Flat Passband Gain Design Algorithm for 2\textsuperscript{nd}-order RC Polyphase Filter

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Contents

- Research Goal
- Roles of RC Polyphase Filter
- Transfer Function of RC Polyphase Filters
- Flat Passband Gain Algorithm for 2\textsuperscript{nd}-order Filter
- Summary
Contents

● Research Goal

● Roles of RC Polyphase Filter

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● Summary
Research Goal

- To establish systematic design and analysis methods of RC polyphase filters.

- To derive flat passband gain algorithm for 2\textsuperscript{nd}-order filter

- To demonstrate usefulness of Nyquist chart
Contents

● Research Goal

● **Roles of RC Polyphase Filter**

● Transfer Function of RC Polyphase Filters

● Flat Passband Gain Algorithm for 2\textsuperscript{nd}-order Filter

● Summary
Features of RC Polyphase Filter

- Its input and output are complex signals.
- Passive RC analog filter
- One of key components in wireless transceiver analog front-end
  - I, Q signal generation
  - Image rejection
- Its design and analysis methods have not been fully developed yet.
First-order RC Polyphase Filter

Differential Complex Input: \[ V_{in} = I_{in} + j Q_{in} \]

Differential Complex Output: \[ V_{out} = I_{out} + j Q_{out} \]
I, Q Signal Generation From Single Sinusoidal Input

\[ I_{\text{in}} = \cos(\omega_{\text{LO}} t) \]

\[ Q_{\text{in}} = 0 \]

Polyphase Filter

\[ I_{\text{out}} = A \cos(\omega_{\text{LO}} t + \theta) \]

\[ Q_{\text{out}} = A \sin(\omega_{\text{LO}} t + \theta) \]

\[ \omega_{\text{LO}} = \frac{1}{R_1 C_1} \]
Cosine, Sine Signals in Receiver

They are used for down conversion
Pure I, Q signal generation

3rd-order harmonics rejection

\[ I_{in} = \cos(\omega_{LO} t) + B \cos^3(\omega_{LO} t) \]

\[ Q_{in} = \sin(\omega_{LO} t) + B\sin^3(\omega_{LO} t) \]

With 3rd-order harmonics

\[ I_{out} = A\cos(\omega_{LO} t + \theta) \]

\[ Q_{out} = A\sin(\omega_{LO} t + \theta) \]

Without 3rd-order harmonics
Simulation of 3\textsuperscript{rd}-order harmonics rejection

\[ I_{in}(t) = \cos(\omega_{LO}t) + B\cos^3(\omega_{LO}t) \]
\[ Q_{in}(t) = \sin(\omega_{LO}t) + B\sin^3(\omega_{LO}t) \]

\[ 3\omega_{LO} = \frac{1}{R_1C_1} \]

\[ I_{OUT}(t) = A\cos(\omega_{LO}t + \theta) \]
\[ Q_{OUT}(t) = A\sin(\omega_{LO}t + \theta) \]
Image Rejection Filter

\[ I_{in} = (A + B) \cos(\omega t) \]

\[ Q_{in} = (A - B) \sin(\omega t) \]

\[ I_{out} = A \cos(\omega t) \]

\[ Q_{out} = A \sin(\omega t) \]

\[ Ae^{j\omega t} + Be^{-j\omega t} \rightarrow Ae^{j\omega t} \]
Problem when $\omega_{LO} \neq \frac{1}{R_1C_1}$

$\omega_{LO} = \frac{1}{R_1C_1}$

$\omega_{LO} = \frac{2}{R_1C_1}$

Amplitudes of I, Q signals differ significantly.
Problem of large amplitude difference between $I_{out}$, $Q_{out}$ can be alleviated.

$$\omega_{LO} = \frac{2}{R_1 C_1}$$
Amplitude difference problem is further alleviated.

\[ \omega_{LO} = \frac{2}{R_1 C_1} \]
Contents

● Research Goal

● Roles of RC Polyphase Filter

● Transfer Function of RC Polyphase Filters

● Flat Passband Gain Algorithm for 2\textsuperscript{nd}-order Filter

● Summary
Transfer Function of RC Polyphase Filter

Complex Signal Theory

- Complex input
- Complex output

Complex Transfer Function

\[
\begin{align*}
V_{in}(j\omega) &= I_{in} + jQ_{in} \\
V_{out}(j\omega) &= I_{out} + jQ_{out} \\
G(j\omega) &= \frac{V_{out}(j\omega)}{V_{in}(j\omega)}
\end{align*}
\]
Transfer Function of 1\textsuperscript{st}-order RC Polyphase Filter

Differential signal

\[ I_{\text{in}}(t) = I_{\text{in}+}(t) - I_{\text{in}-}(t) \]
\[ Q_{\text{in}}(t) = Q_{\text{in}+}(t) - Q_{\text{in}-}(t) \]
\[ I_{\text{out}}(t) = I_{\text{out}+}(t) - I_{\text{out}-}(t) \]
\[ Q_{\text{out}}(t) = Q_{\text{out}+}(t) - Q_{\text{out}-}(t) \]

Complex signal

\[ V_{\text{in}}(t) = I_{\text{in}}(t) + jQ_{\text{in}}(t) \]
\[ V_{\text{out}}(t) = I_{\text{out}}(t) + jQ_{\text{out}}(t) \]
Transfer Function of 1st-order RC Polyphase Filter

Transfer Function

\[ G_1(j\omega) = \frac{1 + \omega RC}{1 + j\omega RC} \]

Gain

\[ |G_1(j\omega)| = \frac{|1 + \omega RC|}{\sqrt{1 + (\omega RC)^2}} \]
Explanation of I, Q Signal Generation by $G_1(j\omega)$

$$Q_{in}(t) \equiv 0, \quad I_{in}(t) = \cos(\omega t)$$

$$V_{in}(t) = I_{in}(t) + j \ Q_{in}(t) = \cos(\omega t) = \frac{1}{2} [e^{j\omega t} + e^{-j\omega t}]$$

$$V_{out}(t) = \frac{1}{2} [|G_1(j\omega)|e^{j(\omega t + \angle G_1(j\omega))} + |G_1(-j\omega)|e^{j(-\omega t + \angle G_1(-j\omega))}]$$

$$= \frac{\sqrt{2}}{2} \cos \left( \omega t - \frac{\pi}{4} \right) + \frac{j\sqrt{2}}{2} \sin(\omega t - \frac{\pi}{4})$$

Here

$$|G_1(j\omega)|_{\omega = \frac{1}{RC}} = 0, \quad |G_1(j\omega)|_{\omega = \frac{1}{RC}} = \sqrt{2}, \quad \angle G_1(j\omega) = -\frac{\pi}{4}$$
Transfer Function of 2\textsuperscript{nd}-order RC Polyphase Filter

Transfer Function

\[ G_2(j\omega) = \frac{(1 + \omega R_1 C_1)(1 + \omega R_2 C_2)}{1 - \omega^2 R_1 C_1 R_2 C_2 + j\omega(C_1 R_1 + C_2 R_2 + 2R_1 C_2)} \]

Derivation is very complicated, so we used "Mathematica."

Gain $|G_2(j\omega)|$ characteristics
Contents

- Research Goal
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Why 2\textsuperscript{nd}-order RC Polyphase Filter?

- 1\textsuperscript{st}-order RC polyphase filter
  - Performance is limited.

- 2\textsuperscript{nd}-order
  - Handy, good performance
  - Widely used.

- 3\textsuperscript{rd}-order
  - Circuit is complicated.
Need for Flat Passband Gain Algorithm of 2\textsuperscript{nd}-order RC Polyphase Filter

Transfer Function

\[
G_2(j\omega) = \frac{(1 + \omega R_1 C_1)(1 + \omega R_2 C_2)}{1 - \omega^2 R_1 C_1 R_2 C_2 + j\omega(C_1 R_1 + C_2 R_2 + 2R_1 C_2)}
\]

We need flat passband gain

Gain $|G_2(j\omega)|$ characteristics

[Graph showing gain characteristics with marked frequency ranges: Stop band, Pass band]
Four Design Parameters

4 parameters: \( R_1, R_2, C_1, C_2 \)

\[
\omega_1 = \frac{1}{R_1 C_1}, \quad \omega_2 = \frac{1}{R_2 C_2}, \quad X = \frac{1}{R_2 C_1}, \quad Y = \frac{1}{R_1 C_2}
\]

4 constraints
Two Constraints from Filter Spec.

2 zeros: \[ \omega_1 = -\frac{1}{R_1 C_1}, \quad \omega_2 = -\frac{1}{R_2 C_2} \]

are given from the filter specification.
We use the third constraint \( X = \frac{1}{R_2C_1} \) for passband gain flattening.

The fourth constraint is left for ease of IC realization.
Nyquist Chart of $G_2(j\omega)$

Gain characteristics $|G_2(j\omega)|$

Nyquist chart of $G_2(j\omega) = X(\omega) + jY(\omega)$

$|G_2(j\omega_1)| = |G_2(j\omega_2)|$

But in general $|G_2(j\omega_1)| = |G_2(j\omega_2)| \neq |G_2(j\sqrt{\omega_1\omega_2})|$
Our Idea for Flat Passband Gain Algorithm

Gain characteristics $|G_2(j\omega)|$

Nyquist chart of $G_2(j\omega) = X(\omega) + jY(\omega)$

If we make $|G_2(j\omega_1)| = |G_2(j\omega_2)| = |G_2(j\sqrt{\omega_1\omega_2})|$, gain would be flat from $\omega_1$ to $\omega_2$. 
Solving Third Constraint

Our algorithm

\[ |G_2(j\omega_1)| = |G_2(j\omega_2)| = |G_2(j\sqrt{\omega_1\omega_2})|, \]

\[ \alpha \omega_{21}^2 + \beta \omega_{21} + \gamma = 0 \]

\[ \omega_{21} = \frac{1}{R_2C_1} \]

We need a positive real solution of \( \omega_{21} \).
Condition for Solution

Third constraint

\[
\omega_{21} = \frac{1}{R_2 C_1} = \frac{-\beta + \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}
\]

\[\alpha = 6\omega_1^2 + 6\omega_2^2 + 4\omega_1\omega_2 - 8\sqrt{\omega_1\omega_2}(\omega_1 + \omega_2)\]

\[\beta = 6\omega_1^3 + 6\omega_2^3 + 10\omega_1\omega_2(\omega_1 + \omega_2) - 8\sqrt{\omega_1\omega_2}(\omega_1 + \omega_2)^2\]

\[\gamma = \omega_1^4 + \omega_2^4 + 2\omega_1\omega_2(\omega_1^2 + \omega_2^2 + 5\omega_1\omega_2) - 4\sqrt{\omega_1\omega_2}(\omega_1^3 + \omega_2^3 + \omega_1^2\omega_2 + \omega_1\omega_2^2)\]

For a positive real \(\omega_{21}\)  \(\Rightarrow\) \(0.79142 < \frac{\omega_1}{\omega_2} < 12.63556\)

This is obtained from numerical calculation.
Numerical Simulation Result of Our Algorithm

Gain characteristics $|G_2(j\omega)|$

Nyquist chart of $G_2(j\omega) = X(\omega) + jY(\omega)$

Passband gain becomes flat.
Image Rejection Ratio (IRR)

\[ |G_2(-j\sqrt{\omega_1\omega_2})| = \frac{(\sqrt{\omega_2} - \sqrt{\omega_1})^2}{K} \]

\[ K = \omega_1 \omega_2 \left( \frac{1}{\omega_1} + \frac{1}{\omega_2} + \frac{2}{\omega_{12}} \right) \]

Image Rejection Ratio = \[ \frac{\text{Passband Gain}}{(\text{Stopband Gain})_{\text{MAX}}} = \left( \frac{\sqrt{\omega_2} + \sqrt{\omega_1}}{\sqrt{\omega_2 - \sqrt{\omega_1}}} \right)^2 \]
Nyquist Chart & Image Rejection Ratio

Nyquist chart visualizes image rejection ratio.

Nyquist chart visualizes image rejection ratio.
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- We have clarified operation of RC polyphase filters using their complex transfer functions.
- We have derived a flat passband gain algorithm for 2nd-order filter
- We have demonstrated usefulness of Nyquist chart for algorithm and image rejection ratio derivation.
Thank you for listening

Nyquist chart is like a compass.