

# Multi-bit Sigma-Delta TDC Architecture with Self-Calibration

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# Outline

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- ▶ **Research Objective**
- ▶ **Single-Bit & Multi-bit  $\Sigma\Delta$  TDCs**
- ▶ **Multi-Bit  $\Sigma\Delta$  TDC with DWA**
- ▶ **Multi-Bit  $\Sigma\Delta$  TDC with Self-Calibration**
- ▶ **Conclusion**

# Outline

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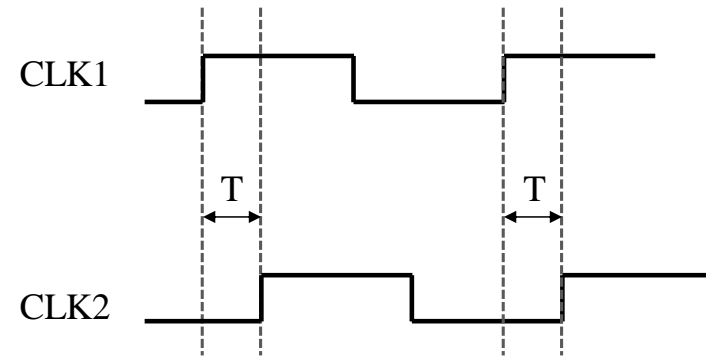
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# Research Purpose

- Testing timing difference between two repetitive digital signals

Ex.

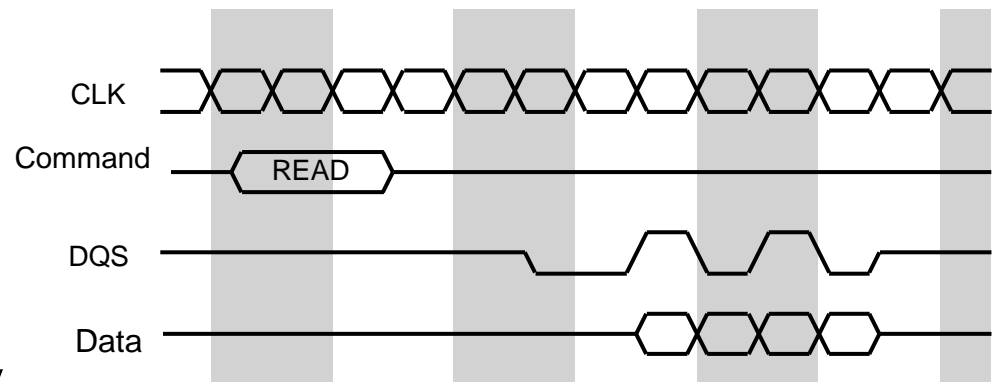
Data and clock  
in Double Data Rate (DDR) memory



- Short testing time
- Good accuracy




Implement with small circuitry



# Our Work

## Focus on Multi-bit $\Delta\Sigma$ Time-to-Digital Converter (TDC)

- Repetitive digital signals  
  $\Sigma\Delta$  TDC can be used
- Simple circuit
- Fine resolution
- Testing time
 

Single-bit $\Sigma\Delta$ TDC	Long
Multi-bit $\Sigma\Delta$ TDC	Short
- Linearity
 

Single-bit $\Sigma\Delta$ TDC	Good
Multi-bit $\Sigma\Delta$ TDC	Bad

due to delay elements mismatches



Two methods for their compensation

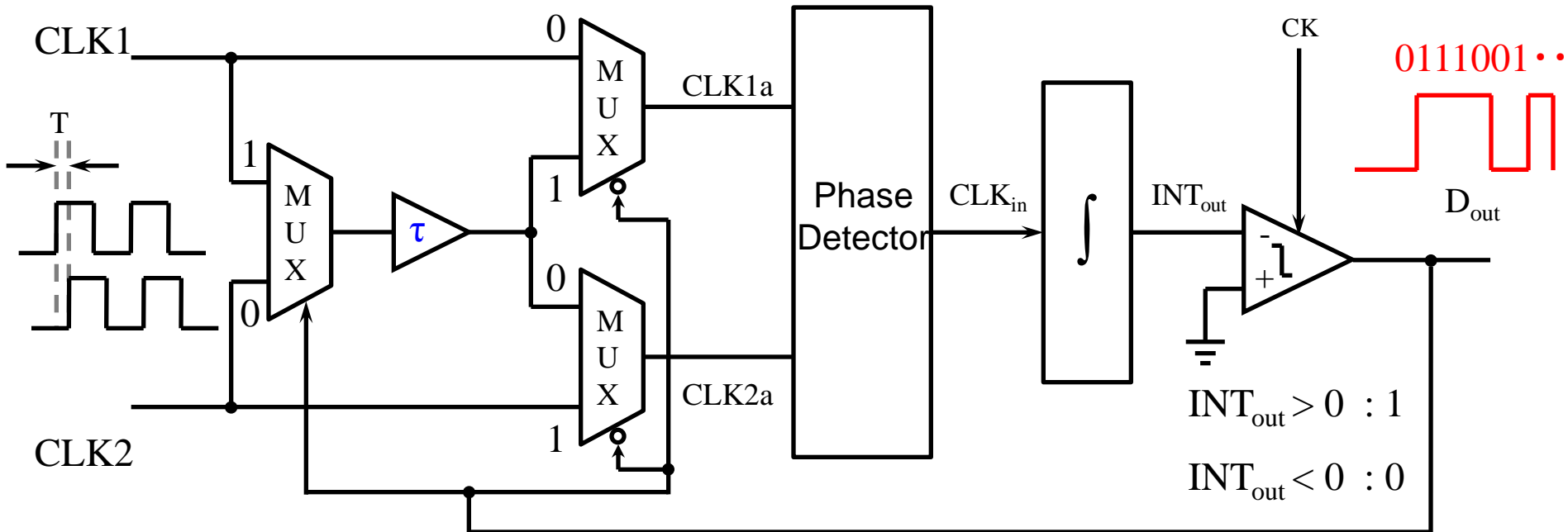
↳ DWA & Self-calibration

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# Single-Bit $\Sigma\Delta$ TDC

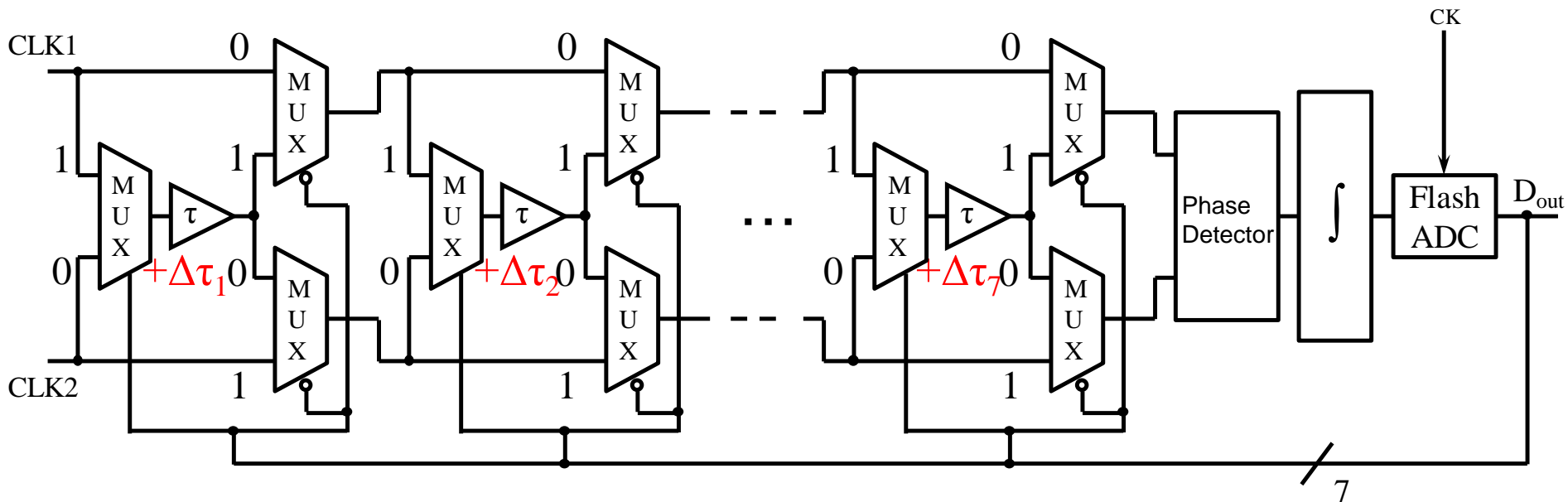


- Measurement of timing  $T$  between repetitive CLK1 and CLK2
- Number of 1's at  $D_{out}$  is proportional to  $T$
- Time resolution becomes finer as measurement time becomes longer

Note:  $\tau$  is not time resolution, but time measurement full range

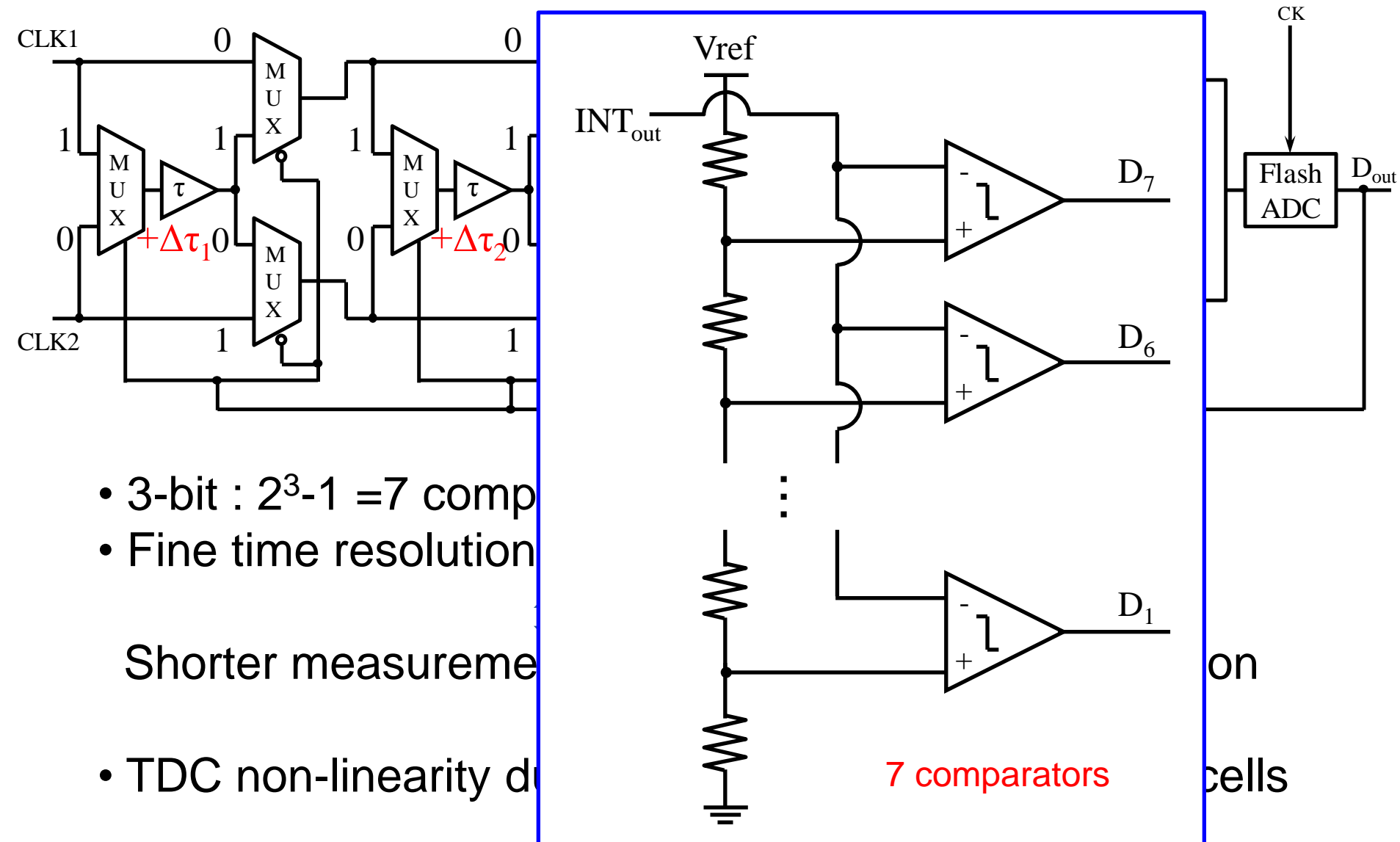


# Multi-Bit $\Sigma\Delta$ TDC



- 3-bit :  $2^3 - 1 = 7$  comparators and delays
  - Fine time resolution with a given measurement time
- ↕
- Shorter measurement time with a given time resolution
- TDC non-linearity due to mismatches among delay cells.

# Multi-Bit $\Sigma\Delta$ TDC

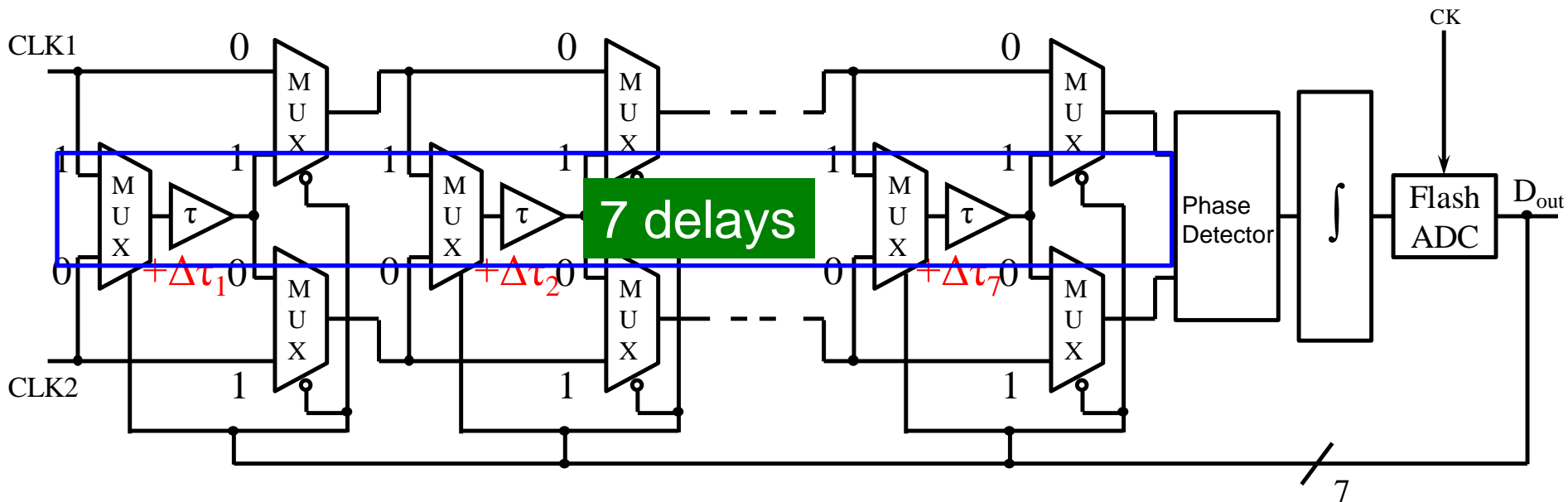


- 3-bit :  $2^3 - 1 = 7$  comp
- Fine time resolution

Shorter measureme

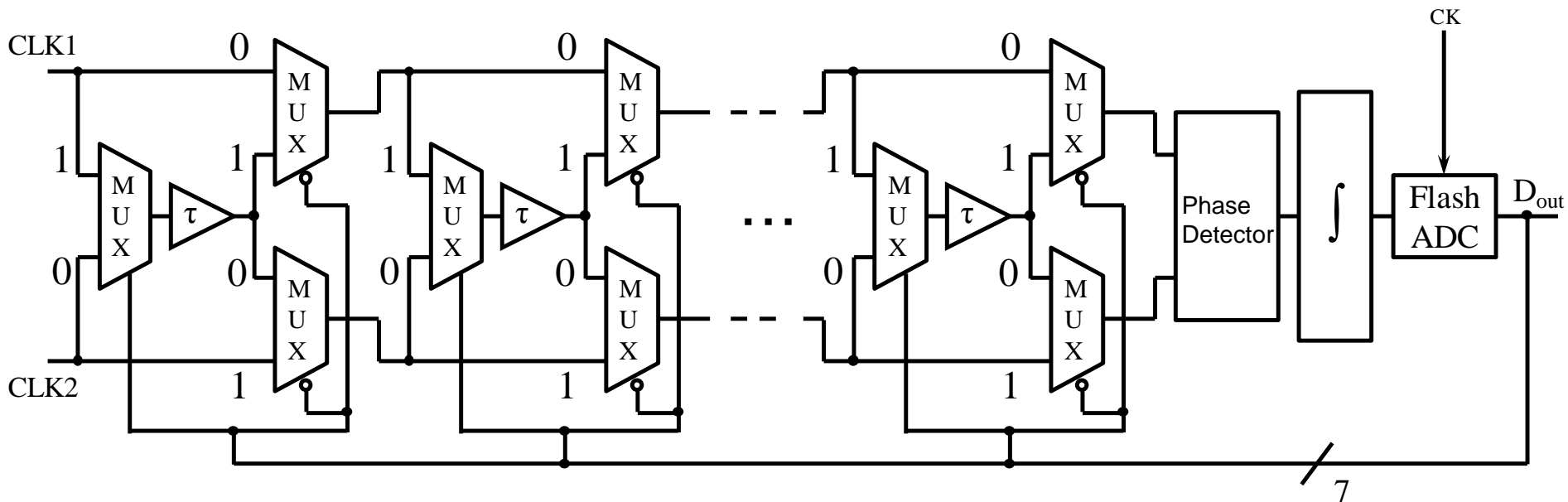
- TDC non-linearity d

# Multi-Bit $\Sigma\Delta$ TDC



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# Multi-Bit $\Sigma\Delta$ TDC



- 3-bit :  $2^3 - 1 = 7$  comparators and delays
- Fine time resolution with a given measurement time



Shorter measurement time with a given time resolution

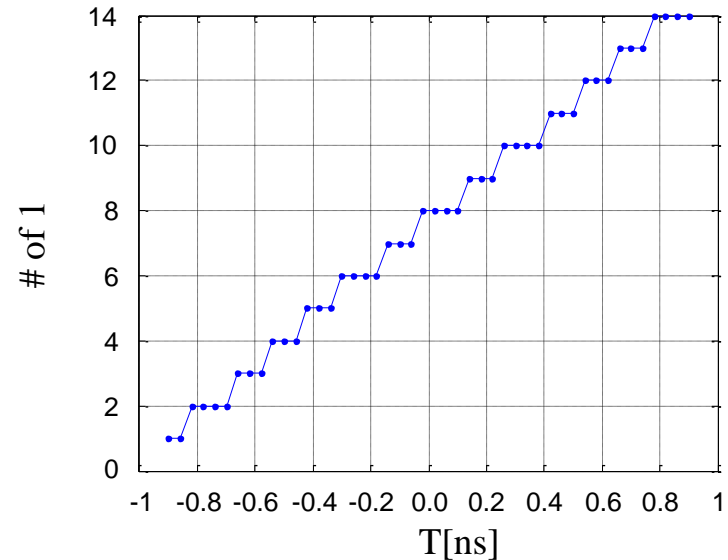
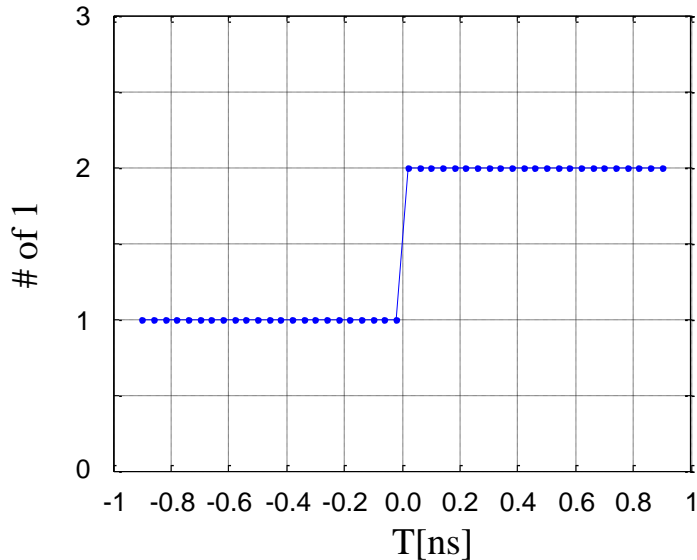
- TDC non-linearity due to mismatches among delay cells

# Difference in Measurement Time

## ● Simulation conditions

	1-bit $\Sigma\Delta$ TDC	3-bit $\Sigma\Delta$ TDC
Rising timing edge difference (T)	-0.9 ~ 0.9[ns] (Resolution : 0.04[ns])	-0.9 ~ 0.9[ns] (Resolution : 0.04[ns])
Delay time ( $\tau$ )	1[ns]	0.145[ns]
The number of digital outputs	2	2

## ■ A rising number of outputs for the interval T

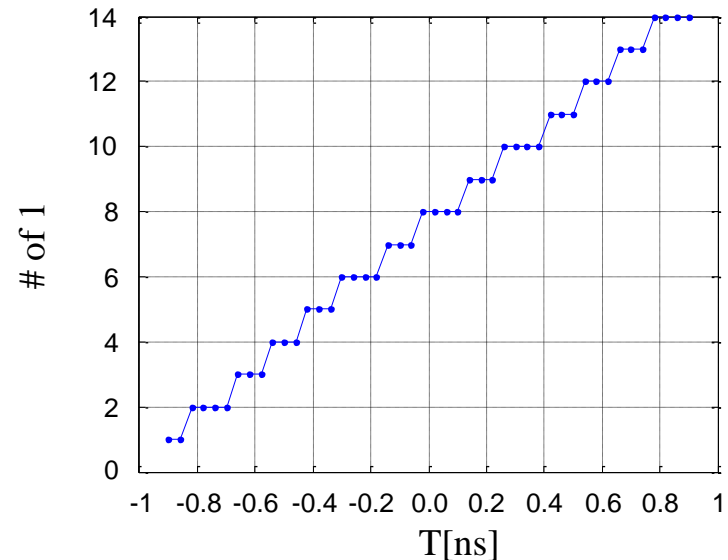
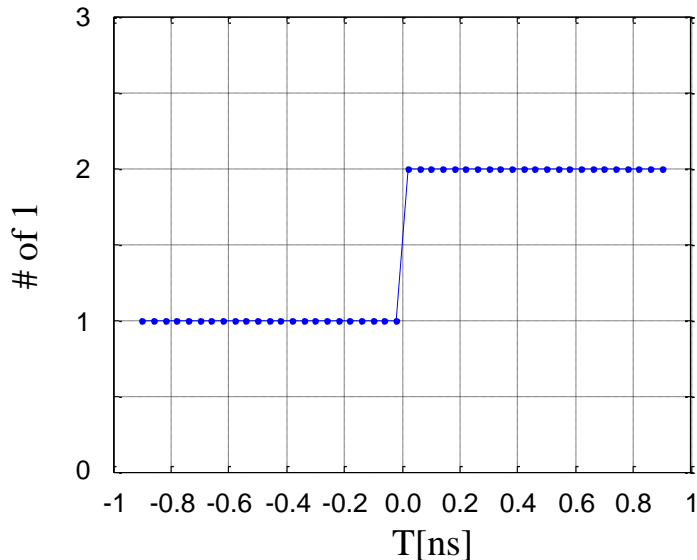


# Difference in Measurement Time

✓ Multi-bit takes short measurement time for a given time resolution



■ A rising number of outputs for the interval T

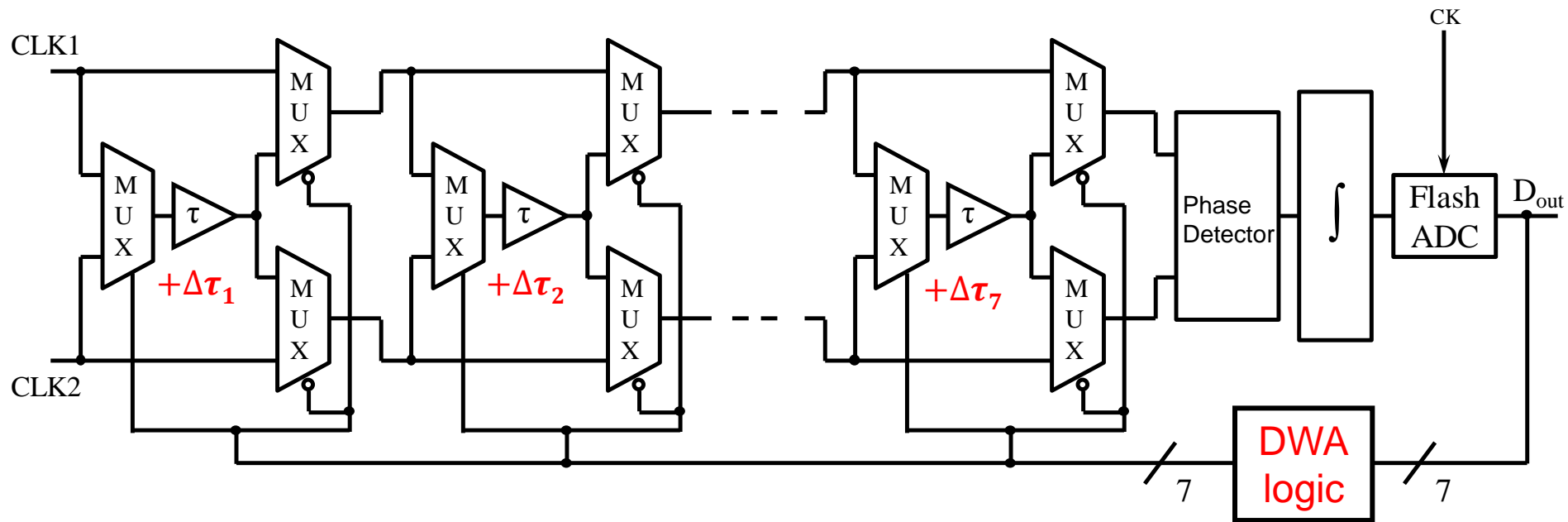


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- ▶ Single-Bit & Multi-bit  $\Sigma\Delta$  TDCs
- ▶ **Multi-Bit  $\Sigma\Delta$  TDC with DWA**
- ▶ Multi-Bit  $\Sigma\Delta$  TDC with Self-Calibration
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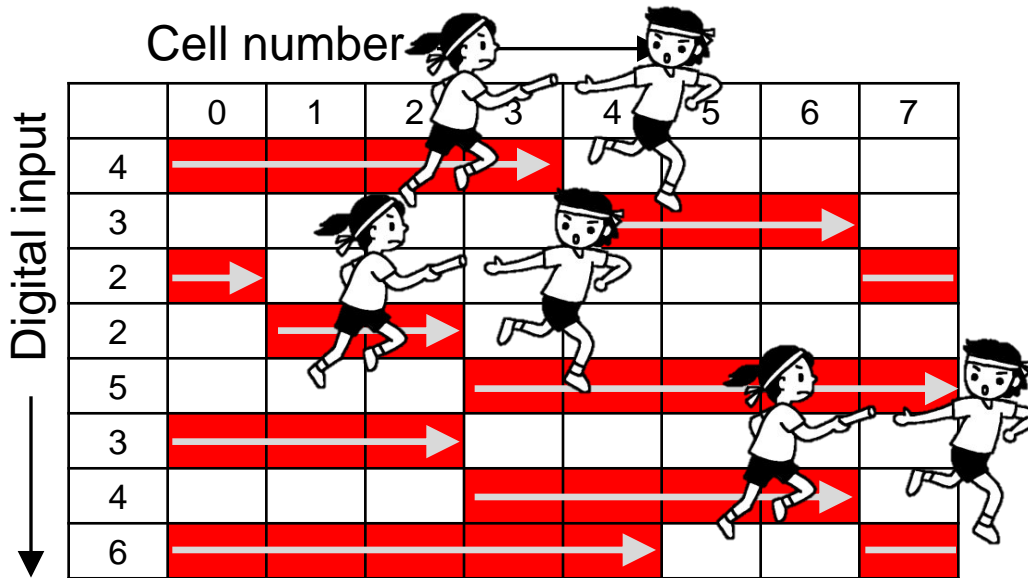
# DWA (Data Weighted Averaging)



- Flash ADC outputs
  - ➔ shuffled by **DWA logic**, fed into MUXs as **select** signals
- Delay mismatch effects
  - ➔ moved to high-frequency (**noise-shaping**)

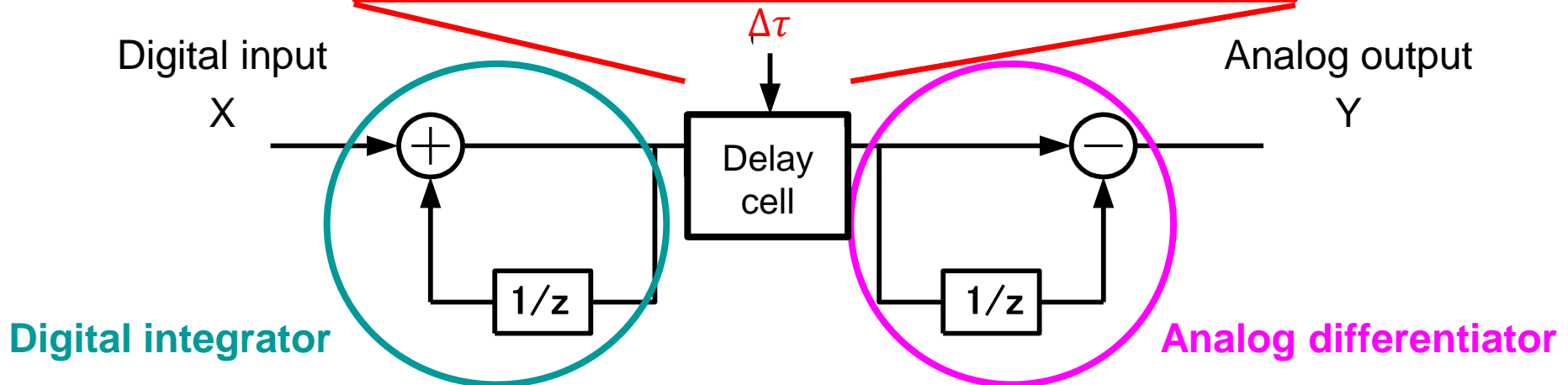
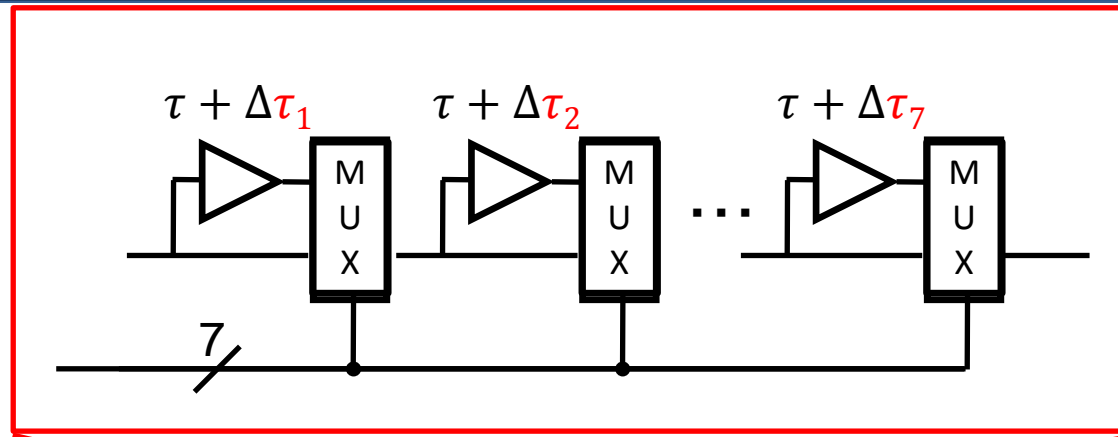


# DWA Operation



Pass a baton in relay race !

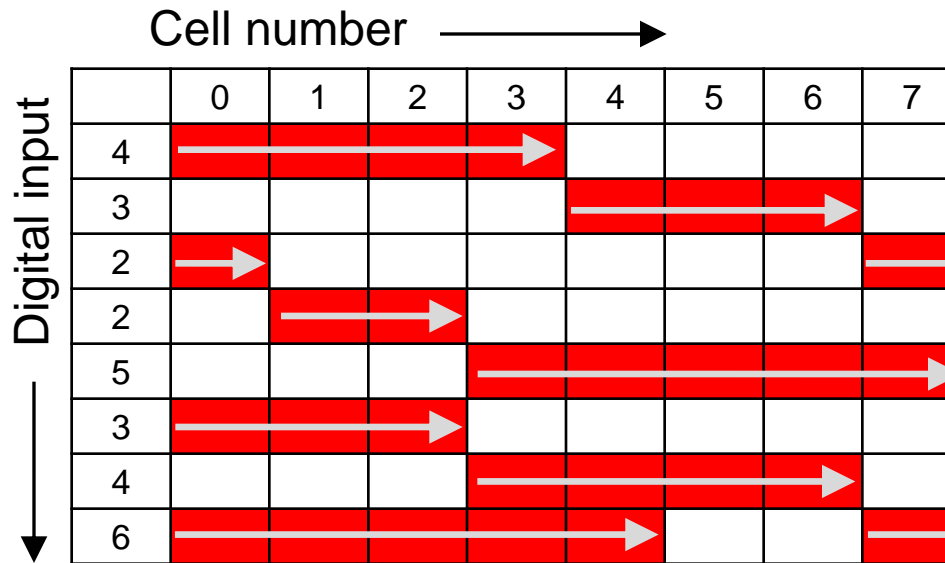
# Noise-Shaping



$$Y(z) = X(z) + \underline{(1 - 1/Z)\Delta\tau(z)}$$

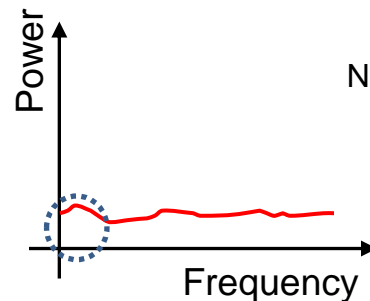
Delay mismatch  $\Delta\tau$  is **first-order noise-shaping**

# DWA & Noise Shaping



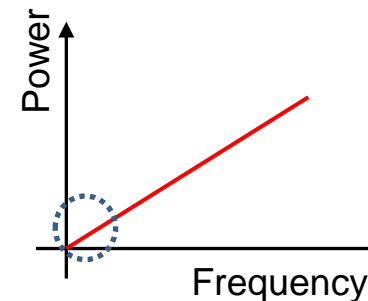
- Delay  $\tau$  : integration & differentiation
- Delay mismatch  $\Delta\tau$  : differentiation

delay cell mismatch effects

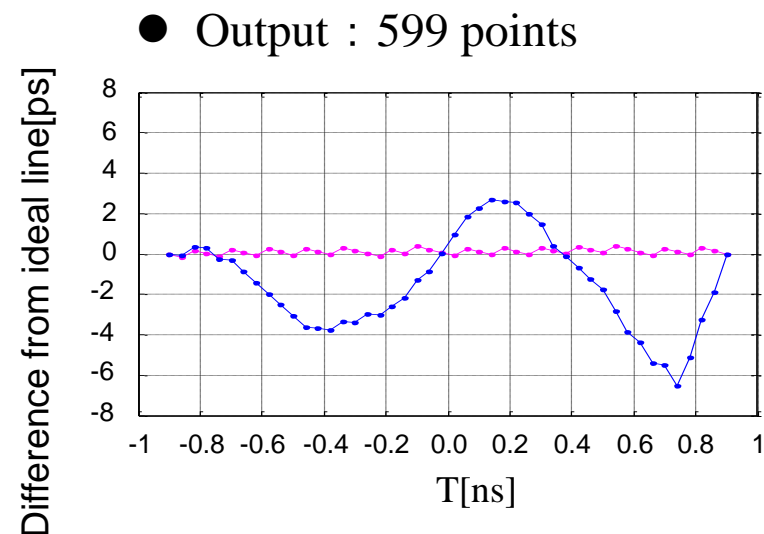
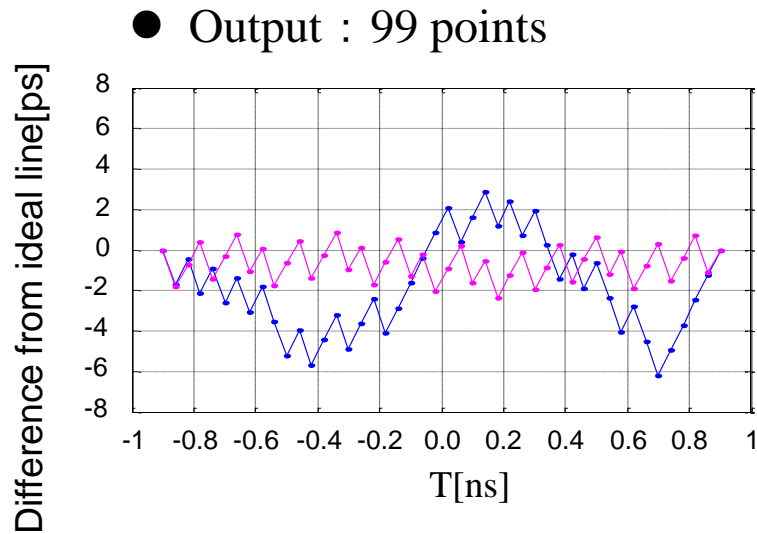


delay cell mismatch effects

Noise Shape  $\longrightarrow$



# Simulation of $\Delta\Sigma$ TDC with DWA

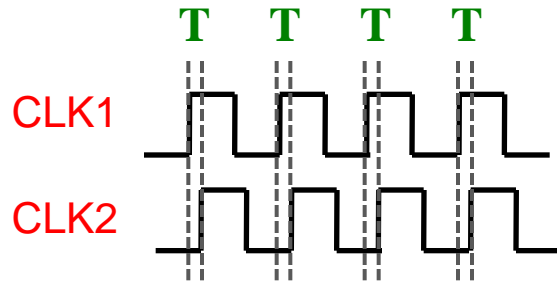


—●—  $\Delta\Sigma$  TDC(with DWA)  
—●—  $\Delta\Sigma$  TDC(without DWA)

✓ Reduce the effect of delay mismatches

$\Sigma\Delta$  TDC linearity is improved

# DWA Effect



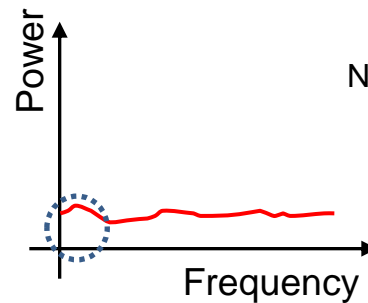
Measure **T**



**T** is DC signal.

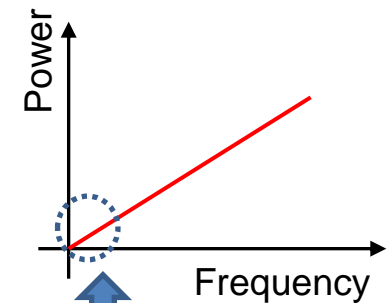
- Delay  $\tau$  : integration & differentiation
- Delay mismatch  $\Delta\tau$  : differentiation

delay cell mismatch effects



delay cell mismatch effects

Noise Shape



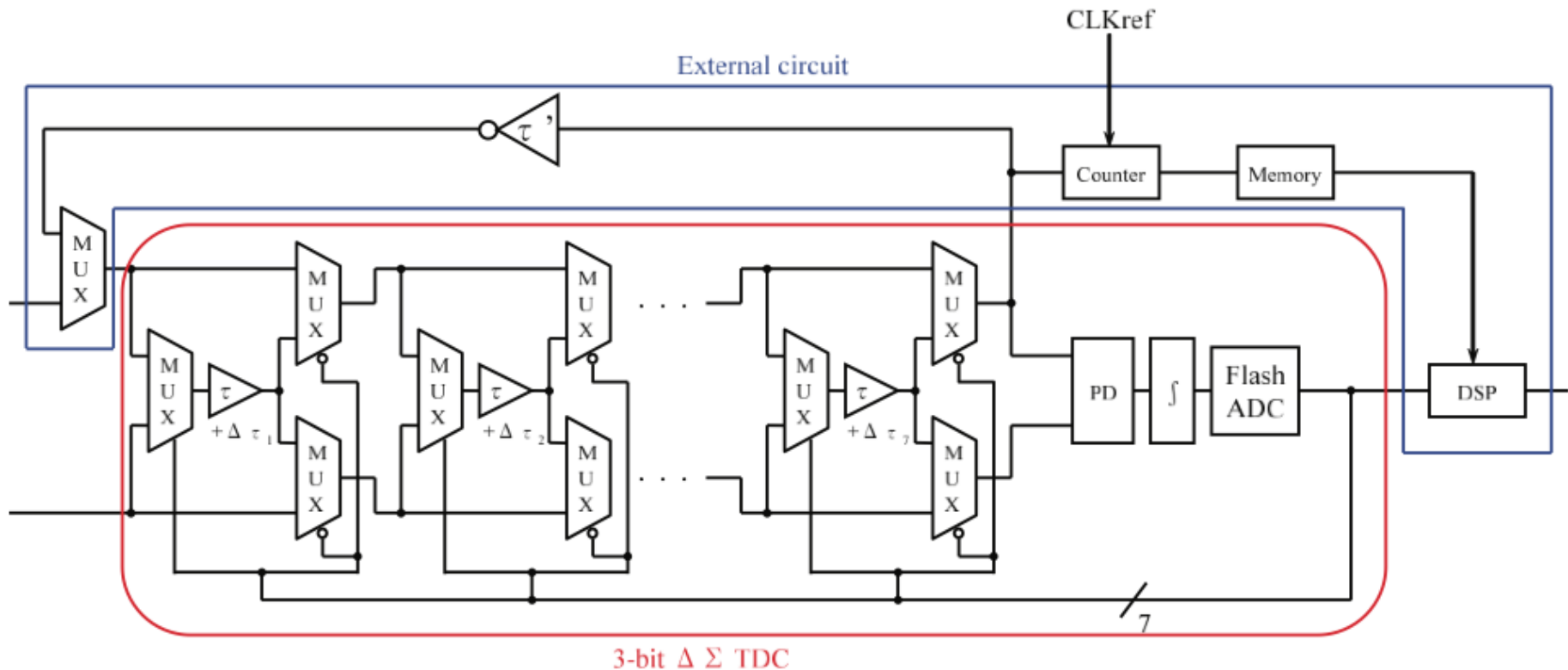
Mismatch effects  
reduction at DC

# Outline

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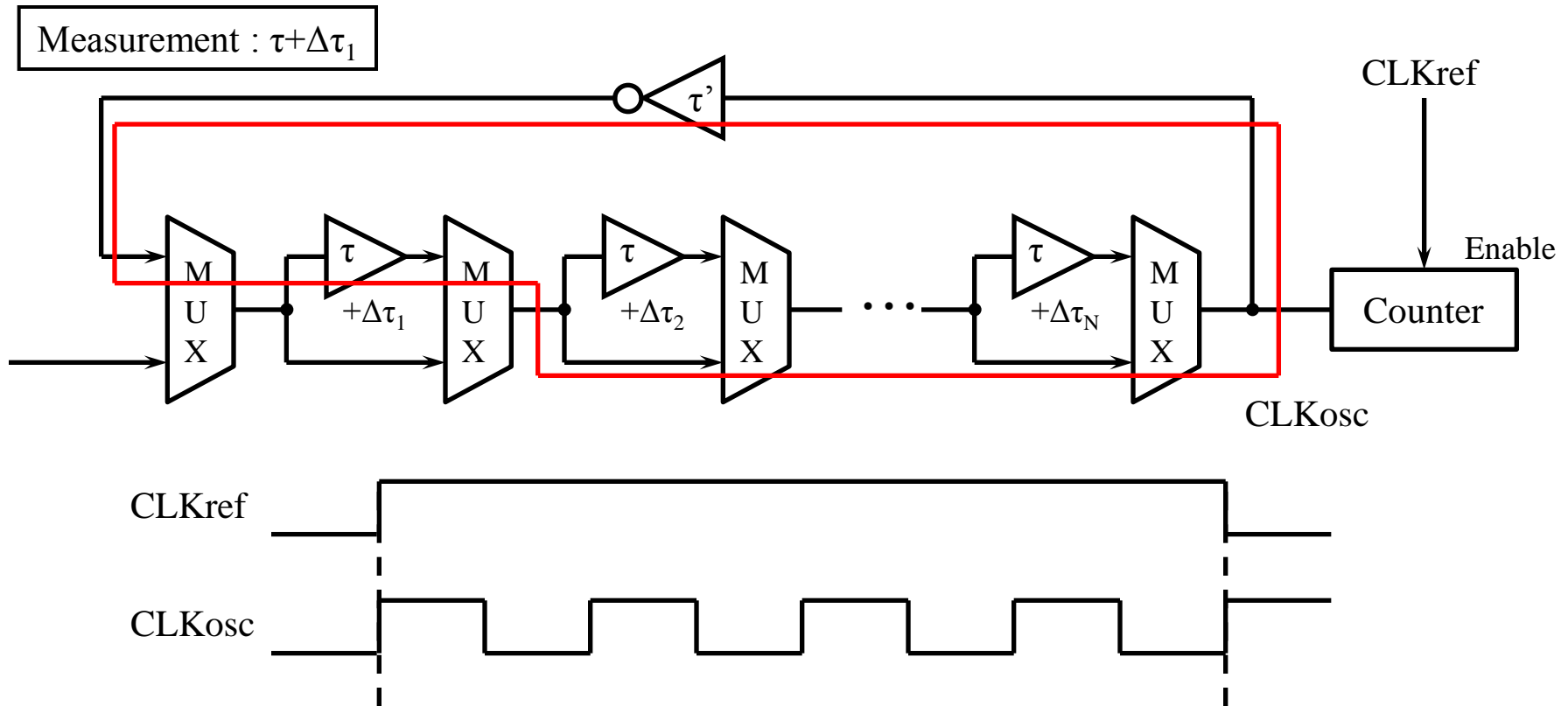
- ▶ Research Objective
- ▶ Single-Bit & Multi-bit  $\Sigma\Delta$  TDCs
- ▶ Multi-Bit  $\Sigma\Delta$  TDC with DWA
- ▶ **Multi-Bit  $\Sigma\Delta$  TDC with Self-Calibration**
- ▶ Conclusion

# $\Sigma\Delta$ TDC with Self-Calibration



- Self-calibration circuit: inverter, MUX, counter, memory
- Measure delay values and store them in memory

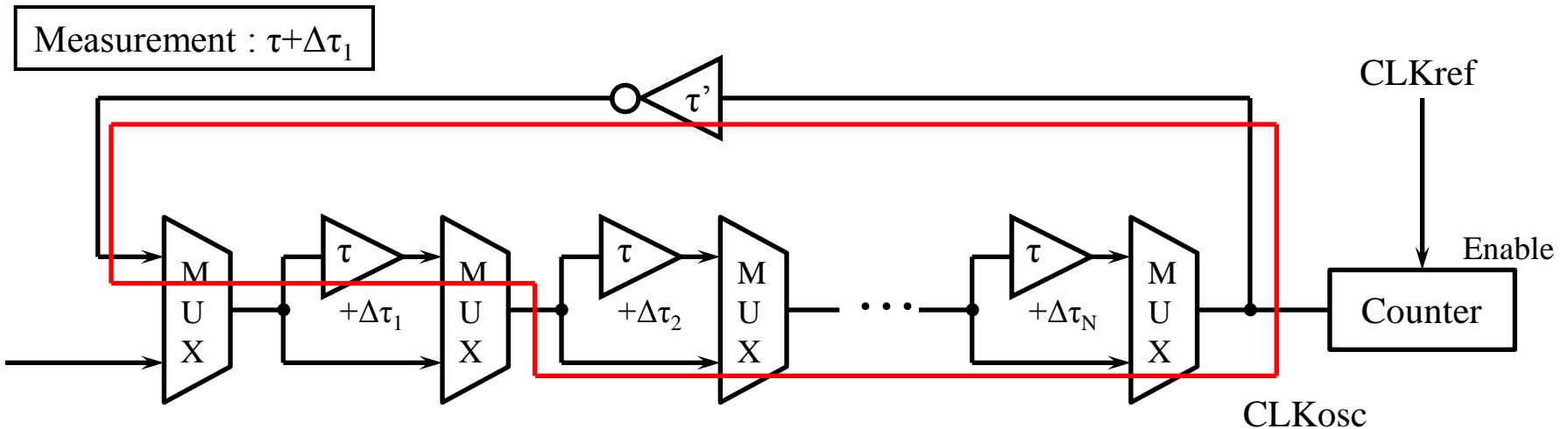
# Self-Measurement of Delay



- Ring oscillator with a delay cell to be measured
- Counter measure the number of the pulses
- $\Delta\tau$  can be calculated
- Measured delay values are stored in memory



# Time Signal & Ring Oscillator

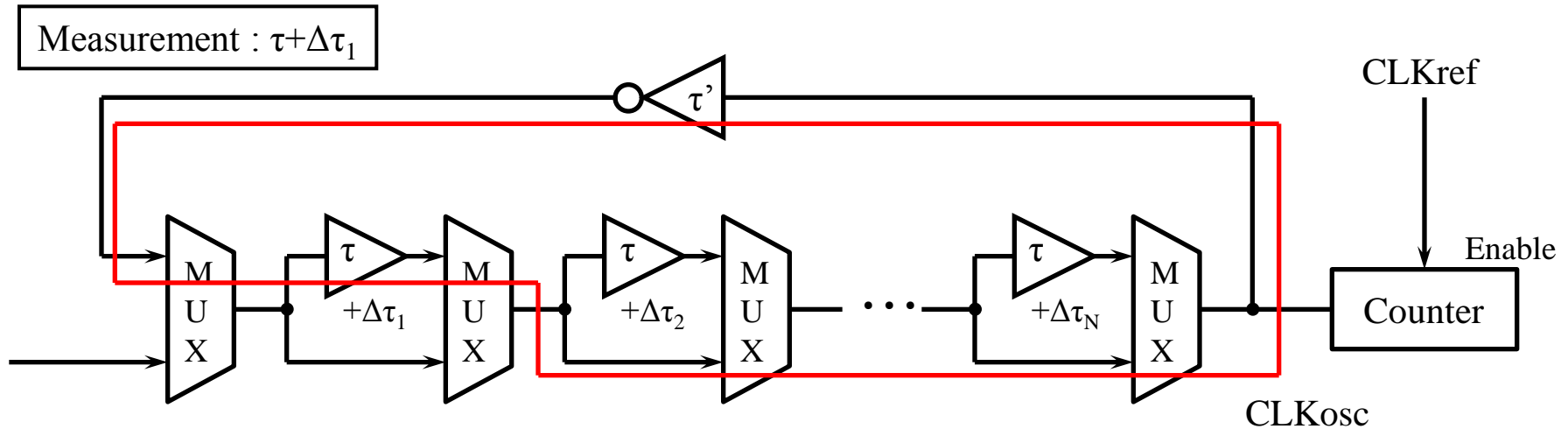


Ring oscillator



Möbius strip

# Self-Measurement of Delay



Oscillation frequency

$$f = \frac{1}{2(\tau' + \tau + \Delta\tau_1)}$$



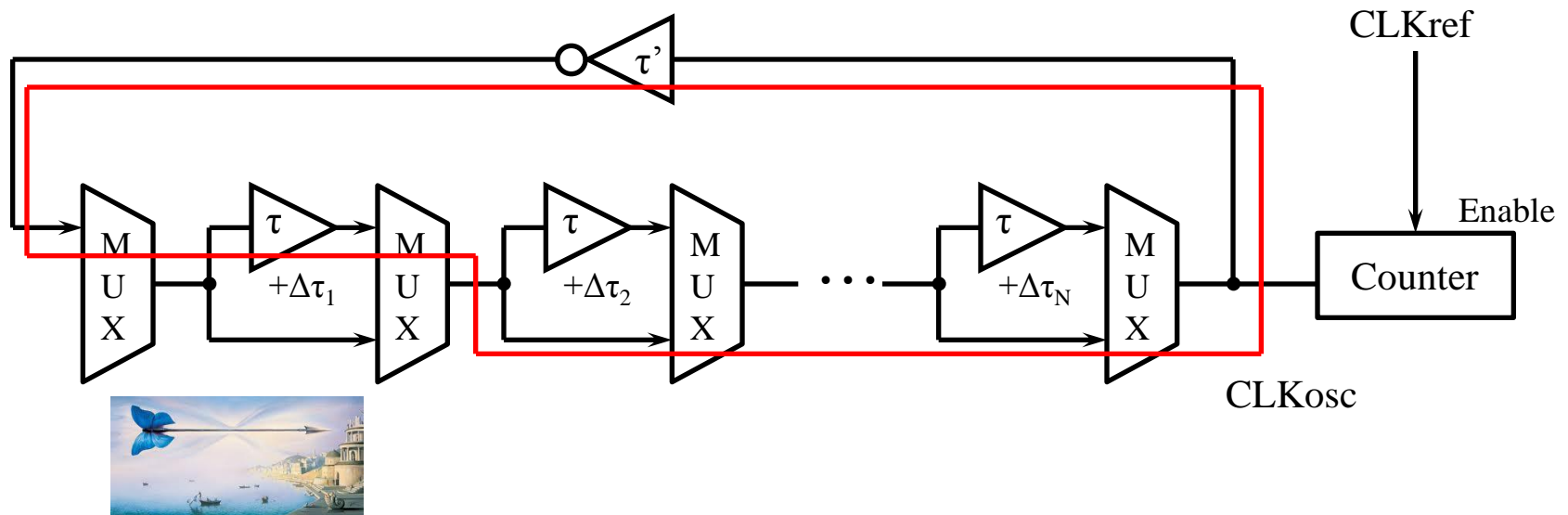
$\Delta\tau_1$  can be calculated from the oscillation frequency

Measure

$\Delta\tau_2, \Delta\tau_3, \Delta\tau_4, \dots, \Delta\tau_N$   
one by one.

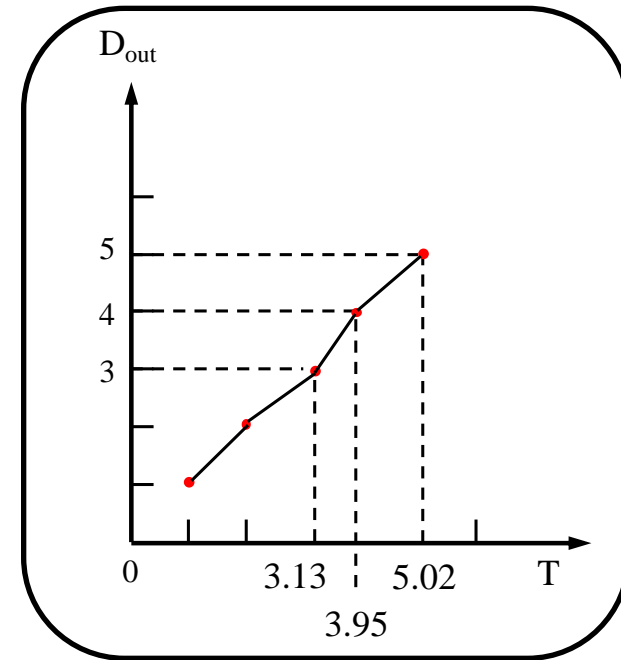
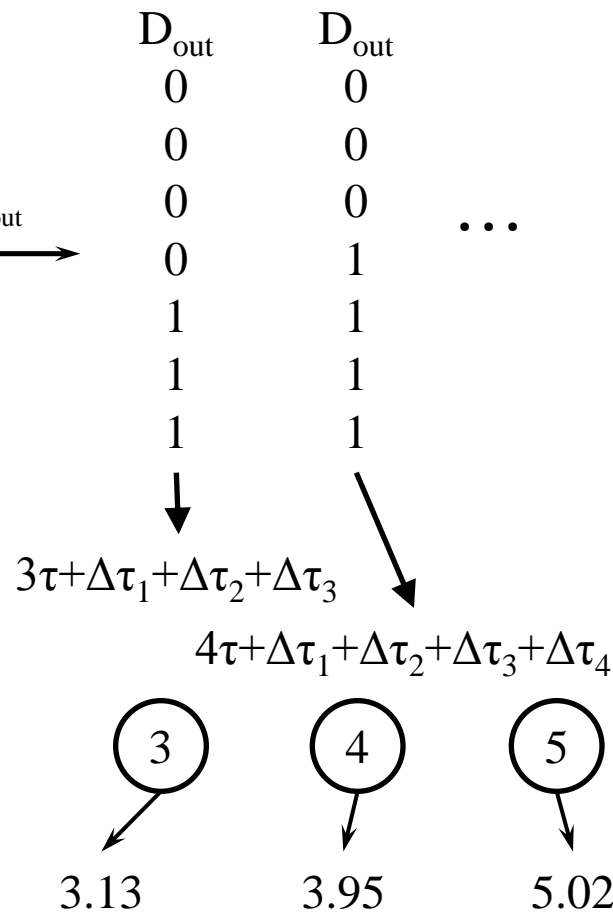
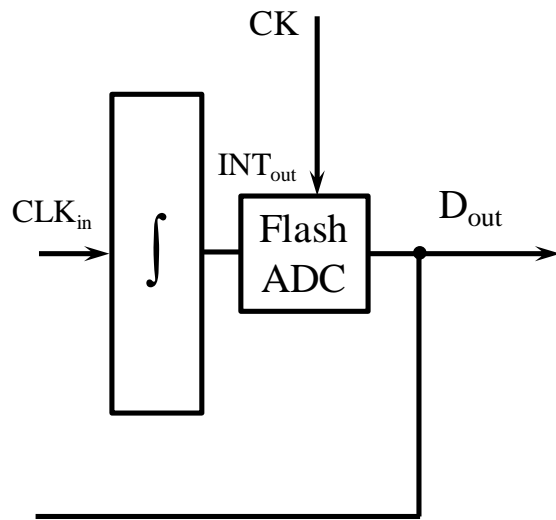
# Essence of Proposed Method

- All operations are done in **digital domain**
  - Signal is **Time** instead of **Voltage**.
- ➔ Easy, accurate measurement of  $\Delta\tau$



Time flies like an arrow!

# Proposed Error Correction Scheme

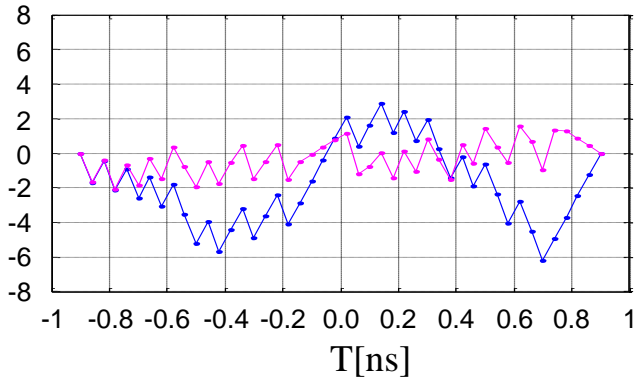


- Obtain TDC raw output ( $D_{out}$ ) for two input clocks
- Read delay values from memory, and compensate for the output based on them

# Simulation of Self-Calibration

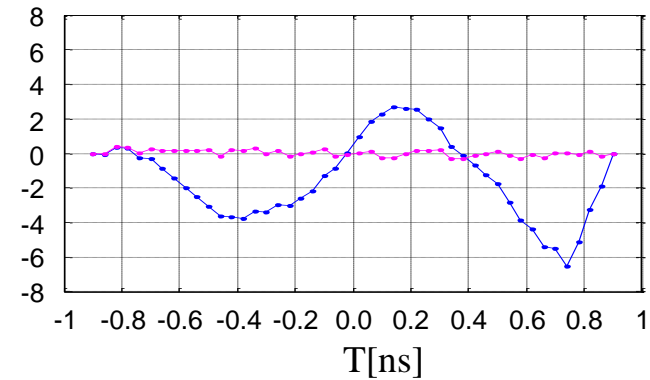
Difference from ideal line[ps]

● Output : 99 points



Difference from ideal line[ps]

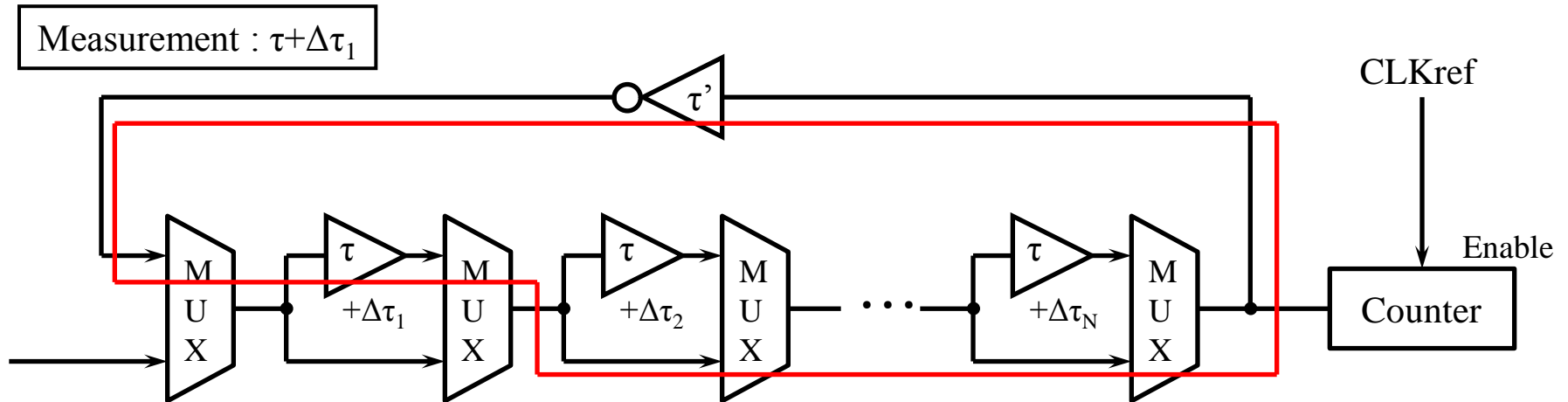
● Output : 599 points



—●—  $\Delta\Sigma$  TDC(with Self-Calibration)  
—●—  $\Delta\Sigma$  TDC(without Self-Calibration)

$\Sigma\Delta$  TDC linearity is improved

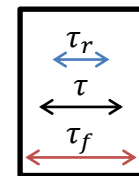
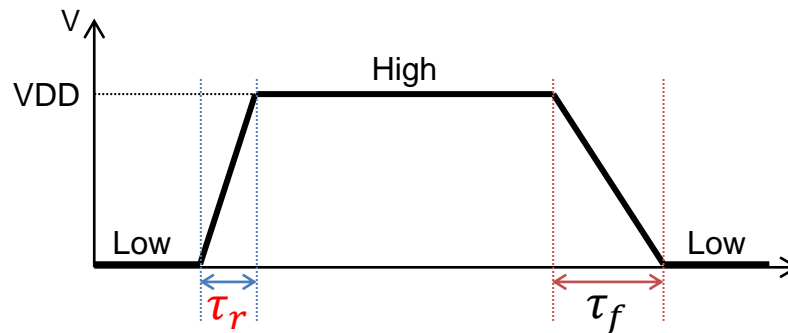
# Problem of Ring Oscillator



$$f = \frac{1}{2(\tau' + \tau + \Delta\tau_1)}$$



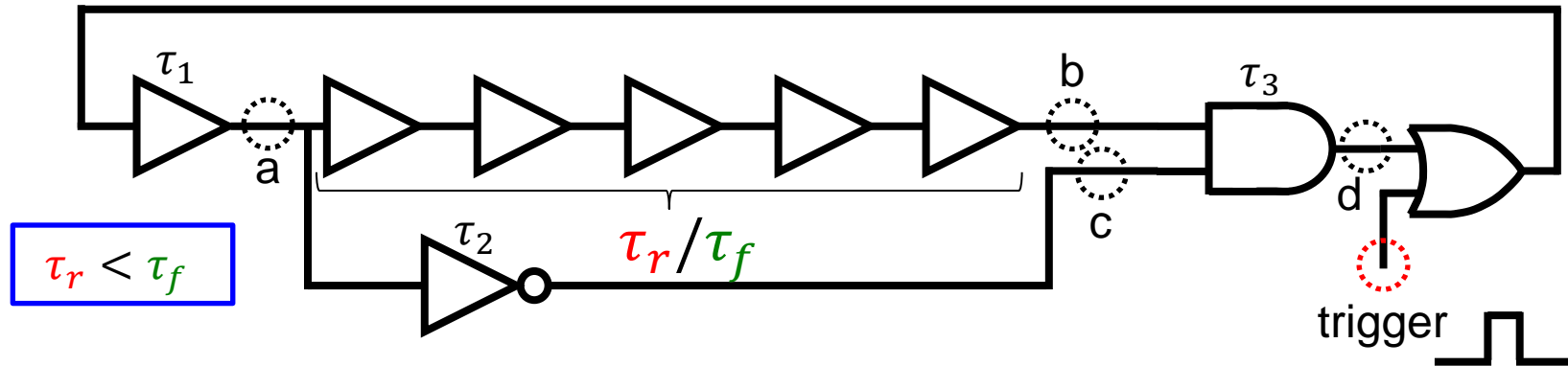
Measured delay



$$\tau = \frac{\tau_r + \tau_f}{2}$$

However, we need the rise delay  $\tau_r$

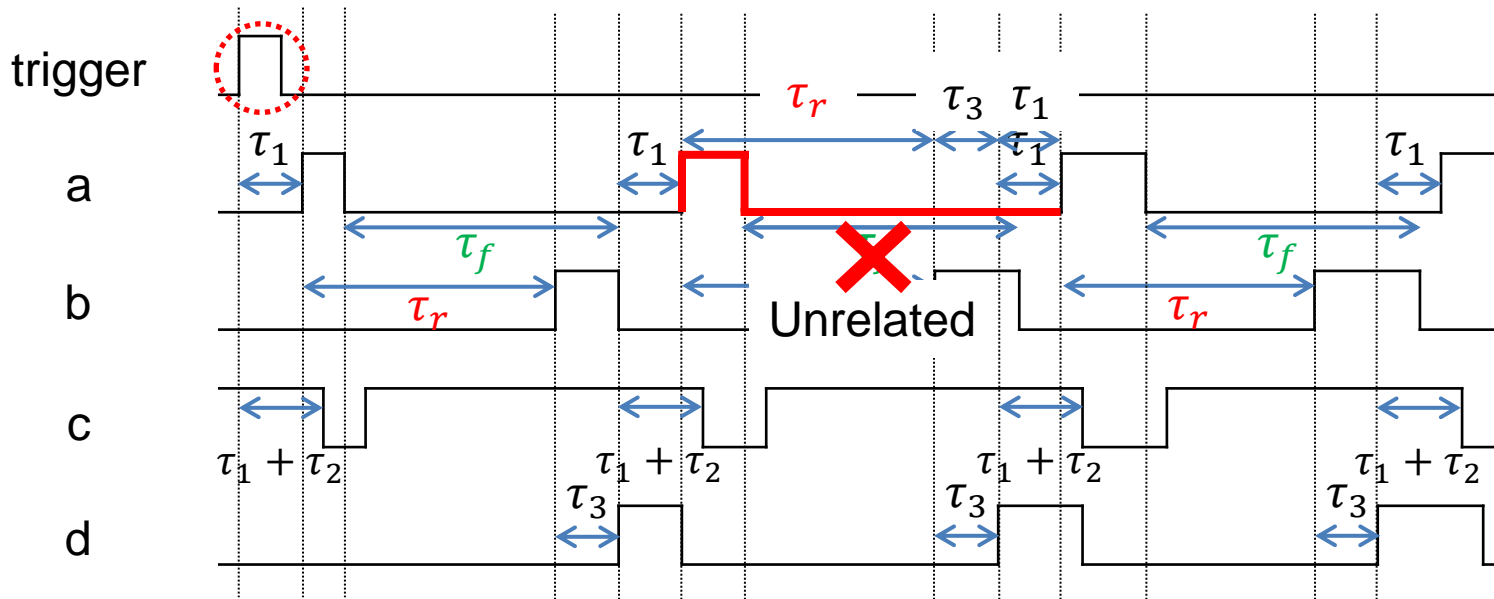
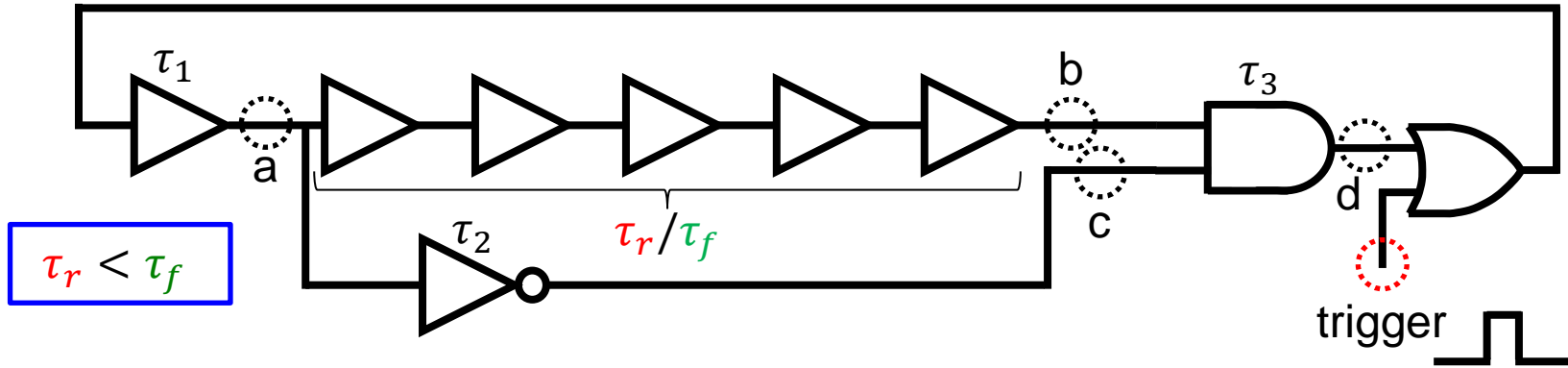
# Improved Delay Measurement Circuit



Oscillator circuit to measure  
the rise delay  $\tau_r$  of the buffer

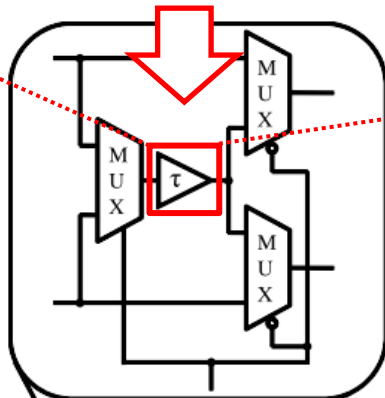
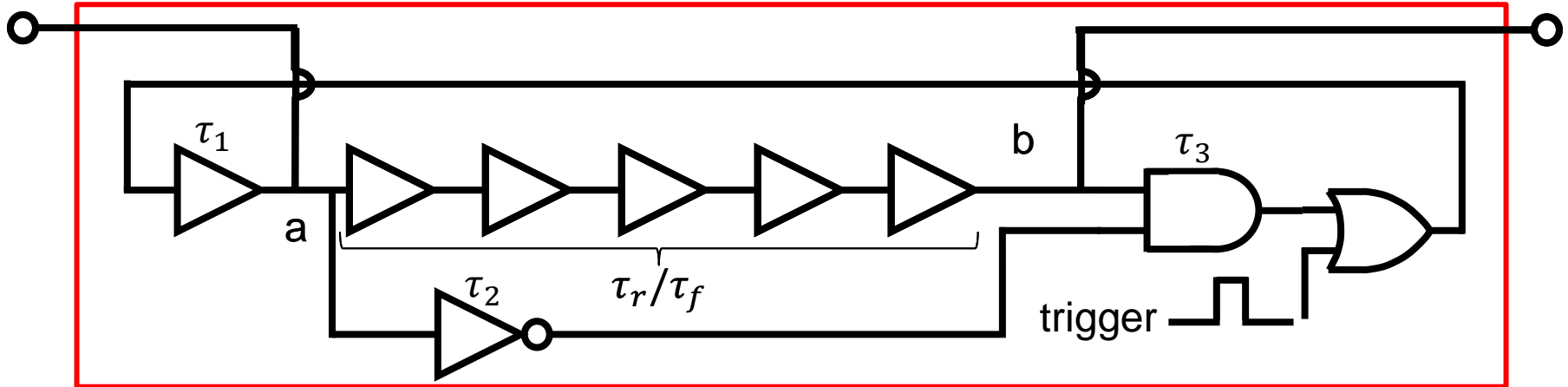
Oscillation period is a function of  $\tau_r$ , but NOT  $\tau_f$

# Oscillation Timing Chart

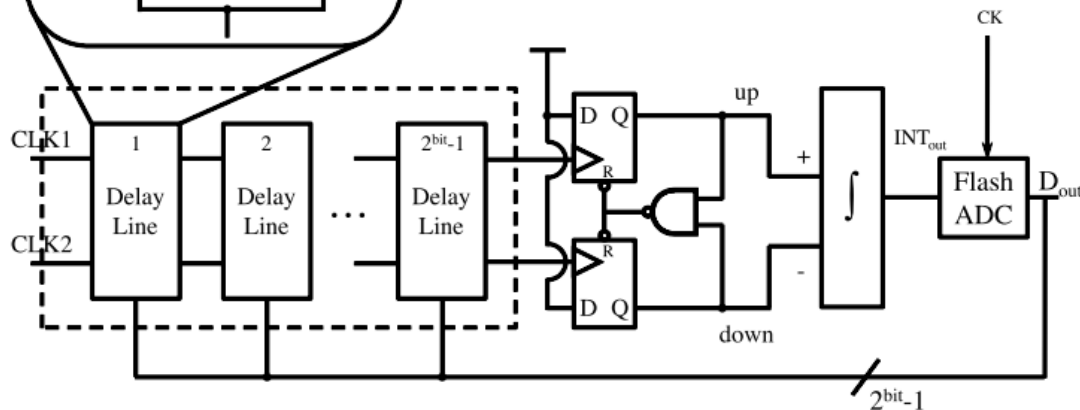




# Delay with Several Buffers



Replace !



# Circuit Performance Comparison

	Flash TDC	1-bit $\Sigma\Delta$ TDC	Multi-Bit $\Sigma\Delta$ TDC (without correction)	Multi-Bit $\Sigma\Delta$ TDC (with correction)
Area	×	◎	○	○
Resolution	×	◎	◎	◎
Accuracy	△	◎	×	◎
Time	◎	×	○	○

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# Conclusion

- We propose to use  $\Sigma\Delta$  TDC for digital signal timing measurement
- Multi-bit  $\Sigma\Delta$  TDC
  - Short measurement time
  - Fine time resolution
  - Non-linearity due to mismatches among delay cells
    - ➡ Two techniques to improve linearity
      - DWA
      - Self-Calibration (signal is “time”)

Low cost, high quality digital timing test can be realized



Kobayashi  
Laboratory



Time makes **GOLD** !!

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# Appendix

# How to Calculate the Delay Time

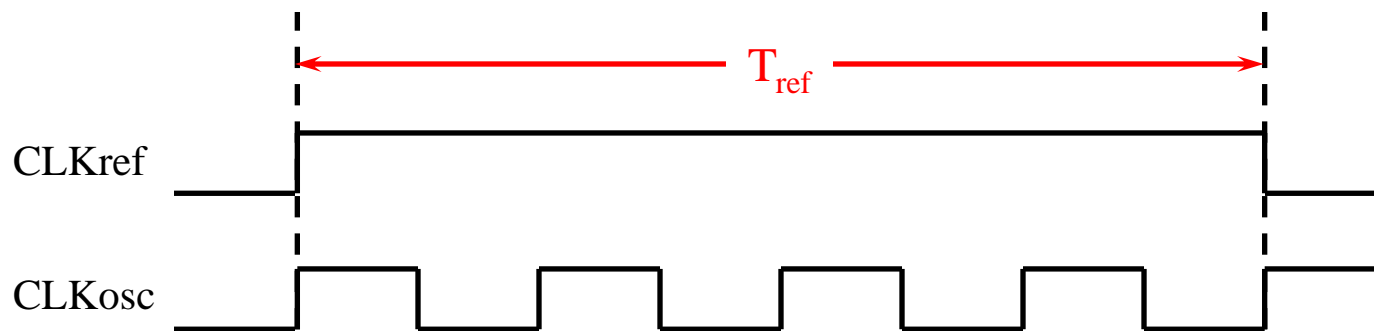
$$f_{OSC}^k \approx \frac{M_k}{T_{ref}} = \frac{1}{2(\tau' + \tau_k)}$$

$$\tau_k = \tau + \Delta\tau_k$$

$$f_{OSC}^0 \approx \frac{M_0}{T_{ref}} = \frac{1}{2\tau'}$$

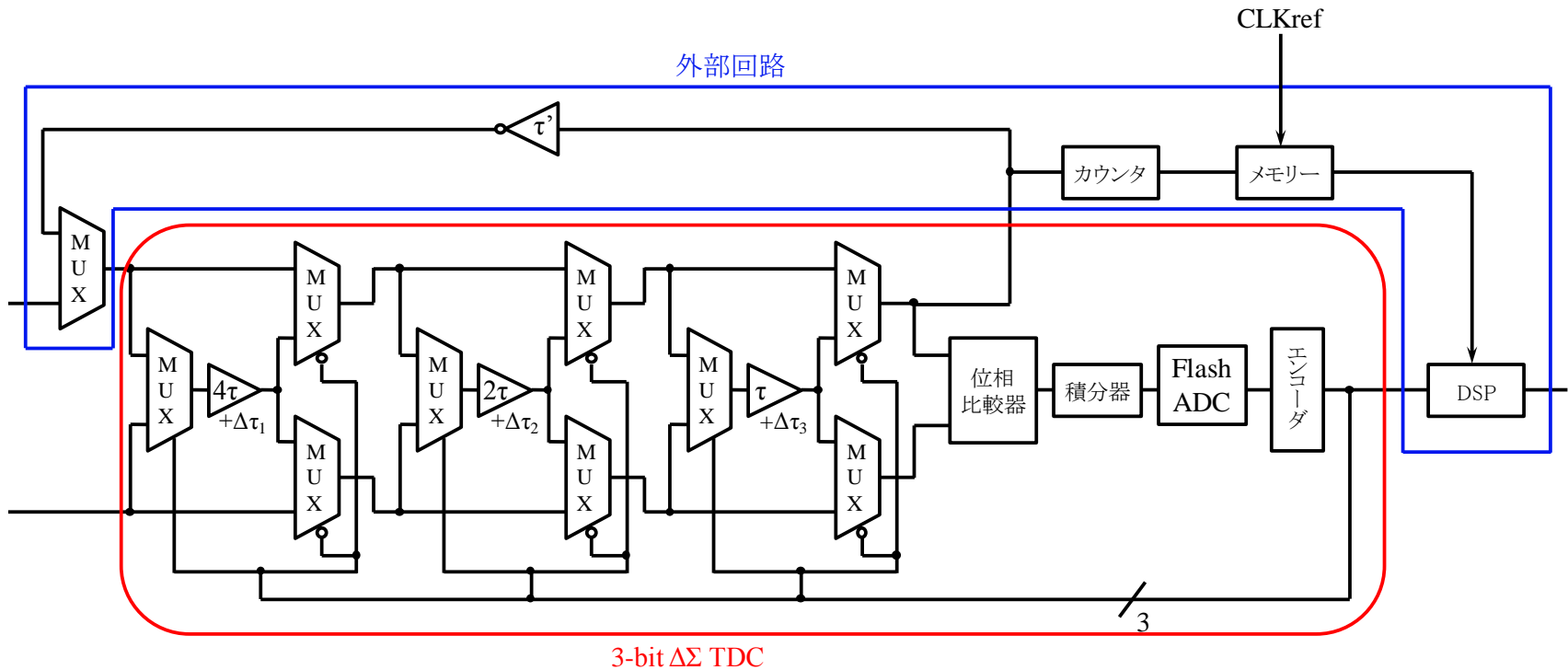
$$\tau_k = \frac{1}{2} \left( \frac{1}{f_k} - \frac{1}{f_0} \right) \approx \frac{T_{ref}}{2} \left( \frac{1}{M_k} - \frac{1}{M_0} \right)$$

$$k=1, 2, \dots, 2^N-1$$



Number of Pulses :  $M_k$

# TDC Circuit with Self-Calibration

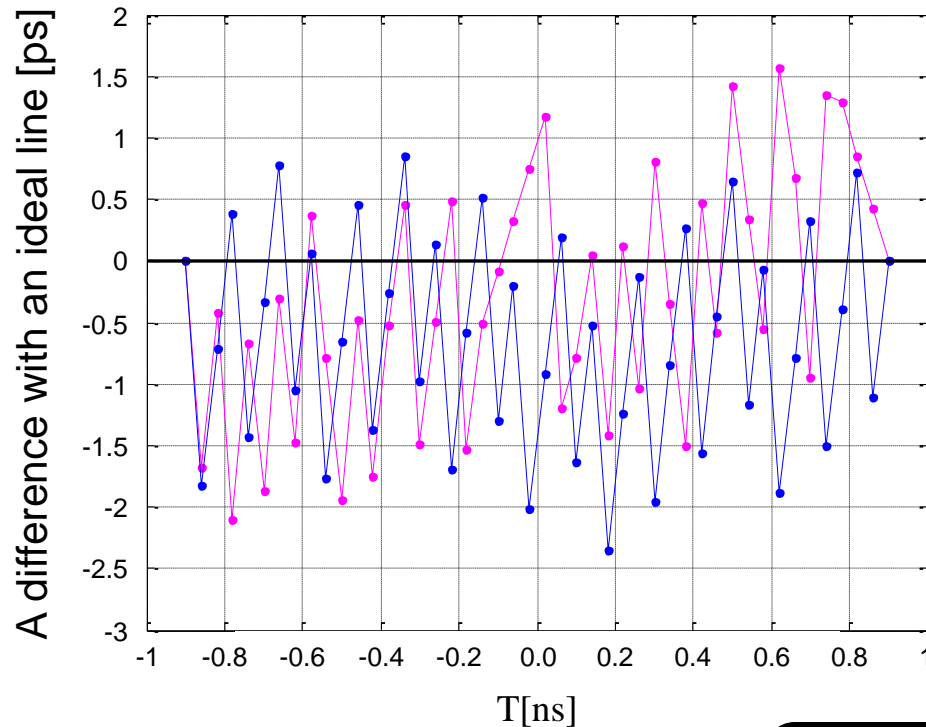


- 各遅延値に重みをもたせる
- 測定にはN-bit で Nステップかかる



# Comparison of Linearity

- 3-bit  $\Delta\Sigma$  TDC (Delay Time(Ideal) :  $\tau=0.145\text{ns}$ )



## 出力数99点において

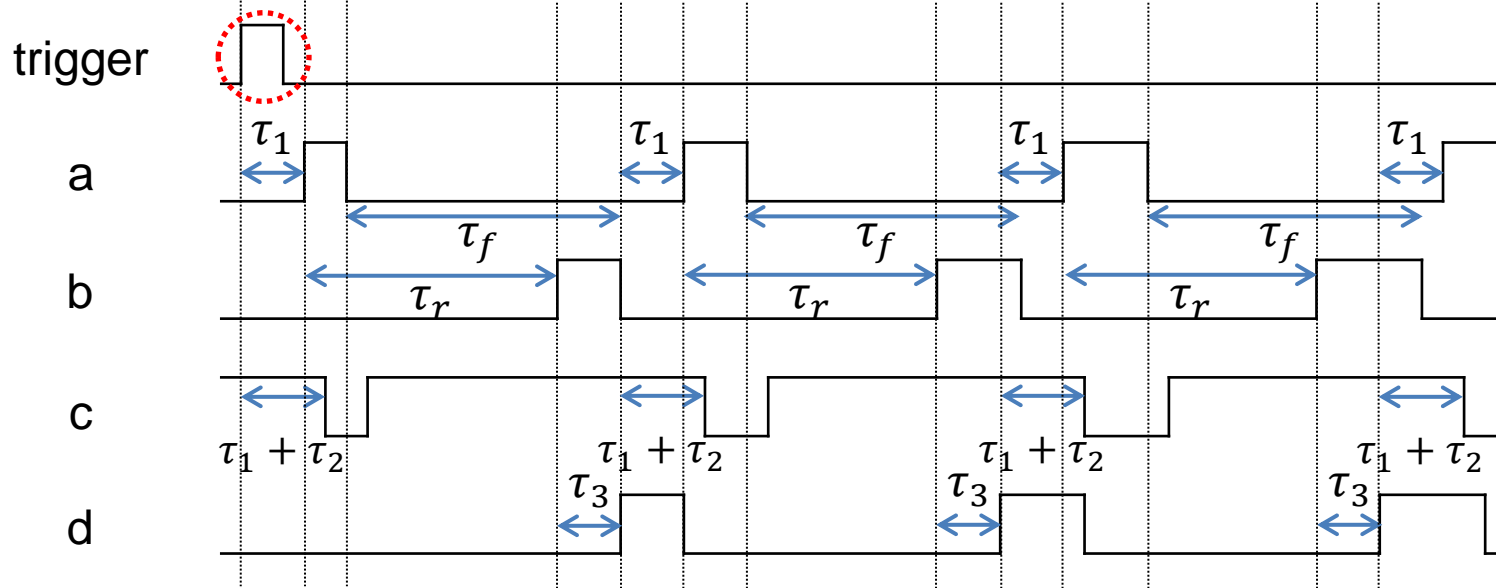
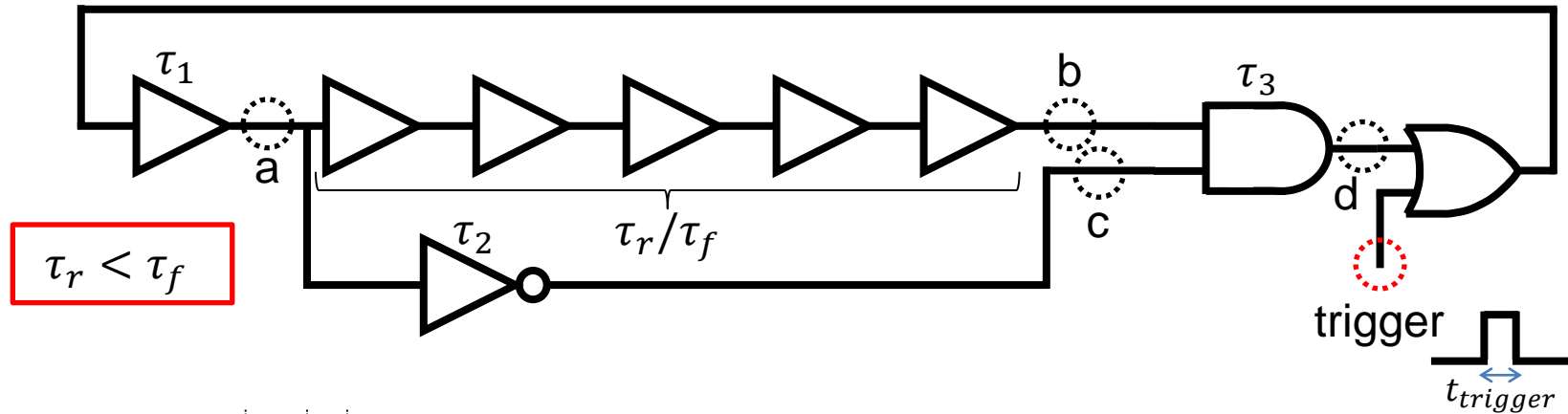
- 理想状態 :  $\pm 2$  ps 以内の差
- 補正後 :  $\pm 2.5$  ps 以内の差
  - 線形性がほぼ理想状態まで改善

- $\Delta\Sigma$  TDC (with Element Rotation)
- $\Delta\Sigma$  TDC (with Self-Calibration)

# Detail of Oscillation timing chart

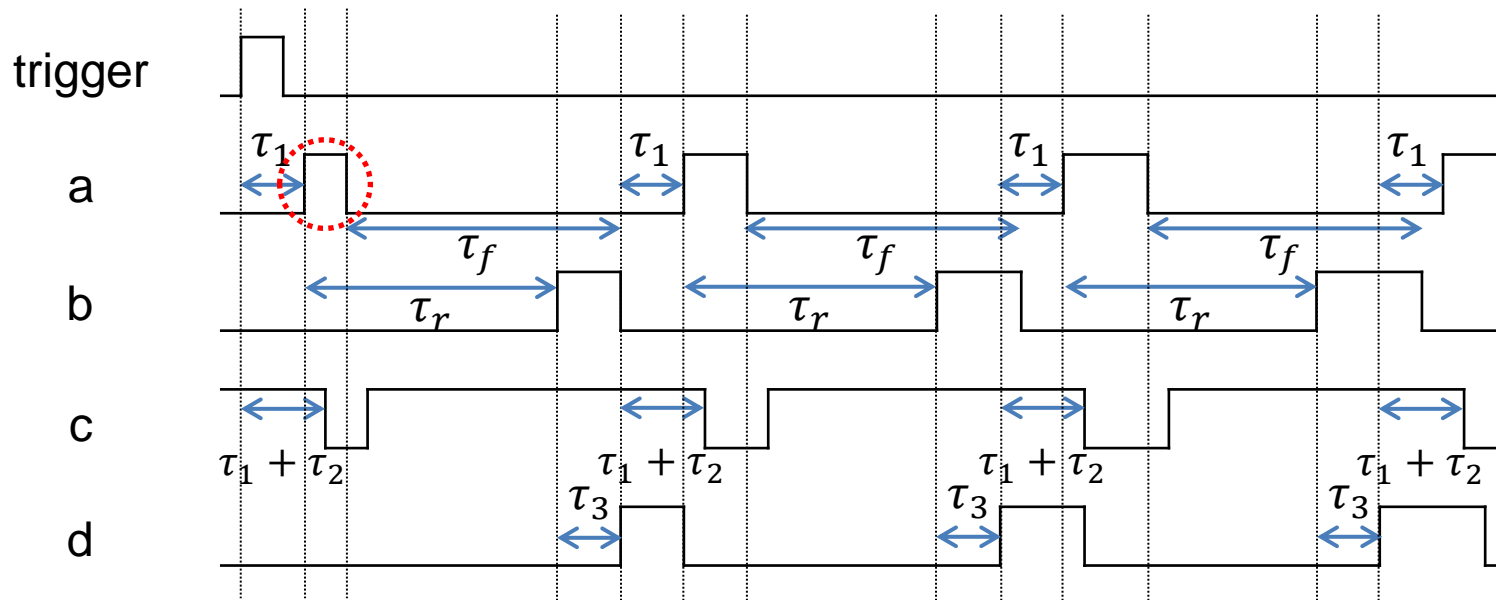
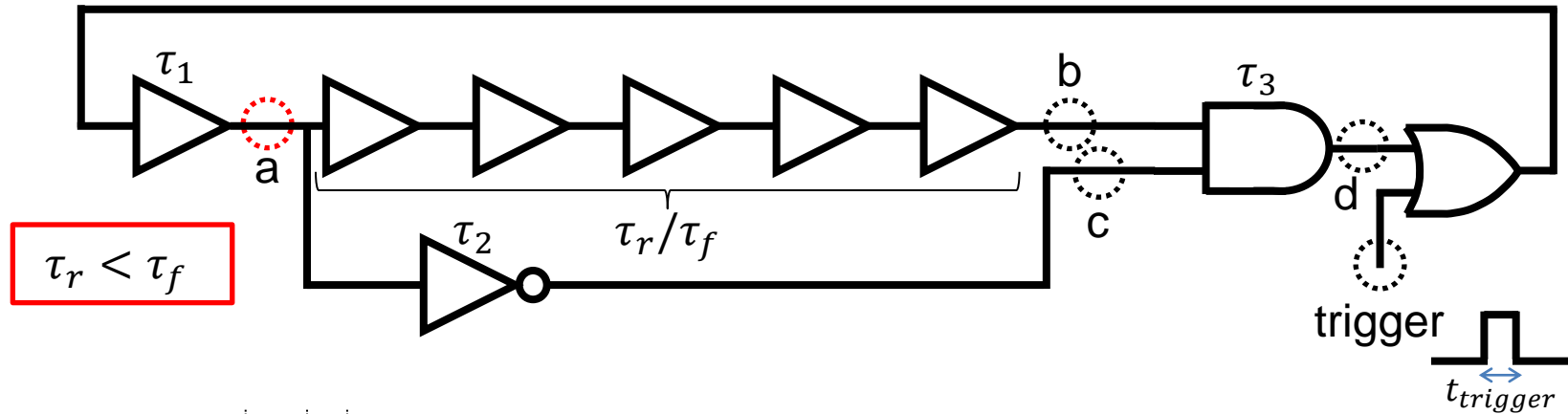
# Timing chart

Input trigger



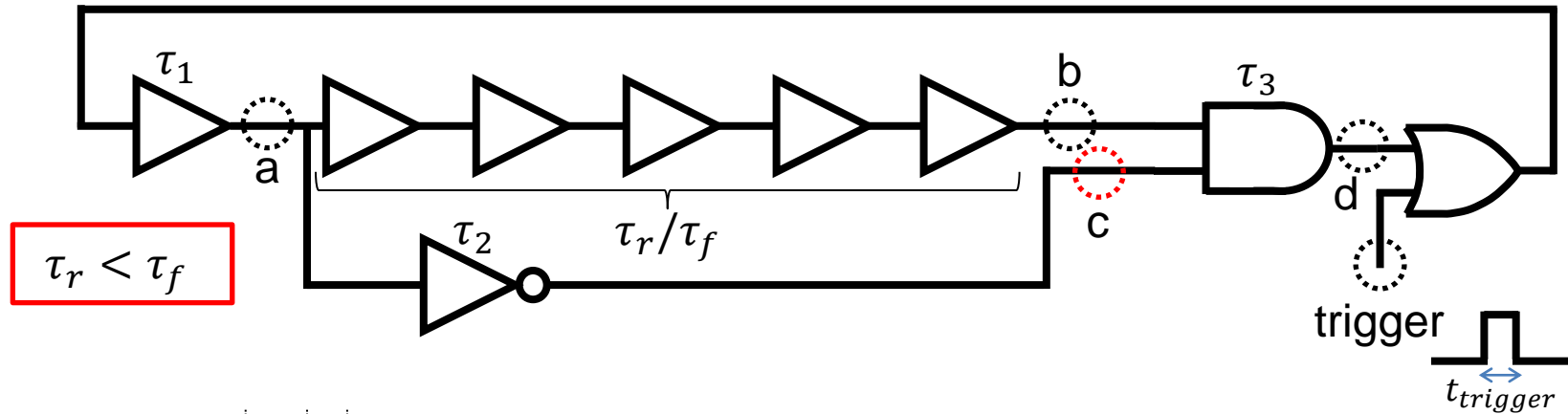
# Timing chart

Buffer out put “a” is rises from low to high level after  $\tau_1$

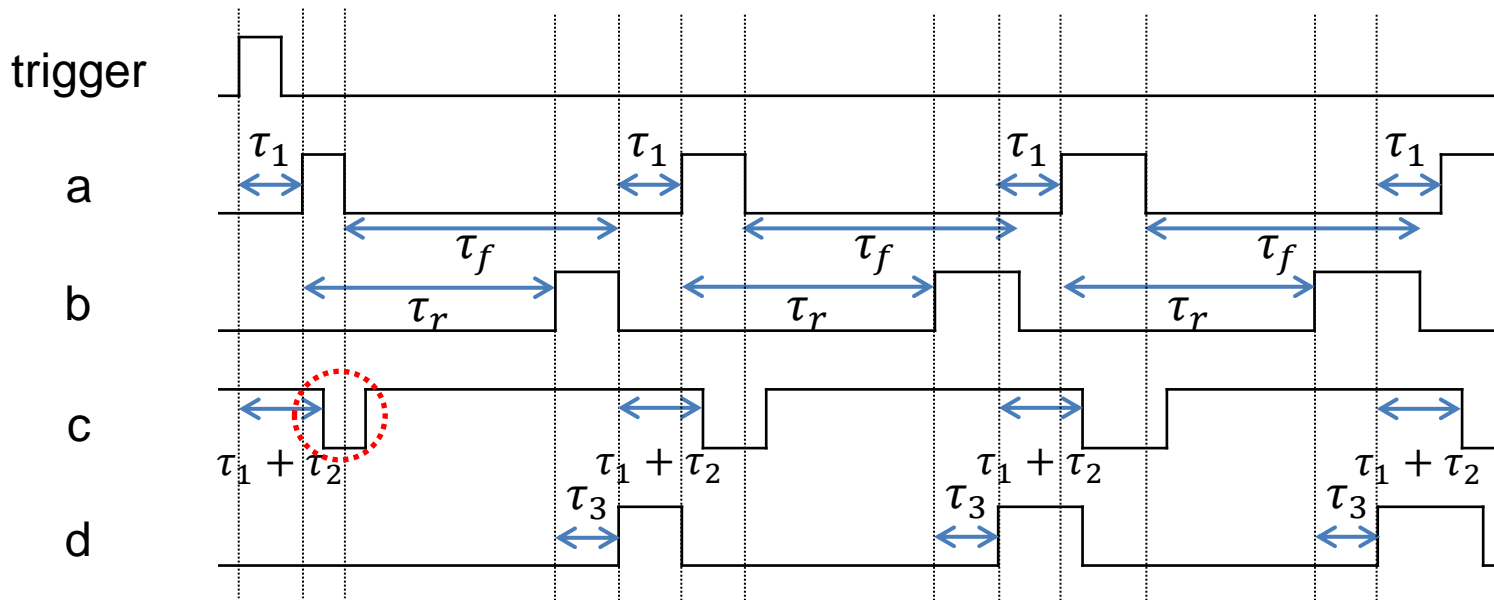


# Timing chart

Inverter out put “c” is falls from high to low level after  $\tau_2$

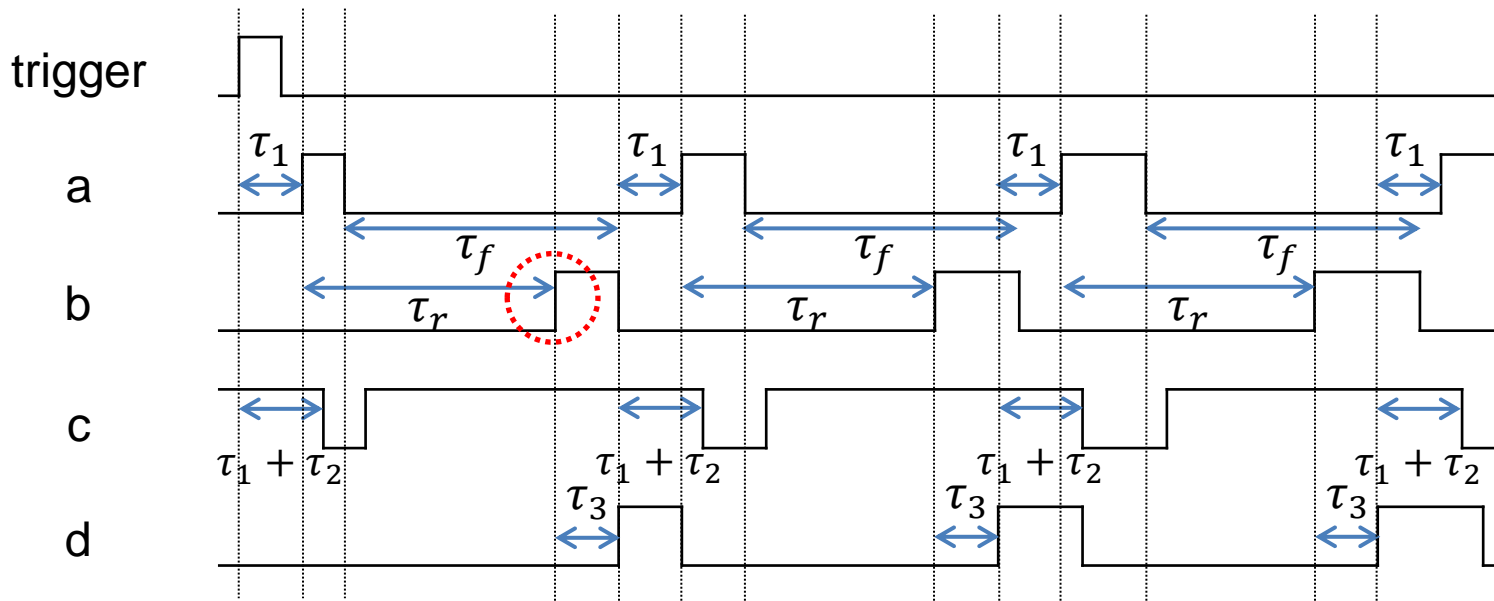
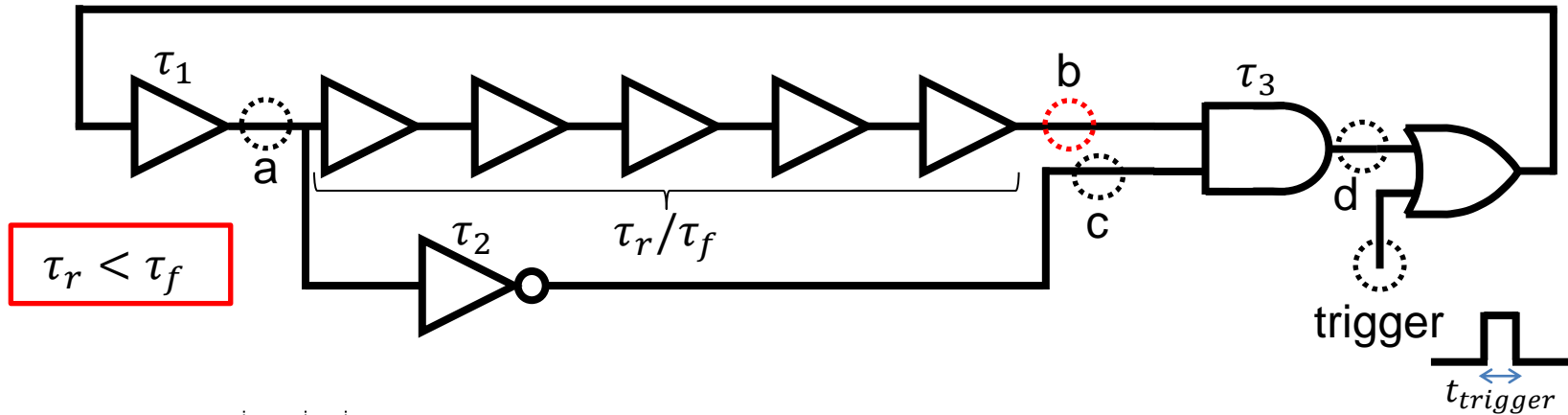


$$\tau_r < \tau_f$$



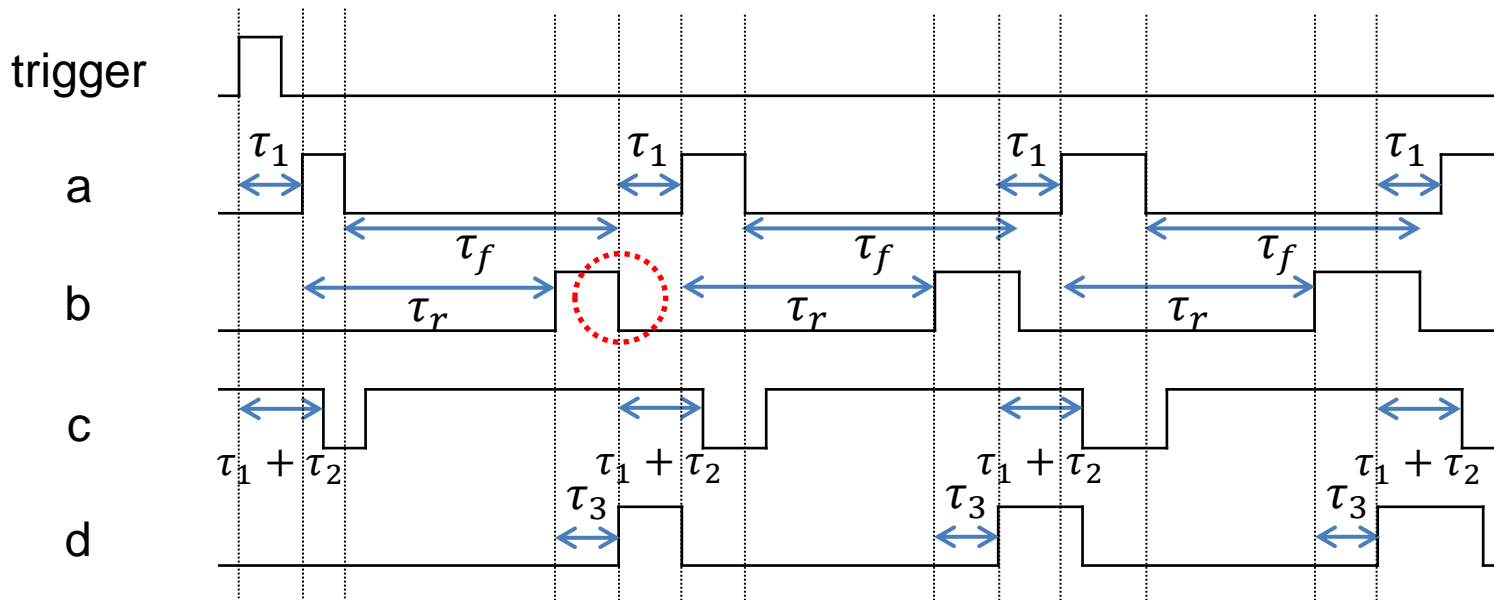
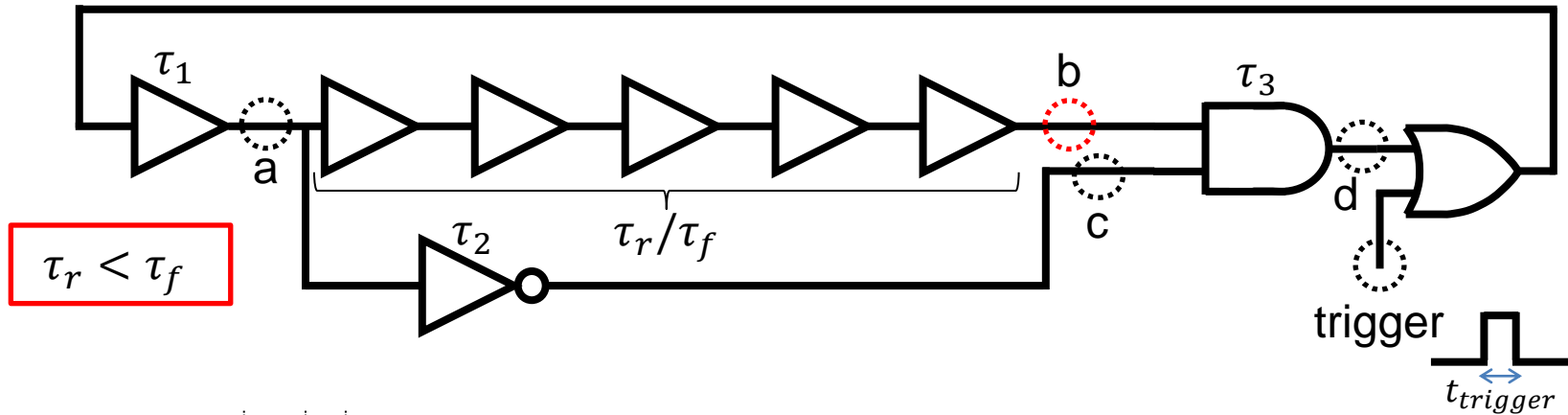
# Timing chart

5 buffers out put "b" is rises from low to high level after  $\tau_r$



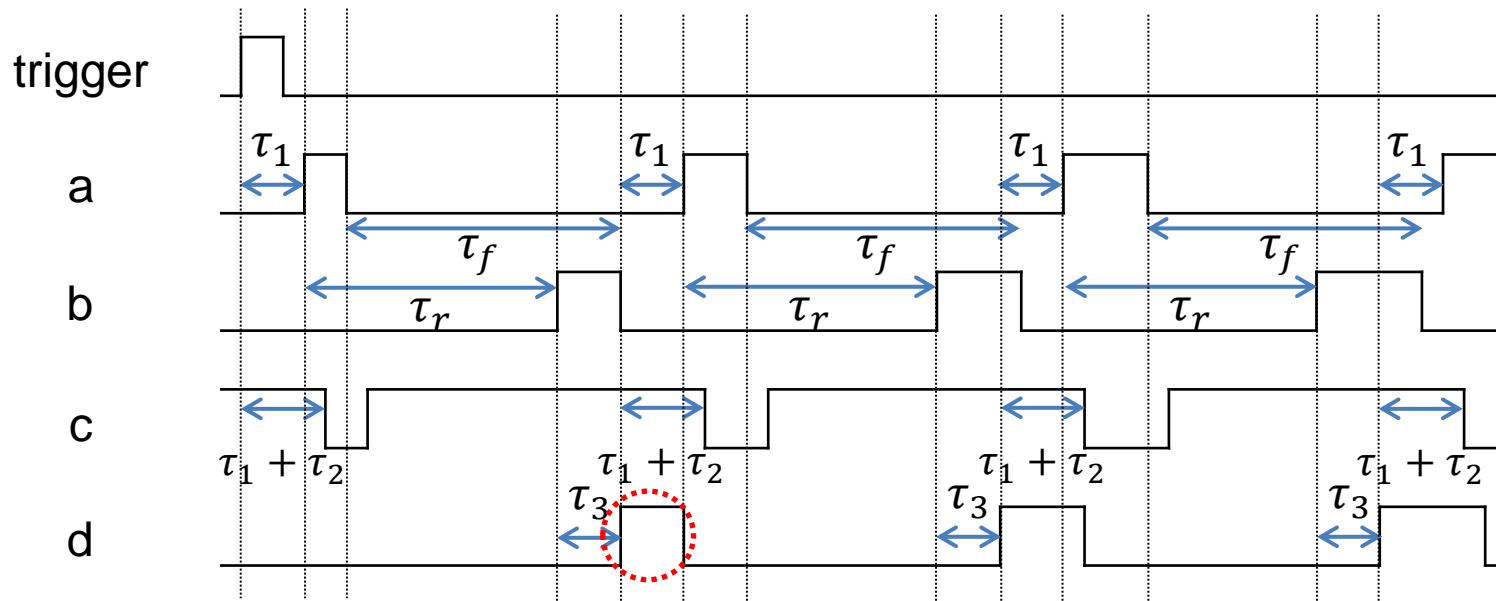
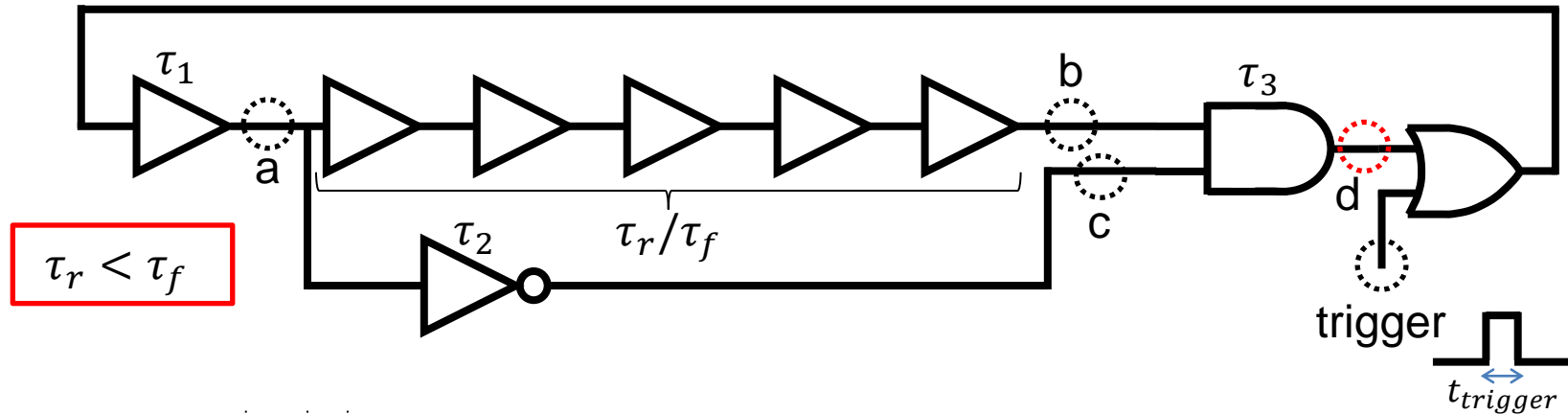
# Timing chart

5 buffers out put "b" is falls from high to low level after  $\tau_f$



# Timing chart

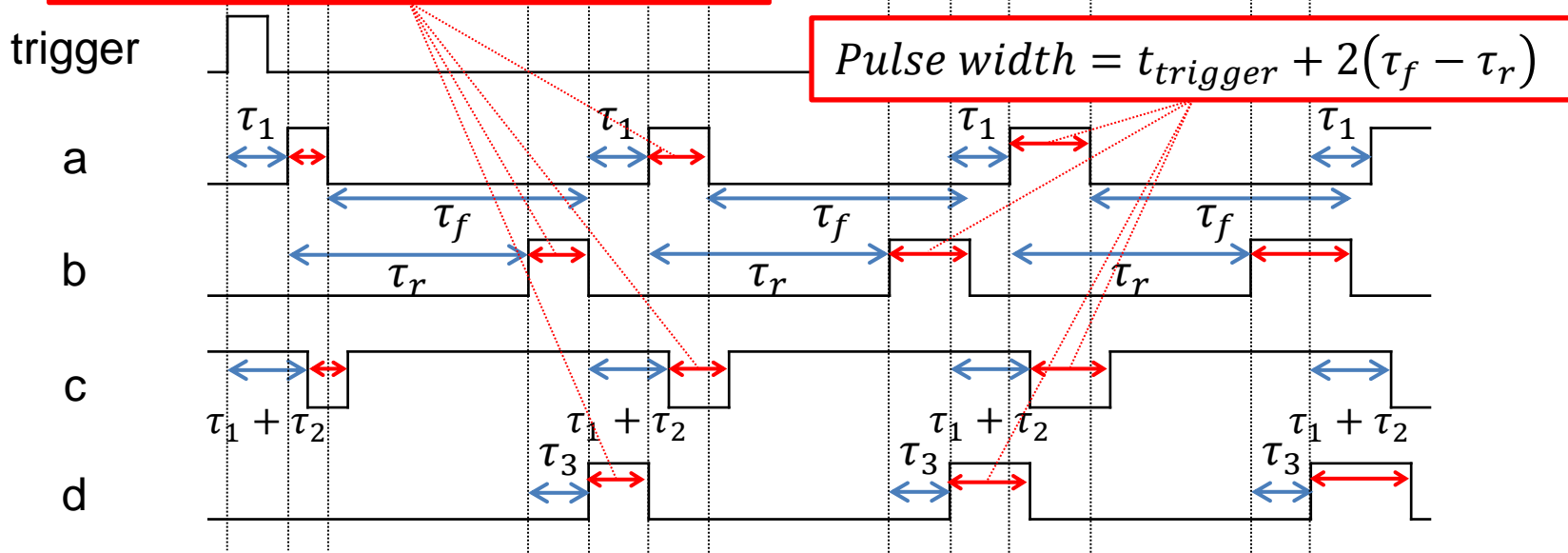
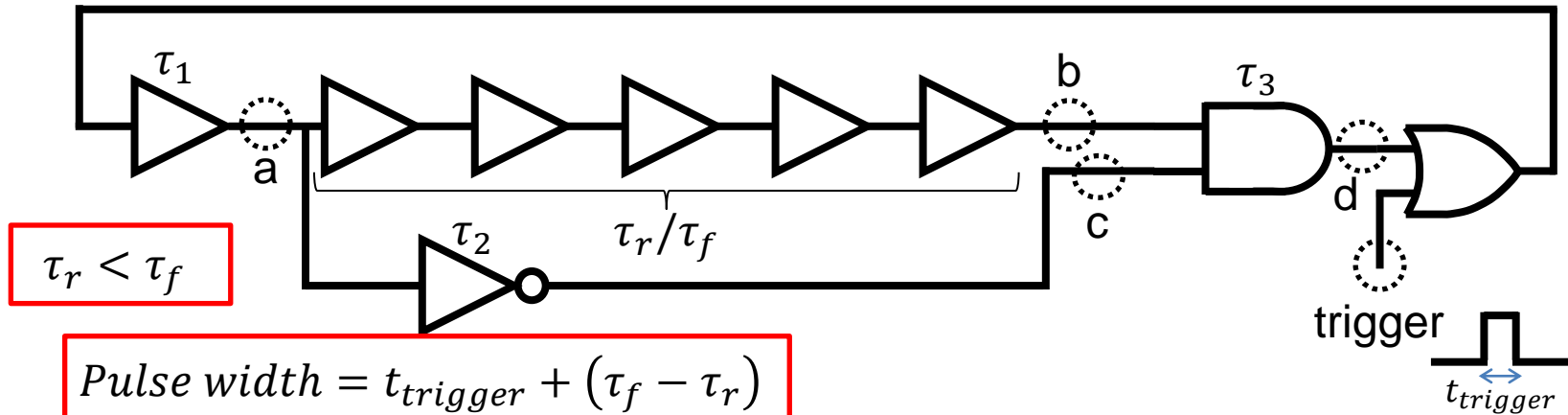
AND out put “d” is rises from low to high level after  $\tau_3$





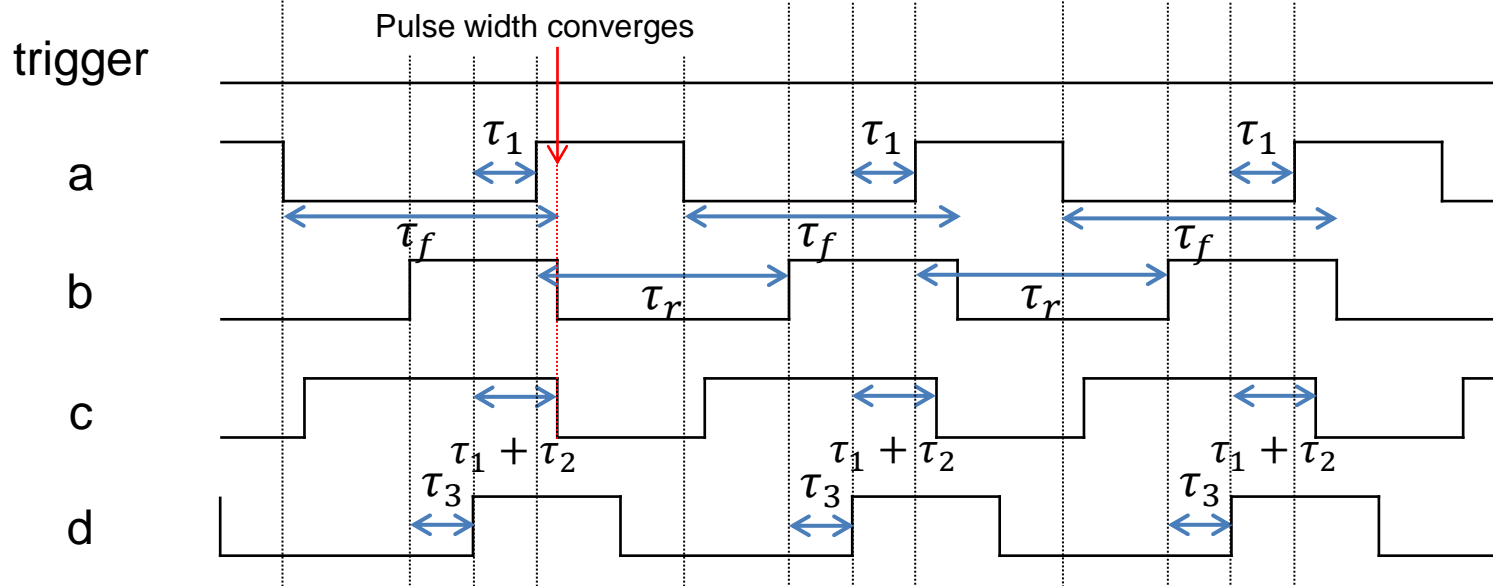
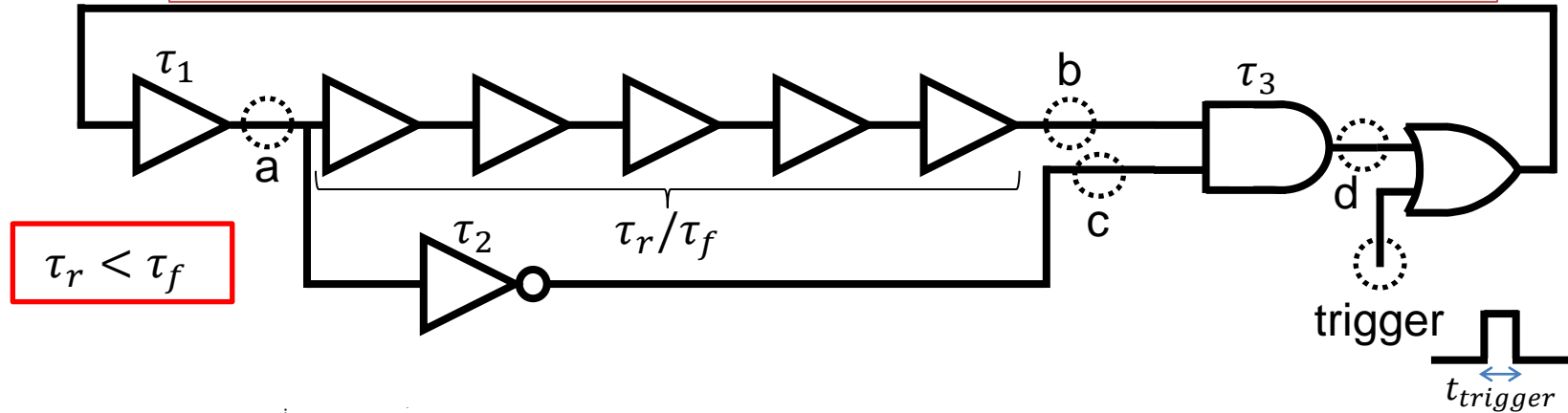
# Timing chart

Over time, Pulse width of each node is increasing by  $(\tau_f - \tau_r)$



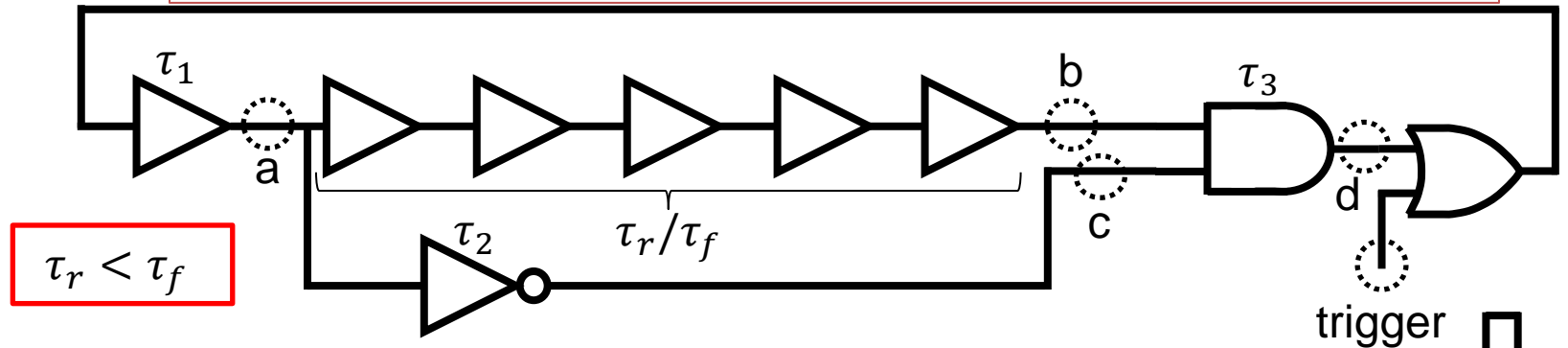
# Timing chart

The timing of the falling edge of node B and C becomes the same, a pulse width converges



# Timing chart

The timing of the falling edge of node B and C becomes the same, a pulse width converges

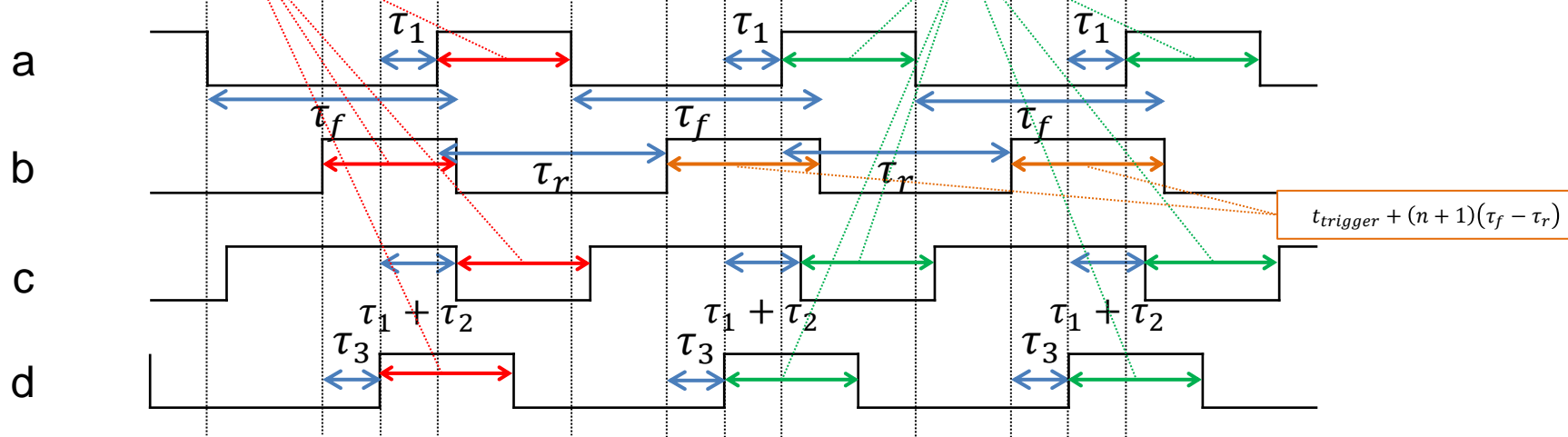


$$\tau_r < \tau_f$$

$$\text{Pulse width} = t_{\text{trigger}} + n(\tau_f - \tau_r)$$

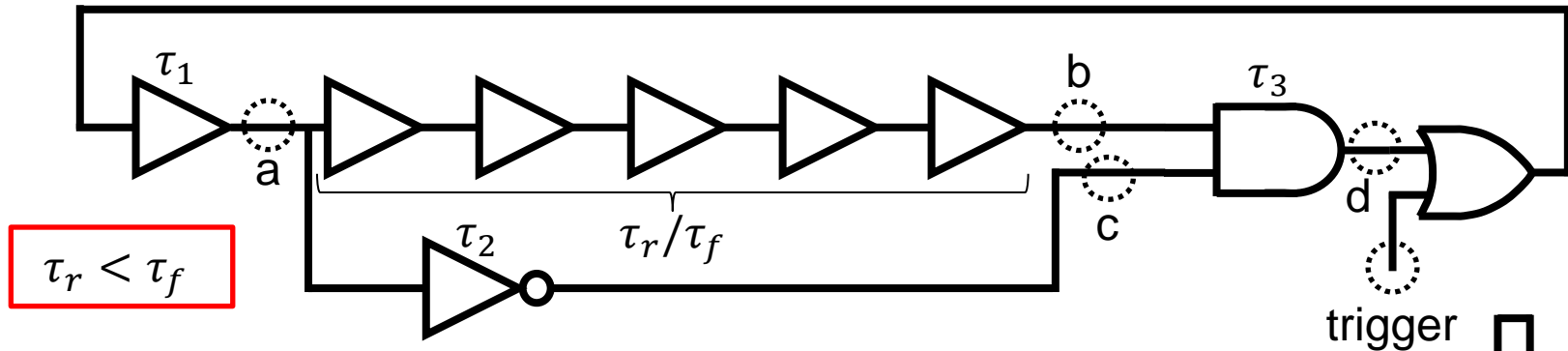
$$\text{Pulse width} = t_{\text{trigger}} + n(\tau_f - \tau_r)$$

trigger



# Timing chart

After convergence, period  $T$  of node "d" is  $\tau_1 + \tau_r + \tau_3$



trigger  $f_{osc} = \frac{1}{\tau_r + (\tau_1 + \tau_3)}$  Obtain the accurate value of  $\tau_r$  with measuring the oscillation frequency !!

