DC-DC Converter with Continuous-Time Feed-Forward Sigma-Delta Modulator Control

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***Abstract*—This paper describes applications of continuous-time feed-forward Sigma-Delta (ΣΔ) modulators to control DC-DC converters as follows. We propose to use continuous-time feed-forward ΣΔ controllers in DC-DC converters, and show that their transient response is faster than discrete-time and/or feedback-type ΣΔ controllers. We also show that second-order ΣΔ controllers have superior performance to first-order ones. SPICE and Matlab simulations substantiate these results.**

Keywords- DC-DC Converter; Continuous-Time; Sigma-Delta Modulator; Feed-Forward

I. INTRODUCTION

Recently the portable power management system landscape has changed due to an explosion in demand for portable devices such as cellular phones, personal digital assistants (PDA) and digital cameras. The DC-DC converter plays a crucial role in maintaining long battery life while providing stable supply and noise isolation. Most DC-DC converters use PWM controllers. However, rapid advances in power MOSFET devices have led to many researchers investigating the feasibility of ΣΔ modulators as controllers [1-5]; the expected advantages over PWM controllers are as follows:

(1) Fast transient response

(2) High efficiency at low load

(3) Spread spectrum of switching noise

(4) Can operate at higher switching frequency, and thus can use smaller L and C.

So far, most ΣΔ modulators proposed as controllers for DC-DC converters have been discrete-time (DT). In this paper, we propose to use continuous-time (CT) feed-forward (FF) ΣΔ modulators, and we compare their performance with that of conventional DT and/or feedback (FB) alternatives. Compared with a DT ΣΔ modulator, the CT ΣΔ has benefits such as low-power and high-speed [6-10]. Also compared with a FB ΣΔ modulator, the FF ΣΔ has better phase performance. These make the CT FF ΣΔ modulator more attractive as a controller for DC-DC converters. This paper describes the theory of operation and shows simulation results.

II. TRANSFER FUNCTION DESIGN OF CT FF ΣΔ MODULATOR

Fig.1 shows the block diagrams of DT and CT ΣΔ modulators, where Q denotes a quantizer. Since both discrete and continuous time signals exist in the CT ΣΔ loop, we use a transformation between the discrete and continuous-time, based on the impulse response invariant transformation. Suppose that the impulse response of the transfer function L1(z) in the DT ΣΔ loop is g(nT) and that of the transfer function L1(jw) is h(t). If we use the impulse response invariant transformation, which requires that the impulse response h(nT) should be equal to g(nT) (where n is an integer), we can calculate Hc(s) (where Hc(s) is the transfer function of low pass filter in the continuous-time ΣΔ *modulator*).

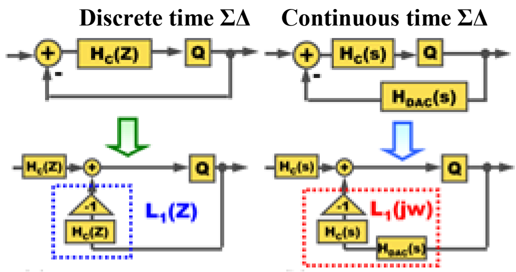

Fig.1 DT and CT ΣΔ modulators.

A. *First-order CT ΣΔ Modulator*

For the first order ΣΔ modulator,

$$L1(z) = -(1/z)/(1-(1/z))$$

Its impulse response g(nT) is given by

$$g(nT) = \begin{cases} 0 & \text{for n} < 0 \\ -1 & \text{for n} >= 0 \end{cases}$$

The impulse response of L1(jw) is obtained as

$$h(t) = hc(t) * h_{DAC}(t)$$

Here * denotes convolution. A non-return-to-zero (NRZ) DAC is used as the DAC inside the ΣΔ modulator that is the controller of the DC-DC converter.

Therefore, $h_{DAC}(t) = u(t) - u(t-T)$,

where

$$u(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t >= 0 \end{cases}$$

So we have $H_{DAC}(s) = (1 - \exp(-sT))/s$.

Suppose Hc(s) = A/s (where A is a constant),

then we have

$$H(s) = Hc(s)\,H_{DAC}(s)$$
$$= (A/s)\,[1-\exp(-sT)]/s.$$

Using the Laplace transform, we have

$$h(t) = \begin{cases} 0 & \text{for } t <= 0 \\ A \cdot T & \text{for } t > 0 \end{cases}$$

Thus,
$$h(nT) = \begin{cases} 0 & \text{for } n <= 0 \\ A \cdot T & \text{for } n > 0 \end{cases}$$

Since the inverse Laplace transform of H(s), h(nT) is equal to g(nT), we can calculate

$$A = -1/T$$

So $\quad Hc(T) = -1/(sT)$

We can design the FB CT ΣΔ as shown in Fig.2.

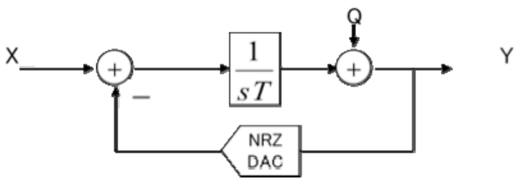

Fig.2 First-order CT FB ΣΔ modulator.

Additionally, we can calculate its signal transfer function as follows:

$$STF(s) = -\,Hc(s)NTF(s)$$
$$= 1/(sT)\,[1-\exp(-sT)]$$

We can design the CT FF ΣΔ as shown in Fig. 3.

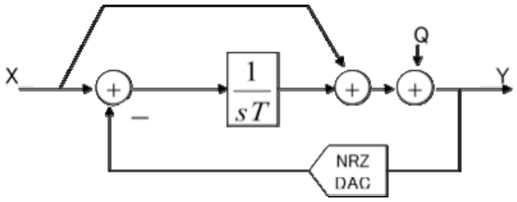

Fig.3 First-order CT FF ΣΔ modulator.

We can calculate its signal transfer function (STF) as follows:

$$STF(s) = [1+ Hc(s)]\,NTF(s)$$
$$= [1+1/(sT)]\,[1-\exp(-sT)]$$

Using Matlab, we obtained Bode plots for the two types of ΣΔ modulators (Fig. 4, Fig. 5).

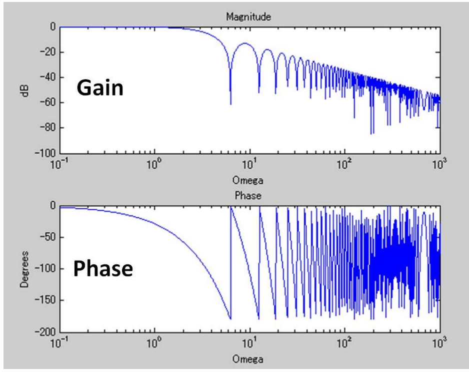

Fig.4. Bode plot of the STF for the first-order CT FB ΣΔ modulator

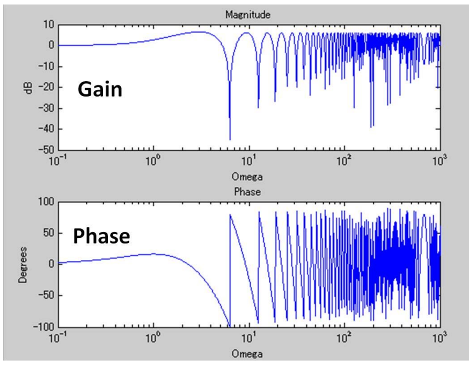

Fig.5. Bode plot of the STF for the first-order CT FF ΣΔ modulator.

The phase delay of the FB ΣΔ modulator increases with angular frequency ω, but that of the FF ΣΔ modulator does not; this is an advantage of the FF ΣΔ modulator as a controller in a feedback system.

B. Second-order CT FF ΣΔ Modulator

We calculate the signal transfer function of second-order CT ΣΔ modulators in a similar manner to that for first-order ΣΔ modulators. For the DT ΣΔ,

$$L1(z) = -\{2+(1/z)/[1-(1/z)]\}\,/[1-(1/z)]$$

Its impulse response g(nT) is given by

$$g(nT) = \begin{cases} 0 & \text{for } n<=0 \\ -(n+1) & \text{for } n>0 \end{cases}$$

The impulse response of L1(jw) is obtained by

$$h(t) = hc(t) * h_{DAC}(t)$$

Since the DAC output is NRZ type,

$$h_{DAC}(t) = u(t) - u(t-T),$$

where

$$u(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t >= 0 \end{cases}$$

So $$H_{DAC}(s) = (1 - \exp(-sT))/s.$$

Suppose $Hc(s) = A/s + B^2/s^2$ (where A and B are constants). Then

$$H(s) = Hc(s)\ H_{DAC}(s)$$

$$= (A/s + B^2/s^2)[1 - \exp(-sT)]/s.$$

Using the inverse Laplace transform, we have

$$h(t) = \begin{cases} 0 & \text{for } t <= 0 \\ A\cdot T & \text{for } t > 0 \end{cases}$$

Thus,

$$h(nT) = \begin{cases} 0 & \text{for } n <= 0 \\ A\cdot T & \text{for } n > 0 \end{cases}$$

Since the inverse Laplace transform of H(s), h(t) at t= nT is equal to g(nT), we find that $A = 3/(2T), B = 1/T^2$.

So $Hc(T) = 3/(2sT) + 1/(sT)^2$.

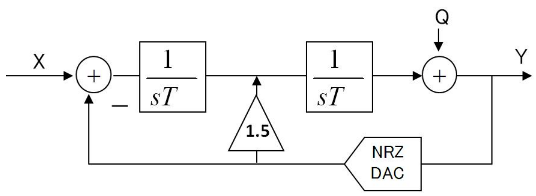

Fig.6 Second-order CT FB ΣΔ modulator.

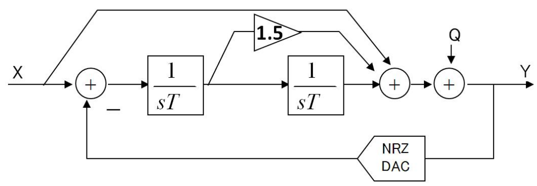

Fig.7 Second-order CT FF ΣΔ modulator.

We can design the second-order FB ΣΔ modulator as shown in Fig.6, and its signal transfer function is given by

$$STF(s) = Hc(s)NTF(s)$$

$$= [2/(sT) + 1/(sT)^2][1 - \exp(-sT)].$$

We can also design a second-order FF ΣΔ modulator as shown in Fig. 7. Its signal transfer function is given by:

$$STF(s) = [1 + Hc(s)]NTF(s)$$

$$= [1 + 3/(2sT) + 1/(sT)^2][1 - \exp(-sT)]$$

Using Matlab, we obtained Bode plots (Fig. 8 and Fig. 9) for these two types of ΣΔ modulator. We see from these figures that the phase delay of FB-type ΣΔ modulator increases with frequency, while that of the FF-type is not delayed; again this is the advantage of the FF-type as a controller in a feedback system.

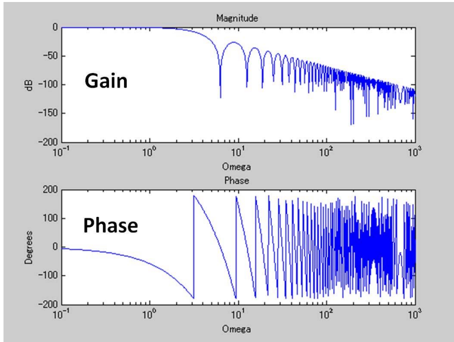

Fig.8. Bode plot of the STF for the second-order CT FB ΣΔ modulator.

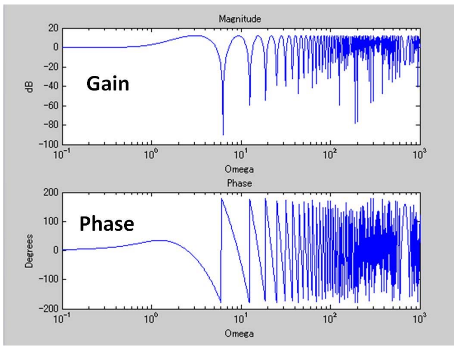

Fig.9 Bode plot of the STF for the second-order CT FF ΣΔ modulator.

Comparing Fig. 4 and Fig. 5, we observe that the second-order CT ΣΔ modulator shows better phase characteristics than the first-order one.

III. SIMULATION RESULTS

In Section II, we showed a theoretical analysis of the CT ΣΔ modulator; we found that the CT FF ΣΔ modulator shows better phase characteristics than the FB one, and the 2nd-order CT ΣΔ is better than the first-order one. We used Simplis 6.00 for simulation to validate the theoretical analysis in Section II, and compared the performance of PWM and various types of ΣΔ modulators as controllers for the buck converter. Table I lists simulation parameters.

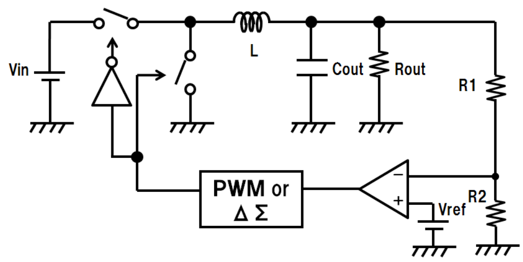

Fig.10. Basic circuit for simulation.

Fig. 11 shows the steady state output voltage waveforms of buck converters controlled by PWM, first and second-order DT, CT FB, and FF ΣΔ modulators. We see that the steady-state output voltage ripple of the buck converter controlled by PWM is smallest. Of the DT ΣΔ controllers, the 2nd-order type was superior to the 1st-order one, and the FF type was superior to the FB type, with less output ripple. However, the steady-state ripple of buck converters controlled by CT ΣΔ was almost identical.

Fig. 12 shows load transient output voltage waveforms of buck converters controlled by various types of modulators. At time 10ms, the output load current is changed from 0.5A to 1.0A. At time 15ms, the current is changed from 1.0A to 0.5A. The different colors stand for different outputs controlled by different modulators; the key to colors is in Table 2.

We see that the output voltage controlled by the CT second-order FF ΣΔ reaches the steady state faster than any other, while the PWM one is the slowest.

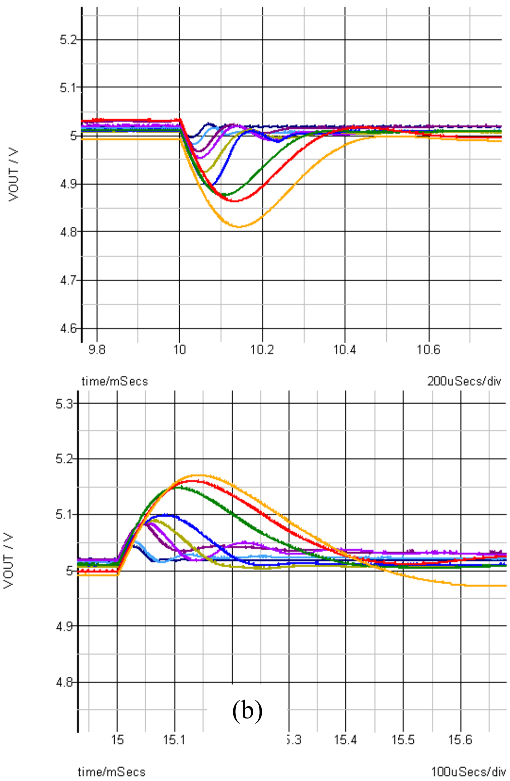

Fig.11. Load transient waveforms.

TABLE I. KEY TO COLORS REPRESENTING OUTPUTS CONTROLLED BY MODULATORS IN FIG.8

Line	Modulator
	PWM
	DT first-order FB ΣΔ
	DT second-order FB ΣΔ
	DT first-order FB ΣΔ
	DT second-order FF ΣΔ
	CT first-order FB ΣΔ
	CT second-order FB ΣΔ
	CT first-order FF ΣΔ
	CT second-order FF ΣΔ

Remark:

(1) Our simulations show that the CT ΣΔ has better transient response than the DT ΣΔ of the same order. This is because there is no delay from the input X(z) to the output Y(z) in the FF ΣΔ.

(2) Our simulations also show that the second-order ΣΔ controller has better output ripple and transient response performance than the first-order one with the same-type ΣΔ. This is because, in the second-order ΣΔ, 2nd-order noise-shaping suppresses low-frequency components of E(z) significantly, and the LC circuit rejects its high-frequency components.

IV. CONCLUSIONS

This paper has proposed using CT FF ΣΔ modulators as controllers for DC-DC converters. Compared with the PWM controller, the ΣΔ modulator can provide faster return to the steady state when the load of the buck converter is changed. We also show that, as DC-DC converter controllers, the CT ΣΔ is superior to the DT modulator, the FF ΣΔ is superior to the FB one, and the second-order ΣΔ is superior to the first-order in fast transient response.

We close this paper by remarking that the STF *gain* characteristics of the CT ΣΔ modulator is often considered for the AD converter application (because the CT ΣΔ modulator can incorporate anti-aliasing filtering function), however we pay attention here to its STF *phase* characteristics for the feedback control applications, which would be a new and interesting theoretical issue for the CT ΣΔ modulator.

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