Equivalence between Nyquist and Routh-Hurwitz Stability Criteria for Operational Amplifier Design

〇王建龍

Gopal Adhikari　築地伸和　平野繭　小林春夫　栗原圭汰

群馬大学大学院　理工学府電子情報部門

長浜顕仁　野田一平　吉井宏治

リコー電子デバイス(株)

群馬大学
GUNMA UNIVERSITY

Contents

Contents

Research Background (Stability Theory)

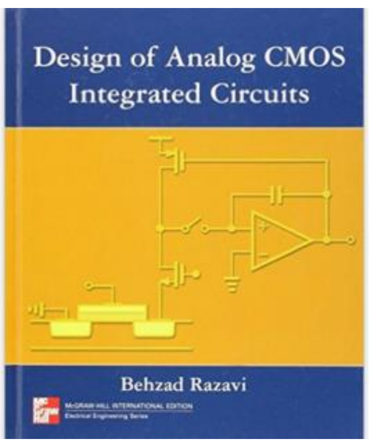
- **Electronic Circuit Design Field**
  - Bode plot (>90% frequently used)
  - Nyquist plot

- **Control Theory Field**
  - Bode plot
  - Nyquist plot
  - Nicholas plot
  - Routh-Hurwitz stability criterion
    ➡ Very popular in control theory field
    but rarely seen in electronic circuit books/papers
  - Lyapunov function method

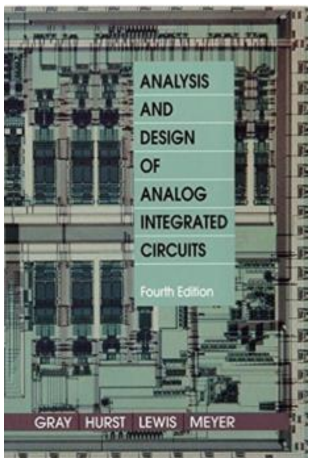
:

Electronic Circuit Text Book

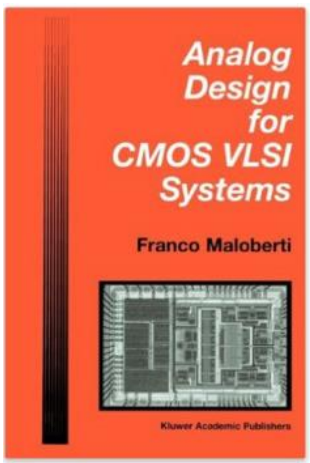
We were NOT able to find out any electronic circuit text book which describes Routh-Hurwitz method
for operational amplifier stability analysis and design !



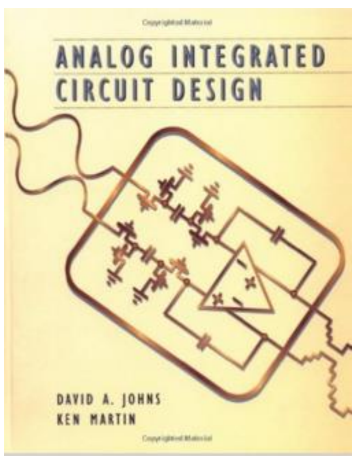
Razavi



Gray



Maloberti

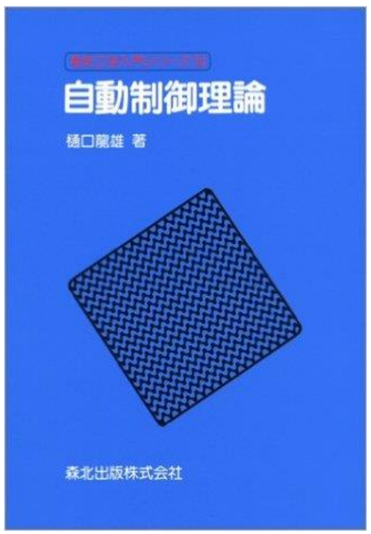


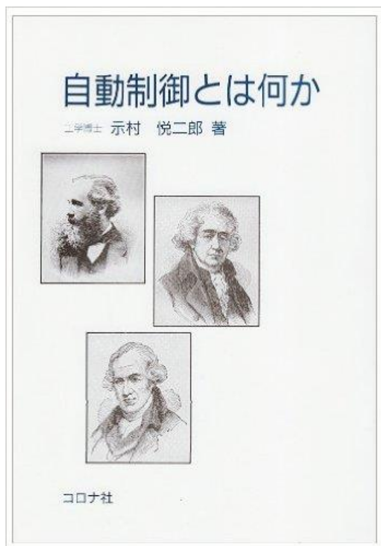
Martin

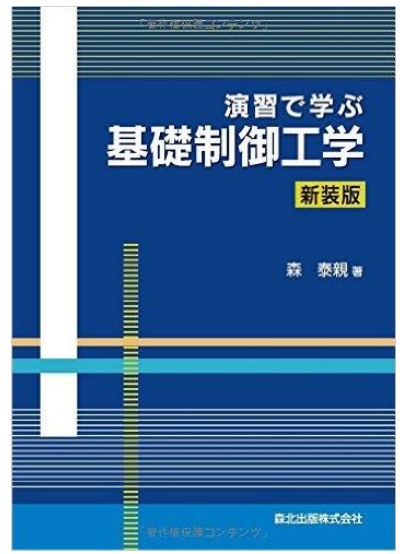
None of the above describes Routh-Hurwitz.
Only Bode plot is used.

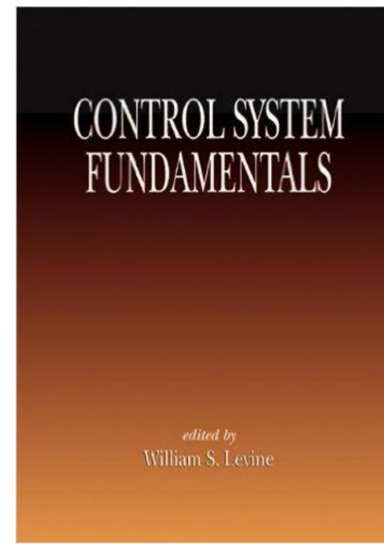
Control Theory Text Book

Most of control theory text books
describe Routh-Hurwitz method
for system stability analysis and design !









Research Objective

Our proposal

For

Analysis and design of operational amplifier stability

Use

Routh-Hurwitz stability criterion

We can obtain

Explicit stability condition for circuit parameters

(which can NOT be obtained only with Bode plot).

We can verify

Equivalence between Nyquist and Routh-Hurwitz Stability Criteria

Contents

- Research Objective & Background

- Stability Criteria
  - Nyquist Criterion
    - Nyquist plot
    - Bode plot
  - Routh-Hurwitz Criterion

- Equivalence at Mathematical Foundations

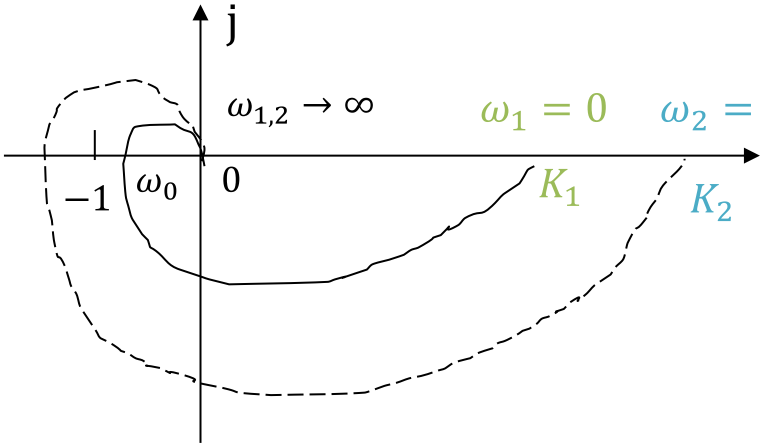
- Simulation Verification

- Discussion & Conclusion

Nyquist plot

- Open-loop frequency characteristic

  ⇨ Closed-loop stability



Nyquist plot of open-loop system

- Necessary and sufficient condition :

$$\text{When } \omega = 0 \to \infty, \qquad N = \frac{P}{2}$$

N : number, Nyquist plot anti-clockwise encircle point (-1,j0).

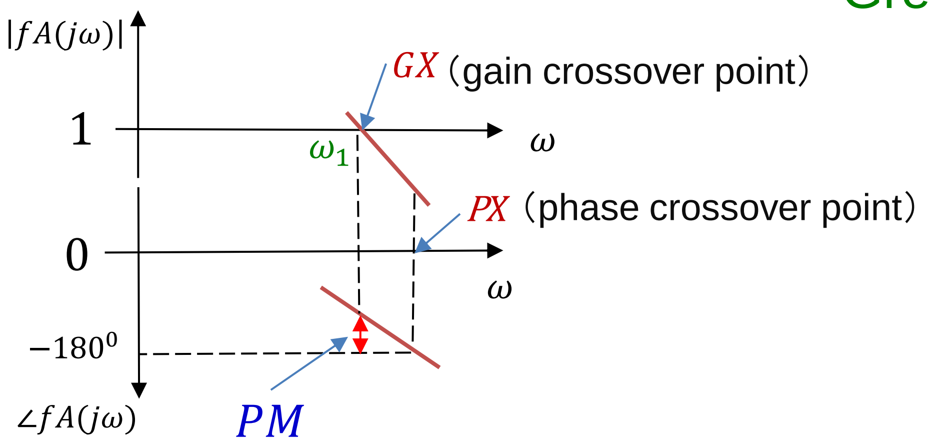
P: number, positive roots of open-loop characteristic equation.

- If the open-loop system is stable(P=0),
  the Nyquist plot mustn't encircle the point (-1,j0).

$$\angle G_{open}(j\omega_0) = -\pi, \left|G_{open}(j\omega_0)\right| < 1$$

Phase Margin from Bode Plot

GX precedes PX ⇨ Feedback system is stable



Greater spacing between GX and PX

⇩

More stable

ω_1: gain crossover frequency

Phase margin : $\mathrm{PM} = 180^0 + \angle fA(\omega = \omega_1)$

Bode plot is useful,
but it does NOT show explicit stability conditions of circuit parameters.

Contents

Routh Stability Criterion

Characteristic equation:

$$D(s) = \alpha_n s^n + \alpha_{n-1} s^{n-1} + \cdots + \alpha_1 s + \alpha_0 = 0$$

Routh table

s^n	α_n	α_{n-2}	α_{n-4}	α_{n-6}	$\cdots$
s^{n-1}	α_{n-1}	α_{n-3}	α_{n-5}	α_{n-7}	$\cdots$
s^{n-2}	$\beta_1 = \frac{\alpha_{n-1}\alpha_{n-2} - \alpha_n\alpha_{n-3}}{\alpha_{n-1}}$	$\beta_2 = \frac{\alpha_{n-1}\alpha_{n-4} - \alpha_n\alpha_{n-5}}{\alpha_{n-1}}$	β_3	β_4	$\cdots$
s^{n-3}	$\gamma_1 = \frac{\beta_1\alpha_{n-3} - \alpha_{n-1}\beta_2}{\beta_1}$	$\gamma_2 = \frac{\beta_1\alpha_{n-5} - \alpha_{n-1}\beta_3}{\beta_1}$	γ_3	γ_4	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s^0	α_0				

Sufficient and necessary condition:

(i) $\alpha_i > 0$ for $i = 0,1,\ldots,n$

&

(ii) All values of Routh table's first columns are positive.

Mathematical test

Determine whether given polynomial has all roots in the left-half plane.

Contents

- Research Objective & Background
- Stability Criteria
  - Nyquist Criterion
  - Routh-Hurwitz Criterion
- Equivalence at Mathematical Foundations
- Simulation Verification
- Discussion & Conclusion

Four Examples

Ex.1 $G(s) = \dfrac{K}{1 + a_1 s + a_2 s^2 + a_3 s^3}$ Zero Zero, Three poles

Ex.2 $G(s) = \dfrac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2}$ One Zero, Two Poles

Ex.3 $G(s) = \dfrac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$ One Zero, Three Poles

Ex.4 $G(s) = \dfrac{K(1 + b_1 s + b_2 s^2)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$ Two Zeros, Three Poles

Four Examples

Ex.1 $$G(s) = \frac{K}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ Zero Zero, Three poles

Ex.2 $$G(s) = \frac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2}$$ One Zero, Two Poles

Ex.3 $$G(s) = \frac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ One Zero, Three Poles

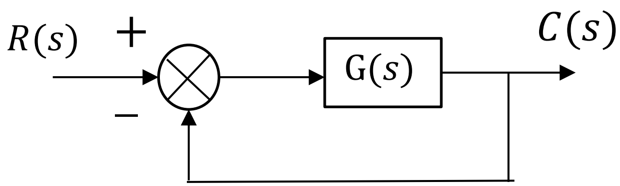
Ex.4 $$G(s) = \frac{K(1 + b_1 s + b_2 s^2)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ Two Zeros, Three Poles

Based on Routh-Hurwitz Criterion

Example 1

Open-loop transfer function:

$$G(s) = \frac{K}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$



Closed-loop transfer function:

$$H(s) = \frac{G(s)}{1 + G(s)} = \frac{K}{1 + K + a_1 s + a_2 s^2 + a_3 s^3}$$

Routh table

S^3	a_3	a_1
S^2	a_2	$1 + K$
S^1	$\frac{a_1 a_2 - a_3(1+K)}{a_2}$	
S^0	$1 + K$	

Based on Routh-Hurwitz criterion:

$$a_3 > 0, a_2 > 0, 1 + K > 0,$$

$$\frac{a_1 a_2 - a_3(1+K)}{a_2} > 0$$

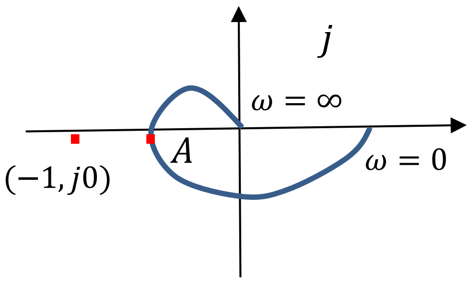
$$\Rightarrow \quad a_1 a_2 - a_3 > K a_3$$

Based on Nyquist Criterion

Frequency domain:

$$G(j\omega) = \frac{K}{1 - a_2\omega^2 + j(a_1\omega - a_3\omega^3)} = \frac{K[(1 - a_2\omega^2) - j(a_1\omega - a_3\omega^3)]}{(1 - a_2\omega^2)^2 + (a_1\omega - a_3\omega^3)^2}$$

Special frequency expressions



sketch chart of Nyquist plot

$$\angle G(j\omega) = -\pi$$

$$\Rightarrow (a_1\omega - a_3\omega^3) = 0$$

$$\Rightarrow \omega^2 = \frac{a_1}{a_3}$$ At point A

$$\Rightarrow |G(j\omega)| = \left|\frac{K\sqrt{(1 - a_2\omega^2)^2 + (a_1\omega - a_3\omega^3)^2}}{(1 - a_2\omega^2)^2 + (a_1\omega - a_3\omega^3)^2}\right| = \frac{K}{\left|1 - a_2\frac{a_1}{a_3}\right|}$$

Stability condition:

$$|G(j\omega)| < 1 \Rightarrow \begin{cases} a_1a_2 - a_3 < Ka_3 < a_3 - a_1a_2 & \text{At condition: } a_3 - a_1a_2 > 0 \\ a_3 - a_1a_2 < Ka_3 < a_1a_2 - a_3 & \text{At condition: } a_3 - a_1a_2 < 0 \end{cases}$$

Four Examples

Ex.1 $$G(s) = \frac{K}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ Zero Zero, Three poles

Ex.2 $$G(s) = \frac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2}$$ One Zero, Two Poles

Ex.3 $$G(s) = \frac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ One Zero, Three Poles

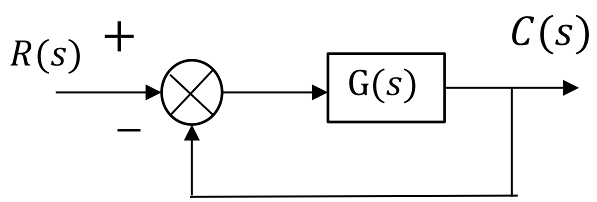
Ex.4 $$G(s) = \frac{K(1 + b_1 s + b_2 s^2)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ Two Zeros, Three Poles

Based on Routh-Hurwitz Criterion

Example 2

Open-loop transfer function:

$$G(s) = \frac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2}$$



Closed-loop transfer function:

$$H(s) = \frac{G(s)}{1 + G(s)} = \frac{K + Kb_1 s}{1 + K + (a_1 + Kb_1)s + a_2 s^2}$$

Routh table

S^2	a_2	$1 + K$
S^1	$a_1 + Kb_1$	0
S^0	$1 + K$	

Based on Routh-Hurwitz criterion:

$$a_2 > 0,\ 1 + K > 0$$

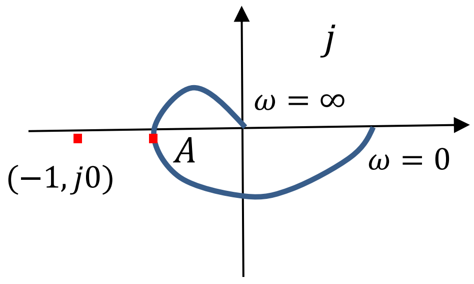
$$a_1 + Kb_1 > 0$$

$$\Rightarrow \begin{cases} K > -\dfrac{a_1}{b_1} & \text{At condition: } b_1 > 0 \\ K < -\dfrac{a_1}{b_1} & \text{At condition: } b_1 < 0 \end{cases}$$

Based on Nyquist Criterion

Frequency domain:

$$G(j\omega)=\frac{K(1+b_1(j\omega))}{1+a_1(j\omega)+a_2(j\omega)^2}=\frac{K\left(1-a_2\omega^2+b_1a_1\omega^2\right)+jK(b_1\omega-a_1\omega-a_2b_1\omega^3)}{(1-a_2\omega^2)^2+a_1^2\omega^2}$$



sketch chart of Nyquist plot

Special frequency expressions

$$\angle G(j\omega)=-\pi$$

$$\Rightarrow\quad b_1\omega-a_1\omega-a_2b_1\omega^3=0$$

$$\Rightarrow\quad \omega^2=\frac{1}{a_2}\left(1-\frac{a_1}{b_1}\right)$$ At point A

$$\Rightarrow\quad |G(j\omega)|=\left|\frac{K1-a_2\omega^2+b_1a_1\omega^2}{(1-a_2\omega^2)^2+a_1^2\omega^2}\right|=\frac{K\left|\frac{a_1}{b_1}+\frac{a_1}{a_2}(b_1-a_1)\right|}{\left|(\frac{a_1}{b_1})^2+\frac{a_1}{a_2}\frac{a_1}{b_1}(b_1-a_1)\right|}=K\left|\frac{b_1}{a_1}\right|$$

Stability condition:

$$|G(j\omega)|<1\quad\Rightarrow\quad\begin{cases}-\dfrac{a_1}{b_1}<K<\dfrac{a_1}{b_1} & \text{At condition: } a_1b_1>0\\[2ex] \dfrac{a_1}{b_1}<K<-\dfrac{a_1}{b_1} & \text{At condition: } a_1b_1<0\end{cases}$$

Four Examples

Ex.1 $$G(s) = \frac{K}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ Zero Zero, Three poles

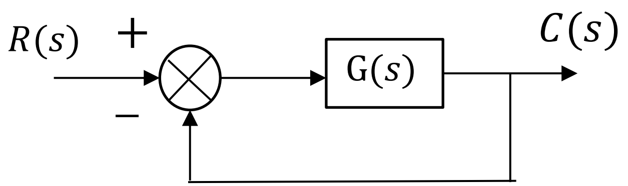
Ex.2 $$G(s) = \frac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2}$$ One Zero, Two Poles

Ex.3 $$G(s) = \frac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ One Zero, Three Poles

Ex.4 $$G(s) = \frac{K(1 + b_1 s + b_2 s^2)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ Two Zeros, Three Poles

Based on Routh-Hurwitz Criterion

Example 3



Open-loop transfer function:

$$G(s) = \frac{K(1+bs)}{1+a_1 s + a_2 s^2 + a_3 s^3}$$

Closed-loop transfer function:

$$H(s) = \frac{G(s)}{1+G(s)} = \frac{K+Kbs}{1+K+(a_1+Kb)s + a_2 s^2 + a_3 s^3}$$

Routh table

S^3	a_3	$a_1 + Kb$
S^2	a_2	$1+K$
S^1	$\frac{a_2(a_1+Kb) - a_3(1+K)}{a_2}$	
S^0	$1+K$	

Based on Routh-Hurwitz criterion:

$a_3 > 0$ $\qquad a_2 > 0$

$1+K > 0$

$$\frac{a_2(a_1+Kb) - a_3(1+K)}{a_2} > 0 \Rightarrow \begin{cases} K > \dfrac{a_3 - a_1 a_2}{a_2 b - a_3} \\ K < \dfrac{a_3 - a_1 a_2}{a_2 b - a_3} \end{cases}$$

At condition: $a_2 b - a_3 > 0$

At condition: $a_2 b - a_3 < 0$

Based on Nyquist Criterion

Frequency domain:

$$G(j\omega) = \frac{K(1+bj\omega)}{1-a_2\omega^2+j(a_1\omega-a_3\omega^3)} = \frac{K[(1-a_2\omega^2+a_1b\omega^2-a_3b\omega^4)+j(b\omega-a_2b\omega^3-a_1\omega+a_3\omega^3)]}{(1-a_2\omega^2)^2+(a_1\omega-a_3\omega^3)^2}$$

Special frequency expressions

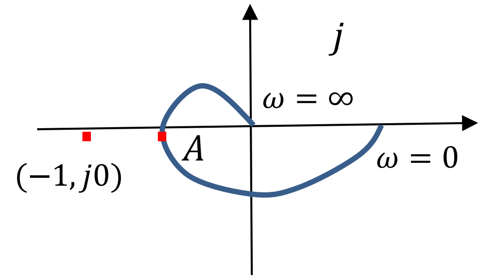
$$\angle G(j\omega) = -\pi$$

$$\Rightarrow \quad b\omega-a_2b\omega^3-a_1\omega+a_3\omega^3=0$$

$$\Rightarrow \quad \omega^2=\frac{a_1-b}{a_3-a_2b}$$

At point A

$$\Rightarrow \quad |G(j\omega)| = \left|\frac{K(1-a_2\omega^2+a_1b\omega^2-a_3b\omega^4)}{(1-a_2\omega^2)^2+(a_1\omega-a_3\omega^3)^2}\right| = K\left|\frac{a_3-a_2b}{a_3-a_1a_2}\right|$$



sketch chart of Nyquist plot

Stability condition:

$$|G(j\omega)|<1 \Rightarrow \begin{cases} \dfrac{a_3-a_1a_2}{a_2b-a_3}<K<\dfrac{a_3-a_1a_2}{a_3-a_2b} & \text{At condition: } (a_3-a_1a_2)(a_3-a_2b)>0 \\[2ex] \dfrac{a_3-a_1a_2}{a_3-a_2b}<K<\dfrac{a_3-a_1a_2}{a_2b-a_3} & \text{At condition: } (a_3-a_1a_2)(a_3-a_2b)<0 \end{cases}$$

Four Examples

Ex.1 $G(s) = \dfrac{K}{1 + a_1 s + a_2 s^2 + a_3 s^3}$ Zero Zero, Three poles

Ex.2 $G(s) = \dfrac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2}$ One Zero, Two Poles

Ex.3 $G(s) = \dfrac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$ One Zero, Three Poles

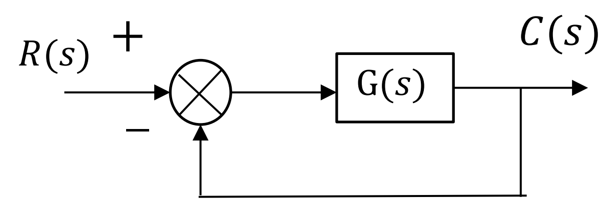
Ex.4 $G(s) = \dfrac{K(1 + b_1 s + b_2 s^2)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$ Two Zeros, Three Poles

Based on Routh-Hurwitz Criterion

Example 4

Open-loop transfer function:

$$G(s) = \frac{K(1 + b_1 s + b_2 s^2)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$



Closed-loop transfer function:

$$H(s) = \frac{G(s)}{1 + G(s)} = \frac{K + Kb_1 s + Kb_2 s^2}{1 + K + (a_1 + Kb_1)s + (a_2 + Kb_2)s^2 + a_3 s^3}$$

Based on Routh-Hurwitz criterion:

$$a_3 > 0,\ a_2 + Kb_2 > 0,\ 1 + K > 0$$

$$\frac{(a_2 + Kb_2)(a_1 + Kb_1) - a_3(1 + K)}{(a_2 + Kb_2)} > 0$$

Routh table

S^3	a_3	$a_1 + Kb_1$
S^2	$a_2 + Kb_2$	$1 + K$
S^1	$\frac{(a_2 + Kb_2)(a_1 + Kb_1) - a_3(1 + K)}{(a_2 + Kb_2)}$	
S^0	$1 + K$	

Based on Routh-Hurwitz Criterion

$$(a_2 + Kb_2)(a_1 + Kb_1) - a_3(1 + K) > 0 \quad (1)$$

Set one function:

$$f(K) = (a_2 + Kb_2)(a_1 + Kb_1) - a_3(1 + K)$$
$$= K^2 b_1 b_2 + K a_1 b_2 + K a_2 b_1 - K a_3 + a_1 a_2 - a_3$$

- Domain of definition $K \in (0, +\infty)$
- Initial value: $f(0) = a_1 a_2 - a_3$
- Derived function: $f'(K) = 2K b_1 b_2 + a_1 b_2 + a_2 b_1 - a_3$

if $\left.\begin{array}{l} f'(K) = 2K b_1 b_2 + a_1 b_2 + a_2 b_1 - a_3 > 0 \\ f(0) = a_1 a_2 - a_3 \geq 0 \end{array}\right\} \Rightarrow \boxed{f(K) > 0} \Rightarrow$ (1) will be established

Stability condition is becoming:

$$2K b_1 b_2 > -a_1 b_2 - a_2 b_1 + a_3$$

at condition: $a_1 a_2 - a_3 \geq 0$

Based on Nyquist Criterion

Frequency domain:

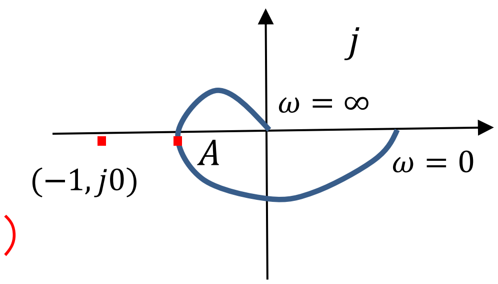
$$G(j\omega)=\frac{K+Kb_1(j\omega)+Kb_2(j\omega)^2}{1-a_2\omega^2+j(a_1\omega-a_3\omega^3)}$$

$$=\frac{K\left(1-a_2\omega^2-b_2\omega^2+a_2b_2\omega^4+a_1b_1\omega^2-a_3b_1\omega^4\right)+jK(a_3\omega^3-a_1\omega+a_1b_2\omega^3-a_3b_2\omega^5+b_1\omega-a_2b_1\omega^3)}{(1-a_2\omega^2)^2+(a_1\omega-a_3\omega^3)^2}$$

Special frequency expressions

$$\angle G(j\omega)=-\pi$$

$$\Rightarrow a_3\omega^3-a_1\omega+a_1b_2\omega^3-a_3b_2\omega^5+b_1\omega-a_2b_1\omega^3=0 \quad (2)$$



sketch chart of Nyquist plot

$$\Rightarrow 1-a_2\omega^2=\frac{(1-b_2\omega^2)(a_1-a_3\omega^2)}{b_1}$$

At point A

$$\Rightarrow |G(j\omega)|=\frac{K|1-a_2\omega^2-b_2\omega^2+a_2b_2\omega^4+a_1b_1\omega^2-a_3b_1\omega^4|}{|(1-a_2\omega^2)^2+(a_1\omega-a_3\omega^3)^2|}$$

$$=\cdots=\frac{Kb_1}{(a_1-a_3\omega^2)}$$

(2) $\Rightarrow a_3b_2\omega^4+(a_2b_1-a_1b_2-a_3)\omega^2+a_1-b_1=0$

$$\Rightarrow \omega^2=\frac{a_3+a_1b_2-a_2b_1\pm\sqrt{(a_2b_1-a_1b_2-a_3)^2-4a_3b_2(a_1-b_1)}}{2a_3b_2}\approx\frac{a_3+a_1b_2-a_2b_1}{2a_3b_2}$$

Based on Nyquist Criterion

Nyquist criterion

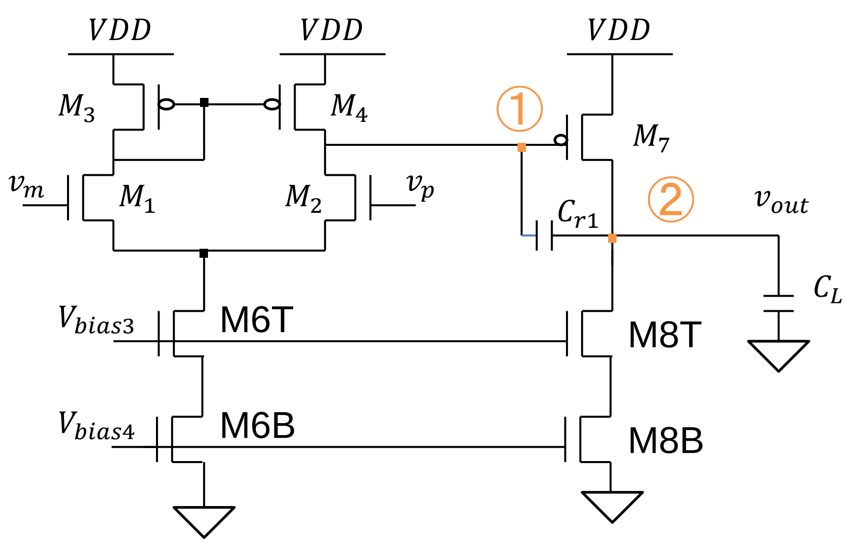
$$|G(j\omega)| = \frac{K|b_1|}{|a_1 - a_3\omega^2|} = \frac{K|b_1|}{\left|a_1 - a_3 \frac{a_3 + a_1 b_2 - a_2 b_1}{2a_3 b_2}\right|} = \frac{K|2b_1 b_2|}{|a_1 b_2 + a_2 b_1 - a_3|} < 1$$

Stability condition:

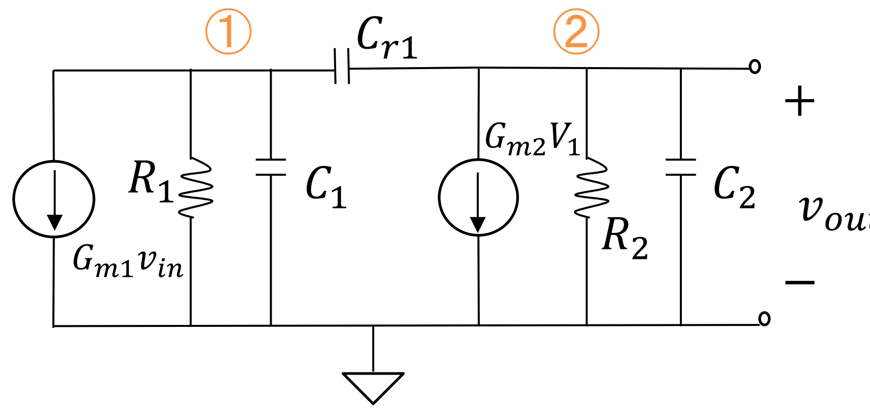
$$\begin{cases} a_3 - a_1 b_2 - a_2 b_1 < 2Kb_1 b_2 < a_1 b_2 + a_2 b_1 - a_3 & \text{At condition:} a_1 b_2 + a_2 b_1 - a_3 > 0 \\ a_1 b_2 + a_2 b_1 - a_3 < 2Kb_1 b_2 < a_3 - a_1 b_2 - a_2 b_1 & \text{At condition:} a_1 b_2 + a_2 b_1 - a_3 < 0 \end{cases}$$

Contents

Amplifier Circuit and Small Signal Model



Transistor level circuit



Small-signal model

Open-loop transfer function from small signal model

$$A(s) = \frac{v_{out}(s)}{v_{in}(s)} = A_0 \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}$$

$$b_1 = -\frac{C_r}{G_{m2}} \qquad A_0 = G_{m1} G_{m2} R_1 R_2$$

$$a_1 = R_1 C_1 + R_2 C_2 + (R_1 + R_2 + R_1 G_{m2} R_2) C_r \qquad a_2 = R_1 R_2 C_2 \left[C_1 + \left(1 + \frac{C_1}{C_2} \right) C_r \right]$$

Routh-Hurwitz method

Closed-loop transfer function:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{A(s)}{1 + fA(s)} = \frac{A_0(1 + b_1 s)}{1 + fA_0 + (a_1 + fA_0 b_1)s + a_2 s^2}$$

Short-channel CMOS parameters:

$R_1 = r_{on}||r_{op} = 111k\Omega$

$R_2 = r_{op}||R_{ocasn} \approx r_{op} = 333k\Omega$

$G_{m1} = g_{mn} = 150\,uA/V$

$G_{m2} = g_{mp} = 150\,uA/V$

$C_1 = C_{dg4} + C_{dg2} + C_{gs7} = 13.6fF$

$C_2 = C_L + C_{gd8} \approx C_L + 1.56fF = 101.56fF \quad (C_L = 100fF)$

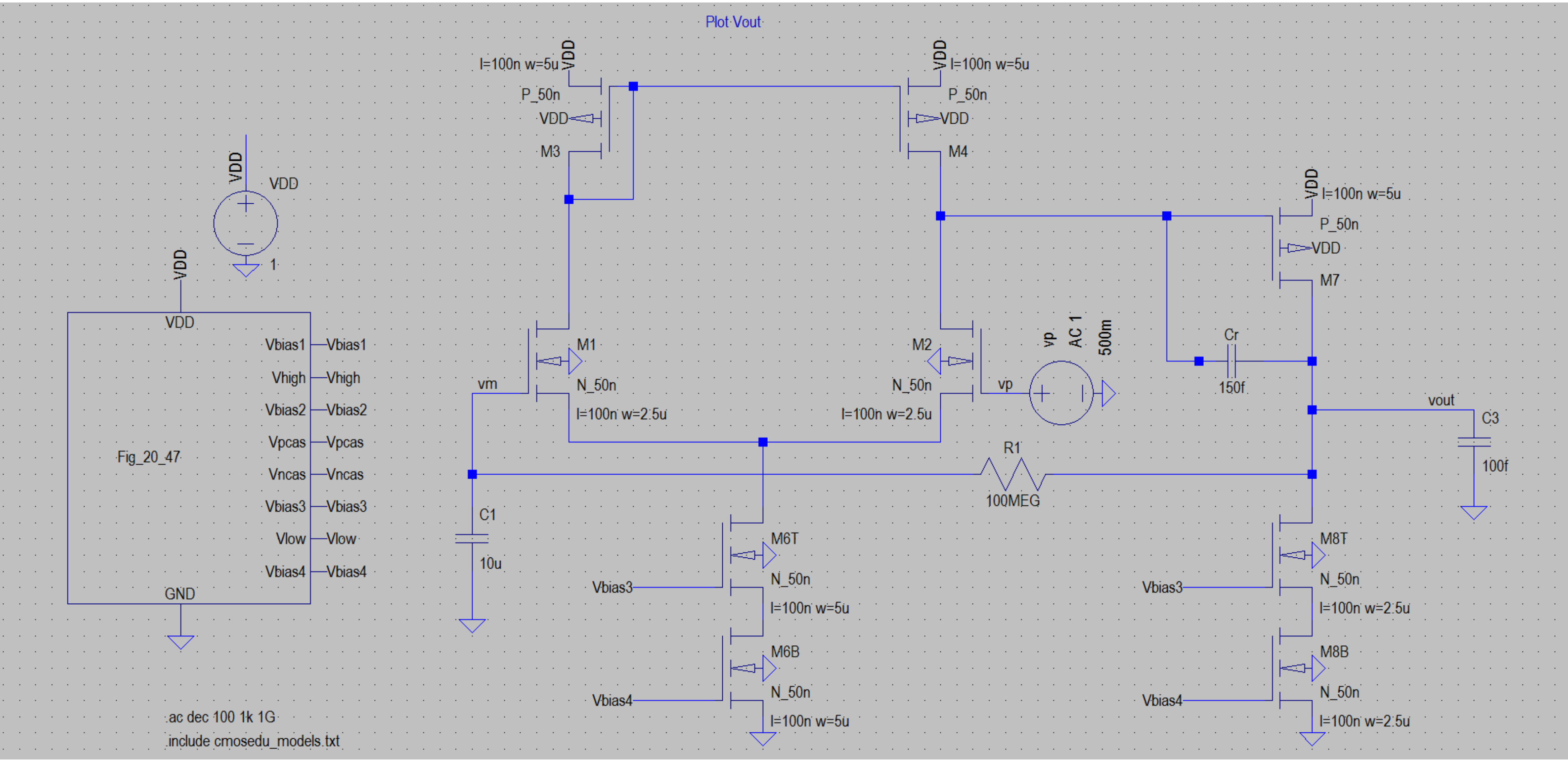
Explicit stability condition of parameters:

$$a_1 + fA_0 b_1 = R_1C_1 + R_2C_2 + (R_1 + R_2)C_r + (G_{m2} - fG_{m1})R_1R_2C_r > 0$$

$\Rightarrow$ $C_r >$ 79.57fF

Based on the different values of the transistor parameters,
We can obtain appropriate C_r.

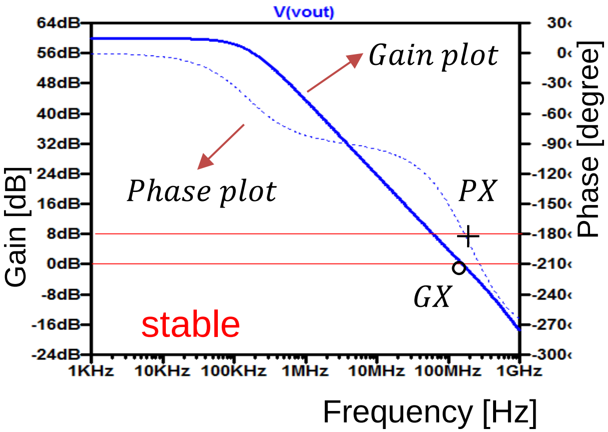
AC analysis

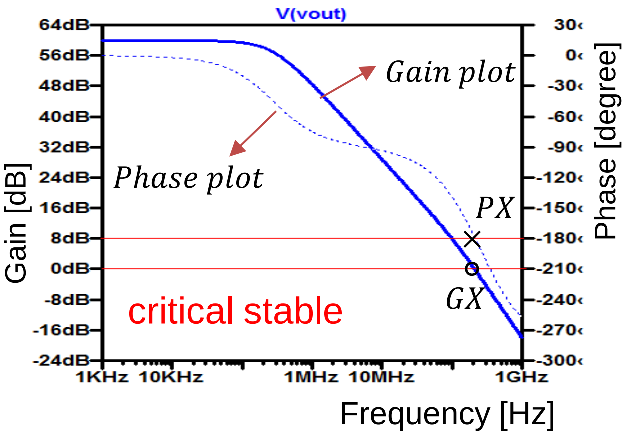


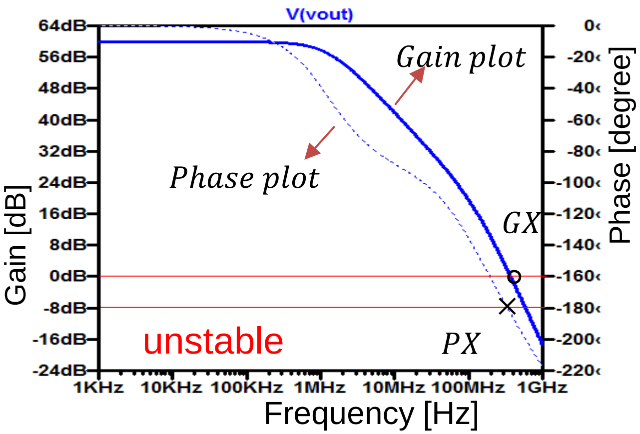
Ltspice simulation circuit

Consistency of Bode Plots and R-H Results

case	C_{r1}	SPICE
(1)	150fF	stable
(2)	79.57fF	critical stable
(3)	10fF	unstable

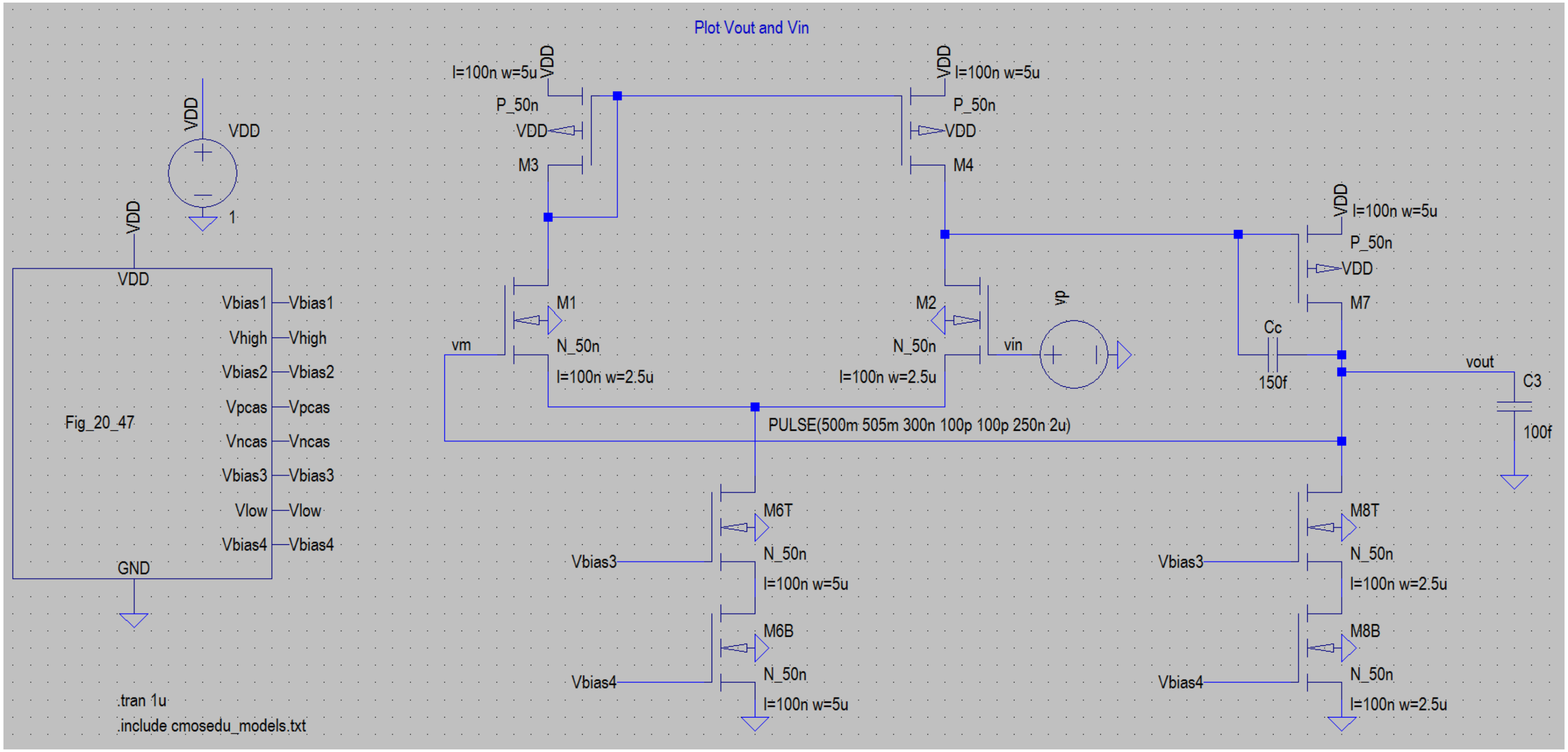






Case (1) $C_r = 150\text{fF}$ Case (2) $C_r = 79.57\text{fF}$ Case (3) $C_r = 10\text{fF}$

Transient Analysis

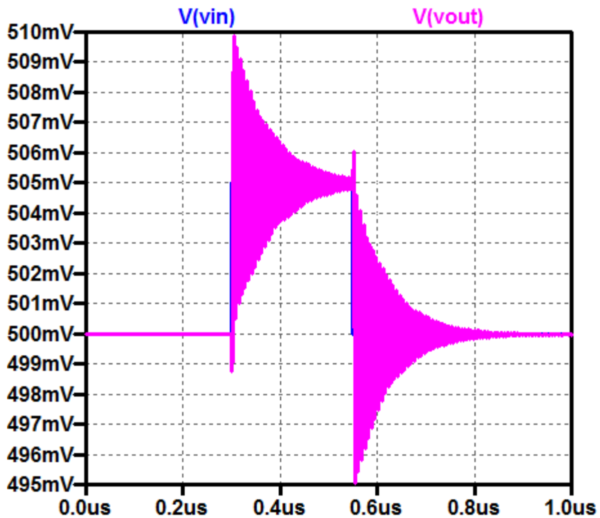


Ltspice simulation circuit

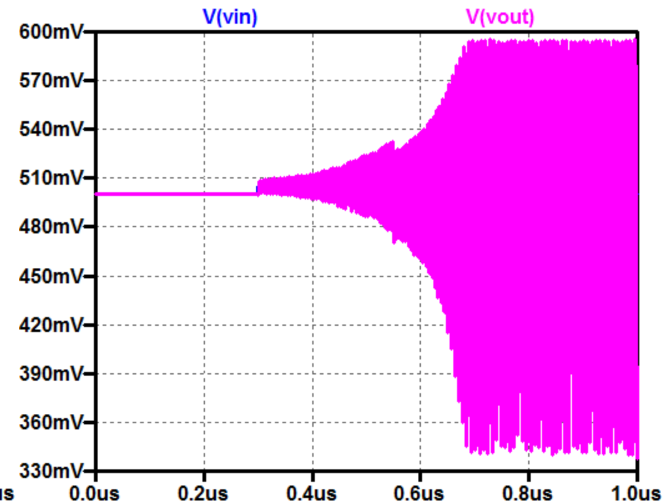
Consistency of Transient Analysis and R-H Results

Pulse response

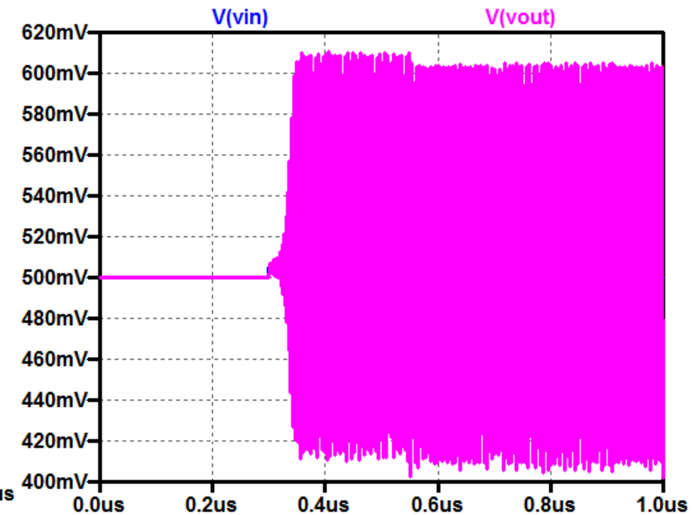
PULSE(500m 505m 300n 100p 100p 250n 2u)



Case (1) $C_r = 150\text{fF}$

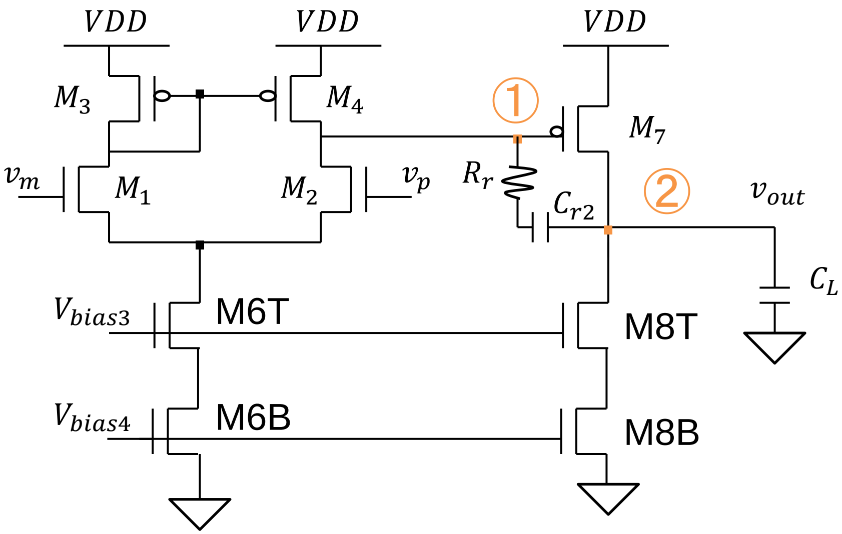


Case (2) $C_r =$79.57fF

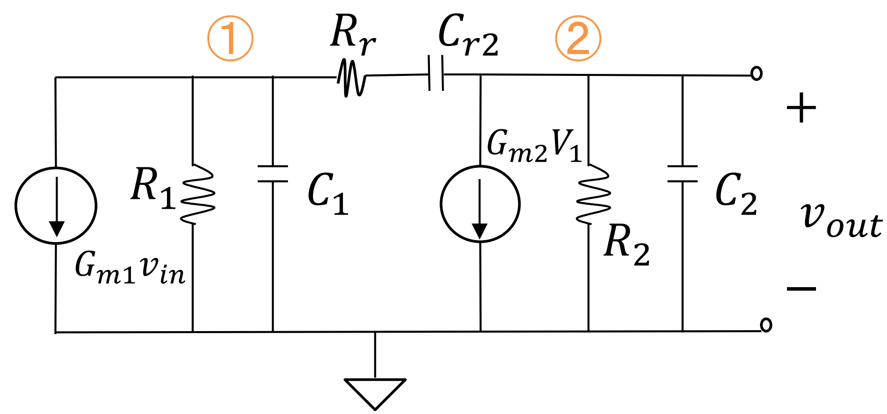


Case (3) $C_r = 10\text{fF}$

Amplifier Circuit and Small Signal Model



Transistor level circuit



Small-signal model

Open-loop transfer function:

$$A(s) = \frac{v_{out}(s)}{v_{in}(s)} = A_0 \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$

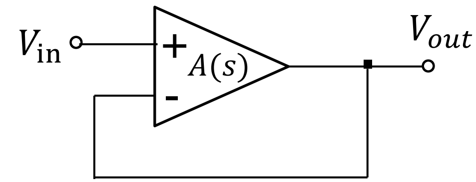
$A_0 = G_{m1}G_{m2}R_1R_2$ $\quad$ $b_1 = -\left(\frac{C_r}{G_{m2}} - R_rC_r\right)$ $\quad$ $a_1 = R_1C_1 + R_2C_2 + (R_1 + R_2 + R_r + R_1R_2G_{m2})C_r$

$a_2 = R_1R_2(C_2C_r + C_1C_2 + C_1C_r) + R_rC_r(R_1C_1 + R_2C_2)$ $\quad$ $a_3 = R_1R_2R_rC_1C_2C_r$

Routh-Hurwitz method

Closed-loop transfer function:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{A(s)}{1 + fA(s)} = \frac{A_0(1 + b_1 s)}{1 + fA_0 + (a_1 + fA_0 b_1)s + a_2 s^2 + a_3 s^3}$$



$f = 1$

Explicit stability condition of parameters:

$$\frac{(a_1 + fA_0 d_1)a_2 - a_3(1 + fA_0)}{a_2} > 0$$

$$R_1C_1 + R_2C_2 + (R_1 + R_2 + R_r)C_r + (G_{m2} - fG_{m1} + fG_{m1}G_{m2}R_r)R_1R_2C_r > \frac{R_1R_2C_1C_2R_rC_r(1 + G_{m1}G_{m2}R_1R_2)}{R_1R_2(C_2C_r + C_1C_2 + C_1C_r) + R_rC_r(R_1C_1 + R_2C_2)}$$

Short-channel CMOS parameters:

$R_1 = r_{on}||r_{op} = 111k\Omega$ $\quad$ $G_{m1} = g_{mn} = 150\,uA/V$

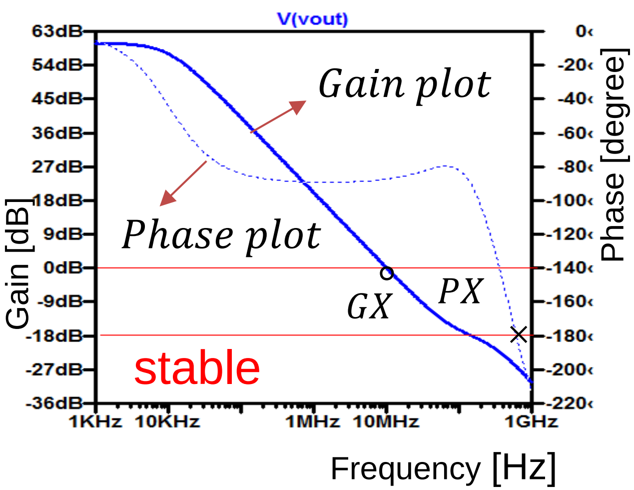
$R_2 = r_{op}||R_{ocasn} \approx r_{op} = 333k\Omega$ $\quad$ $G_{m2} = g_{mp} = 150\,uA/V$

$C_1 = C_{dg4} + C_{dg2} + C_{gs7} = 13.6fF$ $\quad$ $C_2 = C_L + C_{gd8} \approx C_L + 1.56fF = 101.56fF$

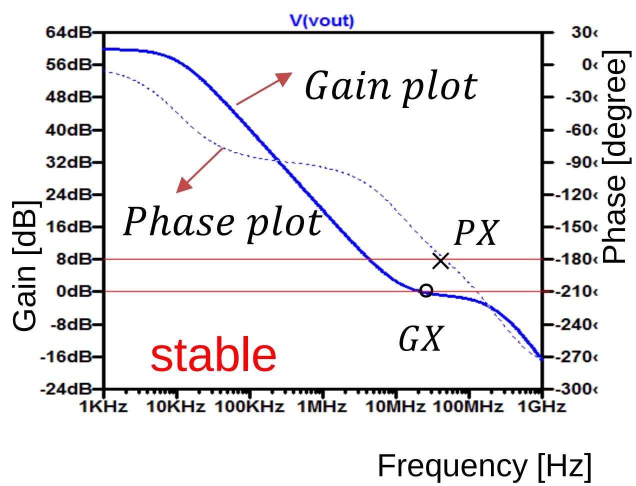
$$\underbrace{3.5 \times 10^{-8} + 3.7 \times 10^{10}C_r + R_rC_r + 831.7R_r}_{X} > \underbrace{\frac{4.3 \times 10^{-8}R_rC_r}{5.1 \times 10^{-17} + 4.3 \times 10^{-3}C_r + 3.5 \times 10^{-8}R_rC_r}}_{Y}$$

Consistency of Bode Plots and R-H Results

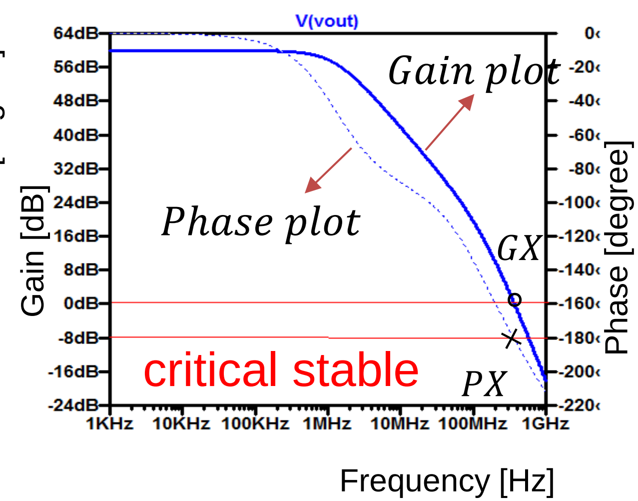
	parameter values				R-H criterion	Bode plot
case	R_r	C_{r2}	X	Y		SPICE simulation
(1)	6.5k	2.4p	1.41×10^{-5}	6.13×10^{-8}	$X > Y$	stable
(2)	1	2.4p	1.10×10^{-6}	9.94×10^{-12}	$X > Y$	stable
(3)	7k	10f	9.8×10^{-8}	3.10×10^{-8}	$X \approx Y$	critical



Case (1) $X > Y$



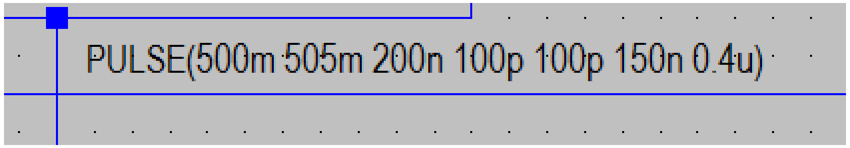
Case (2) $X > Y$



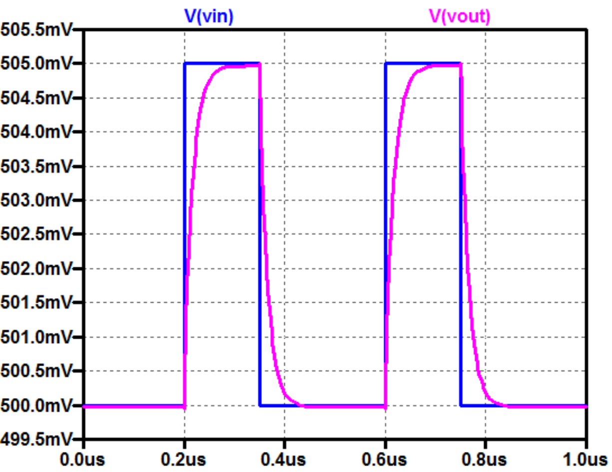
Case (3) $X \approx Y$

Consistency of Transient Analysis and R-H Results

Pulse response

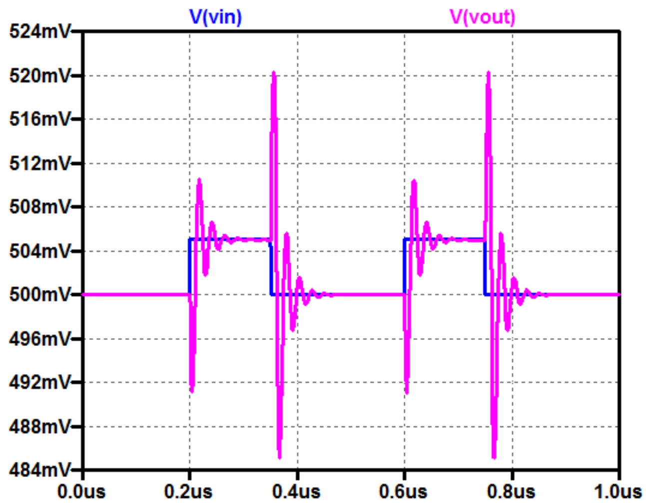


$X > Y$ Stable



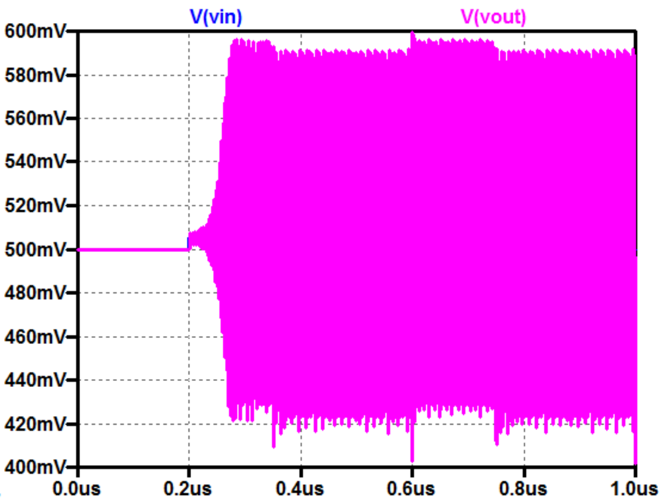
Case (1)

$X > Y$ Stable



Case (2)

$X \approx Y$ critical



Case (3)

Contents

Discussion

Depict small signal equivalent circuit of amplifier

Derive open-loop transfer function

Derive closed-loop transfer function
& obtain characteristics equation

Apply R-H stability criterion
& obtain explicit stability condition

Especially effective for

Multi-stage opamp (high-order system)

Limitation

Explicit transfer function with polynomials of s has to be derived.

Conclusion

- Show the equivalence between Nyquist and R-H stability criteria for analysis and design of the opamp stability under some conditions.
- Equivalency of their mathematical foundations be shown.
- R-H method, explicit circuit parameter conditions can be obtained for feedback stability.
- Consistency with Bode plot method has been confirmed with SPICE simulation.

R-H method can be used
with conventional Bode plot method.

Acknowledgements

We acknowledge
Prof. T. Kitamori
Prof. H. Tanimoto
Dr. Y. Gendai
for their valuable comments.

Thank you
for your kind attention.

Q&A

First teacher

Q1: For stability, margin is important thing, can you show the margin by your proposed method?

A: By Routh-Hurwitz method, we can find out internal connection between circuit parameter and stability. We can obtain explicit circuit parameter condition. this can not obtain from Bode plot method.
About phase margin, according to previous simulation results, in general, the degree that R-H parameter’s value greater than zero is more greater, the phase margin also will be more greater.
But specific expression from R-H method have not been obtained.
I will have a try at this research direction, thank you for your suggestion.

Q&A

Second teacher

Q2: How to use the proposed method to design operational amplifier?

A: This is the work that I am working.

Used opamp simulation circuit come from text book.

Future work, I want to use R-H method to design opamp circuit.

Q3:From open-loop to closed-loop, circuit configuration and corresponding feedback parameter should be noticed.

A: Bode plot based on open-loop transfer function, and R-H method based on closed-loop transfer function.
At this research we selected voltage follower configuration, by analysis simulation results, we can find out Bode plot results and R-H method results is consistent.
I will apply other circuit configuration at next work.
Thank you for your suggestion.