Abstract This paper investigates applications of \( \Delta \Sigma \) control to DC-DC converters as follows: (1) We propose to use a feed-forward-type \( \Delta \Sigma \) controller in DC-DC converter and show that its output voltage ripple is smaller and the transient response is faster compared with a feedback-type \( \Delta \Sigma \) controller case. (2) We also show that a second-order \( \Delta \Sigma \) controller is superior to the first-order one in these performances. (3) As a complicated DC-DC converter application, we show that \( \Delta \Sigma \) controller can be applied to a single-inductor dual-output DC-DC converter; it is expected that its driver is simplified and the cross regulation could be better.

キーワード: DC-DC コンバータ, \( \Delta \Sigma \) 変調, フィードフォワード, 単一インダクタ多出力 (DC-DC Converter, Delta-Sigma Modulation, Feed-forward, Single-Inductor Dual-Output DC-DC Converter)

I. Introduction

A stable direct current (DC) supply is required in every electronic device, and the DC supply is usually derived from an alternating current (AC) source; it is first rectified and filtered and then is passed to the DC-DC converter where the voltage is either stepped up or stepped down. Most DC-DC converter uses PWM controller, however, recently attention is being paid to usage of \( \Delta \Sigma \) modulator (DSM) as DC-DC converter controller [1, 2], due to rapid advancement of power MOSFET devices. Expected advantages of \( \Delta \Sigma \) control over PWM control are as follows: (1) Fast transient response (2) High efficiency at low load (3) Spread spectrum of switching noise (4) Usage of smaller L and C due to higher switching frequency.

These are under investigation by many researchers in this area.

In this paper, we will present the following: (1) We investigate to use a feed-forward-type \( \Delta \Sigma \) controller in DC-DC converter for small ripple and fast response compared to a feedback-type one. (2) The second-order \( \Delta \Sigma \) is better than the first-order \( \Delta \Sigma \) in small ripple and fast response. (3) \( \Delta \Sigma \) control can be applied to a single-inductor dual-output DC-DC converter for simple driver circuit and
fast recovery from the other output load change.
We will provide some theoretical considerations and simulation results.

II Z-domain Analysis of Feedback and Feed-forward ΔΣ Modulators

A. Feedback-type ΔΣ Modulator

Fig. 1 (a) Block diagram of a feedback ΔΣ modulator.
(b) Its functional block diagram.

Fig.2 Schematics of a first-order feedback ΔΣ modulator.

Fig.1 (a) shows a block diagram of a first-order feedback ΔΣ modulator, which is composed of a low pass filter (LPF), an ADC and a DAC. Its z-domain model is demonstrated in Fig.1 (b), and its circuit is shown in Fig.2. The transfer function of the first-order ΔΣ modulator is given by

\[ Y(z) = \frac{H(z)}{1+H(z)} \cdot X(z) + \frac{1}{1+H(z)} \cdot E(z) \]

(1)
The LPF can be realized by switched capacitor circuits and its transfer function is as follows:

\[ H(z) = \frac{z^{-1}}{1-z^{-1}} \]

(2)
We obtain the following from eqs. (1), (2):

\[ Y(z) = z^{-1}X(z) + (1-z^{-1})E(z) \]

(3)
We see that the output \( Y(z) \) has the input \( X(z) \) delayed by one clock and the first-order noise-shaped \( E(z) \). The one clock delay may cause adverse effect on the transient response in DC-DC converter.

B. Feed-forward-type ΔΣ Modulator

Fig.3 (a) Block diagram of a feed-forward ΔΣ modulator.
(b) Its z-domain functional block diagram.

The block diagram of a first-order feed-forward ΔΣ modulator and its z-domain model are shown in Fig.3 [3,4], and its transfer function is given by
Y(z) = X(z) + \frac{1}{1+H(z)} \cdot E(z) \tag{4}

It follows from eqs. (2), (4), that
\[ Y(z) = X(z) + (1 - z^{-1})E(z) \tag{5} \]
Note that the output Y(z) has the input X(z) without delay (which is the advantage for DC-DC converter application) as well as the first-order noise-shaped E(z).

Fig.4 Schematics of a first-order feed-forward ΔΣ modulator.

Fig.6 shows steady state output voltage waveforms of buck converters using PWM, first-order feedback ΔΣ, first-order feed-forward-type ΔΣ and second-order feed-forward-type ΔΣ controllers. We see that the steady-state output voltage ripples of all the three types are very small. But in the field of ΔΣ controllers, the second-order feed-forward type is superior with less output ripples.

III Simulation Results
We have performed simulation using Simplisis 6.00 to validate the effectiveness of the proposed technique using Vin of 12V, Vout of 5V, and Iout from 0.5A to 1A and then 1A to 0.5A, as well as parameters in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vin</td>
<td>12V</td>
<td>R1</td>
<td>1k ohm</td>
</tr>
<tr>
<td>L</td>
<td>22uH</td>
<td>R2</td>
<td>1k ohm</td>
</tr>
<tr>
<td>C</td>
<td>220uF</td>
<td>Vref</td>
<td>2.5V</td>
</tr>
<tr>
<td>R</td>
<td>10 ohm</td>
<td>Frequency</td>
<td>1MHz</td>
</tr>
</tbody>
</table>

The input is not delayed and E(z) is second-order noise-shaped.

Fig.5 shows a second-order feed-forward ΔΣ modulator, and its transfer function is given by
\[ Y(z) = X(z) + (1 - z^{-1})^2E(z) \tag{6} \]
feed-forward $\Delta \Sigma$. At time 8ms, the output load current is changed from 0.5A to 1.0A. At time 10.5ms, the current is changed from 1.0A to 0.5A.

Fig. 6 (a) Output voltage ripple (0.1mV) in case of PWM control.

Fig. 6 (b) Output voltage ripple (1.1mV) in case of a first-order feedback $\Delta \Sigma$ control.

Fig. 6 (c) Output voltage ripple (0.6mV) in case of a first-order feed-forward $\Delta \Sigma$ control.

Fig. 6 (d) Output voltage ripple (0.23mV) in case of a second-order feed-forward $\Delta \Sigma$ control.

We see that the output voltage controlled by the second-order feed-forward $\Delta \Sigma$ reaches the steady state faster than the feedback $\Delta \Sigma$ while the PWM one is the slowest.

**Remark:** (1) Our simulations show that the feed-forward-type $\Delta \Sigma$ is better than the feedback-type $\Delta \Sigma$ with the same order regarding to the output ripple and transient response. This is because there is no delay from the input $X(z)$ to the output $Y(z)$ in the feed-forward $\Delta \Sigma$.

(2) Our simulations also show that using the second-order $\Delta \Sigma$ controller achieves better performance than the first-order one with the same-type $\Delta \Sigma$ regarding to
the output ripple and transient response. This is because in the second-order ΔΣ, second-order noise-shaping suppresses low-frequency components of E(z) significantly and the LC circuit rejects its high-frequency ones.

IV ΔΣ Control of Single-Inductor Dual-Output DC-DC Converter

We have designed ΔΣ control of a single inductor boost-boost converter [5] and performed simulation by cadence to verify its operation, in order to demonstrate application of a second-order feed-forward ΔΣ control to rather complicated DC-DC converters.

Fig. 8 Circuit schematics of a single-inductor boost-boost converter.

Table 2: Operating conditions of a single inductor boost-boost converter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vin</td>
<td>3V</td>
<td>Vref1</td>
<td>700mV</td>
</tr>
<tr>
<td>L</td>
<td>1μH</td>
<td>Vref2</td>
<td>500mV</td>
</tr>
<tr>
<td>C</td>
<td>220μF</td>
<td>Sampling</td>
<td>frequency</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5MHz</td>
<td></td>
</tr>
<tr>
<td>Rout1</td>
<td>500 Ω</td>
<td>R1,3</td>
<td>9K Ω</td>
</tr>
<tr>
<td>Rout2</td>
<td>500 Ω</td>
<td>R2,4</td>
<td>1K Ω</td>
</tr>
</tbody>
</table>

The circuit schematics and simulation parameters are shown in Fig.8 and Table 2. The simulated output waveforms are shown in Fig.9 with output voltage ripple of 0.03V.

Fig.9 Output waveforms of a single inductor boost-boost DC-DC converter controlled by ΔΣ modulator.

PWM could not work at very high switching frequency; we set its frequency as 500 kHz for PWM control and Fig.10 shows its output waveform with output ripple voltage of 0.5V, which is not acceptable.

Other advantage of the DSM control would be simple driver circuit and fast recovery from the load change of the other output.
V. Conclusion
This paper has proposed to use a feed-forward $\Delta\Sigma$ modulator as a DC-DC converter controller, and shown that it is superior to the feedback-type $\Delta\Sigma$ modulator in small output voltage ripple and fast transient response. We have shown that the second-order modulator is better than the first-order modulator in small ripple and fast response. We have demonstrated its applicability to control of a complicated converter, i.e., a single-inductor dual-output DC-DC converter.

References