2016 International Symposium on
VLSI Design, Automation and Test

D10-4 14:30-14:50 Apr. 26 2016 (Tue)

Fundamental Design Consideration of Sampling Circuit

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OUTLINE

- Research Background and Objective
- Sample/Hold Circuit
- Two S/H Circuits
  - Track/Hold Circuit
  - Impulse Sampling Circuit
- Unified S/H Circuit Theory
- Condition of Maximum SNR Under Constant Bandwidth
- Conclusion

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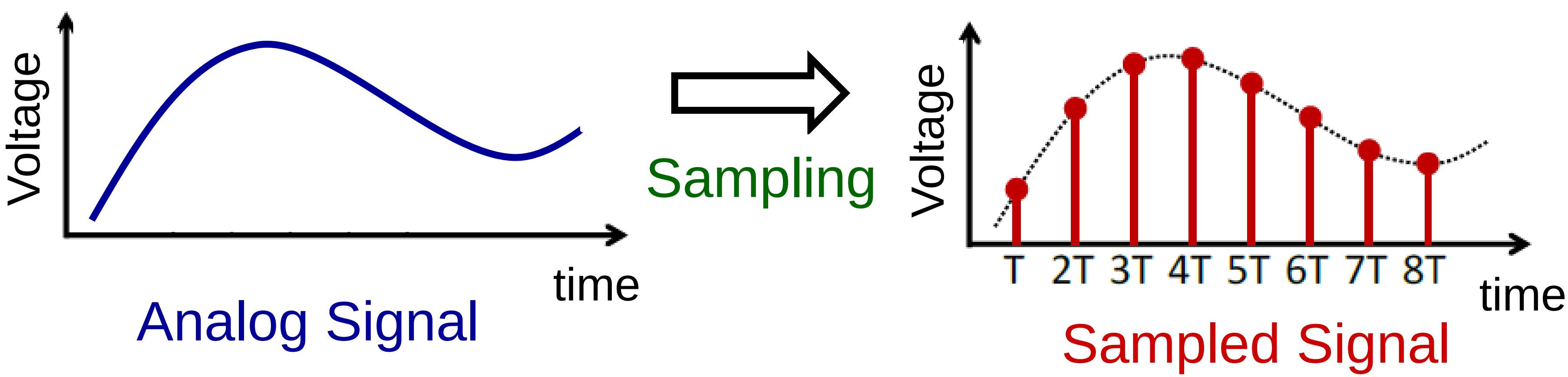
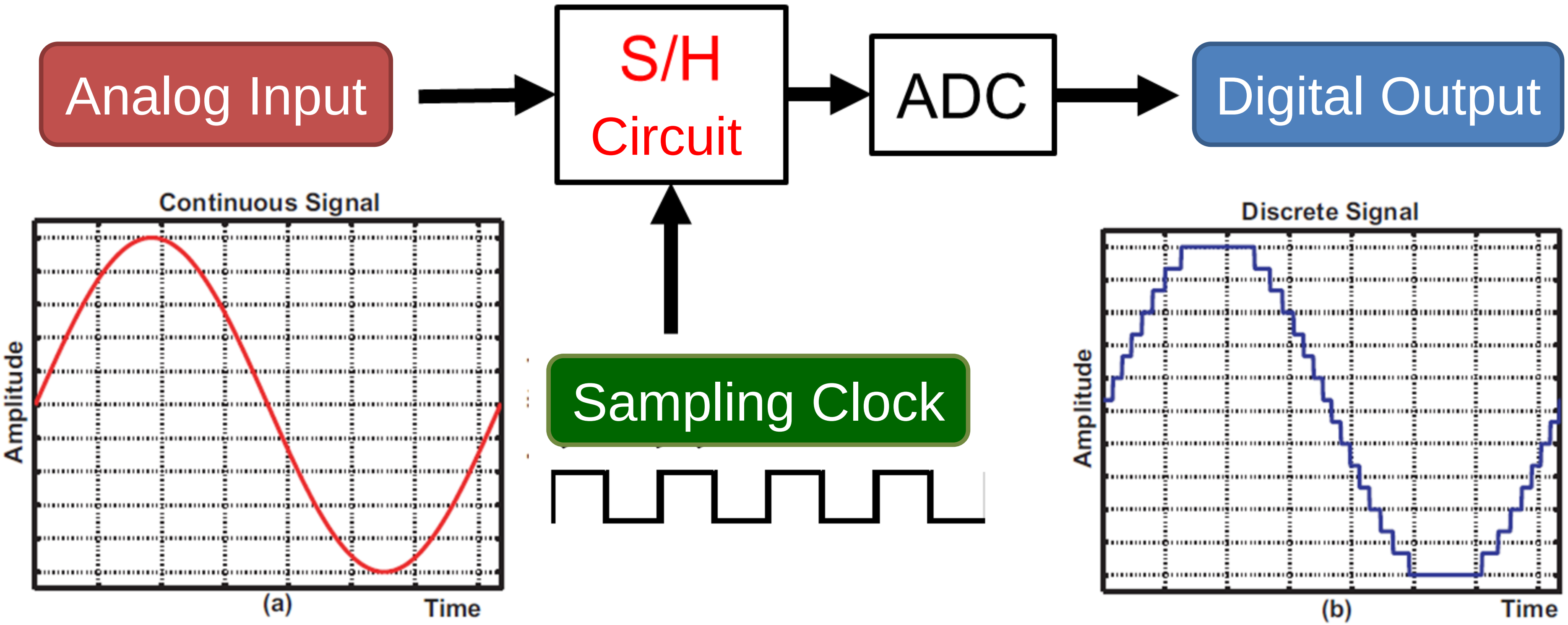
Waveform Sampling

Continuous-time
Continuous amplitude
signal

Discrete-time
signal



Analog-to-Digital Converter



Real world signals

Ex) Radio wave
Voice
Video
Temperature ...

Rounded as integer

Research Background and Objective

Research Background

High-frequency, wideband signals become more utilized
in electronic and communication systems .

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Their acquisition with S/H circuit is very important.

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Fundamental theory of S/H circuit has not been established yet.

Research Objective

Fundamental trade-off clarification of S/H circuit design.

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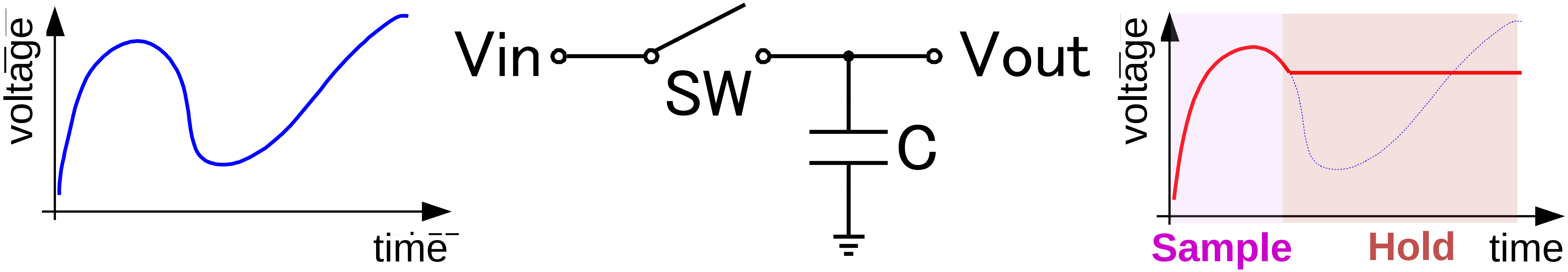
Configuration of S/H Circuit

■Open-loop S/H circuit：

Switch and Capacitor

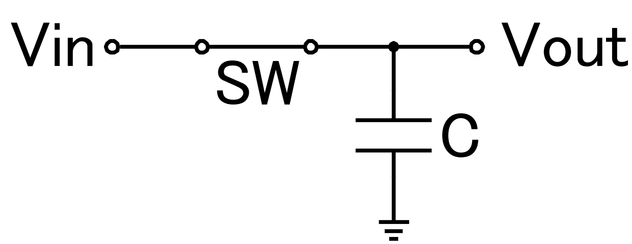


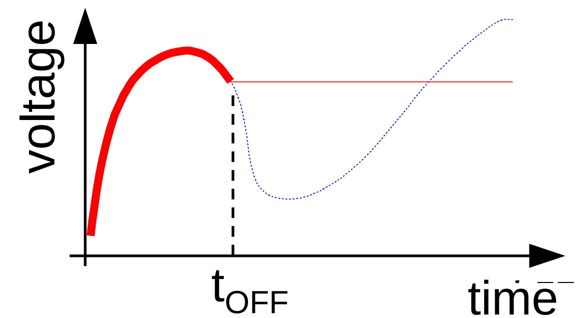
Operation of S/H Circuit

- SW : ON

  Vout(t) = Vin(t)

  Sample mode

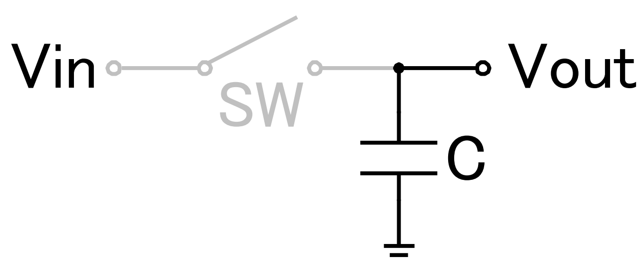


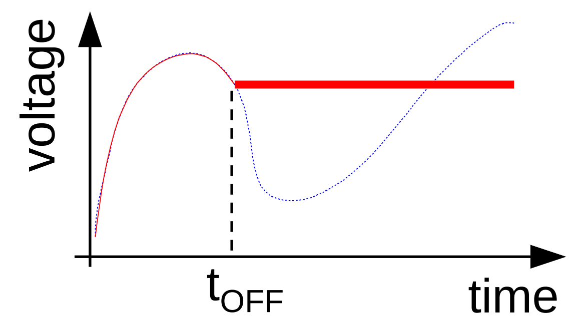


- SW : OFF

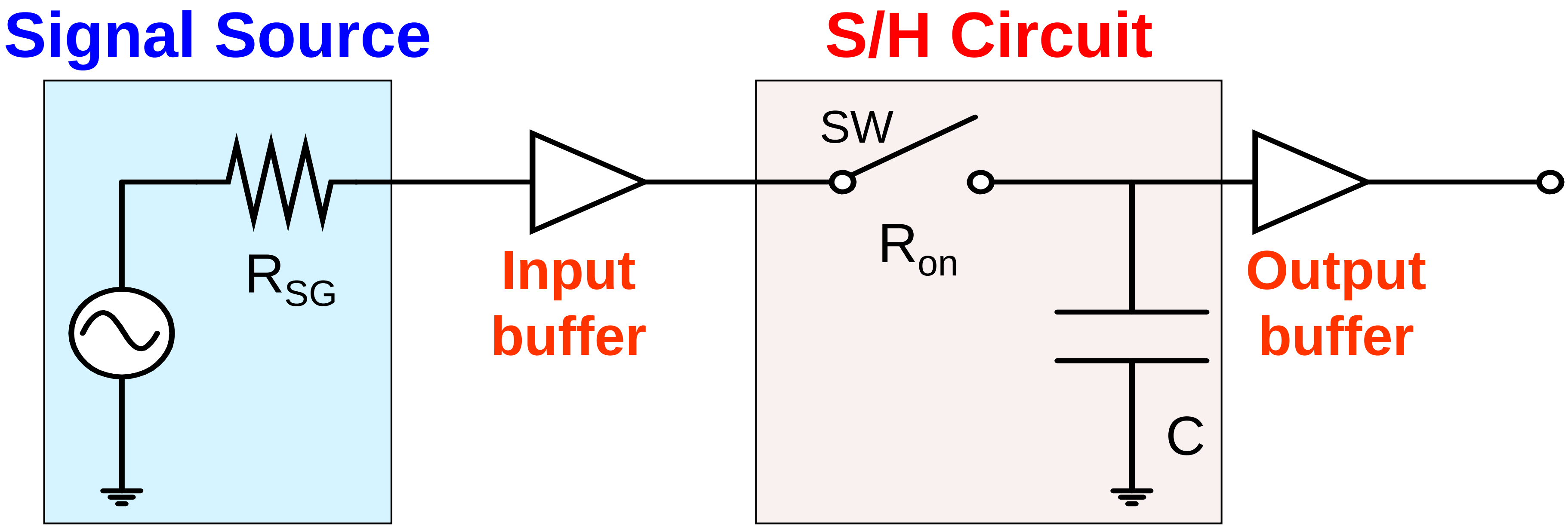
  Vout(t) = Vin(t_{OFF})

  Hold mode



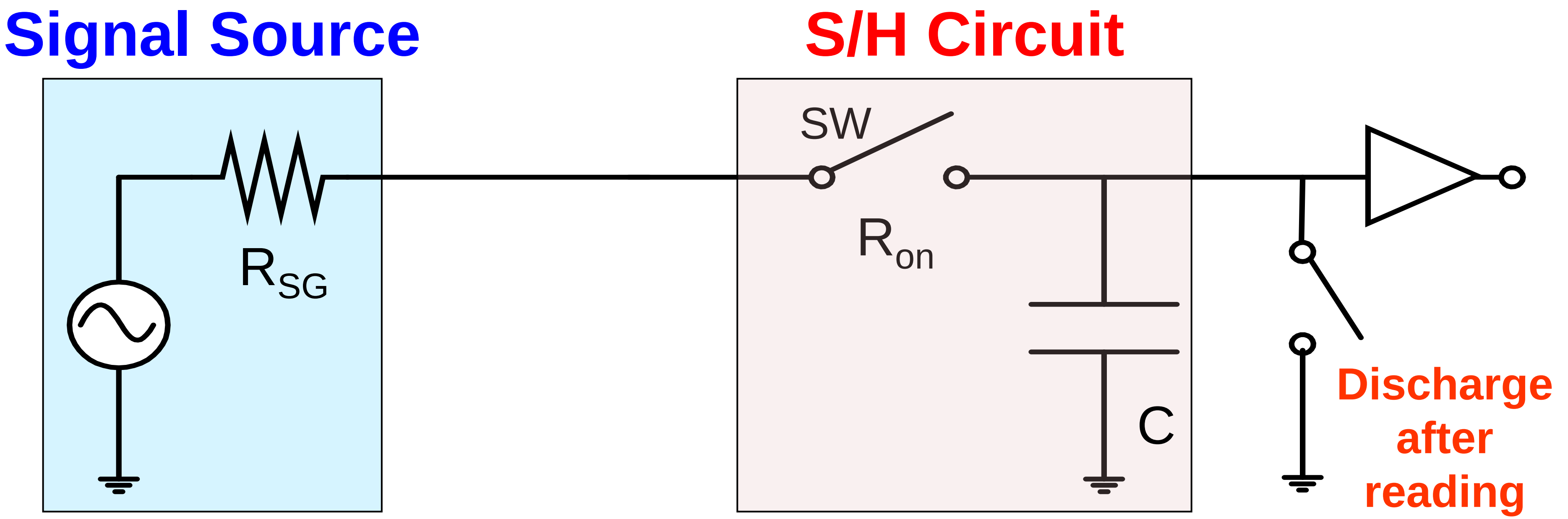


Configuration of Wideband S/H Circuit



Bandwidth-limited by input buffer

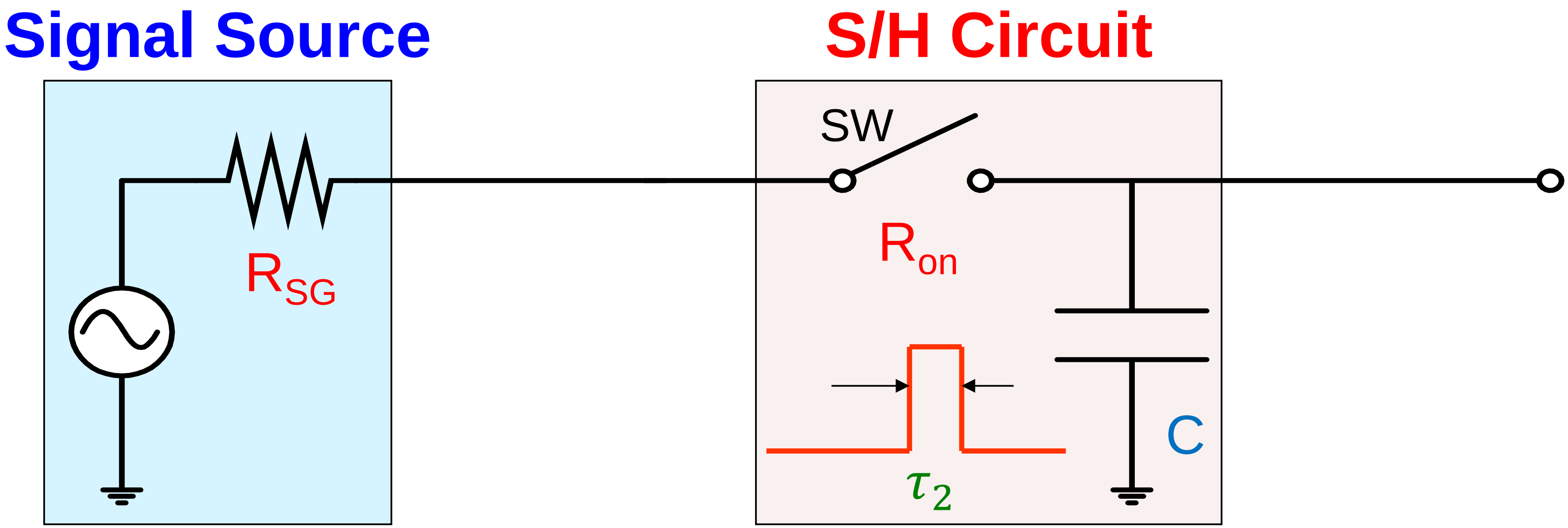
Configuration of Wideband S/H Circuit



Bandwidth-limited by input buffer

Configuration without input buffer

Two Time Constants τ_1, τ_2 in S/H Circuit



- Two Time Constants in S/H Circuit
  - τ_1 : (R_{SG}+ R_{on}) x C
  - τ_2 : Switching time window

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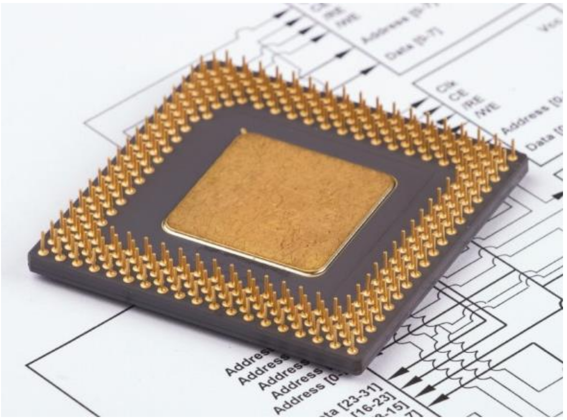
Two S/H Circuits

Track/Hold Circuit

$\tau_2 \gg \tau_1$

Impulse Sampling Circuit

$\tau_2 \ll \tau_1$ (narrow window)



ADC on SoC

Sampling oscilloscope

Currently used in different worlds

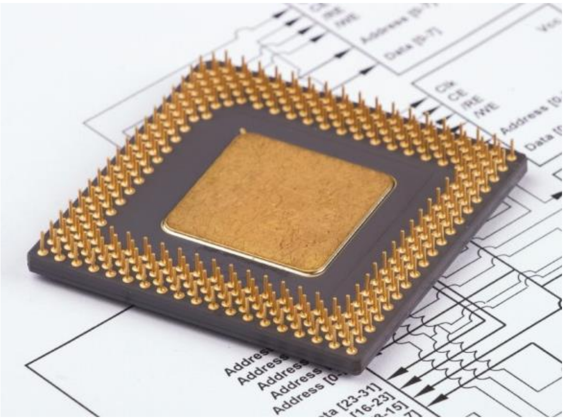
Our Challenge !!

Track/Hold Circuit

$\tau_2 \gg \tau_1$

Impulse Sampling Circuit

$\tau_2 \ll \tau_1$ (narrow window)

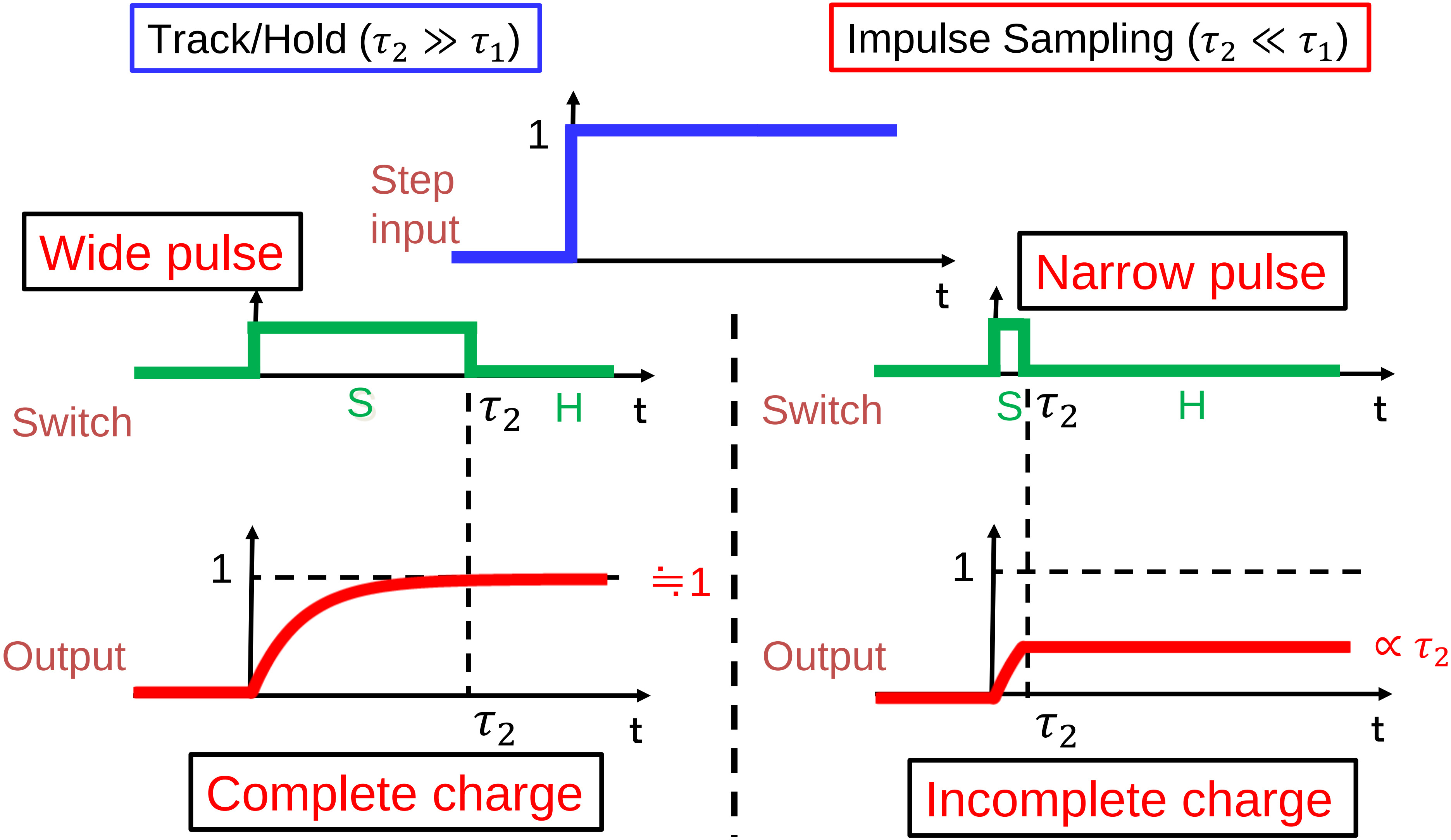


ADC on the SoC

Sampling oscilloscope

Unified Theory

Operation of Two S/H Circuits



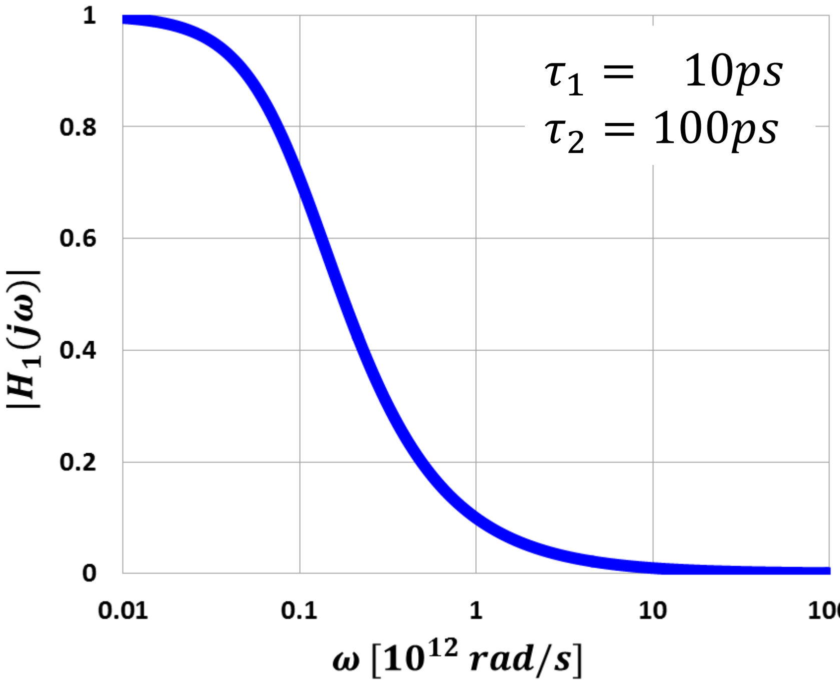
VLSI SINCE 1983

(2016 VLSI-DAT)

Frequency Transfer Function of Two S/H Circuits

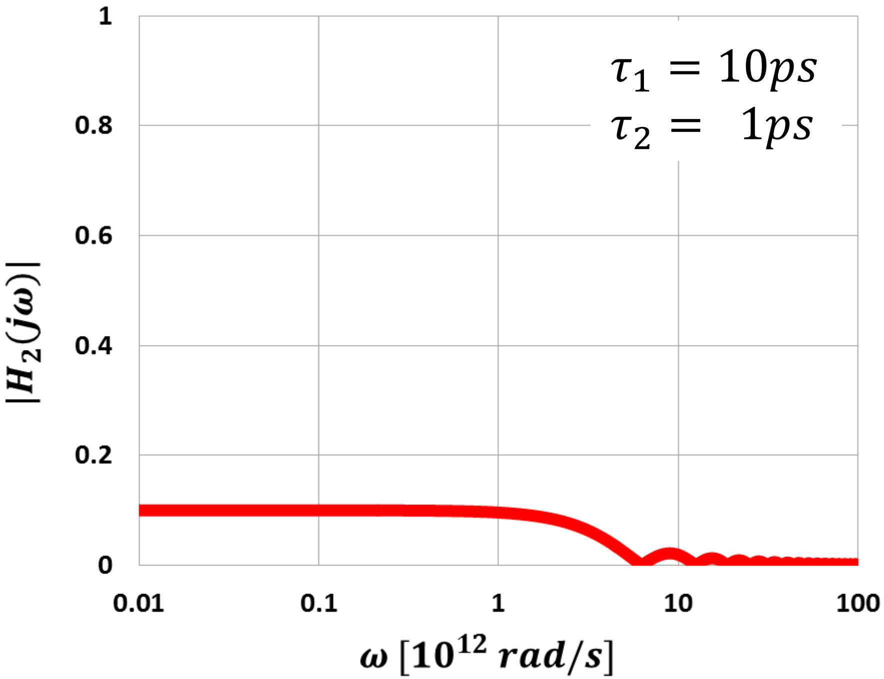
Track/Hold ($\tau_2 \gg \tau_1$)

$$H_1(j\omega) = \frac{1}{1 + j\tau_1\omega}$$



Impulse Sampling ($\tau_2 \ll \tau_1$)

$$H_2(j\omega) = \frac{\tau_2}{\tau_1} sinc\left(\frac{\tau_2}{2}\omega\right) e^{-j\frac{\tau_2}{2}\omega}$$



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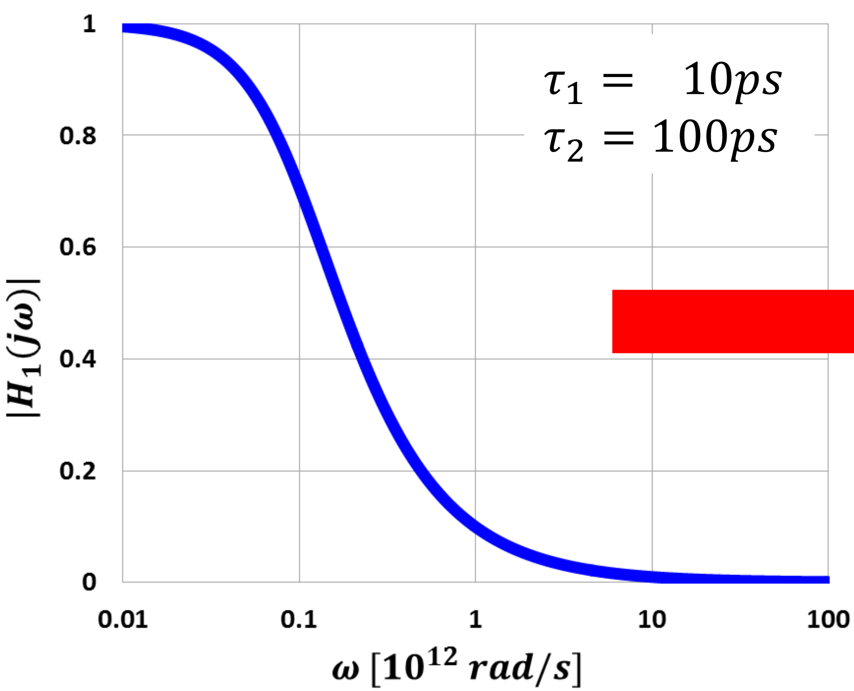

Impulse Sampling ($\tau_2 \ll \tau_1$)

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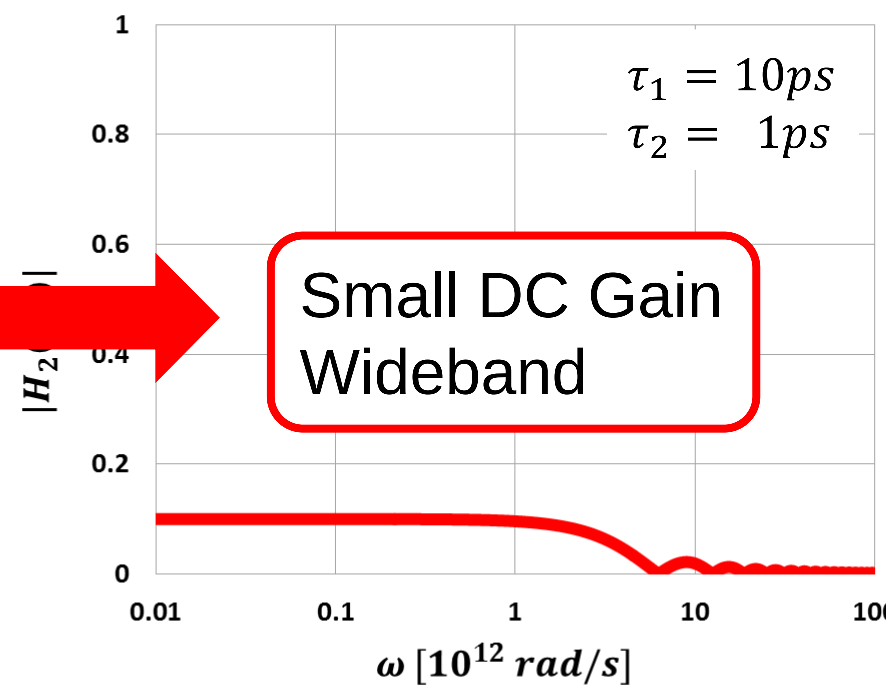
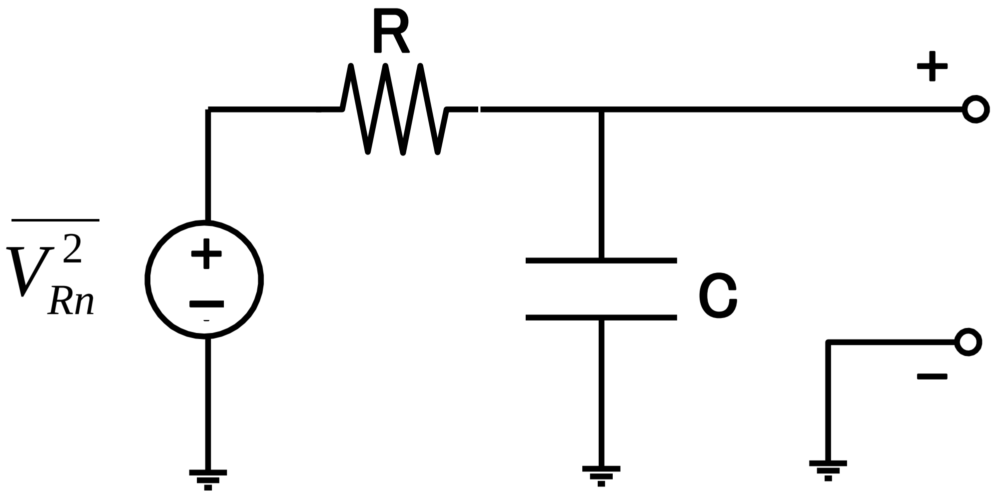
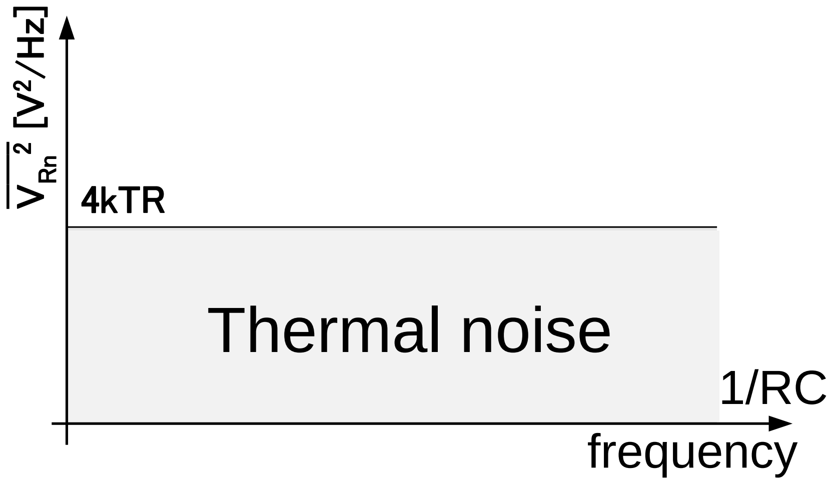


Small DC Gain
Wideband

kT/C Noise in S/H Circuit





Noise power

$$P_{noise} = \int_0^\infty \frac{4k_BTR}{4\pi^2R^2C^2f^2+1}df = \frac{k_BT}{C} = \frac{k_BTR}{\tau_1}$$

$$k_B = 1.38\times10^{-23}\ \mathrm{JK^{-1}}$$
$$T = 300\ \mathrm{K}$$
$$R = 50\ \Omega$$

Bandwidth and SNR of Two S/H Circuits

Track/Hold ($\tau_2 \gg \tau_1$)	Impulse Sampling ($\tau_2 \ll \tau_1$)
$\mathrm{H}_1(j\omega) = \frac{1}{1 + j\tau_1\omega}$	$\mathrm{H}_2(j\omega) = \frac{\tau_2}{\tau_1} sinc\left(\frac{\tau_2}{2}\omega\right) e^{-j\frac{\tau_2}{2}\omega}$
$Bandwidth_1 = 1/\tau_1$	$Bandwidth_2 \approx 2.78/\tau_2$
$SNR_1 \propto \sqrt{\tau_1}$	$SNR_2 \propto \tau_2/\sqrt{\tau_1}$

τ_1 : RC product
τ_2 : Switching time window

Bandwidth and SNR of Two S/H Circuits

Track/Hold ($\tau_2 \gg \tau_1$)

$$H_1(j\omega) = \frac{1}{1 + j\tau_1\omega}$$

$$Bandwidth_1 = 1/\tau_1$$

Trade-off

$$SNR_1 \propto \sqrt{\tau_1}$$

Impulse Sampling ($\tau_2 \ll \tau_1$)

$$H_2(j\omega) = \frac{\tau_2}{\tau_1} sinc\left(\frac{\tau_2}{2}\omega\right) e^{-j\frac{\tau_2}{2}\omega}$$

$$Bandwidth_2 \approx 2.78/\tau_2$$

Trade-off

$$SNR_2 \propto \tau_2/\sqrt{\tau_1}$$

Fundamental Trade-off

τ_1 : RC product
τ_2 : Switching time window

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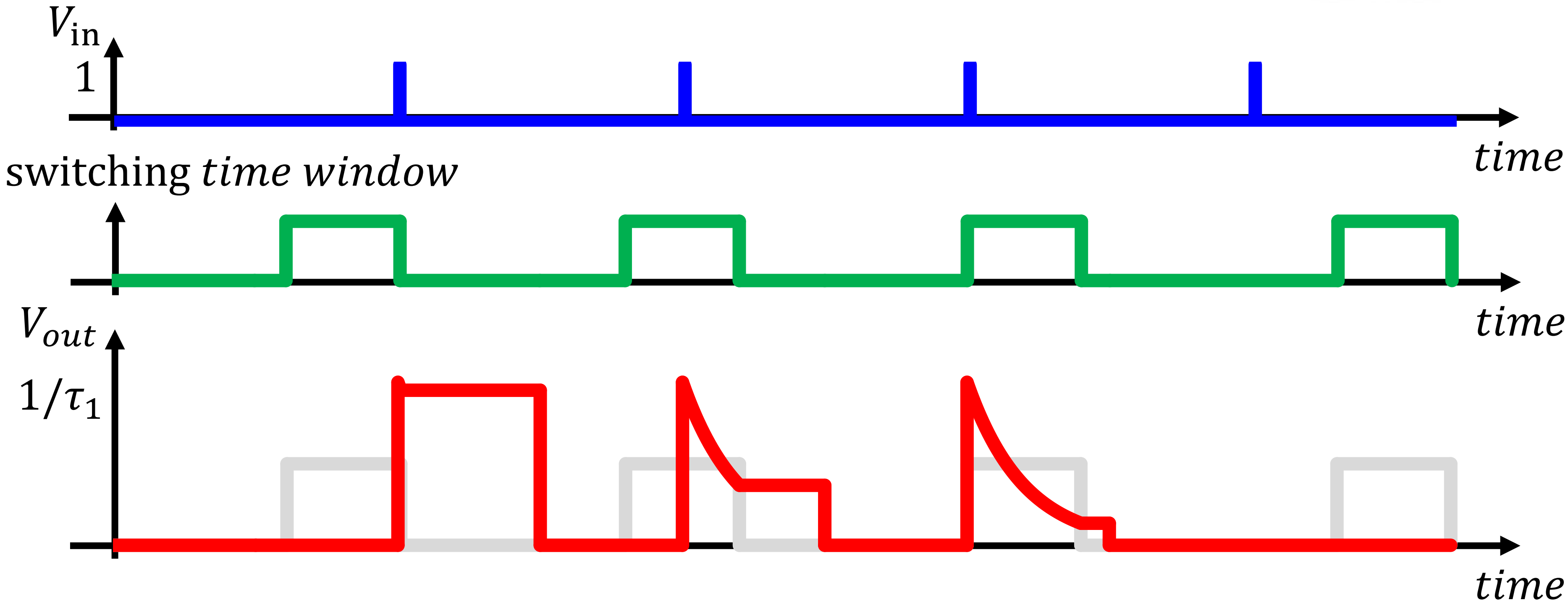
Derivation of Unified Theory (1)
~ Impulse Response by Equivalent Time Sampling ~



Derivation of Unified Theory (1)
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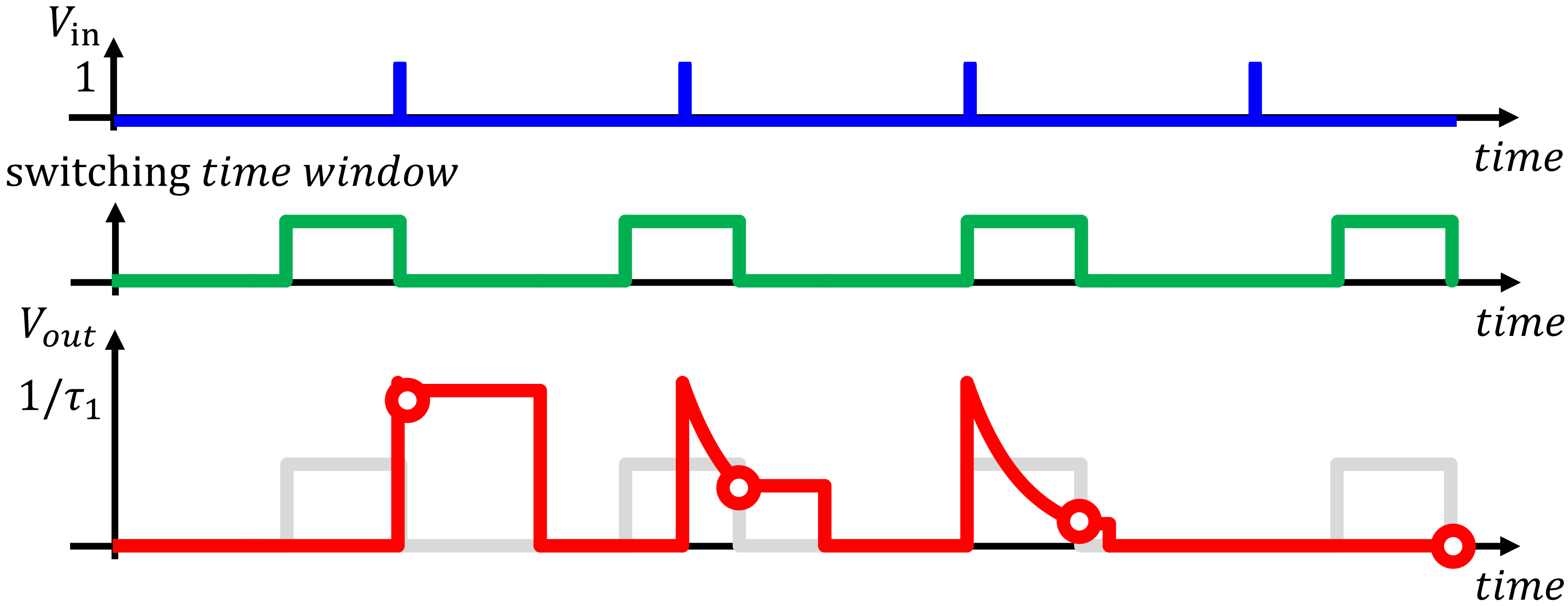
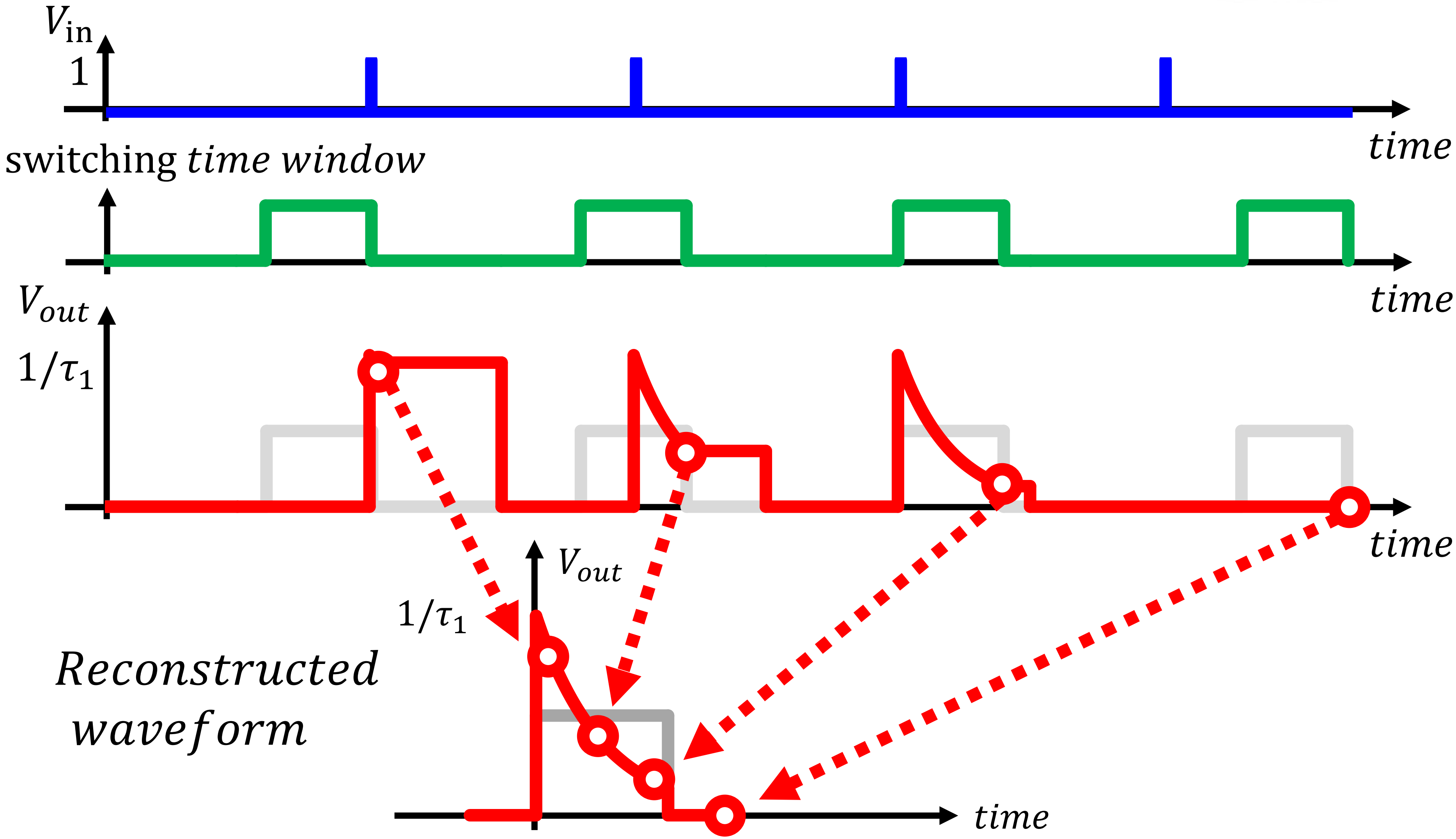
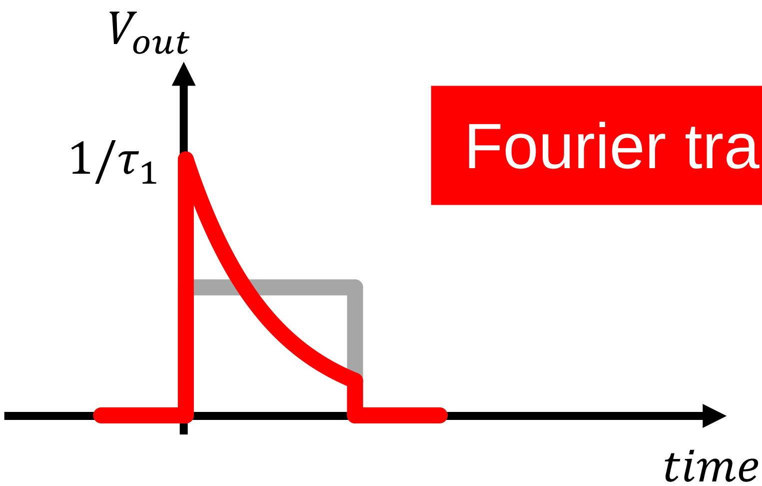


Derivation of Unified Theory (1)
~ Impulse Response by Equivalent Time Sampling ~



Derivation of Unified Theory (2)
~ Fourier Transform of Impulse Response ~



$$h(t) = \begin{cases} 0 & (t < 0) \\ 1/\tau_1 \cdot e^{-\frac{t}{\tau_1}} & (0 \le t < \tau_2) \\ 0 & (\tau_2 \le t) \end{cases}$$

Fourier transform

$$H_3(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

$$= \int_{0}^{\tau_2} \frac{1}{\tau_1} e^{-\frac{1}{\tau_1}t} e^{-j\omega t} dt$$

$$= \frac{1}{\tau_1} \int_{0}^{\tau_2} e^{-\left(\frac{1}{\tau_1}+j\omega\right)t} dt$$

$$= -\frac{1}{\tau_1} \frac{1}{\frac{1}{\tau_1}+j\omega} \left[e^{-\left(\frac{1}{\tau_1}+j\omega\right)t} \right]_0^{\tau_2}$$

$$= \frac{1}{1+j\tau_1\omega} \left\{ 1 - e^{-\frac{\tau_2}{\tau_1}(1+j\tau_1\omega)} \right\}$$

Unified transfer function

Relationship of T/H, Impulse Sampling and Unified S/H Circuits

Unified Theory

$$H_3(j\omega) = \frac{1}{1 + j\tau_1\omega}\left\{1 - e^{-(1+j\tau_1\omega)\frac{\tau_2}{\tau_1}}\right\}$$

$\tau_2 \gg \tau_1$

$$\lim_{\frac{\tau_2}{\tau_1}\to\infty} H_3(j\omega) = \frac{1}{1 + j\tau_1\omega} = H_1(j\omega)$$

(Track/Hold Circuit)

$\tau_2 \ll \tau_1$

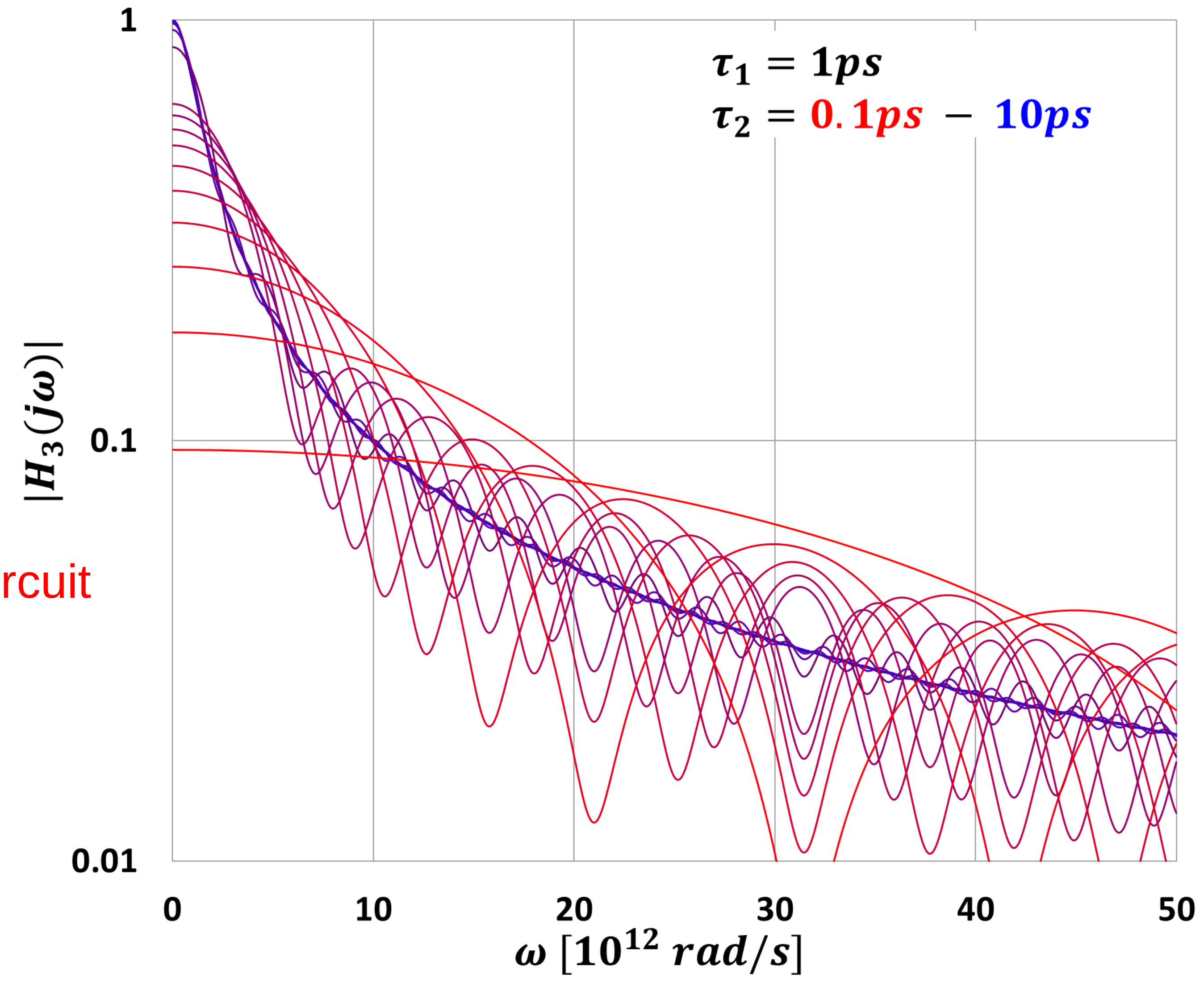
$$\lim_{\substack{\frac{\tau_2}{\tau_1}\to 0 \\ \tau_1\omega\gg 1}} H_3(j\omega) = \frac{\tau_2}{\tau_1} sinc\left(\frac{\tau_2}{2}\omega\right) e^{-j\frac{\tau_2}{2}\omega} = H_2(j\omega)$$

(Impulse Sampling Circuit)

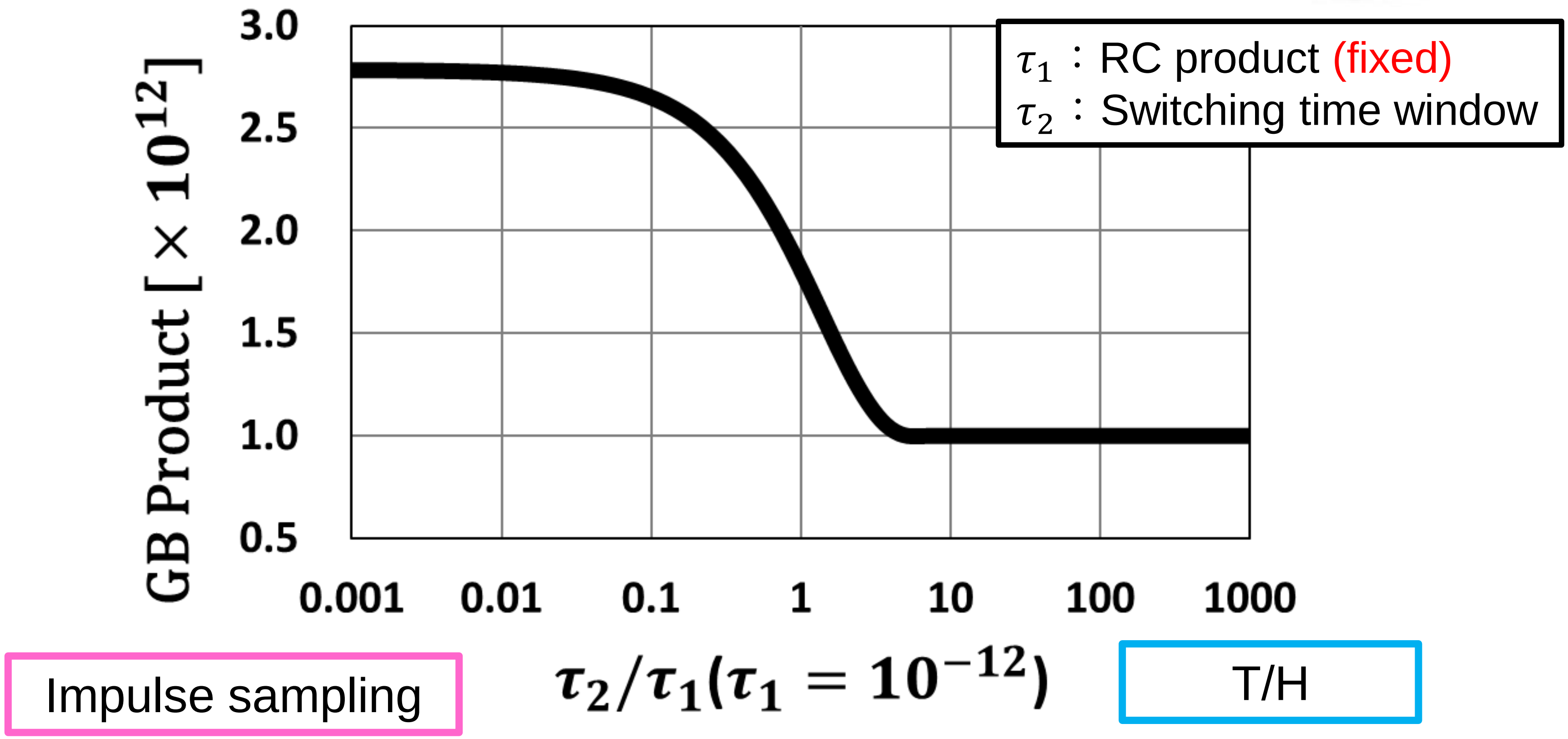
Gain Characteristics of Unified S/H Circuit

Track/Hold Circuit
$(\tau_2/\tau_1 \gg 1\ )$

Impulse Sampling Circuit
$(\tau_2/\tau_1 \ll 1\ )$



GB Product and Switching Time Window τ_2 of Unified S/H Circuit



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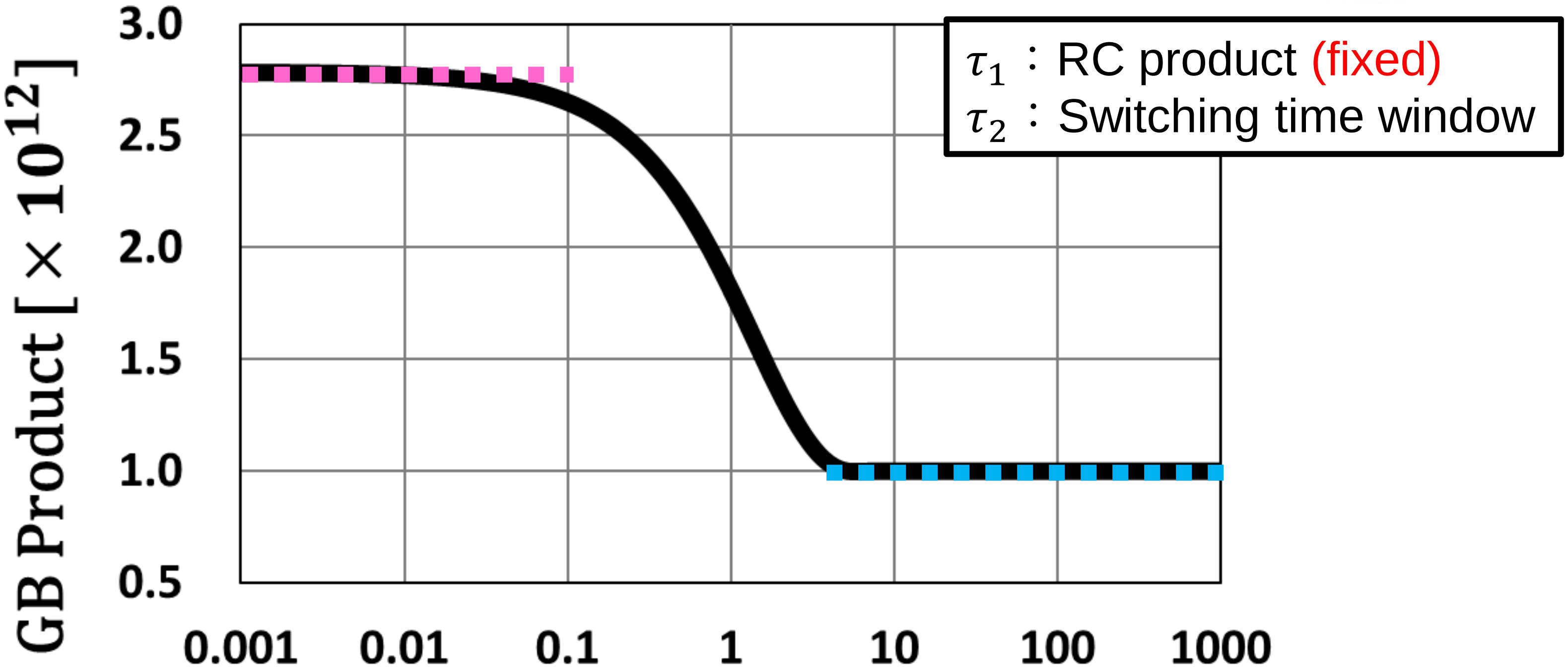


$\tau_2/\tau_1 (\tau_1 = 10^{-12})$

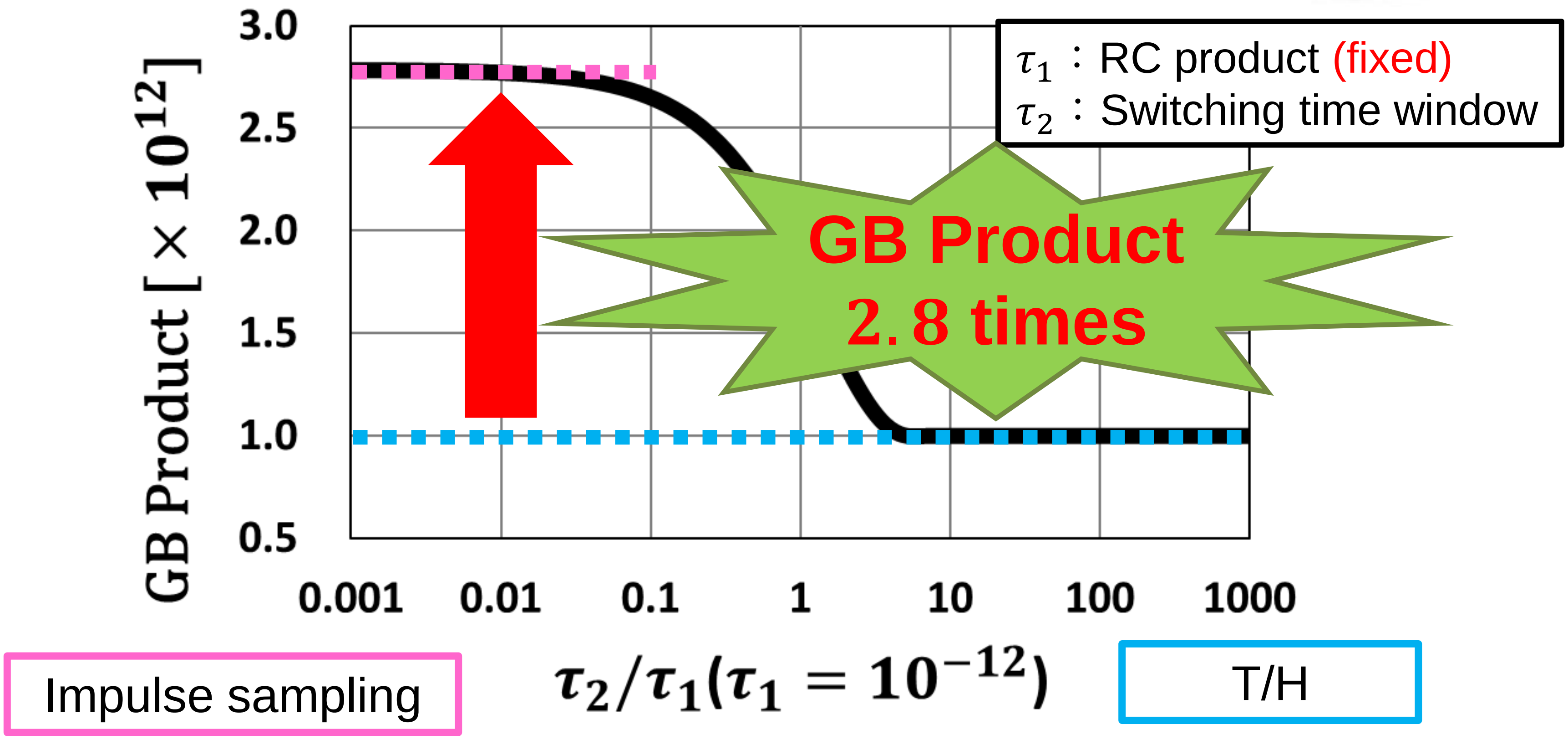
Impulse sampling

T/H

Impulse Sampling Circuit
GB $\text{Product}_2 = 2.8/\tau_1$

Track/Hold Circuit
GB $\text{Product}_1 = 1/\tau_1$

GB Product and Switching Time Window τ_2 of Unified S/H Circuit



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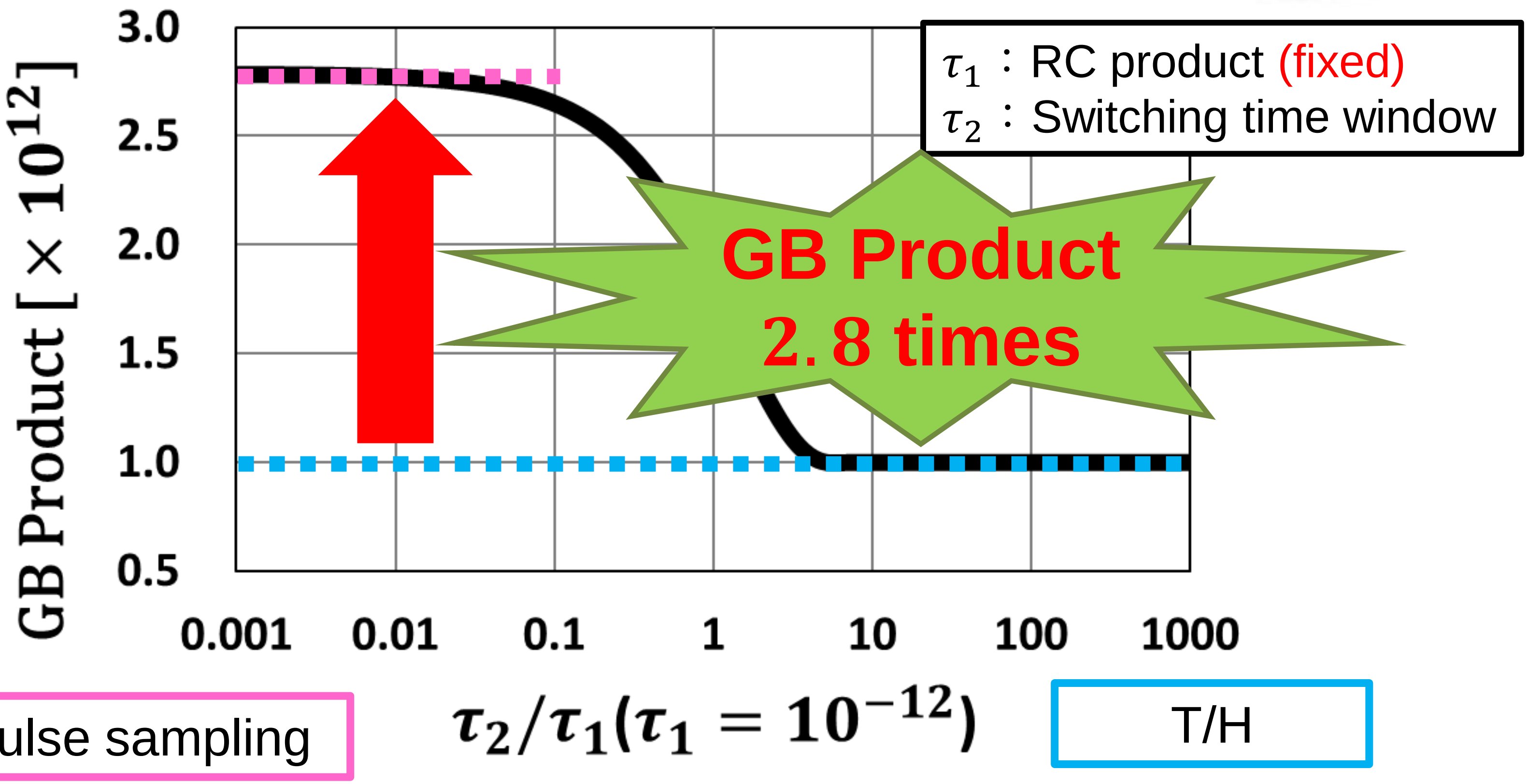
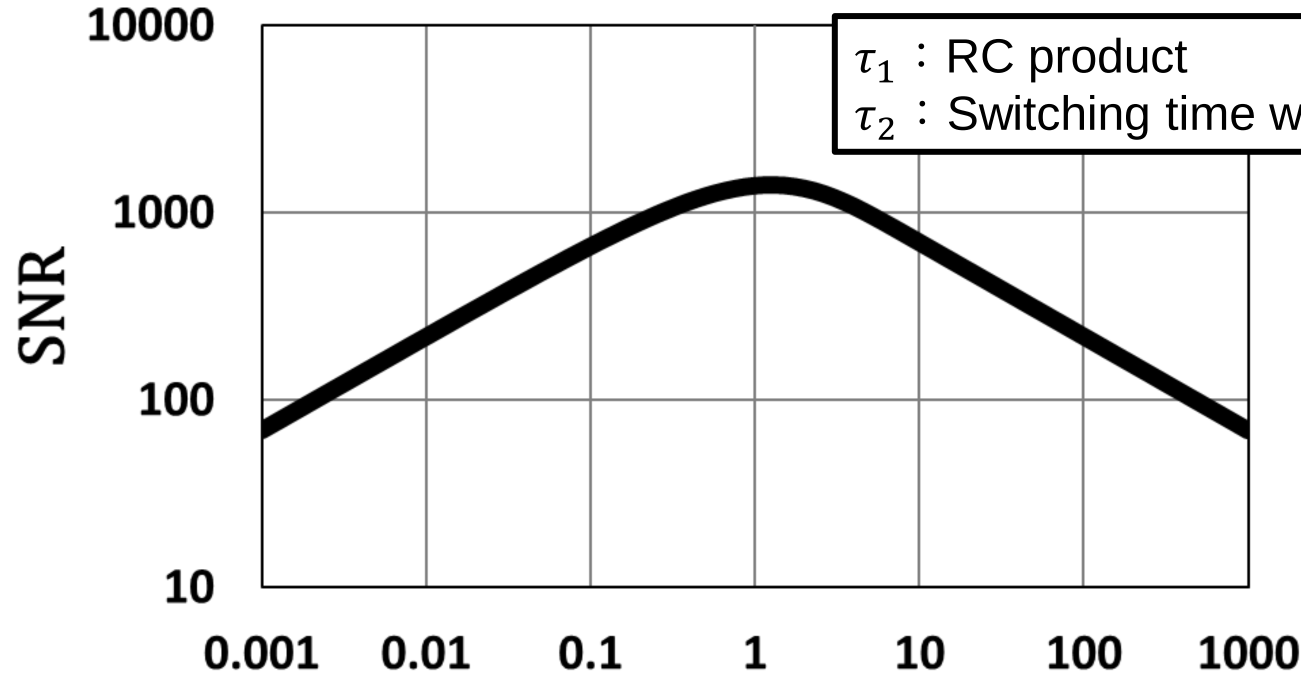


$$\frac{GB\ Product_2}{GB\ Product_1} = \frac{DC\ Gain_2 \cdot Bandwidth_2}{DC\ Gain_1 \cdot Bandwidth_1}$$

$$\approx \frac{(\tau_2/\tau_1) \cdot (2.8/\tau_2)}{(1) \cdot (1/\tau_1)} = 2.8$$

SNR and τ_1 ($\tau_2 = 10^{-12}$) of Unified S/H Circuit



τ_1 : RC product
τ_2 : Switching time window (fixed)

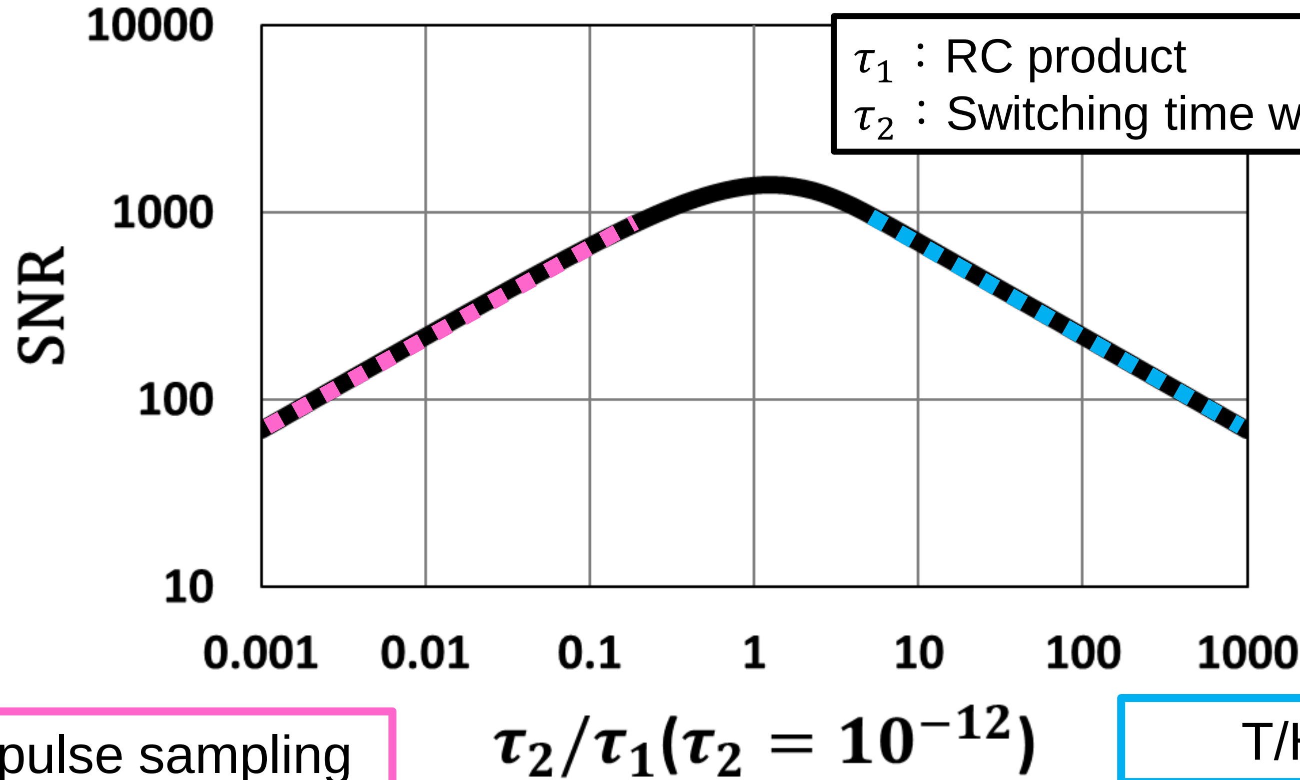
Impulse sampling

$\tau_2/\tau_1(\tau_2 = 10^{-12})$

T/H

SNR and τ_1 ($\tau_2 = 10^{-12}$) of Unified S/H Circuit

τ_1 : RC product
τ_2 : Switching time window (fixed)



Impulse sampling

T/H

Impulse Sampling Circuit
$SNR_2 \propto \tau_2/\sqrt{\tau_1}$

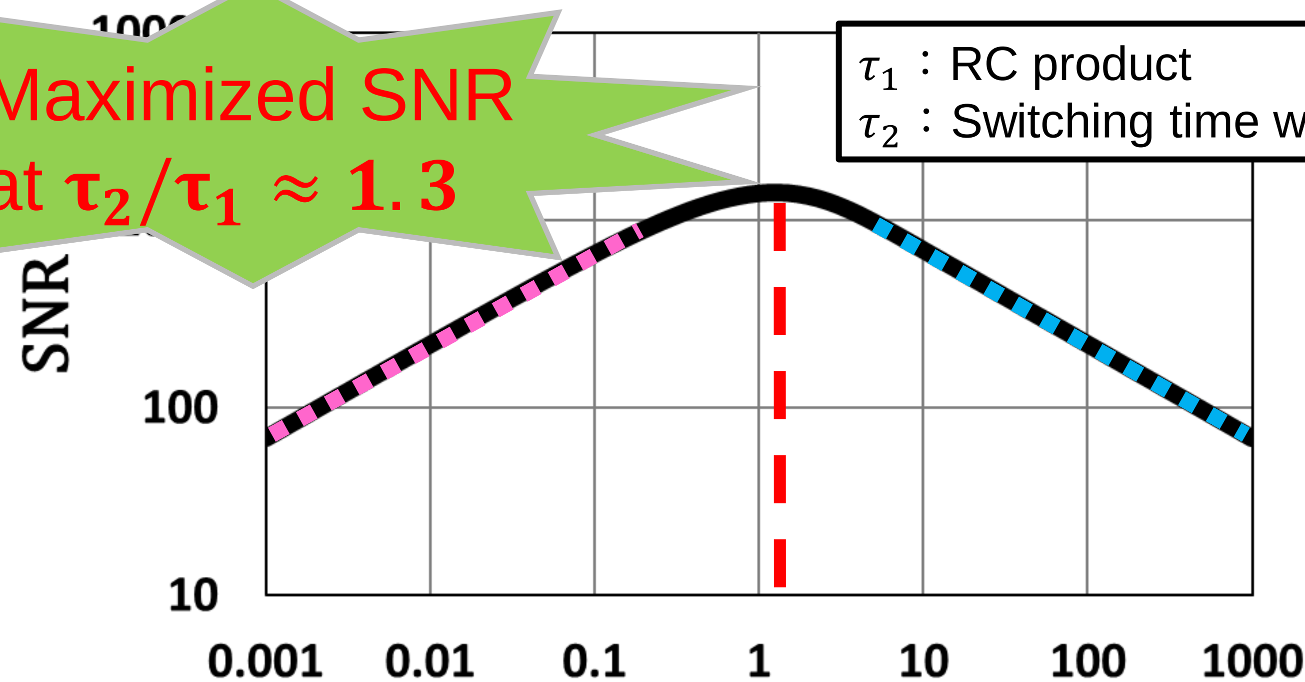
Track/Hold Circuit
$SNR_1 \propto \sqrt{\tau_1}$

VLSI SINCE 1983 (2016 VLSI-DAT)

SNR and τ_1 ($\tau_2 = 10^{-12}$) of Unified S/H Circuit

Maximized SNR at $\tau_2/\tau_1 \approx 1.3$

τ_1 : RC product
τ_2 : Switching time window (fixed)



$\tau_2/\tau_1(\tau_2 = 10^{-12})$

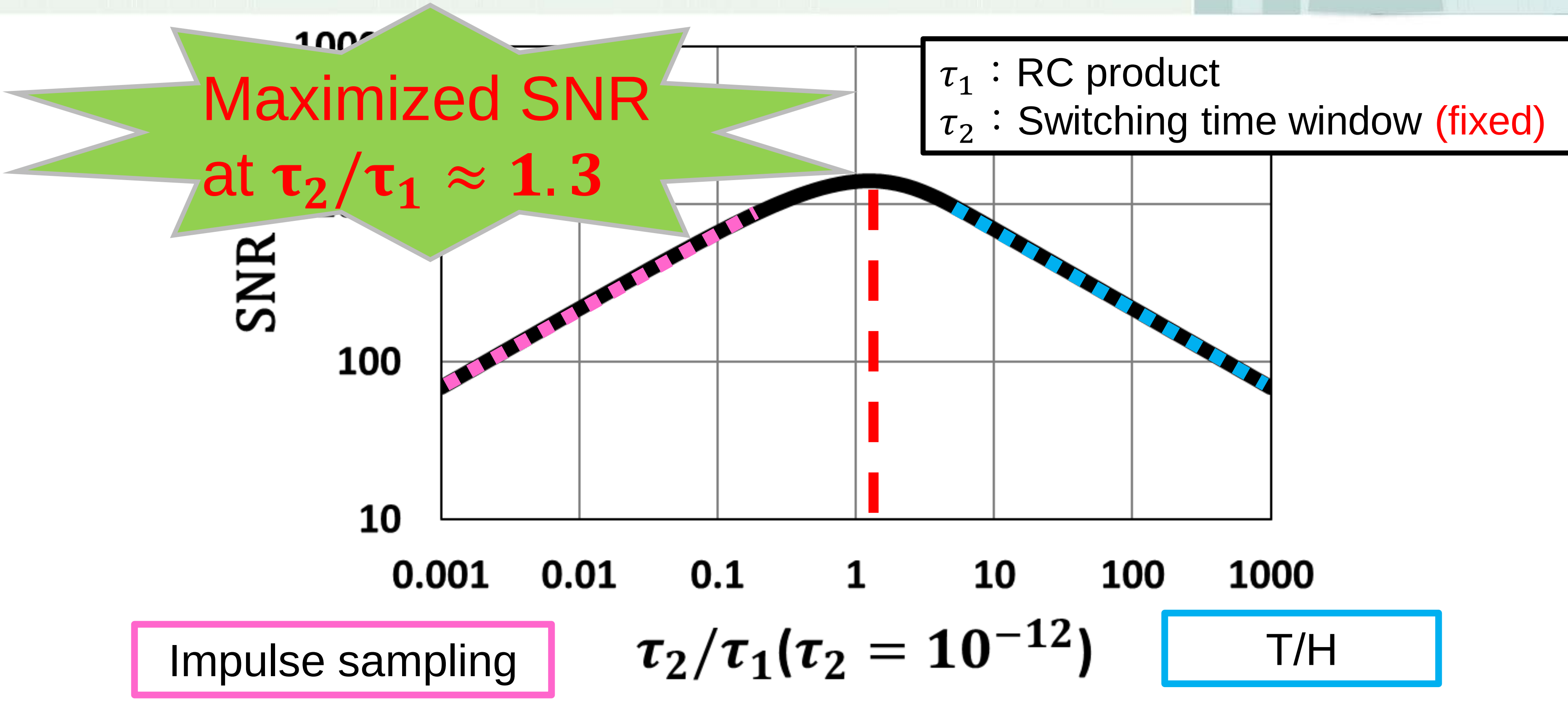
Impulse sampling

T/H

Impulse Sampling Circuit
$SNR_2 \propto \tau_2/\sqrt{\tau_1}$

Track/Hold Circuit
$SNR_1 \propto \sqrt{\tau_1}$

SNR and τ_1 ($\tau_2 = 10^{-12}$) of Unified S/H Circuit



$$SNR_3 = \sqrt{\tau_1/(k_B T R)}\left(1 - e^{-\tau_2/\tau_1}\right)$$

From, $\frac{\partial}{\partial \tau_1} SNR_3 = 0$

$$1 + 2\frac{\tau_2}{\tau_1} = e^{\frac{\tau_2}{\tau_1}}$$

$$\frac{\tau_2}{\tau_1} = 1.26$$

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Rigorous Formula Bandwidth ω_{BW} of Unified S/H Circuit

Transfer function : $H_3(j\omega) = \frac{1}{1+j\tau_1\omega}\left\{1 - e^{-\frac{\tau_2}{\tau_1}(1+j\tau_1\omega)}\right\}$

Bandwidth $\boldsymbol{\omega}_{BW}$: $|H_3(j\omega_{BW3})| = \frac{1}{\sqrt{2}}|H_3(j0)|$

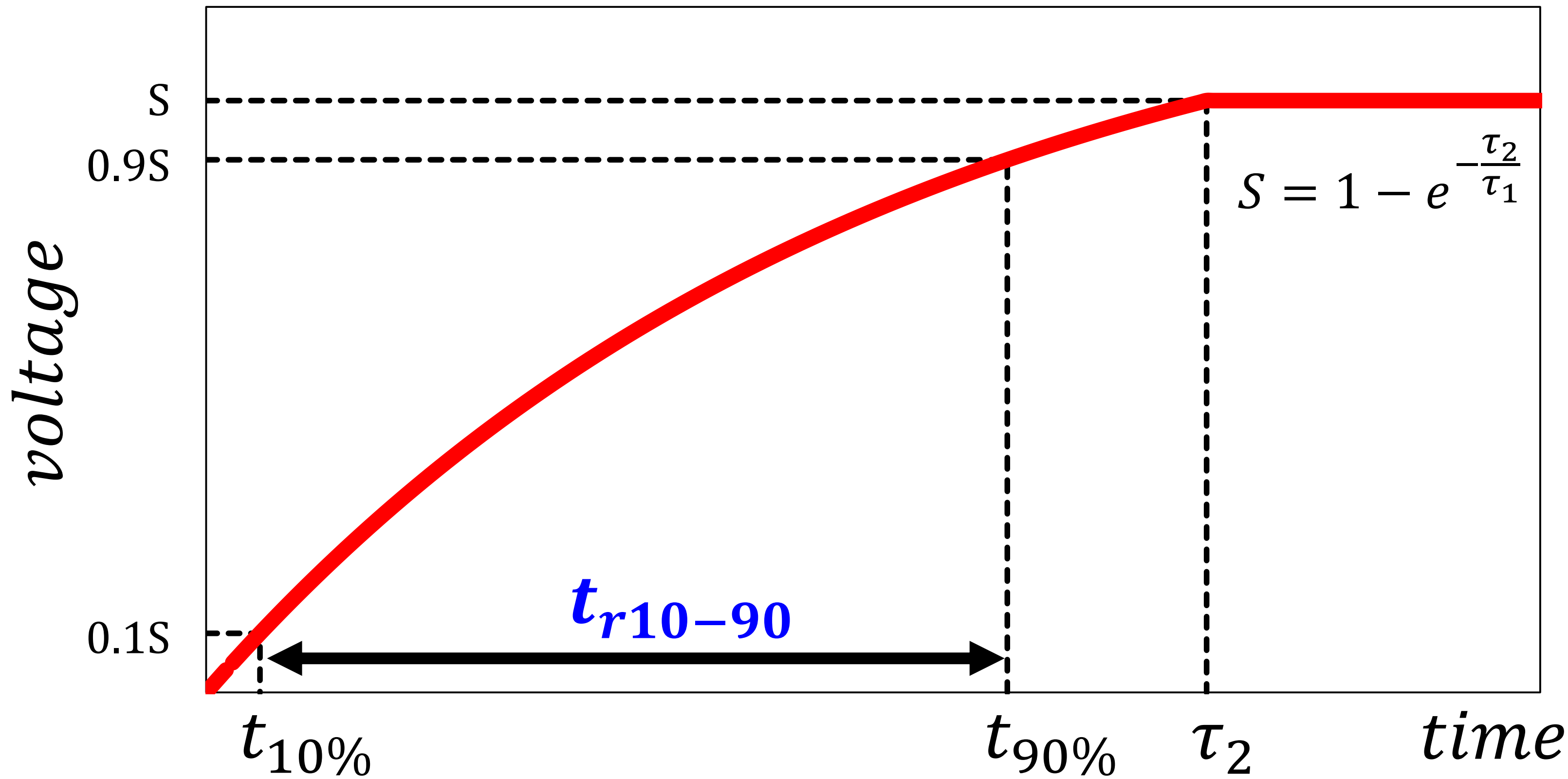
$$\frac{1}{\sqrt{1+\tau_1^2\omega^2}}\sqrt{\left(1 - e^{-\frac{\tau_2}{\tau_1}}\cos(\omega\tau_2)\right)^2 + \left(e^{-\frac{\tau_2}{\tau_1}}\sin(\omega\tau_2)\right)^2} = \frac{1}{\sqrt{2}}\left(1 - e^{-\frac{\tau_2}{\tau_1}}\right)$$

Analytical solution is difficult to obtain.

Approximation Formula Bandwidth ω_{BW} of Unified S/H Circuit

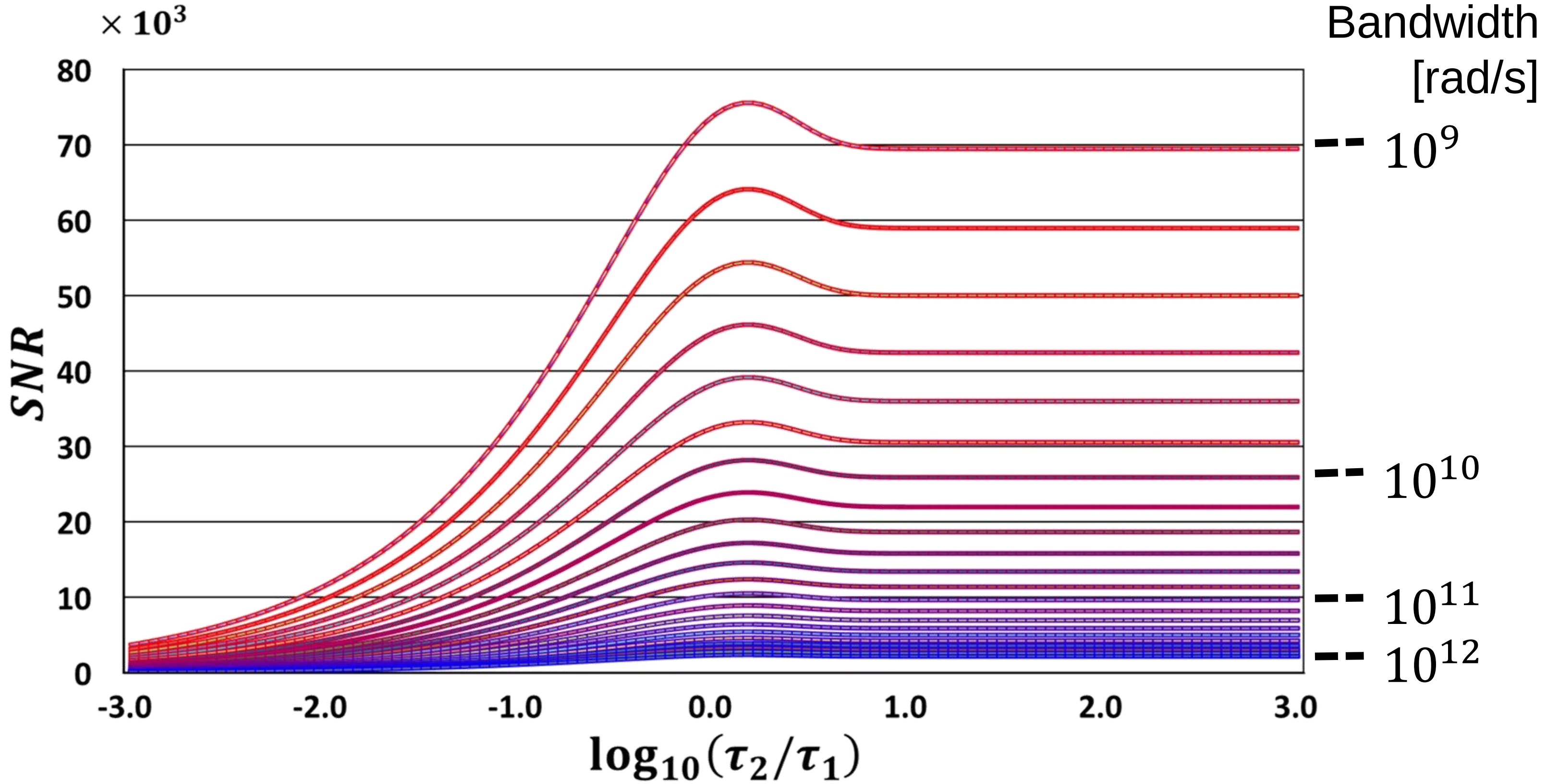
Assume first-order system,
deriving $\boldsymbol{\omega}_{BW}$ from t_{r10-90}

$$\omega_{BW3} \approx \frac{2.20}{\boldsymbol{t_{r10-90}}}$$



SNR Under Constant Bandwidth

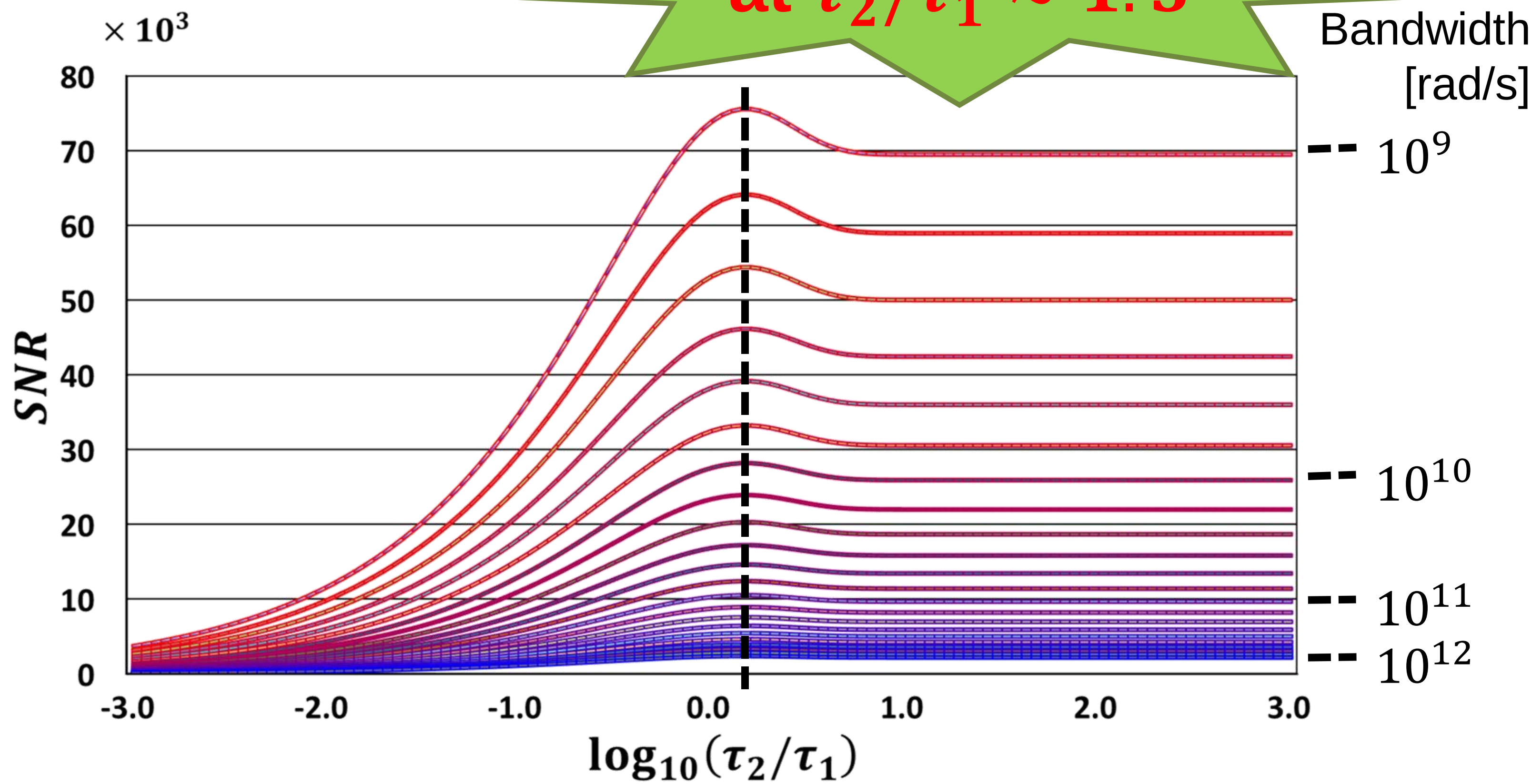
$$SNR = 10\sqrt{\frac{1}{k_B T R}}\sqrt{\tau_1}\frac{1-e^{\frac{t_{r10-90}}{\tau_1}}}{1-9e^{\frac{t_{r10-90}}{\tau_1}}}$$



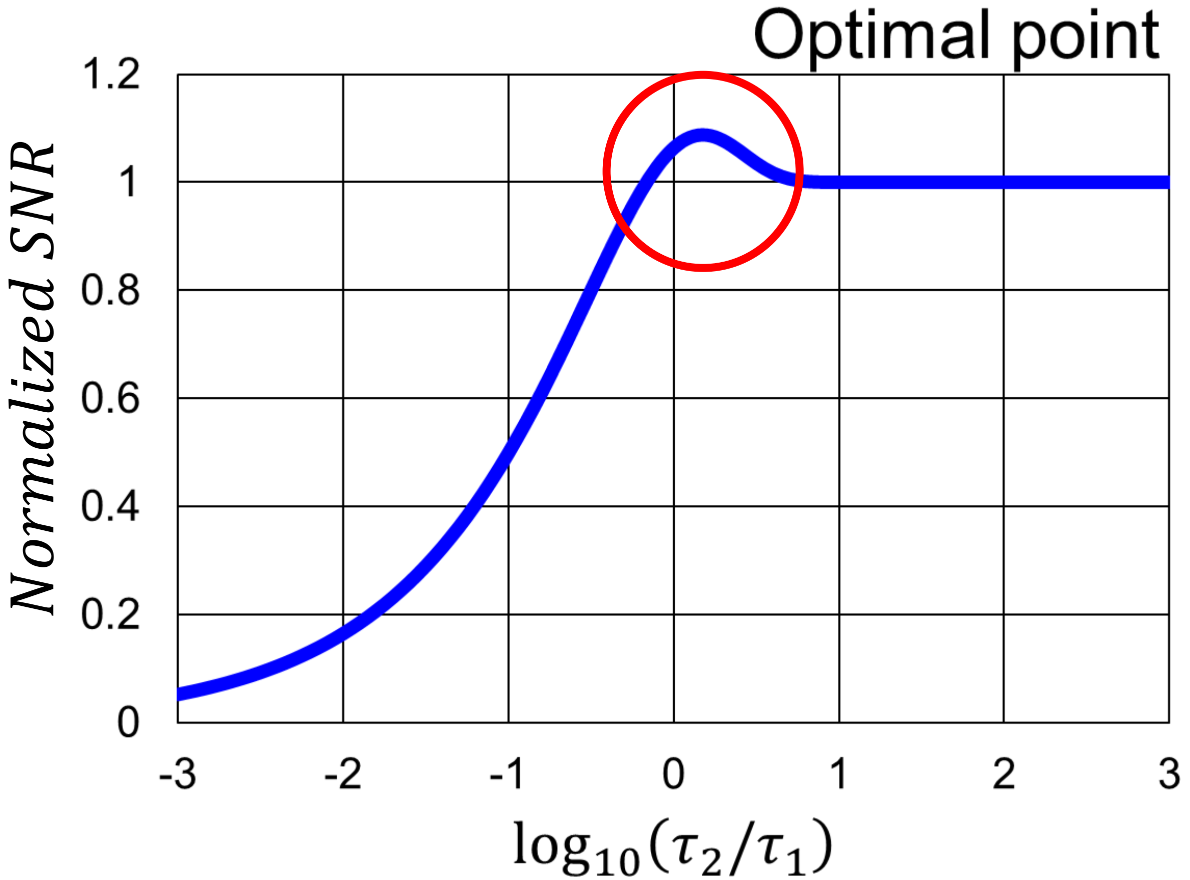
SNR Under Constant Bandwidth

$$SNR = 10\sqrt{\frac{1}{k_B T R}}\sqrt{\tau_1}\,\frac{1 - e^{-\frac{t_{r10-90}}{\tau_1}}}{1 - 9e \cdots}$$

Maximum SNR at $\tau_2/\tau_1 \approx 1.5$



SNR vs τ_2/τ_1 of Unified S/H Circuit



$$SNR_{max} = 1.1 \times SNR_{T/H}$$

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Conclusion

■Two S/H Circuits
- Track/Hold Circuit $(\tau_2 \ll \tau_1)$
- Impulse Sampling Circuit $(\tau_2 \gg \tau_1)$

↓

Bandwidth, SNR

⇒ Trade-off

⇒ Theoretical limitation

■Unified S/H Circuit Theory

↓

- GB Product: Impulse mode is 2.8 times larger than T/H mode
- Maximum SNR condition:

$\tau_2/\tau_1 \approx 1.3$ Under Constant Switching Time Window

$\tau_2/\tau_1 \approx 1.5$ Under Constant Bandwidth

Final Statement

学而不思則罔

Deep consideration
would advance modern technology.

Kobayashi
Laboratory

Appendix

Characteristics of S/H Circuits

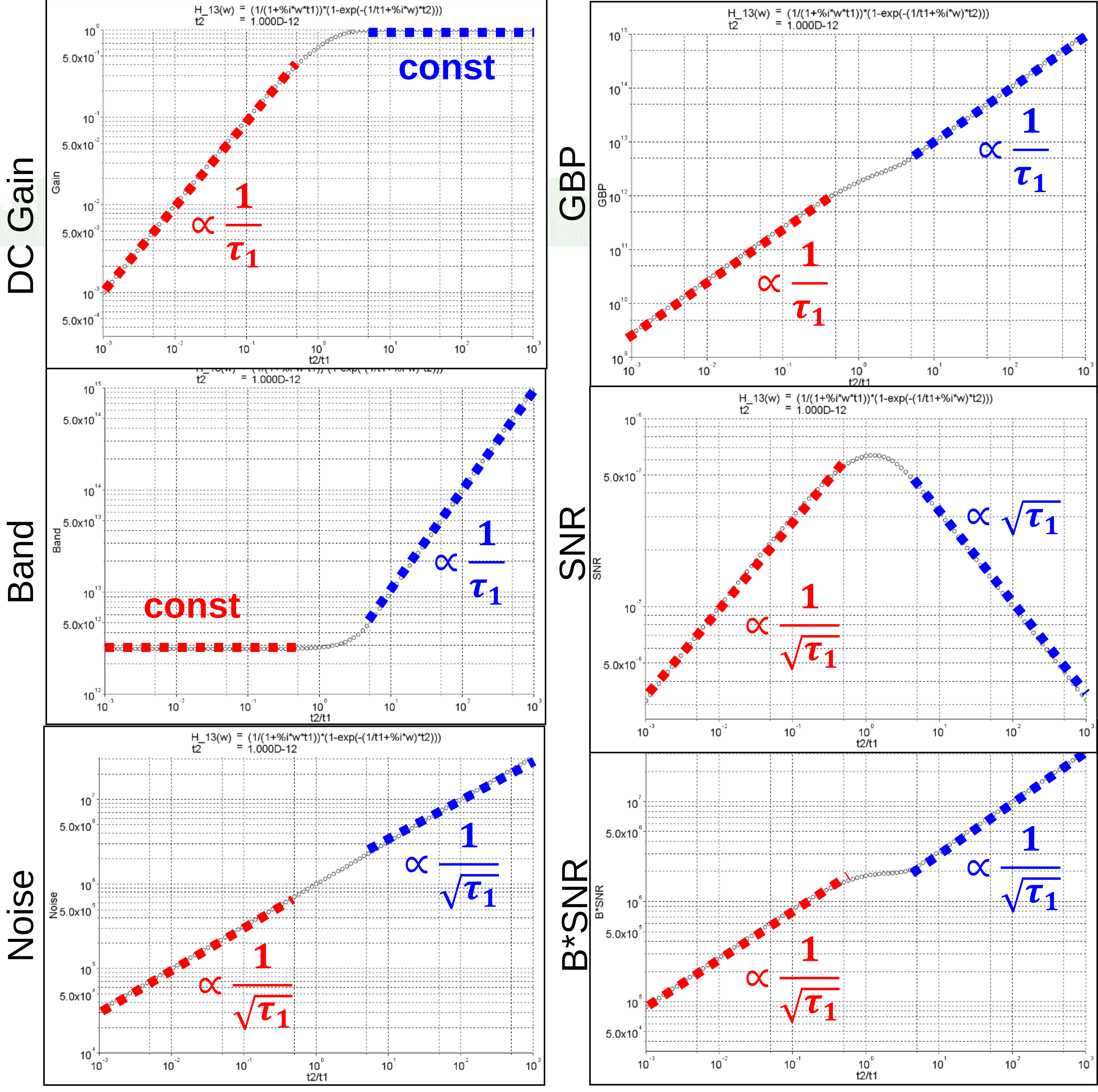
	T/H Circuit	Impulse Sampling Circuit
Transfer Function	$\mathrm{H}_1(j\omega) = \frac{1}{1 + j\tau_1\omega}$	$\mathrm{H}_2(j\omega) = \frac{\tau_2}{\tau_1} sinc\left(\frac{\tau_2}{2}\omega\right) e^{-j\frac{\tau_2}{2}\omega}$
DC Gain	$V_{signal1} = \mathrm{H}_1(0) = 1$	$V_{signal2} = \mathrm{H}_2(0) = \frac{\tau_2}{\tau_1}$
Bandwidth	$\omega_{BW1} = \frac{1}{\tau_1}$	$\omega_{BW2} \approx \frac{2.78}{\tau_2}$
Thermal Noise	$V_{noise} = \sqrt{k_B TR / \tau_1}$	$V_{noise} = \sqrt{k_B TR / \tau_1}$
GB Product	$GBP_1 = \frac{1}{\tau_1}$	$GBP_2 \approx \frac{2.78}{\tau_1}$
SNR	$SNR_1 = \frac{\sqrt{\tau_1}}{\sqrt{k_B TR}} \propto \sqrt{\tau_1}$	$SNR_2 = \frac{1}{\sqrt{k_B TR}} \cdot \frac{\tau_2}{\sqrt{\tau_1}} \propto \frac{\tau_2}{\sqrt{\tau_1}}$

Characteristics of S/H Circuits

τ_1:varied

τ_2:fixed

$(\tau_2 = 10^{-12})$



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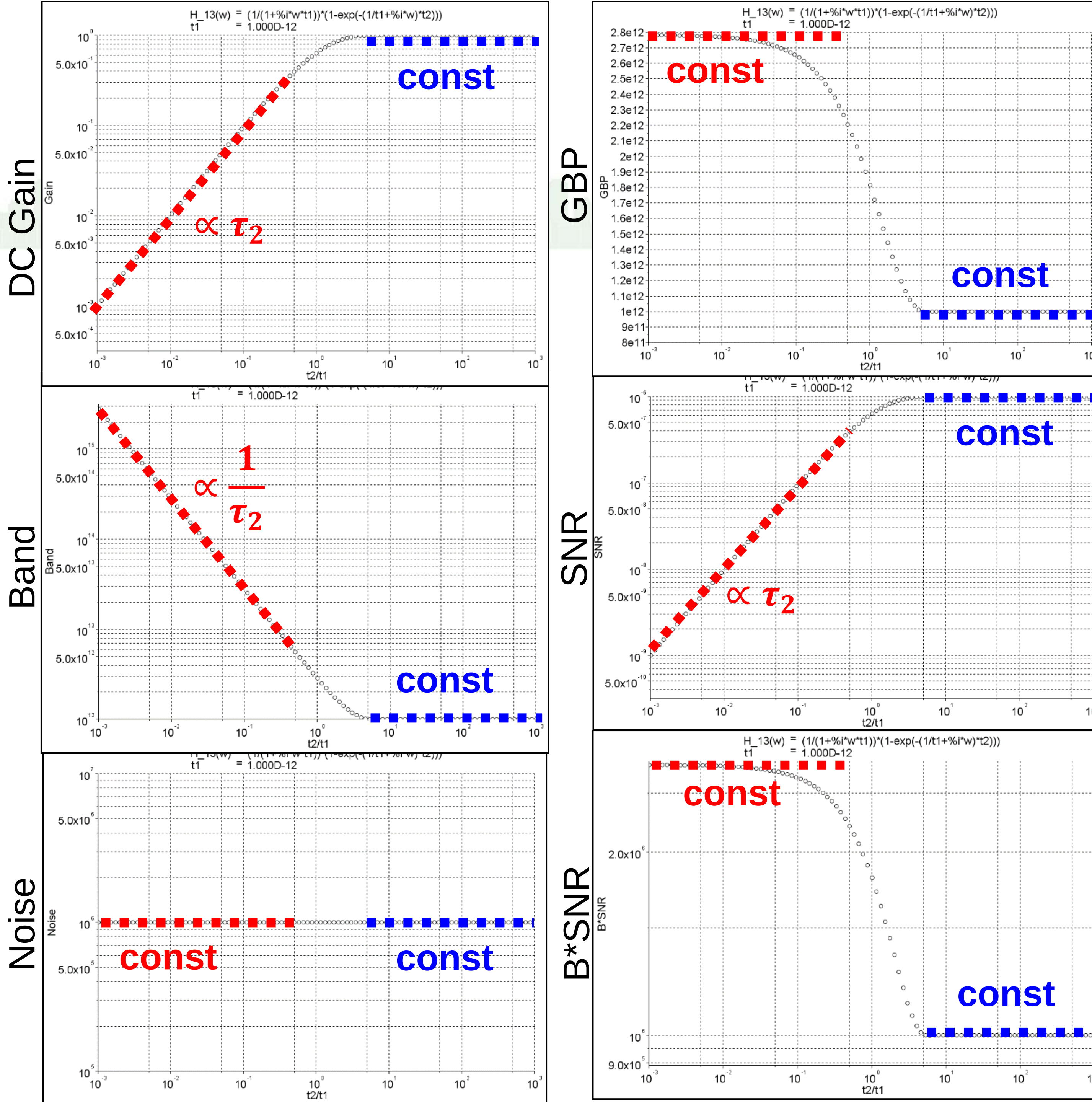

Derivation of the transfer function of the impulse sampling circuit

$$H_2(j\omega) = \int_0^{\infty} V_{out} e^{-j\omega t} dt$$

$$= \int_0^{\tau_2} \frac{1}{\tau_1} e^{-j\omega t} dt$$

$$= \frac{1}{\tau_1} \frac{1}{j\omega} \left(1 - e^{-j\omega\tau_2}\right)$$

$$= \frac{1}{\tau_1} \frac{1}{j\omega} \left(e^{j\frac{\omega\tau_2}{2}} - e^{-j\frac{\omega\tau_2}{2}}\right) e^{-j\frac{\omega\tau_2}{2}}$$

$$= \frac{\tau_2}{\tau_1} sinc\left(\frac{\tau_2}{2}\omega\right) e^{-j\frac{\tau_2}{2}\omega}$$

$$\lim_{\tau_2/\tau_1 \to 0}\left\{1 - e^{-(1+j\tau_1\omega)\frac{\tau_2}{\tau_1}}\right\} = 0 \ ?$$

$$\lim_{\substack{\frac{\tau_2}{\tau_1}\to 0 \\ \tau_1\omega \gg 1}} \mathrm{H}_3(\mathrm{j}\omega)$$

$$= \lim_{\substack{\frac{\tau_2}{\tau_1}\to 0 \\ \tau_1\omega \gg 1}} \frac{1}{1+\mathrm{j}\tau_1\omega}\left\{1 - e^{-(1+j\tau_1\omega)\frac{\tau_2}{\tau_1}}\right\}$$

$$= \frac{1}{\mathrm{j}\tau_1\omega}\left\{1 - e^{-j\tau_2\omega}\right\}$$

$$= \frac{1}{\tau_1}\frac{1}{j\omega}\left(e^{j\frac{\omega\tau_2}{2}} - e^{-j\frac{\omega\tau_2}{2}}\right)e^{-j\frac{\omega\tau_2}{2}}$$

$$= \frac{\tau_2}{\tau_1} sinc\left(\frac{\tau_2}{2}\omega\right)e^{-j\frac{\tau_2}{2}\omega}$$

$$= \mathrm{H}_2(\mathrm{j}\omega)$$

$$\lim_{\tau_2/\tau_1 \to 0}\left\{1 - e^{-(1+j\tau_1\omega)\frac{\tau_2}{\tau_1}}\right\}$$

$$= \lim_{\tau_2/\tau_1 \to 0}\left\{1 - e^{-\frac{\tau_2}{\tau_1}}e^{-j\tau_2\omega}\right\}$$

$$= \left\{1 - e^{-j\tau_2\omega}\right\}$$