

## An Analysis of Stochastic Self-Calibration of TDC Using Two Ring Oscillators

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**Abstract**—This paper presents a theoretical analysis of the stochastic calibration of TDC using two ring oscillators. Designers of TDC with the calibration function have to decide the design parameters to guarantee the convergence of error and valid calibration time. The basic theory of the calibration is useful to decide these parameters and the policy on the calibration design. The performance of the stochastic calibration depends on the design parameters, the frequencies of the two ring oscillators, the number of the stages, the buffer delay, and so on. This work analyzes explicitly the relation between these parameters and the performance of the calibration with simulation-based analysis. Simulation results reveal that the convergence of the calibration is guaranteed when both of the cycles of the two ring oscillators are the prime cycles. The histogram of each bin converges to the corresponding buffer delay value in a well-behaved manner; the *DNL* measurement error decreases monotonically in proportion to the increase of the number of the times of the measurement. In other words, the required number of the measurement times is in proportion to the required accuracy of calibration. This result is applied to the calibration of VDL-based TDC, too.

### I. INTRODUCTION

A Time-to-Digital-Converter (TDC) measures the time interval between two edges, and time resolution of several picoseconds can be achieved when the TDC is implemented with an advanced CMOS process. TDC applications include phase comparators of all-digital PLLs, sensor interface circuits, modulation circuits, demodulation circuits, as well as TDC-based ADCs [1], [2], [3], [4], [5], [6], [7], [8], [9]. The TDC will play an increasingly important role in the nano-CMOS era, because it is well suited to implementation with fine digital CMOS processes; a TDC consists mostly of digital circuitry, and resolution improves as switching speed increases.

Although the resolution of TDC is high, the linearity of TDC is lower. Therefore, self-calibration technique for high linearity is required. Ito et al. proposed the stochastic self-calibration technique of TDC using two ring oscillators [10]. Because this calibration technique is fully digital, it can be easily implemented on SOC fabricated with nanometer technology. Because this technique requires two ring oscillators unlike the method from [11], [12], the cost is lower.

This paper analyzes the stochastic calibration of TDC using two ring oscillators theoretically. Designers of TDC with the calibration function have to decide design parameters to guarantee the convergence of error and valid calibration time. The basic theory of the calibration is useful to decide these parameters and the policy on the calibration design. The convergence of the stochastic calibration depends on the uniformity of the differential delay sequence. The uniformity of the differential delay sequence depends on the design parameters, the frequencies of the two ring oscillators, the number of the stages, the buffer delay and so on. This work analyzes explicitly the relation between these parameters and the performance of the calibration with simulation-based analysis.

The rest of the paper is organized as follows. Section II describes the preliminaries for the following explanation. Section III explains the stochastic calibration using two ring oscillators. Section IV shows the simulation results. Finally, section V concludes the paper.

### II. PRELIMINARIES

In this work, we assume the basic TDC. Figure 1 (a) shows an example of the architecture of the basic TDC with four stages. The TDC is composed of four positive edge triggered D-type flip flops and an upper delay line and a lower clock line. The delay line is inserted four buffers with uniform delay. Each stage of a TDC is composed of a flip flop and a buffer. Suppose the two input signals are START and STOP. The START is the input of the upper delay line. The STOP is the input of the clock line. The delay of the buffer of each stage is  $\tau$ . The TDC measures the time interval between a transition from START and a transition from STOP. The resolution is equal to delay of a buffer.

In case of vernier delay line (VDL), buffers are inserted to the clock line, too. Each stage of VDL is composed of a flip flop, an upper buffer, and a lower buffer [13]. When delay of an upper buffer is  $\tau_1$  and the delay of a lower buffer is  $\tau_2$ , the resolution  $\Delta$  is equal to  $\tau_1 - \tau_2$ . The function of

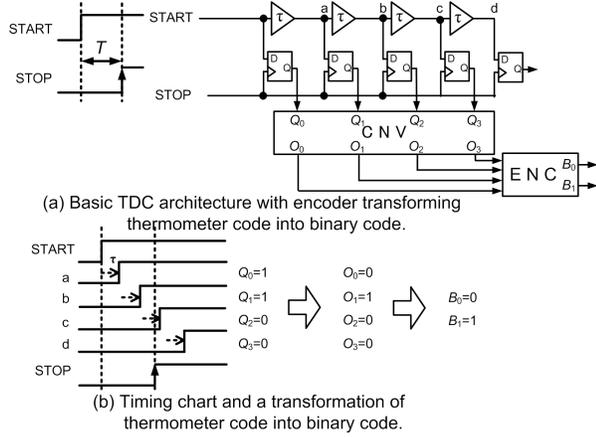


Figure 1. Basic 2-bit TDC.

VDL is equivalent to that of the basic TDC with the buffers with the delay  $\Delta$ .

Figure 1 (b) shows the timing chart of the basic TDC when the time interval between a transition signal from START and a transition signal from STOP is 2. After the measurement, the converter CNV transforms the result of the thermometer code  $Q_0Q_1Q_2Q_3 = 1100$  into the corresponding SW code  $O_0O_1O_2O_3 = 0100$ . The SW code is transformed into the corresponding binary code  $B_0B_1 = 01$  by the encoder ENC.

### III. STOCHASTIC SELF-CALIBRATION USING TWO RING OSCILLATORS

This section explains the stochastic self-calibration of TDC using two ring oscillators. Subsection III-A describes the basics. Subsection III-B shows the TDC with the stochastic self-calibration function. Subsection III-C explains the calibration sequence. Subsection III-D shows the characteristics of the differential delay sequence generated by the two ring oscillators.

#### A. Basics

In the stochastic calibration, the differential delays generated by the two ring oscillators are measured consecutively with TDC. The histogram is constructed with the measurement results. The variation of the delay of the buffer of each stage is estimated with the constructed histogram. Figure 2 shows the basics. All the buffers in the delay line of the TDCs of Fig. 2 (a) and (c) have the uniform delay 1. On the other hand, the buffers of the TDCs of Fig. 2 (b) and (d) have varied delay. The delay of the buffers of the first, the second, the third, and the fourth stages are 2, 0.5, 0.5, 1, respectively. Each delay of the differential delay sequence  $DF = \{1, 2, 3, 4\}$  is applied to the TDC to measure it sequentially one by one. After each measurement, the bin corresponding to the measurement result is incremented.

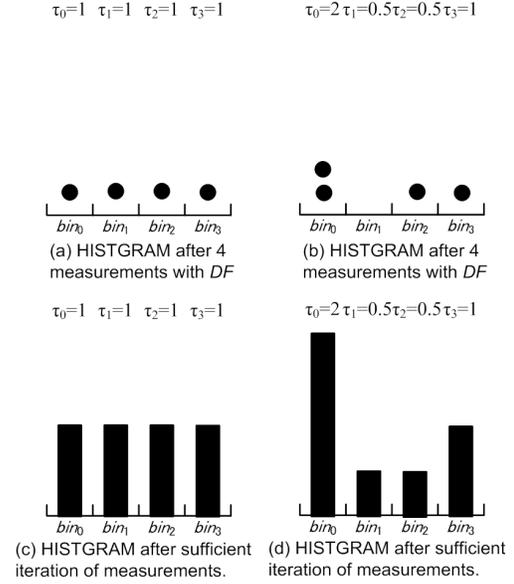


Figure 2. Basics.

case of the ideal TDC, the length of the bins  $bin_0$ ,  $bin_1$ ,  $bin_2$ , and  $bin_3$  become 1 after the four times measurement (Fig. 2 (a)). On the other hand, in case of the TDC with varied delay, the bin of the first stage  $bin_0$  is incremented after the measurement of the differential delays 1 and 2. The bin of the 3rd stage  $bin_2$  is incremented after the measurement of the differential delay 3. The bin of the 4th stage  $bin_3$  is incremented after the measurement of the differential delay 4. Consequently the length of the  $bin_0$ ,  $bin_1$ ,  $bin_2$ , and  $bin_3$  are 2,0,1,1, respectively after the four times measurement. As a buffer delay of a stage is larger, the length of the bin of the stage is longer. After sufficient measurement times with the delay sequence following uniform distribution whose lower limit is 0 and upper limit is 4, the length of the bin is proportional to the amount of the delay of the stage. Accordingly the variation of delay of the buffer of each stage can be estimated from the constructed histogram.

#### B. TDC with Stochastic Self-Calibration Function Using Two Ring Oscillators

Figure 3 shows the 8 stage TDC with stochastic self-calibration function using two ring oscillators. The upper part is TDC, the bottom part is the sub-circuit for the construction of the histogram. The input START is connected to an input of the 2-to-1 multiplexer MUX<sub>0</sub>. The output of MUX<sub>0</sub> is connected to the left-side edge of the delay line. The right-side edge of the delay line is feed-back to another input of MUX<sub>0</sub> through an inverter. The delay of the buffer of the  $i$ th stage ( $0 \leq i \leq 7$ ) is  $\tau_i$ , and the delay of the inverter is  $\tau_u$ .

The input STOP is connected to an input of the 2-to-1



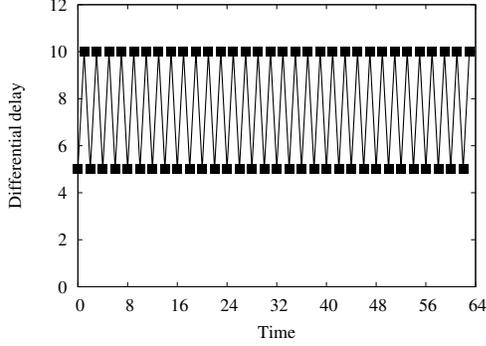


Figure 6. Differential delay sequence ( $N_{STG} = 8, T_0 = 10, T_1 = 5, d = 5$ ).

proposed calibration requires the dedicated counters for the construction of the histogram. The extra area depends on the bit length. In Subsection IV-D, we estimate the bit length of the counters.

#### A. Simulation Setup

In this evaluation, we assume that the ideal delay of a buffer on the delay line is 1. We add variations following gaussian distribution to the buffers. The  $3\sigma$  of the distribution is 10%. The number of the stage  $N_{STG}$  is 8, 16, 32, 64, 128. The differential non-linearity error is defined. When  $b_{ij}$  is the length of the bin of the  $i$ th stage after  $j$  times measurement, the differential non-linearity error of stage  $i$  ( $1 \leq i \leq N_{MEAS} - 1$ ) after  $j$  times measurement  $dnl_{ij}$  is expressed as the following formula.

$$dnl_{ij} = \frac{\tau_i}{\sum_{i=1}^{N_{STG}-2} \tau_i} - \frac{b_{ij}}{\sum_{i=1}^{N_{STG}-2} b_{ij}} \quad (1)$$

The differential non-linearity error after  $j$ th measurement  $DNL_j$  is expressed as the following formula.

$$DNL_j = \max(|dnl_{1j}|, \dots, |dnl_{(N_{MEAS}-2)j}|) \quad (2)$$

In the equations (1) and (2), the first and the last stages are ignored because the length of the histogram of these stages can be illegal value. As calibration process proceeds,  $DNL_j$  is convergent within the target error  $DNL$ . When  $DNL_{N_{MEAS}-1} > DNL$ , the calibration is fail, otherwise the calibration is success. When multiple calibrations are performed, the convergent probability to a target error  $DNL$  is defined as follows.

$$P_E(DNL) = N_{SCAL}/N_{CAL} \times 100.0$$

, where  $P_E(DNL)$  is the convergent probability,  $N_{CAL}$  is the calibration times, and  $N_{SCAL}$  is the times that the calibration is succeeded. Let  $N_E(TDC, T_0, T_1, d, DNL)$  be the required measurement times where  $TDC$  is the target TDC, the  $T_0$  and  $T_1$  are the cycles of the upper and

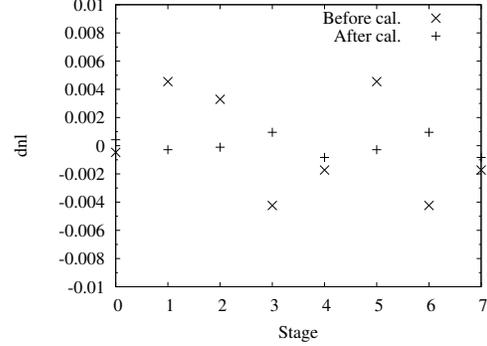


Figure 7.  $dnl$  error distribution of an 3-bit TDC before calibration and after calibration ( $T_0=11.93, T_1 = 2.11, d=0, N_E=699$ ).

the lower ring oscillators respectively,  $d$  is the initial differential delay, and  $DNL$  is the target error. When the calibration is fail,  $N_E(TDC, T_0, T_1, d, DNL) = 0$ . The  $N_E(TDC, T_0, T_1, d, DNL)$  is obtained with the following routine.

Step 1 Initialize  $i \leftarrow 1$ .

Step 2 Execute  $i$ th measurement.

Step 3 Calculate  $DNL_i$ .

Step 4 If  $e > DNL_i$ , the value of  $N_E$  is  $i$ . If  $i$  is equal to  $N_{MEAS}$ , the value of  $N_E$  is 0. Otherwise, increment  $i$  and go to Step 2.

The  $dnl$  error distribution of a 3-bit TDC before calibration and after calibration is shown in Fig. 7.

The set of the prime cycle  $T_{Pni}$  of which integer part is  $i$  is defined as follows.

$$T_{Pni} = \{d | d = p/10^n, 10^n \times i < p < 10^n(i+1), p \in P, n \in N\}$$

, where  $P$  is the set of prime numbers and  $n$  is the decimal digit. The inverse of the prime cycle is defined as the prime frequency. In this evaluation,  $n$  is set to 2. The influence of RMS jitter is ignored for theoretical analysis.

#### B. Specification of Cycle of Lower Ring Oscillator

We evaluate the  $T_1$  specification on the following cases of the combination of the cycles of the upper and the lower ring oscillators.

Case1 Both of  $T_0$  and  $T_1$  are prime cycles.

Case2  $T_0$  is multiple of  $T_1$ .

Case3 Arbitrary values of  $T_0$  and  $T_1$ .

The maximum value of the differential delay generated by the two ring oscillators is equal to the width of  $T_0$ . Therefore,  $T_0$  is fixed. The  $T_1$  is swept up in a range. Let  $TS_1$  be the set of the cycles of the lower ring oscillator. To generate the differential delay sequence following uniform distribution,  $T_0$  should be larger than the largest measurable delay  $1.1 \times N_{STG}$ . We decide the fixed value of  $T_0$  and  $TS_1$

which is the set of  $T_1$  for the calibration on the above three cases as follows.

*Case1*  $T_0 = T_{p0}$ . The  $TS_1$  includes all the prime cycles in the range between  $T_{min}$  and  $T_{max}$ .

*Case2*  $T_0 = T_{max}$ . The  $TS_1$  includes all the dividers in the range between  $T_{min}$  and  $T_{max}$ .

*Case3*  $T_0 = T_{max}$ . The  $TS_1$  includes all the cycles in the range between  $T_{min}$  and  $T_{max}$ .

The  $T_{min}$  and  $T_{max}$  are the upper and the lower limits of the range, respectively. The cycle  $T_{p0}$  is the maximum prime cycle in the range.

Table I shows the parameter setup of this evaluation. 100 multiple TDCs ( $TDC_0 - TDC_{99}$ ) are generated randomly. With all the combination of  $T_0$  and  $T_1$  picked up from  $TS_1$  the calibration is performed to calculate the convergent probability on *Case1*, *Case2*, *Case3* in each TDC. The target error  $DNL$  is  $1/1,024$ . The initial differential delay  $d$  is 0.

Table II shows the result. All the convergent probability of *Case1* is 100 %. All the convergent probability of *Case2* is 0 %. The convergent probability is 100 % when both of  $T_0$  and  $T_1$  are the prime cycles, while the convergent probability is 0 % when  $T_0$  is a multiple of  $T_1$ . In *Case3*, the convergent probability increases as  $T_{STG}$  increases. It means that the convergent probability is better as the number of stages is larger when  $T_1$  is an element picked up from  $TS_1$ . When  $T_{STG} = 128$ ,  $P_{E(1/1024)}$  is 77.7 %. It means that the probability of the convergence of the calibration is 77.7 % when  $T_0 = 132$ , and a lower cycle  $T_1$  is randomly chosen from  $TS_1$ . We conclude that 100 % convergence is guaranteed when both of  $T_0$  and  $T_1$  are the prime cycles, otherwise is not guaranteed.

Next we evaluate the specification of the required number of times of measurement for the convergence to the target error. In this evaluation,  $TS_1 = \{T_{10}, T_{11}, \dots$

Table I  
PARAMETER SETUP OF EVALUATION OF SPECIFICATION OF  $T_1$ .

$N_{STG}$	$T_{min}$	$T_{max}$	$T_{p0}$
8	2	12	11.93
16	2	18	17.89
32	2	36	35.93
64	2	68	67.93
128	2	132	131.87

Table II  
CONVERGENT RATIO ( $d = 0, e = 1/1,024$ ).

Case	$T_{STG}$				
	8	16	32	64	128
<i>Case1</i>	100.0	100.0	100.0	100.0	100.0
<i>Case2</i>	0.0	0.0	0.0	0.0	0.0
<i>Case3</i>	37.5	44.0	57.1	75.9	77.7

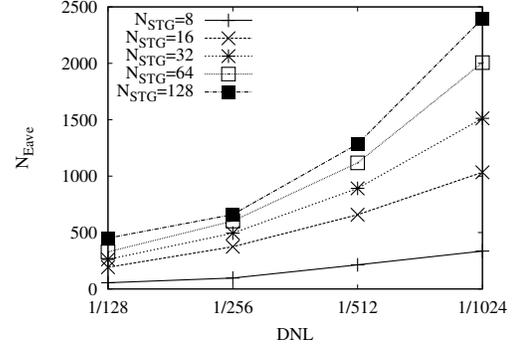


Figure 8.  $DNL-N_{Eave}$  specification (*Case1*).

,  $T_1(N_{TDC}-1)$ . When the combination of the cycles is a pair of  $T_0$  and  $T_{1i}$  ( $0 \leq i \leq N_{TDC}-1$ ), the required measurement  $N_{Ei}$  is expressed as the following formula.

$$N_{Ei} = \begin{cases} \sum_{j=0}^{N_{TDC}} N_E(T_0, T_{1j}, TDC_i, d, e) / N_{SCAL} \\ (N_{SCAL} \neq 0), \\ 0 (N_{SCAL} = 0) \end{cases}$$

, where  $N_{SCAL}$  is the number of TDCs that calibrations are convergent. The average number of the times of the delay measurement  $N_{Eave}$  is expressed as the following formula.

$$N_{Eave} = \begin{cases} \sum_{i=0}^{N_{TDC}} N_{Ei} / N_{STDC} (N_{STDC} \neq 0), \\ 0 (N_{STDC} = 0) \end{cases}$$

, where  $N_{STDC}$  is the sum of TDC that  $N_{Ei} \neq 0$ . The target error  $DNL$  is  $1/128, 1/256, 1/512, 1/1,024$ . The initial differential delay  $d$  is 0. Figure 8 represents the  $DNL-N_{Eave}$  specification in case of *Case1*. The horizontal axis is  $DNL$ . The vertical axis is  $N_{Eave}$ . This graph demonstrates that approximately  $N_{Eave}$  increases in proportion to the decrease of  $DNL$ . As  $N_{STG}$  increases, the curves move up to the vertical direction.

Figure 9 shows the  $DNL - N_E$  specification. The  $T_0$  is fixed to 12.0. Four  $T_1$ s are randomly chosen from  $TS_1$ . Then the four curves  $a, b, c, d$  are plotted. The number of the maximum delay measurement times  $N_{MEAS}$  is 10,000. Accordingly, if a curve sticks to  $N_{MEAS}$ , then the calibration fails on the point. The curve  $a$  fails to converge in  $DNL = 1/1,024$ . The curve  $b$  fails to converge in  $DNL = 1/256, 1/512, 1/1,024$ . The curves  $c$  and  $d$  succeed to converge in  $DNL = 1/128, 1/256, 1/512, 1/1,024$ . Like this, the variance of the curves is large when  $T_0$  and  $T_1$  are not prime cycles.

### C. Specification of Initial Differential Delay

The characteristics of the differential delay sequence depend on the initial differential delay. We evaluate the  $d$  specification of the required measurement  $N_E$ . With the pair of  $T_0$  and  $T_1$  which guarantees the convergence when

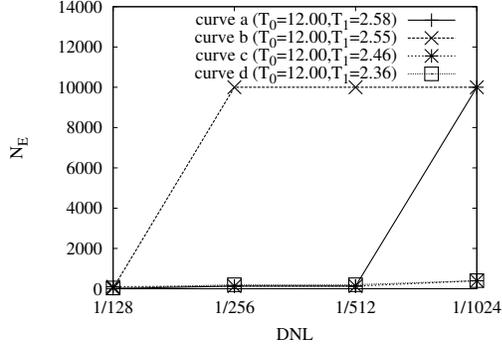


Figure 9.  $DNL-N_E$  specification (*Case2*).

$d = 0$ , we perform the multiple calibrations consecutively with sweeping up  $d$  from 0 to  $T_0$ . Figure 10 shows the curve of  $d$  specification when  $T_{STG} = 64$ ,  $T_{p0} = 67.93$ , and  $T_1$  is 2.11. The horizontal axis is  $d$ . The vertical axis is the required measurement times  $N_E$  for the convergence to  $DNL = 1/128, 1/256, 1/512, 1/1,024$ . This result shows that  $N_E$  does not depend on  $d$ .

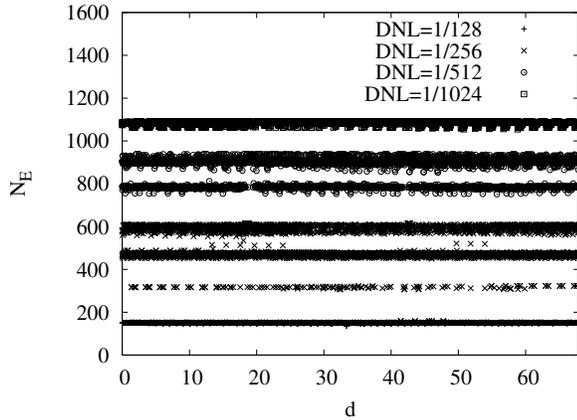


Figure 10.  $d-N_E$  specification ( $N_{STG} = 64, T_0 = 67.93, T_1 = 2.11$ ).

#### D. Bit Length of Counters

When  $3\sigma$  of the distribution of the buffer delays is 0.1, the required bit length of the counter of each stage  $L_{CNT}$  is expressed as the following formula.

$$L_{CNT} = \log_2[1.1 \times N_E / N_{STG}].$$

According to the result of the value of  $N_{Eave}$  shown in Fig. 8, the required bit length of each counter is calculated. The target error  $DNL$  is  $1/1,024$ . Table III shows the result. The required bit length is around 5-7 bit. The required bit length tends to decrease as  $N_{STG}$  increases.

## V. CONCLUSIONS

In this paper, we have analyzed the TDC with the stochastic self-calibration using two ring oscillators. Here, we summarize this study.

- 1) The histogram of each bin converges to the corresponding buffer delay value in a well-behaved manner; the  $DNL$  measurement error decreases monotonically in proportion to the increase of the number of the times of the measurement.
- 2) Accordingly, the required number of the measurement times is in proportion to the required accuracy of the calibration.
- 3) When both of the frequencies of the two ring oscillators are not the prime frequencies, the convergence of the calibration is not guaranteed. The simulation results show that the convergent probability is 77.7 % when  $N_{STG} = 128$ . In other words, 22.3 % are not convergent.
- 4) According to the above results, we get the following conclusions.
  - Both of the frequencies of the two ring oscillators should be the prime frequencies.
  - When both of the frequencies are the prime frequencies, we estimate the required number of the times of measurement from the target error  $DNL$ .

Sometimes, the ring oscillators are infected with the injection lock [15]. In this paper, we ignore this effect for theoretical analysis. We will consider it in the future work. The two frequency generators are implemented by the two ring oscillators. However the waveform include considerable amount of jitter. It gives bad influences on the convergence of the calibration. We will consider the strategy to reuse the existing PLL with lower jitter as a frequency generator in a future work. A SOC usually includes scan design in its logic block. If TDC is implemented around the logic block, we can reuse the scan chains as the counters to reduce the extra area.

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Table III  
BIT LENGTH OF COUNTERS ( $DNL = 1/1,024$ ).

$N_{STG}$	8	16	32	64	128
$N_{Eave}$	336.0	1,033.9	1,512.4	2,006.5	2,396.0
$L_{CNT}$	6	7	6	6	5

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