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Consideration of Uncertainty Principle in Sampling Circuit

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Introduction

Research Objective

■ **Background**

Waveform acquisition ➡ Sampling

Sampling circuit ➡ Many non-idealities

Aperture time

Serious for high frequency signal

■ **Objective**

・Sampling circuit with aperture time

・Uncertainty principle

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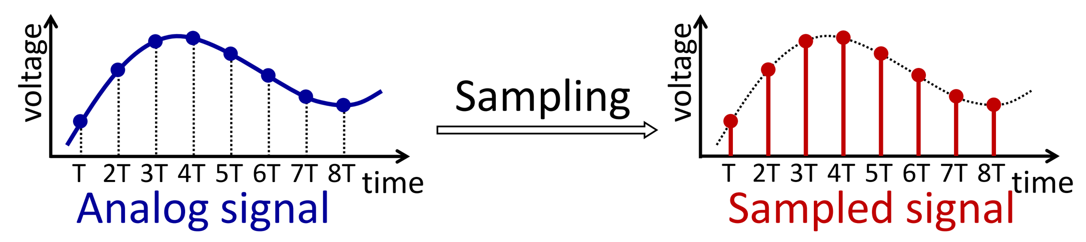
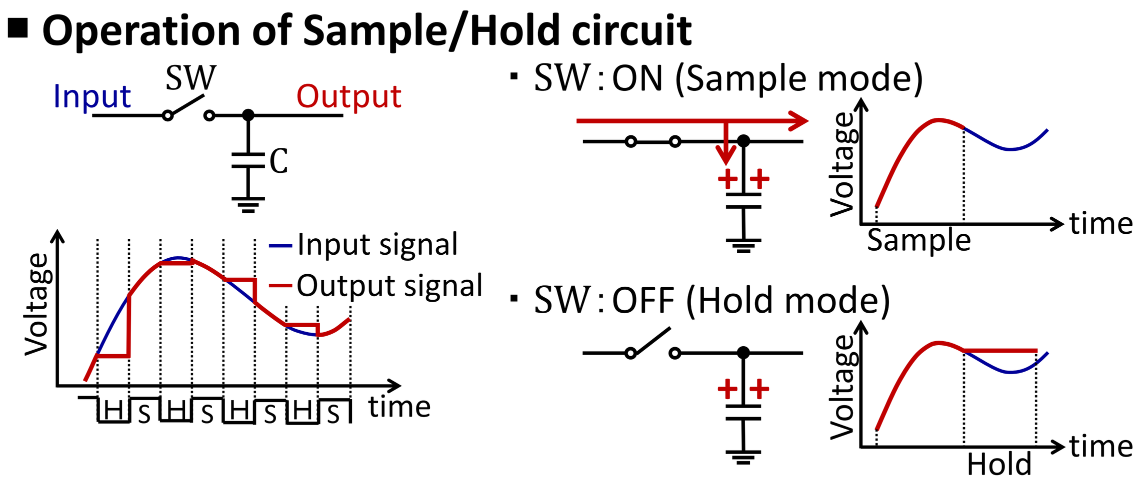
Theoretical foundation for signal acquisition

Sampling

■ **Sampling**



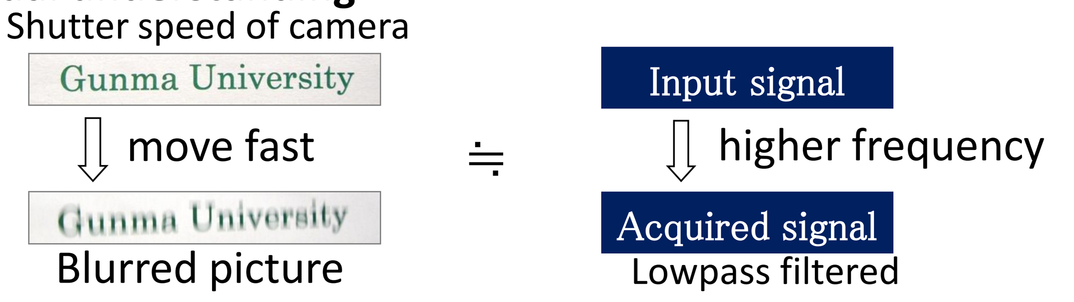
■ **Operation of Sample/Hold circuit**



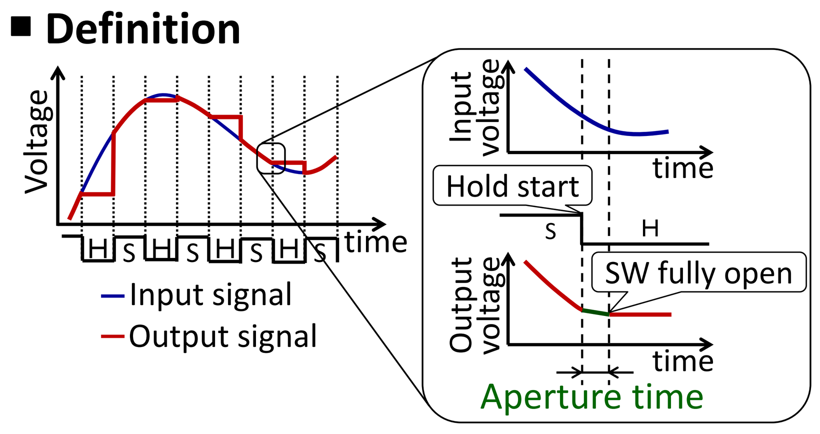
Aperture Time

■ **Visual understanding**

ex) Shutter speed of camera



■ **Definition**



Integration from hold start until switch open

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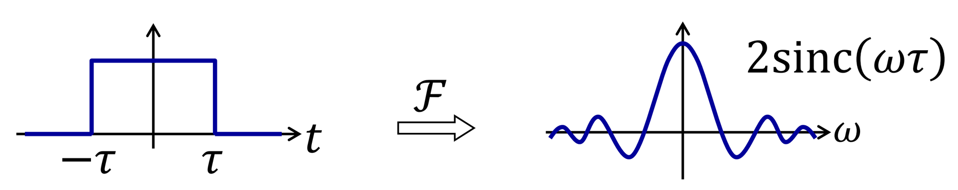
Aperture time

Uncertainty Principle

Uncertainty Principle

Fourier transform of rectangular pulse

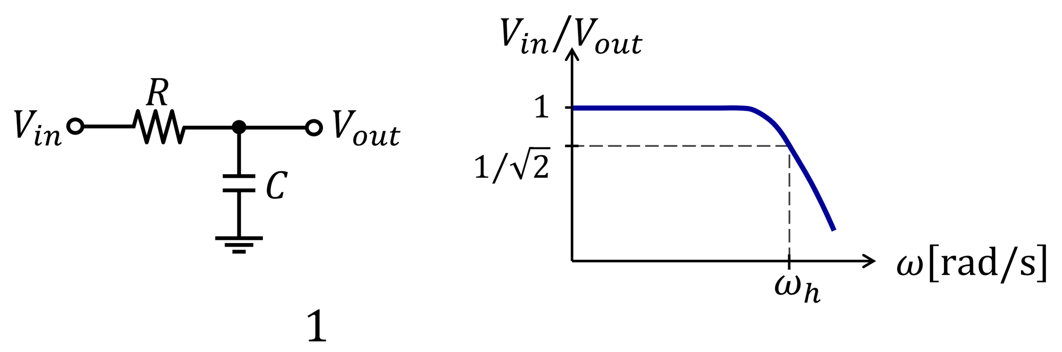


Pulse width : σ_t		Spectrum width : σ_ω
Narrow	⟶	Wide
Wide	⟶	Narrow

$$\sigma_\tau\sigma_\omega \geq \frac{1}{2}$$: Uncertainty Principle

RC Circuit and Transfer function

RC Circuit



$$\omega_h = \frac{1}{\tau_1}$$

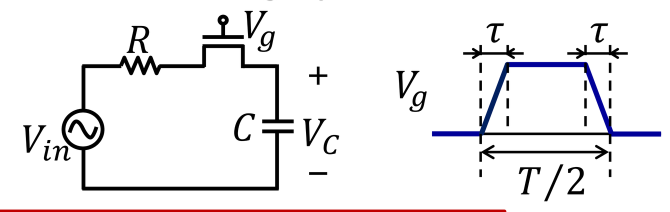
τ_1 : Time constant RC

ω_h : Cutoff frequency

Time constant : τ_1		Pass band : ω_h
Small	⟶	Wide
Large	⟶	Narrow

Transfer Function for RC Sampling Circuit with Aperture Time

■ **Formula considering aperture time**



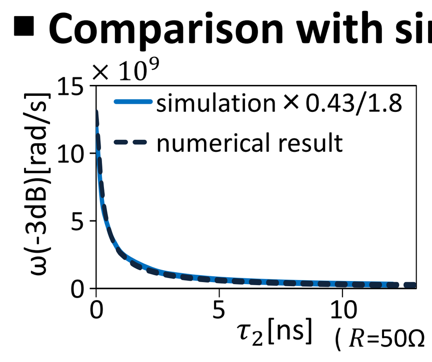
τ : Aperture time

$$\frac{V_C}{V_{in}} = \frac{\mathrm{sinc}(\omega\tau_2)}{\mathrm{sinc}(\omega\tau_2) + j\omega\tau_1}$$

$\tau_1 = RC$

$\tau_2 =$ Aperture time

■ **Comparison with simulation**



(R=50Ω , C=1pF, L=180nm, W=20um, T=0.1us, V_g=0~1.8 V, V_{in}=0~300 mV)

Simulation results ≅ theoretical value

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The derived formula is reasonable

Uncertainty Principle and Sampling Circuit

Derivation Transfer Function of Gain=-3[dB]

$$20\log\left|\frac{V_C}{V_{in}}\right| = 20\log\sqrt{\frac{1}{2}}$$

$$\left|\frac{V_C}{V_{in}}\right| = \left|\frac{\mathrm{sinc}(\omega\tau_2)}{\mathrm{sinc}(\omega\tau_2) + j\omega RC}\right|$$

$$= \sqrt{\frac{\mathrm{sinc}^2(\omega\tau_2)}{\mathrm{sinc}^2(\omega\tau_2) + (\omega RC)^2}} = \sqrt{\frac{1}{2}}$$

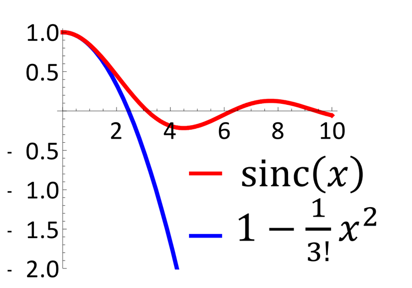
⇩

$$\mathrm{sinc}(\omega\tau_2) = \omega\tau_1$$

Application of Taylor Expansion

■ **Taylor expansion of *sinc* function**

$$\mathrm{sinc}(x) = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n} \cong 1 - \frac{1}{3!}x^2$$



$$\mathrm{sinc}(x) \geq 1 - \frac{1}{3!}x^2$$

$$\therefore\ \omega\tau_1 \geq 1 - \frac{1}{3!}(\omega\tau_2)^2$$

■ **To rewrite the standard derivation**

$\omega \rightarrow \sigma_\omega$

$\tau_1 \rightarrow \sigma_{\tau_1}$

$$\sigma_\omega\sigma_{\tau_1} + \frac{1}{6}(\sigma_\omega\tau_2)^2 \geq 1$$

Consideration of $\sigma_\tau\sigma_\omega \geq 1/2$

■ **LPF Design**

$\sigma_\omega\sigma_\tau \geq 1/2$

σ_ω : bandwidth

σ_τ : time constant

Narrow bandwidth

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Time constant increase

Large R, C chip area

■ **High frequency signal sampling**

$\sigma_\omega\sigma_\tau = 1/2$

σ_ω : bandwidth

σ_τ : aperture time

Wideband

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Short aperture time

Uncertainty Relationship with Aperture Time

$$\sigma_\omega\sigma_{\tau_1} + \frac{1}{6}(\sigma_\omega\tau_2)^2 \geq 1$$

σ_ω : bandwidth

σ_{τ_1} : time constant

τ_2 : aperture time

■ **High frequency signal sampling**

$$\sigma_\omega\sigma_{\tau_1} + \frac{1}{6}(\sigma_\omega\tau_2)^2 = 1$$

RC time constant $\sigma_{\tau_1} > 0$

Obtain the same bandwidth σ_ω

Shorter aperture time τ_2

summary

Conclusion

We have clarified design trade-off among RC time constant, aperture time & bandwidth in sampling circuit with uncertainty principle.

$$\sigma_\omega\sigma_{\tau_1} + \frac{1}{6}(\sigma_\omega\tau_2)^2 = 1$$

σ_ω : bandwidth

σ_{τ_1} : time constant

τ_2 : aperture time

Publications & References

■ **Conference presentation**

[1] A.A.Abidi, M. Arai, et. al., "Finite Aperture Time and Sampling Bandwidth", IEICE General Conference (Mar. 2011)

[2]A.A.Abidi, M. Arai, et. al., "Finite Aperture Time Effects in Sampling Circuit", 24th IEICE Workshop on Circuits and Systems (Aug. 2011)

[3]M. Arai, "Analysis of Non-ideal Factors of T/H Circuit", 51th System LSI Joint Seminar (Jun. 2012)

■ **References**

[1]T.Tuduki, *The Uncertainty Principle – Challenge to Fate*, Kodansha (Sep. 2002)

[2] L. Cohen, *Time-Frequency Analysis*, Prentice Hall (1995).

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