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Flat Passband Gain Design Algorithm for 2nd-order RC Polyphase Filter

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Contents

- Research Goal
- Roles of RC Polyphase Filter
- Transfer Function of RC Polyphase Filters
- Flat Passband Gain Algorithm for 2^{nd}-order Filter
- Summary

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Research Goal

- To establish
  systematic design and analysis methods
  of RC polyphase filters.

- To derive
  flat passband gain algorithm for 2^{nd}-order filter

- To demonstrate usefulness of Nyquist chart

Contents

- Research Goal
- <u>Roles of RC Polyphase Filter</u>
- Transfer Function of RC Polyphase Filters
- Flat Passband Gain Algorithm for 2^{nd}-order Filter
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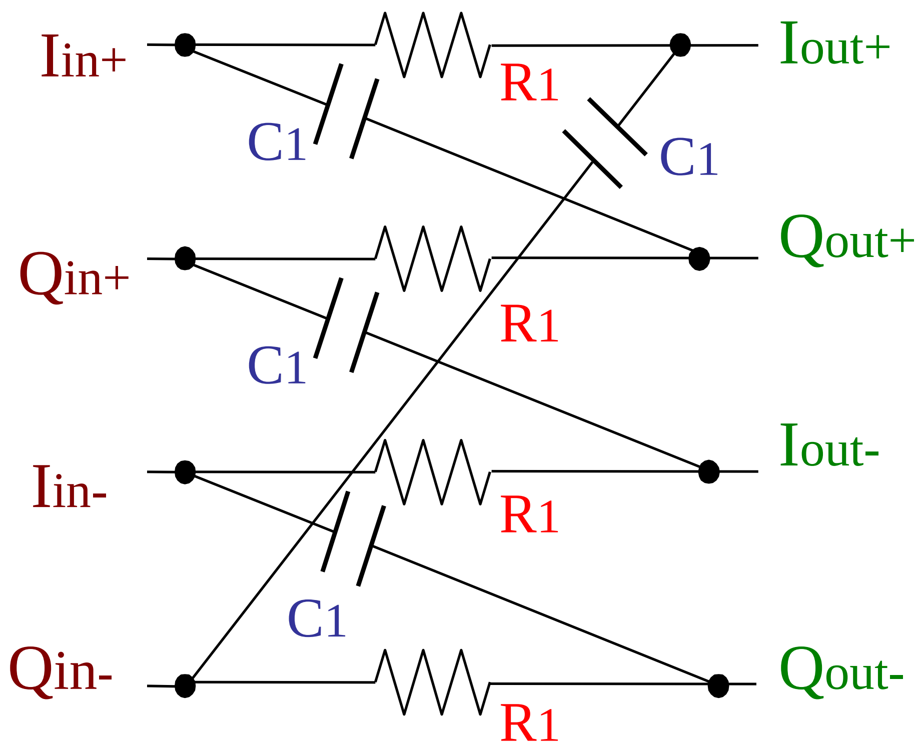
Features of RC Polyphase Filter

- Its input and output are complex signals.
- Passive RC analog filter
- One of key components in wireless transceiver analog front-end
  - I, Q signal generation
  - Image rejection
- Its design and analysis methods have not been fully developed yet.

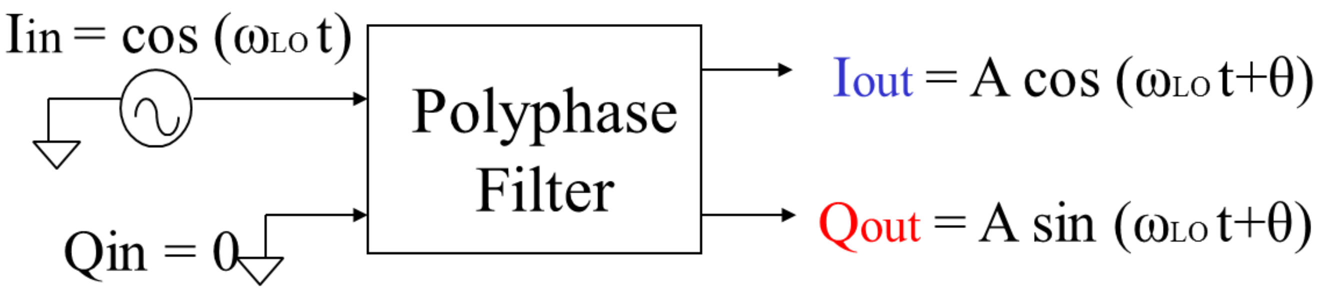
First-order RC Polyphase Filter



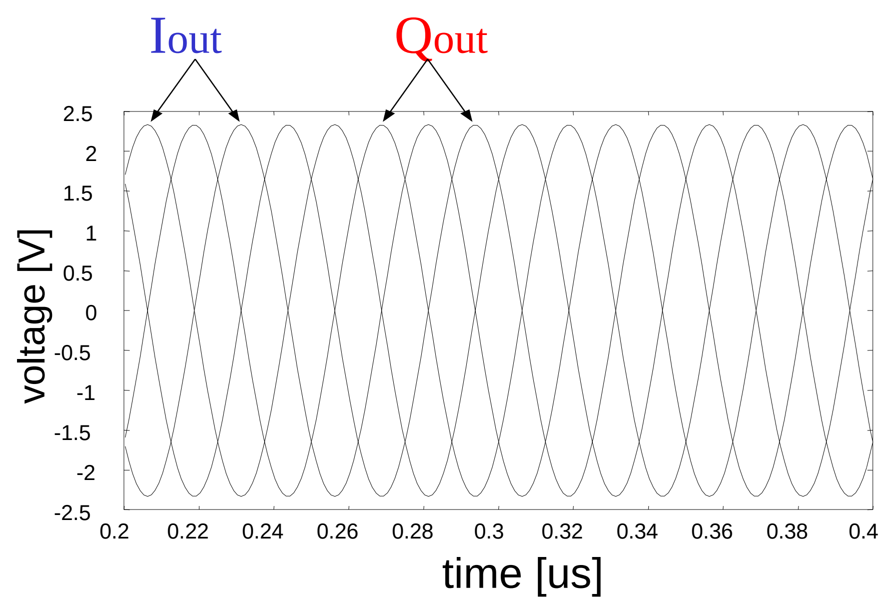
Differential Complex Input: $V_{in} = I_{in} + j\,Q_{in}$

Differential Complex Output: $V_{out} = I_{out} + j\,Q_{out}$

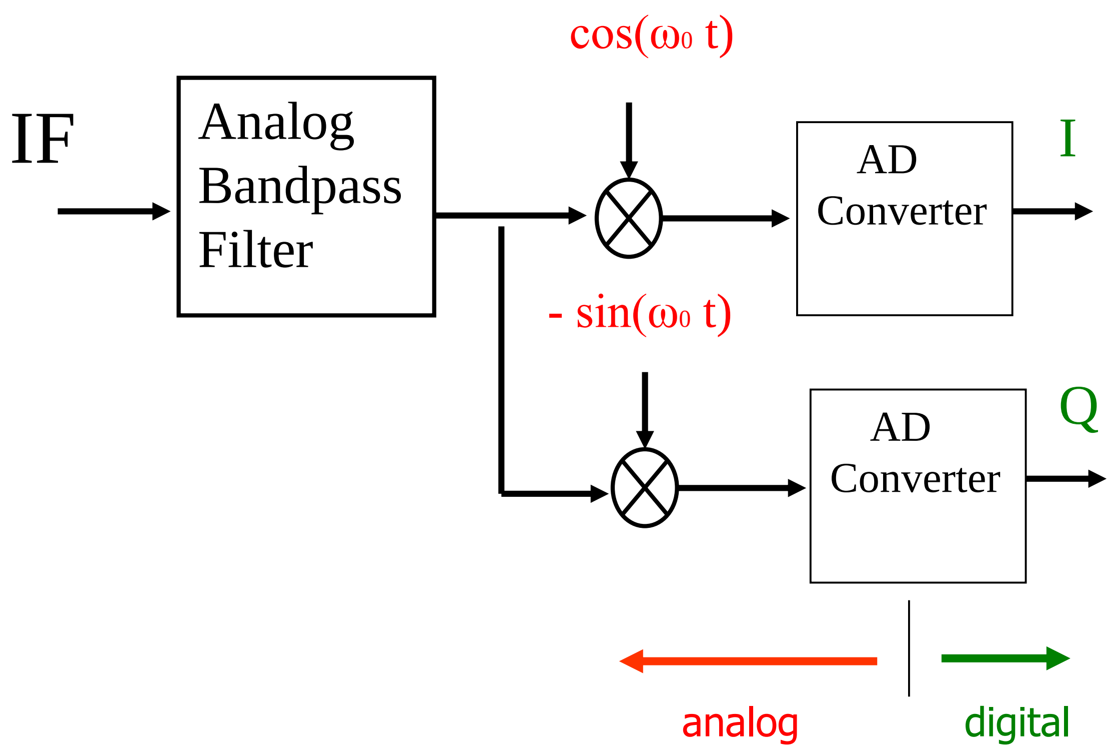
I, Q Signal Generation From Single Sinusoidal Input



$$\omega_{LO} = \frac{1}{R_1 C_1}$$



Cosine, Sine Signals in Receiver

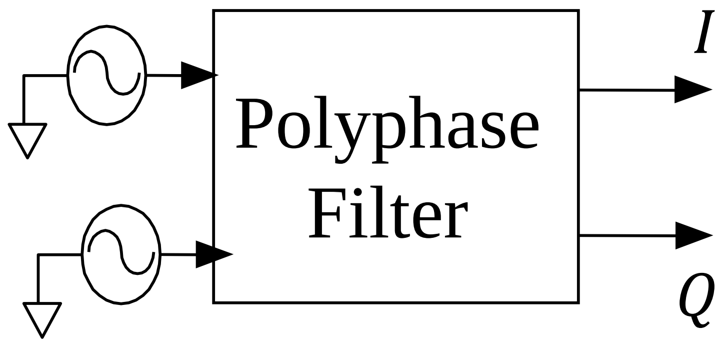


They are used for down conversion

Pure I, Q signal generation

3^{rd}-order harmonics rejection

$$I_{in} = \cos(\omega_{LO}t) + B\cos^3(\omega_{LO}t)$$



$$I_{out} = A\cos(\omega_{LO}t + \theta)$$

$$Q_{out} = A\sin(\omega_{LO}t + \theta)$$

$$Q_{in} = \sin(\omega_{LO}t) + B\sin^3(\omega_{LO}t)$$

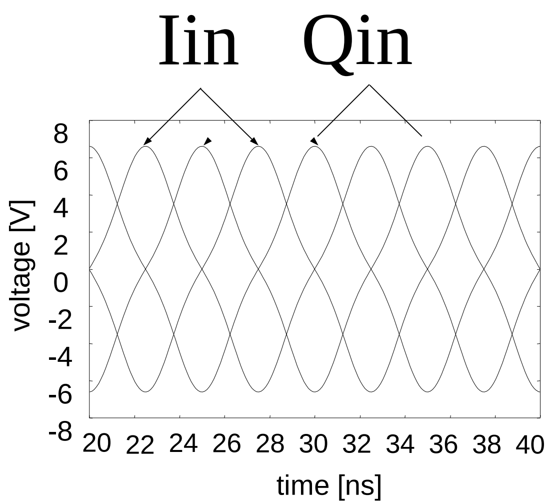
With
3rd-order harmonics

Without
3rd-order harmonics

Simulation of 3rd-order harmonics rejection

$$I_{in}(t) = \cos(\omega_{LO}t) + B\cos^3(\omega_{LO}t)$$

$$Q_{in}(t) = \sin(\omega_{LO}t) + B\sin^3(\omega_{LO}t)$$



$$\boxed{3\omega_{LO} = \frac{1}{R_1C_1}}$$

$$I_{OUT}(t) = A\cos(\omega_{LO}t + \theta)$$

$$Q_{OUT}(t) = A\sin(\omega_{LO}t + \theta)$$

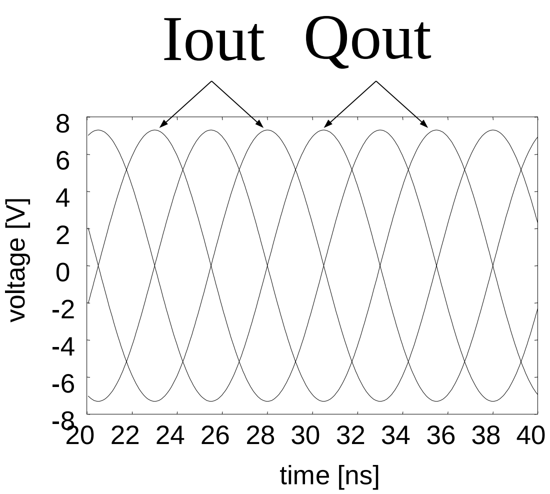
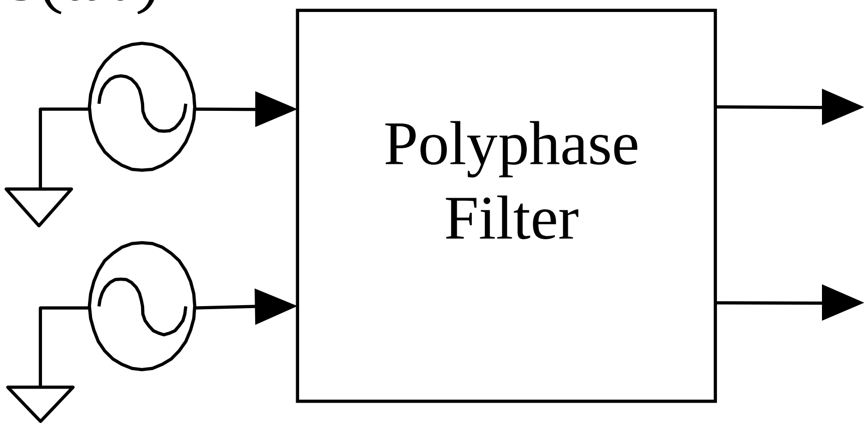


Image Rejection Filter

$$I_{in} = (A + B)\cos(\omega t)$$



$$I_{out} = A\cos(\omega t)$$

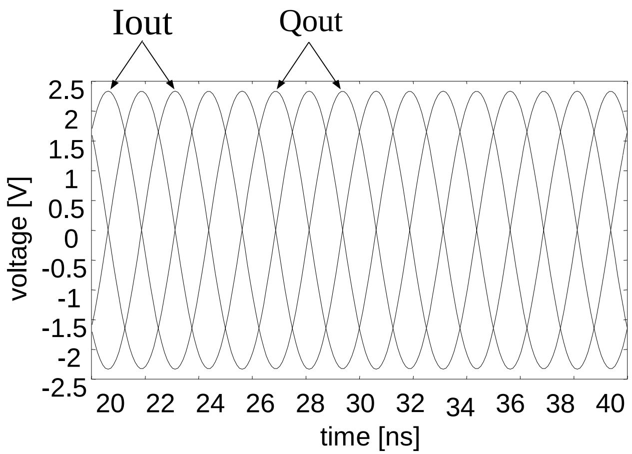
$$Q_{out} = A\sin(\omega t)$$

$$Q_{in} = (A - B)\sin(\omega t)$$

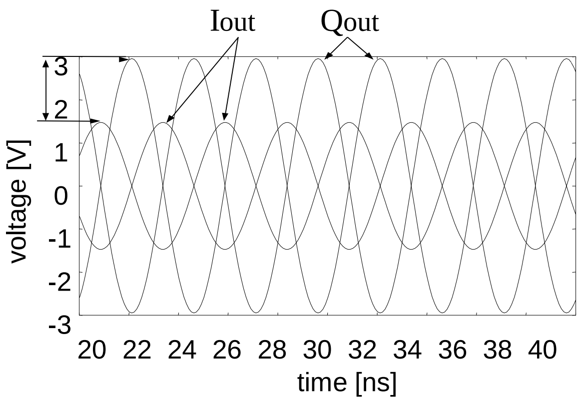
$$\boxed{Ae^{j\omega t} + Be^{-j\omega t}} \Rightarrow \boxed{Ae^{j\omega t}}$$

Problem when $\omega_{LO} \neq \frac{1}{R_1 C_1}$

$$\boxed{\omega_{LO} = \frac{1}{R_1 C_1}}$$



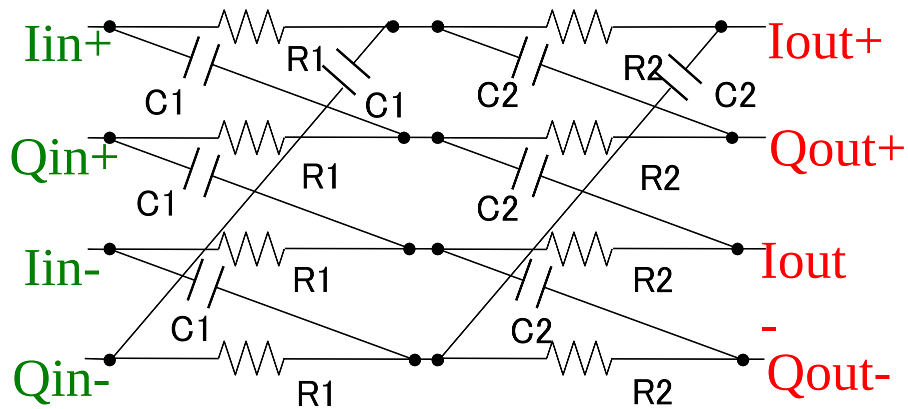
$$\boxed{\omega_{LO} = \frac{2}{R_1 C_1}}$$



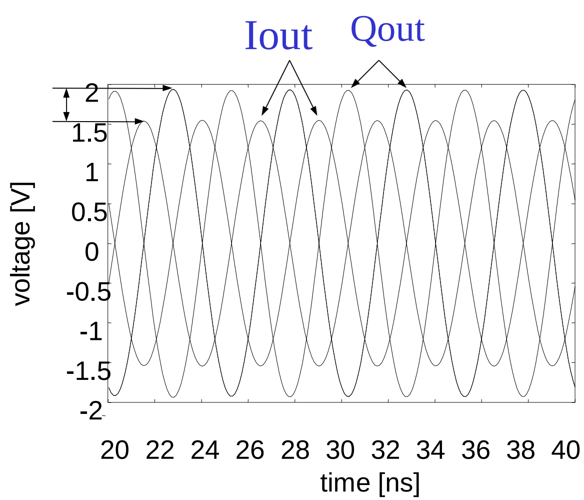
Amplitudes of I,Q signals differ significantly.

2nd-order RC Polyphase Filter

Problem of
large amplitude difference
between Iout, Qout
can be alleviated

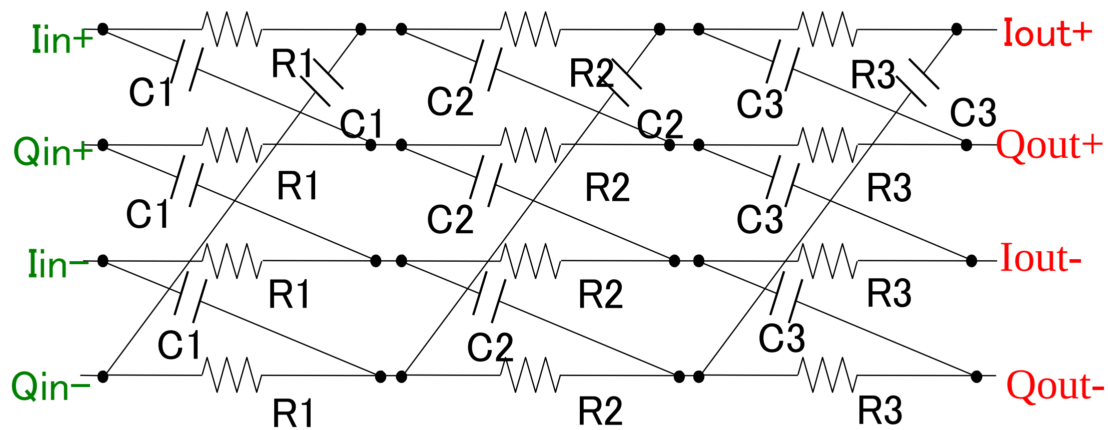


$$\omega_{LO} = \frac{2}{R_1 C_1}$$

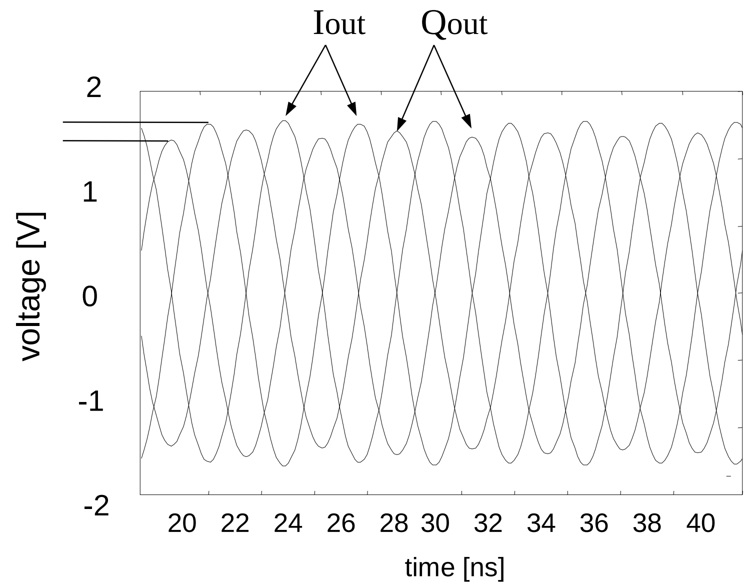


3rd-order RC Polyphase Filter

Amplitude difference problem is further alleviated.



$$\omega_{LO} = \frac{2}{R_1 C_1}$$



Contents

Transfer Function of RC Polyphase Filter

- Complex Signal Theory

- Complex input
- Complex output

$$V_{in}(j\omega) = I_{in} + jQ_{in}$$
$$V_{out}(j\omega) = I_{out} + jQ_{out}$$

- Complex Transfer Function

$$G(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$$

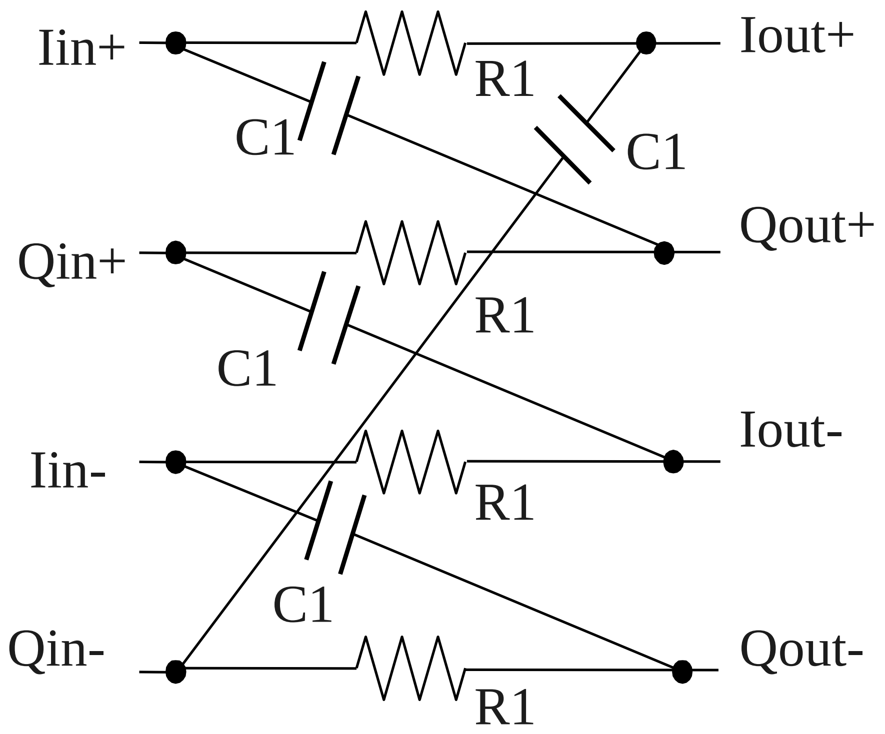
Transfer Function of 1st-order RC Polyphase Filter

Differential signal

$$I_{in}(t) = I_{in+}(t) - I_{in-}(t)$$
$$Q_{in}(t) = Q_{in+}(t) - Q_{in-}(t)$$
$$I_{out}(t) = I_{out+}(t) - I_{out-}(t)$$
$$Q_{out}(t) = Q_{out+}(t) - Q_{out-}(t)$$

Complex signal

$$V_{in}(t) = I_{in}(t) + jQ_{in}(t)$$
$$V_{out}(t) = I_{out}(t) + jQ_{out}(t)$$



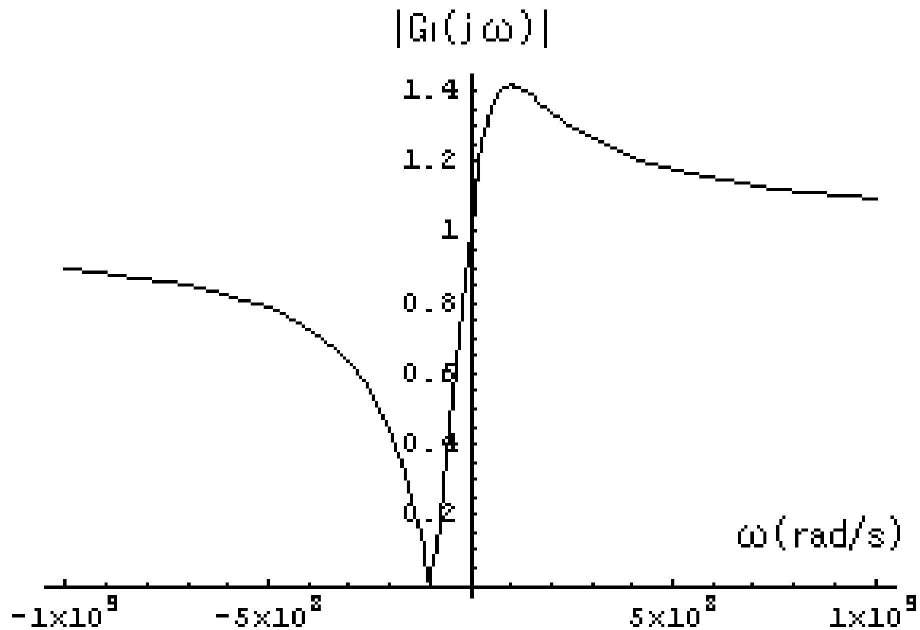
Transfer Function of 1st-order RC Polyphase Filter

Transfer Function

$$G_1(j\omega) = \frac{1+\omega RC}{1+j\omega RC}$$

Gain

$$|G_1(j\omega)| = \frac{|1+\omega RC|}{\sqrt{1+(\omega RC)^2}}$$



$$-\frac{1}{RC}$$

Explanation of I, Q Signal Generation by G1(jω)

$$Q_{in}(t) \equiv 0, \qquad I_{in}(t) = \cos(\omega t)$$

$$V_{in}(t) = I_{in}(t) + j\ Q_{in}(t) = \cos(\omega t) = \frac{1}{2}[e^{j\omega t} + e^{-j\omega t}]$$

$$V_{out}(t) = \frac{1}{2}[|G_1(j\omega)|e^{j(\omega t + \angle G_1(j\omega))} \boxed{+ |G_1(-j\omega)|e^{j(-\omega t + \angle G_1(-j\omega))}]}$$

$$= \frac{\sqrt{2}}{2}\cos\left(\omega t - \frac{\pi}{4}\right) + \frac{j\sqrt{2}}{2}\sin(\omega t - \frac{\pi}{4})$$

$$\boldsymbol{|G_1(-j\omega)|e^{j(-\omega t + \angle G_1(-j\omega))}] = 0}$$

Here

$$|G_1(j\omega)|_{\omega = \frac{1}{RC}} = 0\,, \quad |G_1(j\omega)|_{\omega = \frac{1}{RC}} = \sqrt{2}\,, \quad \angle G_1(j\omega) = -\frac{\pi}{4}$$

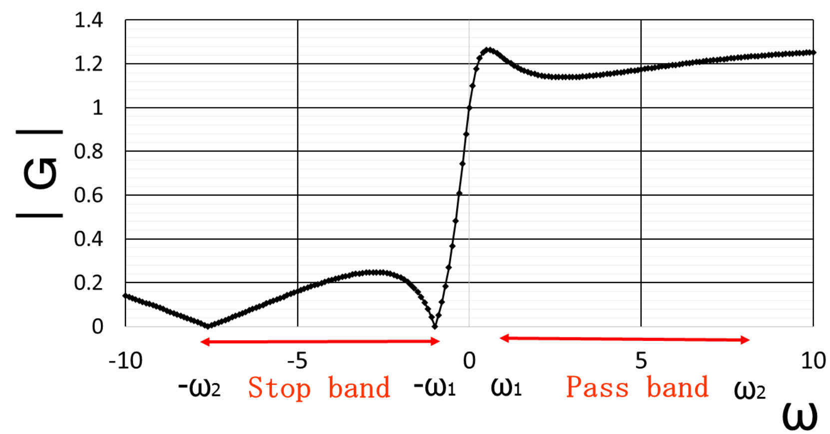
Transfer Function of 2nd-order RC Polyphase Filter

Transfer Function

$$G_2(j\omega) = \frac{(1+\omega R_1 C_1)(1+\omega R_2 C_2)}{1-\omega^2 R_1 C_1 R_2 C_2 + j\omega(C_1 R_1 + C_2 R_2 + 2R_1 C_2)}$$

Derivation is very complicated, so we used "Mathematica."

Gain |G2(jω)| characteristics



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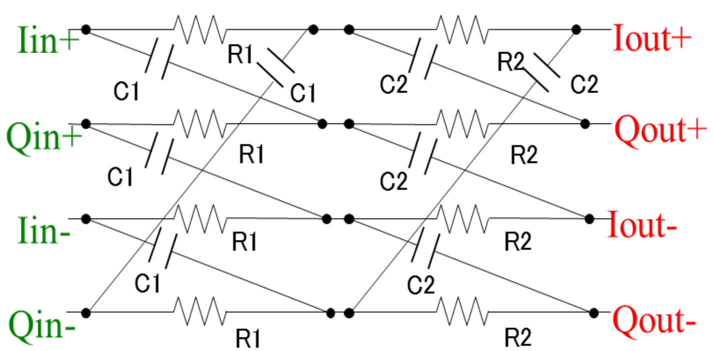
Why 2nd-order RC Polyphase Filter ?

- 1st-order RC polyphase filter
  - ⇨ Performance is limited.
- 2nd-order
  - ⇨ Handy, good performance
  - ⇨ Widely used.
- 3rd-order
  - ⇨ Circuit is complicated.



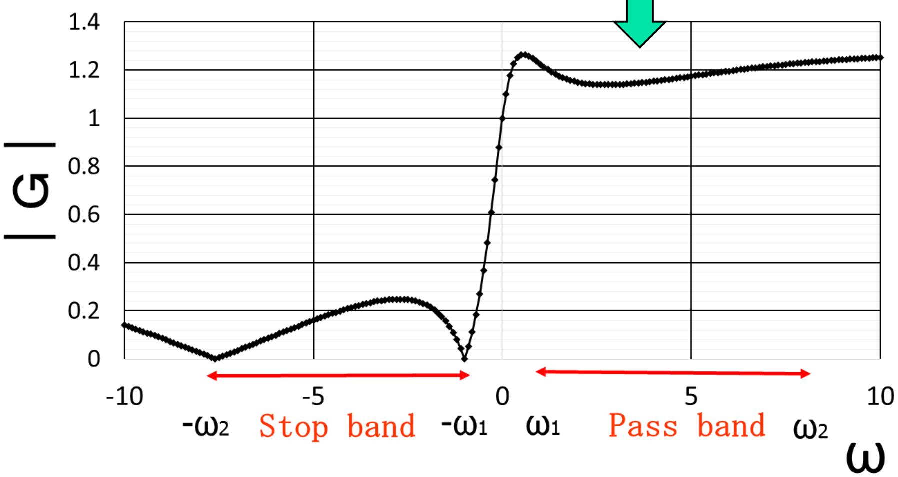
Need for Flat Passband Gain Algorithm of 2nd-order RC Polyphase Filter

Transfer Function

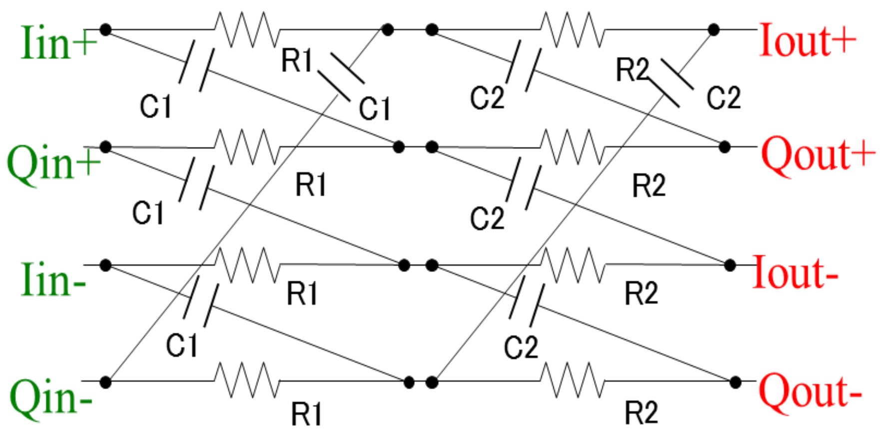
$$G_2(j\omega) = \frac{(1+\omega R_1 C_1)(1+\omega R_2 C_2)}{1-\omega^2 R_1 C_1 R_2 C_2 + j\omega(C_1 R_1 + C_2 R_2 + 2R_1 C_2)}$$

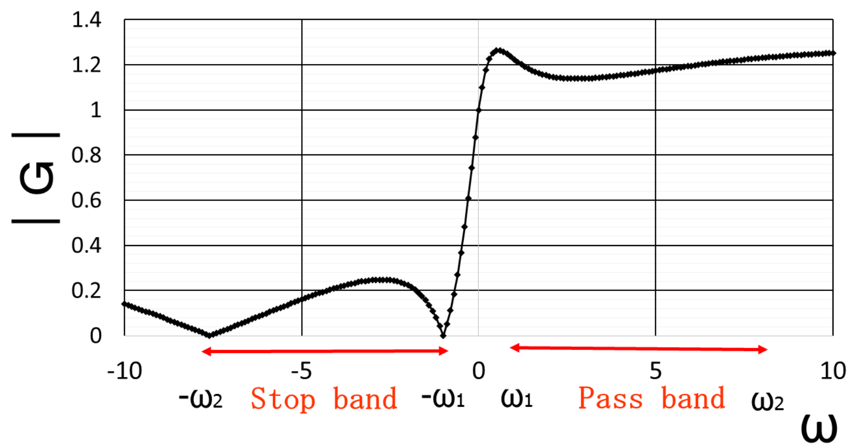
We need flat passband gain

Gain $|G2(j\omega)|$ characteristics



Four Design Parameters



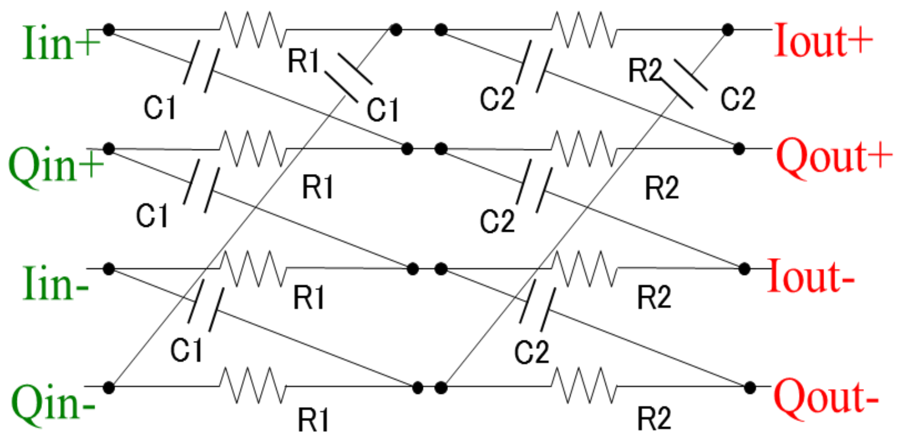


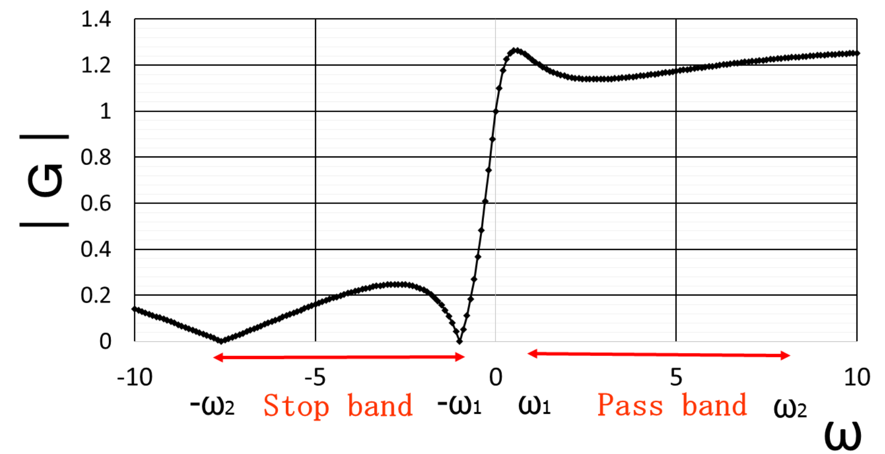
4 parameters : R_1, R_2, C_1, C_2

$$\omega_1 = \frac{1}{R_1 C_1}, \omega_2 = \frac{1}{R_2 C_2}, X = \frac{1}{R_2 C_1}, Y = \frac{1}{R_1 C_2}$$

4 constraints

Two Constraints from Filter Spec.

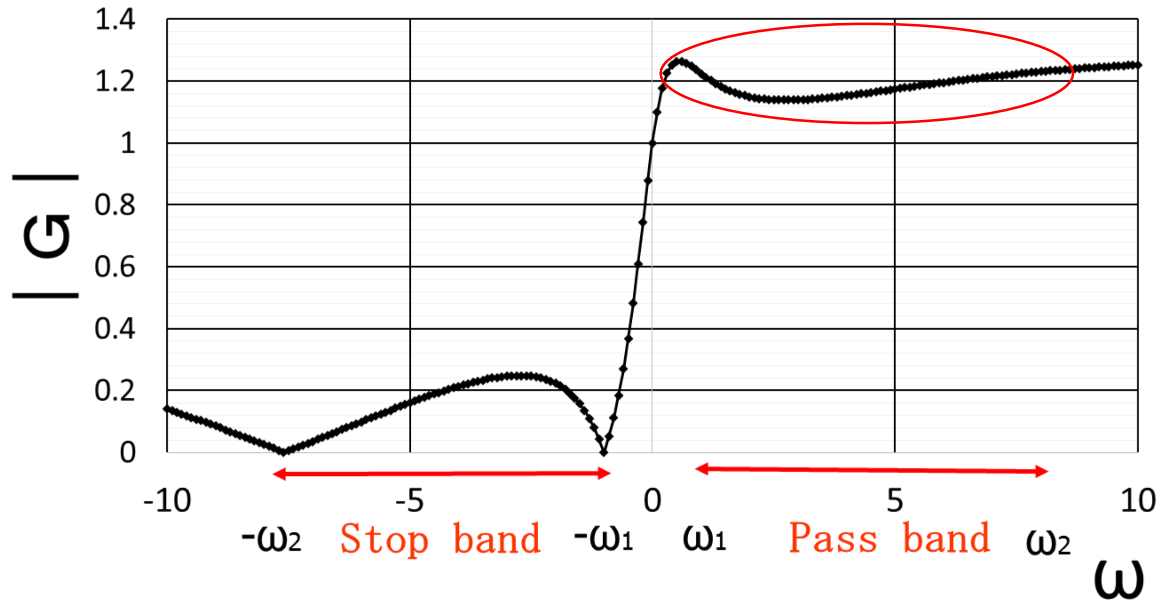




● 2 zeros : $-\omega_1 = \dfrac{-1}{R_1 C_1}$, $-\omega_2 = \dfrac{-1}{R_2 C_2}$

are given from the filter specification.

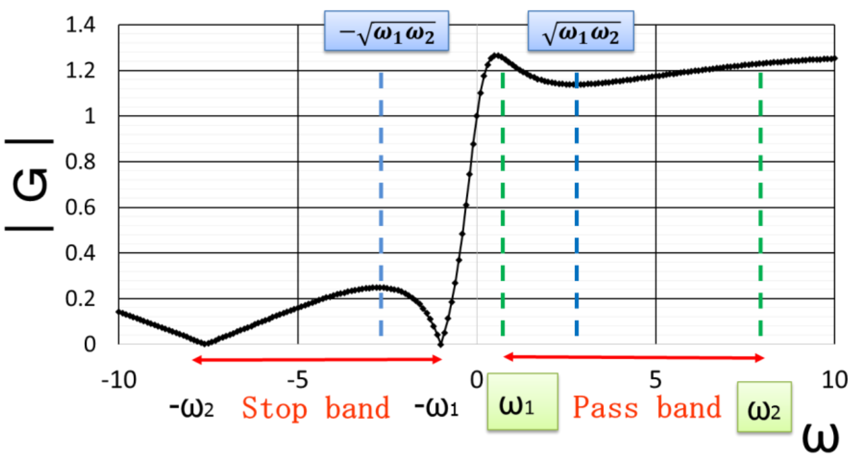
Proposed Algorithm Uses Third Constraint



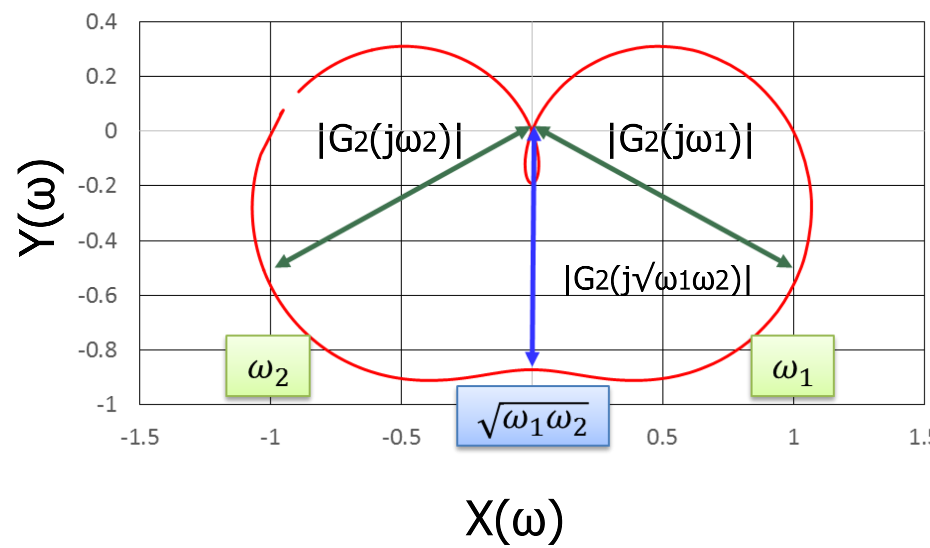
- We use the third constraint $X = \dfrac{1}{R_2 C_1}$ for passpand gain flattening.
- The fourth constraint is left for ease of IC realization.

Nyquist Chart of $G_2(j\omega)$

Gain characteristics $|G_2(j\omega)|$



Nyquist chart of $G_2(j\omega)=X(\omega)+j\,Y(\omega)$

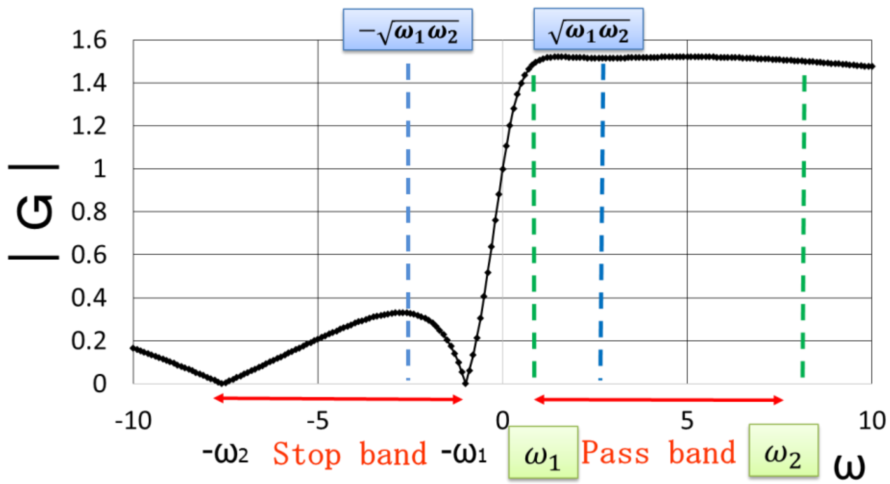


$$|G_2(j\omega_1)|=|G_2(j\omega_2)|$$

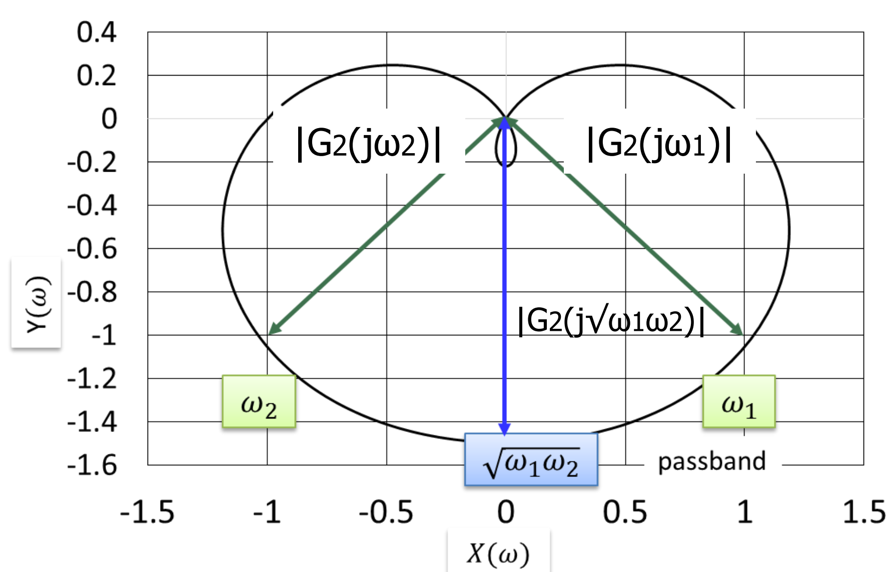
But in general $|G_2(j\omega_1)|=|G_2(j\omega_2)|\neq|G_2(j\sqrt{\omega_1\omega_2})|$

Our Idea for Flat Passband Gain Algorithm

Gain characteristics |G2(jω)|



Nyquist chart of G2(jω)=X(ω)+j Y(ω)



If we make $|G_2(j\omega_1)| = |G_2(j\omega_2)| = |G_2(j\sqrt{\omega_1\omega_2})|$,
gain would be flat from ω_1 to ω_2.

Solving Third Constraint

Our algorithm

$$|G_2(j\omega_1)| = |G_2(j\omega_2)| = |G_2(j\sqrt{\omega_1\omega_2})|,$$

$$\boxed{\alpha\omega_{21}^2 + \beta\omega_{21} + \gamma = 0} \qquad \omega_{21} = \frac{1}{R_2C_1}$$

We need a positive real solution of ω_{21}.

Condition for Solution

Third constraint

$$\boxed{\omega_{21} = \frac{1}{R_2 C_1} = \frac{-\beta + \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}}$$

$$\alpha = 6\omega_1^2 + 6\omega_2^2 + 4\omega_1\omega_2 - 8\sqrt{\omega_1\omega_2}(\omega_1 + \omega_2)$$

$$\beta = 6\omega_1^3 + 6\omega_2^3 + 10\omega_1\omega_2(\omega_1 + \omega_2) - 8\sqrt{\omega_1\omega_2}(\omega_1 + \omega_2)^2$$

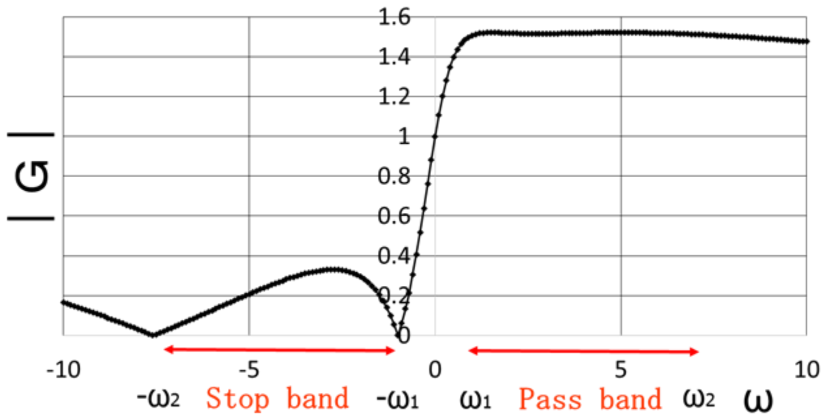
$$\gamma = \omega_1^4 + \omega_2^4 + 2\omega_1\omega_2(\omega_1^2 + \omega_2^2 + 5\omega_1\omega_2) - 4\sqrt{\omega_1\omega_2}(\omega_1^3 + \omega_1^2\omega_2 + \omega_1\omega_2^2)$$

For a positive real ω_{21} ➡ $0.79142 < \frac{\omega_1}{\omega_2} < 12.63556$

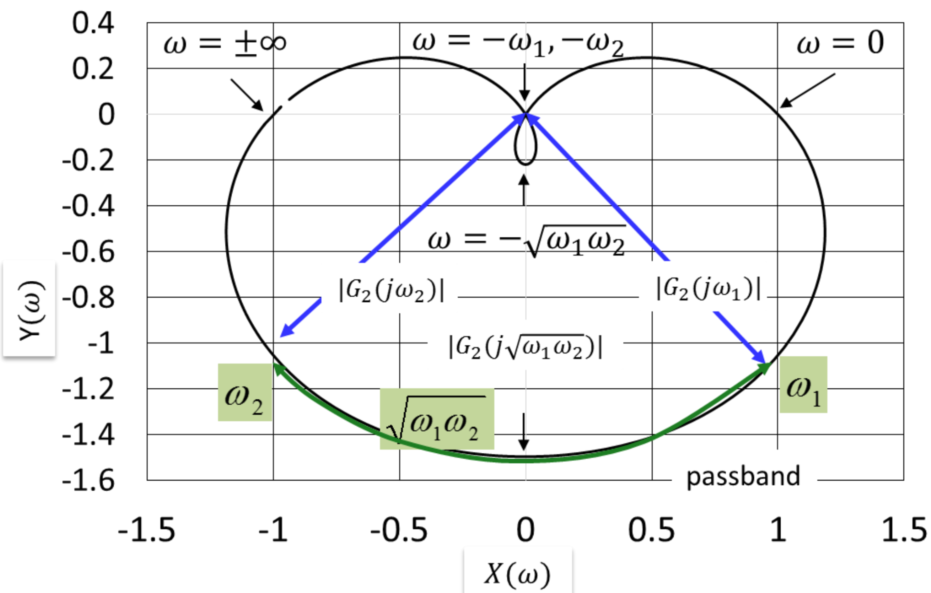
This is obtained from numerical calculation.

Numerical Simulation Result of Our Algorithm

Gain characteristics $|G_2(j\omega)|$



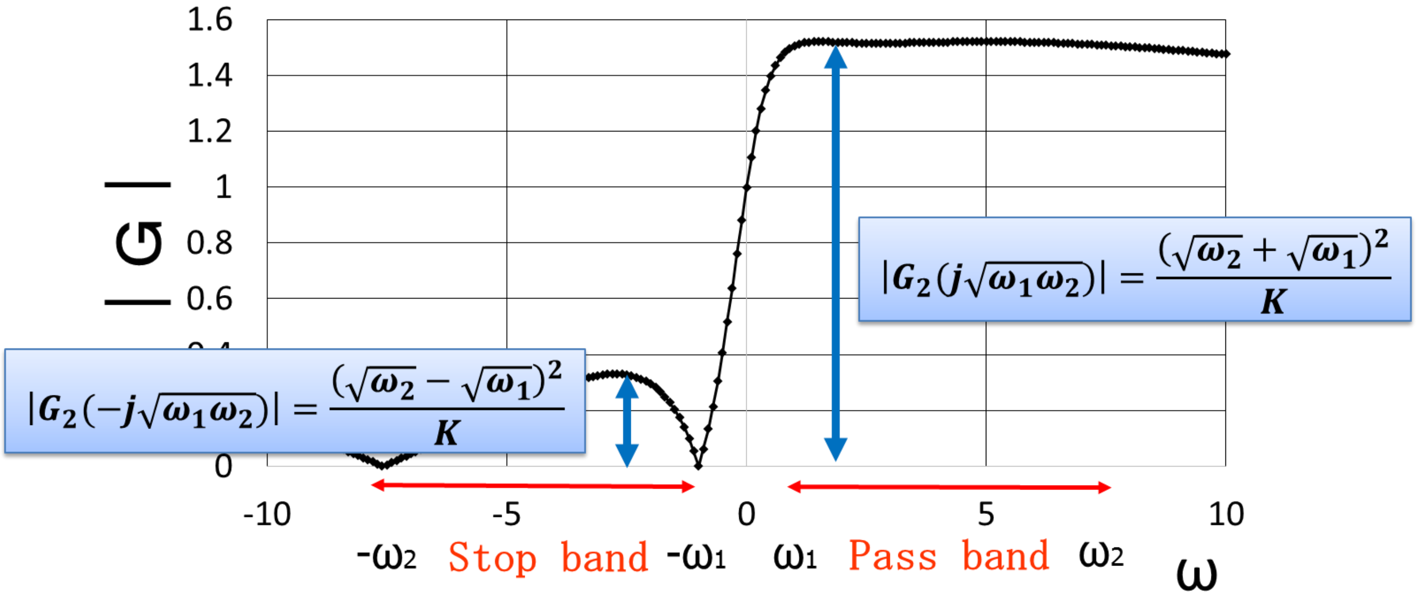
Nyquist chart of $G_2(j\omega)=X(\omega)+j\,Y(\omega)$



$R_1 = 1k$, $C_1 = 10p$ $\quad$ $R_2 = 2k$, $C_2 = 1p$ $\quad$ $\omega_1 = \frac{1}{R_1 C_1}$ $\quad$ $\omega_2 = \frac{1}{R_2 C_2}$

Passband gain becomes flat.

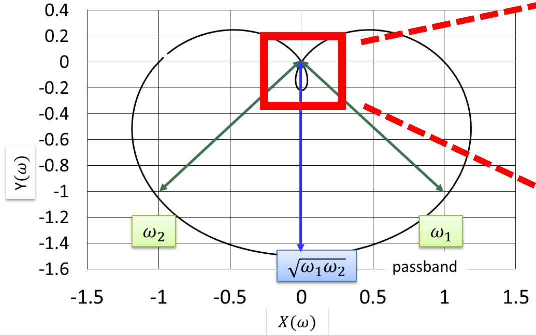
Image Rejection Ratio (IRR)



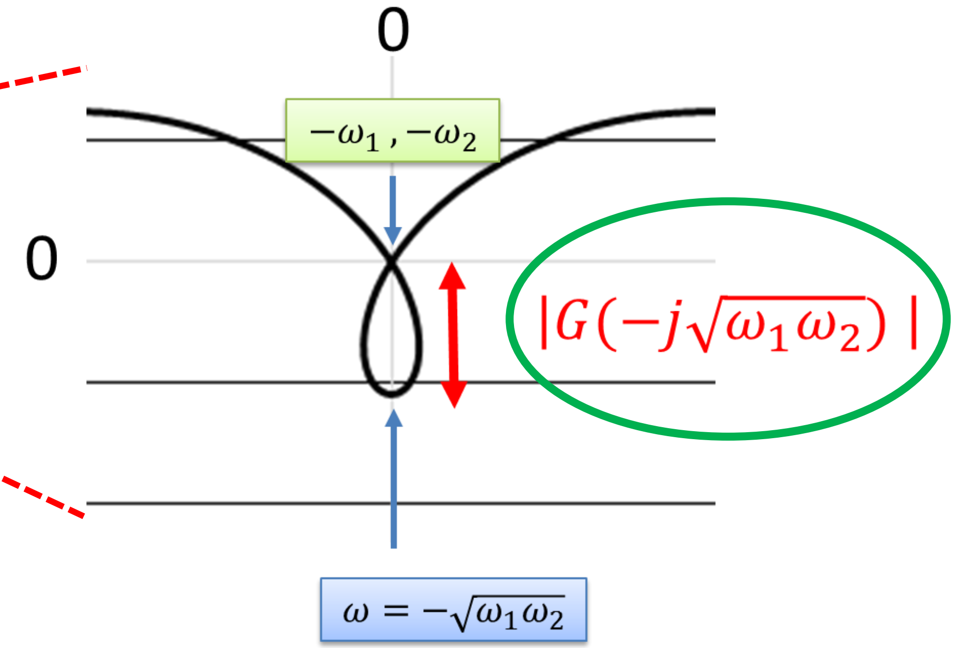
$$K = \omega_1\omega_2\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}+\frac{2}{\omega_{12}}\right)$$

●Image Rejection Ratio = $\dfrac{\textbf{Passband Gain}}{(\textbf{Stopband Gain})_{\textbf{MAX}}} = \left(\dfrac{\sqrt{\omega_2}+\sqrt{\omega_1}}{\sqrt{\omega_2}-\sqrt{\omega_1}}\right)^{2}$

Nyquist Chart & Image Rejection Ratio



Nyquist Chart



Nyquist chart visualizes image rejection ratio.

Contents

Summary

- We have clarified operation of RC polyphase filters using their complex transfer functions.

- We have derived a flat passband gain algorithm for 2^{nd}-order filter

- We have demonstrated usefulness of Nyquist chart for algorithm and image rejection ratio derivation.

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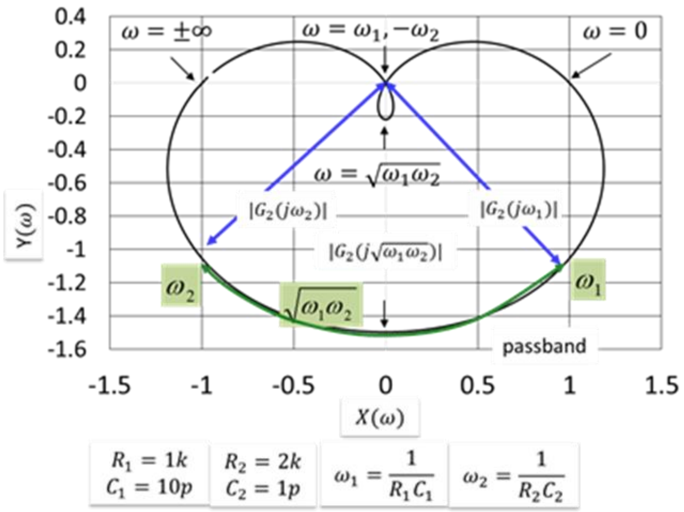
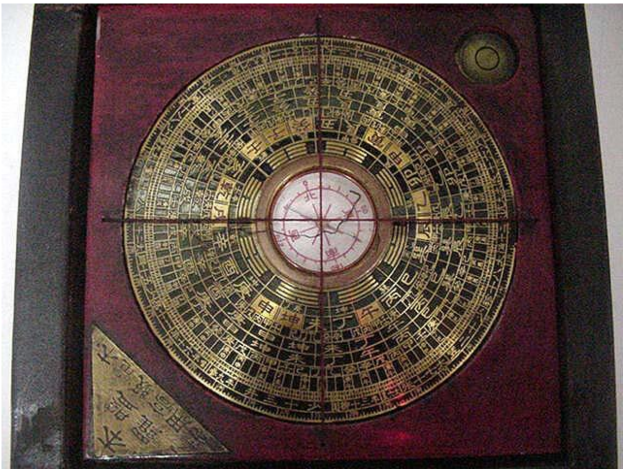
Thank you for listening





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Nyquist chart is like a compass.