

# Noise-Coupled Image Rejection Architecture of Complex Bandpass $\Delta\Sigma$ AD Modulator\*

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**SUMMARY** This paper proposes a new realization technique of image rejection function by noise-coupling architecture, which is used for a complex bandpass  $\Delta\Sigma$ AD modulator. The complex bandpass  $\Delta\Sigma$ AD modulator processes just input I and Q signals, not image signals, and the AD conversion can be realized with low power dissipation. It realizes an asymmetric noise-shaped spectra, which is desirable for such low-IF receiver applications. However, the performance of the complex bandpass  $\Delta\Sigma$ AD modulator suffers from the mismatch between internal analog I and Q paths. I/Q path mismatch causes an image signal, and the quantization noise of the mirror image band aliases into the desired signal band, which degrades the SQNDR (Signal to Quantization Noise and Distortion Ratio) of the modulator. In our proposed modulator architecture, an extra notch for image rejection is realized by noise-coupled topology. We just add some passive capacitors and switches to the modulator; the additional integrator circuit composed of an operational amplifier in the conventional image rejection realization is not necessary. Therefore, the performance of the complex modulator can be effectively raised without additional power dissipation. We have performed simulation with MATLAB to confirm the validity of the proposed architecture. The simulation results show that the proposed architecture can achieve the realization of image-rejection effectively, and improve the SQNDR of the complex bandpass  $\Delta\Sigma$ AD modulator.

**key words:** complex bandpass  $\Delta\Sigma$ AD modulator, noise-coupled architecture, image rejection

## 1. Introduction

The research for complex bandpass  $\Delta\Sigma$ ADCs has become popular for their applications to RF receivers in wireless communication systems. In the RF receiver of communication systems of cellular phones and wireless LANs, low-IF receiver architecture is frequently used so that more receiver functions, such as multi-standard and automatic gain control, can be moved to the digital part to provide more programmability. In conventional low-IF receiver architectures, two real (one input and one output)  $\Delta\Sigma$ AD modulators are used for In-phase (I) and Quadrature (Q) paths. Its disadvantage is that not only input signals but also image signals are converted by ADCs. On the other hand, a complex bandpass  $\Delta\Sigma$ AD modulator can provide superior performance to a pair of real bandpass  $\Delta\Sigma$ AD modulators of the same order. It realizes asymmetric noise-shaped spectra, processes just

input I and Q signals, not image signals, and AD conversion can be realized with low power dissipation. Thus, it is desirable for such low-IF receiver applications [1]. However, the performance of the complex bandpass  $\Delta\Sigma$ AD modulator suffers from the mismatches between internal analog I and Q paths. Mismatches between forward I and Q paths cause an image signal, and the quantization noise of the mirror image band aliases into the desired signal band, which degrades the modulator SQNDR [2]. By providing an extra notch placed at the center of the image band, the SQNDR degradation due to I and Q paths mismatch can be reduced [3], [4]. Although this image-band noise suppression works well for a high-order complex modulator, an additional integrator circuit composed of an operational amplifier is necessary, which will cause the circuit implementation complicated and more power dissipation for the modulator. We propose an image rejection architecture by applying a complex noise-coupled technique [5]. Image band noise suppression can be realized just by adding some passive capacitors and switches, and the additional active integrator circuits are not necessary. Therefore, it can achieve higher SQNDR effectively with low power dissipation.

## 2. Complex Bandpass $\Delta\Sigma$ AD Modulator

A complex bandpass  $\Delta\Sigma$ AD modulator gains its advantage by implementing the poles and zeros of its loop filter without conjugates, which are leaked in the image band for a complex single-side band signal. Figure 1 shows the signal-flow-graph (SFG) of the complex bandpass  $\Delta\Sigma$ AD modulator [6], and Fig. 2 shows its simplified structure, which is composed of a complex bandpass filter, two internal quantizers (ADCs) and two DACs. When input signal  $X(z)$ , output signal  $Y(z)$  and quantizer noise  $E_q(z)$  are given in complex form,

$$X(z) = I_{in} + jQ_{in}$$

$$Y(z) = I_{out} + jQ_{out}$$

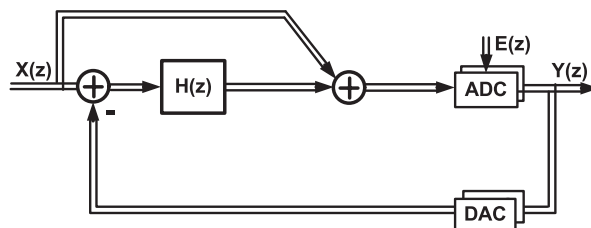


Fig. 1 SFG of complex bandpass  $\Delta\Sigma$ AD modulator.

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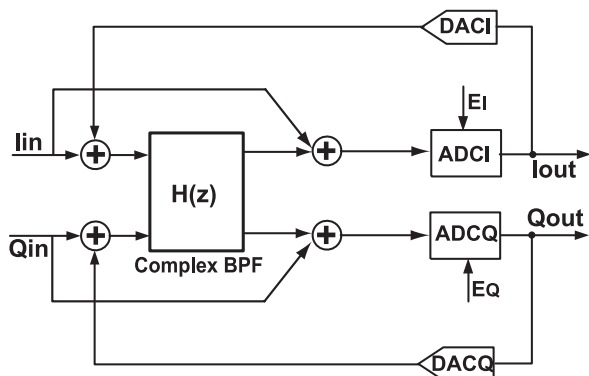


Fig. 2 Complex bandpass  $\Delta\Sigma$ AD modulator structure.

$$E(z) = E_I + jE_Q$$

then, the transfer function of input and output of this complex modulator can be expressed as:

$$I_{out} + jQ_{out} = (I_{in} + jQ_{in}) + \frac{1}{1 + H(z)}(E_I + jE_Q) \quad (1)$$

Here,  $H(z)$  is a complex filter transfer function, and then we have signal transfer function  $STF(z)$  and noise transfer function  $NTF(z)$  as follows:

$$STF(z) = 1 \quad (2)$$

$$NTF(z) = \frac{1}{1 + H(z)} \quad (3)$$

We see from Eq. (1) that the complex bandpass  $\Delta\Sigma$ AD modulator has two inputs and outputs of I and Q signal paths, two analog input signals being modulated in complex form, and getting two digital output signals. Quantization noise of two ADCs  $E(z) = E_I + jE_Q$  is noise shaped in complex form according to  $NTF(z)$  (Eq. (3)) of the modulator. Complex bandpass filter in the modulator has asymmetrical frequency characteristics to the axis of  $\omega = 0$ , which is different from a real bandpass filter. It has opposite frequency characteristics for  $\omega > 0$  and  $\omega < 0$ ; one side is signal-band (passband), the other side is image-band (attenuation band). Therefore, a complex bandpass  $\Delta\Sigma$ AD modulator performs AD conversion effectively only for the positive frequency of I, Q input signals in a low-IF receiver, and hence it can be realized with lower power dissipation than a pair of real bandpass  $\Delta\Sigma$ AD modulators which perform AD conversion for the negative frequency (image signal) as well as the positive frequency.

### 3. Proposed Complex Bandpass $\Delta\Sigma$ AD Modulator with Noise-Coupled Image Rejection

In the complex bandpass  $\Delta\Sigma$ AD modulator, the I/Q channel mismatch becomes a problem because any gain or phase imbalances cause the complex conjugate of frequency response to be aliased into the desired signal band. The image band can be all noise, and degrade the SQNDR of the modulator. In [3] and [4], an image rejection technique is proposed.

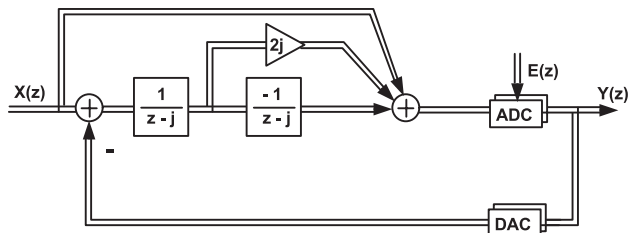


Fig. 3 SFG of the conventional 2nd-order complex bandpass  $\Delta\Sigma$ AD modulator.

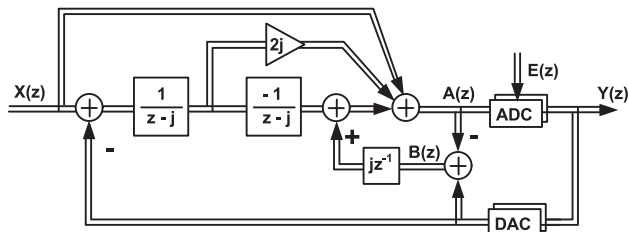


Fig. 4 SFG of the proposed complex bandpass  $\Delta\Sigma$ AD modulator with noise-coupled image rejection.

By providing the way to place an NTF zero (Notch) in the image band, less noise will be aliased into the signal band, and the influence of image signal can be suppressed. This extra complex zero is realized by an additional integrator composed of an operational amplifier, which will cause the circuit implementation more complicated and more power is necessary for the modulator. We propose a noise-coupled architecture, which realizes a notch for image rejection just by adding some passive capacitors and switches, without additional amplifier.

Figure 3 shows the SFG of the conventional 2nd-order complex bandpass  $\Delta\Sigma$ AD modulator, and Fig. 4 shows the SFG of the proposed complex bandpass  $\Delta\Sigma$ AD modulator with noise-coupled image rejection. The proposed modulator is a conventional 2nd-order complex bandpass  $\Delta\Sigma$ AD modulator with an additional complex error-feedback structure of quantization noise. This architecture is an extension of the noise-coupled time-interleaved  $\Delta\Sigma$ AD modulator [7] in complex domain. The complex modulator with two input and output signals and with an error-feedback structure through the filter of  $jz^{-1}$ . Notice the error-feedback structure surrounded by dotted line, we see that:

$$A(z) = Y(z) - E(z),$$

$$B(z) = E(z)$$

which means that the quantization noise  $E(z)$  is obtained by subtracting the internal ADC's input from the DAC output; after though a filter  $jz^{-1}$ , a delayed replica of  $E(z)$  is fed back to the input node of ADC again [8].

The noise transfer function of the conventional 2nd-order  $\Delta\Sigma$ AD modulator shown in Fig. 3 can be written as:

$$NTF(z) = (1 - jz^{-1})^2. \quad (4)$$

Then, the transfer function of input and output of the proposed complex bandpass  $\Delta\Sigma$ AD with noise-coupled image

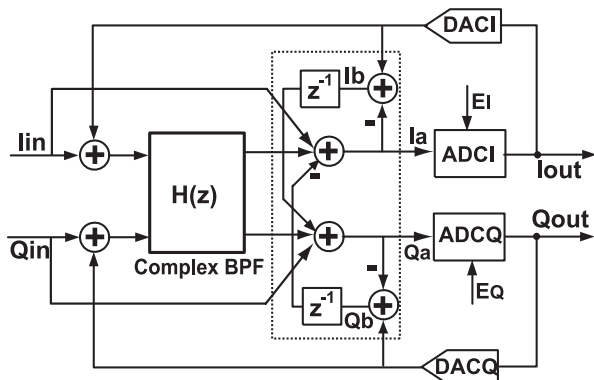


Fig. 5 Structure of the proposed complex bandpass  $\Delta\Sigma$ AD modulator with noise-coupled image rejection.

rejection shown in Fig. 4 can be written as following:

$$\begin{aligned} Y(z) &= X(z) + NTF'(z)E(z), \\ NTF'(z) &= NTF(z)(1 + jz^{-1}). \end{aligned} \quad (5)$$

According to Eq. (4), we obtain that

$$NTF'(z) = (1 - jz^{-1})^2(1 + jz^{-1}). \quad (6)$$

We see from Eqs. (5) and (6) that, by providing this additional noise-coupled structure with the error-feedback topology, the  $NTF'(z)$  of the proposed  $\Delta\Sigma$ AD modulator increased by an extra  $(1 + jz^{-1})$  factor, which has a complex zero at  $z = -j$ . Therefore, the NTF of the proposed modulator is third-order, with two notches distributed at the desired signal band and the third notch is placed at the center of image band. The third notch realized by noise-coupling degrades the effectiveness of noise-shaping somewhat, but provides the necessary image rejection for I and Q path mismatch.

Figure 5 shows the realization structure of the complex bandpass  $\Delta\Sigma$ AD modulator with noise-coupled image rejection. The modulator is a conventional feedforward complex  $\Delta\Sigma$ AD modulator with an additional error-feedback structure in complex domain. Notice the complex error-feedback structure of I and Q paths surrounded by dotted line, we see that:

$$\begin{aligned} I_a &= I_{out} - E_I, & Q_a &= Q_{out} - E_Q \\ I_b &= E_I, & Q_b &= E_Q. \end{aligned}$$

Above equations mean that the quantization noise  $E_I$  and  $E_Q$  of two ADCs are obtained by subtracting the internal ADCs' input from the DACs output, respectively; after though the filter  $z^{-1}$ , delayed replica of the quantization noise  $E_I$  and  $E_Q$  are cross-coupled to the input node of ADCQ and ADCI, but not ADCI and ADCQ, respectively. The cross-coupled error-feedback structure is equivalent to the realization of  $j$  factor to the complex signals (with  $90^\circ$  phase-shifted), then we get the followings:

$$\begin{aligned} I_b + jQ_b &= (-I_a + I_{out}) + j(-Q_a + Q_{out}) \\ &= E_I + jE_Q. \end{aligned}$$

While the noise transfer function of the original complex bandpass  $\Delta\Sigma$ AD modulator without additional error-feedback structure is  $NTF(z)$ , the transfer function of input and output of the proposed complex  $\Delta\Sigma$ AD modulator shown in Fig. 5 can be written as the same as Eq. (5).

According to above equations, we see that in the complex bandpass  $\Delta\Sigma$ AD modulator with noise-coupled image rejection shown in Fig. 4 and Fig. 5, two delayed quantization noise of ADCI and ADCQ are cross-coupled to the different input node of ADCQ and ADCI with different polarities. By providing this additional noise-coupled structure with the error-feedback topology, the  $NTF'(z)$  of the proposed  $\Delta\Sigma$ AD modulator is the increments of the  $NTF(z)$  by an extra  $(1 + jz^{-1})$  factor, which realizes the complex notch of  $z = -j$ , and the image rejection is realized simply.

In a noise-coupled  $\Delta\Sigma$ AD modulator, the injection method of the quantization noise to modulator is similar to the cascade (or MASH) scheme, which provides a higher-order noise shaping using a lower-order loop filter. However, there is no mismatch error of the noise leakage at all. Furthermore, while multi-bit quantizers are used for the modulator, the quantization noise can be assumed under busy signal conditions. Then the injected noise also acts as well as a dither signal, which reducing tones and harmonic spurs. Thus, the noise coupling method can realize the image rejection zero for a noise transfer function, at the same time, and the stability condition of the original modulator is preserved [7].

In the circuit implementation, the proposed structure can be realized only by adding some passive capacitors and switches, the additional complex integrator circuit is not necessary, and the performance of the complex modulator can be effectively raised without more power dissipation.

Same as a lowpass noise-coupled modulator [7], multi-bit ADC/DACs are required for the complex bandpass noise-coupled modulator, so that the additional noise coupling does not have any damage to the stability of modulator. On the other hand, multibit DACs cannot be made perfectly linear and their nonlinearity in the feedback paths are equivalent to errors added directly to the input signals; hence, they may degrade the SQNDR of the  $\Delta\Sigma$ AD modulator. However, data-weighted averaging (DWA) algorithm can be provided for the modulator to suppress nonlinearity effects of multibit DACs for interesting signal band [9], [10].

#### 4. Simulation Results

We have conducted MATLAB simulations to evaluate the effectiveness of the proposed complex bandpass  $\Delta\Sigma$ AD architecture with noise-coupled image rejection. We made the comparison between behavioral models of conventional modulator and proposed modulator. Figure 6 illustrates our behavioral model of the proposed complex bandpass  $\Delta\Sigma$ AD modulator with complex noise coupling image rejection structure. A second-order full-feedforward complex bandpass  $\Delta\Sigma$ AD modulator with 3-bit internal ADCs/DACs is used, and  $z^{-1}$  block shown in Fig. 6 is realized by using a

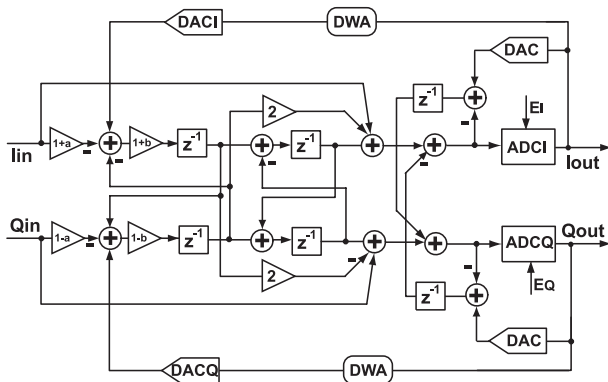


Fig. 6 Behavioral model of the proposed complex bandpass ΔΣAD modulator for MATLAB simulation.

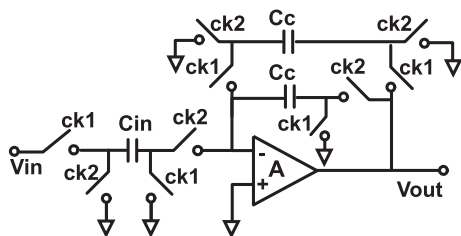


Fig. 7 z<sup>-1</sup> block realization using a switched-capacitor delay cell.

switched-capacitor delay cell which is shown in Fig. 7. We assume that capacitors mismatch between I and Q paths for  $C_{in}$ ,  $C_c$  as random value of  $a$  and  $b$  are about standard deviation of 3% respectively. Also, the capacitors matching accuracy in 3-bit DACs of the first integrator is assumed as random values with standard deviation of 0.5%. To suppress the nonlinearity of DACs in the modulator, data weighed averaging (DWA) blocks which select the unit capacitor segment as a bandpass noise-shaping algorithm [10] shown in Fig. 8 are used for both I and Q path DACs. The DWA block realizes the transfer function of  $H(z) = 1 + z^{-2}$ , and the zeros of  $H(z)$  are placed at  $z = \pm j$ , providing the notches at both of the signal and image bands. In the behavioral model of the conventional modulator, we just eliminate the noise coupling structure from the proposed modulator shown in Fig. 6. For comparison with the proposed modulator, the capacitors matching accuracy is assumed as the same as the proposed modulator, and also the same DWA blocks are used.

Figure 9 shows simulation results comparison of the output power spectrum between behavioral models of the proposed modulator shown in Fig. 6 and the conventional modulator. Around the intermediate frequency (IF) input signal band of  $f_{in} = F_s/4$  and image signal band of  $3F_s/4$ , ( $F_s$  is sampling frequency of ΔΣAD modulator), the signal power of the proposed modulator is the same as conventional one, but the noise floor of two modulators are different. For the conventional complex bandpass modulator, the noise floor around the image signal band is flat, and the noise is aliased into the signal band, so that the noise floor around

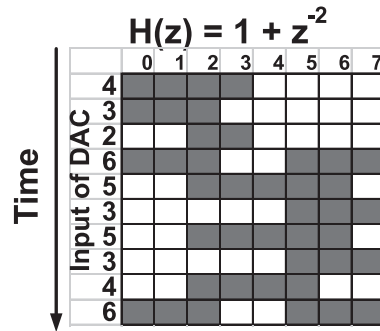


Fig. 8 Bandpass noise-shaping DWA algorithm.

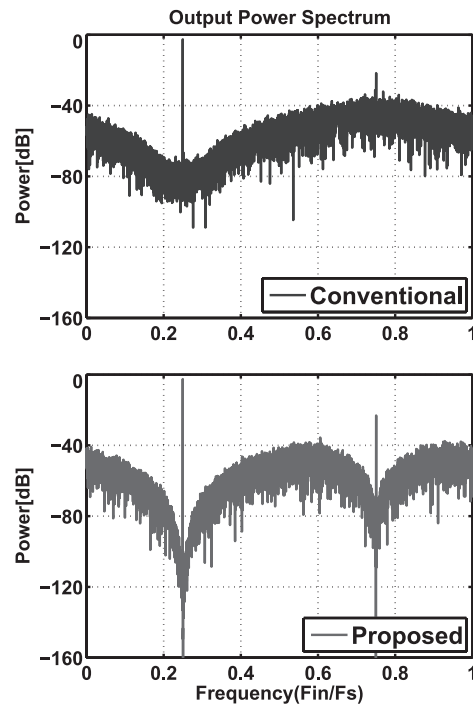


Fig. 9 Comparison of power spectrum ( $F_{in} = F_s/4$ ).

input signal band is raised, and the SQNDR is damaged. However, for the proposed modulator with noise-coupled image rejection, there is a notch placed at the image band, the noise around image band is suppressed, and the noise floor in the input signal band is lower than the conventional architecture, which means that the aliased noise power can be suppressed well in the proposed modulator.

Figure 10 shows simulation results comparison of SQNDR vs. OSR which are calculated from above of their output power spectrum between behavioral models of proposed modulator shown in Fig. 6 and the conventional modulator. For the conventional complex bandpass ΔΣAD modulator, the SQNDR is saturated as OSR is increased. On the other hand, for the proposed complex bandpass ΔΣAD modulator with noise coupled image rejection architecture shown in Fig. 6, the SQNDR increases by 15 dB/Oct as OSR is increased, which shows the 2nd-order characteristics of ΔΣAD modulator. It suggests that the proposed modula-

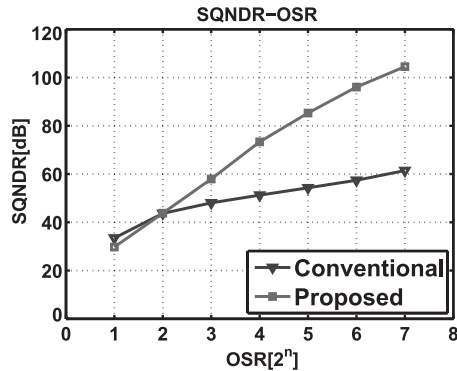


Fig. 10 Simulation results comparison of SQNDR-OSR.

tor can suppress the noise aliased into the signal band by complex noise-coupled image rejection architecture, as a result, it can effectively suppress the noise power of interest band. Cross-coupled image rejection injection provides an efficient way to realize higher-performance complex bandpass  $\Delta\Sigma$ AD modulators. The SQNDR of the proposed complex bandpass  $\Delta\Sigma$ AD modulator can be higher than the conventional one.

## 5. Conclusion

We have proposed a new complex bandpass  $\Delta\Sigma$ AD modulator with noise-coupled image rejection architecture. By providing the cross-coupled quantization noise injection between internal I and Q paths, complex noise-coupled image rejection can be realized, which effectively suppresses the noise aliased into the desired signal band. Proposed complex noise-coupled image rejection structure can be realized just by adding some passive capacitors and switches. As a result, the proposed complex modulator provides one notch of NTF using a lower-order loop filter, the additional integrator circuit which composed of an operational amplifier is not necessary, and the performance of the complex modulator can be effectively raised without more power dissipation. The MATLAB simulation results with behavioral model show that the proposed architecture can effectively improve the SQNDR of the complex bandpass  $\Delta\Sigma$ AD modulator.

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