Digitally-Assisted Compensation Technique for Timing Skew in ATE Systems

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Abstract

This paper describes timing skew adjustment techniques in ATE systems (such as for timing skew compensation in an interleaved ADC system and an SSB signal generation system) using a digital filter with novel linear phase condition proposed in our ITC2010 paper. A conventional linear phase digital filter is an FIR filter with coefficients of odd- or even –symmetry and whose group delay NTs/2 where N is the number of the FIR filter taps and Ts is the sampling period; its group delay time resolution is Ts/2. We have generalized the linear phase condition, and with our novel linear phase condition, the group delay time resolution can be arbitrary small, and the coefficients are not necessarily odd- or even-symmetric. In this paper we discuss several practical issues for applying our digital filter to timing skew compensation in ATE systems, such as truncation of the infinite number of taps, techniques of using window and DC gain adjustment. We also compare our digital filter with the fractional delay digital filter.

Keywords: Digital Filter, Linear Phase, Digitally-Assisted Analog Technology, Timing Skew, ATE, Fractional Delay Digital Filter

1. Introduction

In this paper we describe a digital filter with novel linear phase condition and show that its delay time resolution is arbitrary fine (i.e., its group delay can be set with arbitrary small time resolution), and its practical issues for timing skew adjustment applications in ATE systems. In section 2, conventional linear phase condition for digital filter is explained, and in section 3, our novel linear phase condition is explained based on our ITC2010 paper [1]. In section 4, we investigate realization consideration for our digital filter, and in section 5, comparison with fractional delay filter is shown. Section 6 concludes the paper.

2. Conventional Linear Phase Condition

Linear phase characteristics are important for the digital filter to preserve the signal waveform in time domain. It is well-known in [2-4] that the FIR digital filter with odd or even symmetry coefficients has linear phase characteristics and it is unconditionally stable. The IIR digital filter with odd or even symmetry of both its denominator and numerator has also linear characteristics but it is unstable. Hence in almost all cases, the FIR digital filter with odd or even symmetry coefficients is used where the linear phase is required, and in such cases its group delay is (N/2)Ts where N is the number of the FIR filter taps and T_s is the sampling period; in other words the time resolution of the group delay is $T_s/2$, and this cannot be used for fine timing skew adjustment in ATE systems.

3. Novel Linear Phase Condition

In this section, we describe - based on our ITC2010 paper [1] - the extended linear phase characteristics conditions for the digital filter which has not necessarily odd or even symmetry coefficients, and its time resolution of the group delay is arbitrary small.

First we discuss without consideration of causality, for simplicity. Let us consider the following analog filter (Fig.1):

$$v_{out} = \begin{cases} a_0 v_{in}(t) & \text{in case} -\pi/T_s < \omega < \pi/\mathrm{T_s} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Then its impulse response h(t) is given as follows:

$$h(t) = a_0 T_s \,\mathrm{sinc}(\pi t/T_s). \tag{2}$$

We consider the case that the input $v_{in}(t)$ is band-limited to $-\pi/T_s < \omega < \pi/\mathrm{T_s}$. We sample the above impulse response with a period Ts, and use the following transformation to obtain the digital filter which corresponds to the analog filter in (1):

$$\begin{aligned} &T_s \to 1 \\ &v_{out}(nT_s) \to y(n) \\ &v_{in}(nT_s) \to x(n). \end{aligned} \tag{3}$$

Then we have the following digital filter:

$$y(n) = a_0 x(n) \tag{4}$$

This is because

$$\mathrm{sinc}(\pi n) = \begin{cases} 1 & \text{in case } n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

This digital filter has obviously linear phase characteristics (or rigorously speaking zero phase characteristics).

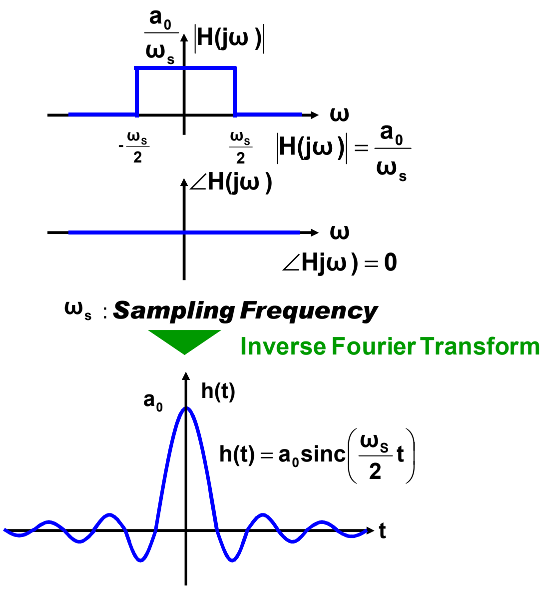

Fig. 1. An ideal analog low pass filter. Gain, phase characteristics, and impulse response.

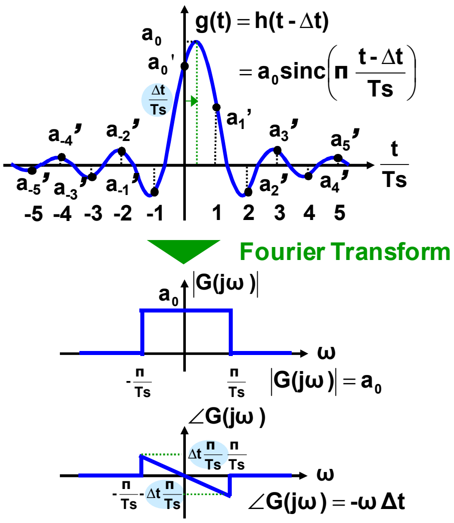

Fig. 2. Sampling timing shift can maintain the linear phase characteristics. Impulse response, and gain, phase characteristics.

Now let us consider to sample $h(t)$ at $t = nT_s + \tau$ (Fig.2), where $0 < \tau < T_s$, and use (3). characteristics). Then we have the following digital filter which corresponds to the analog filter in (1):

$$y(n) = \sum_{k=-\infty}^{\infty} a'_k x(n-k). \qquad (6)$$

Here

$$a'_k = \mathrm{sinc}(\pi(k + \tau/T_s)). \qquad (7)$$

In general a'_k is not necessarily zero and a'_k is not necessarily equal to a'_{-k} or $-a'_{-k}$.

Proposition 1 : The digital filter given by (6), (7) has the linear filter characteristics, and its group delay is τ.

Next we discuss in case of Fig.3, and consider the following analog filter:

$$v_{out} = \begin{cases} a_0 v_{in}(t) + a_1 v_{in}(t - T_s) \\ \quad \text{in case} - \pi/T_s < \omega < \pi/T \\ 0 \quad \text{otherwise.} \end{cases} \qquad (8)$$

Note that its impulse response is given as follows:

$$h(t) = a_0 T_s \mathrm{sinc}(\pi t/T_s) + a_1 T_s \mathrm{sinc}(\pi(t/T_s - 1)). \qquad (9)$$

We assume that the input $v_{in}(t)$ is band-limited to $-\pi/T_s < \omega < \pi/T_s$. Similarly we sample this filter with $t = nT_s$, and we have the following digital filter using (3):

$$y(n) = a_0 x(n) + a_1 x(n-1). \qquad (10)$$

Next we sample (9) with $t = nT_s + \tau$, and we have the following digital filter:

$$y(n) = \sum_{k=-\infty}^{\infty} a'_k x(n-k). \qquad (11)$$

Here

$$a'_k = a_0 \mathrm{sinc}(\pi(k + \tau/T_s)) + a_1 \mathrm{sinc}(\pi((k-1) + \tau/T_s)). \qquad (12)$$

Proposition 2 : The digital filter given by (11), (12) with $a_0 = a_1$ or $a_0 = -a_1$ has the linear phase characteristics and its group delay is $T_s/2 + \tau$. Also the digital filter of (11) has the same gain characteristics as (10). The same argument holds for an N-tap FIR filter.

Proposition 3 : Let us consider an N-tap FIR digital filter with coefficients a_k of odd or even symmetry.

$$y(n) = \sum_{k=0}^{N-1} a_k x(n-k) \qquad (13)$$

Then the following digital filter has the linear characteristics with group delay $(N/2)T_s + \tau$.

$$y(n) = \sum_{k=-\infty}^{\infty} a'_k x(n-k) \qquad (14)$$

Here

$$a'_k = \sum_{l=0}^{N-1} a_l \mathrm{sinc}(\pi((k-l) + \tau/T_s)). \qquad (15)$$

Table 1 shows the frequency characteristics of digital filters with our proposed linear phase condition.

TABLE I Frequency characteristics of the proposed linear phase digital filter

case	N	$h(n)$	$H(e^{j\omega})$
1	odd	even symmetry	$e^{-j\omega(\frac{N-1}{2}+\frac{\tau}{T_s})} \sum_{k=0}^{\frac{N-1}{2}} a_k \cos(k\omega)$
2	even	even symmetry	$e^{-j\omega(\frac{N-1}{2}+\frac{\tau}{T_s})} \sum_{k=0}^{\frac{N}{2}} a_k \cos((k-\frac{1}{2})\omega)$
3	odd	odd symmetry	$e^{-j\omega(\frac{N-1}{2}+\frac{\tau}{T_s})} \sum_{k=0}^{\frac{N-1}{2}} a_k \sin(k\omega)$
4	even	odd symmetry	$e^{-j\omega(\frac{N-1}{2}+\frac{\tau}{T_s})} \sum_{k=0}^{\frac{N}{2}} a_k \sin((k-\frac{1}{2})\omega)$

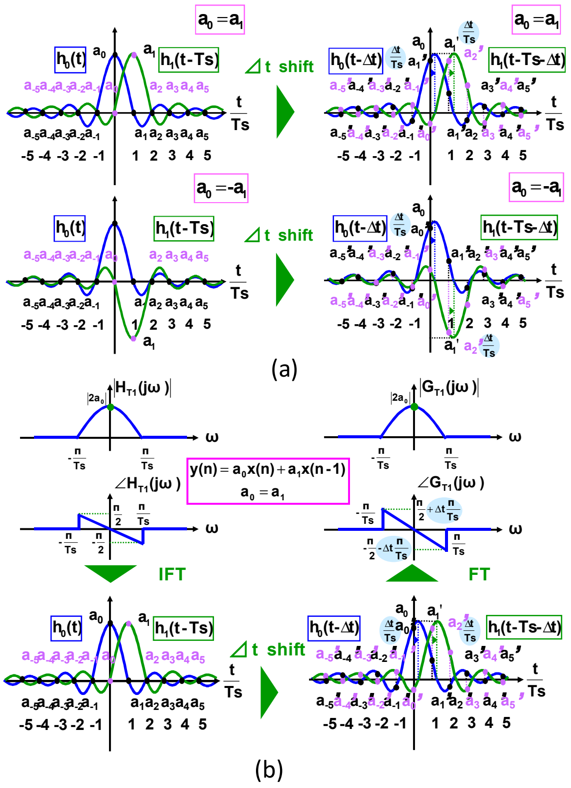

Fig. 3. 2-tap FIR filter without and with sampling timing shift. (a) Impulse response. (b) Gain and phase responses.

Now we will provide the proof for our proposed linear phase digital filter in general case: let us consider an N-tap FIR filter with conventional linear phase condition, and we have the impulse response $\tilde{h}(t)$ with continuous time and its Fourier transform $\tilde{H}(f)$:

$$\tilde{h}(t)=\sum_{n=0}^{N-1}h(nT_s)\delta(t-nT_s). \qquad (16)$$

$$\tilde{H}(f)=H(f)\star\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta(f-\frac{k}{T_s}). \qquad (17)$$

Here $\star$ indicates convolution, $\delta(\cdot)$ denotes a delta function, and

$$H(f)=|H(f)|e^{-j2\pi f\frac{N-1}{2}T_s} \quad (-\frac{1}{2T_s}\le f\le\frac{1}{2T_s}). \qquad (18)$$

When we add a delay τ to the impulse response $h(t)$ and we have its frequency characteristics as follows:

$$H'(f)=|H(f)|e^{-j2\pi f\frac{N-1}{2}T_s}\cdot e^{-j2\pi f\tau}. \qquad (19)$$

We see that the phase characteristics of $H'(f)$ is linear with respect to f. $H'(f)$ can be interpreted as the convolution between $H(f)$ and $S(f)$, where $S(f)$ is the ideal filter with a delay τ:

$$S(f)=e^{-j2\pi f\frac{N-1}{2}\tau} \quad (-\frac{1}{2T_s}\le f\le\frac{1}{2T_s}). \qquad (20)$$

Thus after the sampling operation in time domain, the ideal filter $S(f)$ in Eq.(20) leads to the following $\tilde{S}(f)$:

$$\tilde{S}(f)=S(f)\star\sum_{k=-\infty}^{\infty}\delta(f-\frac{k}{T_s})=\sum_{k=-\infty}^{\infty}S(f-\frac{k}{T_s}). \qquad (21)$$

Next we will consider the effect of the delay τ to the impulse response. The inverse Fourier transform of $\tilde{S}(f)$ is given as follows:

$$\begin{aligned}\tilde{s}(t)&=\mathrm{sinc}(\frac{\pi(t-\tau)}{T_s})\cdot\sum_{n=-\infty}^{\infty}\delta(t-nT_s)\\&=\sum_{n=-\infty}^{\infty}\mathrm{sinc}(\frac{\pi(t-\tau)}{T_s})\delta(t-nT_s).\end{aligned} \qquad (22)$$

We see from (22) that $\tilde{s}(t)$ is asymmetric with respect to $t=0$, and we have the following impulse response:

$$\begin{aligned}&\tilde{h}(t)\star\tilde{s}(t)\\&=\sum_{n=0}^{N-1}h(nT_s)\delta(t-nT_s)\star\sum_{n=-\infty}^{\infty}\mathrm{sinc}(\frac{\pi(kT_s-\tau)}{T_s})\cdot\delta(t-nT_s)\\&=\sum_{n=-\infty}^{\infty}\sum_{n=0}^{N-1}h(nT_s)\mathrm{sinc}(\frac{\pi(kT_s-\tau)}{T_s})\cdot\delta(t-(n-k)T_s). \qquad (23)\end{aligned}$$

Thus the impulse response of time delay τ with continuous time has finite values for $t\to\pm\infty$ due to the *sinc* function effects.

4. Realization Consideration

We sample the input signal with the sampling period Ts and then we consider the band-limited case to $-\pi/T_s<\omega<\pi/T_s$, in order to avoid the aliasing effects. In such case $h(t)$ does not converge to zero as t becomes plus/minus infinity. So the digital filter with our novel linear phase condition has to have the infinite number of taps and this cannot be realized. (Note that in case of $\tau=0$, $h(nT_s)$ can be zero as n becomes large which corresponds to the conventional linear phase FIR digital filter case.) So we consider to truncate the terms for large number of $|k|$ in (15) applying a window function and we approximate the digital filter of (14), (15) with the finite number of taps. We also consider here the DC gain adjustment.

4.1 Approximation with Finite Number of Taps

The ideal digital filter with our proposed linear phase condition needs infinite number of taps. However it is cannot be realized, and hence we have to approximate it as the filter with finite number of taps. We consider here the effects of the truncation to the finite number of taps. We observe from our simulation results so-called Gibbs oscillation at the edges of pass-band of the gain characteristics and also phase characteristics (Fig.4) [2], [3]; Gibbs oscillation for phase characteristics is not observed in many cases, and we have found that this Gibbs oscillation for phase characteristics is due to the asymmetry of the impulse response $h(n)$ with respect to $n=0$.

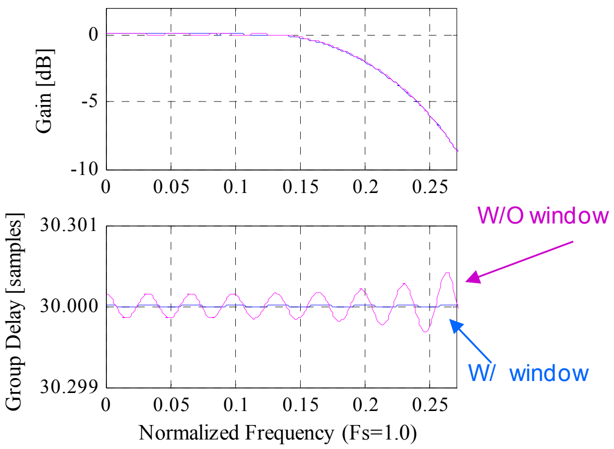

Fig. 4. Gain and phase characteristics of the proposed digital filter (with time shift τ of 0.3Ts) after truncation to finite number (N=61) of filter taps with and without applying Hann window.

4.2 Applying Window Function

Next we investigate to use window functions when we approximate the ideal filter using the one with the finite number of taps. When we use a window function, the Gibbs oscillations for gain and phase are suppressed. Fig.5 shows our simulation result with time-shift τ of 0.3Ts and applying Hann window. We have also found that this Gibbs oscillation for phase can be further suppressed if we use a window function with the time-shift τ, as shown in Fig.5 where we choose the time shift τ of 0.5Ts (which affects phase characteristics significantly) and we use a Hann window.

There can be two methods for applying a window: one is to use the window with symmetry to the Y-axis (Fig.5 (a)) and the other is to use the window with the symmetry to the center of the impulse response (Fig.5 (b)). We have performed simulation and found that the one in Fig.5 (b) is better. The Gibbs oscillation of the group delay is suppressed when the window of time-shift is used for the LPF (Fig.6).

Our proposed linear phase filter is also applicable to a bandpass filter and Fig.7 shows the group delay of the bandpass filter with the bandwidth of 0.1 fs – 0.4fs. We see that the group delay is almost constant in the wider range when the window of time-shift is used (Fig.7 (b)).

4.3. DC Gain Adjustment

The DC gain of our digital filter can be changed by truncation to the finite number of the taps after windowing, and we have to adjust it for the practical use. The DC gain adjustment technique can be described as follows:

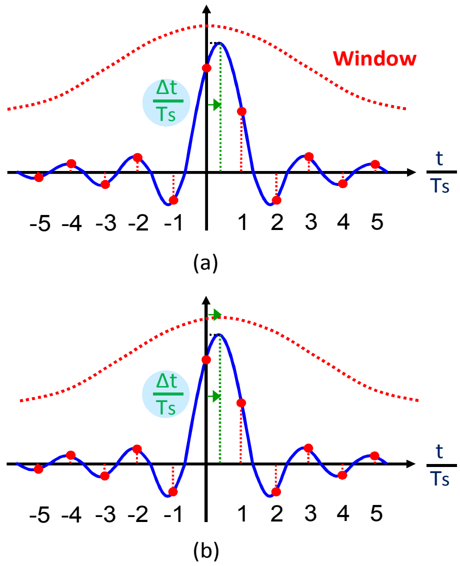

Fig.5. (a) Window with symmetry to the Y-axis (window is not time-shifted). (b) Window with symmetry to the center of the impulse response (window is time-shifted).

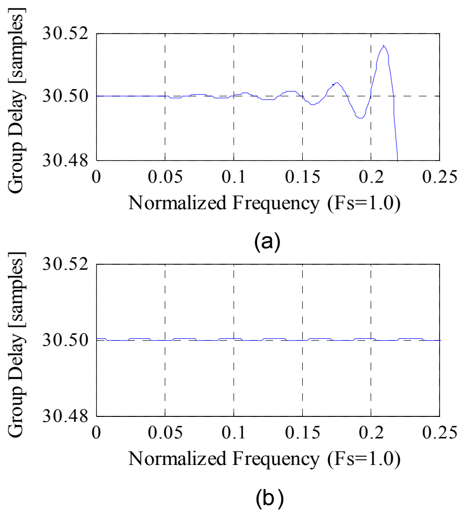

Fig.6. Group delay characteristics of the proposed digital filter (with time shift τ of $0.5T_s$) after truncation to finite number (N = 61) of filter taps. (a) With applying Hann window of no time-shift. (b) With applying Hann window time-shifted by $\tau = 0.5T_s$

Our digital filter without DC gain adjustment

$$g(n) = h(n) \text{ where } n = 0, \pm 1, \pm 2, \pm 3, \ldots \pm N$$

Our digital filter with DC gain adjustment

$$g'(n) = (G_{ideal} / G_{fnt})\, h(n)$$

$$\text{where } n = 0, \pm 1, \pm 2, \pm 3, \ldots \pm N$$

Here DC gain of the ideal filter is given by

$$G_{ideal} = \sum_{n=-\infty}^{\infty} h(n)$$

Also DC gain of the filter after truncation of the finite number (2N+1) taps is given by

$$\text{Gfnt} = \sum_{n=-N}^{N} h'(n)$$

We have performed simulation to demonstrate the effectiveness of the window with time-shift and DC gain adjustment in the single-side band (SSB) signal generation system in Fig,8. We assume that there is timing skew τ in I-path and we use our timing skew compensation digital filter in Q-path. Fig. 11 (a) shows the power spectrum of the output s(t) without timing skew, and Fig.9 (b) shows the one with timing skew τ where spurious components are observed.

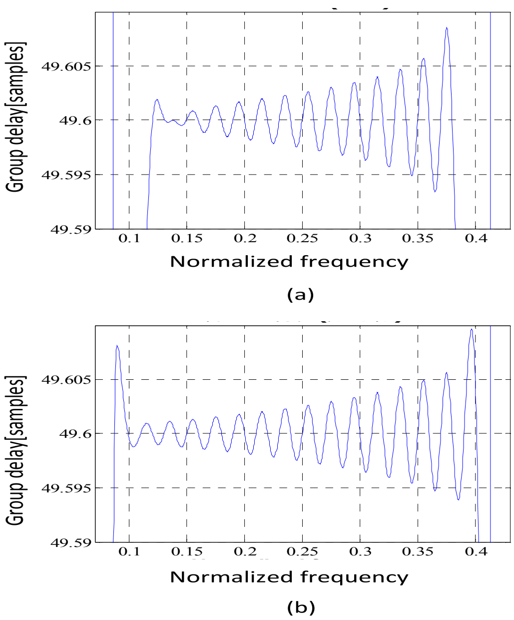

Fig.7.Group delay of bandpass filter with the bandwidth of 0.1fs – 0.4fs. (a) With applying Hann window of no-time shift. (b) With applying Hann window of time-shift.

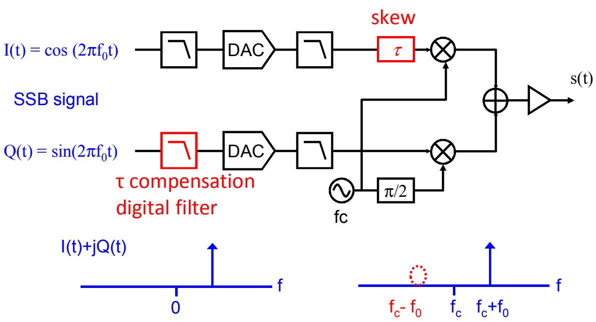

Fig. 8. SSB signal generation system with timing skew τ in I-path and the timing skew compensation digital filter in Q-path.

Fig.10 shows the simulation result using timing skew compensation with our proposed digital filter. Fig.10 (a) is the case that the window of no-time shift are used and DC gain adjustment is not used while Fig.10 (b) is the case that the window of time-shift and DC gain adjustment are used. We see that spurious components are further suppressed when the time-shifted window and DC gain adjustment are used.

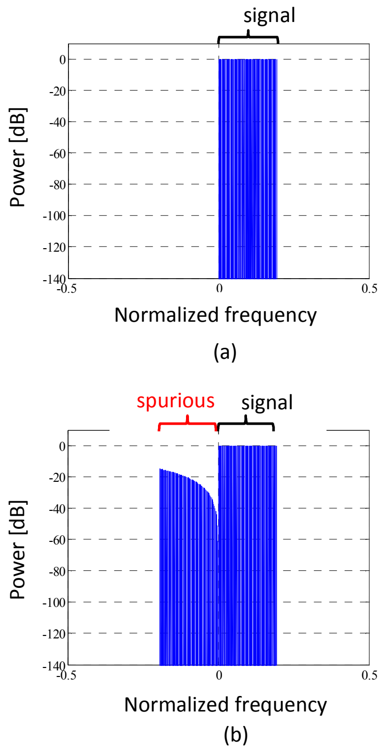

Fig.9. Simulation results of output power spectrum of the SSB signal generation system in Fig.8. (a) Without timing skew. (b) With timing skew τ .

5. Comparison with Fractional Delay Filter

We would like to call the reader's attention that another digital filter with fine time resolution, so-called a fractional delay digital filter has been proposed [5-8], which mainly focuses on the waveform interpolation and reconstruction. However our proposed technique can incorporate filtering characteristics (such as a cosine roll-off filter, a Gaussian filter) as well as fine timing skew adjustment with the clear design method as described above; this is very useful in some electronic manufacturing equipment applications [1]. Furthermore, since our proposed filter is easy to design, we can obtain their coefficient values with small amount of calculation which is desirable for many applications, especially ATE systems where real-time timing calibration is required.

We have performed Matlab simulation and found that our proposed filter can apply for the signal up to the frequency close to the Nyquist rate (in other words, the bandwidth of our proposed filter is close to the Nyquist rate while that of the fractional delay filter is not); this is another advantage of our proposed filter (Fig.11).

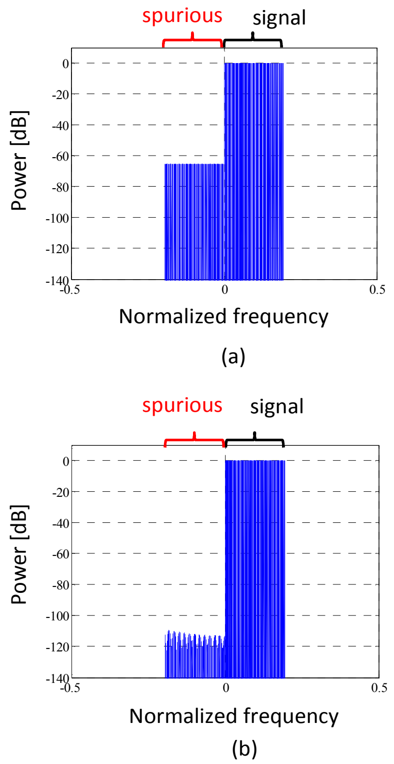

Fig.10. Simulation results of timing skew compensation with our proposed digital filter. (a) With the window of no time-shit and without DC gain adjustment. (b) With the window of time-shift and with DC gain adjustment.

6. Conclusion

We have described the digital filter with novel linear phase characteristics and the time resolution of its group delay is arbitrary small. We have investigated the truncation effects to the finite number of its filter taps, techniques of using window and DC gain adjustment as well as comparison with fractional delay filter. We believe that our proposed digital filter is opening a new research area for digital filters with linear phase and fine resolution of group delay, as well as its applications.

7. Acknowledgement

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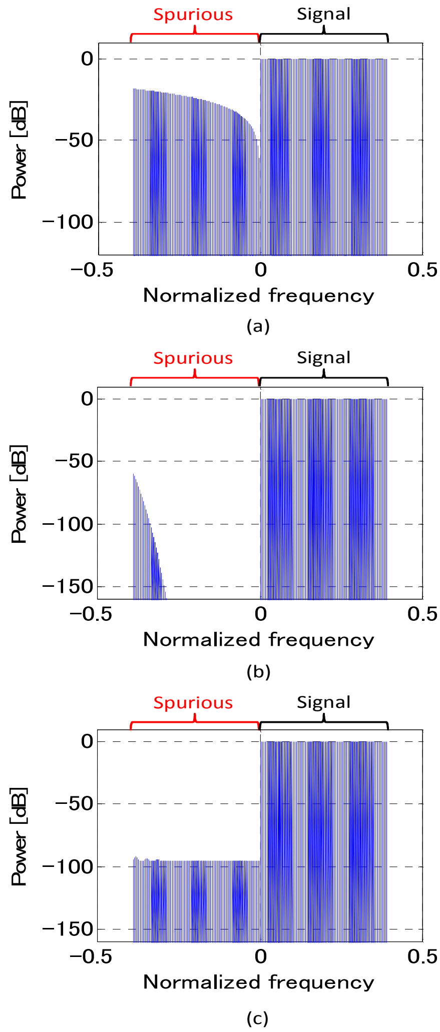

Fig.11: SSB signal power spectrum Matlab simulation results. (a) Without compensation. (b) With compensation using a 301-tap fractional delay filter and a (symmetric) blackman window. (c) With compensation using our proposed 301-tap digital filter and the same blackman window.

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