

A New Procedure for Measuring High-Accuracy Probability Density Functions

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Abstract

This paper proposes a new procedure for calculating high-accuracy PDF estimates, which are free of random error and nearly free of bias error. The procedure is verified experimentally using random jitter and a 16-bit ADC.

1. Introduction

It is well established that jitter in high-speed serial I/O devices, and differential nonlinearities (DNL) in ADCs are effectively tested by characterizing timing or output codes in a probabilistic manner [1], [2], [3]. Fail count or bit error rate (BER) is easily measured by comparing the output voltage from the DUT with a pre-defined voltage of logic one or zero at the strobe. An ATE or an on-chip checker circuit commonly implements a fail counter [4], [5]. These error counts correspond to a cumulative distribution function (CDF). From this, a probability density function (PDF) can be constructed by differentiating the CDF (BER) curve.

Similarly, to measure differential and integral nonlinearities in an ADC, a PDF is constructed directly from the output digital codes of the ADC, into which an analog sinusoid is typically applied. However, the sinusoid spends more time around its maxima and minima, and less time around its zero crossings. Therefore when a sinusoid is captured, less code count is expected around the ADC middle code. Hence, this type of code density test requires that a fairly large number of samples be captured [3]. The higher the ADC resolution, the more difficult it becomes to perform an effective statistical test, gaining sufficient accuracy without utilizing long test times.

A fundamental problem in statistical testing is that a PDF estimate from a sequence of sampled events is an ill-posed measurement [6], and its properties are not well understood. In addition, there is a tradeoff between the bias error and random error of the PDF [7].

Accurate PDF measurements cannot be realized by simply increasing the bandwidth-time (BT) product. If a PDF is measured using high time resolution, its bias

error is reduced. However, this increased time resolution results in increased random error, as shown in Fig. 1(c).

This paper revisits the statistical errors in PDF measurements, and proposes a new procedure to calculate high-accuracy PDF estimates that are nearly free of bias error and random error. This new PDF measurement procedure is validated experimentally using random jitter and a 16-bit ADC.

In Section 2 of this paper, statistical error in PDF estimates is revisited. Existing nonparametric approaches are also discussed. In Section 3, the theory for measuring a high-accuracy PDF is developed, and it is experimentally validated in Section 4. Finally, the advantages and limitations of the proposed procedure are discussed in Section 5.

2. Review of Previous Works

In this section, the two basic parameters of a random variable (RV) used in conjunction with a CDF are defined. Also, statistical errors in PDF estimates are discussed, and existing approaches to PDF estimation are reviewed.

2.1 Probability Fundamentals

Continuous and Discrete RVs [8]. A random variable (RV) \mathbf{t} is a real-valued function of the elements of a sample space, S . Given an experiment E in sample space, S , RV \mathbf{t} maps each possible outcome $x \in S$ to a real number $\mathbf{t}(x)$. If the range of \mathbf{t} is a non-countable infinite number of points, we refer to \mathbf{t} as a *continuous random variable*. On the other hand, if the mapping $\mathbf{t}(x)$ is such that the random variable \mathbf{t} takes on a finite or an infinite but countable set of values in S , then \mathbf{t} is referred to as a *discrete random variable*.

It is important to note that a discrete random variable can be defined on a continuous sample space. For example, a discrete random variable has value 1 for the set of outcomes $\{0 < s < 6\}$ and 0 for $\{6 < s < 13\}$. This establishes a common ground for a fail counter.

Cumulative Distribution Function (CDF) [9]. A non-decreasing function $F(t)$ defined on the whole real line and satisfying the following conditions (1), is called a *cumulative distribution function* of the RV \mathbf{t} .

$$F(t) = \mathbf{P}\{\mathbf{t} \leq t\} \quad (1)$$

$F(t)$ is defined for every \mathbf{t} from $-\infty$ to ∞ . $F(t)$ has the following properties [10]:

A. $\lim_{t \rightarrow -\infty} F(t) = 0$ and $\lim_{t \rightarrow \infty} F(t) = 1$ (2)

B. It is a non-decreasing function of \mathbf{t}

$$F(t_1) \leq F(t_2) \quad t_1 \leq t_2 \quad (3)$$

C. The function $F(t)$ is continuous from the right:

$$F(t^+) = F(t) \quad (4)$$

Probability Density Function (PDF). The PDF $f(t)$ is defined as the *derivative* of the CDF $F(t)$:

$$\begin{aligned} f(t) &= \frac{dF(t)}{dt} = \lim_{W \rightarrow 0} \frac{F(t+W) - F(t)}{W} \\ &= \lim_{W \rightarrow 0} \frac{F(t, W)}{W} \end{aligned} \quad (5)$$

The *mean* of a random variable \mathbf{t} depends only on the distribution of \mathbf{t} . Therefore, if the CDF is known, the mean $E[\mathbf{t}]$ can be expressed as the *integral* function of it with respect to $F(t)$ [9].

$$E[\mathbf{t}] = \int_{-\infty}^{+\infty} t dF = \int_{-\infty}^{+\infty} t f(t) dt \quad (6)$$

The *variance* of a random variable \mathbf{t} also depends only on its distribution. $Var[\mathbf{t}]$ is also expressed as an *integral* function of \mathbf{t}^2 with respect to $F(t)$ [9]:

$$\begin{aligned} Var[\mathbf{t}] &= \int_{-\infty}^{+\infty} t^2 dF - \left(\int_{-\infty}^{+\infty} t dF \right)^2 \\ &= \int_{-\infty}^{+\infty} (t - E[\mathbf{t}])^2 f(t) dt \end{aligned} \quad (7)$$

If a CDF is constant except for a finite number of jump discontinuities (i.e., it is piecewise constant), then \mathbf{t} is said to be a discrete random variable, and its PDF has the general form

$$f(t) = \sum_j p_j \delta(t - t_j)$$

In the case of a discrete RV, the PDF is known as the probability mass function (PMF).

The *mean* of the discrete RV is given from (6) by:

$$E[\mathbf{t}] = \int_{-\infty}^{+\infty} t \sum_j p_j \delta(t - t_j) dt = \sum_j p_j t_j \quad (8)$$

The *variance* of the discrete RV is given from (7) by:

$$\begin{aligned} Var[\mathbf{t}] &= \int_{-\infty}^{+\infty} (t - E[\mathbf{t}])^2 \sum_j p_j \delta(t - t_j) dt \\ &= \sum_j p_j (t_j - E[\mathbf{t}])^2 \end{aligned} \quad (9)$$

Example 1. Gaussian RV [10]. An RV \mathbf{t} is called Gaussian if its PDF is the **normal** curve:

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} \quad (10)$$

and its CDF is given by:

$$F(t) = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{t - \mu}{\sqrt{2}\sigma} \right) \right] \quad (11)$$

where $\operatorname{erf}(z)$ denotes the error function.

Example 2. Sinusoidal RV [7]. An RV \mathbf{t} is called sinusoidal if its PDF is the *bathtub* curve:

$$f(t) = \frac{1}{\pi\sqrt{a^2 - t^2}} \quad (12)$$

and its CDF is given by:

$$F(t) = \begin{cases} 0 & t < -a \\ \frac{1}{2} + \frac{1}{\pi} \sin^{-1} \frac{t}{a} & |t| < a \\ 1 & t \geq a \end{cases} \quad (13)$$

Example 3. Binomial RV [11]. This is a very important random variable for this application. If a coin is tossed n times, and a head (H) turns up with a probability of p , the sample space of this experiment is represented as $S = \{H, T\}^n$. Let \mathbf{x} be the total number of heads. The variable \mathbf{x} takes values in the set $\{0, 1, \dots, n\}$ and is discrete. Exactly $\binom{n}{k}$ points in S gives a total of k heads; and each point occurs with a probability of $p^k q^{n-k} = p^k (1-p)^{n-k}$:

$$f(k) = \binom{n}{k} p^k (1-p)^{n-k} \quad (14)$$

This is known as a *binomial random variable*, and it takes on integer values from 0 to n . It is the sum $\mathbf{x} = \mathbf{y}_1 + \mathbf{y}_2 + \dots + \mathbf{y}_n$ of n Bernoulli variables.

The mean and the variance of the Bernoulli RV are:

$$E[\mathbf{x}] = p \quad (15)$$

$$Var[\mathbf{x}] = pq = p(1-p) \quad (16)$$

Hence, the mean and the variance of the binomial RV are:

$$E[\mathbf{x}] = np \quad (17)$$

$$Var[\mathbf{x}] = npq = np(1-p) \quad (18)$$

It is important to note that binomial random variables occur when the number of errors in a DUT are counted by a fail counter in either an ATE digital channel or in an on-chip circuit.

Histogram [6], [7]. A histogram is a plot of the number of measured values of a quantity that partitions t into distinct bins of width W .

2.2 Statistical Errors in PDF Estimates

The PDF in Fig. 1(a) was obtained by dividing the count in each distinct bin by the total number of observations N_{event} and by the bin (interval) width W [12]. From Fig. 1(c), it can be seen that the high-resolution PDF is noisy. This is an artifact of the bin-edge partitions rather than any property of the distribution of the PDF distribution itself [6]. The noise corresponds to statistical errors.

Theoretically, the total mean squared error of the measured PDF $\hat{f}(t)$ is given by [7]:

$$E\left[\left(\hat{f}(t) - f(t)\right)^2\right] = \frac{c^2 f(t)}{2BTW} + \left[\frac{W^2 f''(t)}{24}\right]^2 \quad (19)$$

where W is a finite length window, and $2BT$ corresponds to the number of independent estimates N_{event} . The first term is the variance of an estimate $\hat{f}(t)$ and the second term is the square of the bias.

It is clear from (19) that there are conflicting requirements on the bin width W in PDF measurements.

In order to suppress the bias error $\frac{W^2 f''(t)}{24}$ a small value of W is needed. However, the estimated PDF becomes very spiky, i.e., it contains more random error, as shown in Fig. 1(c).

Alternatively, to reduce the random error in the PDF, represented by $\frac{c^2 f(t)}{2BTW}$, a large bin width W is desirable. This results in a PDF that is *too smooth* as shown in Fig. 1(a). Hence, it would be easy to miss the multimodal nature of the true distribution in PDF.

2.3 Existing Approaches to Estimating a PDF

Parametric approaches to PDF estimation assume prior knowledge of the distribution, e.g. it is Gaussian, sinusoidal, etc. Also, the values of the distribution parameters, such as mean and variance in the case of Gaussian, are assumed to be determined from measured counts.

The model-based approach for estimating the PDF of discrete RVs was proposed in [13]. This approach

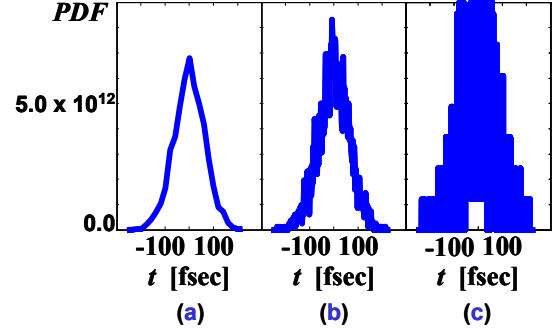


Figure 1. Measured PDFs $f(t)$ from aperture jitter histograms. (a) PDF in time resolution $W = 20$ fs. (b) PDF in $W = 2$ fs. (c) PDF in $W = 0.2$ fs.

assumes that the input histogram consists of equidistant bins. A multi-rate filtering on the input bins can then be performed to obtain an orthogonal projection of the given histogram onto a low-frequency subspace V_0 . It was shown that the model-based PDF estimates are unbiased and have smaller variance than the histogram-based PDF estimates.

However, in order to minimize the variance, a set of PDFs is required as a training set for optimizing the filter coefficients. In Section 4, it will be demonstrated that our new procedure minimizes both the random and bias errors in PDF estimates without requiring a training set.

Nonparametric approaches to PDF estimation make few assumptions about the distributions of the RVs.

One such approach uses the *kernel density estimator*. This type of estimator suffers from the presence of artificial discontinuities at the boundaries of cubes (= D -dimensional bins) however.

A relatively smooth kernel function such as the Gaussian function can be used to obtain a smooth PDF, [6]. In order to obtain a smooth kernel function though, it is implicitly assumed that the input counts N_{bin} are uniformly distributed over t , and that a sufficiently large number of input counts N_{bin} is used. Since test time is negatively impacted by a large N_{bin} , the *kernel density estimator* is inappropriate for production testing.

2.4 Limitations of the Existing Approaches

The limitations of the existing approaches can be summarized as follows:

- If a PDF is directly estimated from a histogram, conflicting requirements on the bin width W are inevitable. To reduce the bias error, a *small* value of W is needed, but to reduce random error, a *large* value of W is required.
- The model-based approach for estimating the PDF requires design of a multi-rate filter. The model-based PDF estimates are unbiased, but a

training set of PDFs is required to minimize error.

- Nonparametric approaches, such as the *kernel density estimator*, suffer from the presence of artificial discontinuities at the boundaries of cubes.

3. Theory of High-Accuracy PDF Estimates

The theory in support of our procedure for calculating high-accuracy convex (e.g., Gaussian) or concave (e.g., sinusoidal) PDFs is now presented. Since the random error in (19) is caused by the difference operation in (5), our procedure performs no difference operations on $F(t, W)$.

3.1 PDF Estimates

The variance of $F(t, W)$ based on N independent sample values at t is given by [7]:

$$\text{Var}[F(t, W)] = \frac{F(t, W)[1 - F(t, W)]}{N} \quad (20)$$

Note that $p = F(t, W)$ and $q = 1 - F(t, W)$ are substituted into (16) to obtain the variance. Substituting (5) into (20), the variance at t is approximated by:

$$\text{Var}[F(t, W)] \approx \frac{W}{N} f(t)[1 - Wf(t)]$$

Hence, PDFs can be computed from the quadratic form:

$$Wf^2(t) - f(t) + \frac{N}{W} \text{Var}[F(t, W)] = 0$$

The real valued roots of the quadratic equation are given by:

$$f_+(t) \approx \frac{N}{W} \text{Var}[F(t, W)] \quad (21)$$

$$f_-(t) \approx \frac{N}{W} \left\{ \frac{1}{N} - \text{Var}[F(t, W)] \right\} \quad (22)$$

Note that, since $f_+(t)$ and $f_-(t)$ have opposite signs, $f_+(t)$ provides a convex PDF while $f_-(t)$ provides a concave PDF. Furthermore, since both (21) and (22) have no difference calculation with respect to W , (19) theoretically predicts a bias-free estimate of the PDF. These properties will be validated by both numerical examples & experiment in Section 4.1 & 4.2, respectively.

If $F(t, W)$ is given as counts of logic <1> from a fail counter, the distribution follows a binomial distribution. Therefore $\text{Var}[F(t, W)]$ at t can be

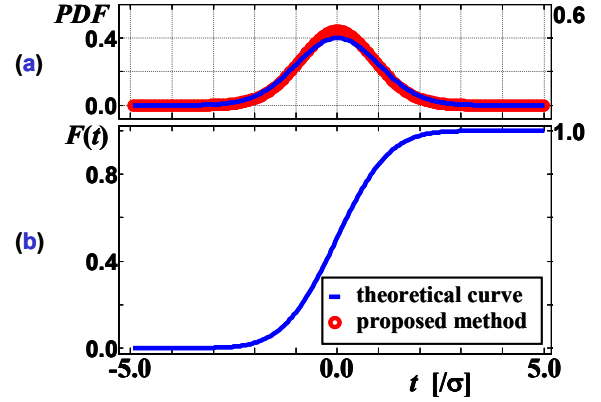


Figure 2. Calculated PDF $f_+(t)$ and theoretical PDF of a Gaussian distribution. (a) PDFs. (b) CDF.

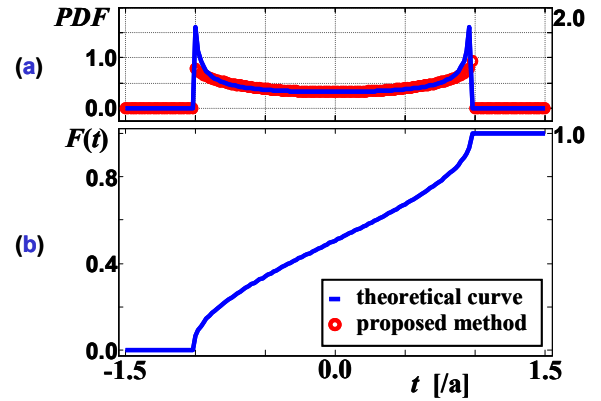


Figure 3. Calculated PDF $f(t)$ and theoretical PDF of a sinusoidal distribution. (a) PDFs. (b) CDF.

calculated using (16).

3.2 PDF Estimation Procedure

The procedure for estimating a PDF from a CDF is outlined in the following steps:

- **Step 1.** Measure a digital CDF.
- **Step 2.** For each t , calculate $\text{Var}[F(t, W)]$ using (16).
- **Step 3.** Determine if the distribution $f(t)$ is convex or concave or flat using the PDF model identifier [14].
- **Step 4.1.** If the distribution is convex, calculate a PDF $f_+(t)$ at t using (21).
- **Step 4.2.** If the distribution is concave, calculate a PDF $f_-(t)$ at t using (22).

- **Step 4.3.** If the distribution is flat, calculate a uniform PDF.

4. Performance Validation and Comparison

Methods for calculating a Gaussian PDF and a sinusoidal PDF are now illustrated using numerical examples. The resulting PDFs are compared with those of the histograms method.

4.1 Numerical Examples

4.1.1 Gaussian Distribution (Convex PDF)

From (11), probabilities p and q of the Gaussian RV \mathbf{t} taking values $\langle 1 \rangle$ and $\langle 0 \rangle$ are given by:

$$p = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{t - \mu}{\sqrt{2}\sigma} \right) \right], \quad q = \frac{1}{2} \operatorname{erfc} \left(\frac{t - \mu}{\sqrt{2}\sigma} \right)$$

Thus, from (21) and a calculated $\operatorname{Var}[F(t, W)]$, the PDF $f_+(t)$ becomes

$$f_+(t) = \frac{1}{4} \left[1 + \operatorname{erf} \left(\frac{t - \mu}{\sqrt{2}\sigma} \right) \right] \operatorname{erfc} \left(\frac{t - \mu}{\sqrt{2}\sigma} \right)$$

The theoretical curve of the Gaussian CDF is plotted in Fig. 2(b). In Fig. 2(a), the theoretical Gaussian PDF is compared with the calculated PDF $f_+(t)$ using (21).

4.1.2 Sinusoidal Distribution

From (13), probabilities p and q of the sinusoidal RV \mathbf{t} taking values $\langle 1 \rangle$ and $\langle 0 \rangle$ are given by:

$$p = \frac{1}{2} + \frac{1}{\pi} \sin^{-1} \frac{t}{a}, \quad q = \frac{1}{2} - \frac{1}{\pi} \sin^{-1} \frac{t}{a}$$

Thus, from (22) and a calculated $-\operatorname{Var}[F(t, W)]$, the PDF $f_-(t)$ becomes

$$f_-(t) = \frac{1}{4} - \frac{1}{\pi^2} \left(\sin^{-1} \frac{t}{a} \right)^2$$

The theoretical curve of the sinusoidal CDF is plotted in Fig. 3(b). In Fig. 3(a), the theoretical sinusoidal PDF is compared with the calculated PDF $f_-(t)$ using (22).

Both the calculated Gaussian PDF and the calculated sinusoidal PDF agree with their theoretical PDF curves, respectively. As being predicted by (19), there is no random error and a very small bias error over the region of $\max|f''(t)|$. These results validate the proposed procedure.

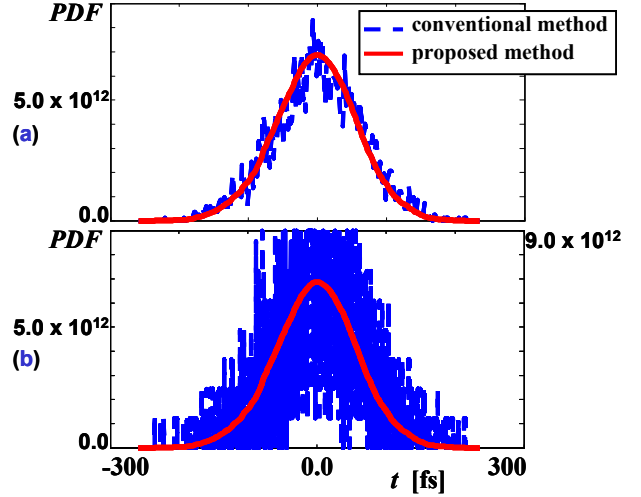


Figure 4. Measured Gaussian PDFs $f(t)$ and $f_+(t)$. (a) PDFs with 2 fs resolution. (b) PDFs with 0.2 fs resolution.

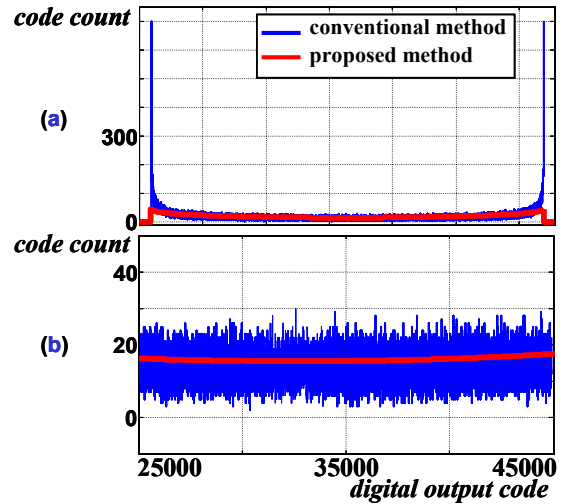


Figure 5. Estimated PDF $f_-(t)$ of the code density. A 16-bit ADC, $f_s = 100.8576$ MS/s, $f_{in} = 200.0008$ MHz, $A = 0.65$ V. (a) PDF. (b) Zoomed PDF.

4.2 Experimental Results

4.2.1 Aperture Jitter PDF

The conventional PDFs and the new PDF estimates $f_+(t)$ of the aperture jitter waveforms were measured by applying our previous method [15] and the currently proposed procedure to an ADC output. The ADC under test was a 16-bit, 130 MS/s, with $V_{DD} = 3.3$ V ADC. An SMA 100A (Rohde & Schwarz) signal generator provided both a sinusoid of amplitude -0.5 dBFS at frequency $f_{in} = 174.8$ MHz and a sampling clock of amplitude $A = 0.6$ V_{pp} at frequency $f_s = 102.4$ MHz.

The aperture jitter PDF is plotted in blue in Fig. 4(a). It follows a Gaussian distribution and was measured with a conventional histogram method with $W = 2$ fs. The

red PDF $f_+(t)$, plotted in red in Fig. 4(a), was calculated from the CDF obtained from the conventional PDF in blue by using the proposed method.

The aperture jitter PDF plotted in blue in Fig. 4(b), was measured with the conventional histogram method, with $W = 0.2$ fs. The PDF $f_+(t)$, plotted in red in Fig. 4(b), was calculated from the CDF obtained from the conventional PDF in blue using the proposed method.

This experimentally validates that our new procedure for measuring a Gaussian PDF, $f_+(t)$, is free of both bias error and random error. Note that, since the proposed procedure performs no difference operation on the CDF using W , this bias-free property of the PDF estimate is theoretically predicted by (19). Note also that an identical value of 59.5 fs for the standard deviation is obtained from the two PDFs $f_+(t)$, even with different values of W .

4.2.2 Code Density PDF

The code density PDFs were measured by applying a sinusoid input to an ADC. The ADC under test was a 16-bit, 130 MS/s, with $V_{DD} = 3.3$ V ADC. An SMA 100A (Rohde & Schwarz) signal generator provided both a sinusoid of amplitude $2A = 1.3$ V_{PP} at frequency $f_{in} = 200.0008$ MHz and a sampling clock at frequency $f_s = 104.8576$ MHz.

The code density PDF is plotted in blue in Fig. 5(a), and follows the sinusoidal distribution. The PDF $f_-(t)$ plotted in red in Fig. 5(a) and Fig. 5(b) was measured from the CDF obtained from the conventional PDF using the proposed procedure.

This experimentally validates the procedure for estimating a high-resolution sinusoidal PDF $f_-(t)$ using Eq. (22). From Fig. 5(b), it is clear that $f_-(t)$ is also free of random error and is nearly free of bias error.

5. Advantages and Limitations of the Proposed Procedure

Advantages

- The PDF estimates obtained from the proposed procedure are free of random error and *nearly* free of bias error.
- Since the new PDF estimates are free of random error, the resolution of the estimates is effectively enhanced, as illustrated in Fig 4.
- The proposed procedure requires no PDF training set.
- Test times are shortened by at least 10x, since the PDF estimates don't require large numbers of samples.

Limitations

- Different calculations are required depending on the

shape of the PDF; convex, concave and flat.

6. Conclusion

A new procedure for calculating high-accuracy PDF estimates was introduced. The procedure was verified numerically, and was also validated experimentally using random jitter and a 16-bit ADC. It was shown that the procedure provides $f_+(t)$ or $f_-(t)$, both of which are free of random error and nearly free of bias error.

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