

# Linearity Improvement Techniques of Multi-bit Sigma-Delta TDC for Timing Measurement

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TDC: Time-to-Digital Converter

# Outline

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- ▶ **Research Objective**
- ▶ **Single-Bit & Multi-bit  $\Sigma\Delta$  TDCs**
- ▶ **Multi-Bit  $\Sigma\Delta$  TDC with DWA**
- ▶ **Multi-Bit  $\Sigma\Delta$  TDC with Self-Calibration**
- ▶ **Multi-Bit  $\Sigma\Delta$  TDC with Sorting**
- ▶ **Conclusion**

# Outline

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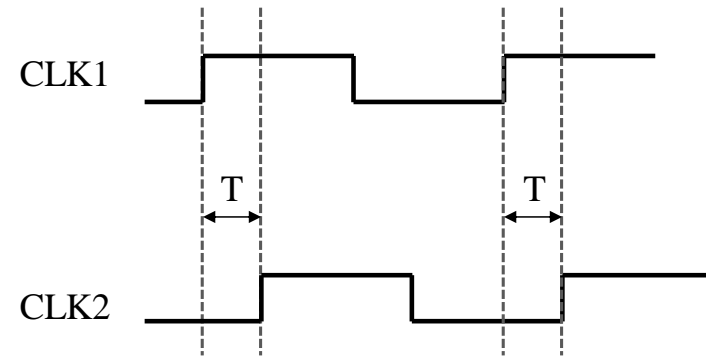
- ▶ **Research Objective**
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# Research Objective

- Testing timing difference between two repetitive digital signals

Ex.

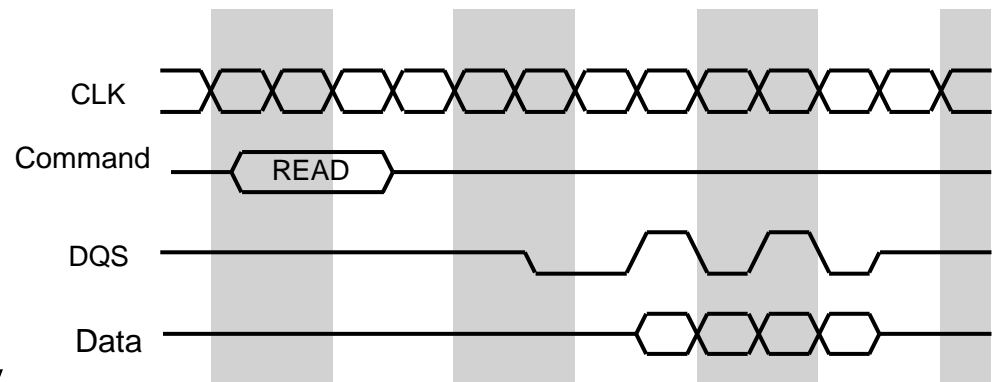
Data and clock  
in Double Data Rate (DDR) memory



- Short testing time
- Good accuracy



Implement with small circuitry



# Our Work

## Focus on Multi-bit $\Sigma\Delta$ Time-to-Digital Converter (TDC)

- Repetitive digital signals  
  $\Sigma\Delta$  TDC can be used

- Simple circuit
- Fine resolution
- Testing time

Single-bit $\Sigma\Delta$ TDC	Long
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Multi-bit $\Sigma\Delta$ TDC	Short
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- Linearity

Single-bit $\Sigma\Delta$ TDC	Good
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Multi-bit $\Sigma\Delta$ TDC	Bad	due to delay elements mismatches
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Three methods for their compensation



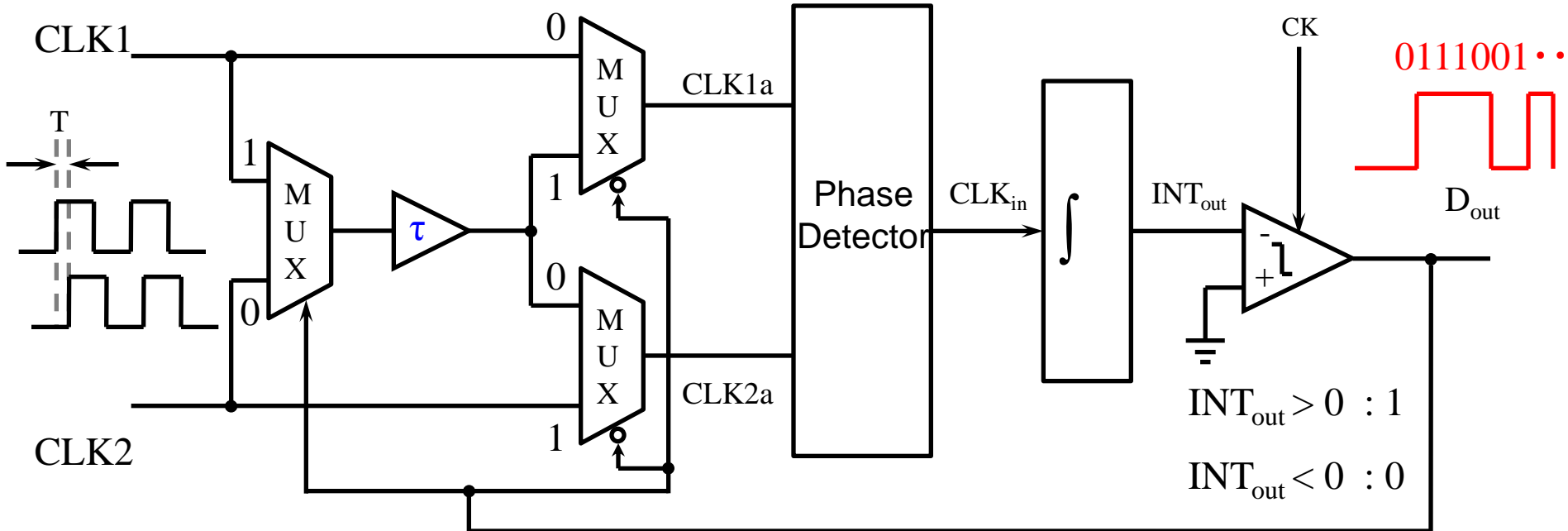
DWA, Self-calibration, Delay cell sorting

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# Single-Bit $\Sigma\Delta$ TDC



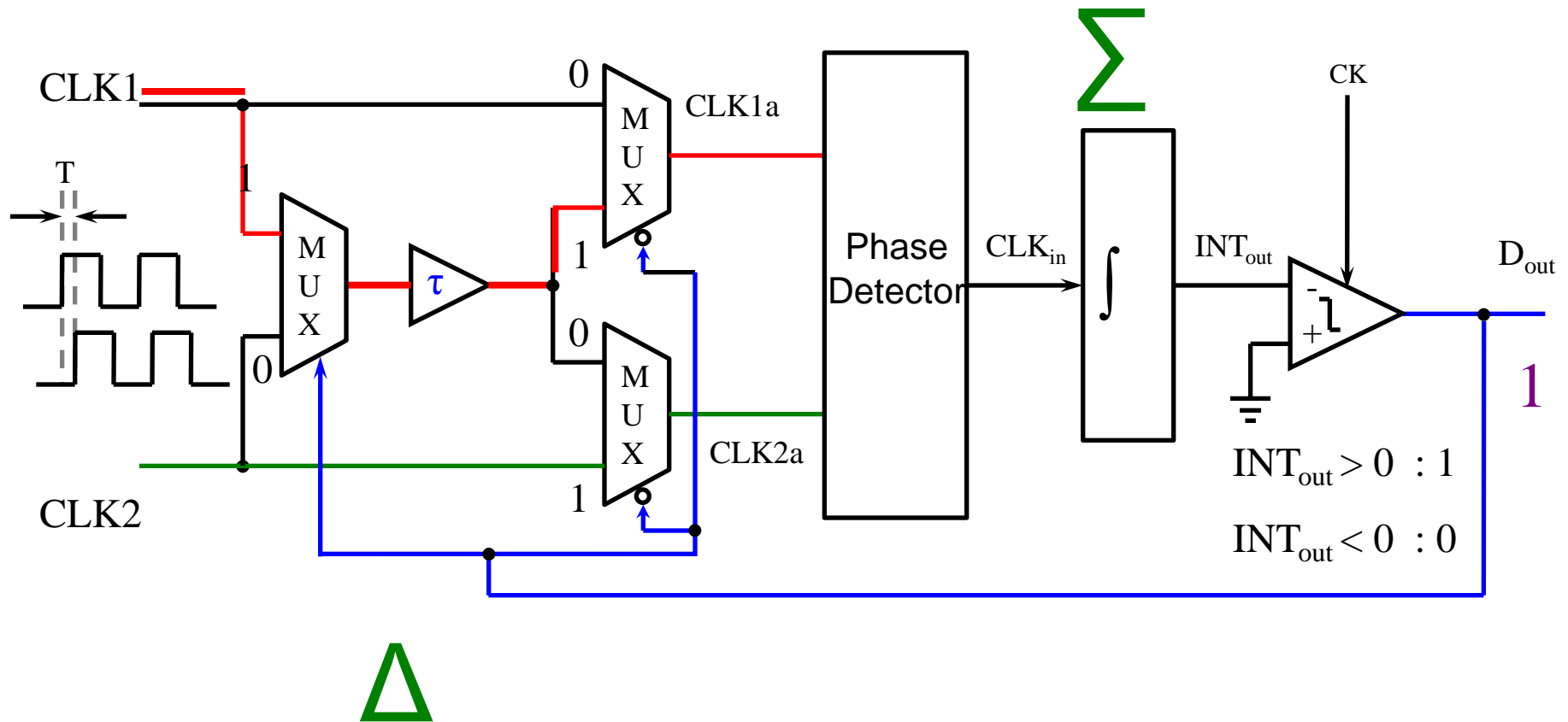
- Measurement of timing  $T$  between repetitive  $CLK1$  and  $CLK2$
- Number of 1's at  $D_{out}$  is proportional to  $T$
- Time resolution becomes finer as measurement time becomes longer

Note:  $\tau$  is not time resolution, but time measurement full range

The delay line with 1bit digital input is inherently linear because it is 1-bit.

# Operation of Single-Bit $\Sigma\Delta$ TDC

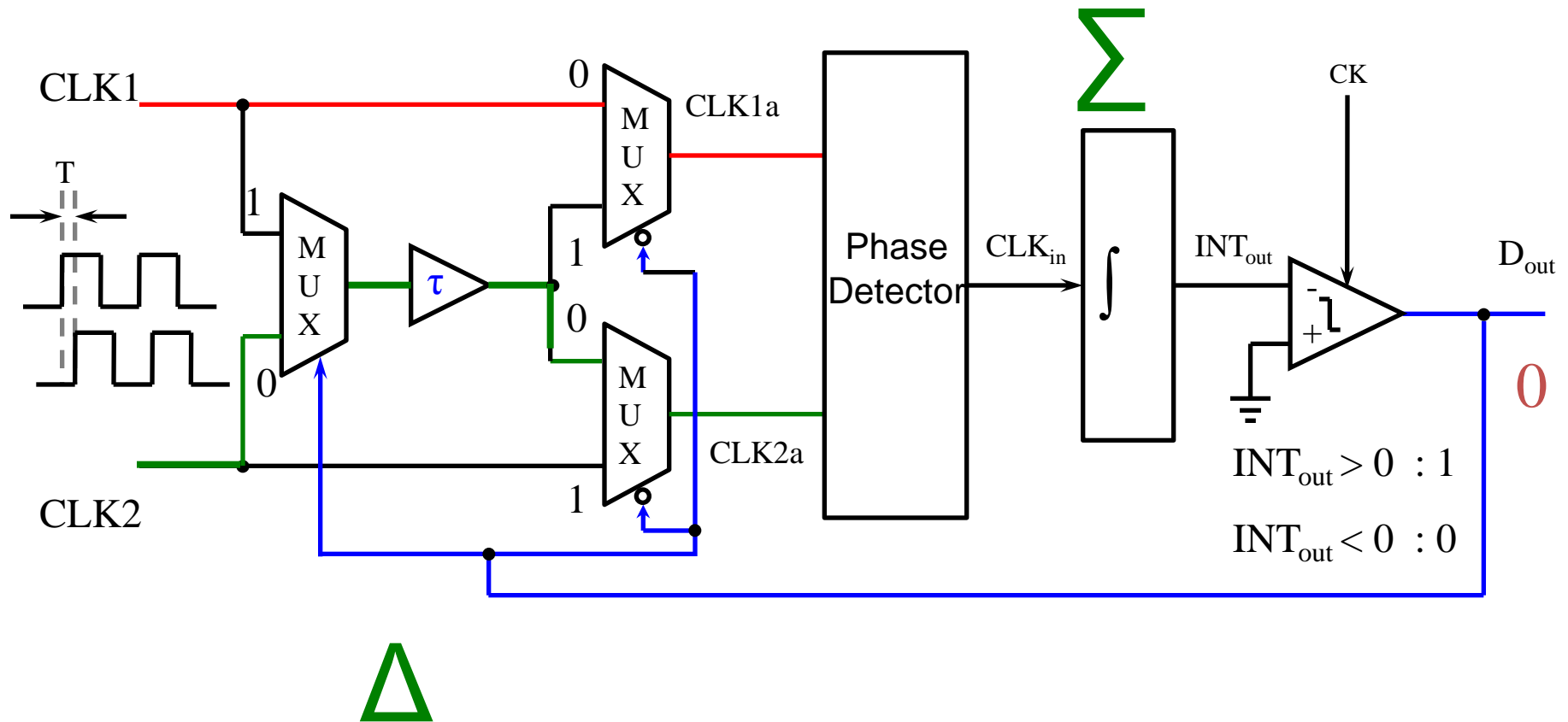
In case  $D_{out} = 1$



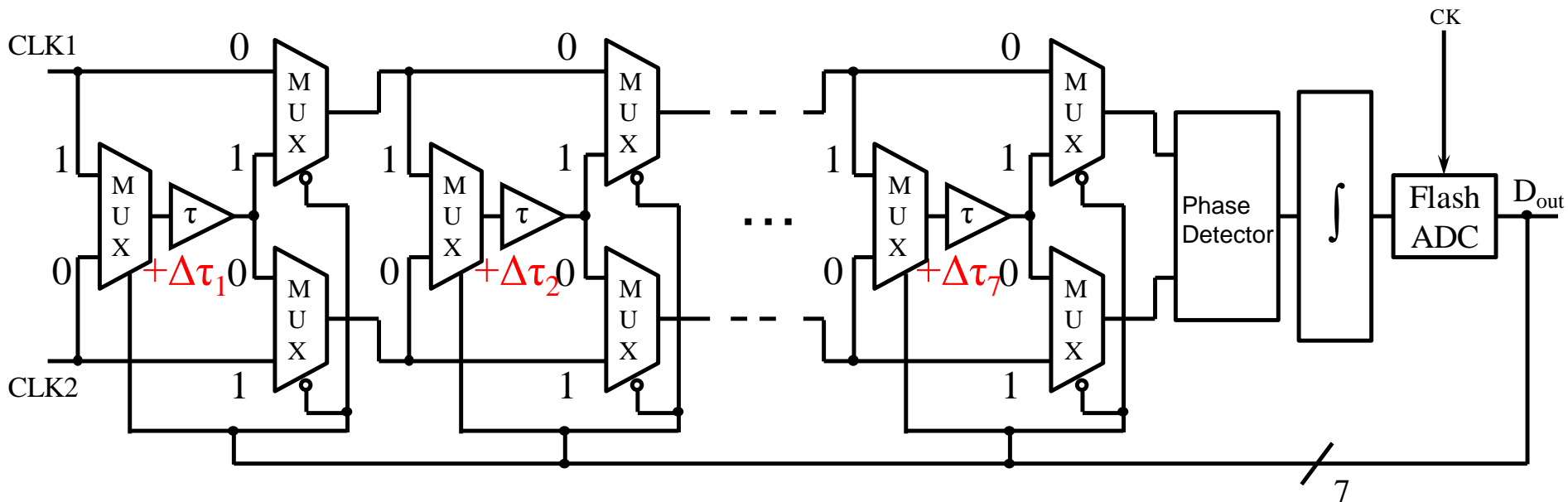


# Operation of Single-Bit $\Sigma\Delta$ TDC

In case  $D_{out} = 0$

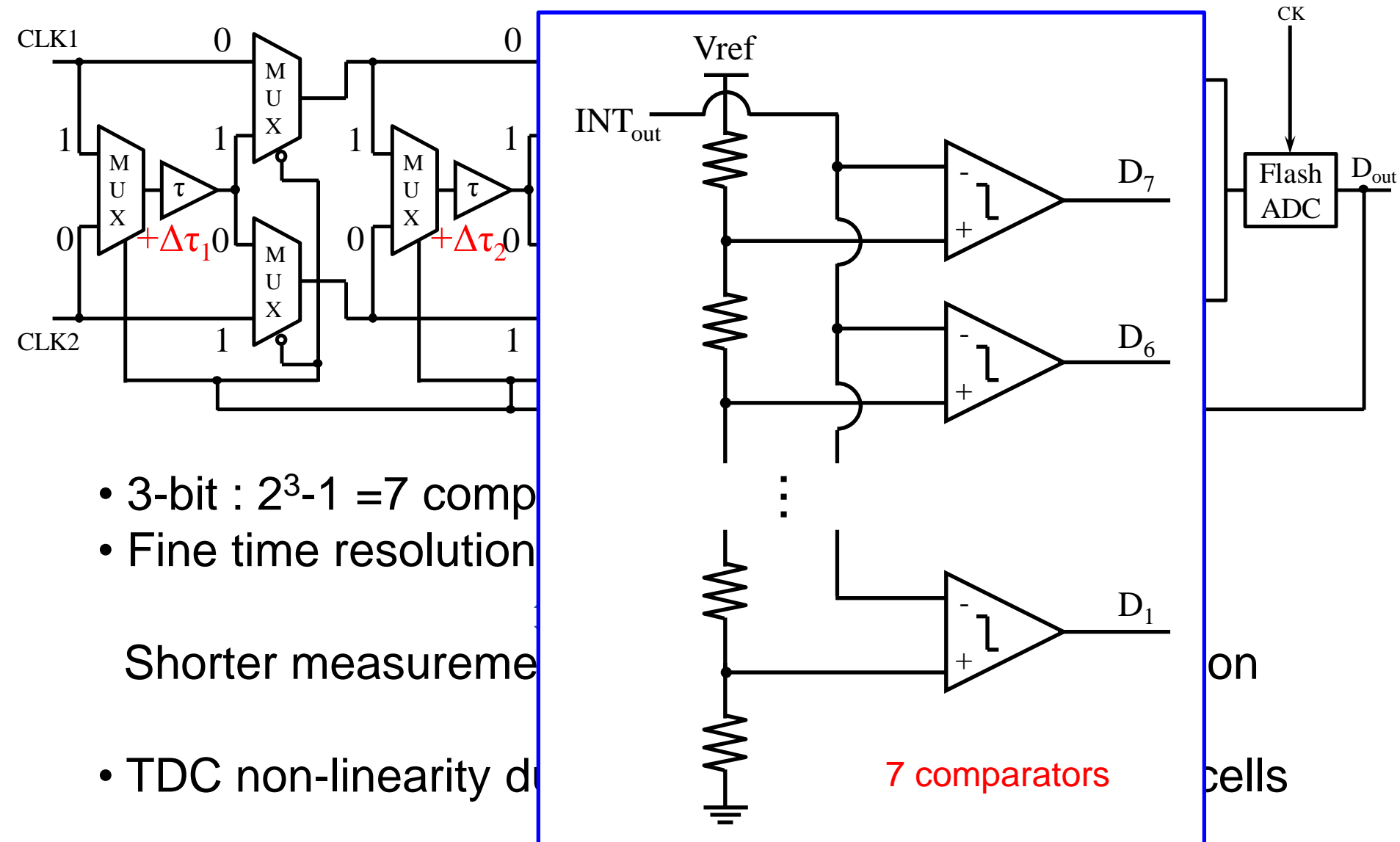


# Multi-Bit $\Sigma\Delta$ TDC



- 3-bit :  $2^3 - 1 = 7$  comparators and delays
  - Fine time resolution with a given measurement time
- ↕
- Shorter measurement time with a given time resolution
- TDC non-linearity due to mismatches among delay cells.

# Multi-Bit $\Sigma\Delta$ TDC



- 3-bit :  $2^3 - 1 = 7$  comp
- Fine time resolution

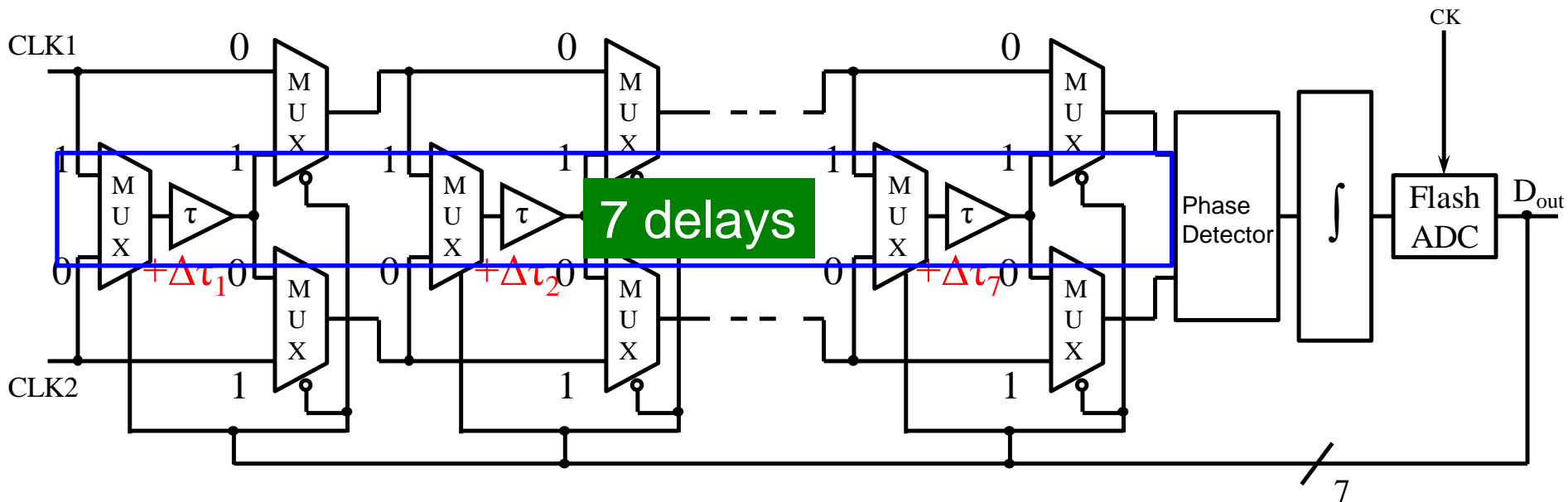
Shorter measureme

- TDC non-linearity d

on

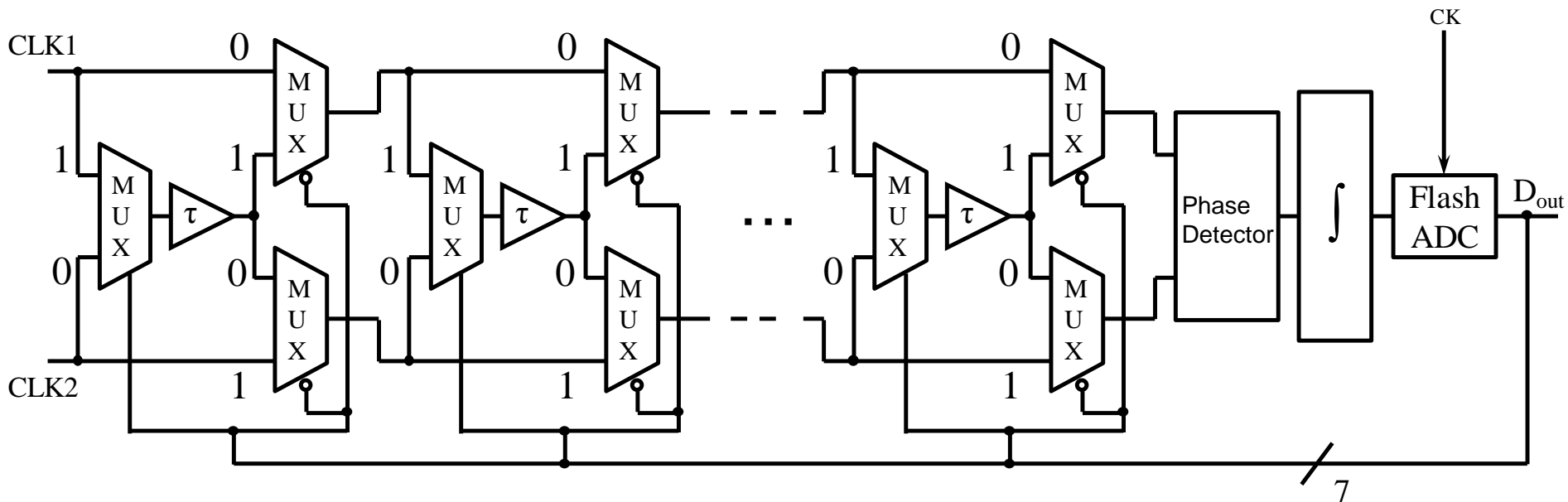
cells

# Multi-Bit $\Sigma\Delta$ TDC



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# Multi-Bit $\Sigma\Delta$ TDC



- 3-bit :  $2^3-1 = 7$  comparators and delays
- Fine time resolution with a given measurement time



Shorter measurement time with a given time resolution

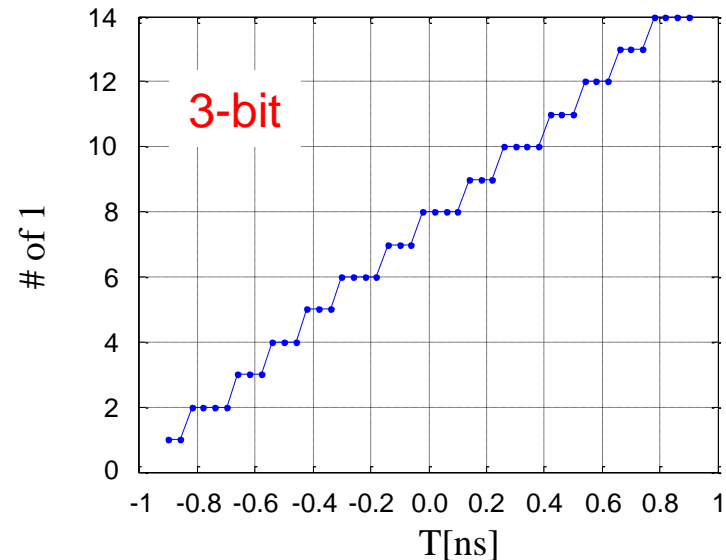
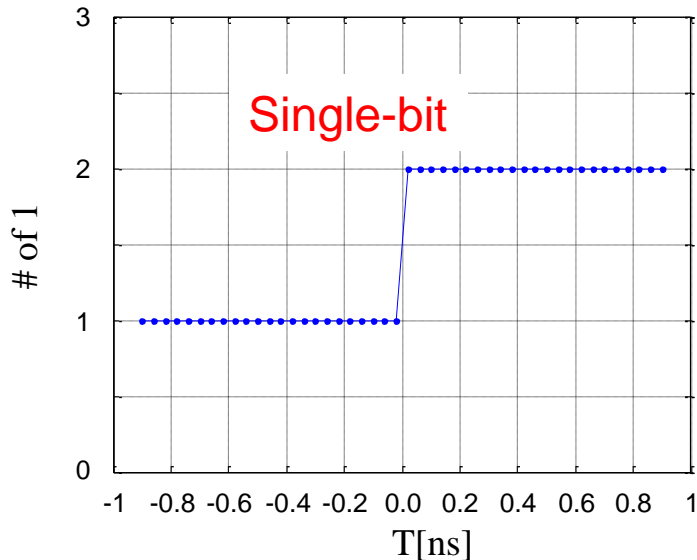
- TDC non-linearity due to mismatches among delay cells

# Time Resolution Comparison

## ● Simulation conditions

	1-bit $\Sigma\Delta$ TDC	3-bit $\Sigma\Delta$ TDC
Rising timing edge difference (T)	-0.9 ~ 0.9[ns] (Resolution : 0.04[ns])	-0.9 ~ 0.9[ns] (Resolution : 0.04[ns])
Delay time ( $\tau$ )	1[ns]	0.145[ns]
The number of digital outputs	2	2

## ■ A rising number of outputs for the interval T

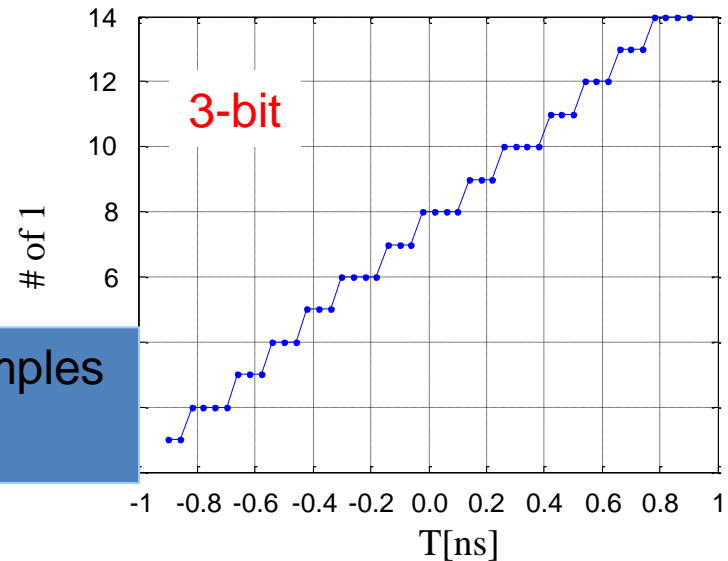
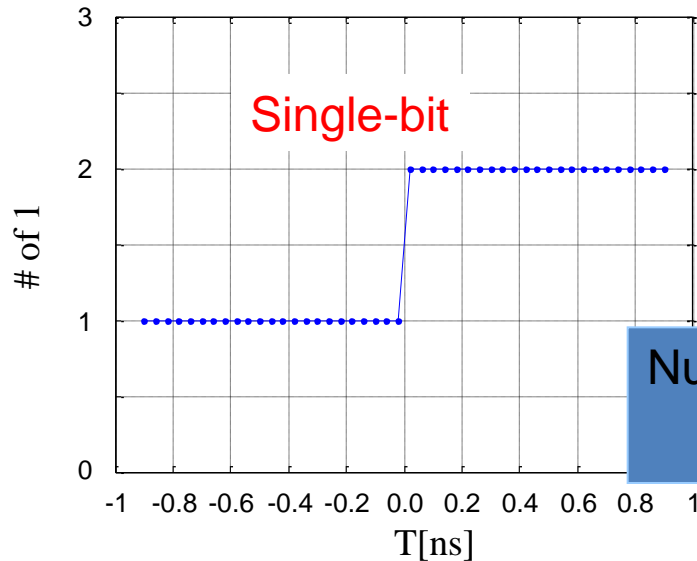


# Measurement Time Comparison

✓ Multi-bit takes short measurement time for a given time resolution



■ A rising number of outputs for the interval T



Number of samples  
is 2

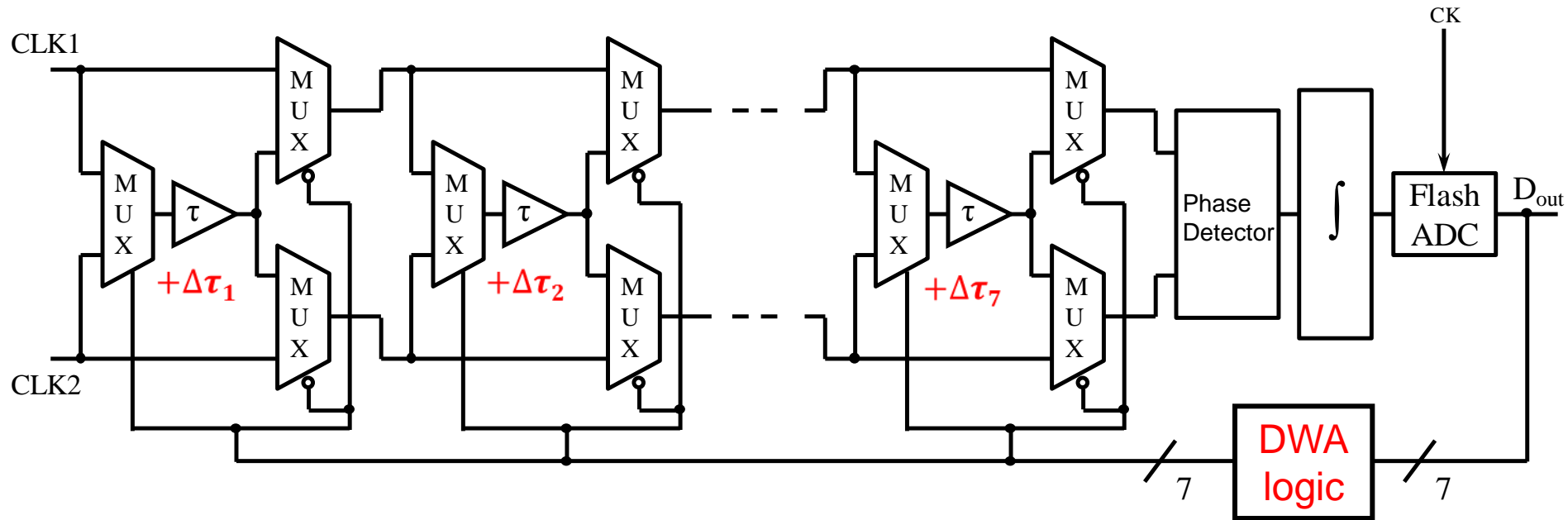
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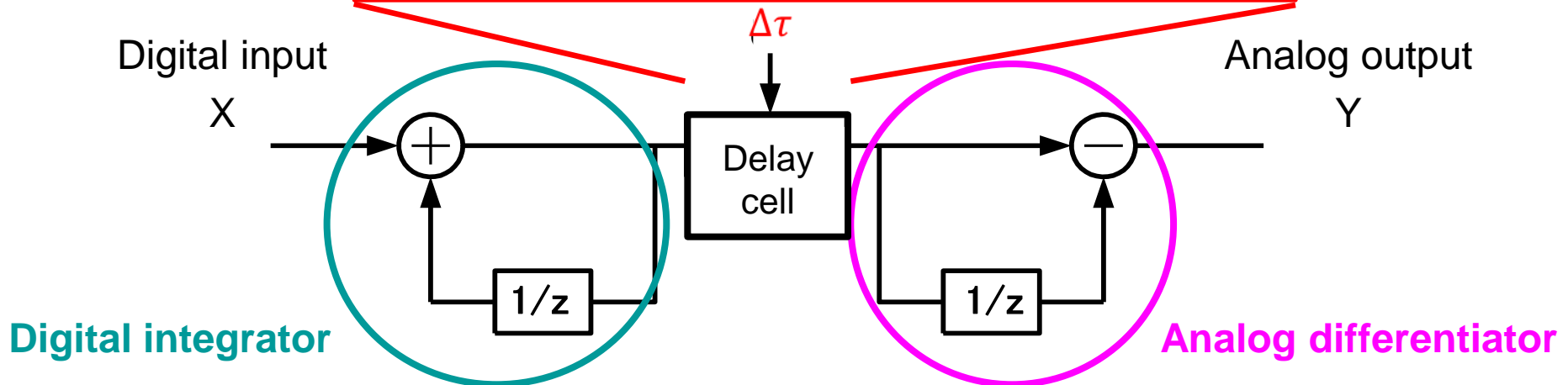
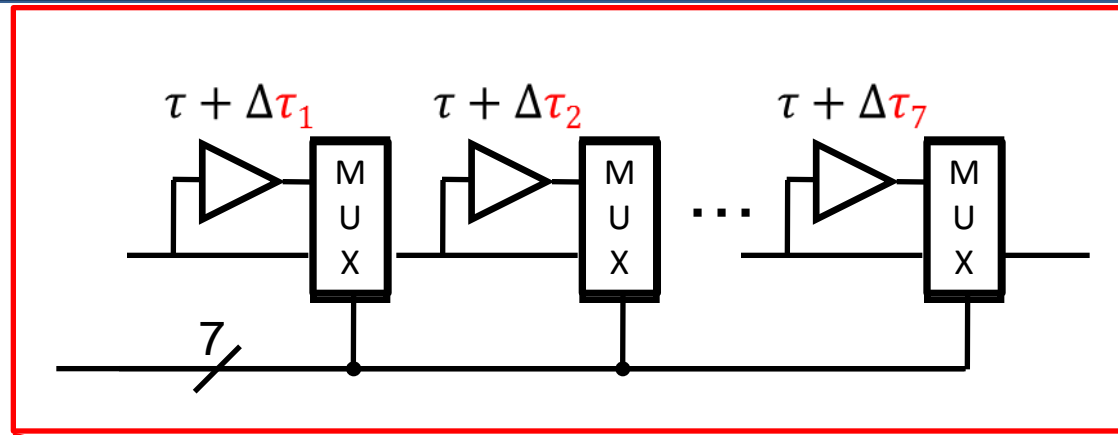


# DWA (Data Weighted Averaging)



- Flash ADC outputs
  - ➔ shuffled by **DWA logic**, fed into MUXs as **select** signals
- Delay mismatch effects
  - ➔ moved to high-frequency (**noise-shaping**)

# Noise-Shaping

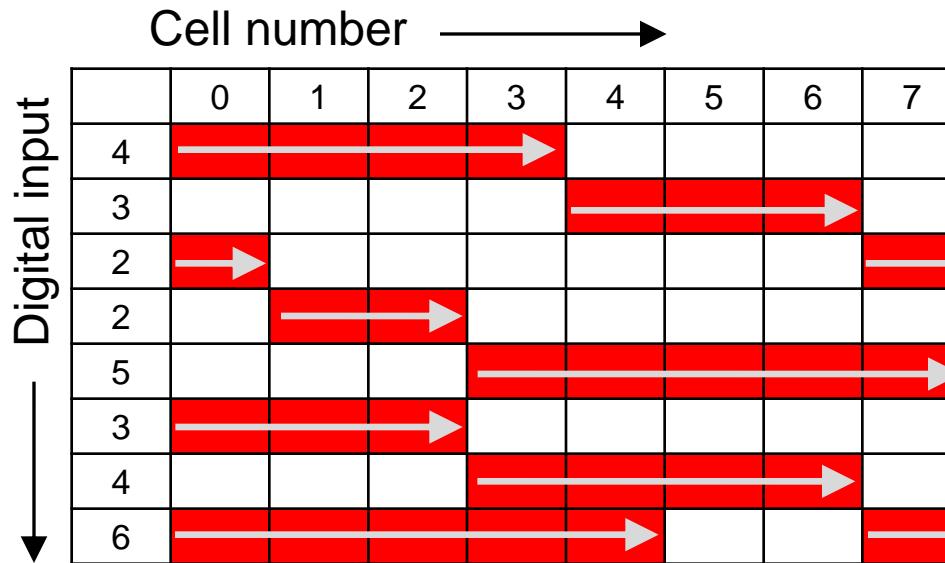


$$Y(z) = X(z) + \underline{(1 - 1/Z)\Delta\tau(z)}$$



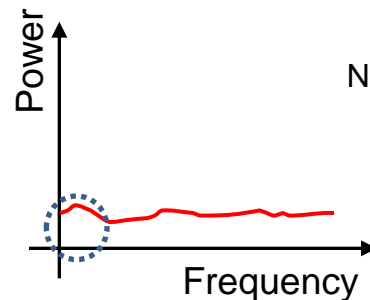
Delay mismatch  $\Delta\tau$  is **first-order noise-shaped**.

# DWA & Noise Shaping



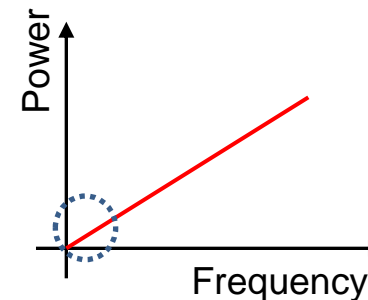
- Delay  $\tau$  : integration & differentiation
- Delay mismatch  $\Delta\tau$  : differentiation

delay cell mismatch effects

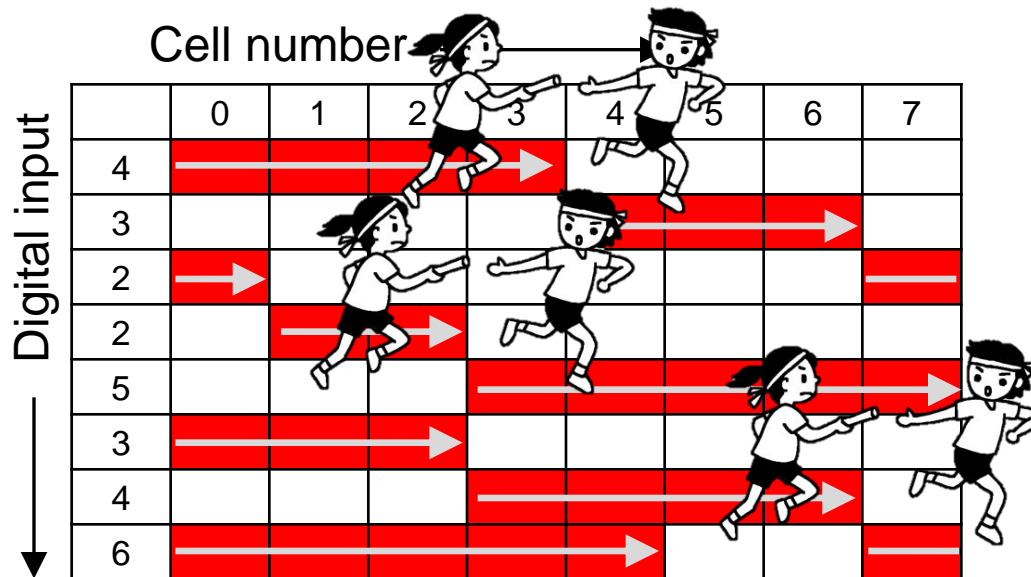


delay cell mismatch effects

Noise Shape

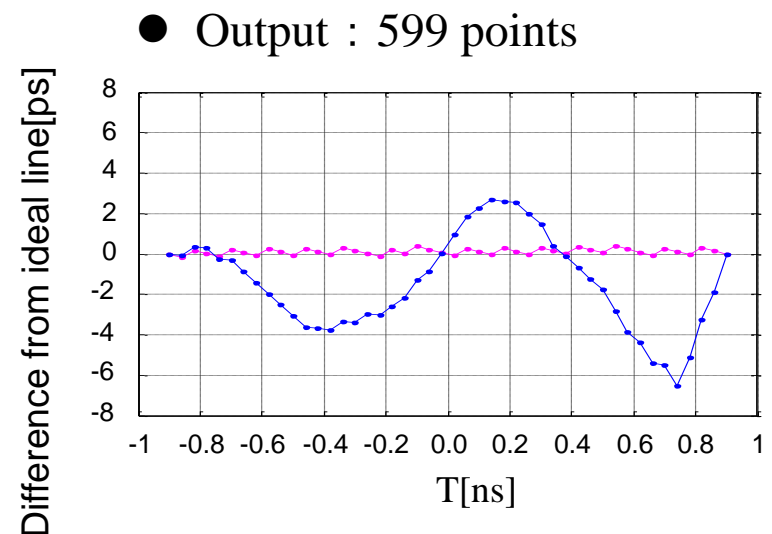
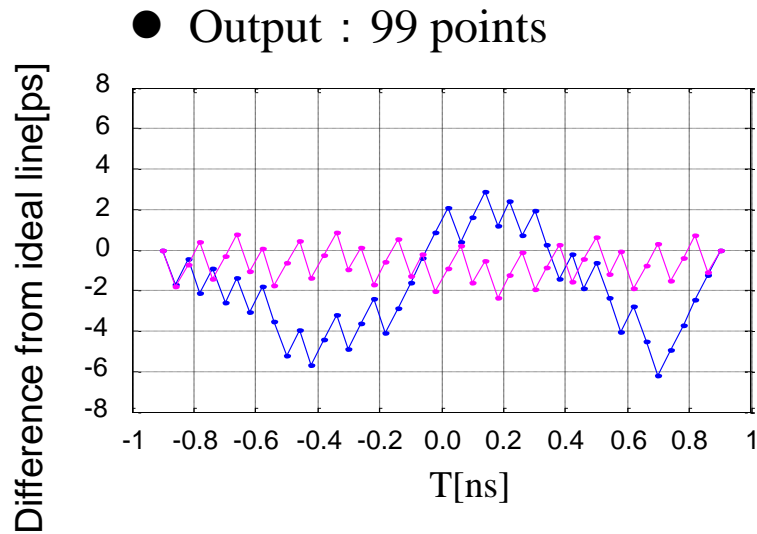


# DWA Operation



Pass a baton in relay race !

# Simulation of $\Delta\Sigma$ TDC with DWA

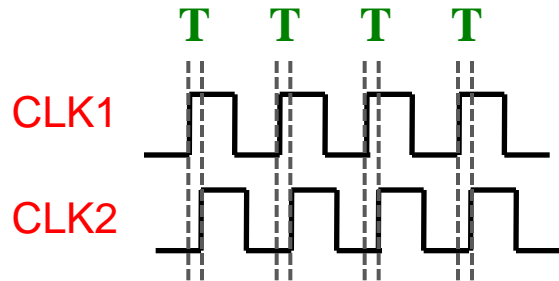


—●—  $\Delta\Sigma$  TDC(with DWA)  
 —●—  $\Delta\Sigma$  TDC(without DWA)

✓ Reduce the effect of delay mismatches

$\Sigma\Delta$  TDC linearity is improved

# DWA Effectiveness



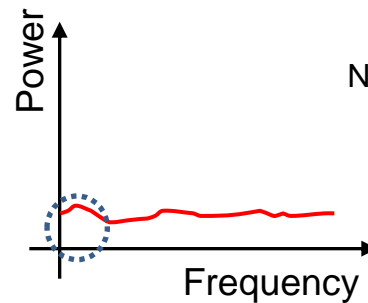
Measure **T**



**T** is DC signal.

- Delay  $\tau$  : integration & differentiation
- Delay mismatch  $\Delta\tau$  : differentiation

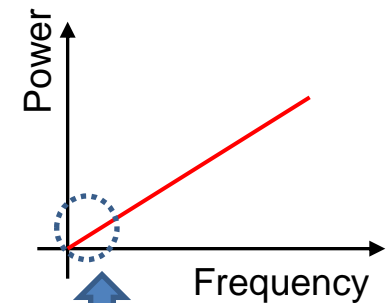
delay cell mismatch effects



Noise Shape



delay cell mismatch effects



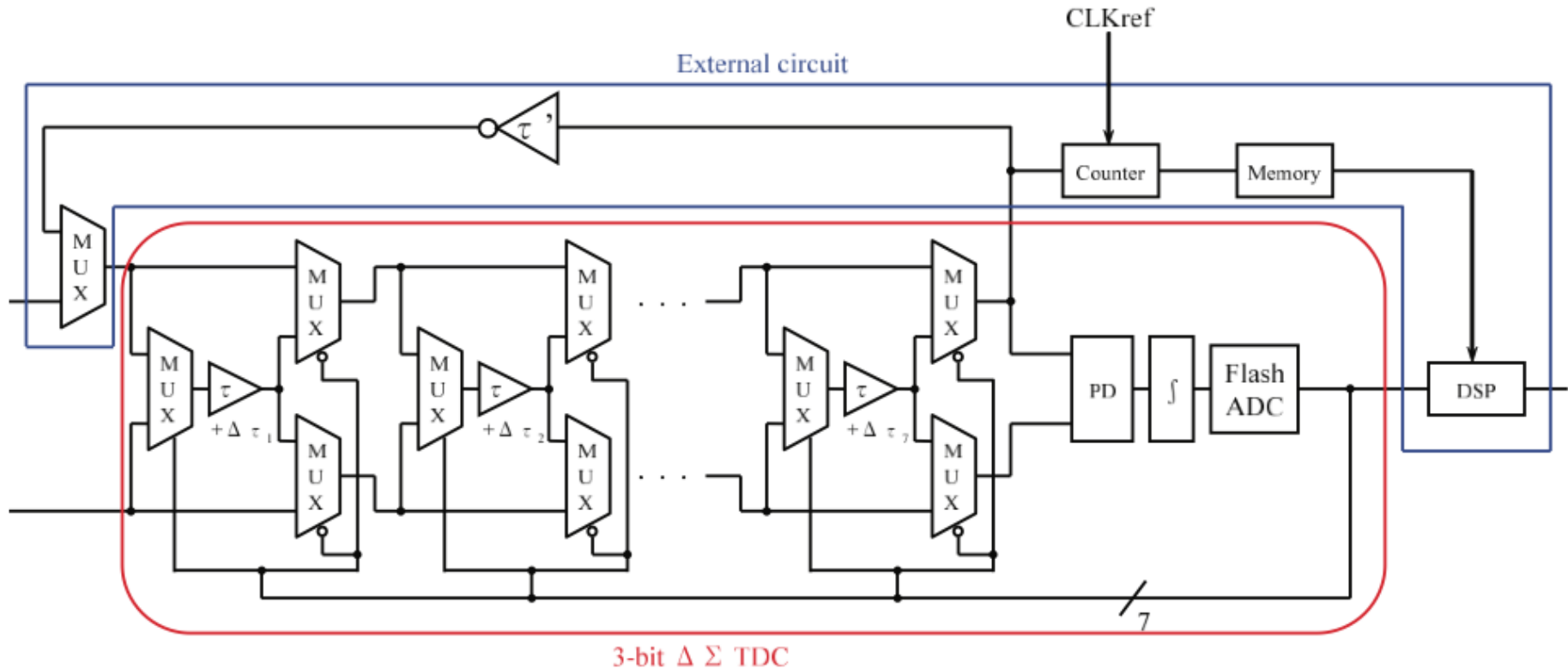
Mismatch effects  
reduction at DC

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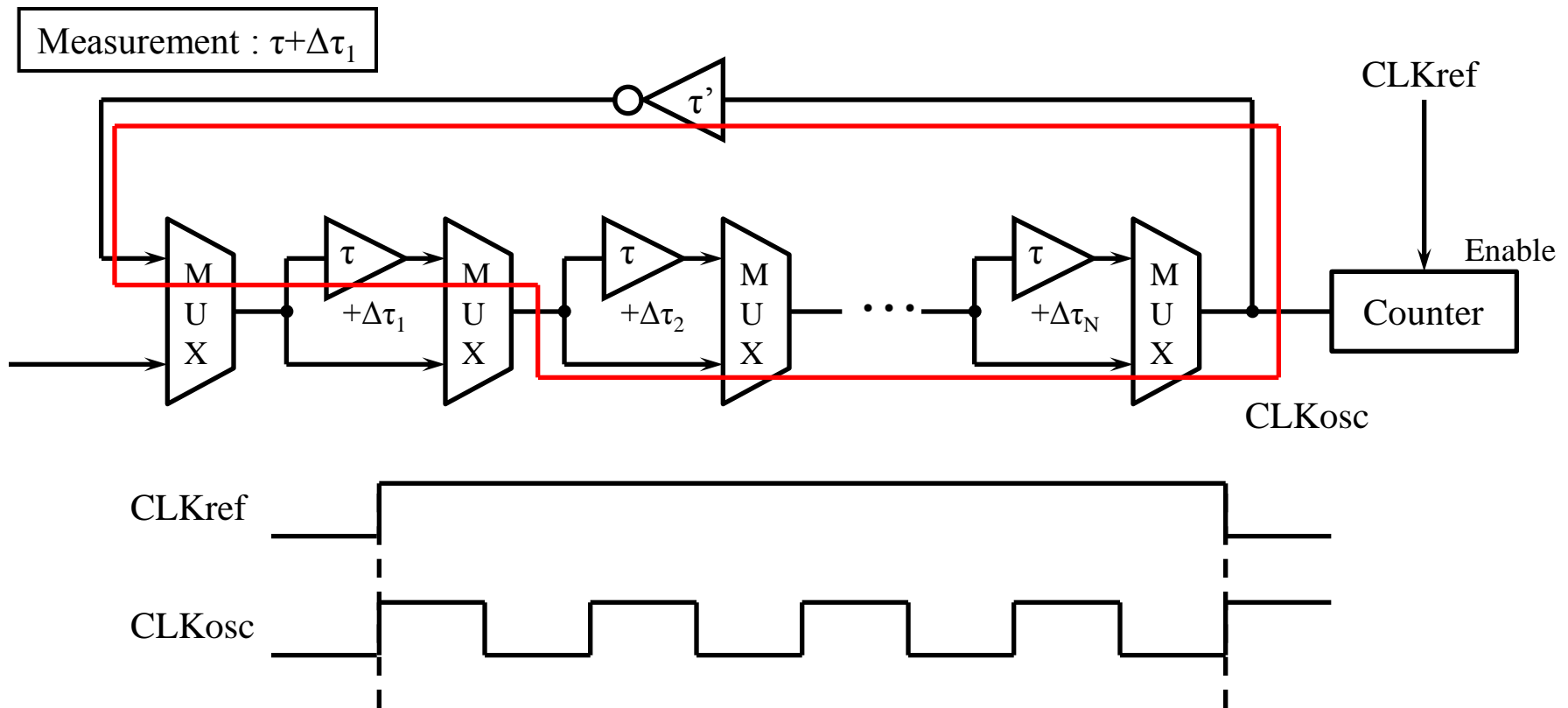
# $\Sigma\Delta$ TDC with Self-Calibration



- Self-calibration circuit: inverter, MUX, counter, memory
- Measure delay values and store them in memory

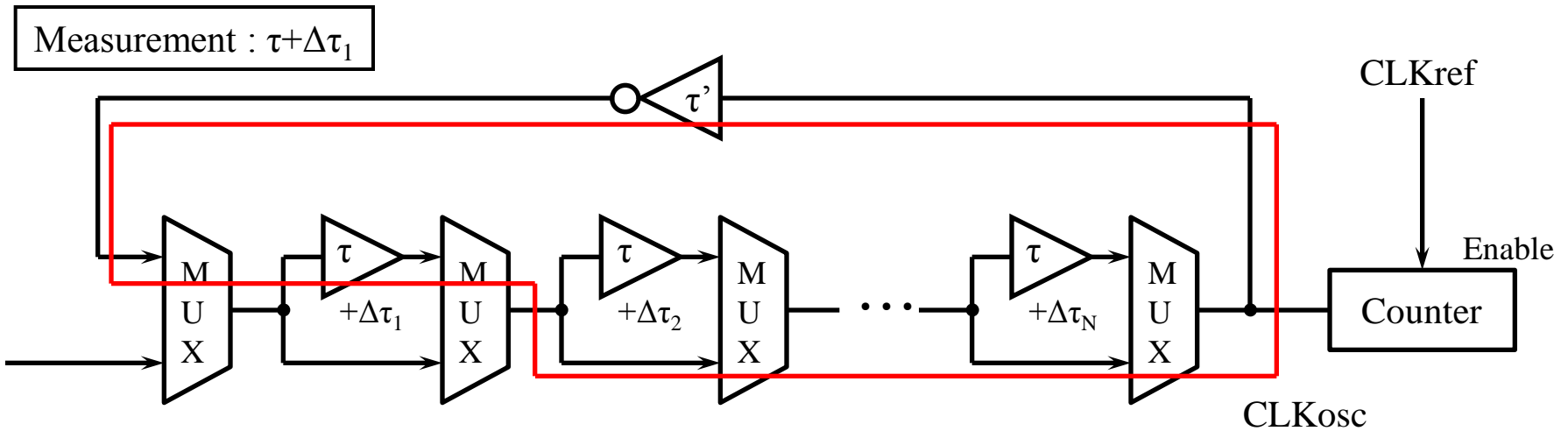


# Self-Measurement of Delay

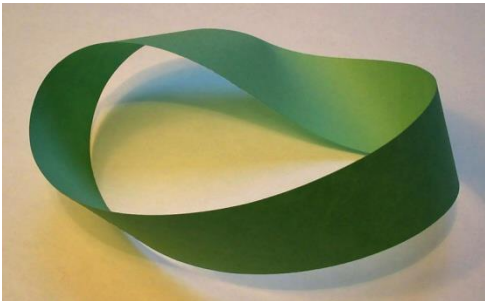


- Ring oscillator with a delay cell to be measured
- Counter measure the number of the pulses
- $\Delta\tau$  can be calculated
- Measured delay values are stored in memory

# Time Signal & Ring Oscillator

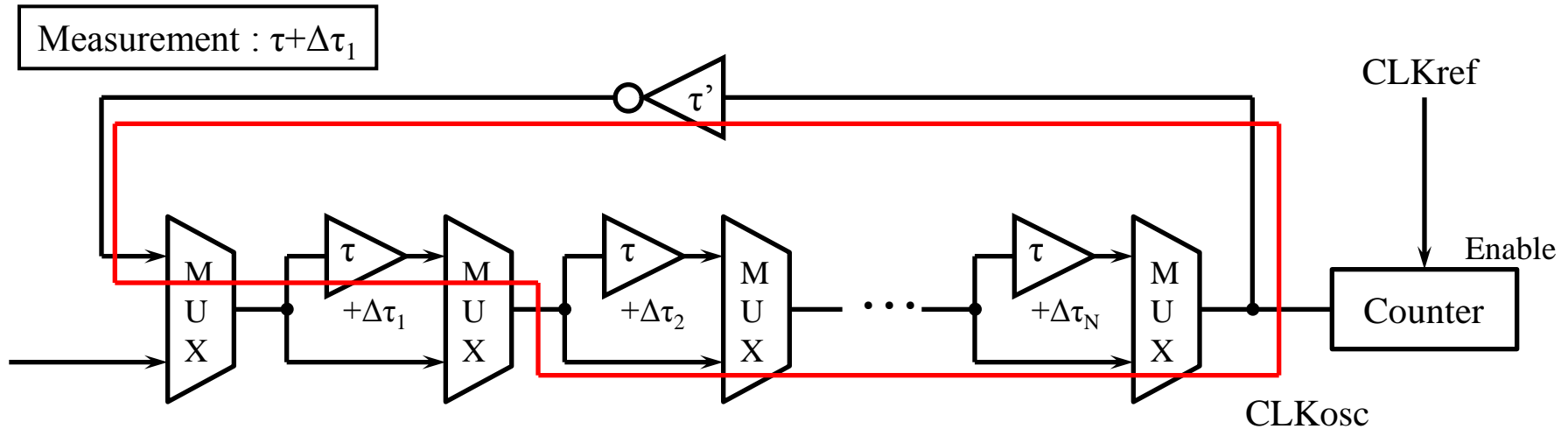


Ring oscillator



Möbius strip

# Self-Measurement of Delay



Oscillation frequency

$$f = \frac{1}{2(\tau' + \tau + \Delta\tau_1)}$$

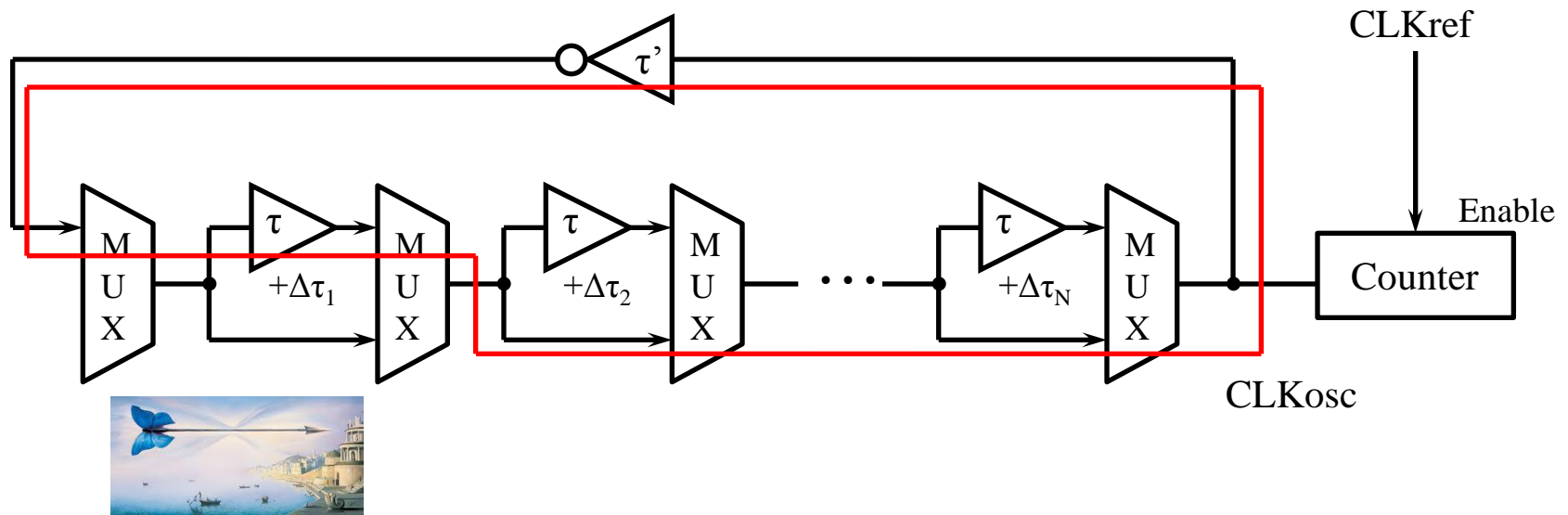
Measure

$\Delta\tau_2, \Delta\tau_3, \Delta\tau_4, \dots, \Delta\tau_N$   
one by one.

$\Delta\tau_1$  can be calculated from the oscillation frequency

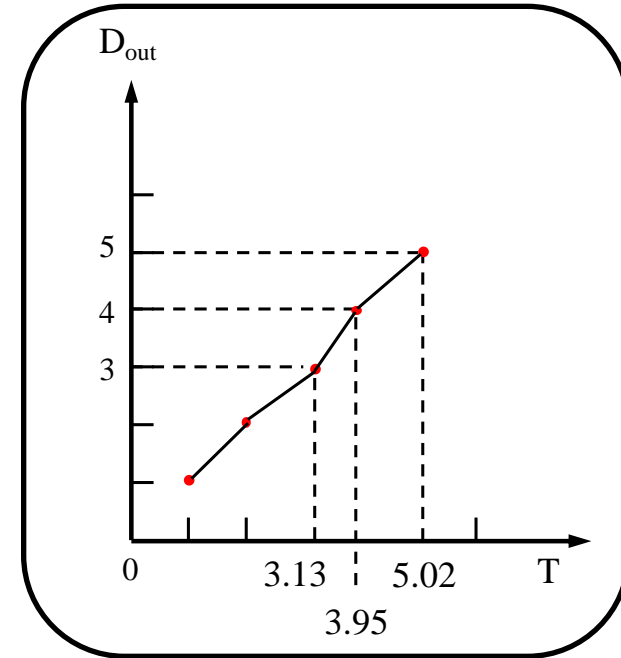
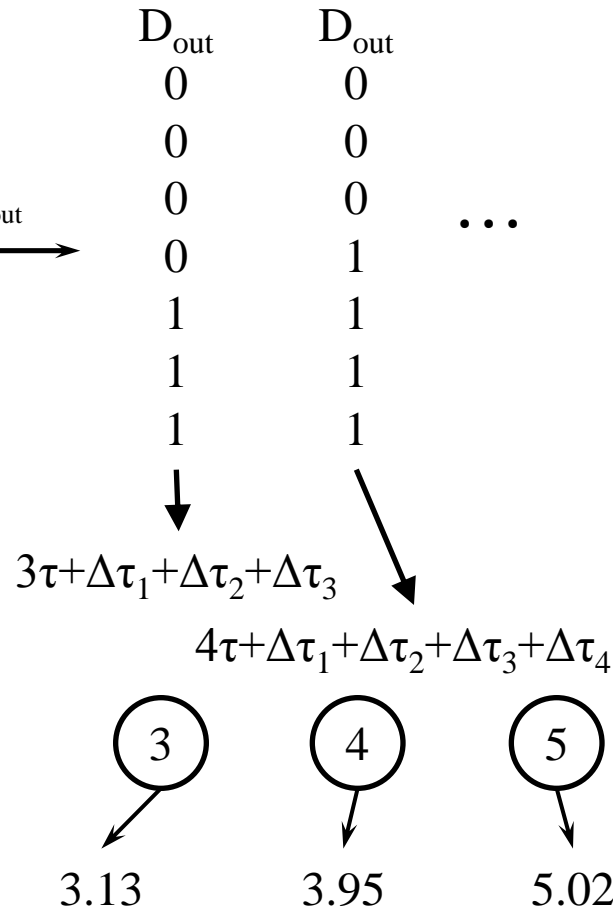
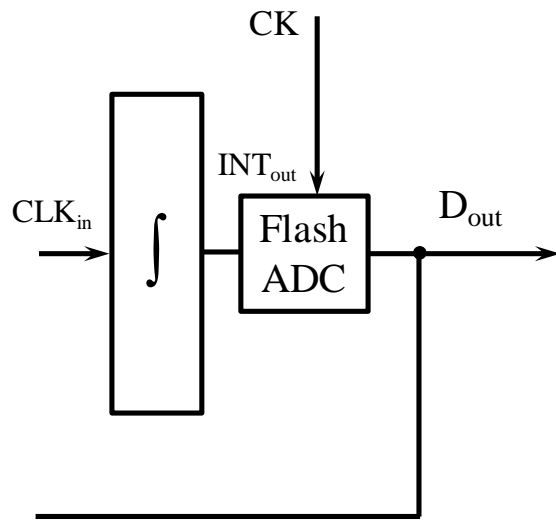
# Essence of Proposed Method

- All operations are done in **digital domain**
  - Signal is **Time** instead of **Voltage**.
- ➔ Easy, accurate measurement of  $\Delta\tau$



Time flies like an arrow!

# Proposed Error Correction Scheme

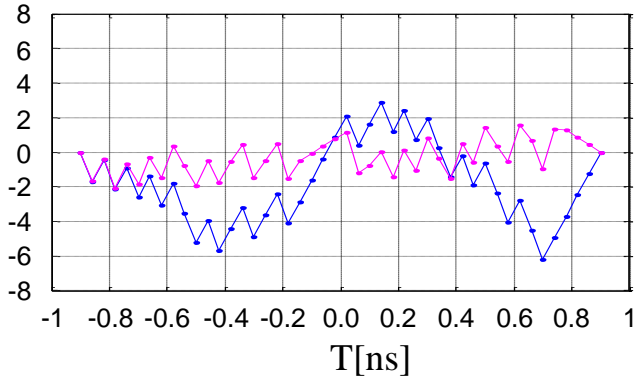


- Obtain TDC raw output ( $D_{out}$ ) for two input clocks
- Read delay values from memory, and compensate for the output based on them

# Simulation of Self-Calibration

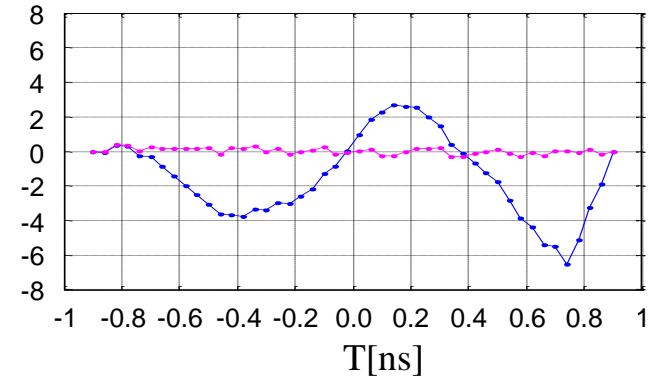
Difference from ideal line[ps]

● Output : 99 points



Difference from ideal line[ps]

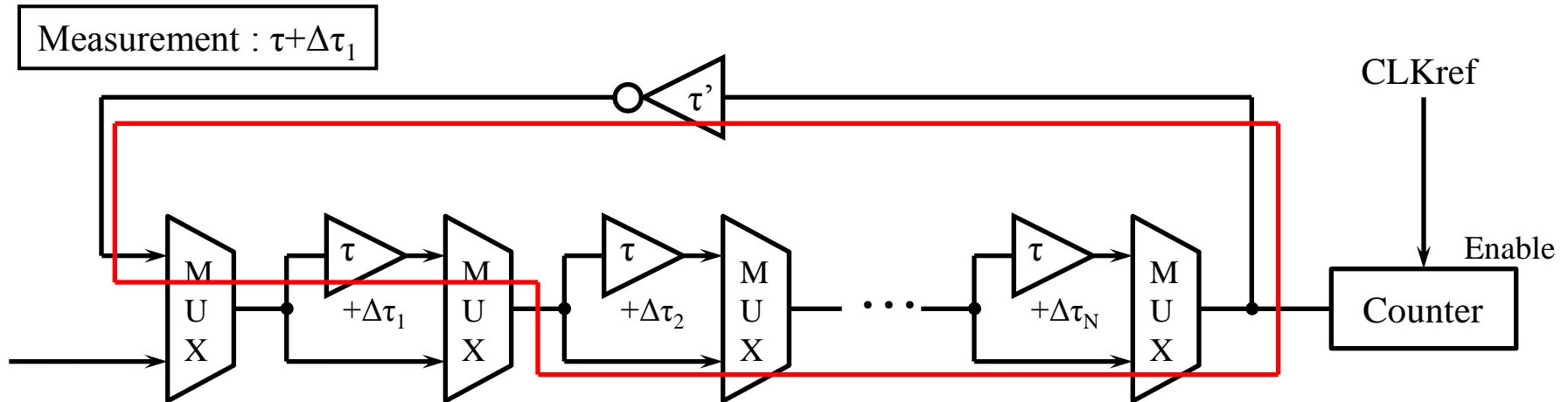
● Output : 599 points



—●—  $\Delta\Sigma$  TDC(with Self-Calibration)  
—●—  $\Delta\Sigma$  TDC(without Self-Calibration)

$\Sigma\Delta$  TDC linearity is improved

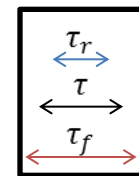
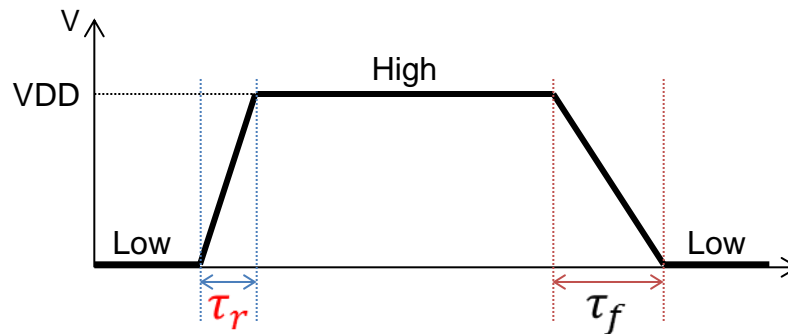
# Problem of Ring Oscillator



$$f = \frac{1}{2(\tau' + \tau + \Delta\tau_1)}$$



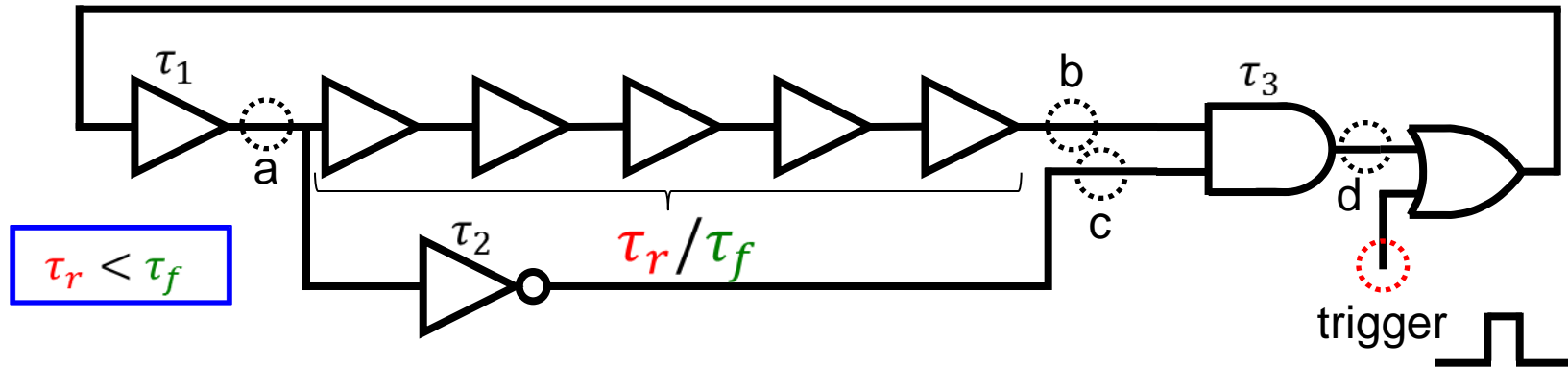
Measured delay



$$\tau = \frac{\tau_r + \tau_f}{2}$$

However, we need the rise delay  $\tau_r$

# Improved Delay Measurement Circuit

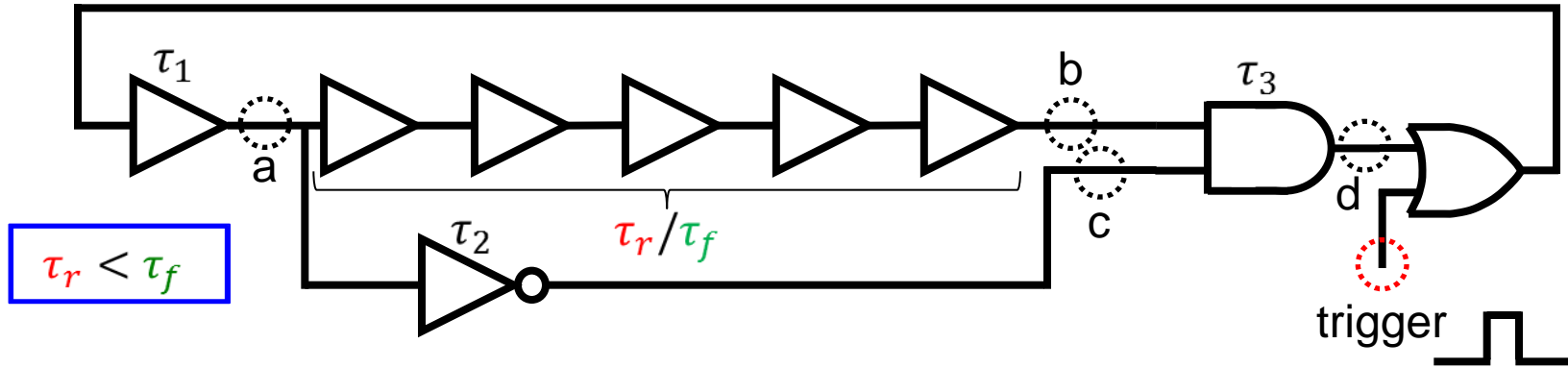


Oscillator circuit to measure  
the rise delay  $\tau_r$  of the buffer

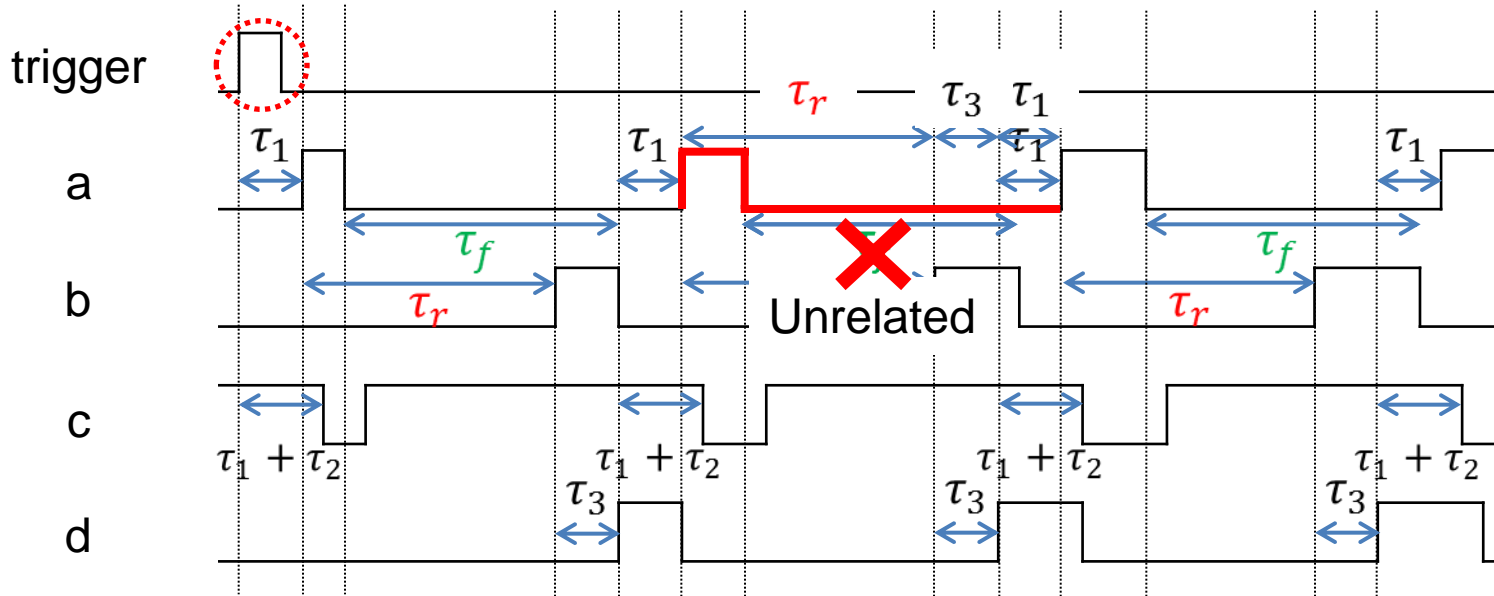
Oscillation period is a function of  $\tau_r$ , but NOT  $\tau_f$



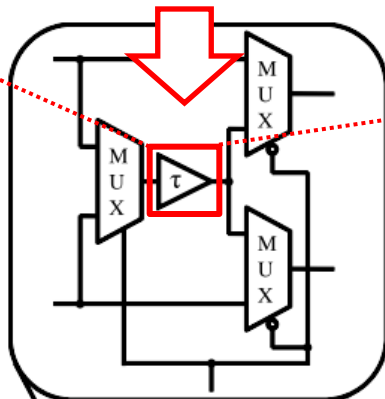
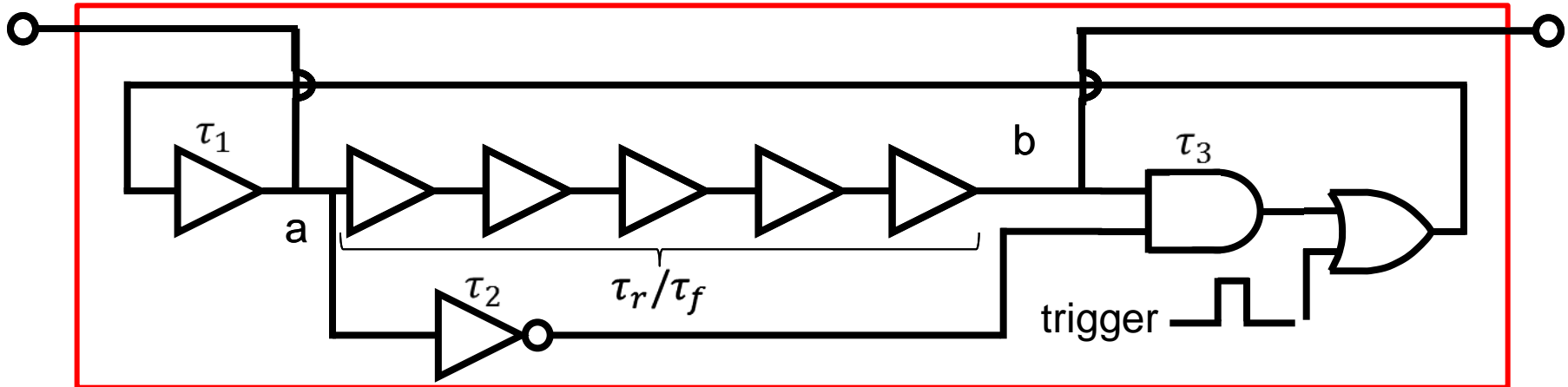
# Oscillation Timing Chart



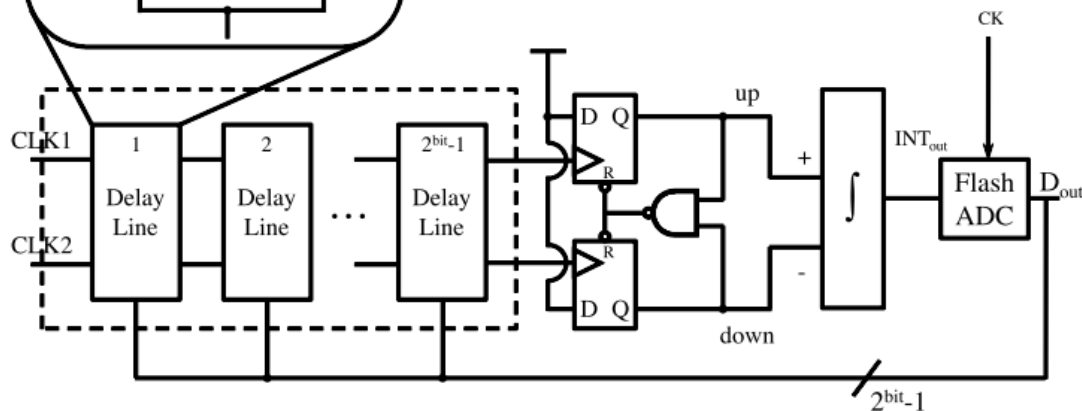
Period:  $\tau_1 + \tau_3 + \tau_r$



# Delay with Several Buffers



Replace !

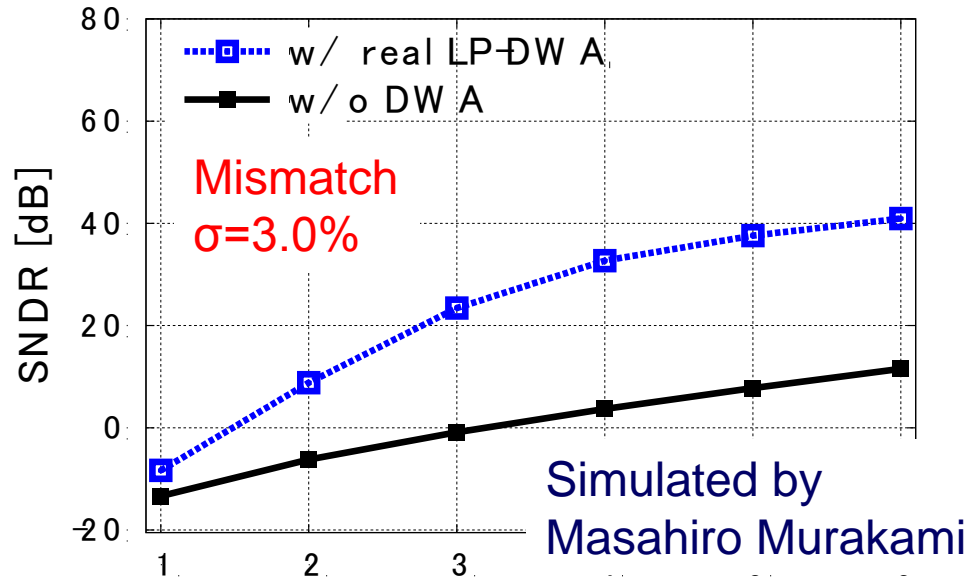
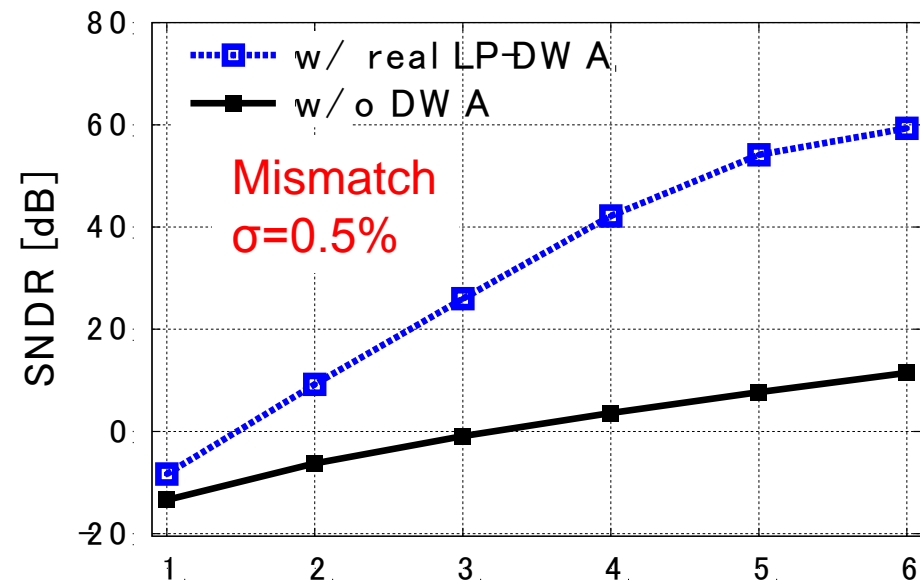
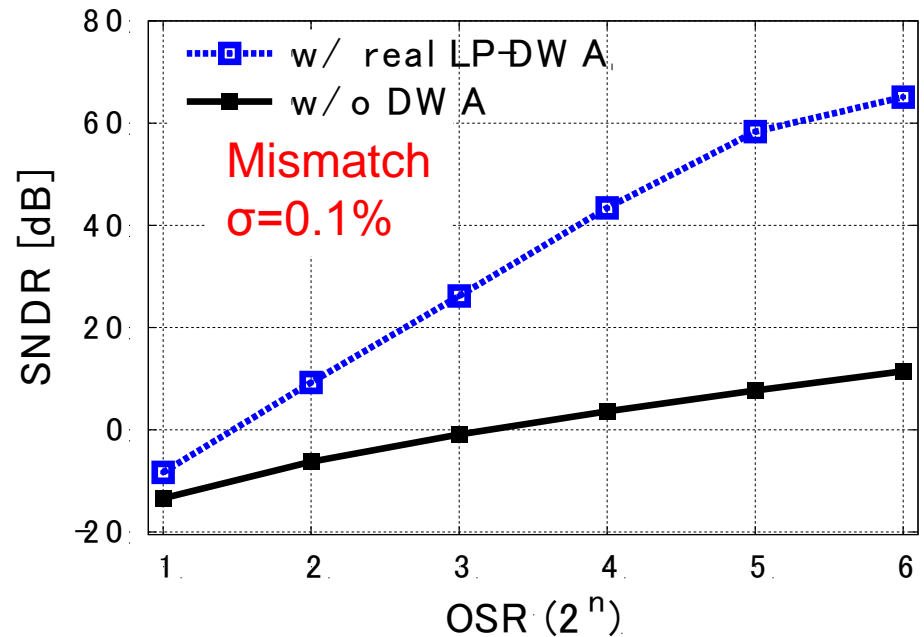
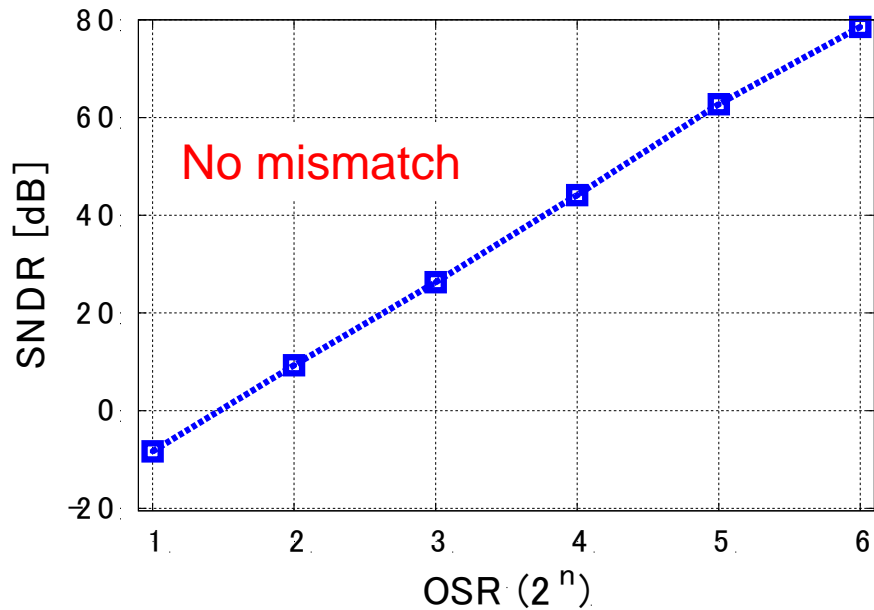


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- ▶ **Multi-Bit  $\Sigma\Delta$  TDC with Sorting**
- ▶ Conclusion

# DWA & Mismatches



# Possible Problem in Multi-bit TDC

For  $\Sigma\Delta$  **ADC**,

mismatch  $\sigma_{\text{dac}}$  in internal DAC  
is as small as 0.1 %.

 DWA is effective.

For  $\Sigma\Delta$  **TDC**,

mismatch  $\sigma_{\text{delay}}$  in internal delay cells  
can be much larger than 0.1 %  
depending on spec. & delay cell design.

 DWA may not be effective.

# Delay Cell Sorting (Step 1, 2)

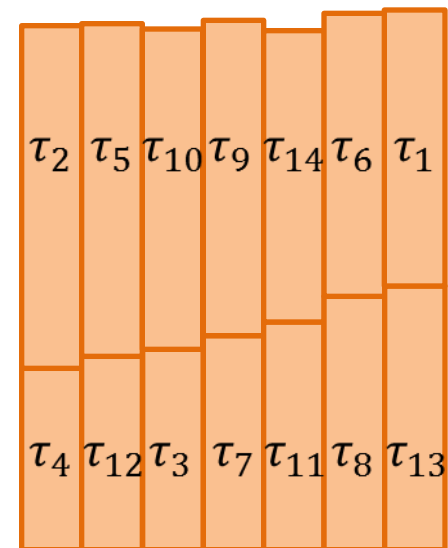
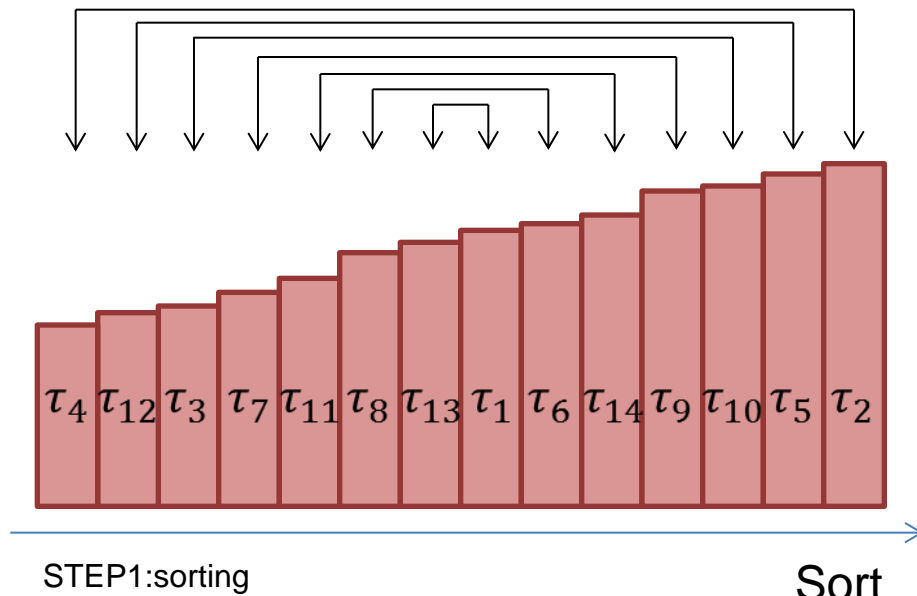
Step1

Measurement of each  
delay elements

Sorting Delay Cell

Step2

The combination of the  
14 delay elements using  
the internal CPU



# Delay Cell Sorting (Step 3, 4)

Step3

Measurement of  
combined delay elements

Sorting Combined Delay Cell

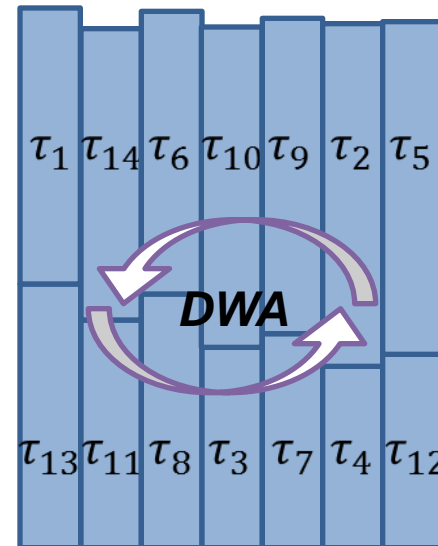
Largest  
smallest  
2<sup>nd</sup> Largest  
2<sup>nd</sup> smallest  
3<sup>rd</sup> Largest  
3<sup>rd</sup> smallest  
medium



STEP3:sorting

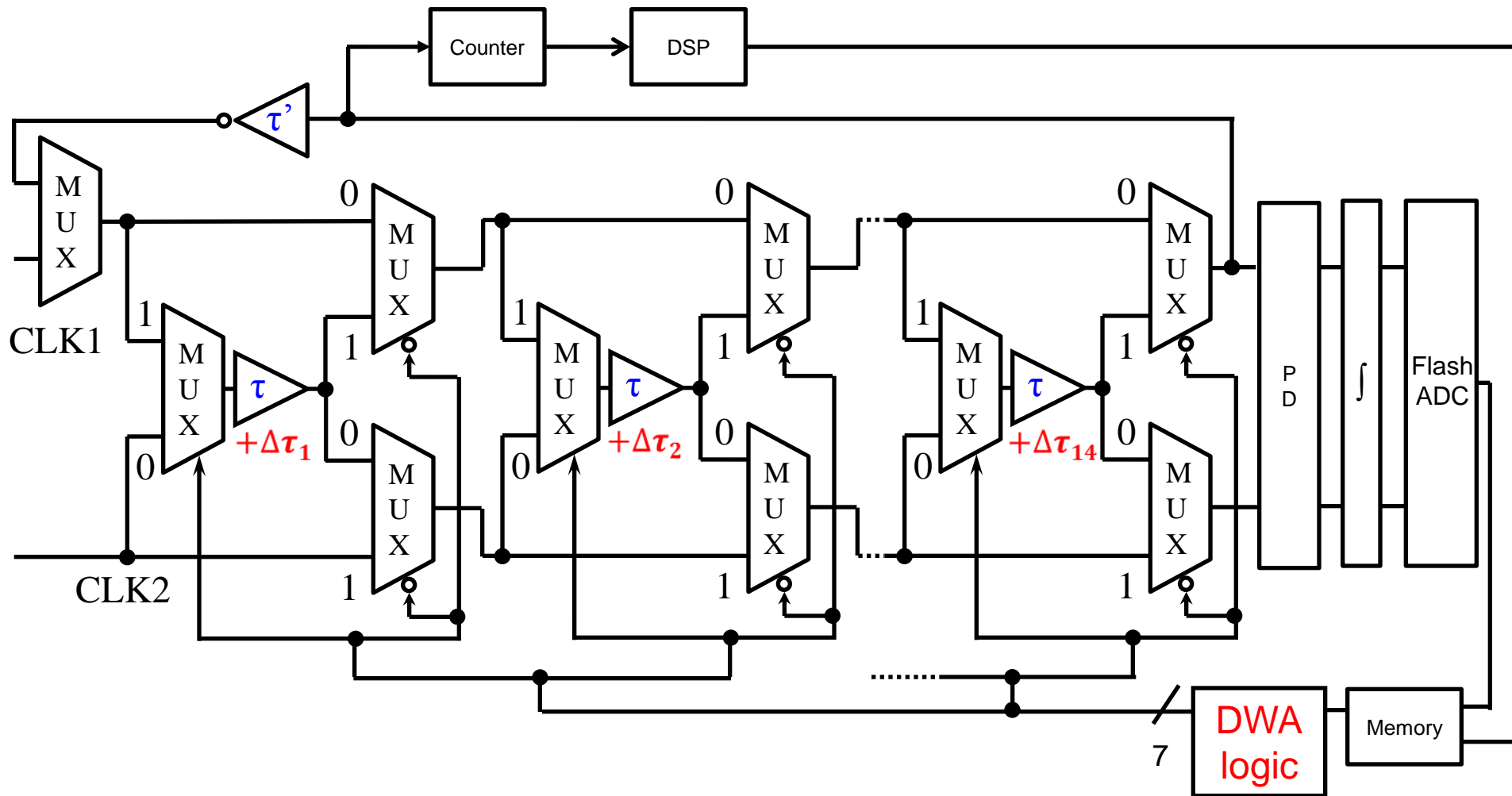
Step4

Randomize 7 combined delay  
cells to develop linearity using  
Data Weighted Averaging



STEP4:DWA

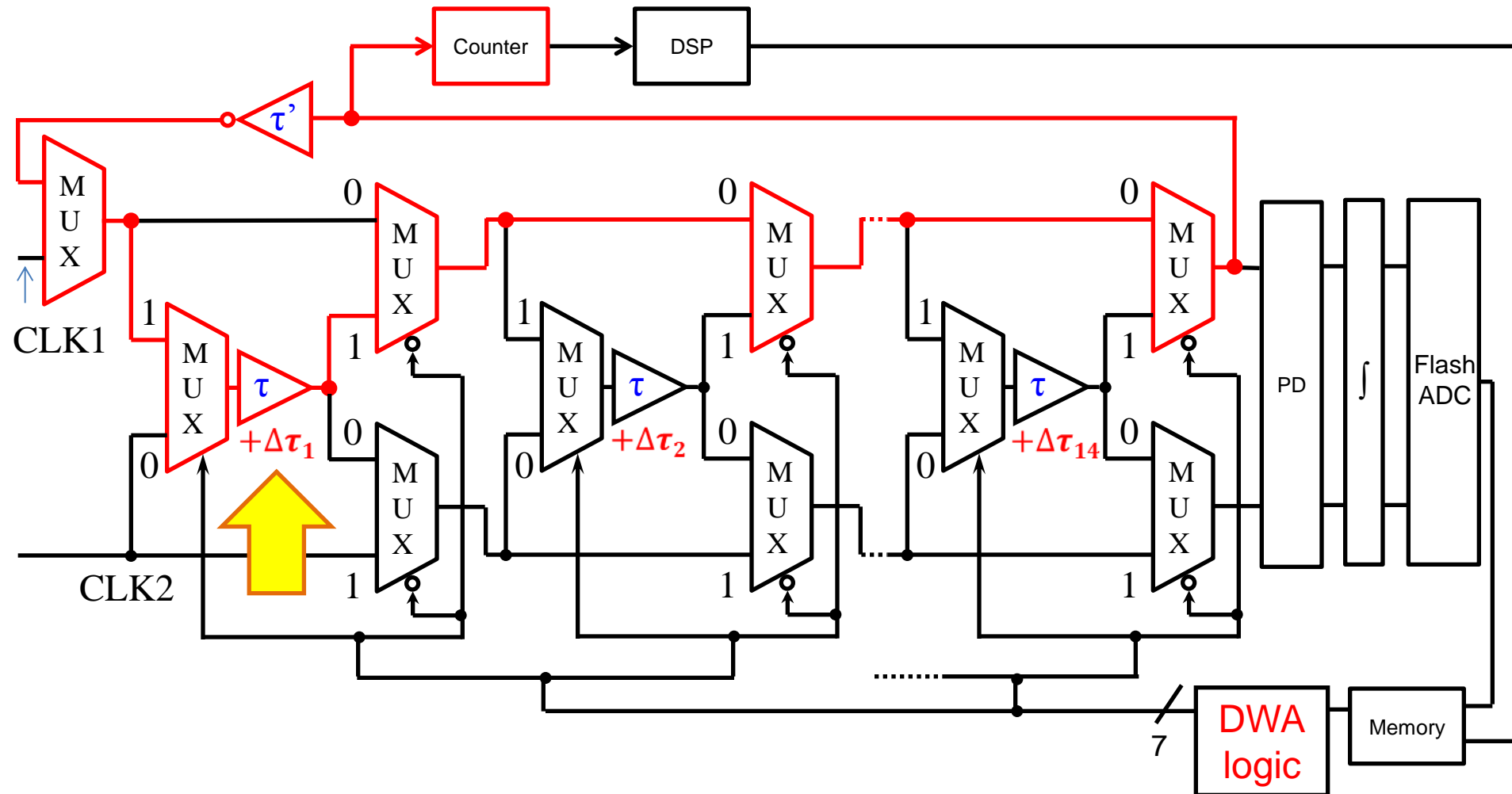
# Multi-Bit $\Sigma\Delta$ TDC with Sorting Technique





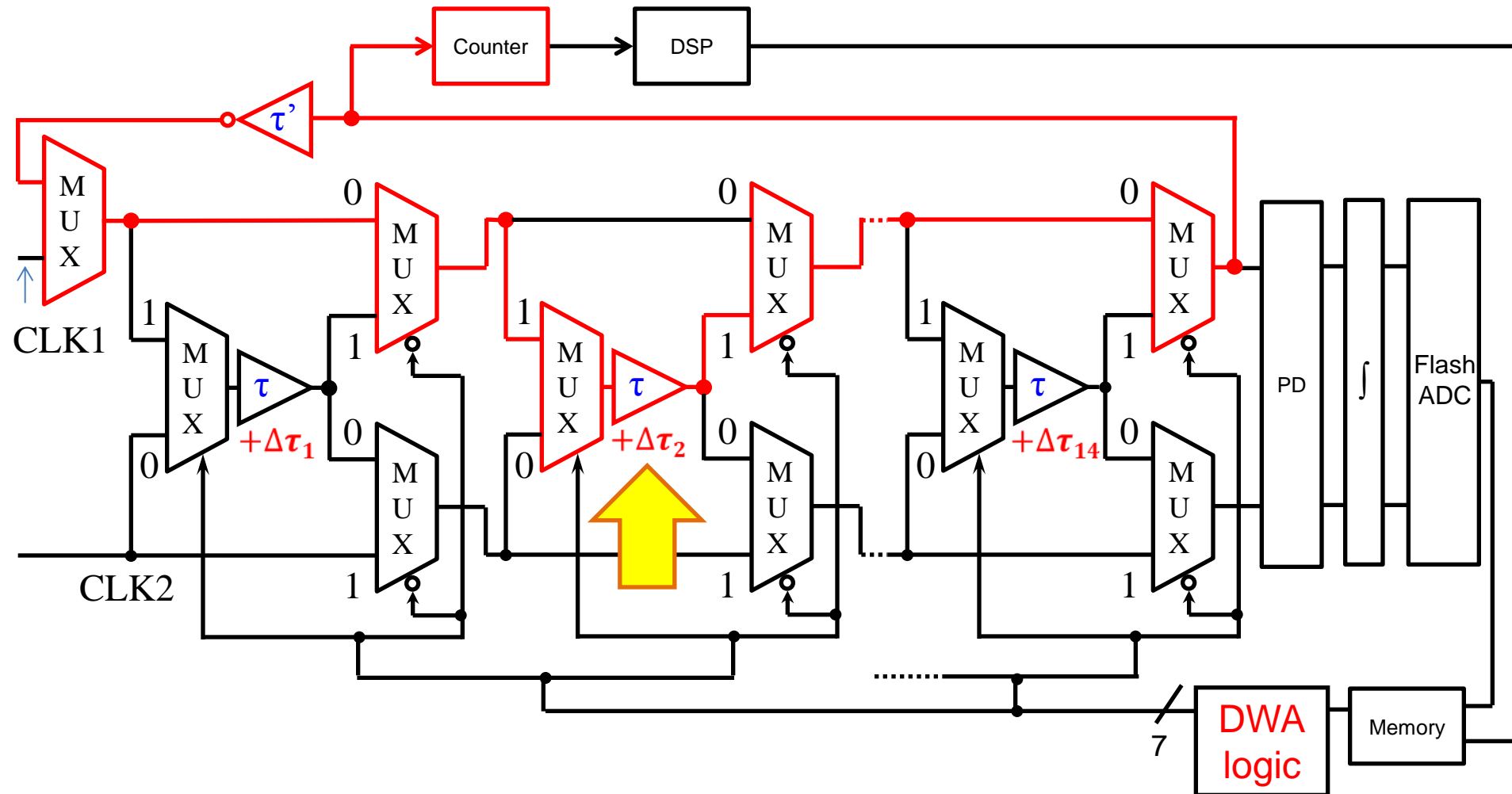
# Measure $\tau + \Delta\tau_1$

$\Delta\tau_1$  can be measured digitally.

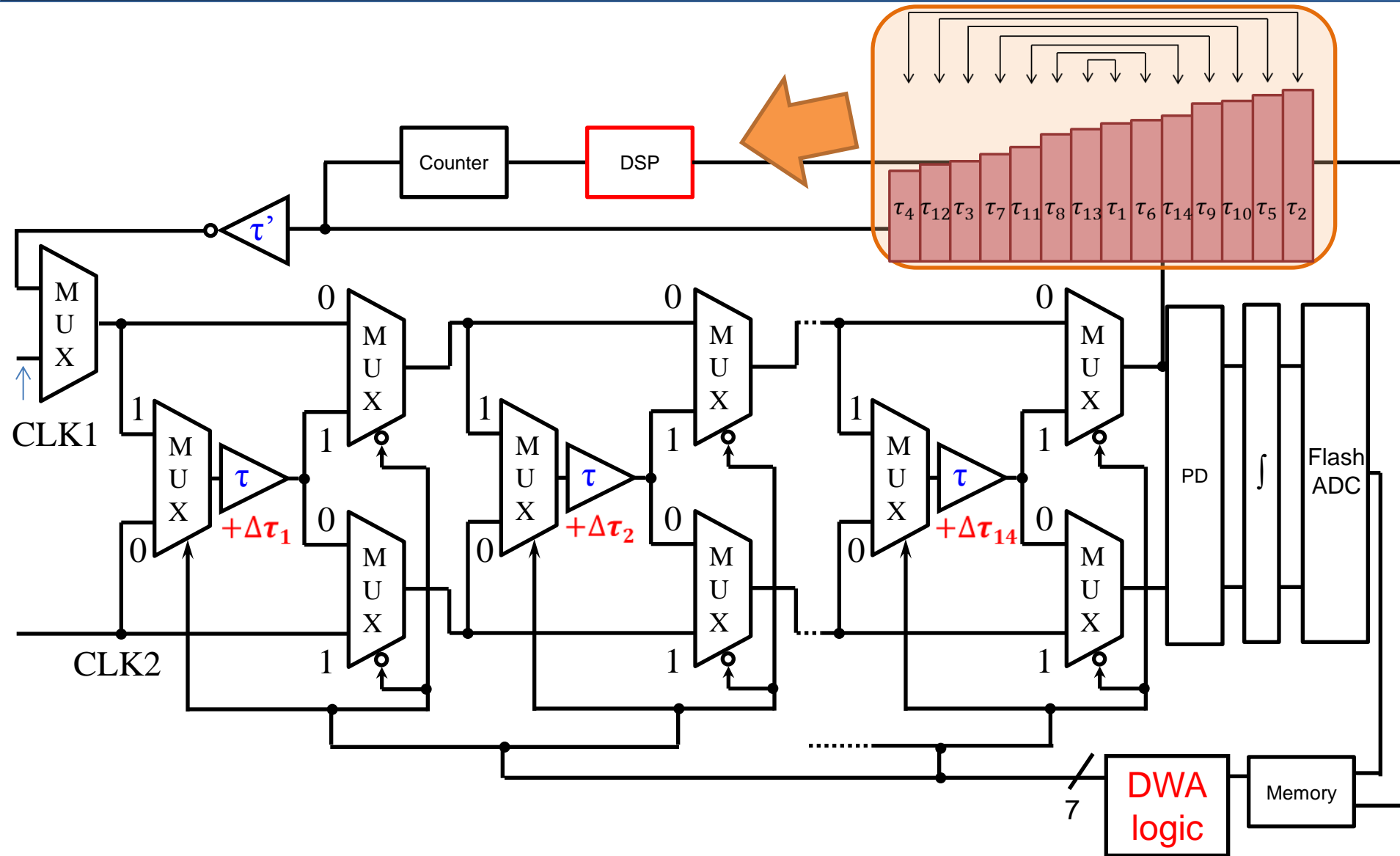


# Measure $\tau + \Delta\tau_2$

$\Delta\tau_2$  can be measured digitally.



# Multi-Bit $\Sigma\Delta$ TDC with Sorting Technique



# Delay Cell Sorting (Step 3, 4)

Step3

Measurement of each  
combined delay elements

Sorting Combined Delay Cell

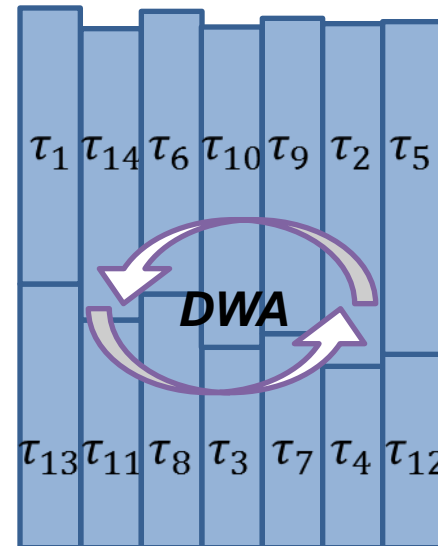
Largest  
smallest  
2<sup>nd</sup> Largest  
2<sup>nd</sup> smallest  
3<sup>rd</sup> Largest  
3<sup>rd</sup> smallest  
medium



STEP3:sorting

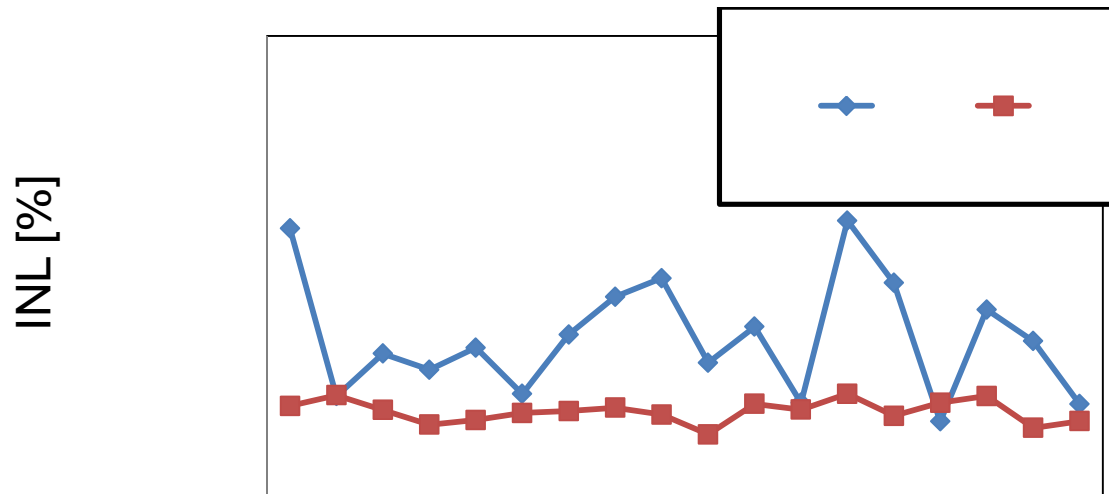
Step4

Randomize 7 combined delay  
cell to develop linearity using  
Data Weighted Averaging



STEP4:DWA

# Matlab Simulation Result



Delay cell variation patterns

Delay variation  $\sigma_{\text{delay}}$  : up to 5%.

Number of TDC output data: 1K points

For large delay variation,  
Sorting is effective, but only DWA may NOT be effective enough.

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- ▶ Research Objective
- ▶ Single-Bit & Multi-bit  $\Sigma\Delta$  TDCs
- ▶ Multi-Bit  $\Sigma\Delta$  TDC with DWA
- ▶ Multi-Bit  $\Sigma\Delta$  TDC with Self-Calibration
- ▶ Multi-Bit  $\Sigma\Delta$  TDC with Sorting
- ▶ **Conclusion**

# Circuit Performance Comparison

	Flash TDC	1-bit $\Sigma\Delta$ TDC	Multi-Bit $\Sigma\Delta$ TDC (without correction)	Multi-Bit $\Sigma\Delta$ TDC (with correction)
Area	×	◎	○	○
Resolution	×	◎	◎	◎
Accuracy	△	◎	×	◎
Time	◎	×	○	○

# Conclusion

- We propose to use  $\Sigma\Delta$  TDC for digital signal timing measurement
  
- Multi-bit  $\Sigma\Delta$  TDC
  - Short measurement time
  - Fine time resolution
  - Non-linearity due to mismatches among delay cells
    - ➡ Three techniques to improve linearity
      - DWA
      - Self-Calibration (signal is “time”)
      - Sorting

Low cost, high quality digital timing test can be realized



# Final Remark

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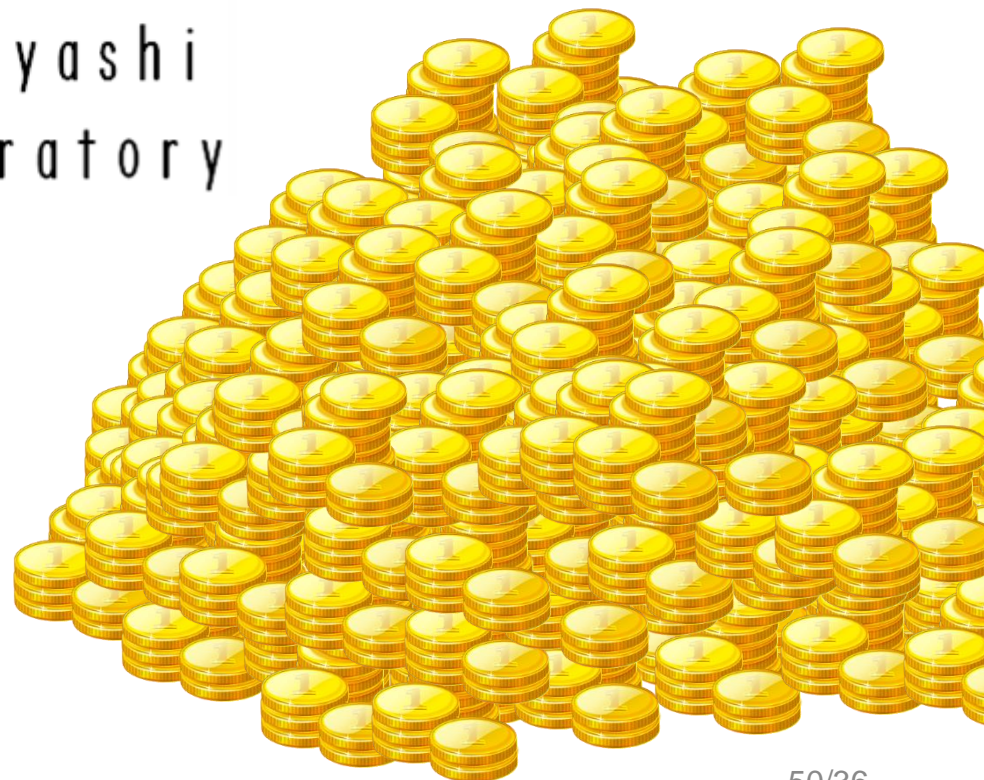
$\Sigma\Delta$ TDC can also measure  
phase noise power spectrum.

Presented by

“Phase Noise Measurement with Sigma-Delta TDC”,  
IEEE International Test Conference, Poster Session,  
Poster No. 3, Anaheim, CA (Sept. 11, 2013)



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Time is *GOLD* !!

$\Sigma\Delta$ TDC is a key.