S0380 S63-2 Analog Circuits IV
14:00-14:15
Oct.28

# RC Polyphase Filter as Complex Analog Hilbert Filter


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# Outline

- Research Objective
  - RC Polyphase Filter
  - Hilbert Filter
- Analysis Method of RC Polyphase Filter Characteristics
- Relevance of RC Polyphase Filter and Hilbert Filter
- Conclusion

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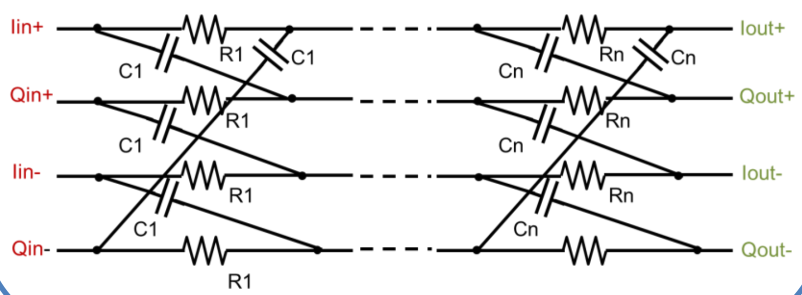
# Research Objective

RC Polyphase Filter

Analog

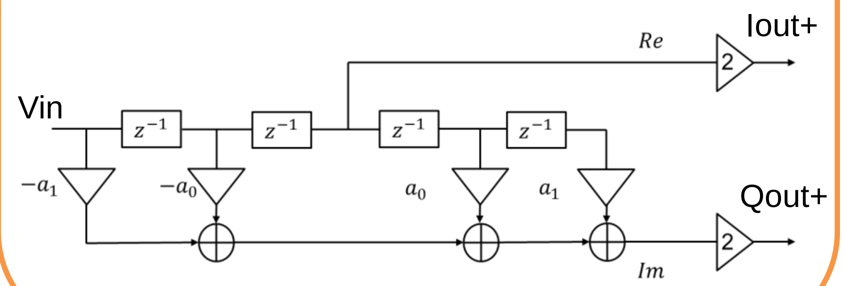
Complex (I, Q) input




Hilbert Filter

Digital

Real part (Vin) input




Analyze RC polyphase filter

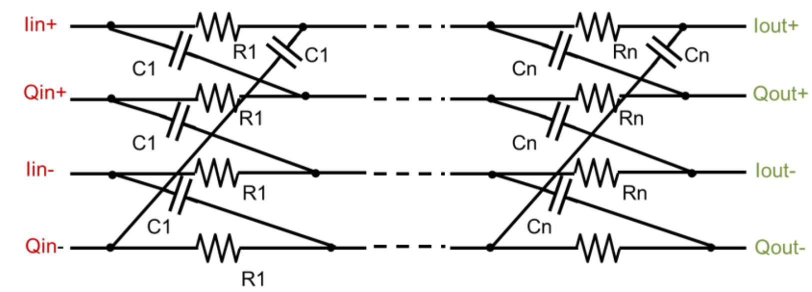
**We found that relevance between**

**RC polyphase filter and Hilbert filter**

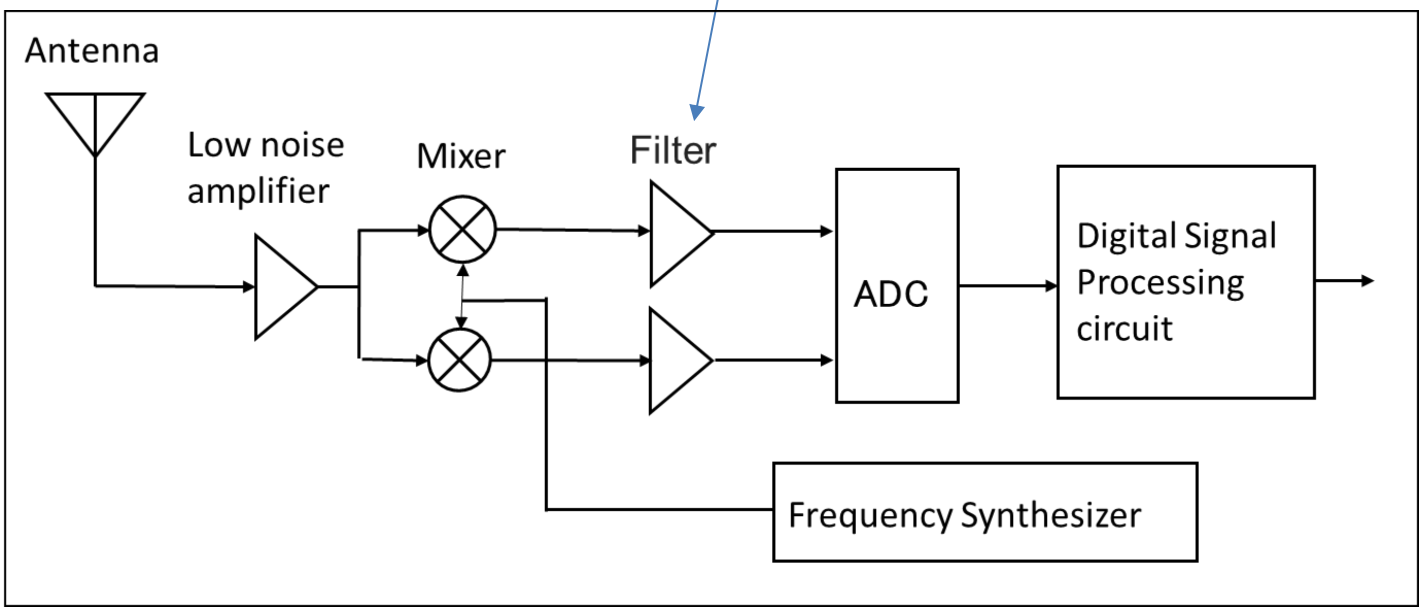
# RC Polyphase Filter

Passive analog bandpass filter

Complex signal processing




n-th order RC polyphase filter

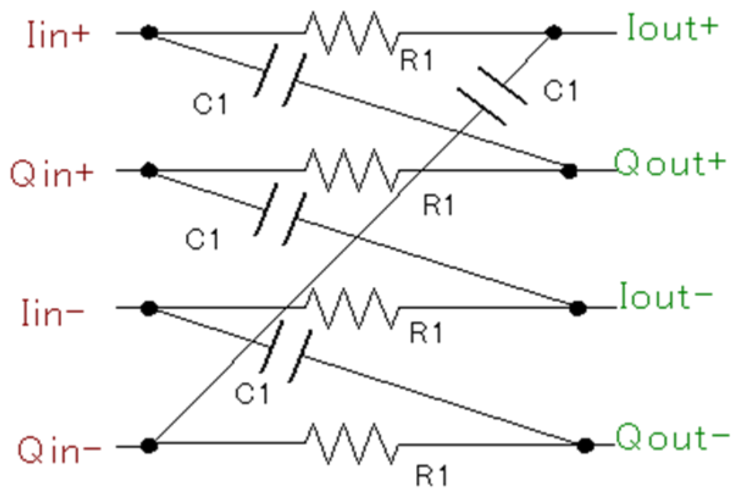



Wireless communication receiver

# What is Polyphase?

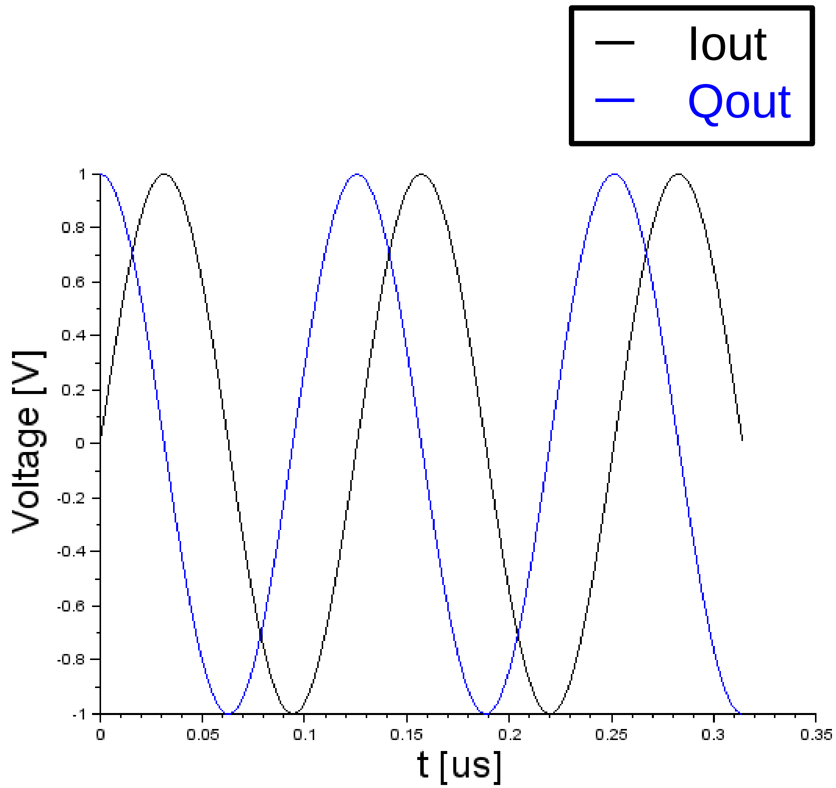
I/Q signal input and output

The same frequency, different phases




1st order RC polyphase filter

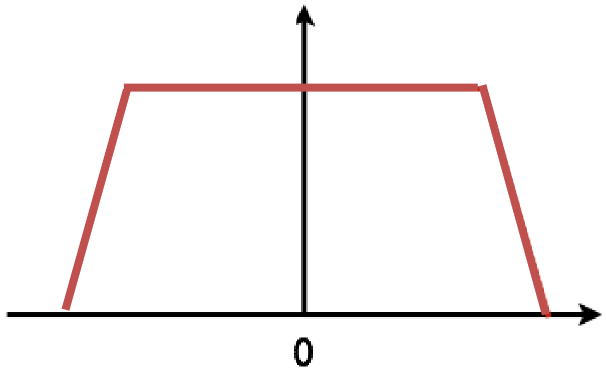
I(t):In-phase  Q(t):Quadrature



# Roles of RC Polyphase Filter

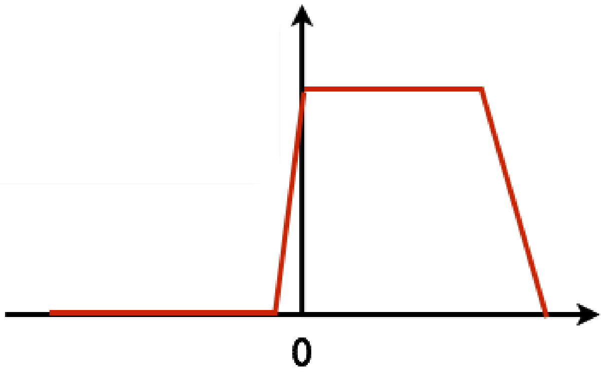
- Orthogonal waveform generation
- Image signal rejection

→ Considering negative frequency




Real filter

Only positive frequencies




Complex filter

positive and negative frequencies

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- Analysis Method of RC Polyphase Filter Characteristic
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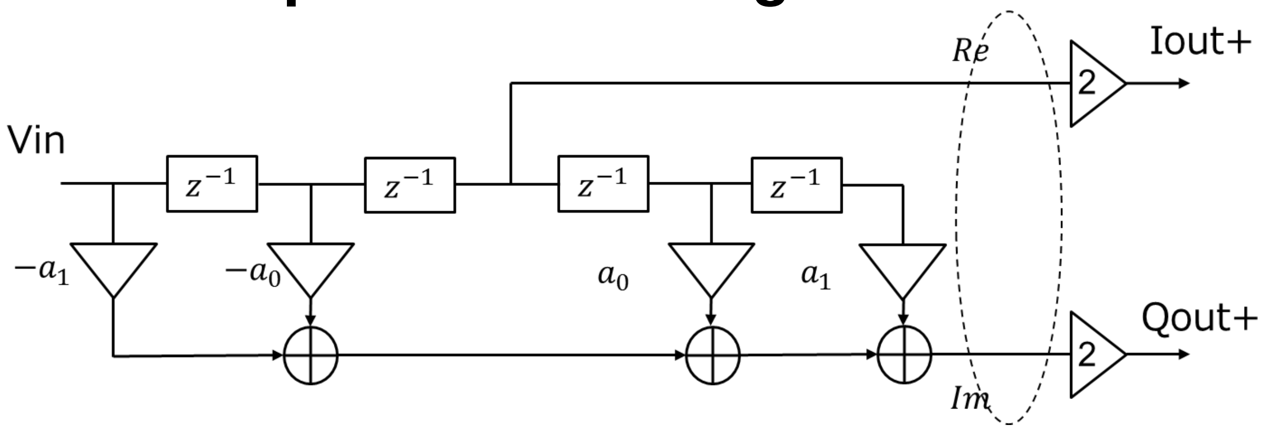
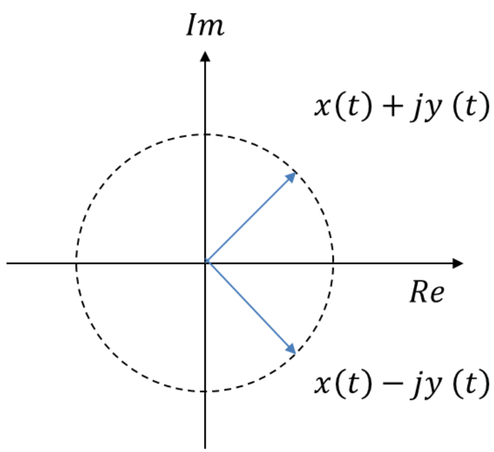
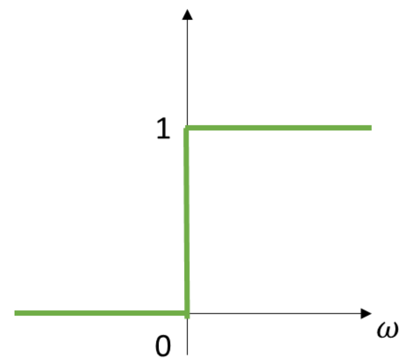
# Hilbert Filter

- Characteristics
  - Hilbert transform
  - 1 input and 2 outputs
  - **It is often implemented in digital filter**

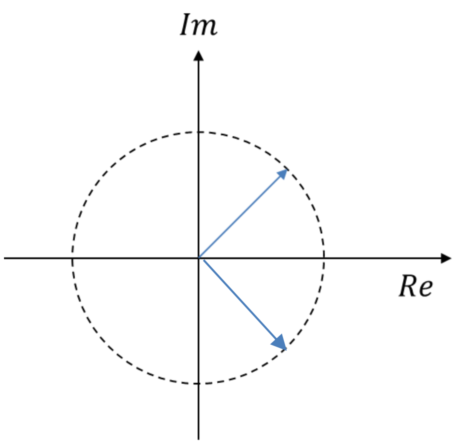



Phase




Gain



# Hilbert Transform

Complex signal from real signal $x\,(t)$

$$x(t) \rightarrow x(t) + jy\,(t)$$

Hilbert transform

$$y(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}d\tau = x(t) * \frac{1}{\pi t}$$

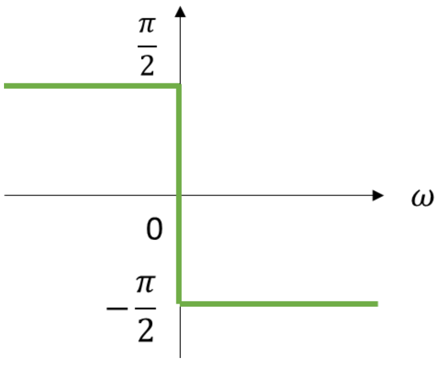
David Hilbert
1862-1943

Impulse response Fourier Transform

$$h(t) = \frac{1}{\pi t} \quad \overset{Fourier}{\Longleftrightarrow} \quad \boldsymbol{H(\omega)} = \begin{cases} -\boldsymbol{j} & (\boldsymbol{\omega} \geq \boldsymbol{0}) \\ \boldsymbol{j} & (\boldsymbol{\omega} < \boldsymbol{0}) \end{cases}$$

Frequency characteristic $H\,(\omega)$

$$\boldsymbol{Y(\omega)} = \boldsymbol{H(\omega)X(\omega)} = \begin{cases} -\boldsymbol{jX(\omega)} & (\boldsymbol{\omega} \geq \boldsymbol{0}) \\ \boldsymbol{jX(\omega)} & (\boldsymbol{\omega} < \boldsymbol{0}) \end{cases}$$




Phase

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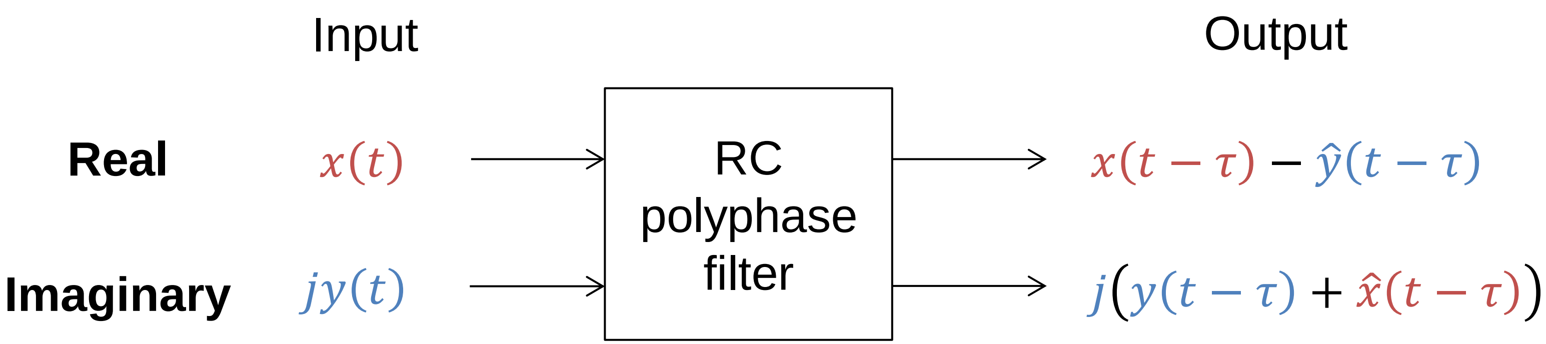
# System Model : Time Domain




$$h(t) = h_{re}(t) + jh_{im}(t)$$

Impulse response

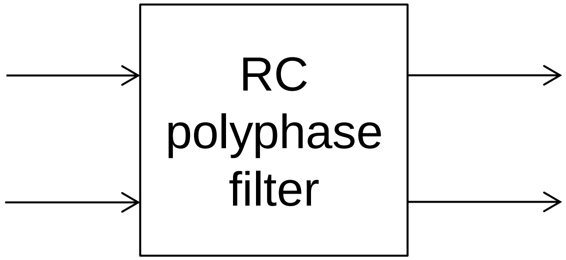
Filter output

$$\big(x(t) + jy(t)\big) \otimes \big(h_{re}(t) + jh_{im}(t)\,\big)$$
$$= \big(x(t) \otimes h_{re}\,(t) - y(t) \otimes h_{im}\,(t)\big) + j\big(y(t) \otimes h_{re}(t) + x(t) \otimes h_{im}(t)\,\big)$$

$\otimes$ : Convolution

# System Model : Frequency Domain

| | Input | | Output |
|---|---|---|---|
| **Real** | $X(j\omega)$ | → RC polyphase filter → | $X(j\omega)H_{re}(j\omega) - Y(j\omega)H_{im}(j\omega))$ |
| **Imaginary** | $jY(j\omega)$ | → RC polyphase filter → | $j\big(Y(j\omega)H_{re}(j\omega) + X(j\omega)H_{im}(j\omega)\big)$ |




$$H(j\omega) = H_{re}(j\omega) + jH_{im}(j\omega)$$

Fourier transform

Filter output

$$\big(X(j\omega) + jY(j\omega)\big) \cdot (H_{re}(j\omega) + jH_{im}(j\omega)\ )$$
$$= \big(X(j\omega)H_{re}(j\omega) - Y(j\omega)H_{im}(j\omega)\big) + j(Y(j\omega)H_{re}(j\omega) + X(j\omega)H_{im}(j\omega)\ )$$

$X(j\omega), Y(j\omega), H_{re}(j\omega), H_{im}(j\omega)$ :Complex function

# Proposed Method

In general, transfer function of analog filter : $H\ (j\omega) = \dfrac{V_{out}}{V_{in}}$

Frequency transfer function

$$H\ (j\omega) = H_{re}(j\omega) + jH_{im}(j\omega)$$

Divide

$$\begin{cases} H_{re}(j\omega) = \dfrac{1}{2}\left(H(j\omega) + H^*(-j\omega)\right) \\ jH_{im}(j\omega) = \dfrac{1}{2}\left(H(j\omega) - H^*(-j\omega)\right) \end{cases}$$

**Simulate $\boldsymbol{H(j\omega)}$ using $\boldsymbol{H_{re}(j\omega)}$ and $\boldsymbol{jH_{im}(j\omega)}$**

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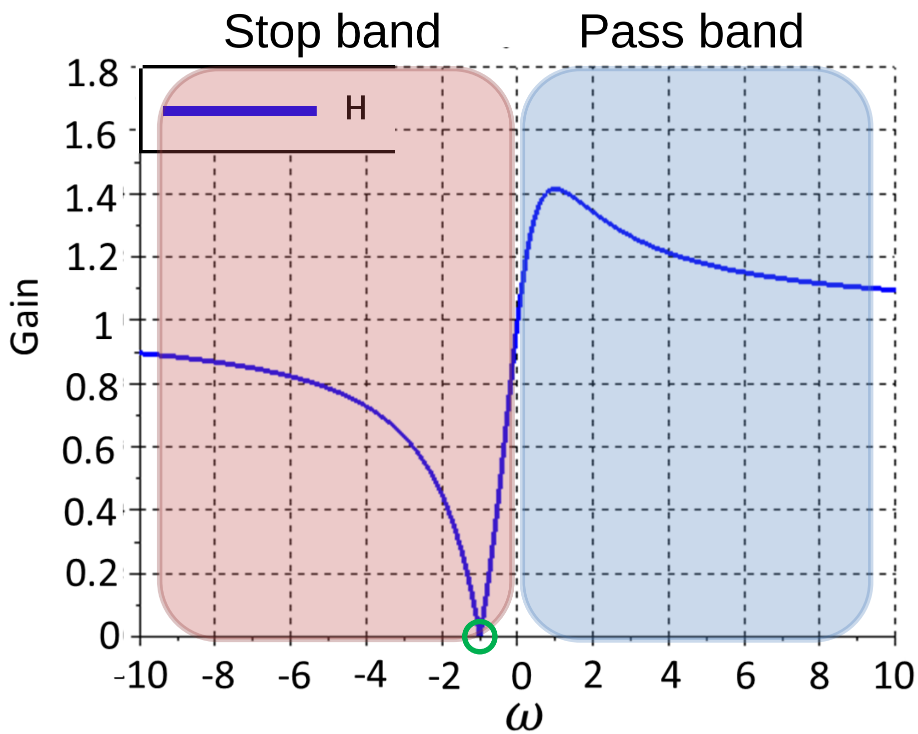
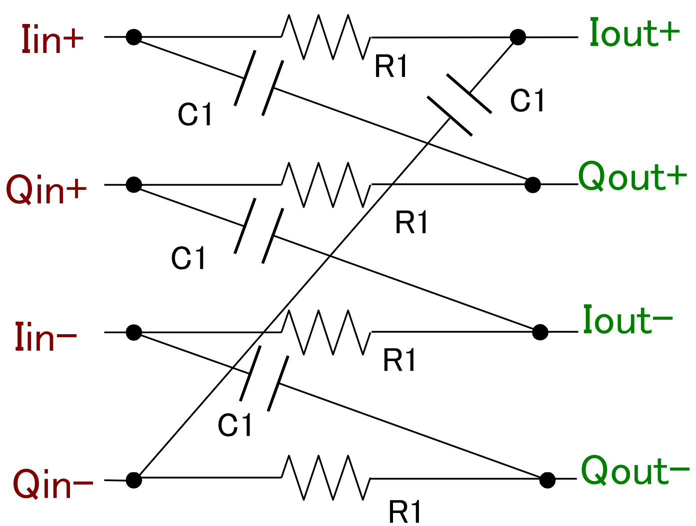
# 1st order RC Polyphase Filter : Analysis

$$H_1(j\omega) = \frac{1+\omega R_1 C_1}{1+j\omega R_1 C_1}$$ : Transfer function



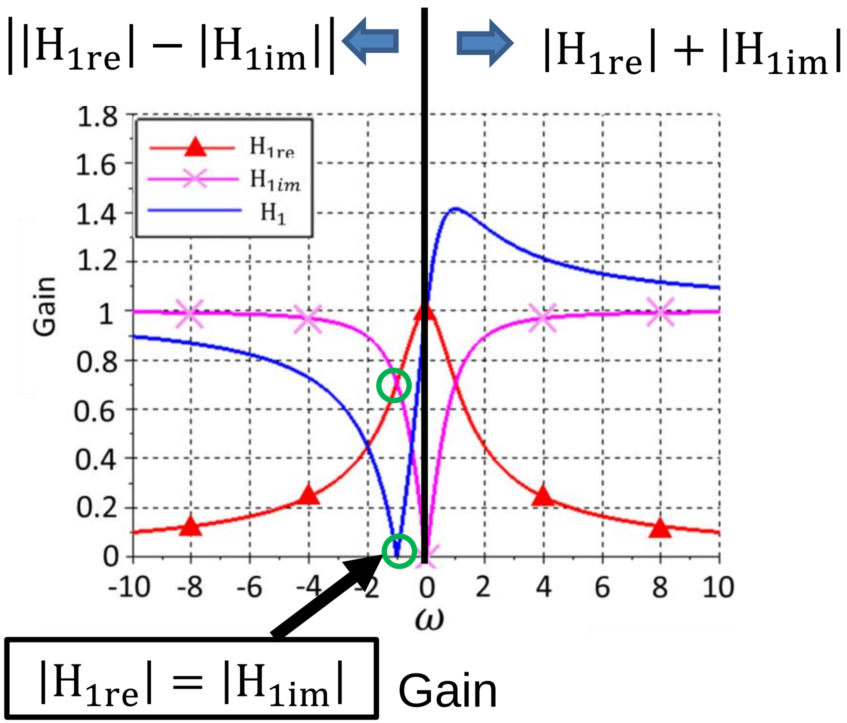

**Zero:** $\omega_k = \frac{1}{R_k C_k}$

# $1^{st}$ order RC Polyphase Filter : Gain and Phase

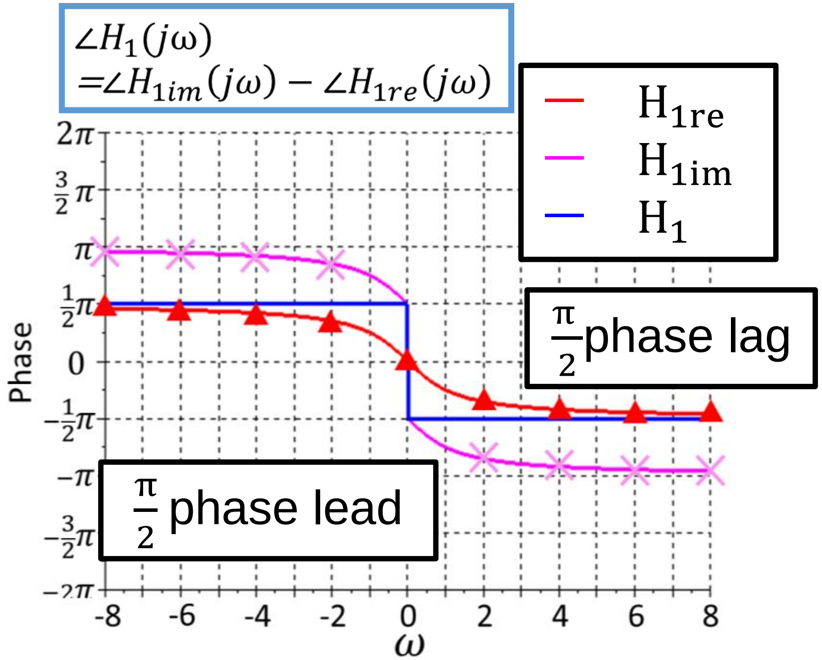
$$H_1(j\omega) = H_{1re}(j\omega) + jH_{1im}(j\omega)$$

$$H_{1re}(j\omega) = \frac{H_1(j\omega) + H_1^*(-j\omega)}{2} = \frac{1}{1 + j\omega R_1 C_1}$$

$$H_{1im}(j\omega) = \frac{H_1(j\omega) - H_1^*(-j\omega)}{2} = -j\frac{\omega R_1 C_1}{1 + j\omega R_1 C_1}$$




Gain



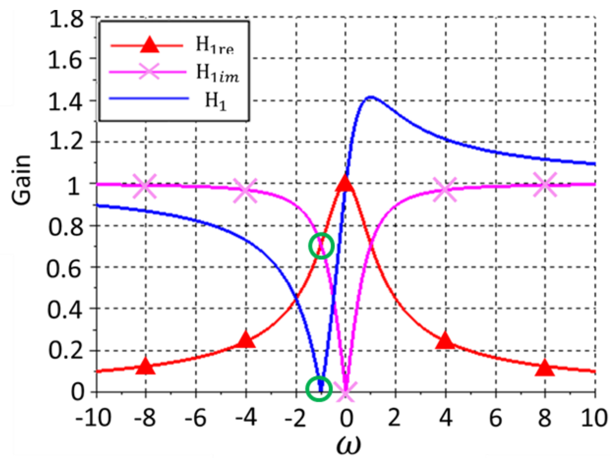

Phase

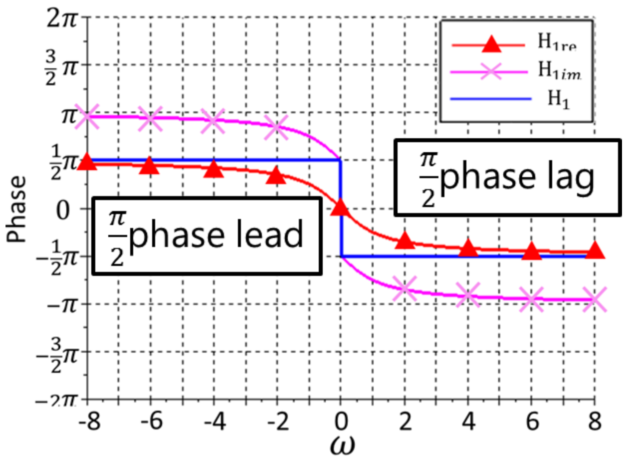
# 1st order case Analysis Results

Gain : Hilbert filter only at zero

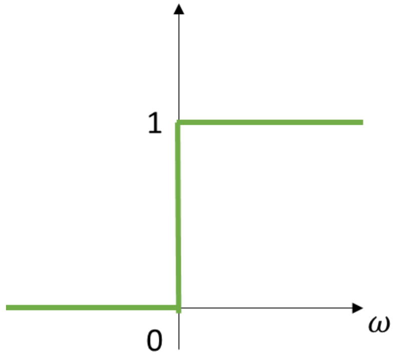
Phase : Completely Hilbert filter

RC Polyphase Filter

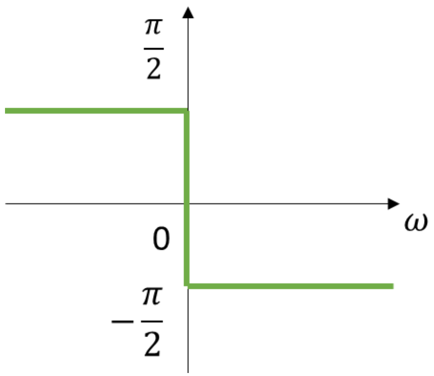



Hilbert filter




Gain




Phase

# Results : 2nd to 4th RC Polyphase Filter

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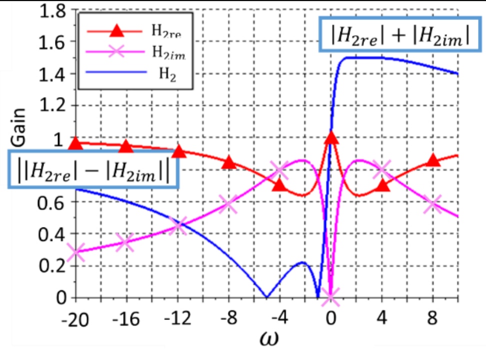
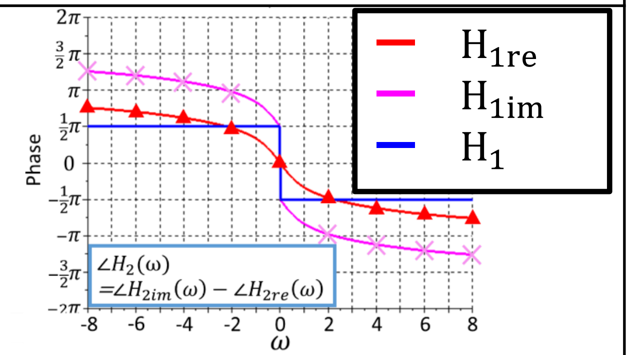
<tr><td>2nd</td><td>
</td><td>
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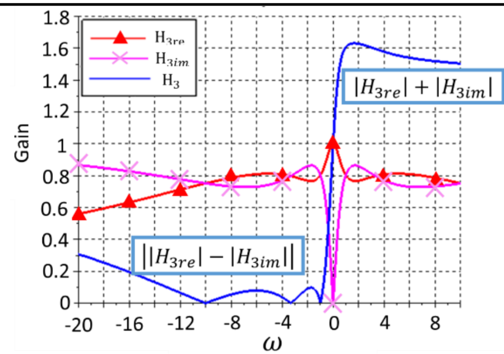
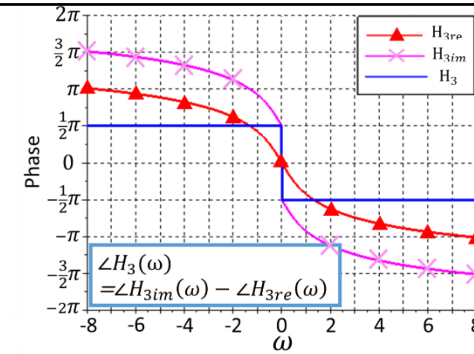
<tr><td>3rd</td><td>
</td><td>
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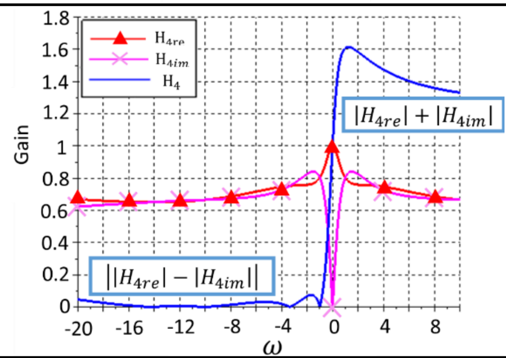
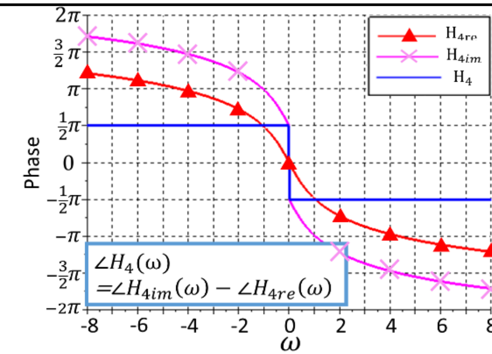
<tr><td>4th</td><td>
</td><td>
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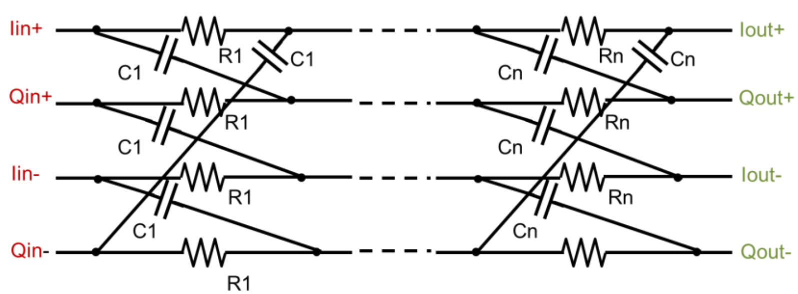
# Analysis Results and Consideration

1st to 4th order RC Polyphase Filter Analysis results

Gain : Hilbert filter only at zero

Phase : Completely Hilbert filter

Prove for general n-th order case
(n = 1, 2, 3, 4, 5, ...)



# n-th order case : Estimate of Transfer Function

Numerator → n zeros

Denominator → n-th Polynomial (j$\omega$)

$$\mathrm{H}_{(\mathrm{n})}(\mathrm{j}\omega) = \frac{(1+\omega \mathrm{R}_1\mathrm{C}_1)(1+\omega \mathrm{R}_2\mathrm{C}_2)\cdots(1+\omega \mathrm{R}_\mathrm{n}\mathrm{C}_\mathrm{n})}{1+(\mathrm{j}\omega)^1 \mathrm{a}_1+\cdots+(\mathrm{j}\omega)^\mathrm{n}\mathrm{a}_\mathrm{n}} = \frac{\mathrm{N}_{(\mathrm{n})}(\mathrm{j}\omega)}{\mathrm{D}_{(\mathrm{n})}(\mathrm{j}\omega)}$$

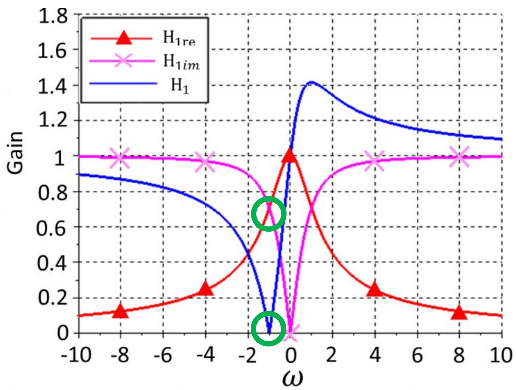
$$\mathrm{H}_1(\mathrm{j}\omega) = \mathrm{H}_{1\mathrm{re}}(\mathrm{j}\omega) + \mathrm{j}\mathrm{H}_{1\mathrm{im}}(\mathrm{j}\omega)$$

$$\omega_\mathrm{k} = 1/\mathrm{R}_\mathrm{k}\,\mathrm{C}_\mathrm{k} \quad (k = 1,2,\ldots,n)$$

# n-th order RC Polyphase Filter : Gain

$$\left|H_{(n)}(j\omega)\right| = \frac{\prod_{k=1}^{n}\left|\left(1 + j\frac{\omega}{\omega_k}\right)\right|}{\left|D_{(n)}(j\omega)\right|} = \left|\frac{N_{(n)}(j\omega)}{D_{(n)}(j\omega)}\right|$$

$$\omega_k = 1/R_k C_k \quad (k = 1,2,\ldots,n)$$




Negative frequency domain

$$N_{(n)}(-j\omega_k) = N_{(n)re}(-j\omega_k) + jN_{(n)im}(-j\omega_k)$$
$$= 0$$

Positive frequency domain

$$N_{(n)}(j\omega_k) = N_{(n)re}(j\omega_k) + jN_{(n)im}(j\omega_k)$$

$$\left|N_{(n)}(-j\omega_k)\right| = \left|N_{(n)re}(-j\omega_k)\right| - \left|jN_{(n)im}(-j\omega_k)\right|$$
$$= 0$$

$$\left|N_{(n)}(j\omega_k)\right| = \left|N_{(n)re}(j\omega_k)\right| + \left|N_{(n)im}(j\omega_k)\right|$$

# n-th order RC Polyphase Filter : Gain in Zeros

$$\left|H_{(n)}(j\omega)\right| = \frac{\prod_{k=1}^{n}\left|\left(1 + j\frac{\omega}{\omega_k}\right)\right|}{\left|D_{(n)}(j\omega)\right|} = \left|\frac{N_{(n)}(j\omega)}{D_{(n)}(j\omega)}\right|$$

$$\omega_k = 1/R_k C_k \quad (k = 1,2,\ldots,n)$$

$$\left|N_{(n)}(j\omega)\right| = \left|N_{(n)re}(j\omega)\right| + \left|N_{(n)im}(j\omega)\right| \qquad (\omega > 0)$$

$$\left|N_{(n)}(j\omega)\right| = \left|N_{(n)re}(j\omega)\right| - \left|N_{(n)im}(j\omega)\right| \qquad (\omega < 0)$$

$D_{(n)} \rightarrow$ polynomial of $(j\omega)$, $\left|D_{(n)}(j\omega)\right| = \left|D_{(n)}(-j\omega)\right|$

$$\left|H_{(n)re}(-j\omega_k)\right| = \left|H_{(n)im}(-j\omega_k)\right| \qquad (\omega = -\omega_k : zeros)$$

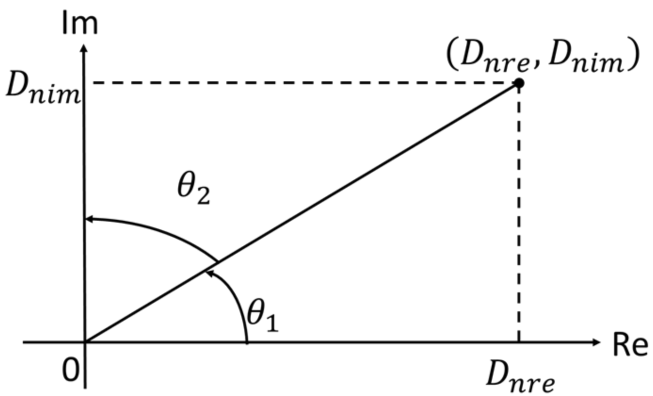
# n-th order RC Polyphase Filter : Phase

$$H_{(n)re}(j\omega) = \frac{N_{(n)re}(j\omega)}{D_{(n)re}(j\omega) + jD_{(n)im}(j\omega)}$$

$$H_{(n)im}(j\omega) = \frac{N_{(n)im}(j\omega)}{D_{(n)re}(j\omega) + jD_{(n)im}(j\omega)}$$

$$\tan\left(\angle H_{(n)re}(j\omega)\right) = -\frac{D_{(n)im}(j\omega)}{D_{(n)re}(j\omega)}$$

$$\tan\left(\angle H_{(n)im}(j\omega)\right) = \frac{D_{(n)re}(j\omega)}{D_{(n)im}(j\omega)}$$




$$\theta_1 = \tan^{-1}\left(\frac{D_{nim}}{D_{nre}}\right), \theta_2 = \tan^{-1}\left(\frac{D_{nre}}{D_{nim}}\right), \theta_1 + \theta_2 = \frac{\pi}{2}$$

$$\angle H_{(n)im}(j\omega) - \angle H_{(n)re}(j\omega)$$

$$= \tan^{-1}\left(\frac{D_{(n)re}(j\omega)}{D_{(n)im}(j\omega)}\right) + \tan^{-1}\left(\frac{D_{(n)im}(j\omega)}{D_{(n)re}(j\omega)}\right)$$

$$= \begin{cases} \frac{\pi}{2} & (\omega > 0) \\ -\frac{\pi}{2} & (\omega < 0) \end{cases}$$

90 ° Phase difference

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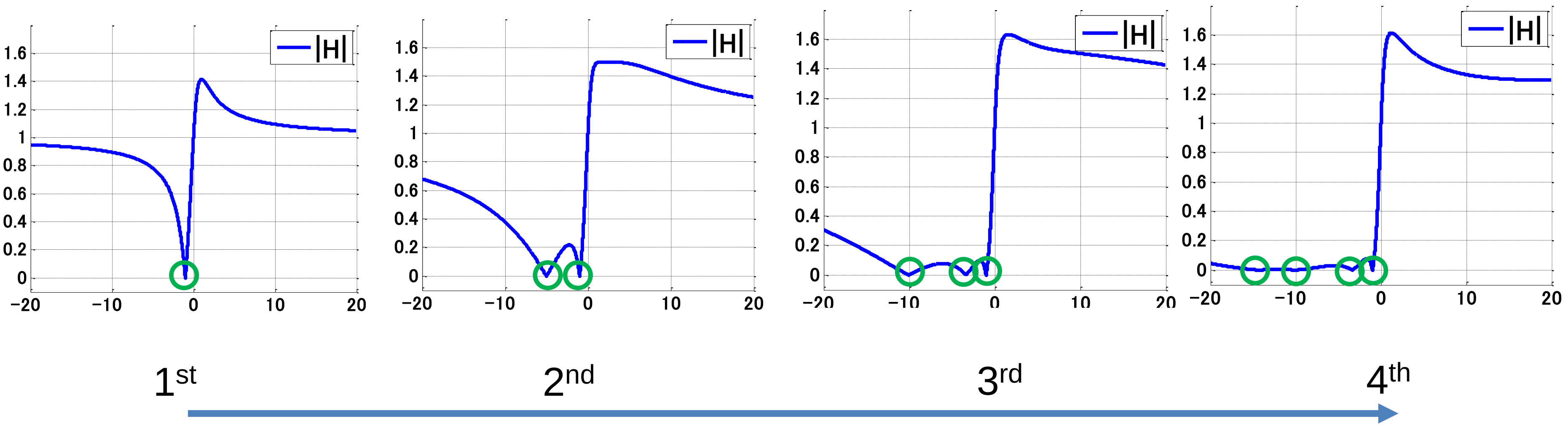
# Order and Gain



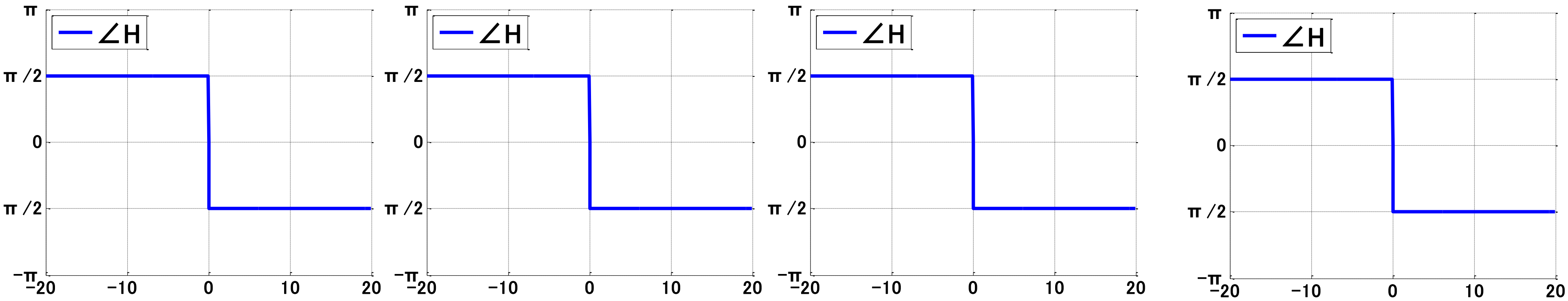

The higher orders,

Increase number of zeros

$|Hre|$ and $|Him|$ becomes close in wide range

**Close to ideal Hilbert transform**

# Order and Phase




$1^{st}$ $2^{nd}$ $3^{rd}$ $4^{th}$

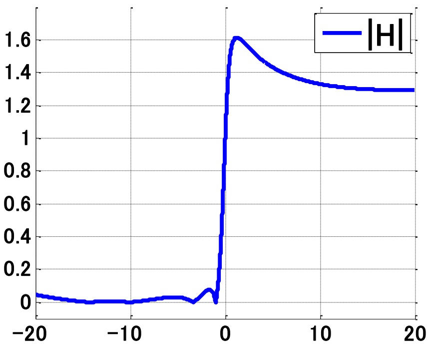
Phase characteristic is not changed

There is always 90° phase difference

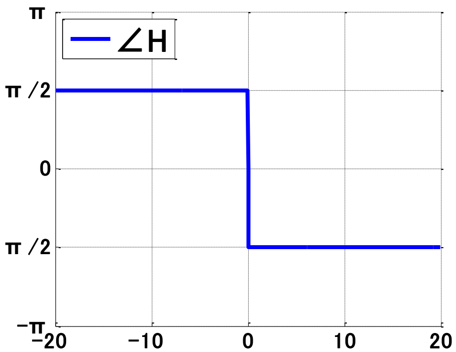
Fulfill Hilbert transform in full range

# Conclusion

## RC Polyphase Filter

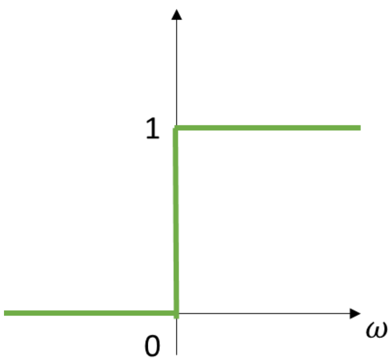



Gain

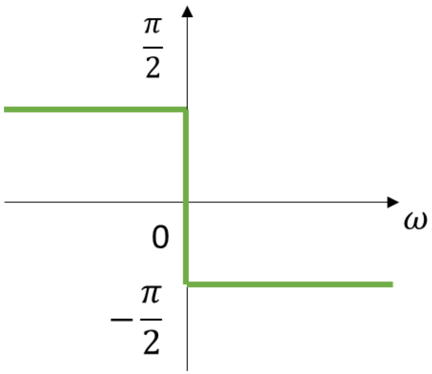



Phase

## Hilbert Filter




Gain

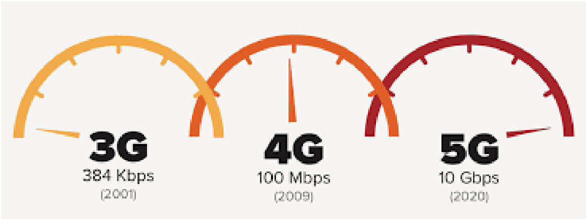



Phase

**RC polyphase filter is approximation of ideal Hilbert filter for complex input signal**

# Applications of This Research

RC polyphase filter characteristics are approximated to Hilbert transform





RC polyphase filter can be used an analog Hilbert filter for high-frequency signal, which DSP cannot handle

# Regression to Analog Filter Theory

Analog filter theory, several of transistor circuit
It had been recognized as completed field ...

**Looking for a yet unseen island!**

Marco Polo