

DAC Linearity Improvement With Layout Technique Using Latin and Magic Squares

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Contents

- Research Objective
- Segment Type DA Converter
- Characteristic of Variation in Circuit Element
- Proposed Layout Method
 - Magic Square
 - Latin Square
- Summary

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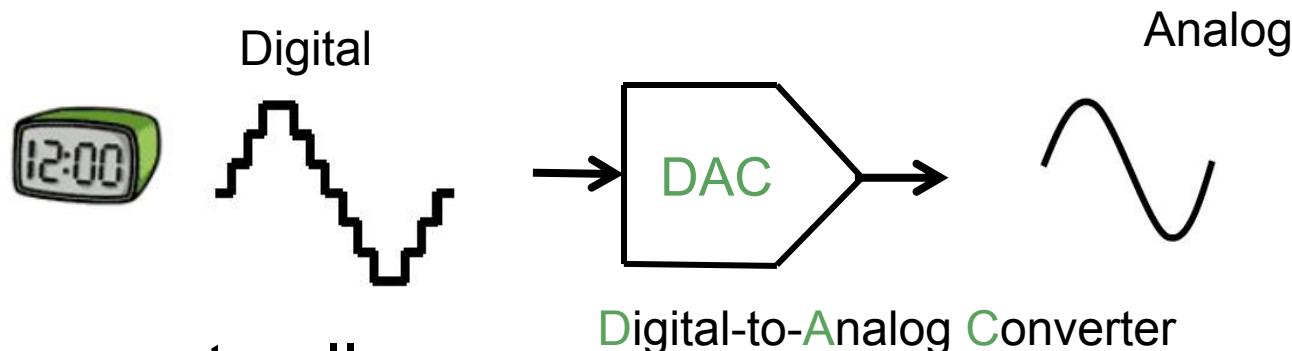
Research Objective

Research Background

- Requirements for electronic equipment
 - High speed
 - High precision

Our Approach

- Improvement the linearity of a segmented DAC

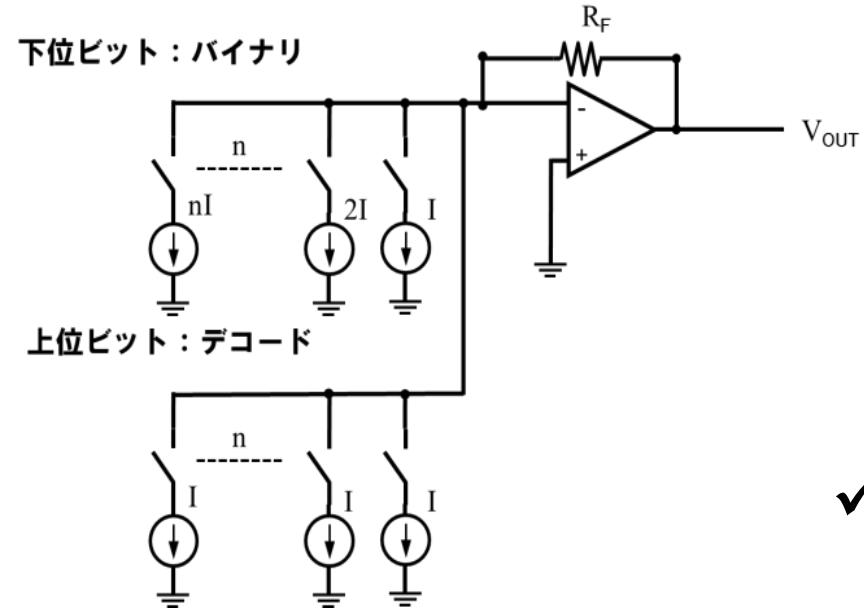


- Unit current cells.
- Layout based on **Magic and Latin Squares**

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Segment Type DAC Configuration



Segmented DAC

✓ Binary (Lower bits)

- Small circuit
- Large glitch
- Large mismatch effect & Large nonlinearity

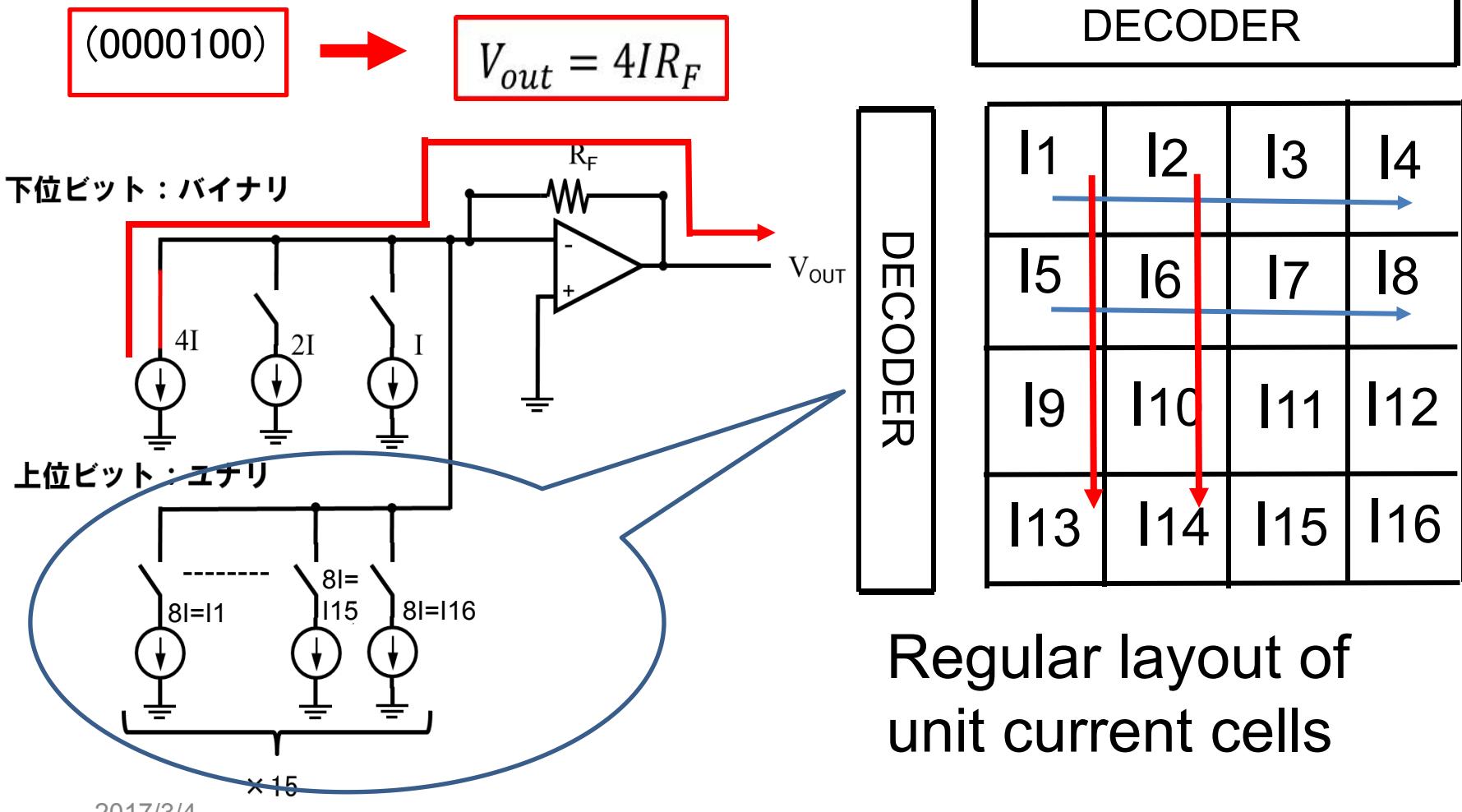
✓ Unary (Upper bits)

- Large circuit
- Small glitch
- Small mismatch effect & modest linearity

Segment Type DAC (7-bit case)

ex

In case digital input =4



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Characteristic of Variation in Circuit Element

Manufacturing variation



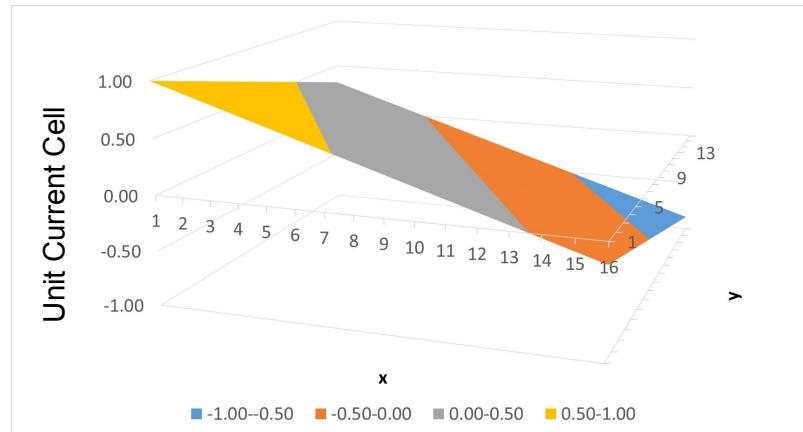
All unit current cells
 $I_1, I_2 \dots I_{16}$ are NOT identical

Systematic variations

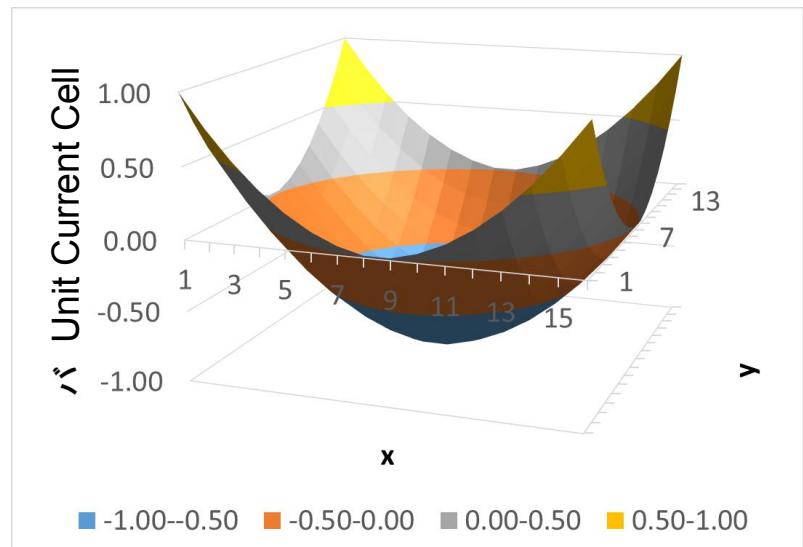


Unit current value depends
on its layout position

Linear error case



Quadratic error case



Characteristic of Variation in Circuit Element

Linear Error

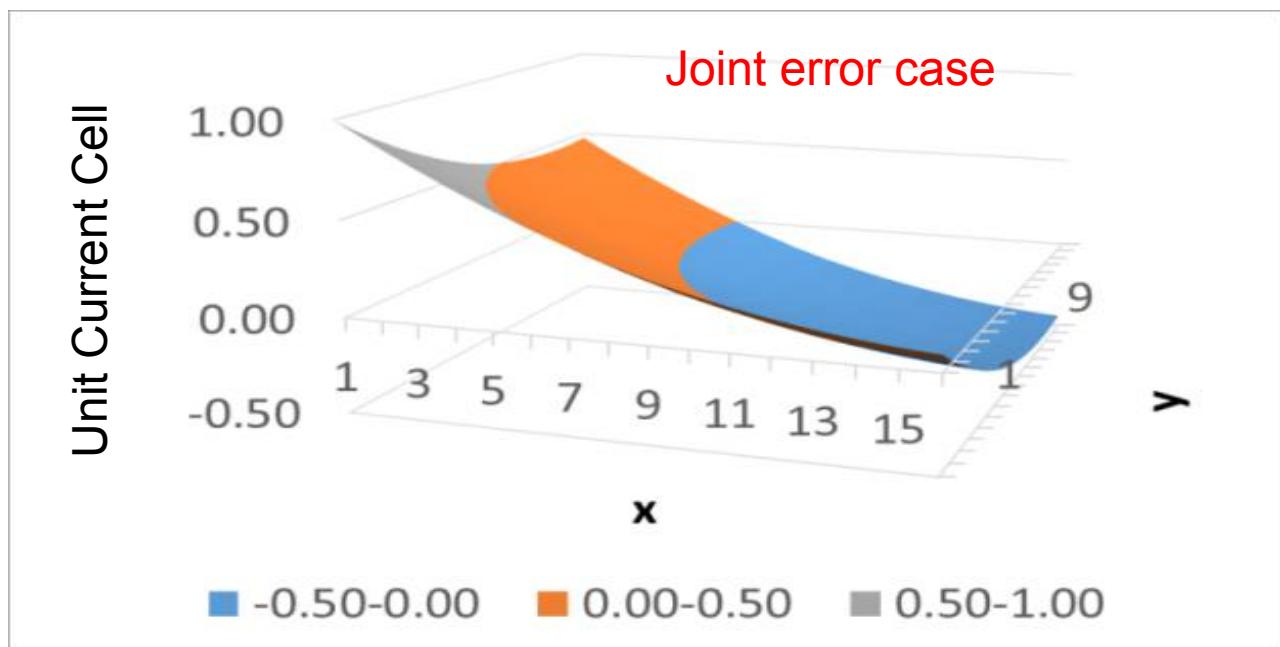
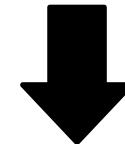
$$\varepsilon_l(x, y) = g_l * \cos \theta * x + g_l * \sin \theta * y$$

Quadratic Error

$$\varepsilon_q(x, y) = g_q * (x^2 + y^2) - a_0$$

Joint Errors

$$\varepsilon_j(x, y) = \varepsilon_l(x, y) + \varepsilon_q(x, y)$$

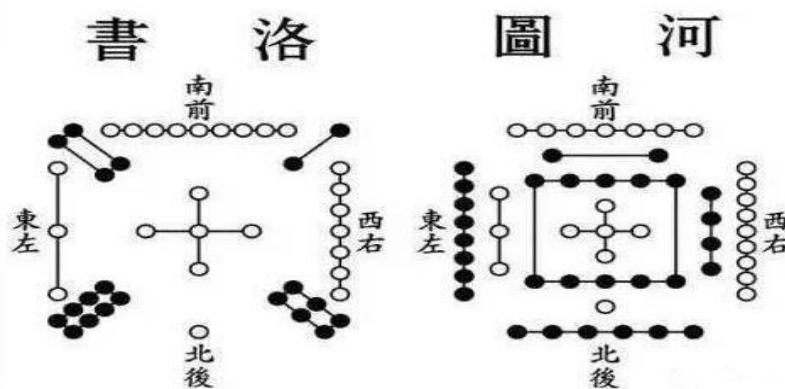


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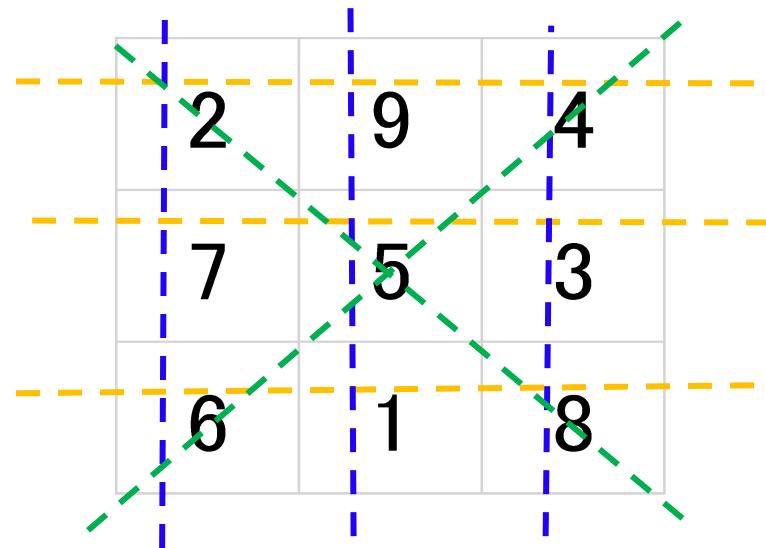
What is Magic Square ?

- Classical mathematics
- Origin from Chinese academia
- “Constant sum” characteristics
- Varieties of magic squares



Constant Sum
Row, column, diagonal

3 × 3 Magic Square



Magic Square has good balance
to cancel the systematic
mismatch effects of current cells

16x16 Current Cells Layout

◆ Concentric Magic Square

Even if one side is removed from the outside, it does not lose compatibility

A1	B1
B2	A2

A: Magic square
of the left figure
B: 45 ° counterclockwise
rotation

Current cell systematic error

59	5	4	62	63	1	8	58	58	56	10	11	53	52	14	6
9	18	17	19	50	42	19	56	8	19	45	21	22	41	47	57
55	20	28	33	29	40	45	10	1	42	40	26	27	37	23	64
54	44	38	31	35	26	21	11	63	50	29	35	34	32	15	2
12	43	39	30	34	27	22	53	62	19	33	31	30	36	16	3
13	24	25	36	32	37	41	52	4	17	28	38	39	25	48	61
51	46	48	16	15	23	47	14	5	18	20	44	43	24	46	60
7	60	61	3	2	64	57	6	59	9	55	54	12	13	51	7
58	56	10	11	53	52	14	6	59	5	4	62	63	1	8	58
8	19	45	21	22	41	47	57	9	18	17	19	50	42	19	56
1	42	40	26	27	37	23	64	55	20	28	33	29	40	45	10
63	50	29	35	34	32	15	2	54	44	38	31	35	26	21	11
62	19	33	31	30	36	16	3	12	43	39	30	34	27	22	53
4	17	28	38	39	25	48	61	13	24	25	36	32	37	41	52
5	18	20	44	43	24	46	60	51	46	48	16	15	23	47	14
59	9	55	54	12	13	51	7	7	60	61	3	2	64	57	6

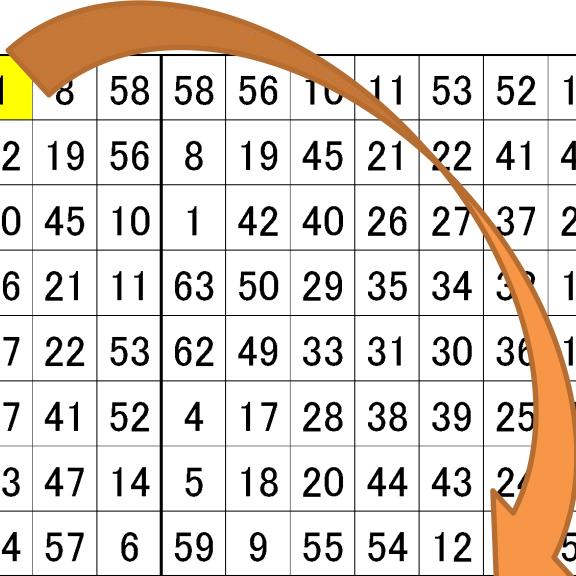
Random selection of current cells
based on Magic Square layout

16x16 Current Cell Layout Details

◆ Concentric Magic Square

- algorithm

A1	B1
B2	A2



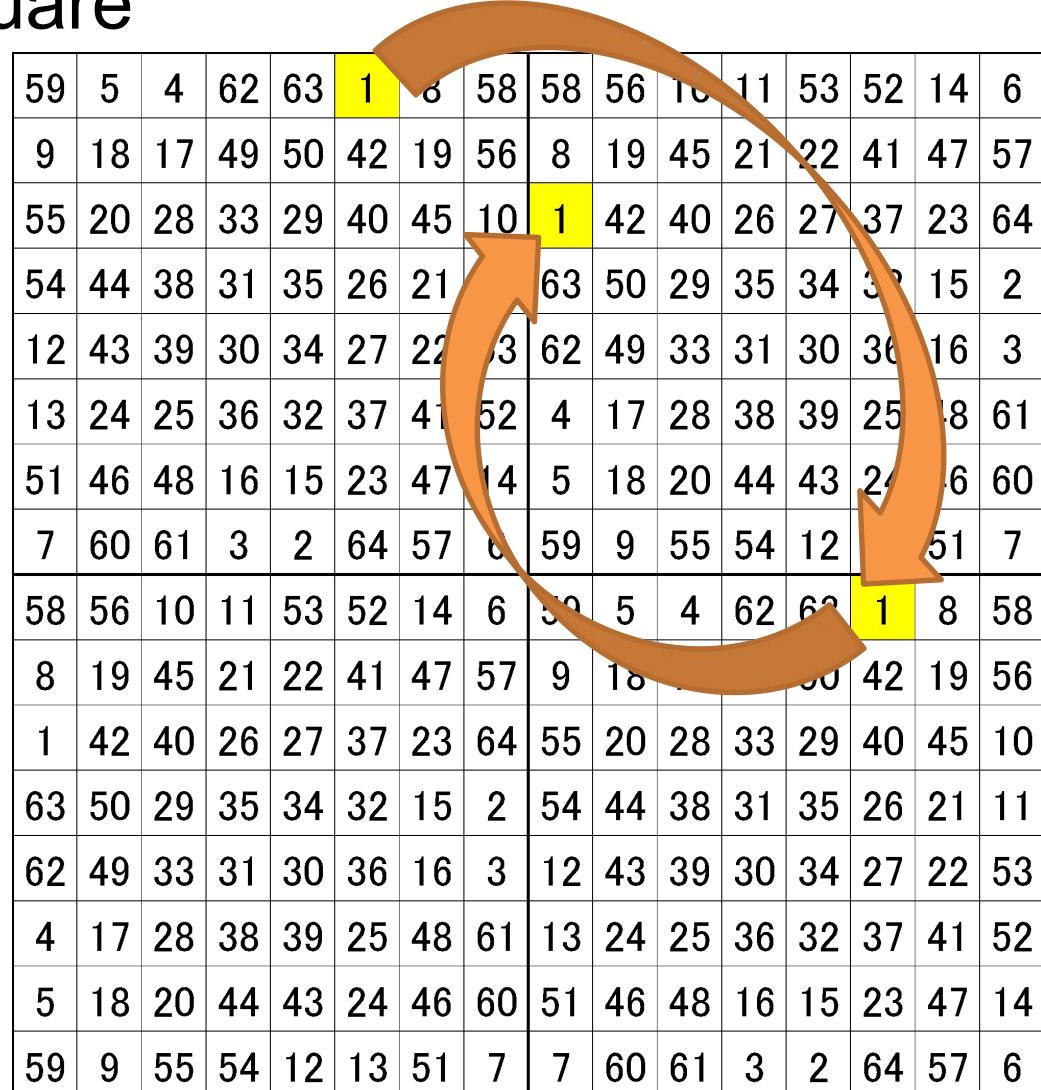
59	5	4	62	63	1	8	58	58	56	10	11	53	52	14	6
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54	44	38	31	35	26	21	11	63	50	29	35	34	32	15	2
12	43	39	30	34	27	22	53	62	49	33	31	30	36	16	3
13	24	25	36	32	37	41	52	4	17	28	38	39	25	8	61
51	46	48	16	15	23	47	14	5	18	20	44	43	24	6	60
7	60	61	3	2	64	57	6	59	9	55	54	12	51	7	
58	56	10	11	53	52	14	6	59	5	4	62	63	1	8	58
8	19	45	21	22	41	47	57	9	18	17	49	50	42	19	56
1	42	40	26	27	37	23	64	55	20	28	33	29	40	45	10
63	50	29	35	34	32	15	2	54	44	38	31	35	26	21	11
62	49	33	31	30	36	16	3	12	43	39	30	34	27	22	53
4	17	28	38	39	25	48	61	13	24	25	36	32	37	41	52
5	18	20	44	43	24	46	60	51	46	48	16	15	23	47	14
59	9	55	54	12	13	51	7	7	60	61	3	2	64	57	6

16x16 Current Cell Layout Details

◆ Concentric Magic Square

- algorithm

A1	B1
B2	A2



A 16x16 grid of numbers from 1 to 256. The grid is labeled with concentric arrows indicating the path of the magic square algorithm. The numbers are arranged in a specific pattern where the sum of the numbers in each row, column, and diagonal is the same.

59	5	4	62	63	1	8	58	58	56	10	11	53	52	14	6
9	18	17	49	50	42	19	56	8	19	45	21	22	41	47	57
55	20	28	33	29	40	45	10	1	42	40	26	27	37	23	64
54	44	38	31	35	26	21	63	50	29	35	34	32	15	2	
12	43	39	30	34	27	22	3	62	49	33	31	30	36	16	3
13	24	25	36	32	37	41	52	4	17	28	38	39	25	18	61
51	46	48	16	15	23	47	14	5	18	20	44	43	24	6	60
7	60	61	3	2	64	57	6	59	9	55	54	12	51	7	
58	56	10	11	53	52	14	6	59	5	4	62	63	1	8	58
8	19	45	21	22	41	47	57	9	18	20	44	43	24	19	56
1	42	40	26	27	37	23	64	55	20	28	33	29	40	45	10
63	50	29	35	34	32	15	2	54	44	38	31	35	26	21	11
62	49	33	31	30	36	16	3	12	43	39	30	34	27	22	53
4	17	28	38	39	25	48	61	13	24	25	36	32	37	41	52
5	18	20	44	43	24	46	60	51	46	48	16	15	23	47	14
59	9	55	54	12	13	51	7	7	60	61	3	2	64	57	6

16x16 Current Cell Layout Details

◆ Concentric Magic Square

- algorithm

A1	B1
B2	A2

The grid shows a 16x16 magic square with the following properties:

- Sum of rows:** 340
- Sum of columns:** 340
- Sum of diagonals:** 340
- Sum of anti-diagonals:** 340
- Sum of concentric layers:** 340

The current cell is highlighted in yellow at position (1, 1). The orange arrows show the path of the current cell as it moves through the grid.

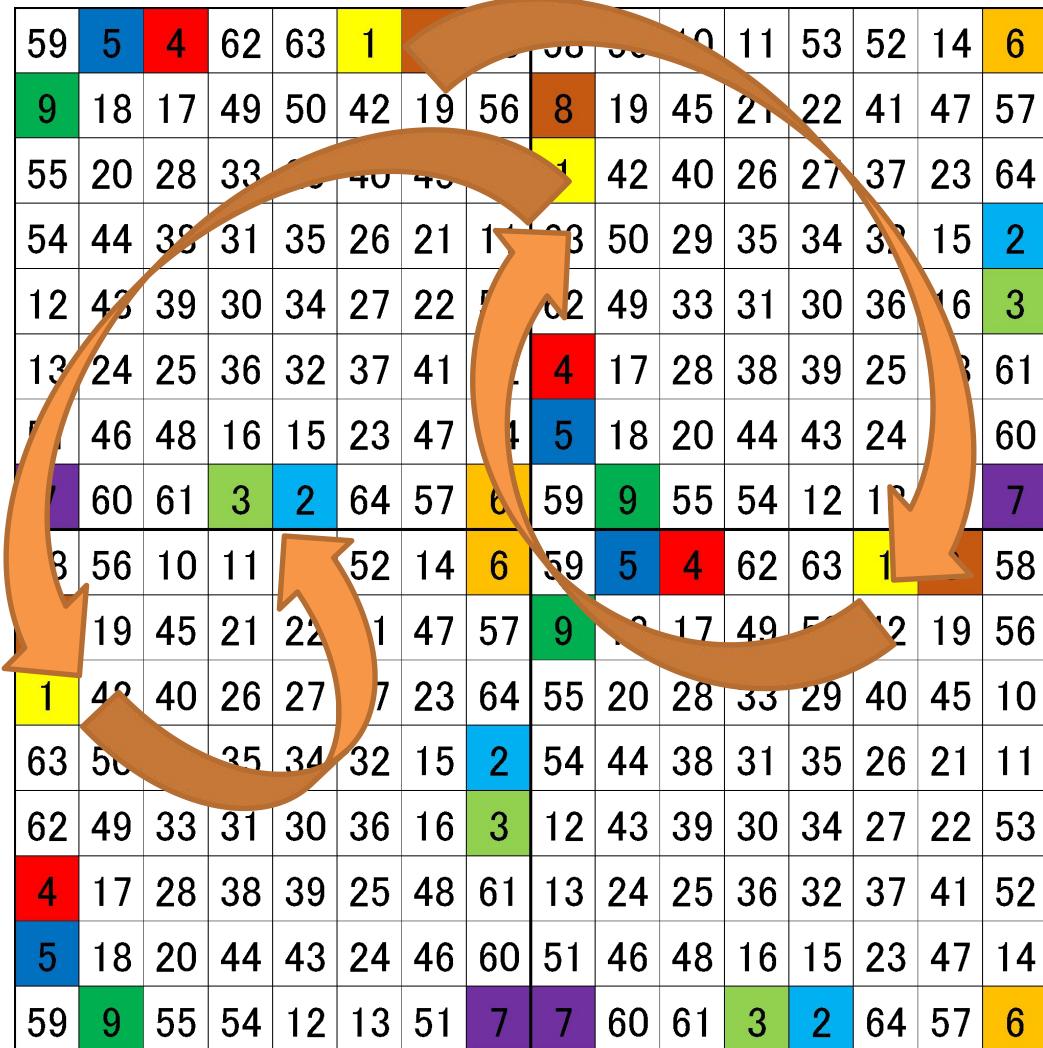
59	5	4	62	63	1	8	58	58	56	10	11	53	52	14	6
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54	44	38	31	35	26	21		63	50	29	35	34	32	15	2
12	43	39	30	34	27	22	13	62	49	33	31	30	36	16	3
3	24	25	36	32	37	41	52	4	17	28	38	39	25	18	61
51	46	48	16	15	23	47	14	5	18	20	44	43	24	6	60
7	60	61	3	2	64	57	6	59	9	55	54	12	51	7	
3	56	10	11	53	52	14	6	59	5	4	62	60	1	8	58
3	19	45	21	22	41	47	57	9	18	17	50	42	19	56	
1	42	40	26	27	37	23	64	55	20	28	33	29	40	45	10
63	50	29	35	34	32	15	2	54	44	38	31	35	26	21	11
62	49	33	31	30	36	16	3	12	43	39	30	34	27	22	53
4	17	28	38	39	25	48	61	13	24	25	36	32	37	41	52
5	18	20	44	43	24	46	60	51	46	48	16	15	23	47	14
59	9	55	54	12	13	51	7	7	60	61	3	2	64	57	6

16x16 Current Cell Layout Details

◆ Concentric Magic Square

- algorithm

A1	B1
B2	A2



The grid is a 16x16 magic square with the following properties:

- Color Cycles:** The cells are colored in cycles of 16 colors: Red, Yellow, Orange, Green, Blue, Purple, Magenta, and Black.
- Central Value:** The central cell is 1.
- Sum:** Every row, column, and diagonal sum to 340.
- Arrows:** Orange arrows show a path starting from the center (1) and moving through several cells, including 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64.

59	5	4	62	63	1	8	19	45	21	22	41	47	57	6	
9	18	17	49	50	42	19	56	8	19	45	21	22	41	47	57
55	20	28	33	35	40	45	1	42	40	26	27	37	23	64	
54	44	39	31	35	26	21	11	23	50	29	35	34	32	15	2
12	43	39	30	34	27	22	5	32	49	33	31	30	36	16	3
13	24	25	36	32	37	41	1	4	17	28	38	39	25	3	61
5	46	48	16	15	23	47	4	5	18	20	44	43	24	60	
7	60	61	3	2	64	57	6	59	9	55	54	12	13	7	
8	56	10	11	52	14	6	59	5	4	62	63	1	58		
19	45	21	22	1	47	57	9	13	17	49	51	12	19	56	
1	42	40	26	27	7	23	64	55	20	28	33	29	40	45	10
63	50	35	34	32	15	2	54	44	38	31	35	26	21	11	
62	49	33	31	30	36	16	3	12	43	39	30	34	27	22	53
4	17	28	38	39	25	48	61	13	24	25	36	32	37	41	52
5	18	20	44	43	24	46	60	51	46	48	16	15	23	47	14
59	9	55	54	12	13	51	7	7	60	61	3	2	64	57	6

Simulation Results (Linear Error Case)

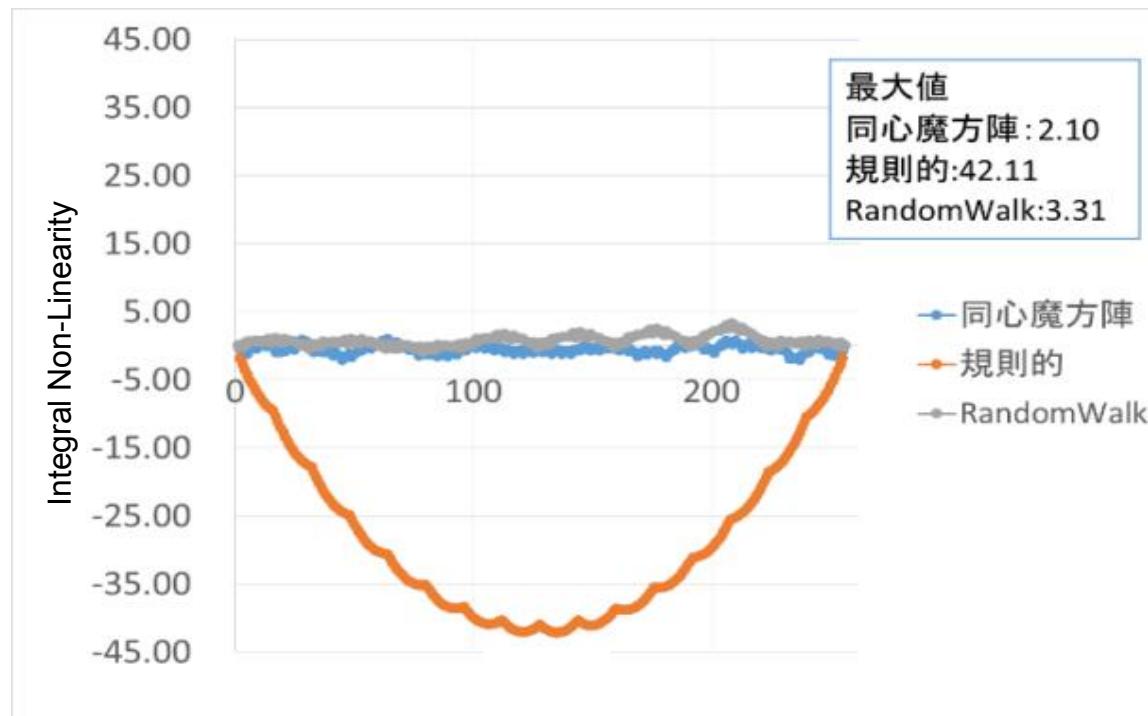
◆ Concentric Magic Square

- ✓ Linear Error (Current Cell Systematic Mismatch)

$$\varepsilon_l(x, y) = g_l * \cos \theta * x + g_l * \sin \theta * y$$

$$\theta = 30^\circ$$

$$g_l = 1$$



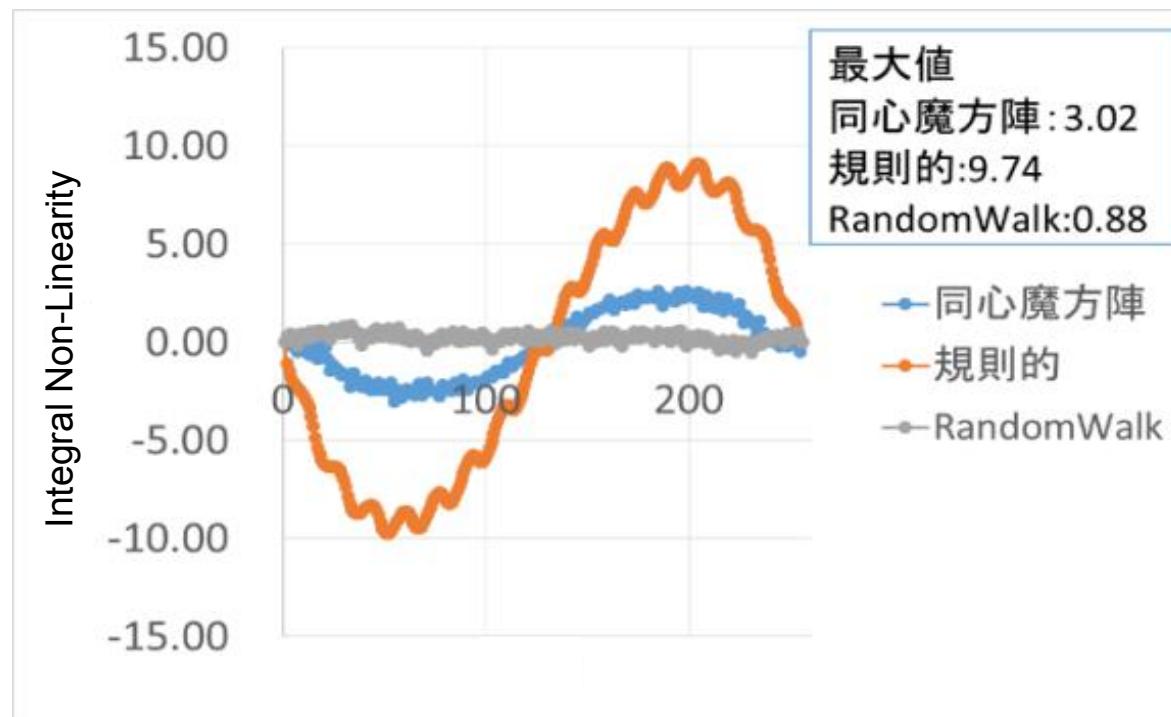
Simulation Results (Quadratic Error Case)

◆ Concentric Magic Square

✓ Quadratic Error (Current Cell Systematic Mismatch)

$$\varepsilon_q(x, y) = g_q * (x^2 + y^2) - a_0$$

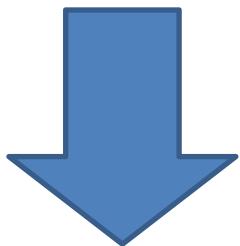
$$g_q = 1, a_0 = 0$$



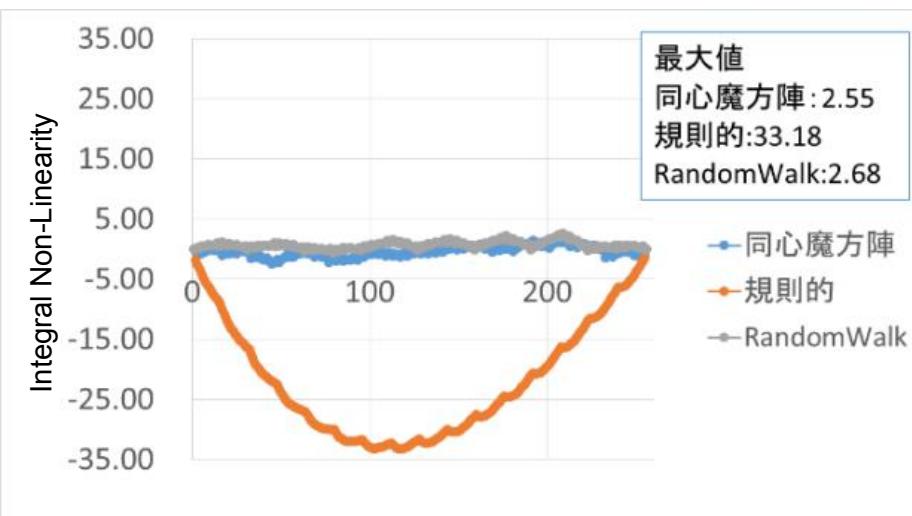
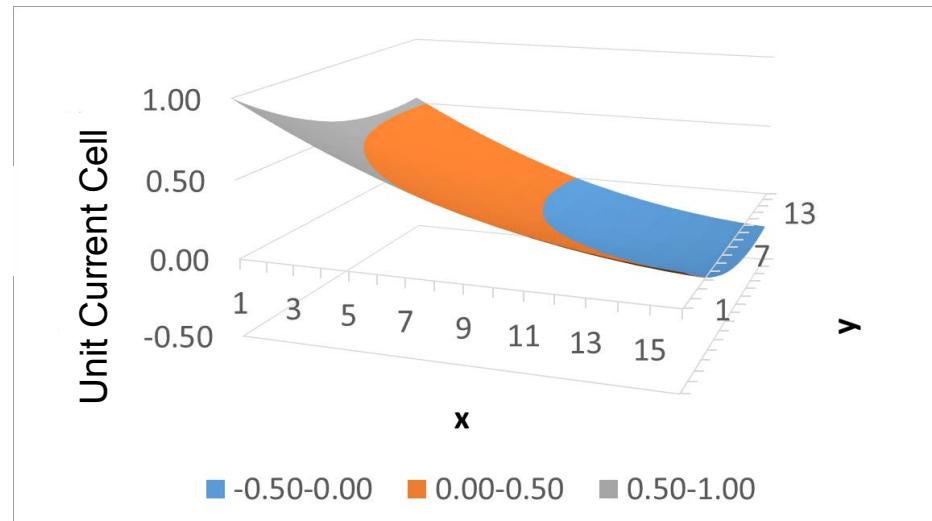
Simulation Results (Joint Error Case)

- ◆ Concentric Magic Square
- ✓ Joint Error

Linear >Quadratic case



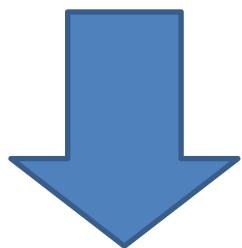
Magic square is better



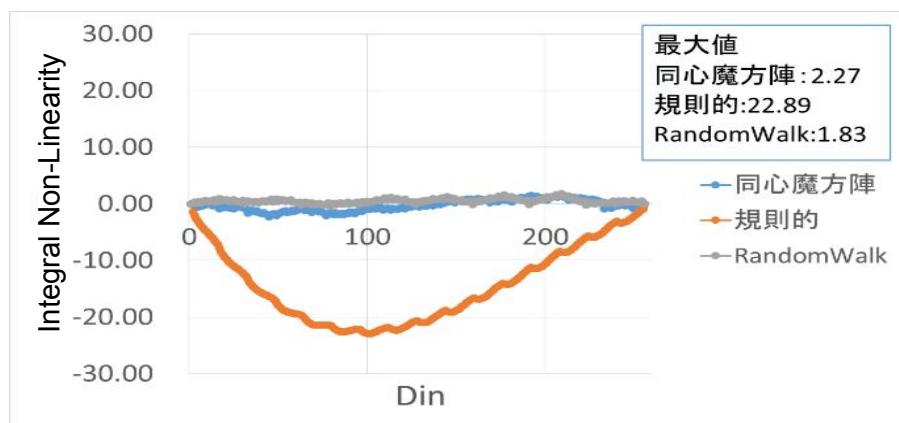
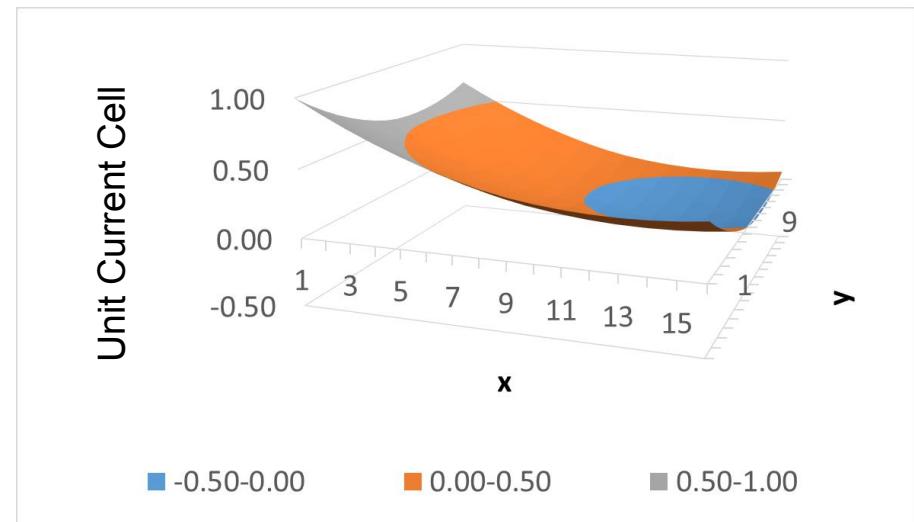
Simulation Results (Joint Error Case)

- ◆ Concentric Magic Square
- ✓ Joint Error

Linear < Quadratic case



Random Walk is better



What is Latin Square ?

- Each symbols occurring exactly once in each row and column
- In the Latin square , If the first row is $(1,2,3,\dots,n)$ and the first column is $(1,2,3,\dots,n)$, it is a **standard Latin square**

Example:



1	2	3	4
2	3	4	1
3	4	1	2
4	1	2	3

4×4 Latin square

1	2	3	4	5	6	7	8
2	3	4	5	6	7	8	1
3	4	5	6	7	8	1	2
4	5	6	7	8	1	2	3
5	6	7	8	1	2	3	4
6	7	8	1	2	3	4	5
7	8	1	2	3	4	5	6
8	1	2	3	4	5	6	7

8×8 standard Latin square

Leonhard Euler(1707-1783)
Swiss mathematician, physicist

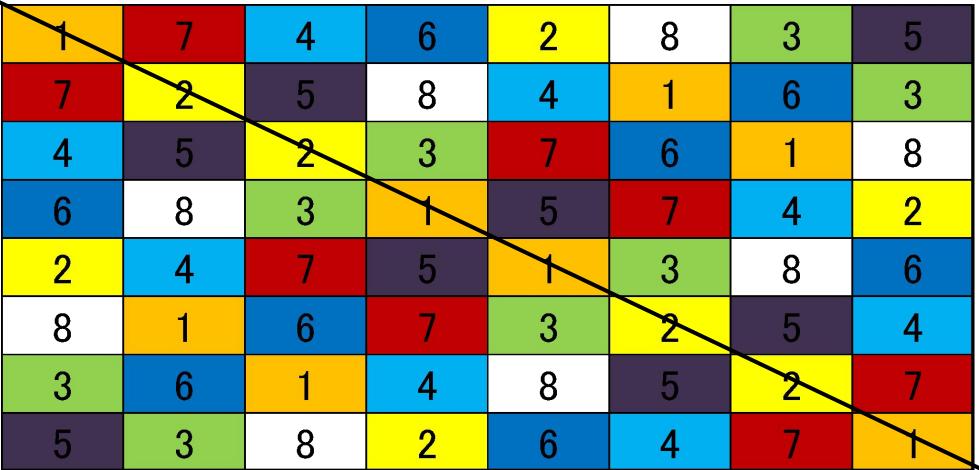
Latin Square Layout Algorithm

◆ algorithm

- Latin Square
- Common Centroid
- Unary Layout

◆ Compare

- Linear Error
- Quadratic Error



A 8x8 Latin square grid where each row and column contains the numbers 1 through 8 exactly once. The cells are colored in a repeating pattern of yellow, red, blue, green, purple, orange, light blue, and dark blue. A thick black diagonal line starts from the top-left cell (1) and ends at the bottom-right cell (8). A red cross highlights the main diagonal.

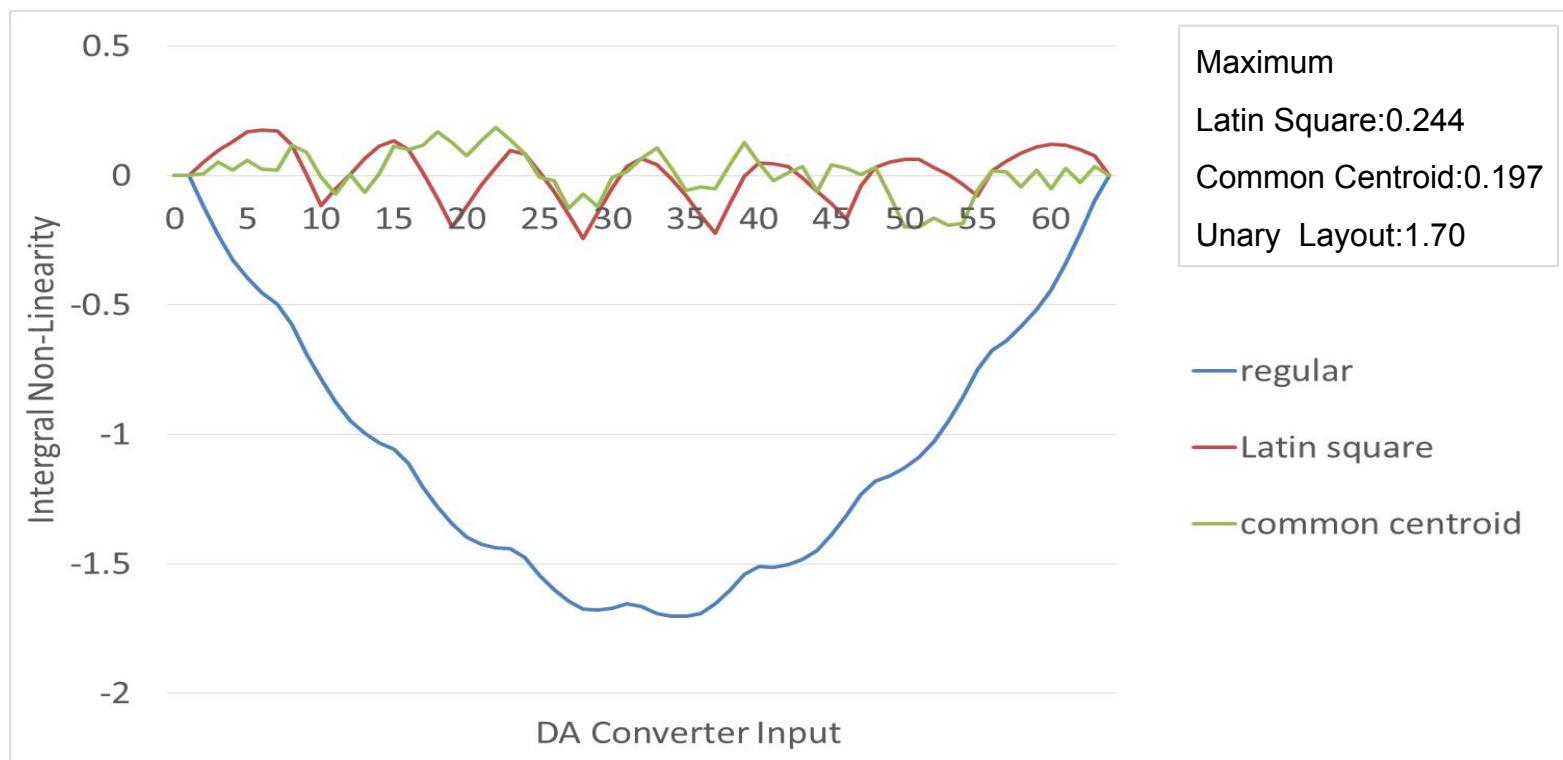
1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

Simulation Results (Linear Error Case)

- ◆ Standard Latin square
 - ✓ Linear Error

$$\varepsilon_l(x, y) = g_l * \cos \theta * x + g_l * \sin \theta * y$$

$$\theta = 30^\circ$$
$$g_l = 1$$



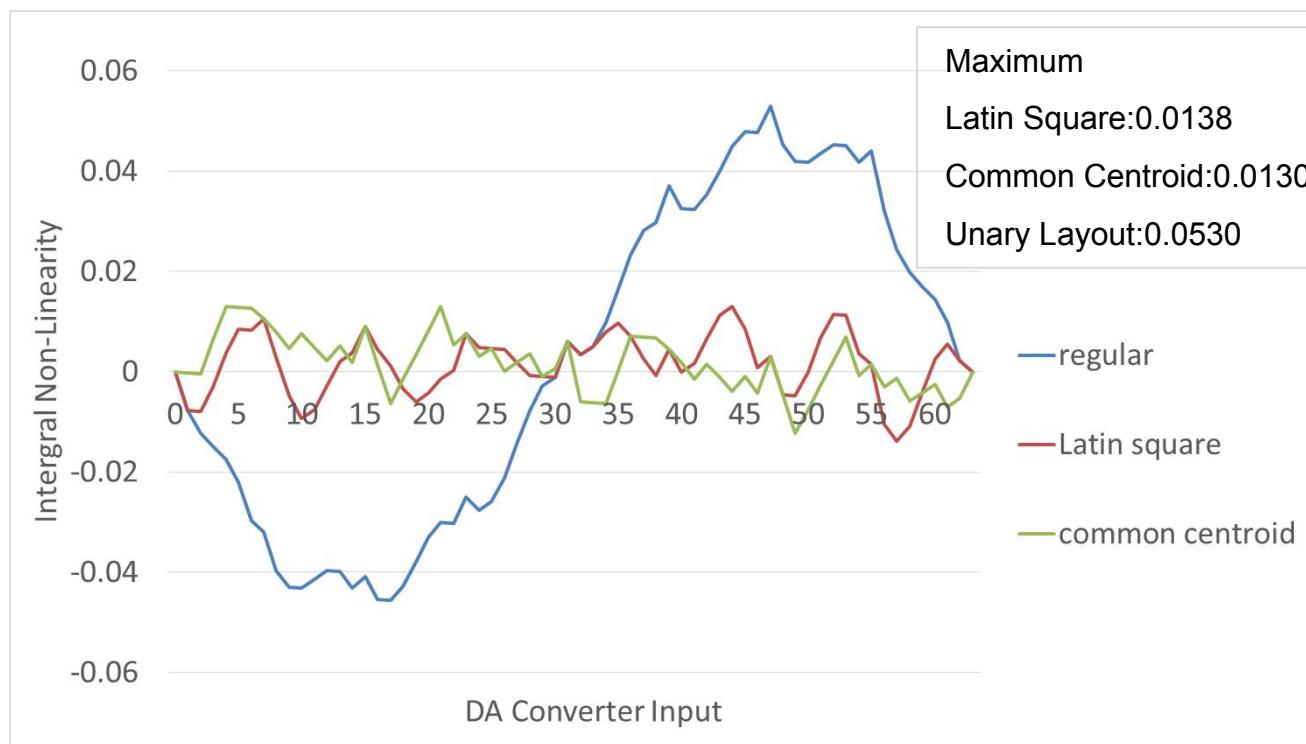
Simulation Results (Quadratic Error Case)

◆ Standard Latin square

- ✓ Quadratic Error

$$\varepsilon_q(x, y) = g_q * (x^2 + y^2) - a_0$$

$$g_q = 1, a_0 = 0$$



Summary

- Research Objective
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 - Latin Square
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Summary

Unary DAC linearity improvement

- Magic square layout
 - Linear error is larger,
Magic square is suitable.
 - Quadratic error is larger,
random walk is suitable.
- Latin square layout
 - Latin square can improve selecting current cells of common centroid.

Final Statement

温故知新

Classical mathematics can contribute
modern technology.



Leonhard Euler(1707~1783)

