}$$
(11)

Note that its impulse response is given as follows:

$$h(t) = a_0 T_s \operatorname{sinc}(\pi t/T_s) + a_1 T_s \operatorname{sinc}(\pi (t/T_s - 1)).$$
(12)

We assume that the input $v_{in}(t)$ is band-limited to $-\pi/T_s < \omega < \pi/T_s$. Similarly we sample this filter with $t = nT_s$, and we have the following digital filter using (3):

$$y(n) = a_0 x(n) + a_1 x(n-1).$$
(13)

Next we sample (12) with $t = nT_s + \tau$, and we have the following digital filter:

$$y(n) = \sum_{k=-\infty}^{\infty} a'_k x(n-k).$$
⁽¹⁴⁾

Here

$$a'_{k} = a_{0} \operatorname{sinc}(\pi(k + \tau/T_{s})) + a_{1} \operatorname{sinc}(\pi((k-1) + \tau/T_{s})).$$
 (15)

Proposition 2 : The digital filter given by (14), (15) with $a_0 = a_1$ or $a_0 = -a_1$ has the linear phase characteristics and its group delay is $T_s/2 + \tau$. Also the digital filter of (14) has the same gain characteristics as (13).

Proof : We consider the case of $a_0 = a_1$. The inverse Fourier transform of (14) is given by

$$Y(j\omega) = H(j\omega)X(j\omega).$$
(16)

It follows from (15) that in case of $\tau = 0$,

$$H(j\omega)|_{\tau=0} = 2a_0 \cos(\omega T_s)e^{-j\omega T_s/2} \qquad (17)$$

Then we have the following for a given τ ($0 < \tau < T_s$):

$$H(j\omega) = 2a_0 \cos(\omega T_s) e^{-j\omega(T_s/2+\tau)}.$$
 (18)

Thus the digital filter given by (14), (15) has the linear filter characteristics, and its group delay is $(T_s/2 + \tau)$.

Also it follows from (17), (18) that

$$\left|H(j\omega)\right|_{\tau=0} = \left|H(j\omega)\right|_{0<\tau< T_s} = \left|2a_0\cos(\omega T_s)\right|.$$
(19)



Fig. 3. 2-tap FIR filter without and with sampling timing shift. (a) Impulse response. (b) Gain and phase responses.

Then the digital filter of (14) has the same gain characteristics as (13).

Similar argument is valid in case of $a_0 = -a_1$. (Q. E. D.)

The same argument holds for the 3-tap FIR filter case (Fig.4), and also in general for an N-tap FIR filter as Propositions 1, 2.

Proposition 3 : Let us consider an N-tap FIR digital filter

with coefficients a_k of odd or even symmetry.

$$y(n) = \sum_{k=0}^{N-1} a_k x(n-k)$$
 (20)

Then the following digital filter has the linear characteristics with group delay $(N/2)T_s + \tau$.

$$y(n) = \sum_{k=-\infty}^{\infty} a'_k x(n-k)$$
(21)

Here

$$a'_{k} = \sum_{l=0}^{N-1} a_{l} \operatorname{sinc}(\pi((k-l) + \tau/T_{s})).$$
(22)



Fig. 4. 3-tap FIR filter without and with sampling timing shift. (a) Without sampling time shift. (b) Gain, phase and impulse responses with sampling timing shift.

Proposition 3 can be proved similarly in Proposition 1, 2 cases.

We have performed MATLAB simulation and checked that the above digital filter with time shift τ have linear phase characteristics and have the same gain characteristics as the one without time shift for N=1, 2, and 3.

Table 1 shows the frequency characteristics of digital filters with the conventional linear phase conditions, and Table 2 shows the ones with our proposed linear phase conditions derived from the corresponding conventional linear phase digital filter.

TABLE I

FREQUENCY CHARACTERISTICS WITH CONVENTIONAL LINEAR PHASE CONDITIONS

case	N	h(n)	$H(e^{j\omega})$
1	odd	even symmetry	$e^{-j\omega \frac{N-1}{2}} \sum_{k=0}^{\frac{N-1}{2}} a_k \cos(k\omega)$
2	even	even symmetry	$e^{-j\omega \frac{N-1}{2}} \sum_{k=0}^{\frac{N}{2}} a_k \cos((k-\frac{1}{2})\omega)$
3	odd	odd symmetry	$e^{-j\omega \frac{N-1}{2}} \sum_{k=0}^{\frac{N-1}{2}} a_k \sin(k\omega)$
4	even	odd symmetry	$e^{-j\omega \frac{N-1}{2}} \sum_{k=0}^{\frac{N}{2}} a_k \sin((k-\frac{1}{2})\omega)$

TABLE II FREQUENCY CHARACTERISTICS WITH PROPOSED LINEAR PHASE CONDITIONS

case	N	h(n)	$H(e^{j\omega})$
1	odd	even symmetry	$e^{-j\omega(\frac{N-1}{2}+\frac{\tau}{T_s})}\sum_{k=0}^{\frac{N-1}{2}}a_k\cos(k\omega)$
2	even	even symmetry	$e^{-j\omega(\frac{N-1}{2}+\frac{\tau}{T_s})}\sum_{k=0}^{\frac{N}{2}}a_k\cos((k-\frac{1}{2})\omega)$
3	odd	odd symmetry	$e^{-j\omega(\frac{N-1}{2}+\frac{\tau}{T_s})}\sum_{k=0}^{\frac{N-1}{2}}a_k\sin(k\omega)$
4	even	odd symmetry	$e^{-j\omega(\frac{N-1}{2}+\frac{\tau}{T_s})}\sum_{k=0}^{\frac{N}{2}}a_k\sin((k-\frac{1}{2})\omega)$

Now we will provide the proof for our proposed linear phase digital filter in general case:

Let us consider an N-tap FIR filter with conventional linear phase condition, and we have the impulse response $\tilde{h}(t)$ with continuous time and its Fourier transform $\tilde{H}(f)$:

$$\widetilde{h}(t) = \sum_{n=0}^{N-1} h(nT_s) \delta(t - nT_s).$$
⁽²³⁾

$$\widetilde{H}(f) = H(f) \star \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta(f - \frac{k}{T_s}).$$
(24)

Here \star indicates convolution, $\delta(\cdot)$ denotes a delta function, and

$$H(f) = |H(f)|e^{-j2\pi f \frac{N-1}{2}T_s} \quad (-\frac{1}{2T_s} \le f \le \frac{1}{2T_s}).$$
(25)

When we add a delay τ to the impulse response h(t) and we have its frequency characteristics as follows:

$$H'(f) = |H(f)|e^{-j2\pi f \frac{N-1}{2}T_s} \cdot e^{-j2\pi f \tau}.$$
 (26)

We see that the phase characteristics of H'(f) is linear with respect to $f \cdot H'(f)$ can be interpreted as the convolution between H(f) and S(f), where S(f) is the ideal filter with a delay τ :

$$S(f) = e^{-j2\pi f \frac{N-1}{2}\tau} \quad (-\frac{1}{2T_s} \le f \le \frac{1}{2T_s}).$$
(27)

Thus the ideal filter $\tilde{S}(f)$ for (24) is given by

$$\widetilde{S}(f) = S(f) \star \sum_{k=-\infty}^{\infty} \delta(f - \frac{k}{T_s}) = \sum_{k=-\infty}^{\infty} S(f - \frac{k}{T_s}). \quad (28)$$

Next we will consider the effect of the delay τ to the impulse response. The inverse Fourier transform of $\tilde{s}(f)$ is given as follows:

$$\widetilde{s}(t) = \operatorname{sinc}(\frac{\pi(t-\tau)}{T_s}) \cdot \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$

$$= \sum_{n=-\infty}^{\infty} \operatorname{sinc}(\frac{\pi(t-\tau)}{T_s}) \delta(t-nT_s).$$
(29)

We see from (29) that $\tilde{s}(t)$ is asymmetric with respect to t = 0, and we have the following impulse response:

$$\widetilde{h}(t) \star \widetilde{s}(t)$$

$$= \sum_{n=0}^{N-1} h(nT_s) \delta(t - nT_s) \star \sum_{n=-\infty}^{\infty} \operatorname{sinc}(\frac{\pi(kT_s - \tau)}{T_s}) \cdot \delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} \sum_{n=0}^{N-1} h(nT_s) \operatorname{sinc}(\frac{\pi(kT_s - \tau)}{T_s}) \cdot \delta(t - (n - k)T_s). \quad (30)$$

Thus the impulse response of time delay τ with continuous time has finite values for $t \to \pm \infty$ due to the *sinc* function effects.

4. Realization Consideration

Here we sample the input signal with the sampling period Ts and then we consider the band-limited case to $-\pi/T_s < \omega < \pi/T_s$, in order to avoid the aliasing effects. In such case h(t) does not converge to zero as t becomes plus/minus infinity. So the digital filter with our novel linear phase condition has to have the infinite number of taps and this cannot be realized. (Note that in case of $\tau = 0$, $h(nT_s)$ can be zero as n becomes large which corresponds to the conventional linear phase FIR digital filter case.) So we consider to truncate the terms for large number of |k| in

(22) applying a window function and we approximate the digital filter of (21), (22) with the finite number of taps.

4.1 Approximation with Finite Number of Taps

The ideal digital filter with our proposed linear phase condition needs infinite number of taps. However it is cannot be realized, and hence we have to approximate it as the filter with finite number of taps. We consider here the effects of the truncation to the finite number of taps. We observe from our simulation results so-called Gibbs oscillation at the edges of pass-band of the gain characteristics and also phase characteristics (Fig.5) [1], [2]; Gibbs oscillation for phase characteristics is not observed in many cases, and we have found that this Gibbs oscillation for phase characteristics is due to the asymmetry of the impulse response h(n) with respect to n = 0.



Fig. 5. Gain and phase characteristics of the proposed digital filter (with time shift τ of 0.3Ts) after truncation to finite number (N=61) of filter taps with and without applying Hann window.

4.2 Applying Window Function

Next we investigate to use window functions when we approximate the ideal filter using the one with the finite number of taps. When we use a window function, the Gibbs oscillations for gain and phase are suppressed. Fig.5 shows our simulation result with time-shift τ of 0.3Ts and applying Hann window. We have also found that this Gibbs oscillation for phase can be further suppressed if we use a window function with the time-shift τ , as shown in Fig.6 where we choose the time shift τ of 0.5Ts (which affects phase characteristics significantly) and we use a Hann window.



Fig. 6. Phase characteristics of the proposed digital filter (with time shift τ of $0.5T_s$) after truncation to finite number (N = 61) of filter taps. (a) With applying Hann window of no time-shift. (b) With applying Hann window time-shifted by $\tau = 0.5T_s$

5. Digital Filter Application for Timing Skew Compensation

5.1 Interleaved ADC System

A time-interleaved ADC system is an effective way to implement a high-sampling-rate ADC with relatively slow circuits (Fig.7) [4], [5], and is widely used in ATE systems. In the ADC system, several channel ADCs operate at interleaved sampling times as if they were effectively a single ADC operating at a much higher sampling rate. However, mismatches among channel ADCs - such as offset, gain and bandwidth mismatches as well as timing skew of the clocks distributed to the channels - degrade SNDR and SFDR of the ADC system as a whole.

Here we consider the timing skew problem in the interleave ADC system. Suppose that the clocks *CK1*, *CK2*, ... *CKM* have skews $dt_1, dt_2, \cdots dt_M$ (Fig.7) [4-7]. If the input signal Vin(t) is sampled at time t+dt instead of time t, we have the sampling time error e(t) :

e(t) = Vin(t+dt) - Vin(t)

which can be approximated by

$$e(t) \rightleftharpoons [dVin(t))/dt] dt$$

This skew causes so-called pattern noise in the ADC system, and in the time domain the largest error occurs when the input signal has the largest slew rate. The timing skew effect in the time-interleaved ADC system is serious

for high frequency analog signal measurements, because its slew rate (dVin(t)/dt) becomes high.



Fig. 7. Interleaved ADC system and timing skew.

Proposed Timing Skew Compensation Method 1 :

We propose to compensate for the timing skew effects using our linear phase digital filter *directly* as shown in Fig.8 (a) in the two-channel case. We have performed MATLAB simulation and obtained the result in Fig.8 (b); we see that the spurious tone is suppressed by our proposed digital filter. However this method is only applicable for the input frequency from 0 to fs/2 where fs is the channel ADC sampling frequency; this method is NOT applicable for the input frequency of the whole interleaved ADC system and Fs=2fs in the two-channel case.

Proposed Timing Skew Compensation Method 2 :

Next we will describe a more sophisticated timing skew compensation method which also uses our proposed linear phase digital filter and is applicable for the input frequency from DC to the whole interleaved ADC sampling frequency Fs/2.

The timing skew effect in the time-interleaved ADC system is serious for high frequency analog signal measurements [4]. We present here its frequency domain compensation method based on [6], [7]. We design and apply a digital filter for the timing skew compensation so that spurious due to the timing skew is cancelled. Its principle is as follows: Let us consider the two channel case for simplicity. The output spectrum for channel 1 and 2 without mismatches are given as follows:



Fig. 8. Proposed timing skew compensation method 1. (a) Timing skew effect compensation in the 2-channel interleave ADC system with our novel linear phase digital filters. (b) Simulation results of the timing skew compensation method 1 with our digital filter.

$$X_{1}(f) = \frac{1}{2T_{s}} \sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{2T_{s}}\right)$$
(31)

$$X_{2}(f) = \frac{1}{2T_{s}} \sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{2T_{s}}\right) e^{-j\pi k}.$$
 (32)

Then we have the output spectrum of the interleaved ADC system:

$$X_{1,2}(f) = X_1(f) + X_2(f)$$

= $\frac{1}{2T_s} \sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{2T_s}\right) (1 + e^{-j\pi k}).$ (33)

Here

$$1 + e^{-j\pi k} = \begin{cases} 2, & k : even \\ 0, & k : odd \end{cases}$$
(34)

and we have the rewritten output power spectrum of (33).

$$X_{1,2}(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{2T_s}\right).$$
 (35)

Now we assume here that the timing skew between channel 1 and 2 is τ , and we have the power spectrum of the channel ADCs and also the whole interleaved ADC system:

$$X_1(f) = \frac{1}{2T_s} \sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{2T_s}\right)$$
(36)

$$X_{2}(f) = \frac{1}{2T_{s}} \sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{2T_{s}}\right) e^{-j2\pi t (f - k/(2T_{s}))} e^{-j\pi k}$$
(37)
$$X_{1,2}(f) = \frac{1}{2T_{s}} \sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{2T_{s}}\right) \left(1 + e^{-j2\pi \tau (f - k/(2T_{s}))} e^{-j\pi k}\right).$$
(38)

For 2-channel case, we have to consider only in k=0, 1, 2 cases because the signal band is from DC to 2 (fs/2), and we have the following:

$$\begin{aligned} X_{1}(f) &= \frac{1}{2T_{s}} \sum_{k=0}^{2} X\left(f - \frac{k}{2T_{s}}\right) \\ X_{2}(f) &= \frac{1}{2T_{s}} \sum_{k=0}^{2} X\left(f - \frac{k}{2T_{s}}\right) e^{-j2\pi\tau(f - k/(2T_{s}))} e^{-j\pi k} . \\ X_{1}(f) &= \frac{1}{2T_{s}} \sum_{k=0}^{2} X\left(f - \frac{k}{2T_{s}}\right) \\ &= \frac{1}{2T_{s}} \left[X(f) + X\left(f - \frac{1}{2T_{s}}\right) + \left(f - \frac{1}{T_{s}}\right) \right] \\ X_{2}(f) &= \frac{1}{2T_{s}} \sum_{k=0}^{2} X\left(f - \frac{k}{2T_{s}}\right) e^{-j2\pi\tau(f - k/(2T_{s}))} e^{-j\pi k} \\ &= \frac{1}{2T_{s}} \left[X(f) e^{-j2\pi\tau f} + X\left(f - \frac{k}{2T_{s}}\right) e^{-j2\pi\tau(f - 1/(2T_{s}))} e^{-j\pi} \\ &+ X\left(f - \frac{k}{2T_{s}}\right) e^{-j2\pi\tau(f - 1/(2T_{s}))} e^{-2j\pi} \right] \end{aligned}$$
(39)

We see from (39) that we can cancel the spurious component for k=1 by multiplying

$$H_{2}(f) = e^{j2\pi\tau(f - 1/(2T_{s}))}.$$
(40)

Then we compensate for the timing skew effect using the following filters:

$$H_1(f) = e^{-j2\pi\xi}$$

$$H_2(f) = e^{-j2\pi\xi} e^{j2\pi\tau(f-1/(2T_s))}.$$
 (41)

This compensation method can be realized with two ways:

(1) Frequency domain approach: We perform FFT to each channel ADC output signal and apply the above filter $H_1(f)$, $H_2(f)$ respectively.

(2) Time domain approach: We apply the digital filter $h_1(n)$, $h_2(n)$ for each channel ADC output which implements $H_1(f)$ and $H_2(f)$ respectively.

We have investigated their compensation accuracy and calculation complexity. These methods are effective over the input frequency range from DC up to $M \cdot (fs/2)$, where M is the number of channels and fs is the channel ADC sampling frequency; such performance was very difficult to realize with conventional methods.

We have performed simulation by applying our methods to a two-channel time-interleaved ADC system with timing skew and validated their effectiveness. Figure 9 shows $h_1(n)$, $h_2(n)$ filter characteristics used for the timing skew compensation method 2, and we see that in both cases, their group delays are constant (phases are linear) with respect to the input frequency. Figure 10 shows the simulation results and we see that spurious signals are suppressed with our proposed method 2.

5.2 SSB Signal Generation

An ATE for communication IC testing incorporates SSB signal generation function (Fig.11) [8], [9], and we consider here to generate a SSB signal with a 2-channel arbitrary waveform generator. When the timing skew between I and Q-path exists, the negative frequency component is not zero. We propose to use our linear phase digital filter to compensate for the timing skew (Fig.12). Fig.13 shows our MATLAB simulation results, and we see that the negative frequency components due to the timing skew are suppressed.

We close this section by remarking that another digital timing skew compensation technique with fine time resolution, so-called a fractional delay digital filter [10], [11], [12], [13], [14] has been proposed, which mainly focuses on the waveform interpolation and reconstruction. However our proposed technique can incorporate filtering characteristics (such as a cosine roll-off filter) as well as fine timing skew adjustment with the clear design method as described above; this is very useful in LSI testing technology applications. Furthermore, since our proposed filter is easy to design, we can obtain their coefficient values with small amount of calculation which is desirable for ATE and LSI testing technologies where real-time timing calibration is required.

6. Conclusion

We have proposed the digital filter with novel linear phase characteristics and the time resolution of its group delay is arbitrary small. Also we have shown its application for timing skew compensation in interleaved ADC systems and SSB signal generation systems. We have performed simulation to validate these results. We believe that our proposed technique will open a new research area for digital filters with linear phase and fine resolution of group delay, and give a significant impact on its application in electronic systems as digital compensation technique for timing skew and frequency characteristics, which is reliable and stable as well as suitable for fine CMOS implementation.



Fig. 9. $h_1(n)$ (shown in purple) and $h_2(n)$ (shown in blue) filter characteristics used for the timing skew compensation method 2. (Top) Impulse response. (Middle) Gain characteristics. (Bottom) Group delay characteristics. For $h_1(n)$ Impulse response is symmetric but for $h_2(n)$ it is asymmetric.

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Fig. 10. Simulation results of the timing skew compensation method 2 for the QPSK input (whose signal band is within Fs/4 - Fs/2) with applying the Blackman window function for the digital filter tap truncation. (a) Without compensation. (b) With compensation.



Fig. 11. Single-side band (SSB) signal.



Fig. 12. Timing skew compensation for the SSB signal with our linear phase digital filter.



Fig. 13. Simulation results of timing skew compensation for the SSB signal with our novel linear phase digital filter for wideband input signal. (a) Without compensation. (b) With compensation using our proposed linear phase digital filter.