

DAC Linearity Improvement With Layout Technique Using Latin and Magic Squares

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Contents

- Research Objective
- Segment Type DA Converter
- Characteristic of Variation in Circuit Element
- Proposed Layout Method
 - Magic Square
 - Latin Square
- Summary

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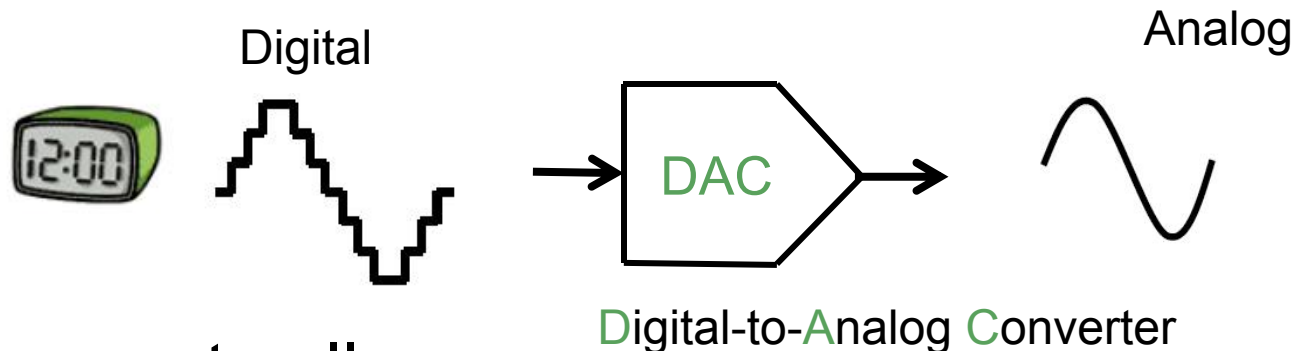
Research Objective

Research Background

- Requirements for electronic equipment
 - High speed
 - High precision

Our Approach

- Improvement the linearity of a segmented DAC



-Unit current cells.

-Layout based on **Magic and Latin Squares**

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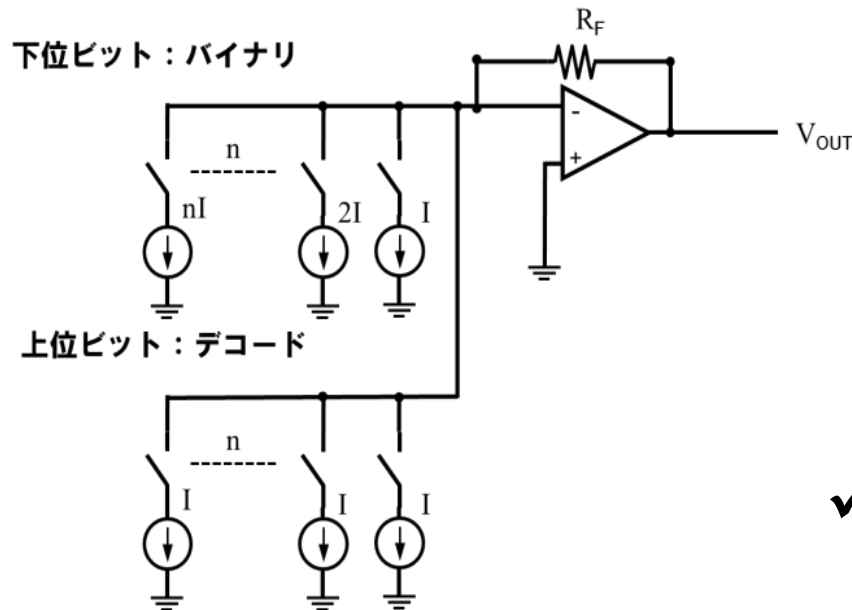
Segment Type DAC Configuration

✓ Binary (Lower bits)

- Small circuit
- Large glitch
- Large mismatch effect & Large nonlinearity

✓ Unary (Upper bits)

- Large circuit
- Small glitch
- Small mismatch effect & modest linearity



Segmented DAC

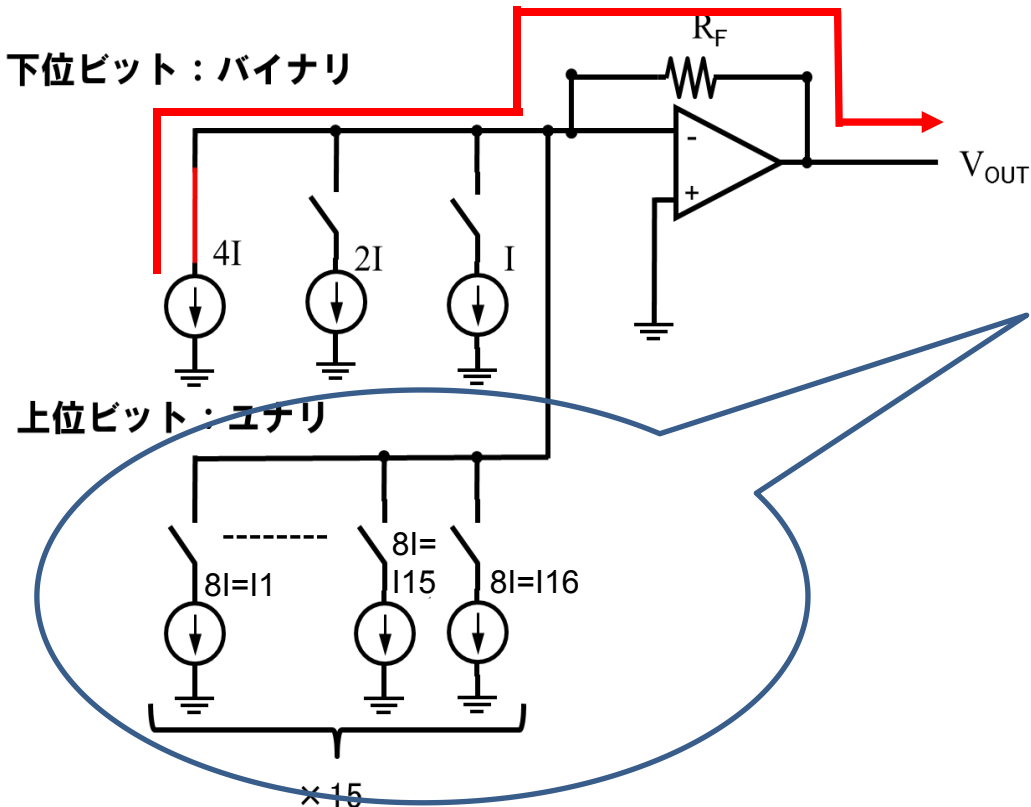
Segment Type DAC (7-bit case)

ex

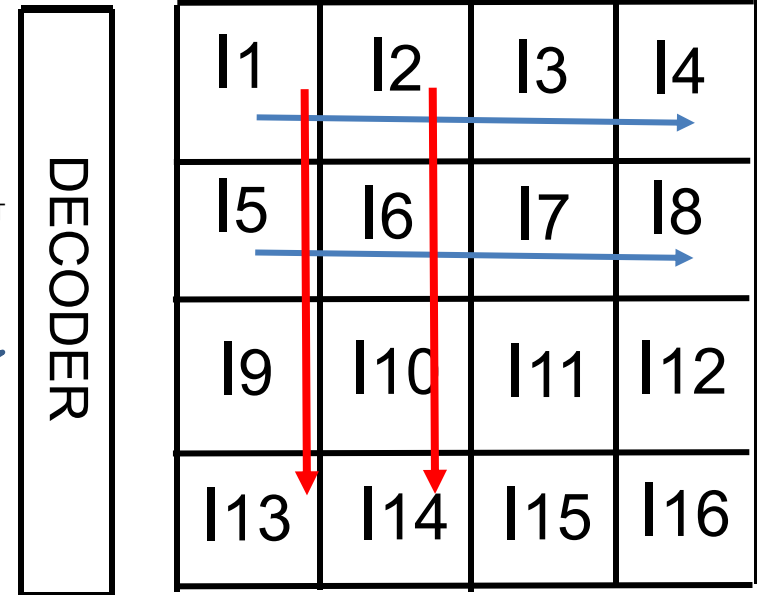
In case digital input = 4

(0000100)

$$V_{out} = 4IR_F$$



DECODER



Regular layout of
unit current cells

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Characteristic of Variation in Circuit Element

Manufacturing variation



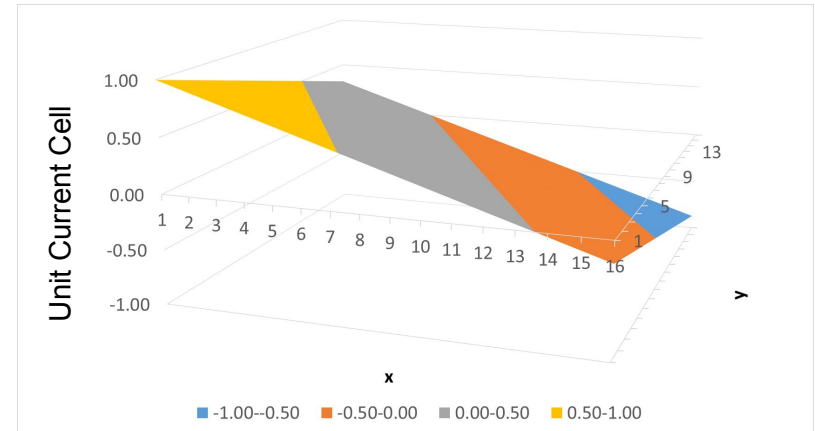
All unit current cells
I₁, I₂... I₁₆ are NOT identical

Systematic variations

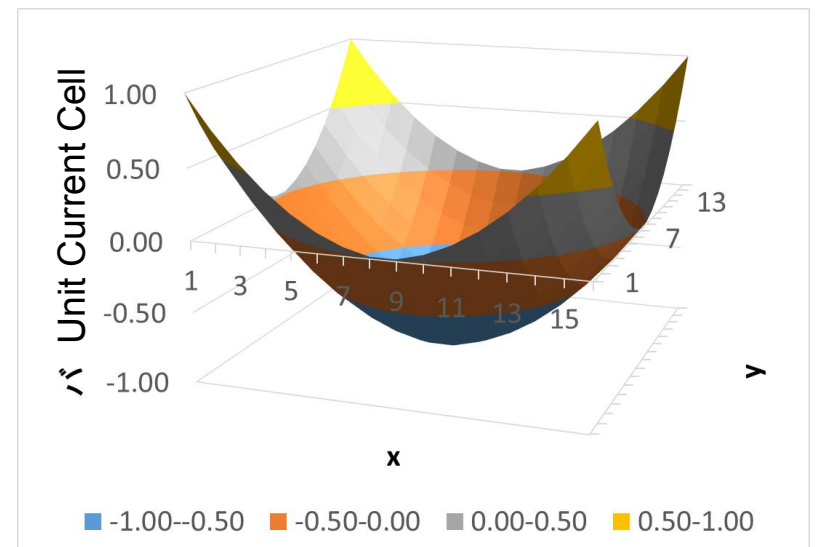


Unit current value depends
on its layout position

Linear error case



Quadratic error case



Characteristic of Variation in Circuit Element

Linear Error

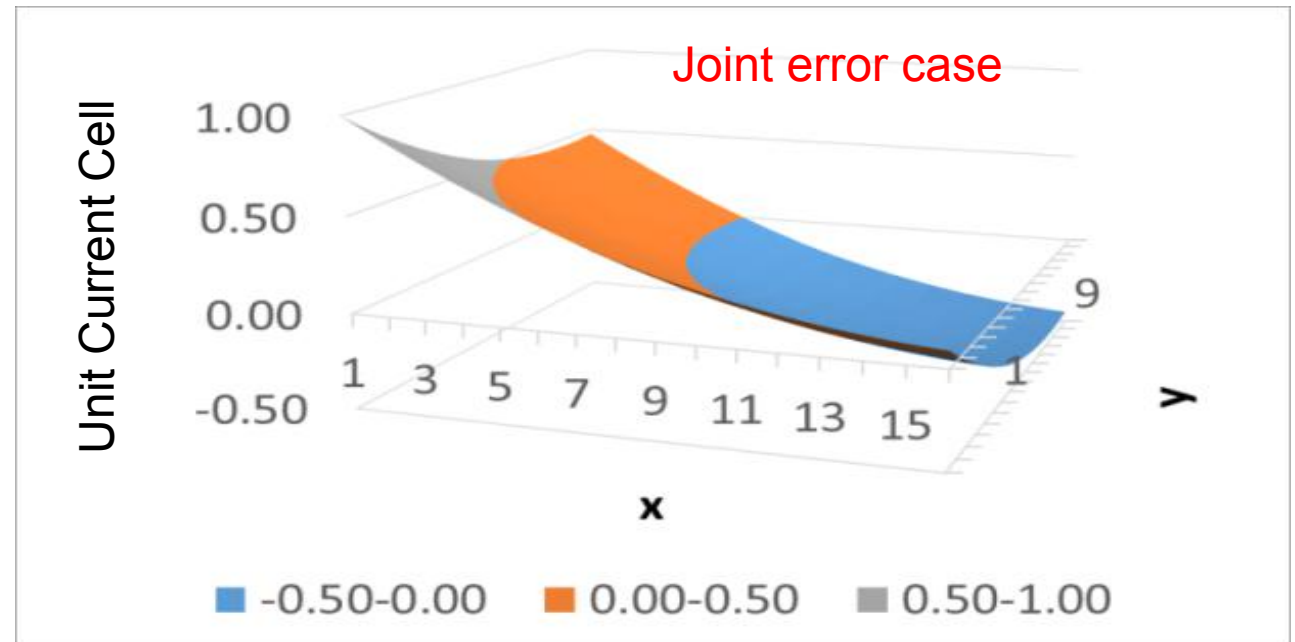
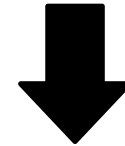
$$\varepsilon_l(x, y) = g_l * \cos \theta * x + g_l * \sin \theta * y$$

Quadratic Error

$$\varepsilon_q(x, y) = g_q * (x^2 + y^2) - a_0$$

Joint Errors

$$\varepsilon_j(x, y) = \varepsilon_l(x, y) + \varepsilon_q(x, y)$$



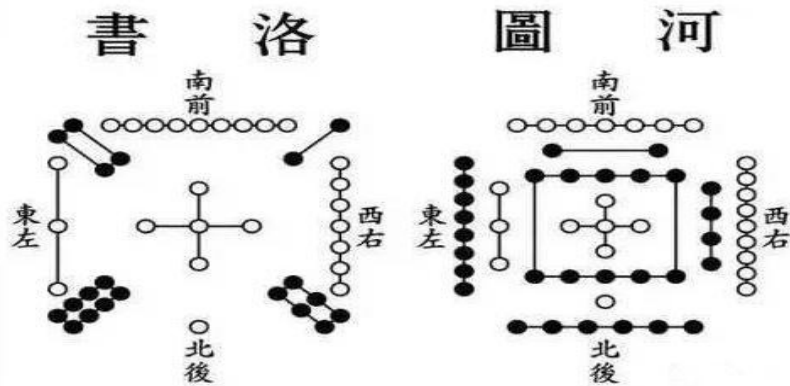
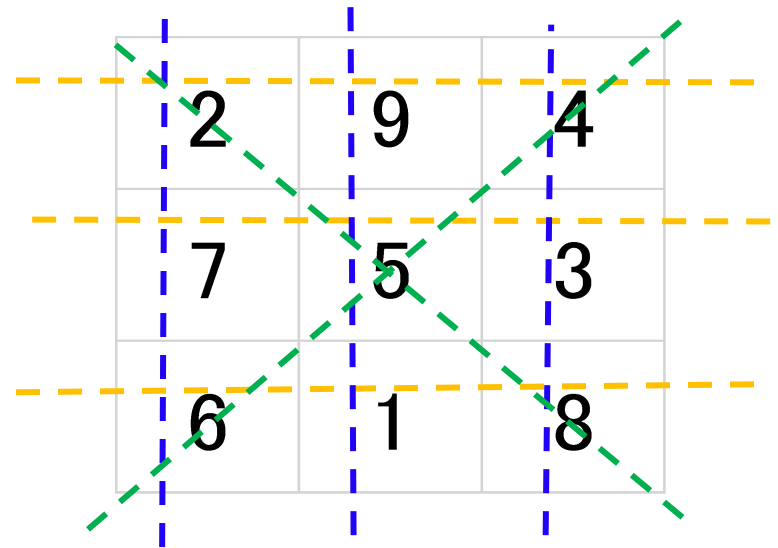
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What is Magic Square ?

- Classical mathematics
- Origin from Chinese academia
- “Constant sum” characteristics
- Varieties of magic squares

3 × 3 Magic Square



Constant Sum
Row, column, diagonal

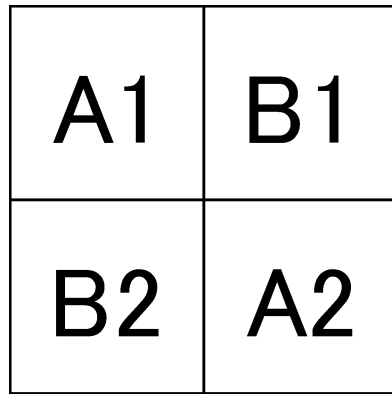


Magic Square has good balance
to cancel the systematic
mismatch effects of current cells

16x16 Current Cells Layout

◆ Concentric Magic Square

Even if one side is removed from the outside, it does not lose compatibility



A: Magic square of the left figure
B: 45 ° counterclockwise rotation

Current cell systematic error

Current cell systematic error

59	5	4	62	63	1	8	58	58	56	10	11	53	52	14	6
9	18	17	19	50	42	19	56	8	19	45	21	22	41	47	57
55	20	28	33	29	40	45	10	1	42	40	26	27	37	23	64
54	44	38	31	35	26	21	11	63	50	29	35	34	32	15	2
12	43	39	30	34	27	22	53	62	19	33	31	30	36	16	3
13	24	25	36	32	37	41	52	4	17	28	38	39	25	48	61
51	46	48	16	15	23	47	14	5	18	20	44	43	24	46	60
7	60	61	3	2	64	57	6	59	9	55	54	12	13	51	7
58	56	10	11	53	52	14	6	59	5	4	62	63	1	8	58
8	19	45	21	22	41	47	57	9	18	17	19	50	42	19	56
1	42	40	26	27	37	23	64	55	20	28	33	29	40	45	10
63	50	29	35	34	32	15	2	54	44	38	31	35	26	21	11
62	19	33	31	30	36	16	3	12	43	39	30	34	27	22	53
4	17	28	38	39	25	48	61	13	24	25	36	32	37	41	52
5	18	20	44	43	24	46	60	51	46	48	16	15	23	47	14
59	9	55	54	12	13	51	7	7	60	61	3	2	64	57	6

Random selection of current cells based on Magic Square layout

16x16 Current Cell Layout Details

◆ Concentric Magic Square

- algorithm

A1	B1
B2	A2

59	5	4	62	63	1	8	58	58	56	10	11	53	52	14	6
9	18	17	49	50	42	19	56	8	19	45	21	22	41	47	57
55	20	28	33	29	40	45	10	1	42	40	26	27	37	23	64
54	44	38	31	35	26	21	11	63	50	29	35	34	33	15	2
12	43	39	30	34	27	22	53	62	49	33	31	30	36	16	3
13	24	25	36	32	37	41	52	4	17	28	38	39	25	48	61
51	46	48	16	15	23	47	14	5	18	20	44	43	24	46	60
7	60	61	3	2	64	57	6	59	9	55	54	12	13	51	7
58	56	10	11	53	52	14	6	59	5	4	62	63	1	8	58
8	19	45	21	22	41	47	57	9	18	17	49	50	42	19	56
1	42	40	26	27	37	23	64	55	20	28	33	29	40	45	10
63	50	29	35	34	32	15	2	54	44	38	31	35	26	21	11
62	49	33	31	30	36	16	3	12	43	39	30	34	27	22	53
4	17	28	38	39	25	48	61	13	24	25	36	32	37	41	52
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16x16 Current Cell Layout Details

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12	43	39	30	34	27	22	62	49	33	31	30	36	16	3	
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1	42	40	26	27	37	23	64	55	20	28	33	29	40	45	10
63	50	29	35	34	32	15	2	54	44	38	31	35	26	21	11
62	49	33	31	30	36	16	3	12	43	39	30	34	27	22	53
4	17	28	38	39	25	48	61	13	24	25	36	32	37	41	52
5	18	20	44	43	24	46	60	51	46	48	16	15	23	47	14
59	9	55	54	12	13	51	7	7	60	61	3	2	64	57	6

16x16 Current Cell Layout Details

◆ Concentric Magic Square

- algorithm

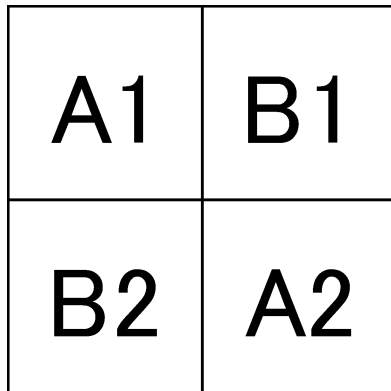
A1	B1
B2	A2

59	5	4	62	63	1	8	58	58	56	10	11	53	52	14	6
9	18	17	49	50	51	52	56	8	19	45	21	22	41	47	57
55	20	29	33	29	40	45	10	1	42	40	26	27	37	23	64
54	44	38	31	35	26	21	63	50	29	35	34	33	15	2	
12	43	39	30	34	27	22	62	49	33	31	30	36	16	3	
13	24	25	36	32	37	41	52	4	17	28	38	39	25	48	61
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7	60	61	3	2	64	57	6	59	9	55	54	12	51	7	
53	56	10	11	53	52	14	6	59	5	4	62	63	1	8	58
8	19	45	21	22	41	47	57	9	18	20	44	43	24	46	60
1	42	40	26	27	37	23	64	55	20	28	33	29	40	45	10
63	50	29	35	34	32	15	2	54	44	38	31	35	26	21	11
62	49	33	31	30	36	16	3	12	43	39	30	34	27	22	53
4	17	28	38	39	25	48	61	13	24	25	36	32	37	41	52
5	18	20	44	43	24	46	60	51	46	48	16	15	23	47	14
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16x16 Current Cell Layout Details

◆ Concentric Magic Square

- algorithm



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12	43	39	30	34	27	22	51	62	49	33	31	30	36	16	3
13	24	25	36	32	37	41	52	4	17	28	38	39	25	8	61
5	46	48	16	15	23	47	44	5	18	20	44	43	24	13	60
7	60	61	3	2	64	57	6	59	9	55	54	12	13	7	7
58	56	10	11	52	14	6	59	5	4	62	63	1	58	58	
19	45	21	22	1	47	57	9	19	17	49	51	12	19	56	
1	42	40	26	27	7	23	64	55	20	28	33	29	40	45	10
63	50	35	34	32	15	2	54	44	38	31	35	26	21	11	
62	49	33	31	30	36	16	3	12	43	39	30	34	27	22	53
4	17	28	38	39	25	48	61	13	24	25	36	32	37	41	52
5	18	20	44	43	24	46	60	51	46	48	16	15	23	47	14
59	9	55	54	12	13	51	7	7	60	61	3	2	64	57	6

Simulation Results (Linear Error Case)

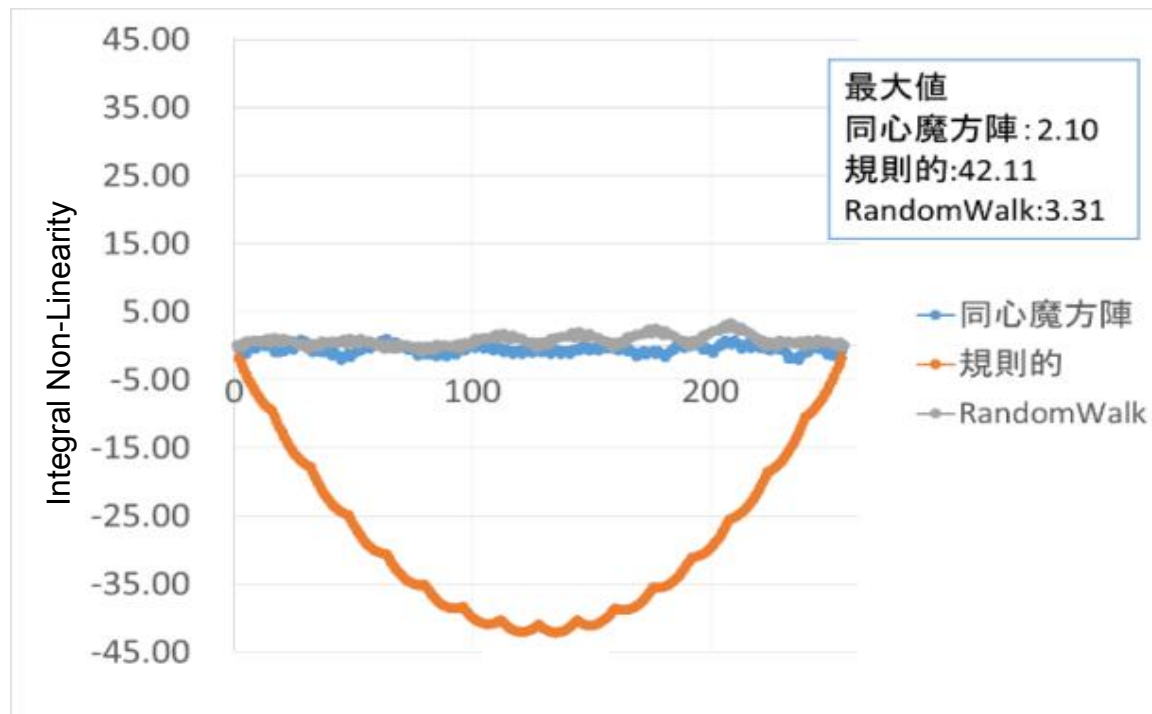
◆ Concentric Magic Square

✓ Linear Error (Current Cell Systematic Mismatch)

$$\varepsilon_l(x, y) = g_l * \cos \theta * x + g_l * \sin \theta * y$$

$$\theta = 30^\circ$$

$$g_l = 1$$



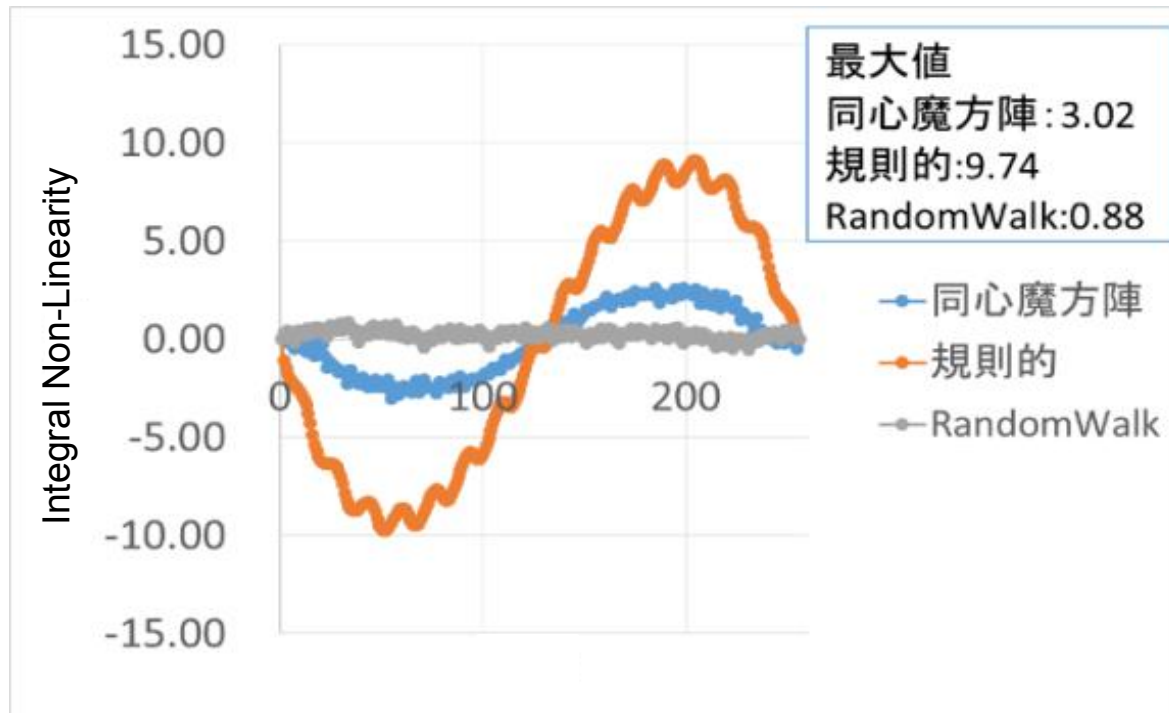
Simulation Results (Quadratic Error Case)

◆ Concentric Magic Square

✓ Quadratic Error (Current Cell Systematic Mismatch)

$$\varepsilon_q(x, y) = g_q * (x^2 + y^2) - a_0$$

$$g_q = 1, a_0 = 0$$

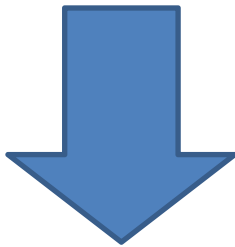


Simulation Results (Joint Error Case)

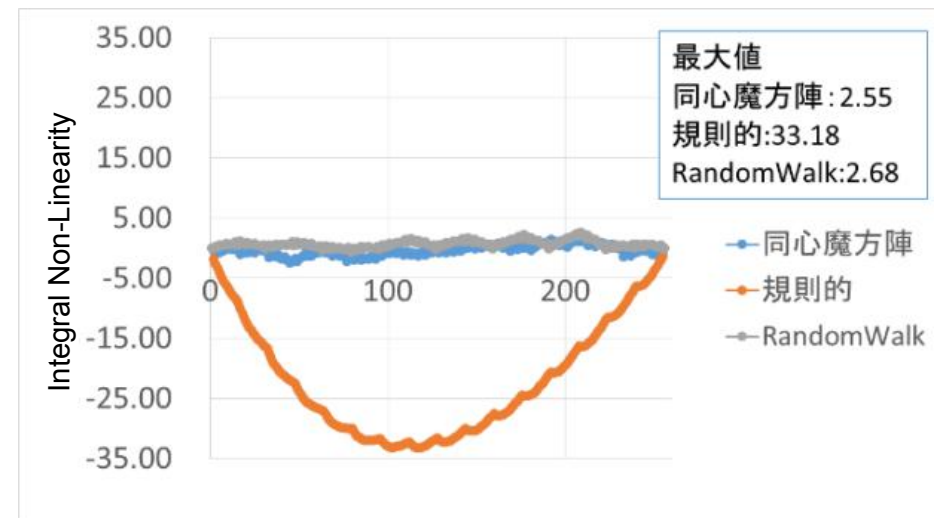
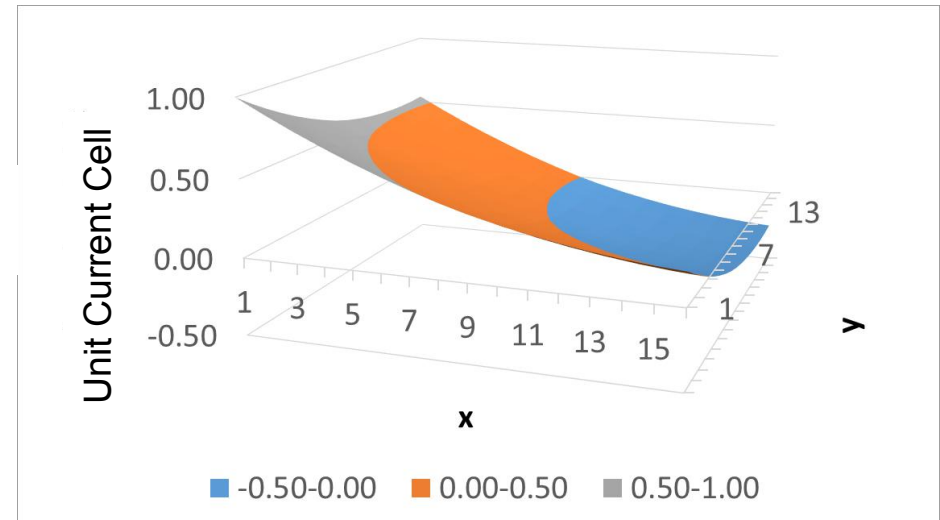
◆ Concentric Magic Square

✓ Joint Error

Linear > Quadratic case



Magic square is better

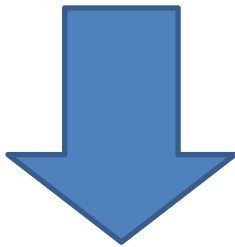


Simulation Results (Joint Error Case)

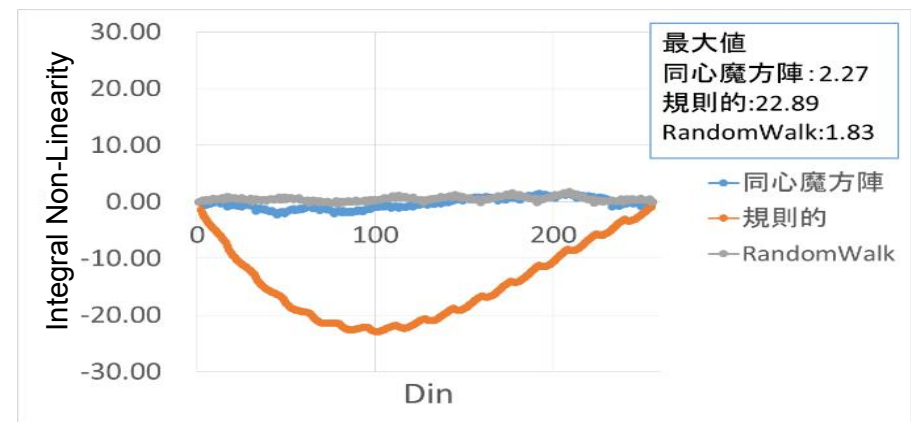
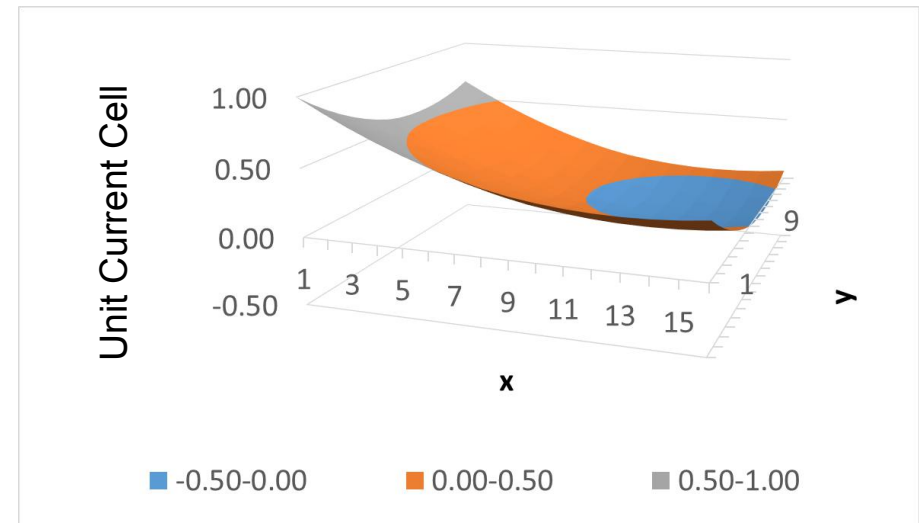
◆ Concentric Magic Square

✓ Joint Error

Linear < Quadratic case



Random Walk is better



What is Latin Square ?

- Each symbols occurring exactly once in each row and column
- In the Latin square , If the first row is(1,2,3,,n) and the first column is (1,2,3,,n), it is a **standard Latin square**



Example:

1	2	3	4
2	3	4	1
3	4	1	2
4	1	2	3

4 × 4 Latin square

1	2	3	4	5	6	7	8
2	3	4	5	6	7	8	1
3	4	5	6	7	8	1	2
4	5	6	7	8	1	2	3
5	6	7	8	1	2	3	4
6	7	8	1	2	3	4	5
7	8	1	2	3	4	5	6
8	1	2	3	4	5	6	7

8 × 8 standard Latin square

Leonhard Euler(1707-1783)
Swiss mathematician, physicist

Latin Square Layout Algorithm

◆ algorithm

- Latin Square
- Common Centroid
- Unary Layout

◆ Compare

- Linear Error
- Quadratic Error

1	7	4	6	2	8	3	5
7	2	5	8	4	1	6	3
4	5	2	3	7	6	1	8
6	8	3	1	5	7	4	2
2	4	7	5	1	3	8	6
8	1	6	7	3	2	5	4
3	6	1	4	8	5	2	7
5	3	8	2	6	4	7	1

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

Simulation Results (Linear Error Case)

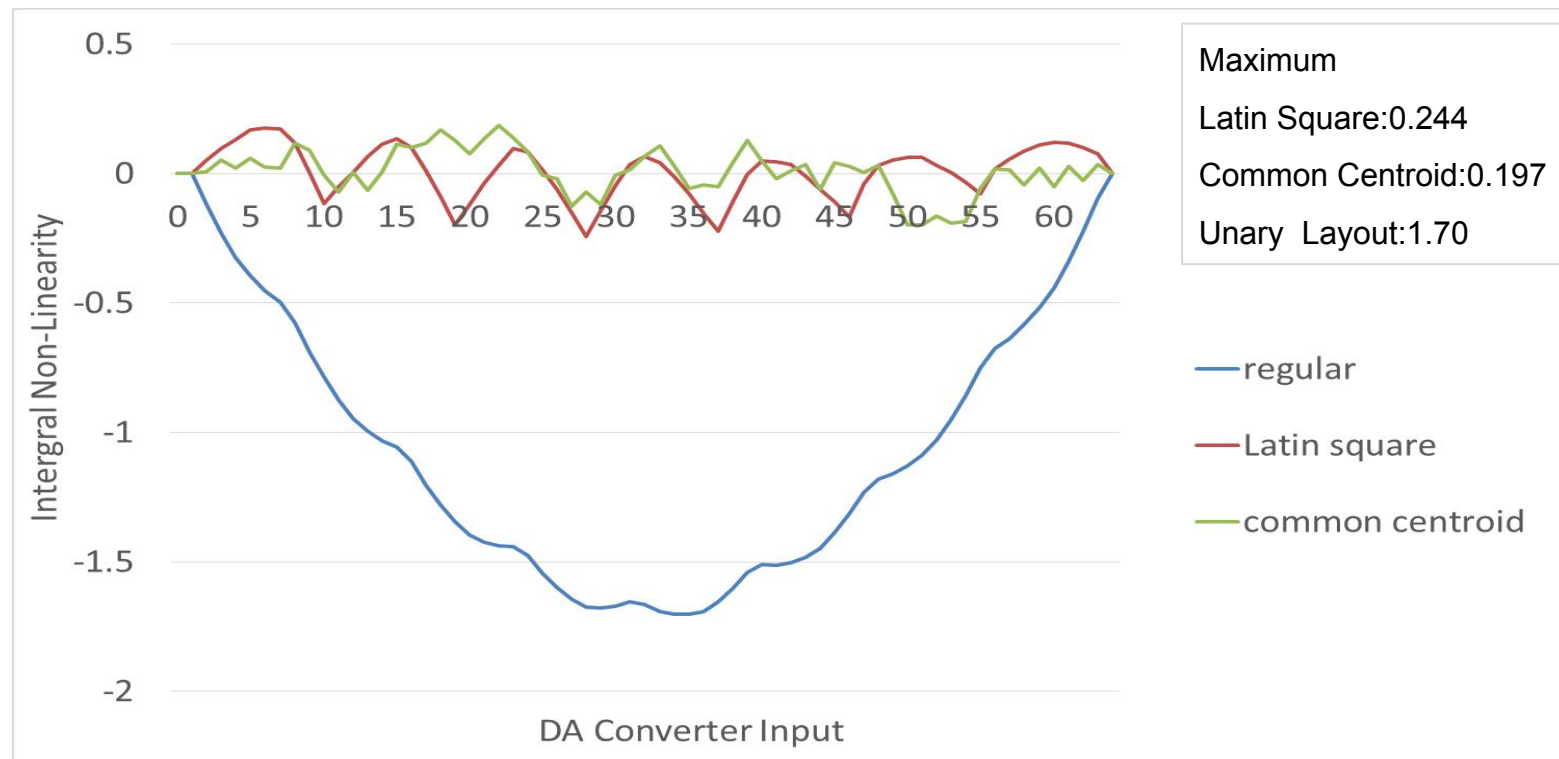
◆ Standard Latin square

✓ Linear Error

$$\varepsilon_l(x, y) = g_l * \cos \theta * x + g_l * \sin \theta * y$$

$$\theta = 30^\circ$$

$$g_l = 1$$



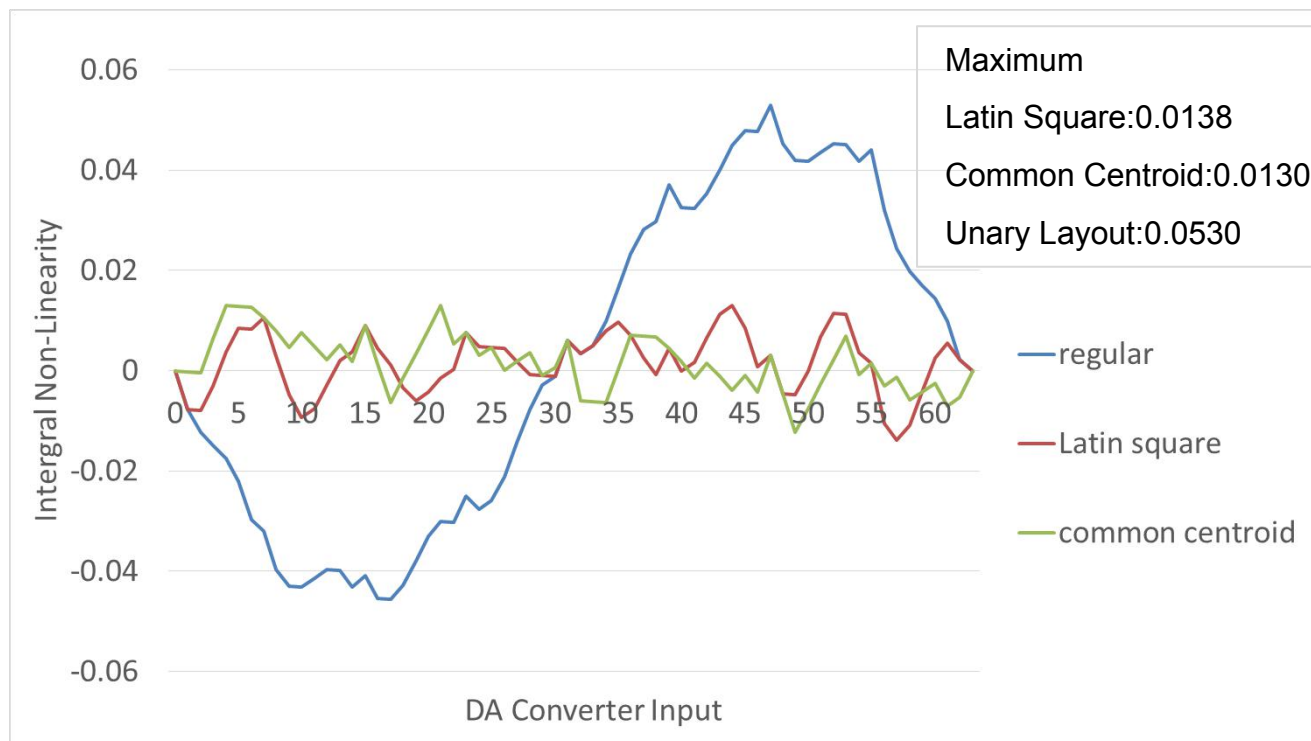
Simulation Results (Quadratic Error Case)

◆ Standard Latin square

✓ Quadratic Error

$$\varepsilon_q(x, y) = g_q * (x^2 + y^2) - a_0$$

$$g_q = 1, a_0 = 0$$



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Summary

Unary DAC linearity improvement

- Magic square layout
 - Linear error is larger,
Magic square is suitable.
 - Quadratic error is larger,
random walk is suitable.
- Latin square layout
 - Latin square can improve selecting current cells of common centroid.

Final Statement

温故知新

Classical mathematics can contribute modern technology.



5	1	6	9	2	7	4	8	3	
9	4	3	1	6	5	2	7	8	
3	8	2	5	7	4	6	1	9	
7	5	1	4	8	6	3	9	2	
6	3	9	7	5	1	8	2	4	
8	7	4	2	9	3	5	6	1	
4	2	8	3	1	9	7	5	6	
1	6	7	8	3	2	9	4	5	
2	9	5	6	4	8	1	3	7	



Leonhard Euler (1707~1783)

