Consideration of Uncertainty Principle in Sampling Circuit

Miho Arai, Isao Shimizu, Haruo Kobayashi
Department of Electronics, Graduate School of Engineering, Gunma University
1-5-1 Tenjin-cho, Kiryu 376-8515, Japan k_haruo@el.gunma-u.ac.jp

A sampling technique is important for waveform acquisition and a sampling circuit performs its operation. However, actual sampling circuit suffers from many non-idealities, and we consider here one of its non-ideality, the aperture time [1], from the viewpoint of uncertainty principle between time and frequency [2,3], and provide some new insights.

The uncertainty relation is well known in Fourier transform theorem; it shows that narrow waveform yields wide spectrum while wide waveform yields narrow spectrum, and both the time waveform and frequency spectrum cannot be made arbitrarily small simultaneously. The time-bandwidth product theorem, or uncertainty principle, is a fundamental statement regarding Fourier transform pairs.The uncertainty relation in the following equation:

$$\sigma_t \sigma_\omega \geq \frac{1}{2} \qquad (1)$$

$$\begin{cases} \sigma_t : \text{duration} \\ \sigma_\omega : \text{bandwidth} \end{cases}$$

Fig.1(a) shows ideal sampling circuit with zero on-resistance in track mode. A capacitor requires finite aperture time τ for charging and also actual clock has finite value of τ (Fig.1(b)). This τ corresponds to σ_t of Eq.(1), which means that the sampling circuit is restricted by the uncertainty principle.

The actual sampling circuit has finite on-resisitance R in track mode as well as the finite turn-off time of the switch (Fig. 2), which affects the circuit response (Fig2(b)).We have derived a Eq.(2) as a transfer function [2].

$$\frac{V_C}{V_{in}} = \frac{sinc(\omega\tau)}{sinc(\omega\tau) + j\omega RC} \qquad (2)$$

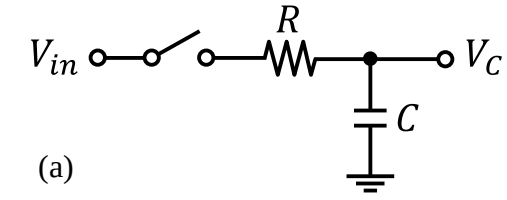
We consider to have some relationships between Eqs.(1), (2) and expect that Eq.(2) yields to Eq.(3).

$$\sigma_t \sigma_\omega + \alpha \geq \frac{1}{2} \qquad (3)$$

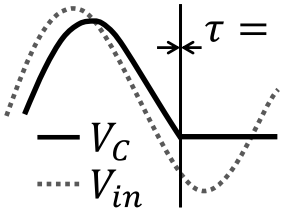
α :non-ideal characteristic

The parameter α (as well as σ_t) restricts the bandwidth of the sampling circuit.

We note that the circuit performance consideration from the uncertainty principle viewpoint will give us many interesting insights in analog circuit design as the performance approaches to the utlimate.

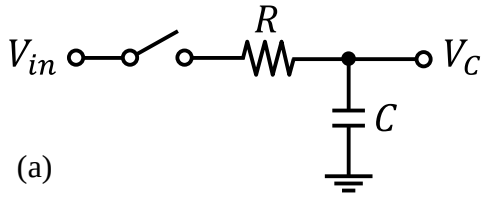


(a)

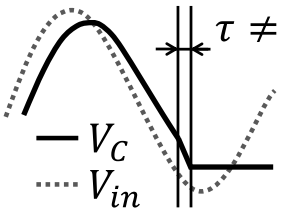


(b)

Fig.1 ideal
(a) Sampling circuit with on-resitance and without finite apertute time.
(b) Circuit response.



(a)



(b)

Fig.2 real
(a) Sampling circuit with on-resitance and finite apertute time.
(b) Circuit response.

[1]A.A.Abidi, M. Arai, et. al., "Finite Aperture Time Effects in Sampling Circuit", 24th IEICE Workshop on Circuits and Sysems (Aug. 2011)
[2]T.Tuduki,*The Uncertainty Principle – Challenge to Fate*, Kodansha (Sep. 2002)
[3] L.Cohen, *Time-Frequency Analysis*, Printice Hall (1995).