

# 通信用 IC テスト用 I, Q 信号発生のための 複素マルチバンドパス $\Delta\Sigma$ DA 変調器の検討(1)

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## A Study of Complex Multi-Bandpass $\Delta\Sigma$ DA Modulator for I-Q Signal Generation (1)

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This paper describes application of a complex bandpass  $\Delta\Sigma$  DA modulator to I-Q signal generation for communication IC testing as well as transmitter. We show that the complex bandpass  $\Delta\Sigma$  DA modulator is superior to two real-bandpass  $\Delta\Sigma$  DA modulators regarding to noise-shaping characteristics (hence the trade-off between bandwidth and sampling speed is better for the complex bandpass  $\Delta\Sigma$  DA modulator.) We examine the characteristics of the complex bandpass  $\Delta\Sigma$  DA modulator and its extension - a complex multi-bandpass modulator. We present their theoretical analysis and simulation results.

(Keywords: Complex Signal, Complex Filter, Complex Bandpass, Multi-band,  $\Delta\Sigma$  DA Modulator, I-Q Signal)

### 1. INTRODUCTION

Demands for low cost, low power and high performance of a digital-to-analog converter (DAC) (Fig.1) are significantly increased especially in communication applications. Since communication devices become inexpensive and more sophisticated, the DAC circuits in their transmitter parts (which often generate I-Q signals) become more complicated and challenging. Thanks to the advancement of VLSI process technology, VLSI fabrication cost is reduced. A successful market of the portable devices has contributed to this matter. Most of these devices are requiring for a low power DAC owing to their operating voltage which mostly uses battery.

On the other hand, their testing cost increases due to the circuit complexity and high specification requirements. The testing of the communication ICs requires high quality I-Q signal at low cost, and in many cases their DACs are required for I-Q signal generation.

This paper discusses applicability of a complex bandpass  $\Delta\Sigma$  D/A modulator to generate I-Q signals with digital rich configuration.



Fig.1 DAC with its input and output signals.

### 2. DAC FOR I-Q SIGNAL GENERATION

This section discusses pros and cons of the architectures for I-Q signal generation. The architecture can be classified as follows:

(1) Analog method

(2) Digital method

(DSP + DAC, or Direct Digital Synthesizer)

2-1) DSP + 2 Nyquist-rate DACs + 2 analog filters

2-2) DSP + 2 real-bandpass  $\Delta\Sigma$  DACs + 2 analog filters

2-3) DSP + 1 complex-bandpass  $\Delta\Sigma$  DAC

+ 1 analog complex filter

- As the VLSI technology progresses, digital method becomes much easier to design.
- The method 2-1) requires relatively large Nyquist-rate DACs and analog filters.
- The method 2-2) uses two digital  $\Delta\Sigma$  modulators (whose circuits are negligible in fine CMOS LSI) and two 1-bit DACs (which are also negligible), and also requirements for two analog filters can be relaxed due to the oversampling.
- The same arguments hold for the method 2-3) as those of the method 2-2).

Fig.2 shows block diagrams for the methods 2-2) and 2-3).

Up-conversion mixers with local oscillators may follow the analog filters in the digital methods.[1]

### 3. COMPLEX BANDPASS $\Delta\Sigma$ DA MODULATOR FOR I-Q SIGNAL GENERATION

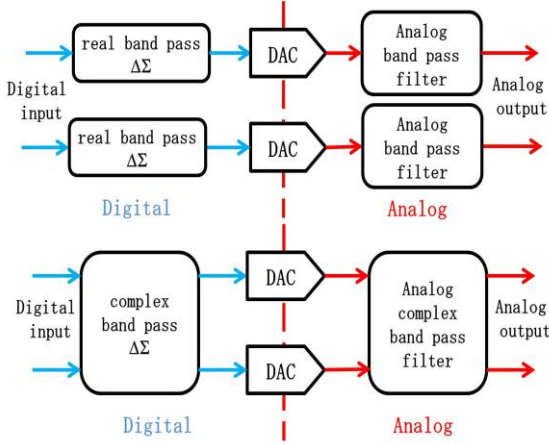


Fig. 2 I-Q signal generation with  $\Delta\Sigma$  DA modulation  
(a) Two real-bandpass modulators (method 2-2)  
(b) One complex bandpass modulator (method 2-3)

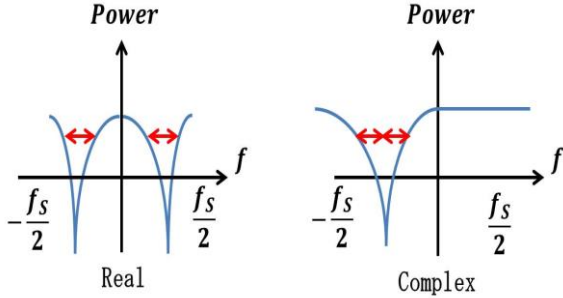


Fig.3 Noise-shaping characteristics.

(Left) Real bandpass modulator. (Right) Complex bandpass modulator.

Now let us compare the methods 2-2) and 2-3). Suppose that the center of the I-Q signal band is  $-f_s/4$ . Then as Fig.3 shows, the noise-shaping characteristics for the complex modulator around  $-f_s/4$  is better than that of the real bandpass modulators (in other words, the quantization noise in the signal band is lower in complex modulator case). See Appendix for simulation results.

The complexity of two real analog filters and one analog complex filter would be comparable.

Hence the method 2-3) (which uses complex signal processing) would be better than the method 2-2).

**Remark:** One might argue that better noise-shaping characteristics could be obtained with higher-order real bandpass modulators and digital modulators are *free* in fine CMOS LSI. However, higher-order modulators require higher-order analog filters following the modulators, and hence comparison of complex and real bandpass modulators with the same order would be fair.

### 4. COMPLEX BANDPASS $\Delta\Sigma$ DA MODULATOR

This section describes the complex bandpass  $\Delta\Sigma$  DA modulator in details. Fig. 2 and Fig. 3 show the illustration of advantages of a complex bandpass  $\Delta\Sigma$  D/A modulator compared to two real bandpass  $\Delta\Sigma$  D/A modulators. By using this type of modulator, larger bandwidth (or better SNR) can be obtained due to its asymmetric behavior with respect to  $\omega_s = 0$ . In contrast, the real bandpass modulator has a symmetric behavior with respect to  $\omega_s = 0$  with 2 poles at two different points and it provides only a half bandwidth for each pole. [1-5]

(a) Complex bandpass filter

Fig.4 shows the structure of a basic complex filter.

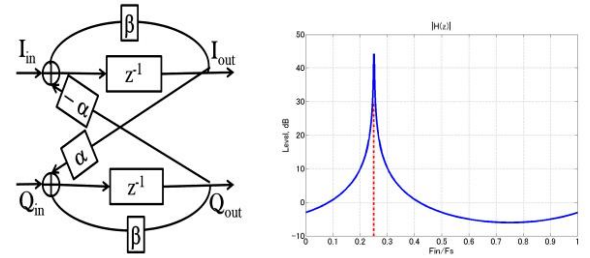


Fig. 4 Complex filter and its gain characteristics.

From the above structure, the gain of the system can be determined by obtaining their transfer function. First, we need to derive the equations from inputs to outputs as follows:

$$I_{out}(n) = I_{in}(n-1) - \alpha Q_{out}(n-1) + \beta I_{out}(n-1) \quad (1)$$

$$Q_{out}(n) = Q_{in}(n-1) + \alpha I_{out}(n-1) + \beta Q_{out}(n-1) \quad (2)$$

We define complex input  $V_{in}(n)$  and complex output  $V_{out}(n)$  as follows:

$$V_{in}(n) = I_{in}(n) + jQ_{in}(n) \quad (3)$$

$$V_{out}(n) = I_{out}(n) + jQ_{out}(n) \quad (4)$$

Then, we define its transfer function  $H(z)$  as follows:

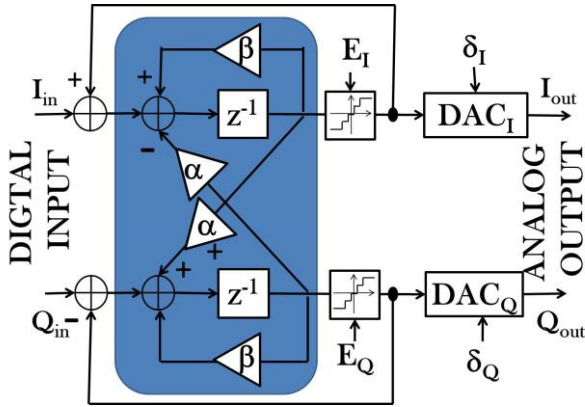
$$H(z) = \frac{V_{in}(z)}{V_{out}(z)} \quad (5)$$

We obtain the following:

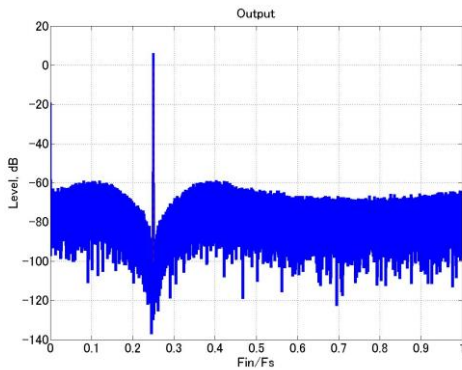
$$H(z) = \frac{1}{z - (\beta + j\alpha)} \quad (6)$$

(b) Complex bandpass  $\Delta\Sigma$  DA modulator

Fig.5 (a) and Fig.6 (b) show first-order and second-order complex bandpass  $\Delta\Sigma$  DA modulators with the center frequency  $-f_s/4$  of the signal band. Fig. 5(b) and Fig.6(b) show their output spectrum for the complex sinusoidal signal input around  $-f_s/4$ , and we see that the quantization noise is shaped at  $-f_s/4$ . Here we use  $\alpha=1$ ,  $\beta=0$ .



(a) Block diagram.



(b) Output power spectrum.

Fig.5 First-order complex bandpass ΔΣ DA modulator.

## 5. COMPLEX MULTI-BANDPASS ΔΣ DA MODULATOR

This section describes complex multi-bandpass ΔΣ DA modulators for multi-tone I-Q signal generation. [6]

### (a) Complex multi-band pass filter

Fig. 5 shows a first-order complex multi-bandpass filter, and Fig.6 shows its gain characteristics for n=2, n=4. Its transfer function is given as follows:

$$H(z) = \frac{1}{z^n - (\beta + j\alpha)} \quad (7)$$

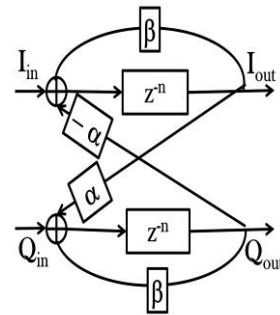
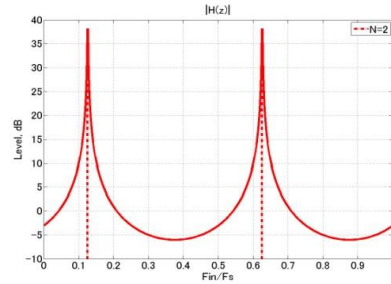
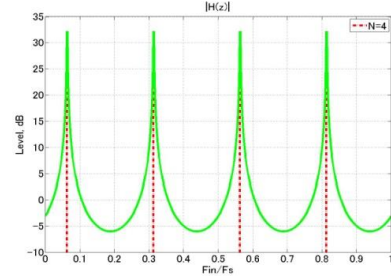


Fig. 7 Complex multi-bandpass filter.

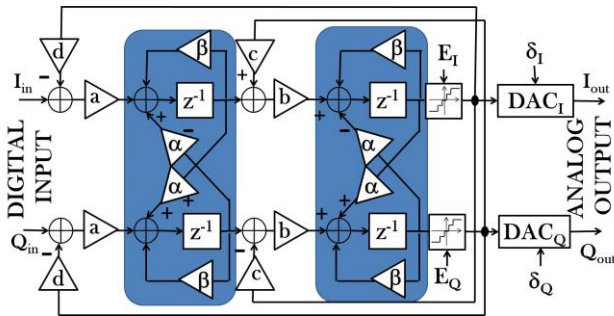


(a) n=2 (α=1, β=0)

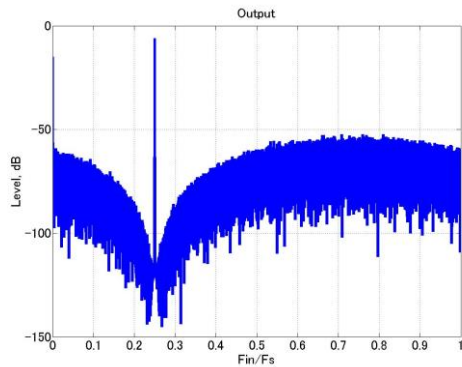


(b) n=4 (α=1, β=0)

Fig.8 Gain characteristics of multi-band complex filters.



(a) Block diagram.



(b) Output power spectrum.

Fig.6 Second-order complex bandpass ΔΣ DA modulator.

### (b) Complex multi-bandpass ΔΣ DA modulator

Fig.5 and Fig.6 show first-order and second-order complex multi-bandpass ΔΣ DA modulators, and Fig.11 shows the simulated output power spectrum for the second-order ones with n=2 and n=4. Also Fig.12 shows SNDR versus OSR (oversampling ratio).

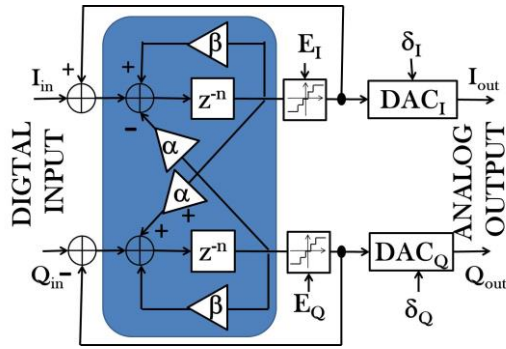


Fig. 9 First-order complex multi-bandpass  $\Delta\Sigma$  DA modulator.

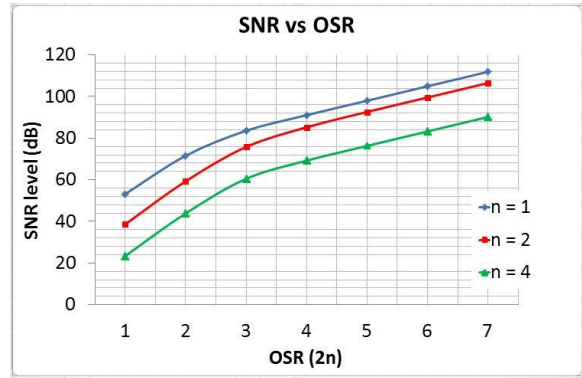


Fig.12 Simulated SNR versus OSR for second-order complex multi-bandpass  $\Delta\Sigma$  DA modulators.

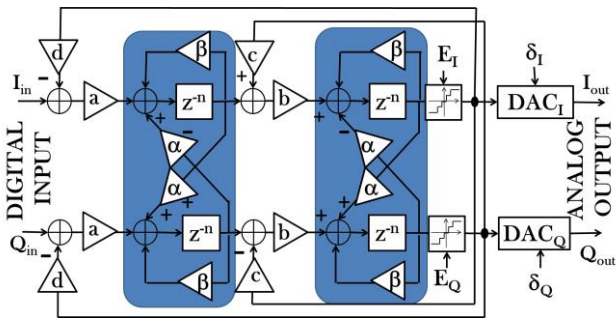
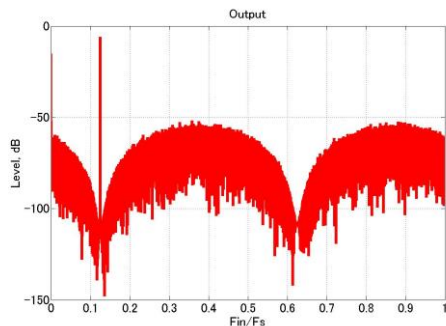
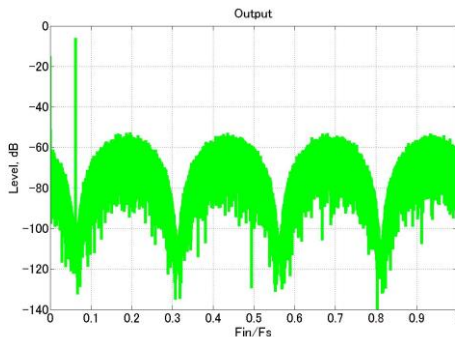


Fig. 10 Second-order complex multi-bandpass  $\Delta\Sigma$  DA modulator.



(a)  $n=2$



(b)  $n=4$ .

Fig. 11 Second-order complex multi-bandpass  $\Delta\Sigma$  DA modulator output spectrum.

## 6. CONCLUDING REMARKS

This paper described that a complex bandpass  $\Delta\Sigma$  DA modulators would be suitable architecture for I-Q signal generation, and multi-bandpass one is also suitable for multi-tone I-Q signal generation. We conclude this paper by remarking the following:

- ① A first-order complex bandpass  $\Delta\Sigma$  DA modulator has one pole between  $-fs/2$  to  $fs/2$ .
- ② A first-order real bandpass  $\Delta\Sigma$  DA modulator has two poles between  $-fs/2$  to  $fs/2$ .
- ③ A first-order complex multi-bandpass  $\Delta\Sigma$  DA modulator has  $n$  poles between  $-fs/2$  to  $fs/2$ .

Then for a given OSR, complex bandpass one has the best SNR, and then real bandpass ones and lastly, n-bandpass one ( $n>2$ ). Clarification of these relationships with analytical equations would be an interesting future work.

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## Appendix

This appendix shows comparison between complex and real bandpass modulators with simulation. We use a second-order complex modulator in Fig.10, and a second-order bandpass modulator in Fig.13. Our simulation results show noise-shaping behaviors in Fig. 14, and we obtain OSR versus SNR in Fig. 15: we see that the complex modulator has better SQNDR by 10dB.

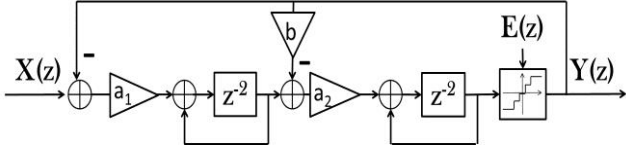


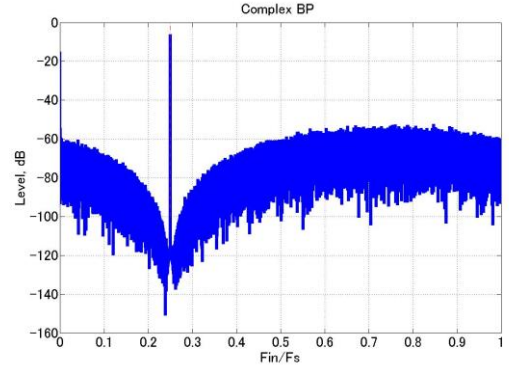
Fig.13 A second-order real bandpass  $\Delta\Sigma$  modulator used for simulation.

$$\mathbf{a_1 = 1, a_2 = 1, b = 2}$$

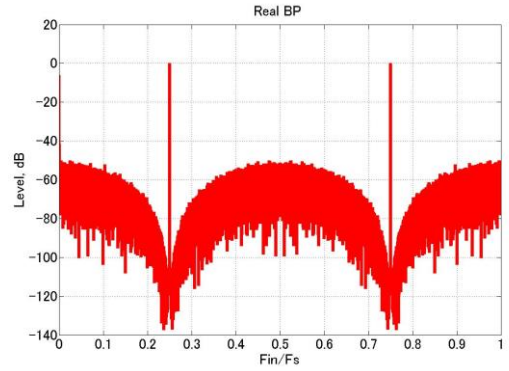
$$STF(z) = \frac{a_1 a_2 z^{-2}}{D(z)} \quad (1)$$

$$NTF(z) = \frac{(1-z^{-1})^2}{D(z)} \quad (2)$$

$$D(z) = (1-z^{-1})^2 + a_1 b z^{-1} (1-z^{-1}) + a_1 a_2 z^{-2} \quad (3)$$



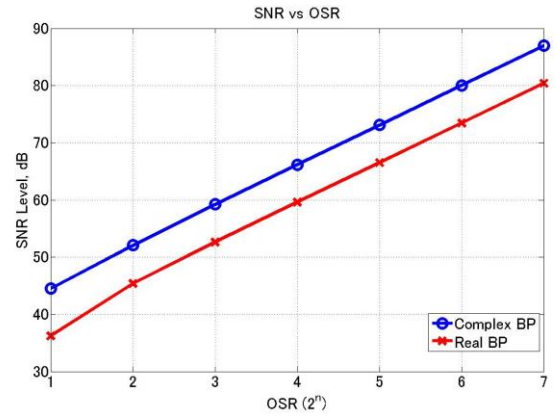
(a)



(b)

Fig.14 Output spectrum comparison.

- (a) The second-order complex bandpass modulator.
- (b) The second-order real bandpass modulator.



(c)

Fig. 15 SNR versus OSR comparison between second-order complex and real bandpass  $\Delta\Sigma$  modulators.