Finite Aperture Time Effects in Sampling Circuit

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Abstract - This paper presents analysis of sampling circuit for high-frequency and high-precision waveform acquisition. We have analyzed effects finite aperture time, and we have shown derived formula for the bandwidth limitation due to its low-pass filter effects. We have checked that our theoretical calculation and SPICE simulation results agree well. We also have focused on the trade-off among bandwidth, aperture time, and time constant, and we have derived their relationships as an extension of the uncertainty relationship between time and frequency; we believe that such analyses would be new in circuit design area.

Keywords: Sampling Circuit, Aperture Time, Bandwidth, Uncertainty Relationship

1. Introduction

Sampling is an important technique for waveform acquisition, but high-frequency sampling is adversely affected by sampling circuit non-idealities such as finite aperture time [1-6]. This paper discusses the effect of finite aperture time, the lowpass action of an RC sampling circuit modeled by a switch with non-zero turn-off time (Fig.1). We have derived the explicit formula for the bandwidth-limiting effects of such a circuit, and show that SPICE simulation results agree well with our derived theoretical results. We also discuss the trade-off among bandwidth, aperture time, and time constant, and derive their relationships as an extension of the uncertainty relationship between time and frequency.

2. Theoretical Result of Finite Aperture Time Effects

The following rigorous transfer function for the sampling circuit with aperture time in Fig.1 was derived (See Appendix I for its proof):

$$\frac{V_{out}}{V_{in}} = \frac{sinc(\omega\tau_2)}{sinc(\omega\tau_2) + j\omega\tau_1} \qquad (1)$$

$$(\tau_1 = RC \ , \ \tau_2 = \tau)$$

Eq.(1) shows a lowpass action in track mode imposed by the RC pole, and another lowpass action given by a sinc function whose null lies at a frequency equal to the inverse of τ_1. The lower of the two cutoffs dominates. Figures 2, 3 show characteristics obtained from eq. (1).

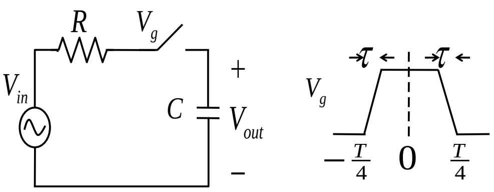


Fig.1. Sampling circuit with aperture time τ.

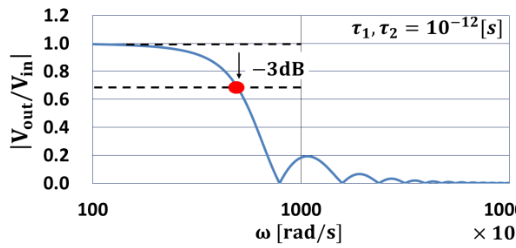


Fig. 2 .Gain characteristics obtained from eq.(1).

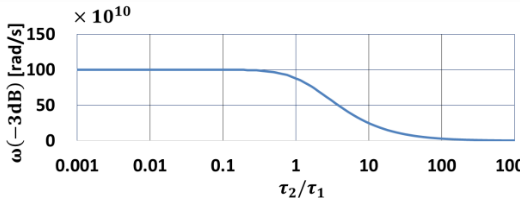
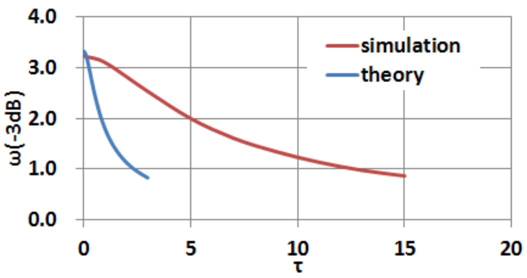


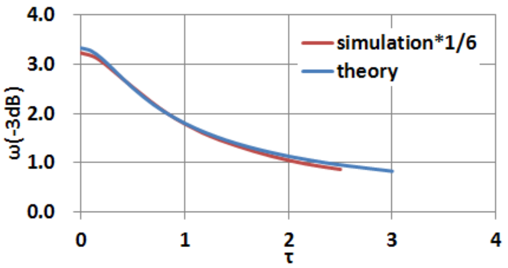
Fig. 3. Sampling circuit bandwidth versus τ_2/τ_1

3. SPICE Simulation Results of Finite Aperture Time Effects

We show results of SPICE simulation using an NMOS switch with TSMC 0.18um CMOS in Fig.1. Fig.4 shows a comparison between the theoretical and simulation results for 3dB bandwidth; Fig.4 (a) shows a large discrepancy, but when the aperture time is multiplied by 1/6 in simulation as in Fig.4 (b), there is good agreement. This is because the sampling FET transitions from ON to OFF over 1/6-th of the fall time of the gate voltage, which swings between Vdd and 0 (Figs. 5, 6). This fraction will change with FET dimensions. Appendix II explains how the effective time is derived empirically.



(a) 3dB bandwidth comparison



(b) 1/6th aperture time in simulation.

Fig. 4. Simulation result with NMOS switch.

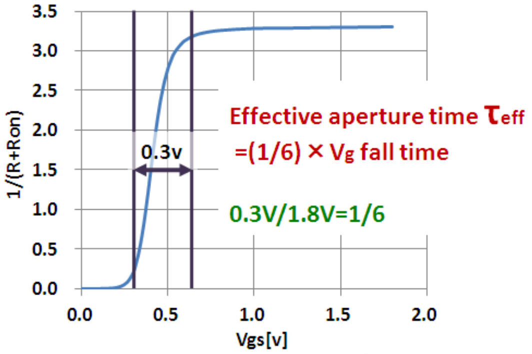


Fig.5. On-resistance with respect to Vg.

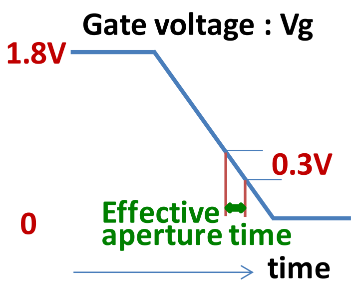


Fig.6. Effective aperture time

4. Uncertainty Relationship in Sampling Circuit

Uncertainty relationship between time and frequency states that time domain waveform and frequency power spectrum cannot be short simultaneously (See Fig. 7) [7].

$$\sigma_\tau \sigma_\omega \geq \frac{1}{2} \cdots\cdots\cdots\cdots (2)$$

In eq. (2), the equality holds in case of Gaussian distribution for power spectrum and time domain waveform distributions, and we consider this case.

$$\sigma_\tau \sigma_\omega = \frac{1}{2} \cdots\cdots\cdots\cdots (3)$$

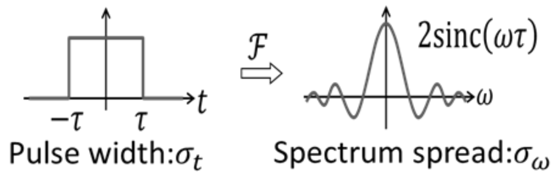


Fig.7. Uncertainty relationships between time and frequency

First-order RC circuit time constant τ_1 and -3dB band frequency ω_h have the following relationships:

$$\omega_h = \frac{1}{\tau_1} \cdots\cdots\cdots\cdots (4)$$

-3dB band frequency ω_h is the one where the gain is $\sqrt{1/2}$, and it can be obtained from the transfer function (eq.(1)). Then we will derive the uncertain relationship for the finite aperture time of a sampling circuit.

We obtain -3dB band frequency from eq. (1):

$$\left|\frac{V_C}{V_{in}}\right| = \left|\frac{\mathrm{sinc}(\omega_h \tau_2)}{\mathrm{sinc}(\omega_h \tau_2)} + j\omega_h \tau_1\right|$$

$$= \sqrt{\frac{\mathrm{sinc}^2(\omega_h \omega \tau_2)}{\mathrm{sinc}^2(\omega_h \tau_2) + (\omega_h \omega \tau_1)^2}} = \sqrt{\frac{1}{2}}$$

$$\mathrm{sinc}(\omega_h \tau_2) = \omega_h \tau_1 \cdots\cdots\cdots\cdots (5)$$

We use Taylor expansion of a *sinc* function with a second order term.

$$\mathrm{sinc}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n}$$

$$\cong 1 - \frac{1}{3!} x^2 \cdots\cdots\cdots\cdots (6)$$

We have the following from eq. (4) and Fig.8:

$$\mathrm{sinc}(x) \geq 1 - \frac{1}{3!} x^2 \cdots\cdots\cdots\cdots (7)$$

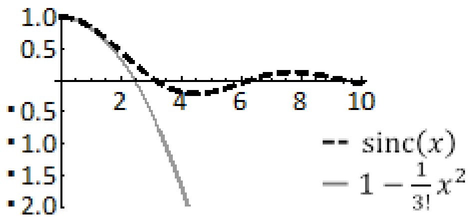


Fig.8. Sinc function and its Taylor expansion approximation.

It follows from eqs. (4) and (6) that

$$\omega_h \tau_1 \geq 1 - \frac{1}{3!}(\omega_h \tau_2)^2 \cdots\cdots\cdots\cdots \text{(8)}$$

Then we have the following by rewriting the bandwidth and RC time constant:

$$\sigma_\omega \sigma_{\tau_1} + \frac{1}{6}(\sigma_\omega \tau_2)^2 \geq 1 \cdots\cdots \text{(9)}$$

$$\begin{cases} \omega_h \rightarrow \sigma_\omega \\ \tau_1 \rightarrow \sigma_{\tau_1} \end{cases}$$

Eq. (9) is an extension of the uncertainty principle between time and frequency, for a sampling circuit with RC time constant and finite aperture time.

5. Conclusion

We have shown a simple formula for finite aperture time effect in a sampling circuit, and confirmed its accuracy with SPICE simulation.

We have also explained a trade-off among RC time constant, aperture time and bandwidth clearly. In other words, we have shown uncertainty relationships in a sampling circuit.

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References

[1] M. Uemori, K. Kobayashi, M. Kono, K. Shimizu, H. Kobayashi, T. Tobari, "Wideband and Large Dynamic Range Sampling Method," IEICE Trans. vol. J90-C, no.9, pp.625-633 (Sept. 2007).

[2] H. Kobayashi, K. Kobayashi, M. Morimura, Y. Onaya, Y. Takahashi, K. Enomoto, H. Kogure: "Samplig Jitter and Finite Aperture Time Effects in Wideband Data Acquisition Systems", IEICE Trans. Fundamentals, Vol.IE85-A, no.2, pp.105-117 (Feb. 2001)

[3] M. Kimura, K. Kobayashi and H. Kobayashi, "A Quasi-Coherent Sampling Method for Wideband Data Acquisition", IEICE Trans. on Fundamentals, vol. E85-A, no. 4, pp.757-763, (April 2002).

[4] M. Kimura, A. Minegishi, K. Kobayashi, and H. Kobayashi, "A New Coherent Sampling System with a Triggered Time Interpolation", IEICE Trans. on Fundamentals, vol. E84-A, no. 3, pp.713-719 (March 2001).

[5] M. Uemori, K. Kobayashi, M. Kono, K. Shimizu, H. Kobayashi, T. Tobari, "Wideband and Larger Dynamic Range Sampling Method", IEICE Trans. vol. J90-C, no.9, pp.625-633（Sept. 2007）.

[6] N. Hayasaka, H. Kobayashi, "Input-Dependent Sampling-Time Error Effects in MOS Samplers", IEICE Trans. on Electronics, E87-C, no. 6, pp.1015-1021 (June 2004).

[7] L. Cohen, Time-Frequency Analysis, Prentice Hall (1995)

Appendix I: Derivation of Finite Aperture Time Effects in a Sampling Circuit

In this appendix we will derive eq.(1).

A. In case of zero aperture time

Let us consider the ideal case as shown in Figs.A1, A2. The output voltage change during a sampling period is expressed by

$$V_{out}\left(\left(n+\frac{1}{2}\right)T\right) - V_{out}(nT)$$

$$= \int_{nT}^{\left(n+\frac{1}{2}\right)T} \frac{V_{in}(t) - V_{out}(t)}{RC} dt$$

$$= \int_{-\infty}^{\infty} \frac{V_{in}(t) - V_{out}(t)}{RC} \cdot W(t - nT) dt . \quad \text{(A1)}$$

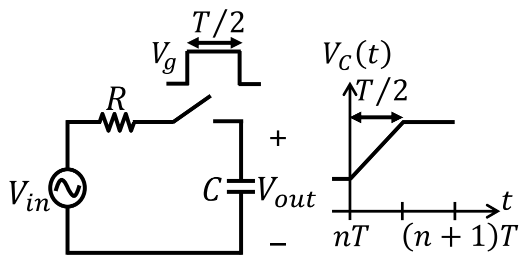
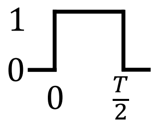


Fig. A1. Sampling circuit of RC model.



$$w(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq \frac{T}{2} \\ 0 & \text{for all other } t \end{cases}$$

Fig. A2. Ideal sampling pulse.

By applying Fourier transform to eq. (A1), we have the following:

$$V_{out}(f)\left(1 - e^{-j\frac{\omega T}{2}}\right)$$

$$= \frac{1}{RC}\{V_{in}(f) - V_{out}(f)\} \cdot \frac{T}{2} sinc\left(f\frac{T}{2}\right) \cdot e^{-j\frac{\omega T}{4}} \quad \text{(A2)}$$

B. In case of finite aperture time

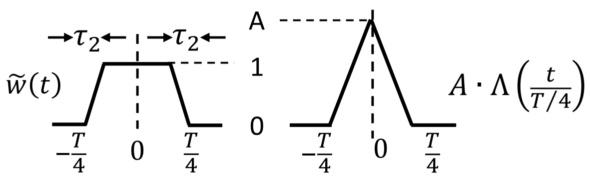
Let us consider to use a sampling pulse as shown in Fig. A3 (a), and we consider a falling time τ_2 as an aperture time. Trapezoid wave sampling pulse $\widetilde{w}(t)$ can be obtained by subtraction from the large triangular in Fig. A4 (a) to the small triangular in Fig. A4 (b):

$$\widetilde{\mathrm{W}}(t) = \frac{T}{4\tau}\Lambda\left(\frac{t}{T/4}\right) - \left(\frac{T}{4\tau} - 1\right)\Lambda\left(\frac{t}{T/4 - \tau}\right) \quad \text{(A3)}$$

By applying Fourier Transform to eq.(A3), we have the following:

$$\widetilde{\mathrm{W}}(f) = \frac{1}{\tau}\left\{\frac{T^2}{16}\mathrm{sinc}\left(\frac{fT}{4}\right) - \left(\frac{T}{4} - \tau\right)^2 \mathrm{sinc}^2\left(f\left(\frac{T}{4} - \tau\right)\right)\right\}. \quad \text{(A4)}$$

Then we have the Fourier transform of the trapezoid sampling wave $\widetilde{w}(t)$ as $\widetilde{\mathrm{W}}(f)$.



(a) Trapezoid wave $\widetilde{\boldsymbol{w}}(t)$ (b) Triangular wave $\Lambda(X)$

Fig. A3. Trapezoid and triangular wave sampling pulses.

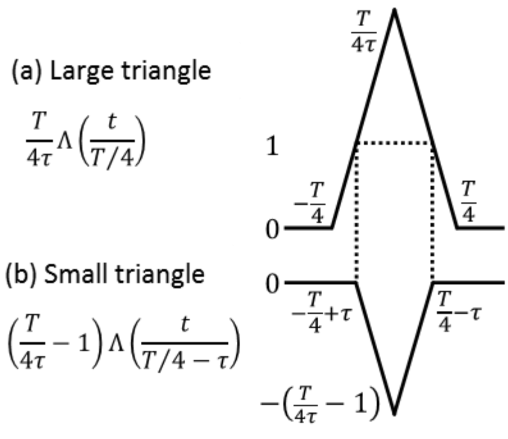


Fig. A4. Derivation of the trapezoid wave.

Now let us consider the ideal case. Eq. (A2) is for a square wave and the square wave component in eq. (A2) is included in the third term in the right hand. Then by replacing the term with the trapezoid term (eq.(A4)), eq. (A2) can be transformed into eq.(A5) which corresponds to the trapezoid waveform.

$$V_{out}(f)\left(1 - e^{-j\frac{\omega T}{2}}\right) = \frac{1}{RC}\{V_{in}(f) - V_{out}(f)\} \cdot \widetilde{W}(f) \cdot e^{-j\frac{\omega T}{4}} \quad \text{(A5)}$$

By transforming of eq. (A3), eq. (1) has been derived, which is a transfer function taking the finite aperture time into account.

$$\frac{V_{out}}{V_{in}} = \frac{sinc(\omega\tau_2)}{sinc(\omega\tau_2) + j\omega\tau_1} \quad (1)$$

$$(\tau_1 = RC\ ,\ \tau_2 = \tau)$$

Appendix II: Empirical Derivation Method of Effective Finite Aperture Time

Fig. A5 explains our empirical derivation method of the effective aperture time. We obtain the point A as a cross point between the tangent of the log (1/Ron) in the subthreshold region and the thresh voltage (see Fig. A5). We also obtain the point B using the tangent of the log(1/Ron) in the saturation region from the point A. Then we have the value 0.43 in the case of Fig.A5 and the effective aperture time is obtained by (0.43/1.8) x [Gate voltage falling time τ_2 from 1.8V to 0.0]. Fig. A6 shows that when the coefficient of (0.43/1.8) is taken into account, the SPICE simulation and the formula (eq.(1)) calculation results agree well.

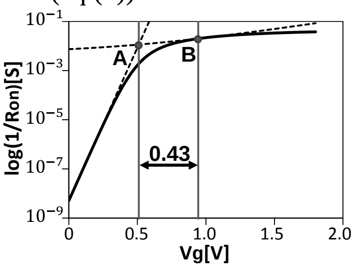
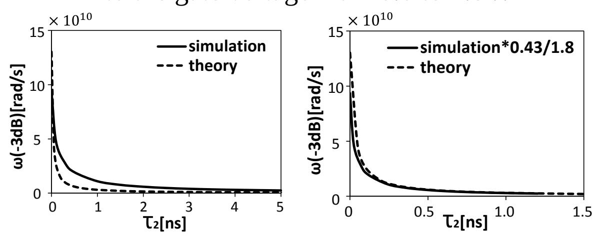


Fig. A5. Sampling switch on-resistance with respect to the gate voltage from 0.0 to 1.8V.



(Left) Effective aperture time is NOT considered.
(Right) Effective aperture time is considered.
Fig. A6. Simulation result comparison for Vdd=1.8V.