C2-4 16:45-17:00 Nov. 4, 2015 (Wed)

Finite Aperture Time Effects in Sampling Circuit

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Research Objective

- To establish fundamental theory of sampling circuit for high-frequency and high-precision waveform acquisition

- Especially, to clarify finite aperture time effect.

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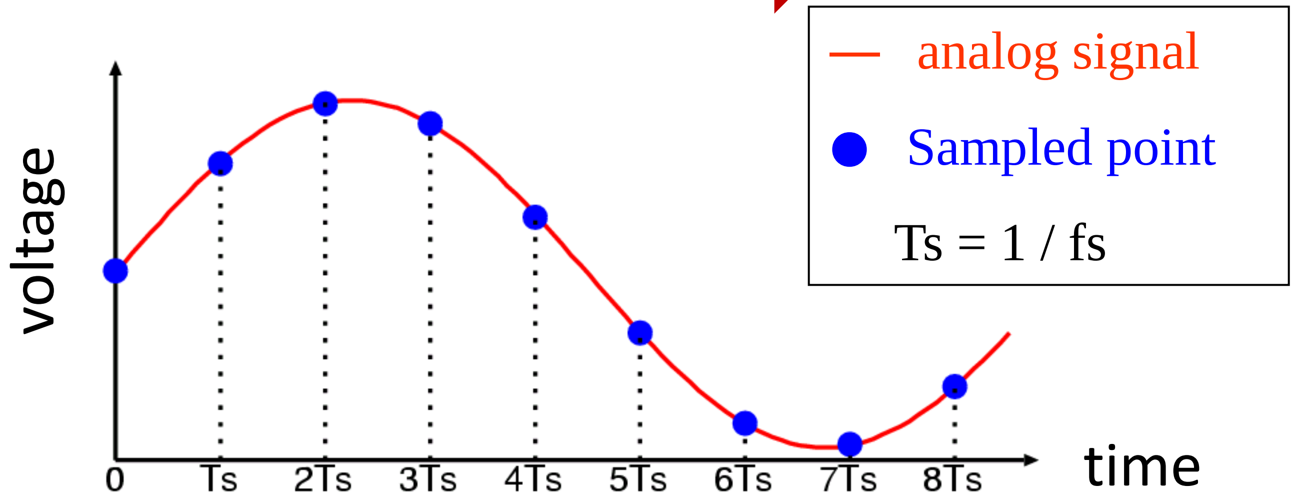
Waveform Sampling

Waveform acquisition ⇨ Sampling

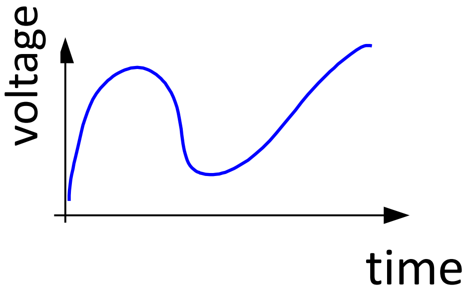


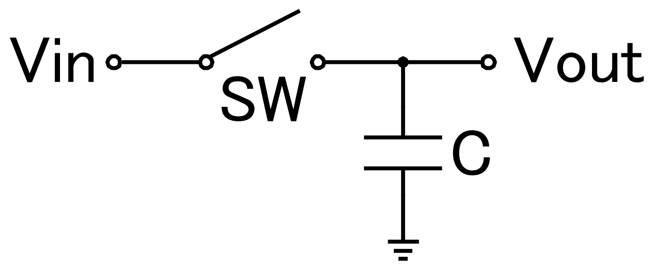
High-frequency signal sampling

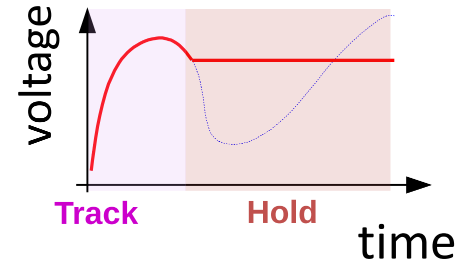
suffers from ⇩

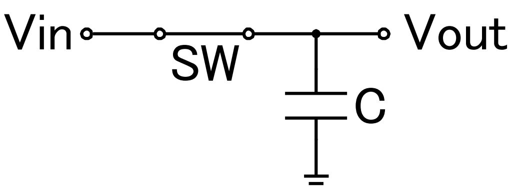
- Finite aperture time (non-zero turn-off time)
- Aperture jitter

Sampling Circuit



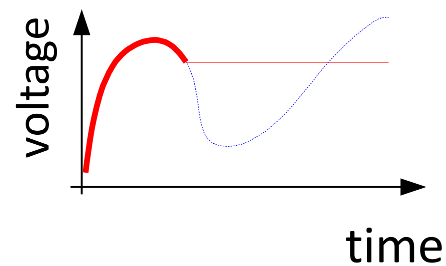


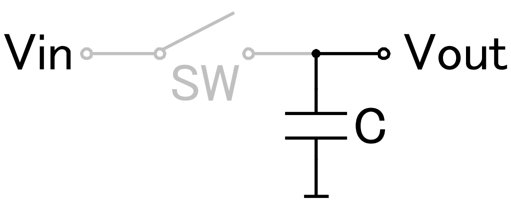




- SW: ON
  - Vout(t) = Vin(t)

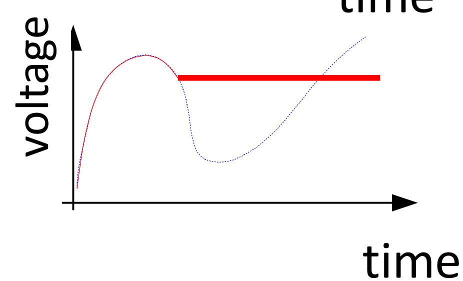
Track mode





- SW: OFF
  - Vout(t) = Vin(t_{OFF})

Hold mode



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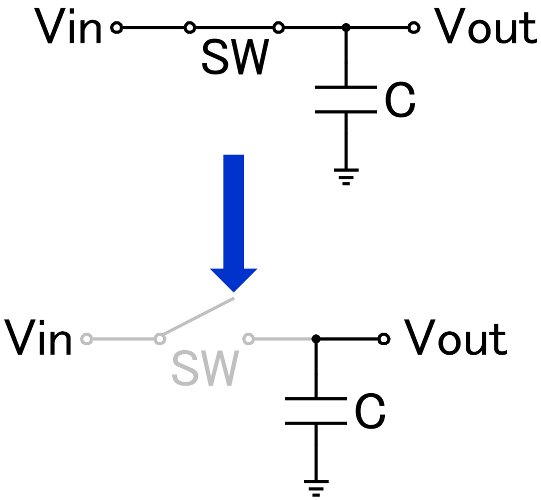
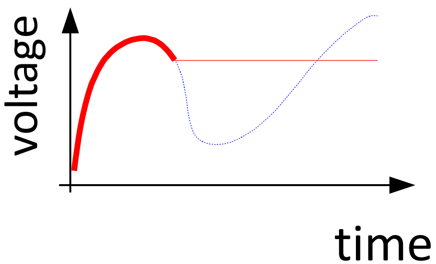
Finite Aperture Time



• SW: ON

  • $V_{out}(t) = V_{in}(t)$

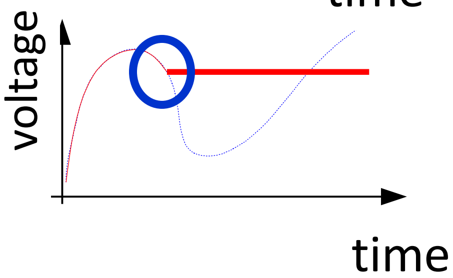
  Track mode



• SW: OFF

  • $V_{out}(t) = V_{in}(t_{OFF})$

  Hold mode



Finite transition time from track to hold modes

Analogy with Camera Shutter Speed

Camera: Finite Shutter Speed

⇩ Moving Object

Blurred

Sampling Circuit:
Finite Aperture Time

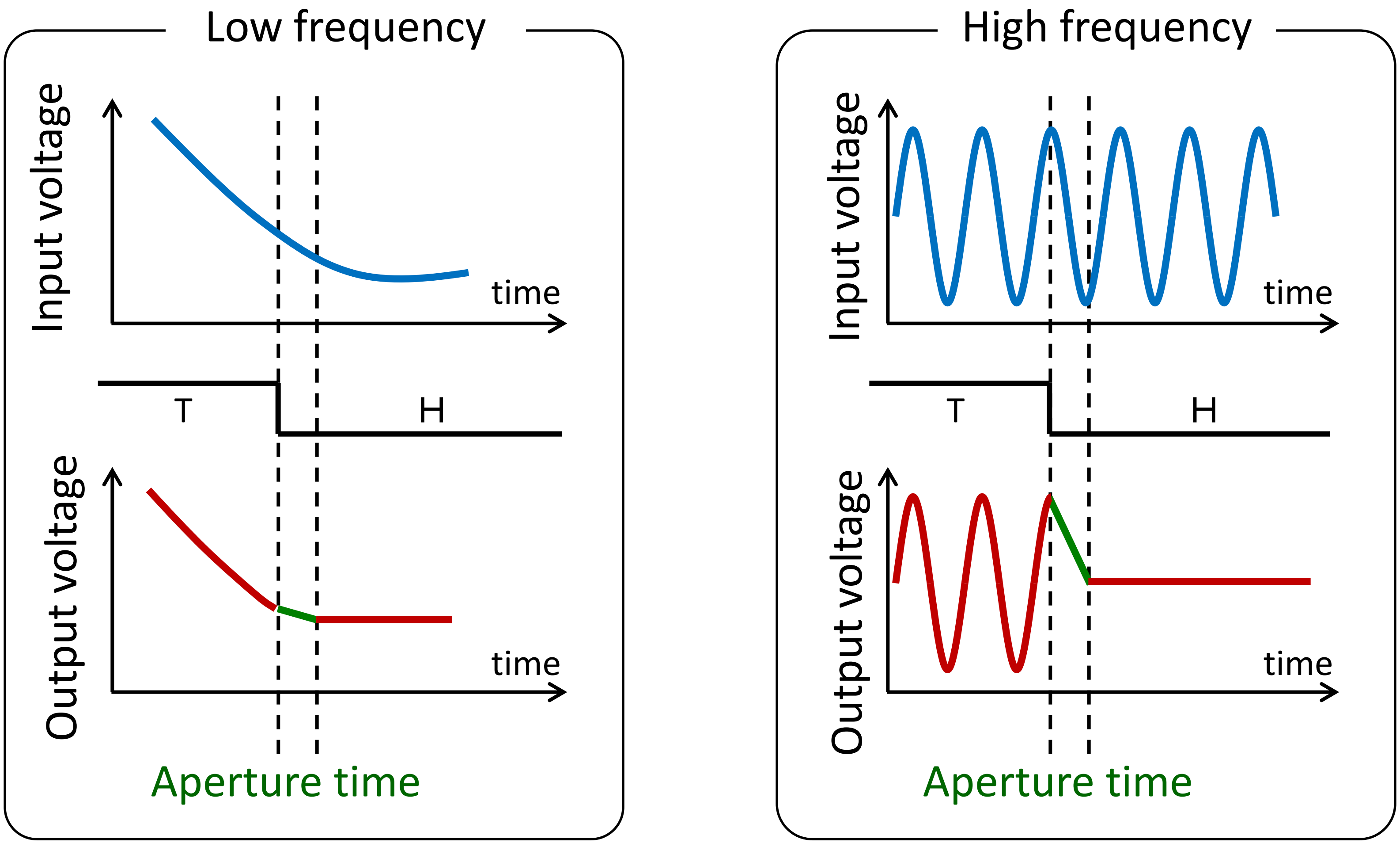
Input signal

⇩ High frequency

Acquired signal

Low pass filtered

Signal Frequency and Aperture Time



Higher frequency signal ⇒ More affected by finite aperture time

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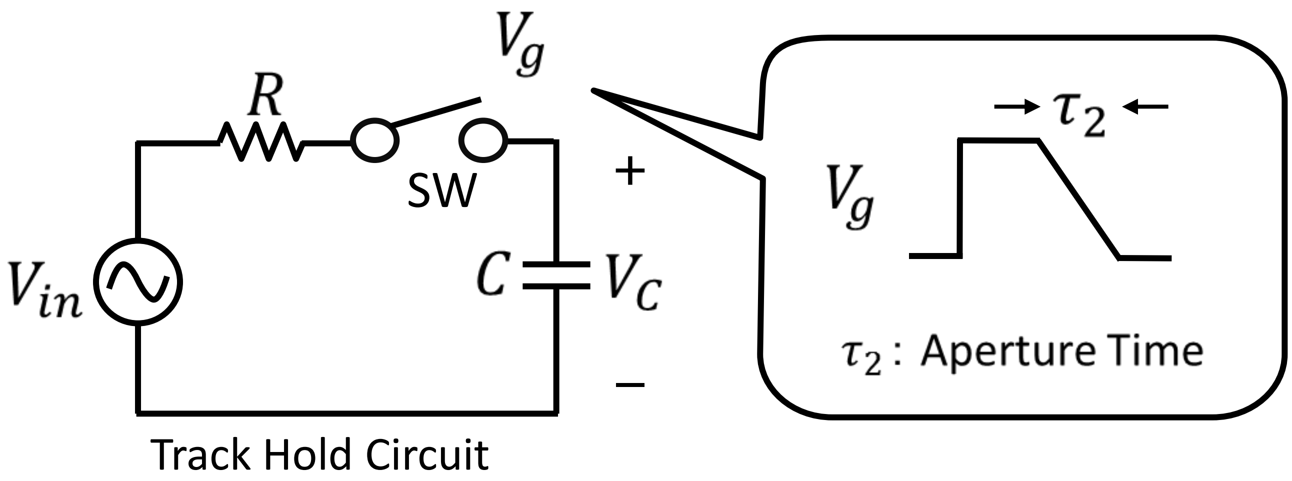
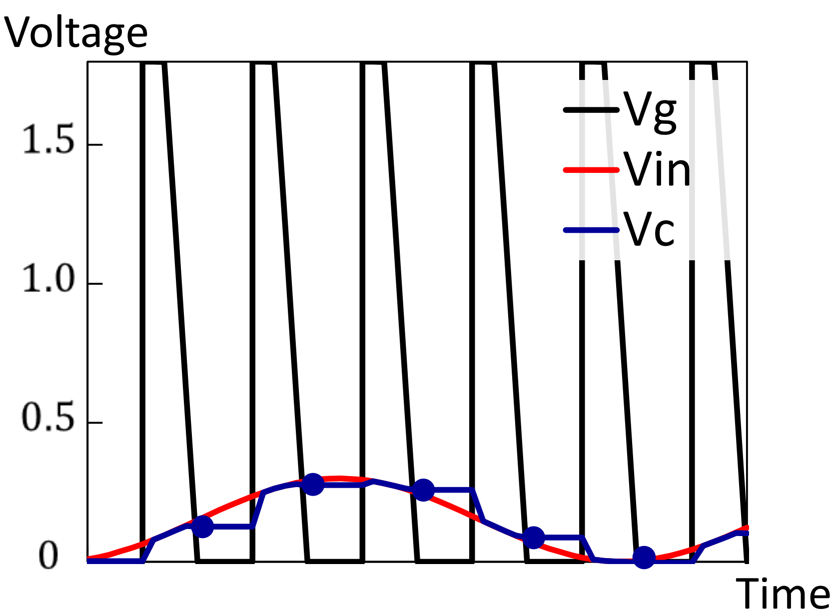
Transfer Function Derivation





Obtain values of ●

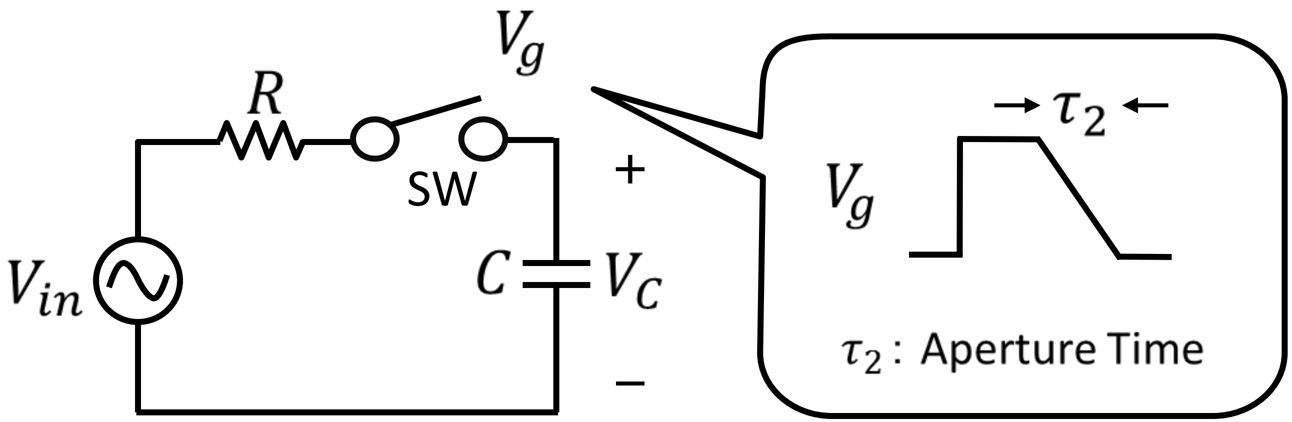
Equivalent time sampling

Obtain gain, phase for each frequency

Frequency transfer function

Contents

Derived Transfer Function



Track Hold Circuit

$$\frac{V_C}{V_{in}} = \frac{sinc(\omega\tau_2)}{sinc(\omega\tau_2) + j\omega\tau_1}$$

$\tau_1 = RC$

Transfer function in case of finite aperture time

The formula derivation was done by Prof. Asad Abidi.

[1] A. Abidi, M. Arai, K. Niitsu, H. Kobayashi, "Finite Aperture Time Effects in Sampling Circuits," 24th IEICE Workshop on Circuits and Systems, Awaji Island, Japan (Aug. 2011)

Consistency with Zero Aperture Time Case

$$\frac{V_C}{V_{in}} = \frac{sinc(\omega\tau_2)}{sinc(\omega\tau_2) + j\omega\tau_1}$$

$$(\tau_1 = RC\ ,\ \tau_2 = \tau\ )$$

Transfer function in case of finite aperture time

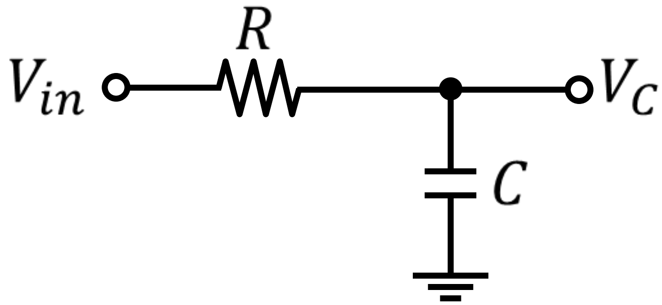
$\tau_2 \to 0$

$sinc(\omega\tau_2) \to 1$

$$\frac{V_C}{V_{in}} = \frac{1}{1 + j\omega\tau_1}$$

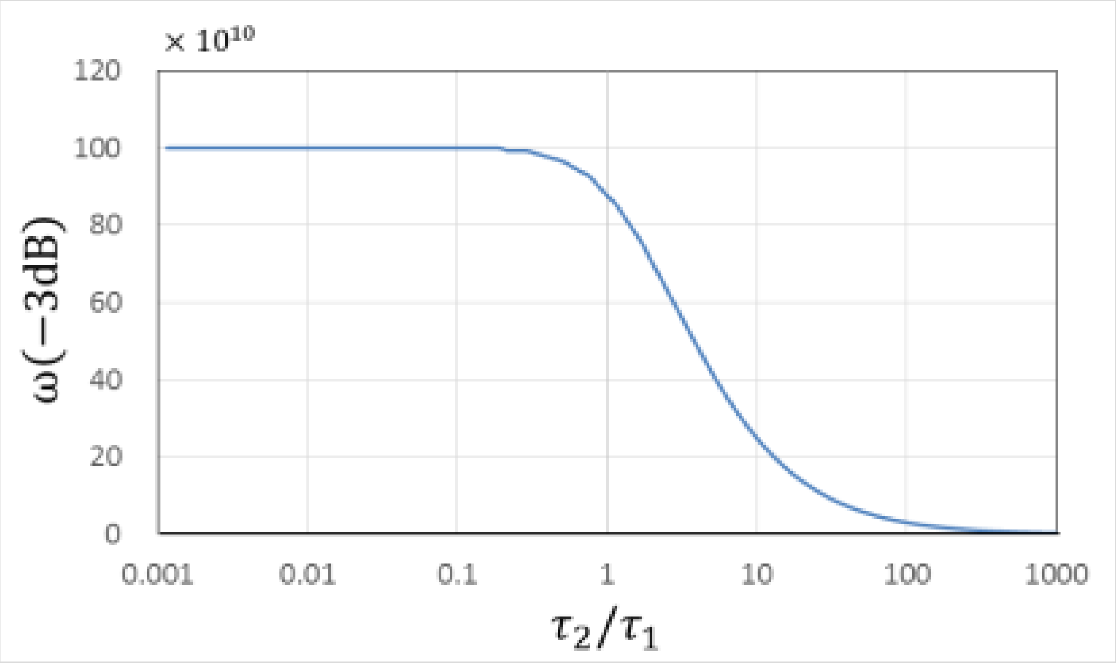
$$(\tau_1 = RC\ )$$

Transfer function in case of zero aperture time



τ_1, τ_2 Effects to Bandwidth

Numerical calculation from the derived transfer function



τ_1 (= R C) : fixed

τ_2 (aperture time) : varied

Bandwidth starts to decrease at $\tau_2 / \tau_1 = 1$

τ_1, τ_2 effects to bandwidth are comparable.

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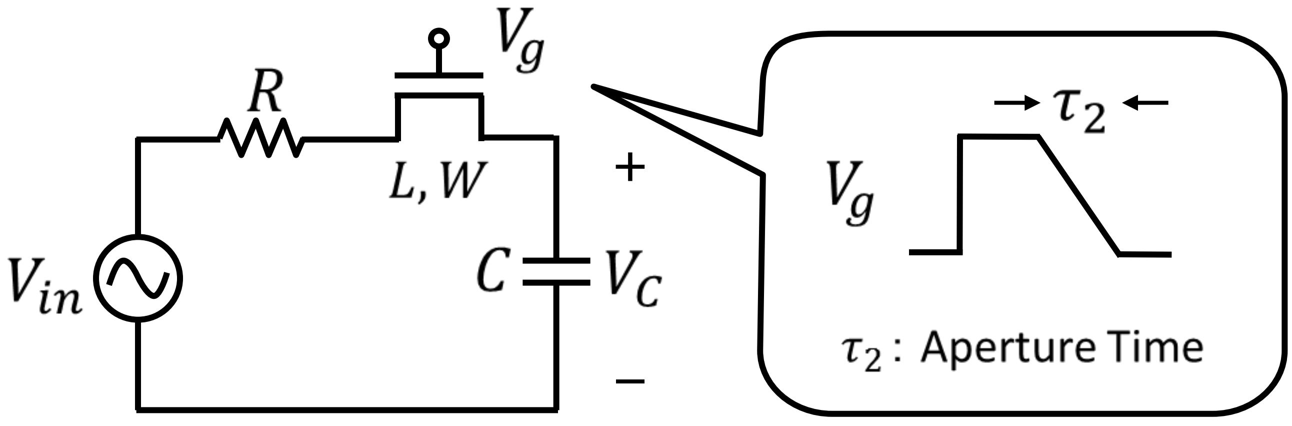
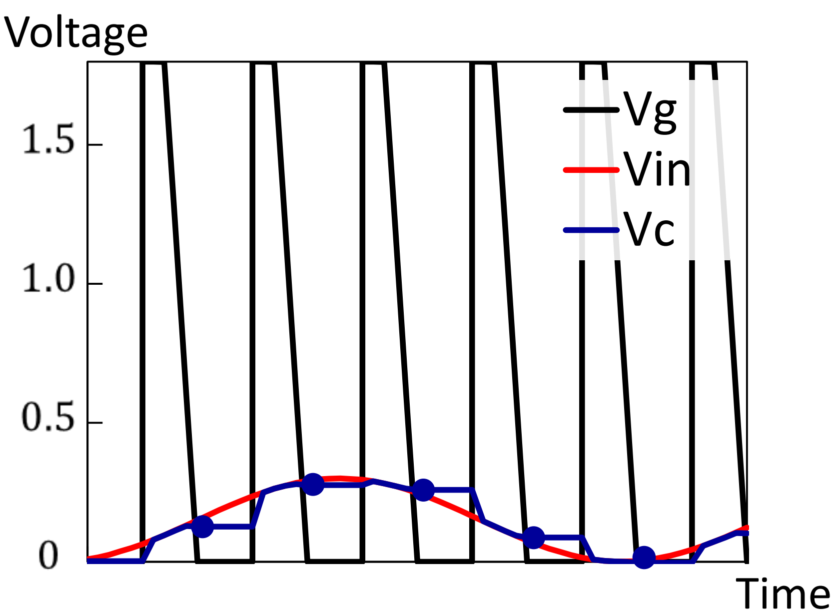
SPICE Simulation Verification



Track Hold Circuit



Obtain values of ●

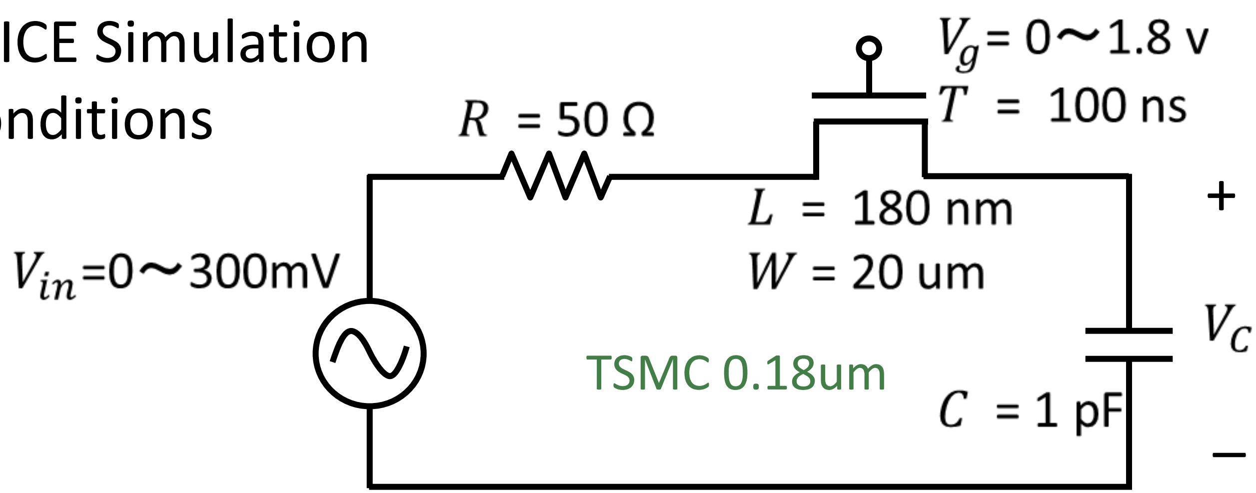
Equivalent time sampling

Obtain gain, phase for each frequency

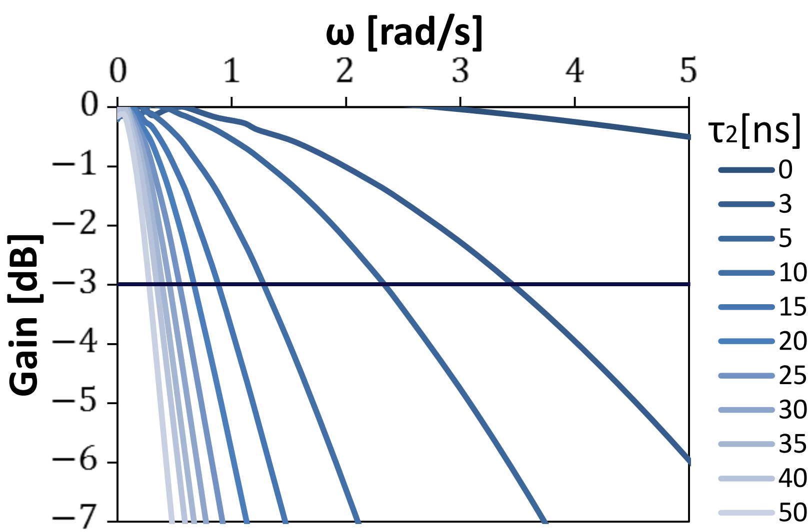
Frequency transfer function

Results

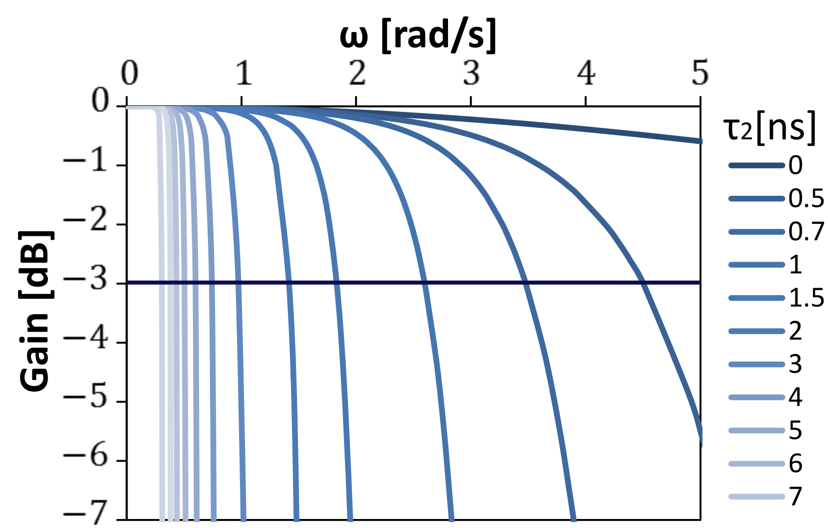
- SPICE Simulation Conditions



- Results



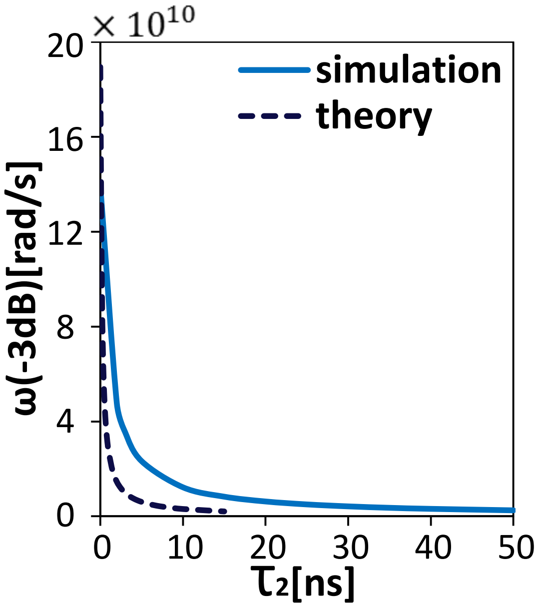
SPICE Simulation Results



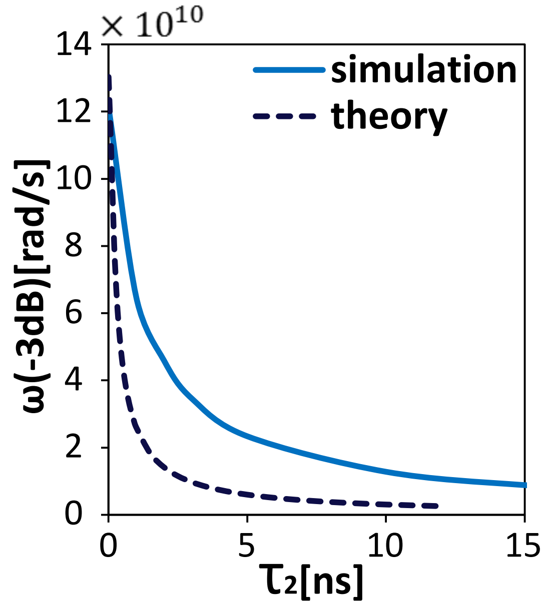
Theory
(Derived Transfer Function)

Comparison of -3dB Bandwidth

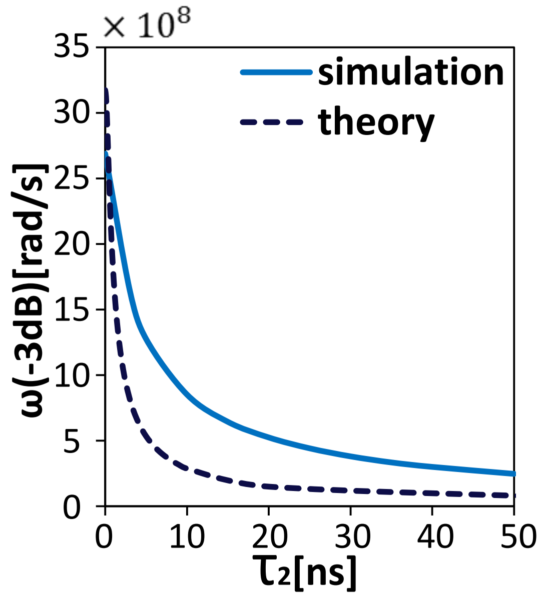
W=200um



W=20um



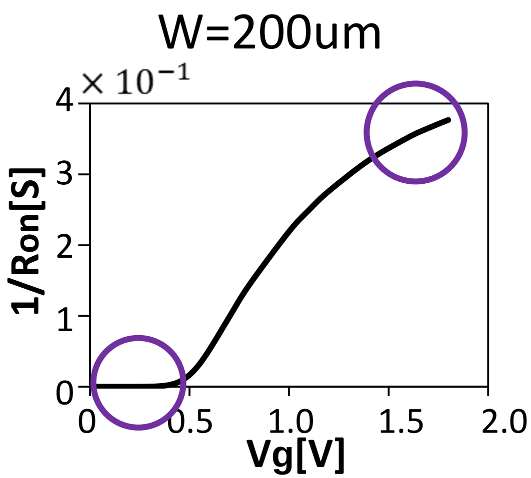
W=2um

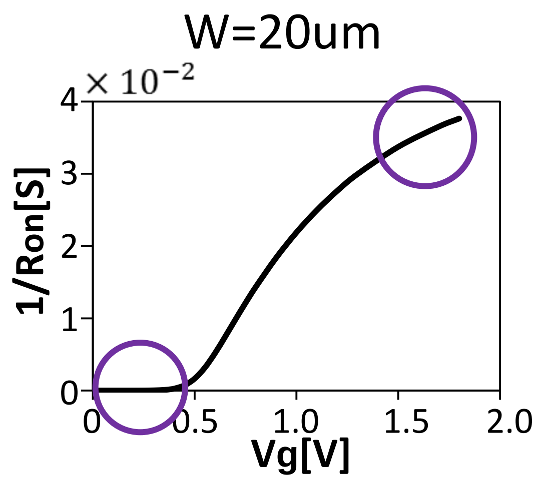


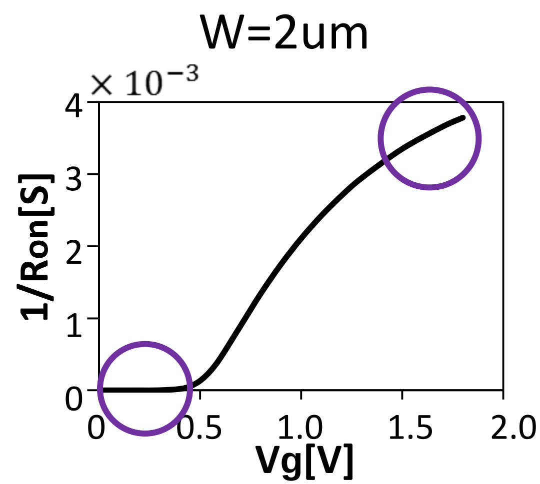
Simulation ≠ Theory

Large discrepancies !

NMOS ON-Conductance Nonlinearity







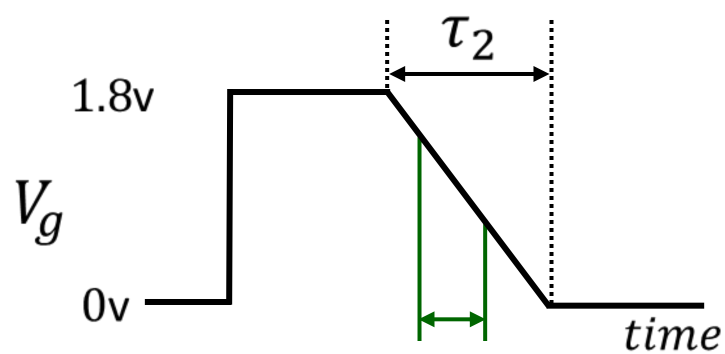
○ region

⇩

Strong nonlinearity of 1/Ron

⇩

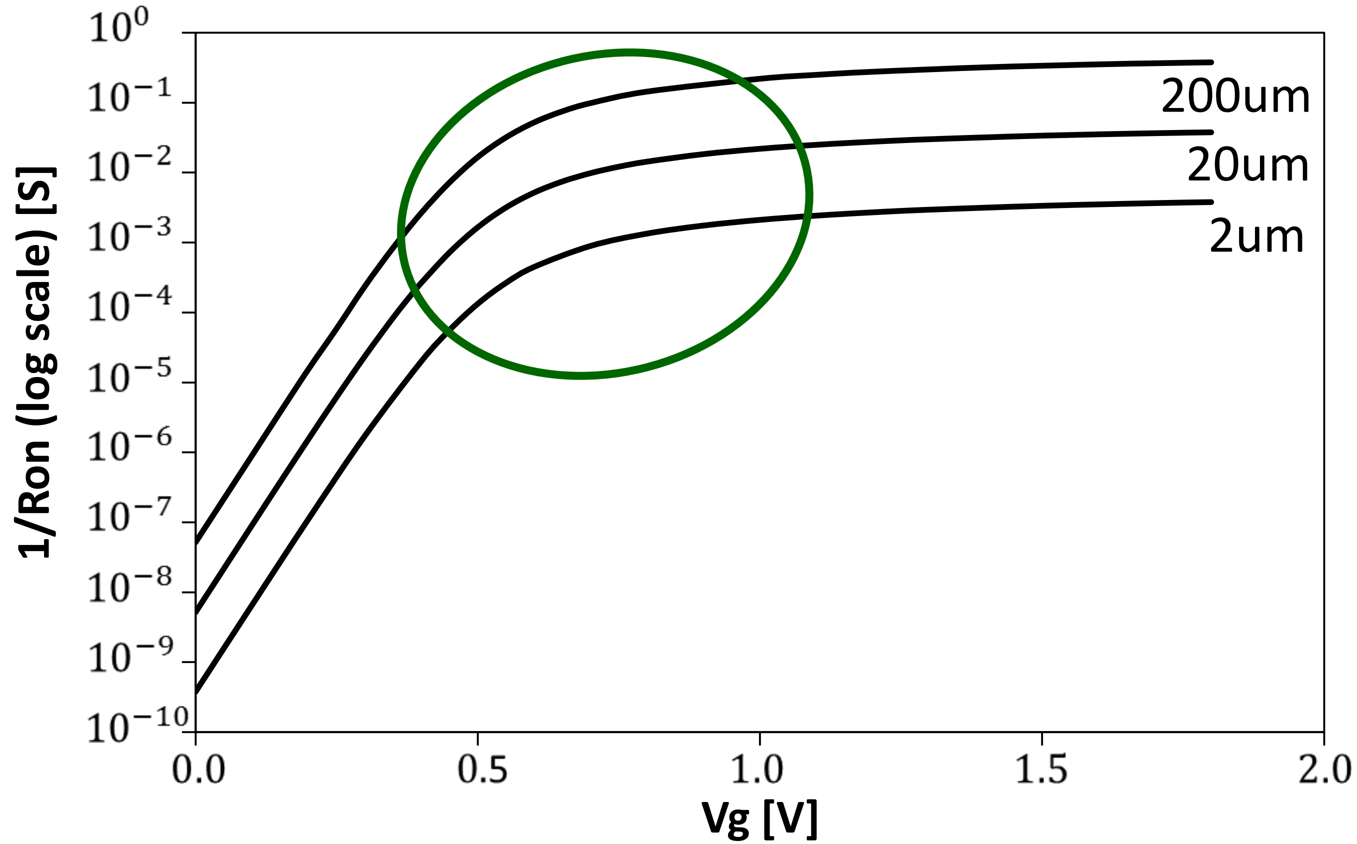
Define effective aperture time τ_{eff}



Effective aperture time

τ_{eff}

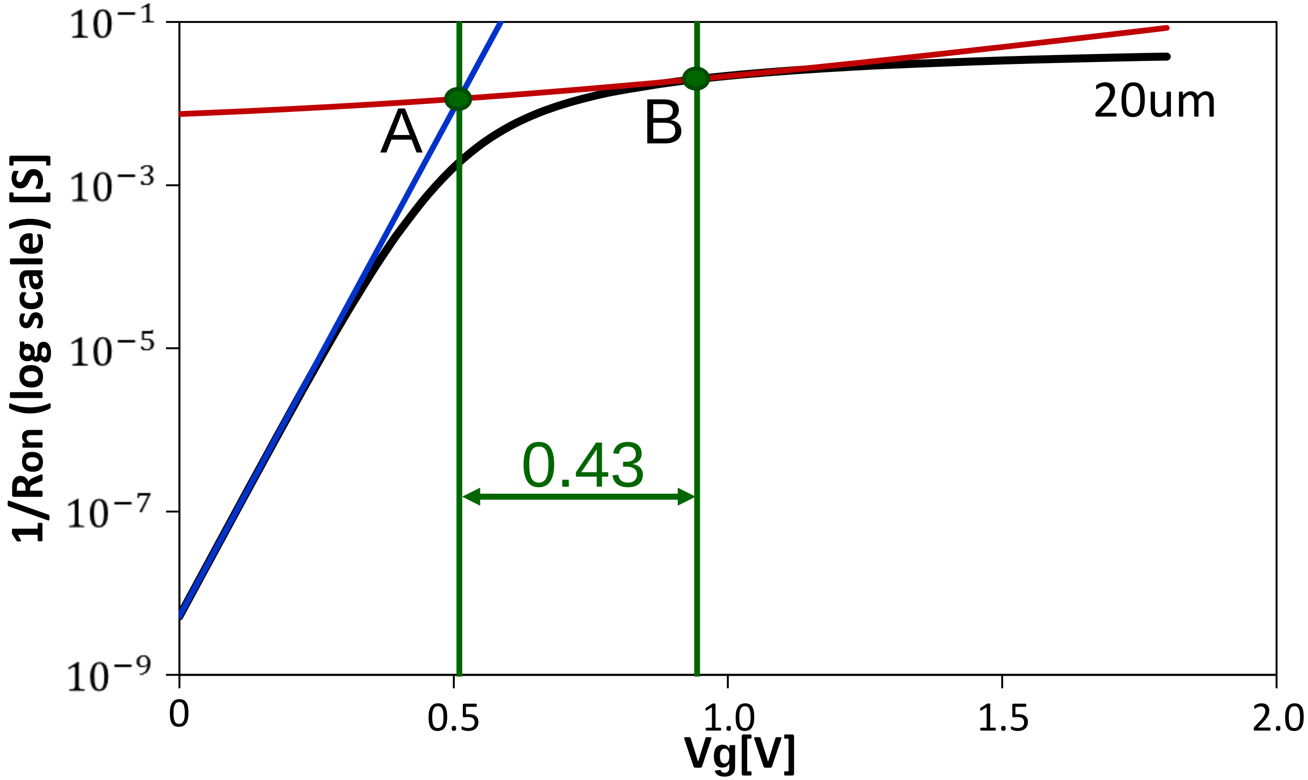
ON-Conductance and Effective Aperture Time



Region ⇒ Effective aperture time

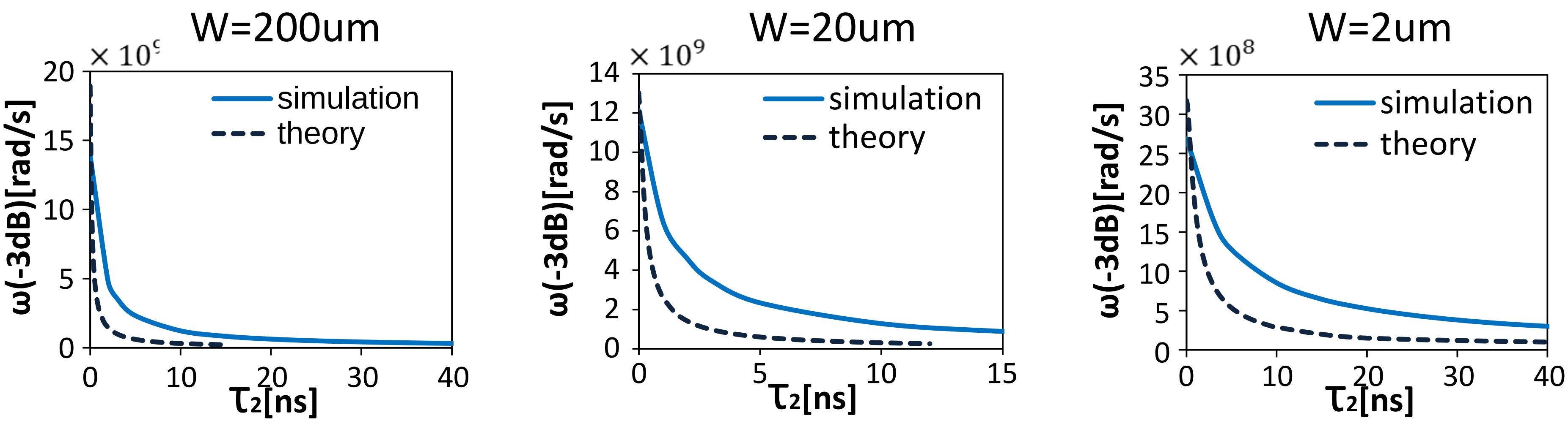
Empirical Effective Aperture Time Derivation

$$y = \left(y_{V_{th}} - 9\times10^{-8}\frac{W}{L}\cdot V_{th}\right)e^{\left(\frac{x}{V_{th}}-1\right)} + 9\times10^{-8}\frac{W}{L}\cdot V_{th}$$

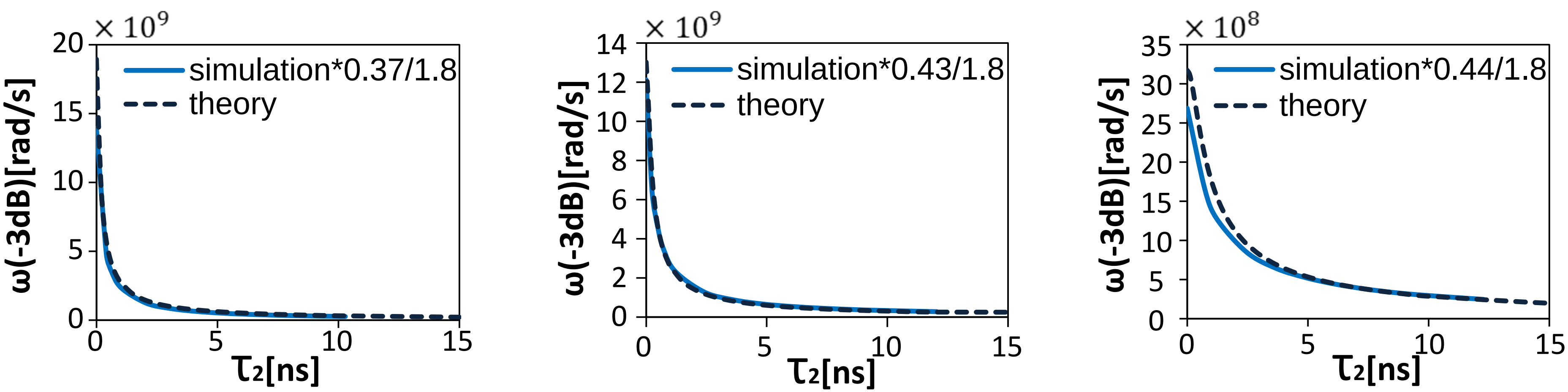


$\therefore$ Effective Aperture Time $\tau_{\text{eff}} = \frac{0.43}{1.8} \times \tau_2$

Discussion Again



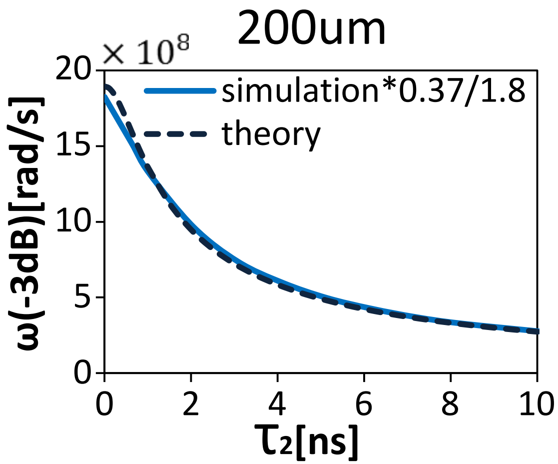
⇩ X 0.37/1.8 ⇩ X 0.43/1.8 ⇩ X 0.44/1.8

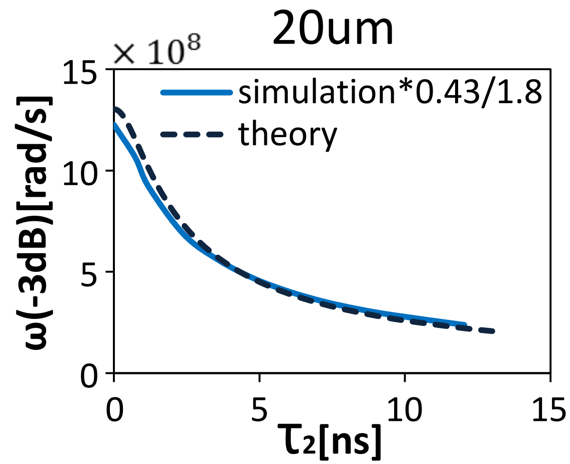


SPICE simulation results ≅ Theory

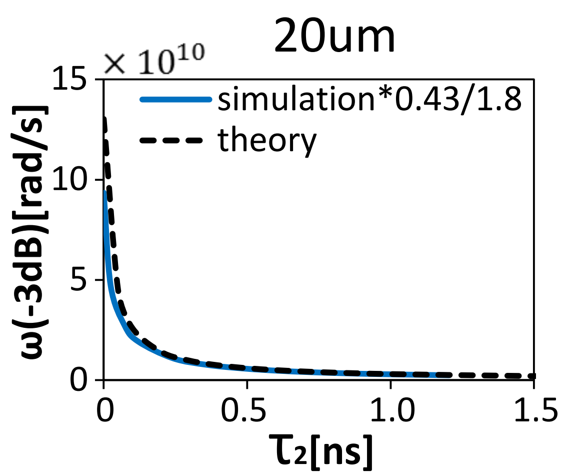
Various Values for RC, W

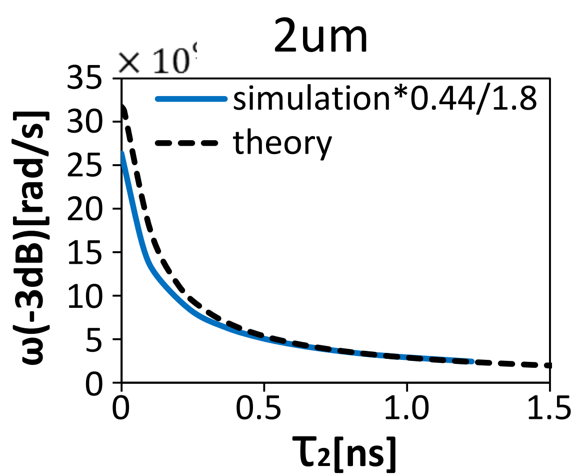
R=50Ω, C=10pF





R=50Ω, C=0.1pF



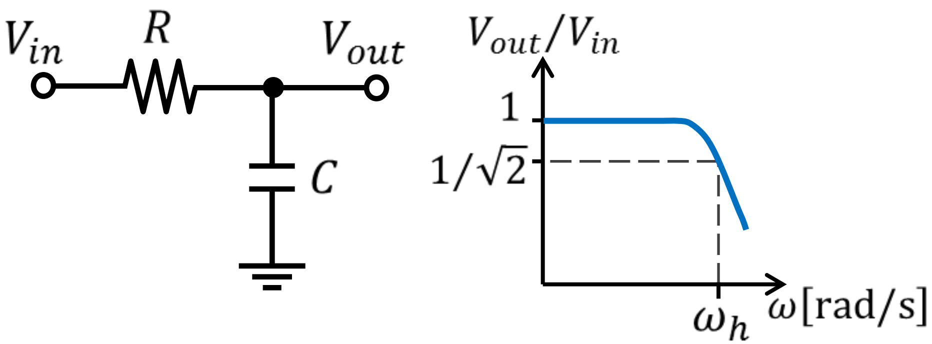


SPICE simulation results ≅ Theory

Contents

Trade-off of Time Constant and Bandwidth

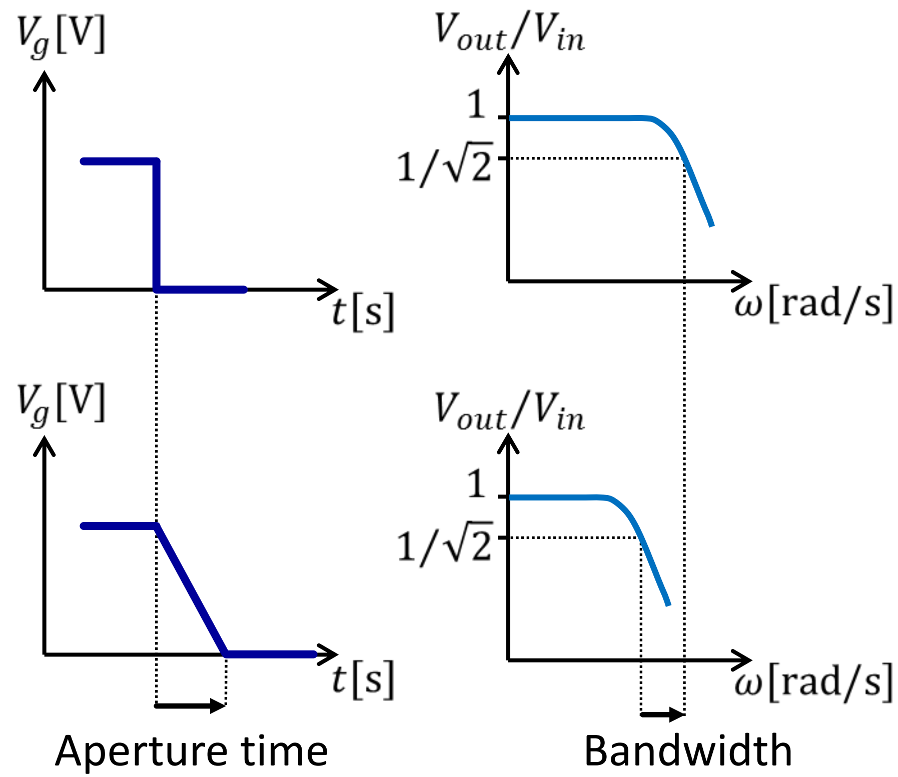
- RC time constant and bandwidth



$$\omega_h = \frac{1}{\tau_1}$$

τ_1 : RC
ω_h : bandwidth

- Aperture time and bandwidth



Time		Band : ω_h
Short	⟶	Wide
Long	⟶	Narrow

-3dB Bandwidth

Transfer function

$$\frac{V_C}{V_{in}} = \frac{sinc(\omega\tau_2)}{sinc(\omega\tau_2) + j\omega\tau_1}$$

($\tau_1 = RC$, $\tau_2 =$ aperture time)

In case of -3dB bandwidth

$$\sqrt{\frac{\mathrm{sinc}^2(\omega\tau_2)}{\mathrm{sinc}^2(\omega\tau_2) + (\omega RC)^2}} = \sqrt{\frac{1}{2}}$$

$$\Downarrow$$

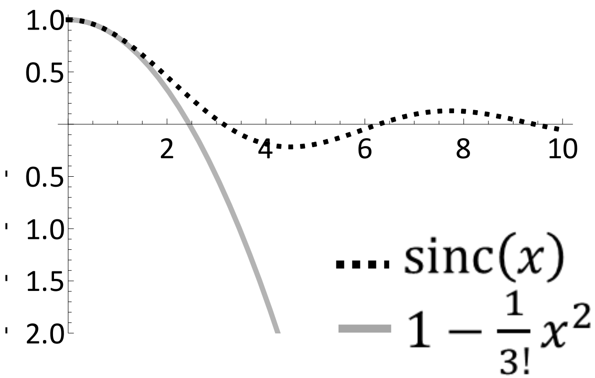
$$\boxed{\mathrm{sinc}(\omega\tau_2) = \omega\tau_1}$$

Uncertainty Relationship

$$\mathrm{sinc}(\omega\tau_2) = \omega\tau_1$$

Taylor expansion of Sinc function

$$\mathrm{sinc}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n} \cong 1 - \frac{1}{3!}x^2$$



$$\mathrm{sinc}(x) \geq 1 - \frac{1}{3!}x^2$$

$$\therefore\ \omega\tau_1 \geq 1 - \frac{1}{3!}(\omega\tau_2)^2$$

Uncertainty Relationship Formula

$\omega \rightarrow \sigma_\omega$

$\tau_1 \rightarrow \sigma_{\tau_1}$

$\Rightarrow$

$$\sigma_\omega \sigma_{\tau_1} + \frac{1}{6}(\sigma_\omega \tau_2)^2 \geq 1$$

- σ_ω : bandwidth
- σ_{τ_1} : RC time constant
- τ_2 : aperture time

Uncertainty Principle and Relationship

- Uncertainty Principle
  - Quantum mechanics

Prof. W. K. Heisenberg

$$\Delta\chi\Delta\rho \geq \frac{\hbar}{2}$$

Impossible to know exactly and simultaneously
- Where the object is
- How fast it is moving

Can NOT be proved

- Uncertainty Relationship
  - Sampling circuit

$$\sigma_\omega\sigma_{\tau_1} + \frac{1}{6}(\sigma_\omega\tau_2)^2 \geq 1$$

Can be proved

Contents

Summary

- Derived explicit transfer function
  of sampling circuit
  with finite aperture time effect.

- Verified it with SPICE simulation

- Introduced concept of effective finite aperture time

- Showed uncertainty relationship
  between time constants and bandwidth
  in sampling circuit.

Thank you for your kind attention

謝謝