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Consideration on Fundamental Performance Limitation of Analog Electronic Circuits Based on Uncertainty Principle

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My First Research

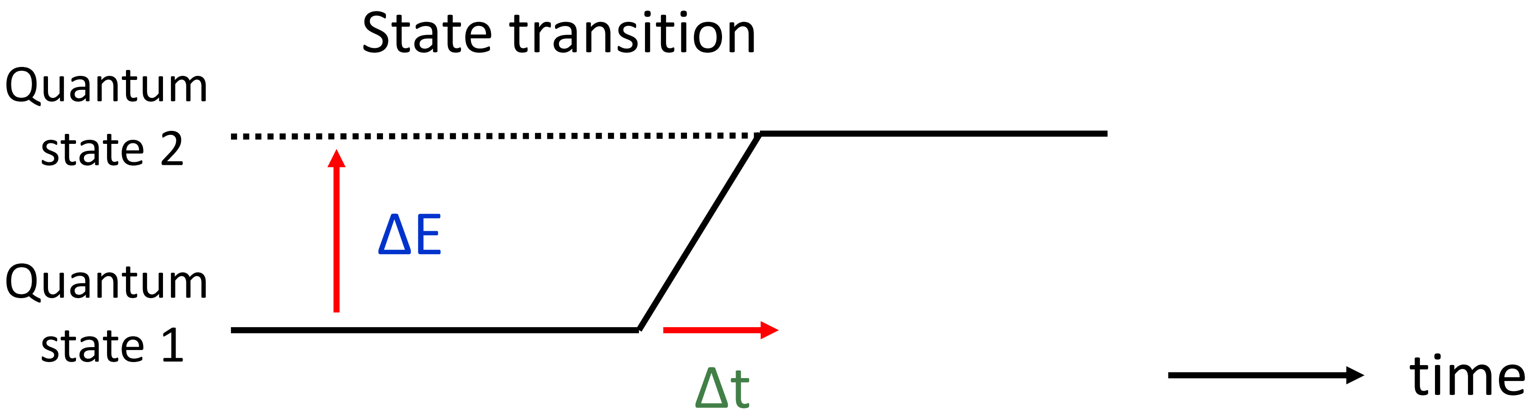
Computer with Superconductor (Josephson Device)

Under supervision of Prof. Ko Hara (原　宏)
at University of Tokyo
Physicist

Undergraduate (Bachelor) course, 4^{th} year

[1] K. Hara, H. Kobayashi, S. Takagi, F. Shiota, "Simulation of a Multi-Josephson Switching Device'', Japanese J. of Applied Physics (1980).

Research Motivation of This Paper



$\Delta E \, \Delta t \geq h/(4\pi)$ Uncertainty principle

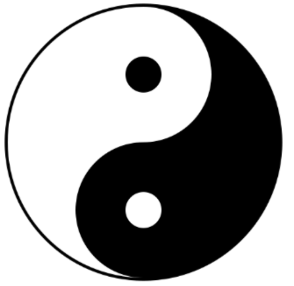
My strong impression :

Transition time Δt ⇨ Time uncertainty

Our Statement

Uncertainty relationships are everywhere in electronic circuits

Our conjecture



陰陽思想
太極図

Ultimately, some would converge to Heisenberg uncertainty principle in quantum physics.

Contents

Contents

Research Objective

- Our Objective

In analog electronic circuits

- Clarify tradeoff among their performance indices
- Provide their fundamental limitation

- Our Approach

Based on

- Uncertainty principle in quantum mechanics
- Uncertainty relationship in signal processing

Contents

Uncertainty Principle in Quantum Mechanics

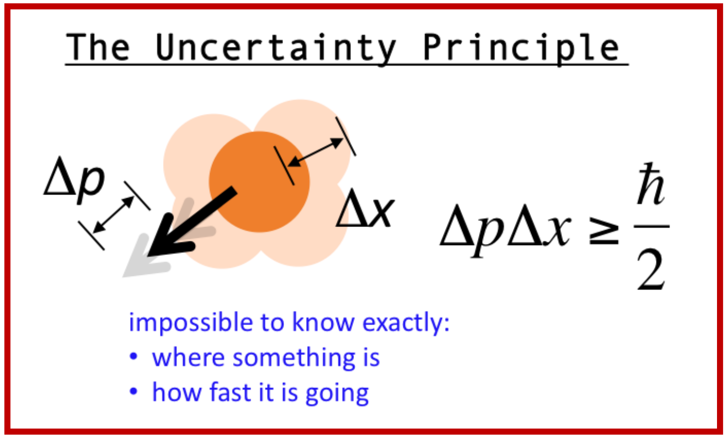
W. K. Heisenberg

$$\Delta t \, \Delta E \geq \quad h/(4\pi)$$

t: time, *E*: energy

$$\Delta x \, \Delta p \geq \quad h/(4\pi)$$

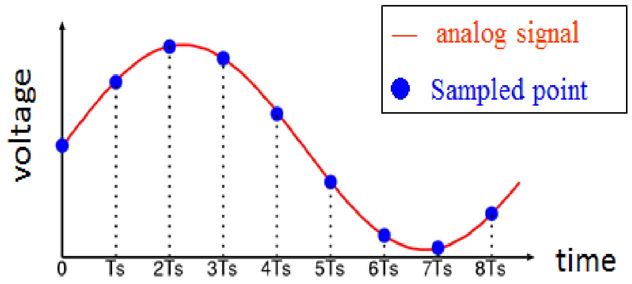
x: position, *p*: momentum.

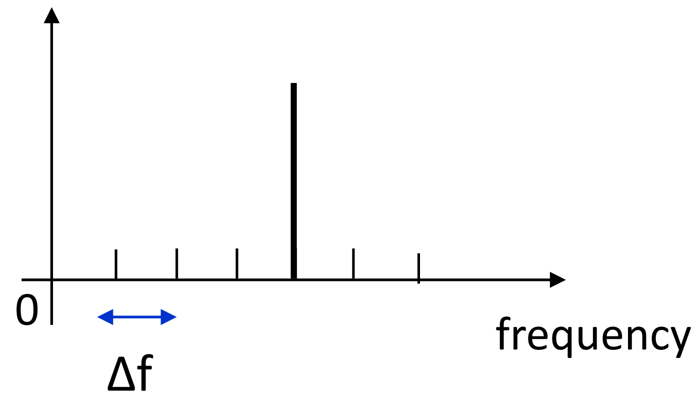


These cannot be proved ⇨ *principle.*

Uncertainty Relationship in Signal Processing (1)

- Discrete Fourier Transform (DFT)





Sampling frequency : fs
Sampling period: Ts (= 1/fs)

Number of DFT points :N

$\Delta f = fs/N = 1/(Ts\ N)$

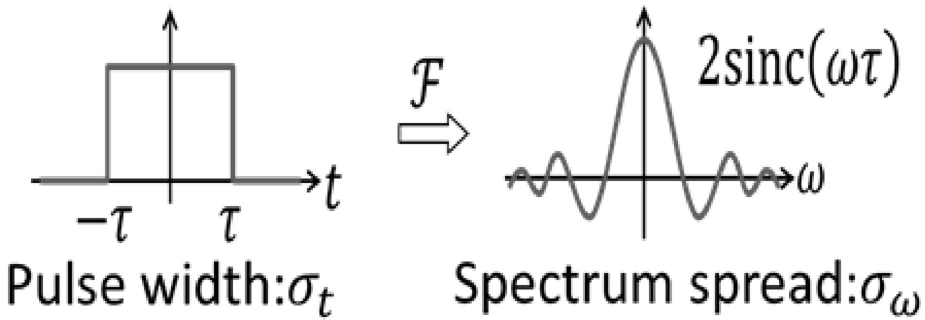
Time & frequency resolution

$\Delta f\ Ts = 1/N$

This can be proved mathematically ⇨ *Relationship*

Uncertainty Relationship in Signal Processing (2)

- Uncertainty Relationship
  between Time & Frequency of Continuous Waveform



$$\sigma_\tau \sigma_\omega \geq \frac{1}{2}$$

This can be proved mathematically ⇨ *Relationship*

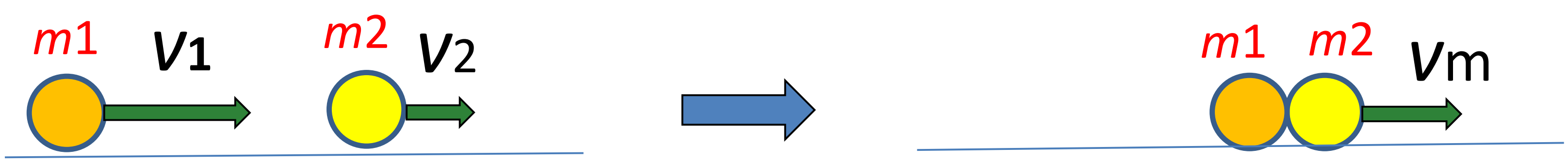
Contents

Importance of Invariant (1)

Invariant quantity ⇨
clarify phenomena & characteristics

Conservation Law in Physics :

- Energy conservation law
- Mass conservation law
- Momentum conservation law
- Charge conservation law



$p1 = m1\ v1,\quad p2 = m2\ v2$

$p1' = m1\ v_m,\quad p2' = m2\ v_m$

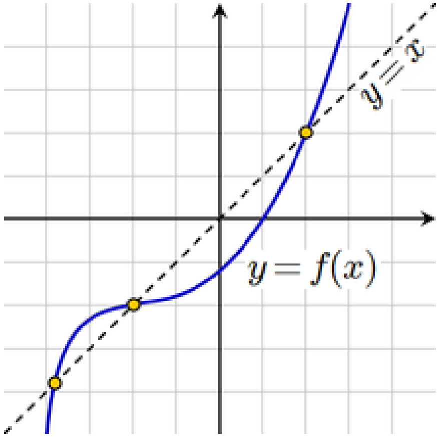
$$p1 + p2 = p1' + p2'$$

Importance of Invariant (2)

Invariant quantity ⇨
clarify phenomena & characteristics

Fixed-Point in Mathematics :

$$f(x) = x$$



Utility for Voyage

Compass

Polaris

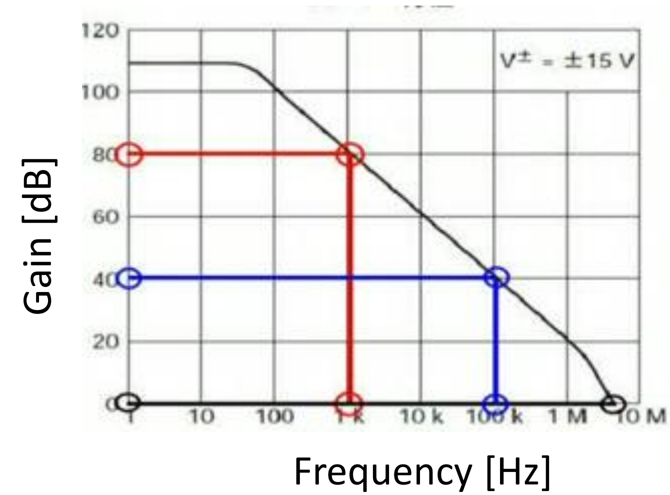
Contents

Gain, Signal Band and Power

- For a given amplifier

Gain・bandwidth = constant

Gain → large, bandwidth → narrow



- Amplifier Performance

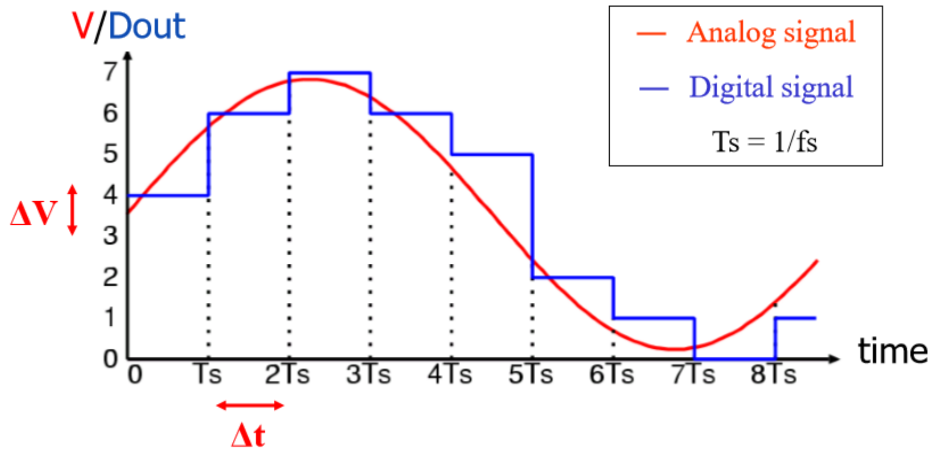
$$\text{FOM} = \frac{\text{Power}}{\text{Gain} \cdot \text{Bandwidth}}$$

Technology constant

→ Converge to uncertainty principle

conjecture

ADC Sampling Speed, Resolution and Power



Sampling period: Δt

Resolution: $V_{full} / \Delta V = 2^n$

Power: P

$$FOM = \Delta t \cdot \Delta V \cdot P / V_{full}$$
$$= \Delta t \cdot P / 2^n$$

→ Converge to uncertainty principle

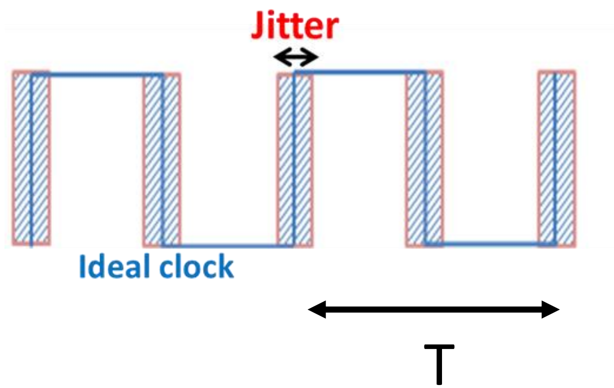
conjecture

$$FOM = \frac{\text{Voltage Resolution} \cdot \text{Power}}{\text{Sampling Speed}}$$

Technology constant

FOM → Smaller, ADC → Better

Clock Jitter, Power



Clock jitter: Δt

Clock generator energy : E

power : P

Design tradeoff

$\Delta t \cdot E \geq K_1$ ⟷ $(\Delta t / T) P \geq K_1$

Power → larger, Jitter → smaller

Noise, Capacitor

Analogy

p (momentum) ⇔ Q (charge)

v (velocity) ⇔ V (voltage)

m (mass) ⇔ C (capacitor)

Momentum conservation law

⇔ Charge conservation law

Uncertainty principle

$\Delta x \, \Delta p \geq K$ ⇔ $\Delta V \; f \, \Delta Q \geq K$

⇔ $C \; \Delta V^2 \; f \geq K$

⇕

Noise bandwidth: f

Noise power $\Delta V^2 = kT/C$

C → large, Noise → small

Noise, Capacitor (2)

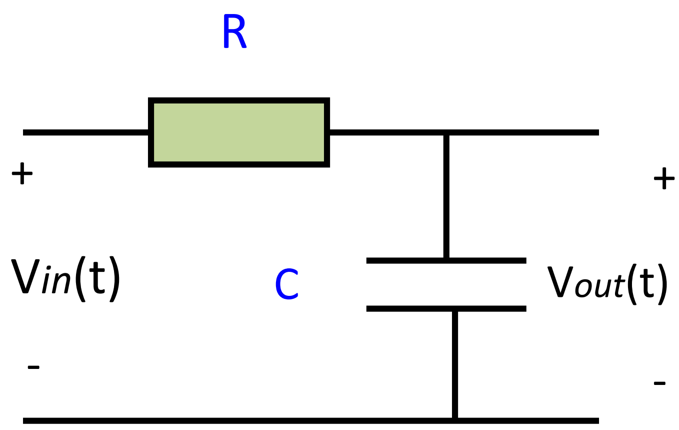
- For a given T=RC
  the same gain & phase characteristics
  for different (R_1, C_1), (R_2, C_2), ...
  with $R_1 C_1 = R_2 C_2 = \dots = T$

- For a given V_{out}

$$E_c = (1/2)\, C\, V_{out}^2$$

$$V_{noise}^2 = kT/C$$

C → large, R → small
⇨ Same gain & phase characteristics
Low noise
Large energy



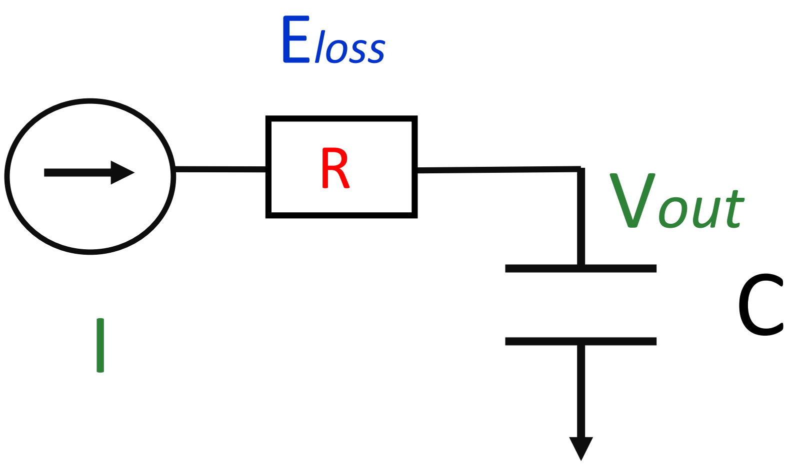
Transfer function

$$G(s) = 1/(1 + sRC)$$

Capacitor Charge & Loss

$E_{loss} = (R \cdot I) \cdot I \cdot T$
$= R \cdot C \cdot V \cdot I$

$V_{out} = I \cdot T / C$



I : Charge Current
T: Charge Duration

$$E_{loss} \cdot T = R \cdot C \cdot V_{out}$$

Uncertainty relationship

For given R, C, V_{out}

I → small, T → long ⇨ E_{loss} → small

Waveform Sampling Circuit

- Research Objective

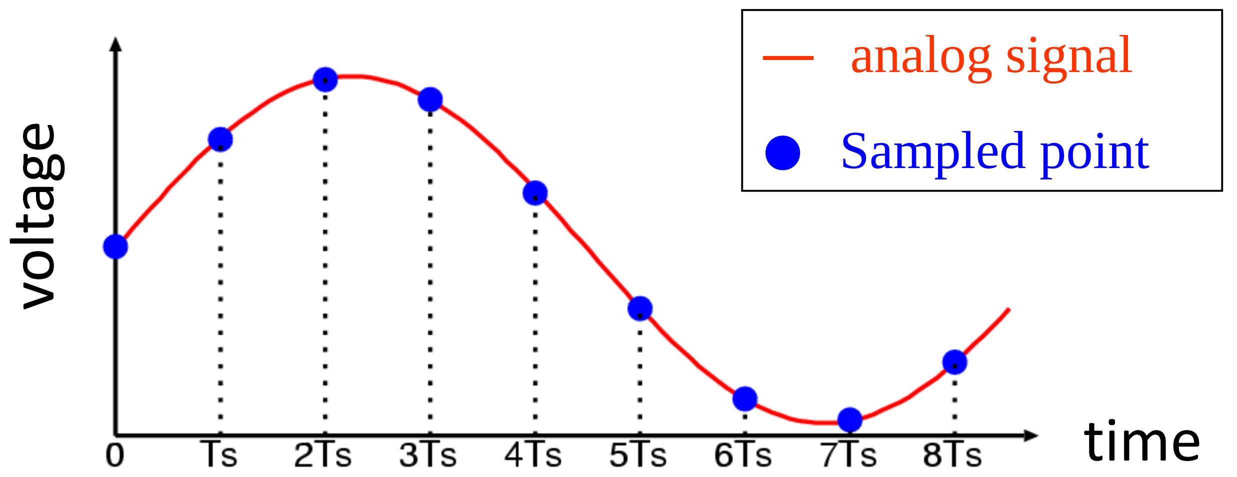
- Uncertainty Principle and Relationship

Example of
Uncertainty Relationship
In Signal Processing

nce Analogy
to Uncertainty Relationship and Invariant

[2] M. Arai , H. Kobayashi , et. al.,“Finite Aperture Time Effects in Sampling Circuit,” IEEE 11th International Conference on ASIC, Chengdu (Nov. 2015).

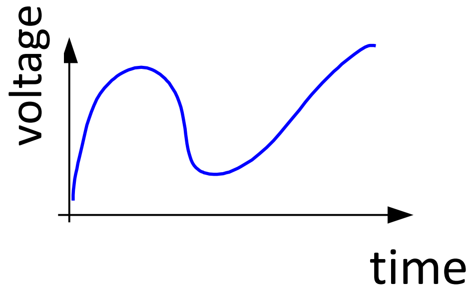
Waveform Sampling

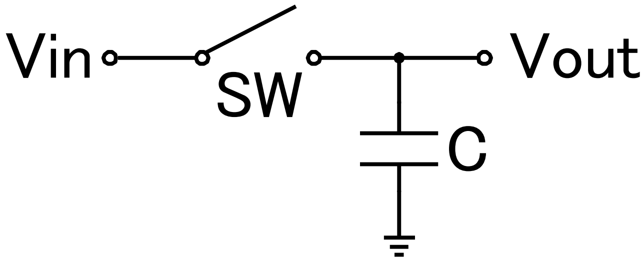


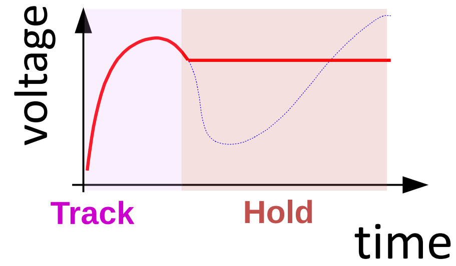
suffers from

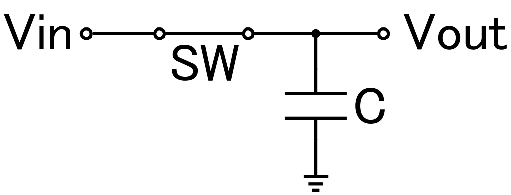
- Finite aperture time (non-zero turn-off time)
- Aperture jitter

Sampling Circuit



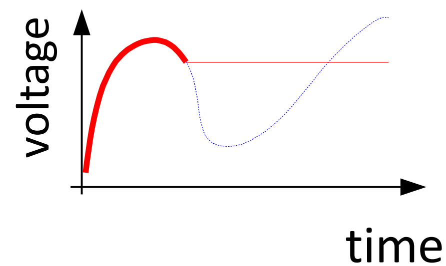


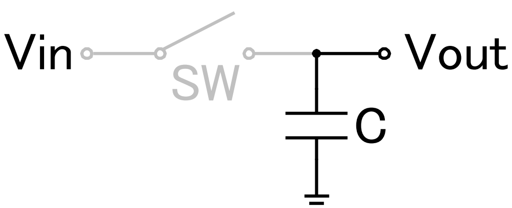




- SW: ON
  - Vout(t) = Vin(t)

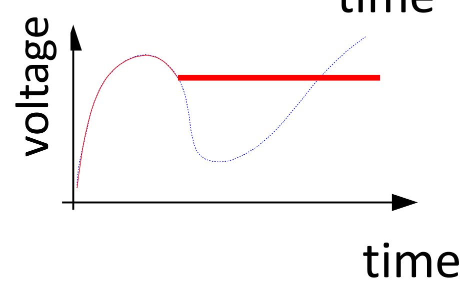
  Track mode



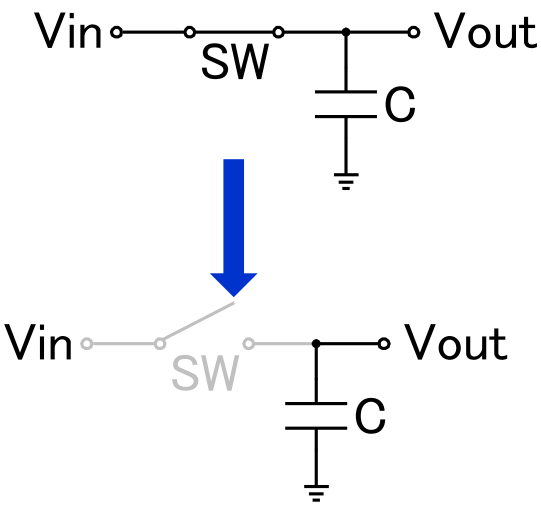


- SW: OFF
  - Vout(t) = Vin(t_{OFF})

  Hold mode



Finite Aperture Time

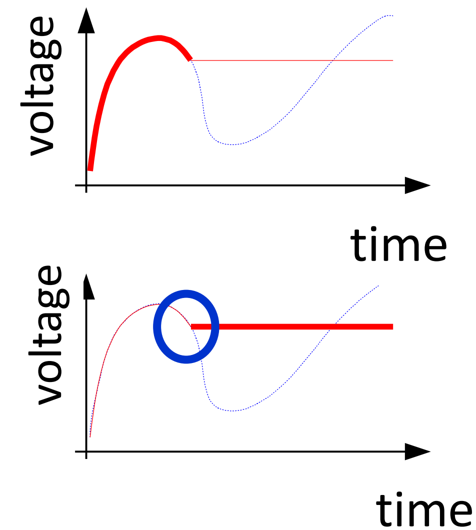


- SW: ON
  - Vout(t) = Vin(t)

  Track mode

- SW: OFF
  - Vout(t) = Vin(t_{OFF})

  Hold mode



Finite transition time from track to hold modes

Analogy with Camera Shutter Speed

Camera: Finite Shutter Speed

Moving Object

Blurred

Sampling Circuit:
Finite Aperture Time

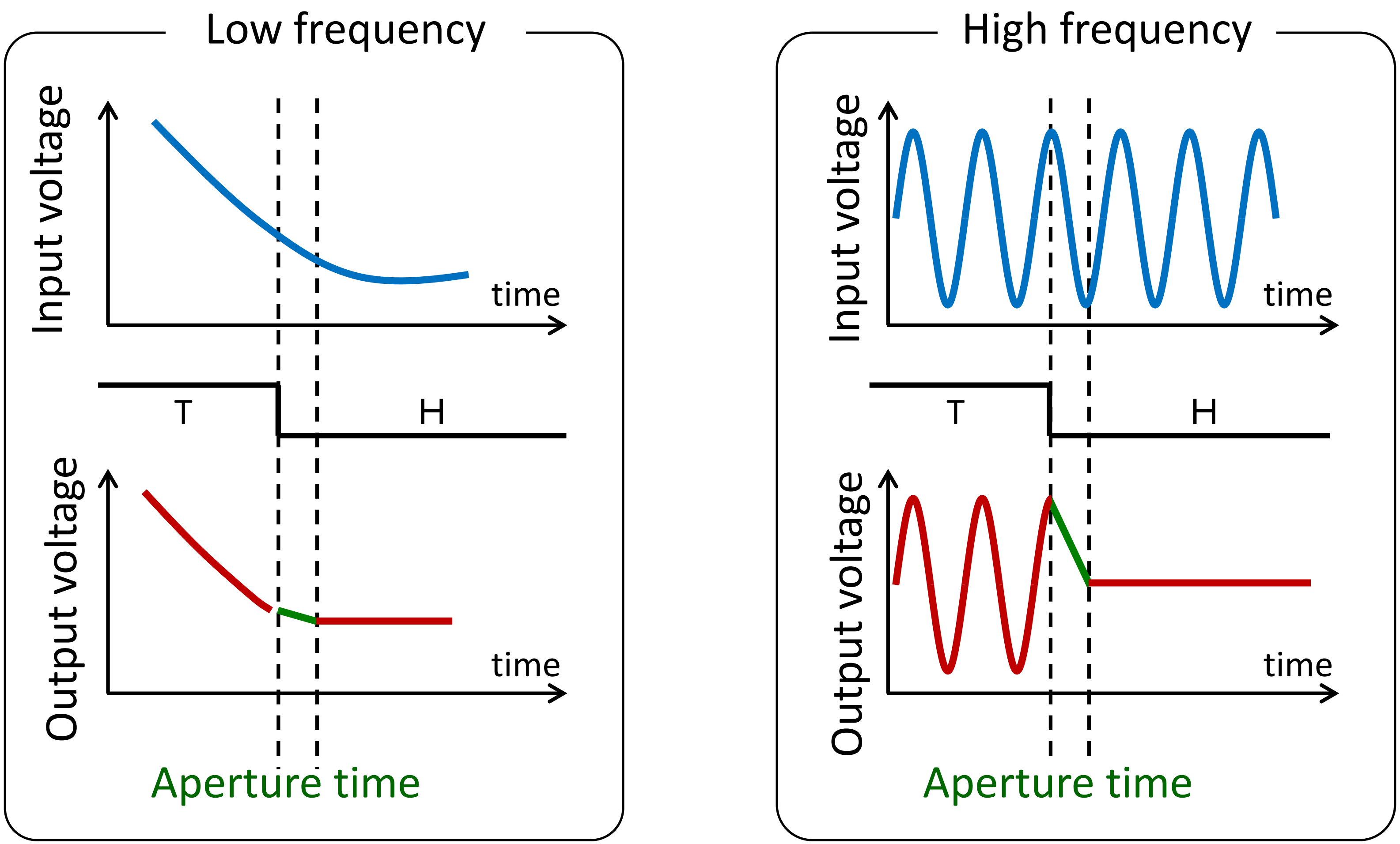
Input signal

High frequency

Acquired signal

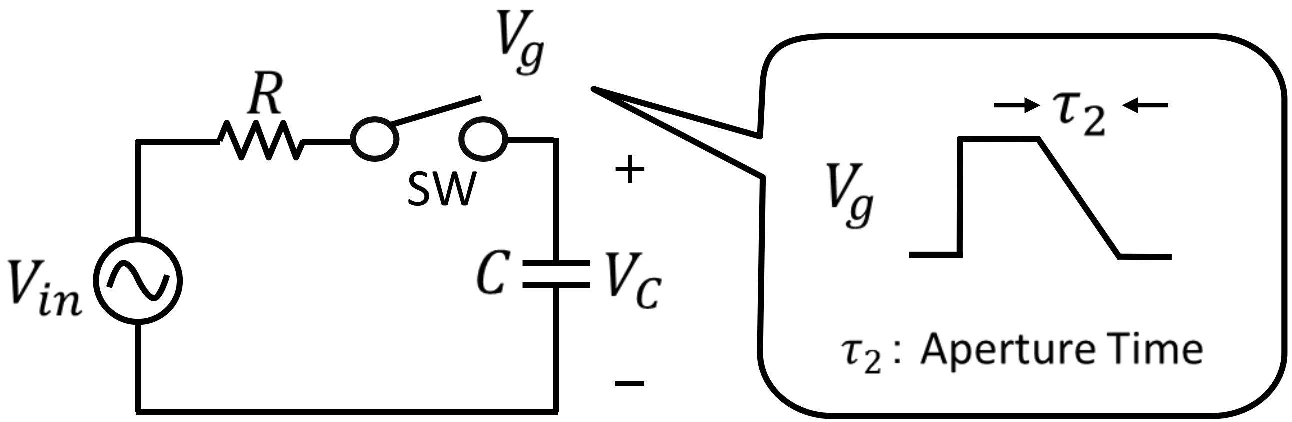
Low pass filtered

Signal Frequency and Aperture Time



Higher frequency signal ⇒ More affected by finite aperture time

Derived Transfer Function



Track Hold Circuit

$$\frac{V_C}{V_{in}} = \frac{sinc(\omega\tau_2)}{sinc(\omega\tau_2) + j\omega\tau_1}$$

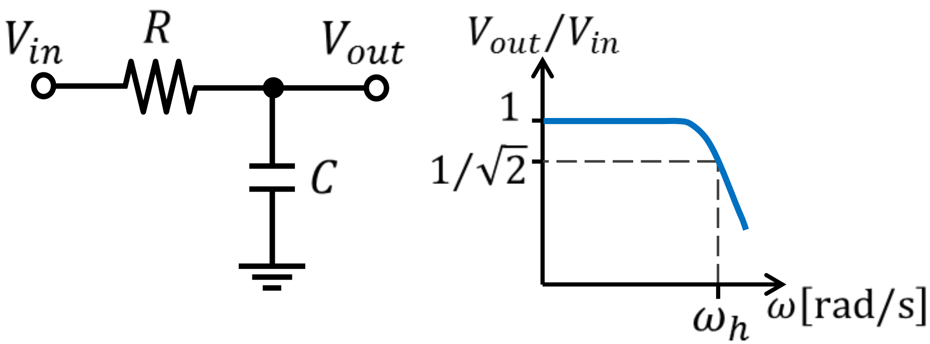
$$\tau_1 = R\,C$$

Transfer function in case of finite aperture time

[3] A. Abidi, M. Arrai, K. Niitsu, H. Kobayashi, "Finite Aperture Time Effects in Sampling Circuits," 24th IEICE Workshop on Circuits and Systems, Awaji Island, Japan (Aug. 2011)

Trade-off of Time Constant and Bandwidth

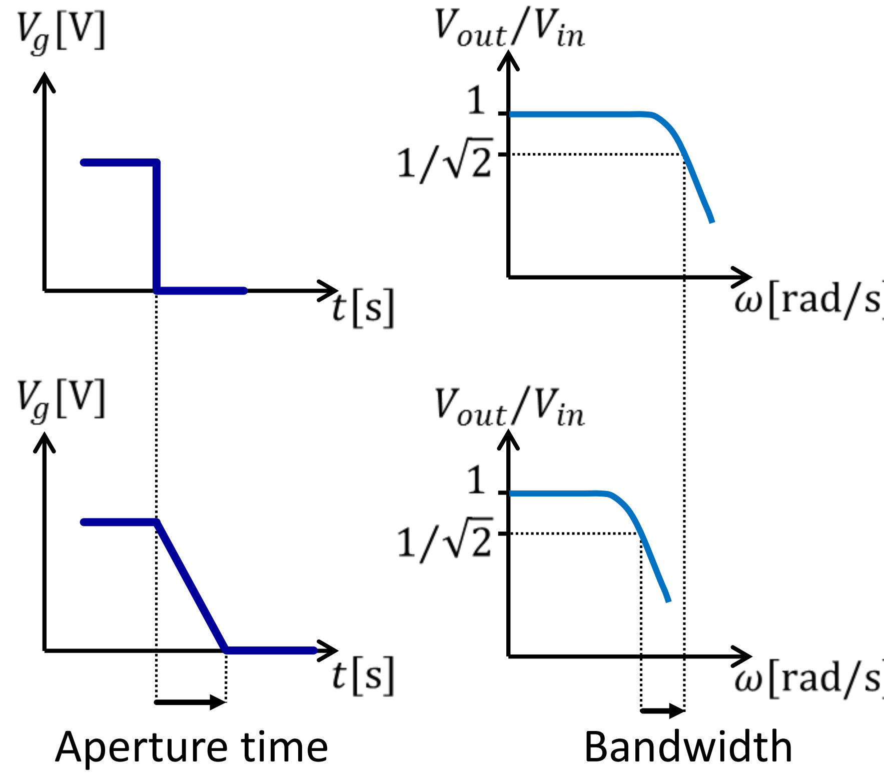
- RC time constant and bandwidth



$$\omega_h = \frac{1}{\tau_1}$$

$$\begin{cases} \tau_1 : \text{RC} \\ \omega_h : \text{bandwidth} \end{cases}$$

- Aperture time and bandwidth



Time		Band : ω_h
Short	⟶	Wide
Long	⟶	Narrow

Analog Electronic Circuits

Performance tradeoffs are everywhere in circuits

$$\Delta a \, \Delta b \geq K$$

- In some cases, these can be proved.

  Uncertainty relationship

- In other cases, these can NOT be proved.

  For a given technology

$$\Delta a \, \Delta b = K$$

K: Technology constant

Technology → advance ⇨ K → smaller

Conjecture: this converges to uncertainty principle

Analog Circuit and Quantum Mechanics

Myth

- Real world signals → analog
- Computer world signals → digital.

Truth

- quantum mechanics →
  signals in nature → digital (discrete).

- Current → average of electrons' moves
- Electronic noises → their variation.

Conjecture

- Analog electronic circuit performance
  → Limited by quantum mechanics

Analogy

In Physics, analogy is just a coincidence,
NOT inevitable.

Analogy

p (momentum) ⇔ Q (charge)

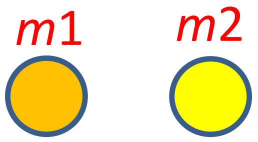
v (velocity) ⇔ V (voltage)

m (mass) ⇔ C (capacitor)

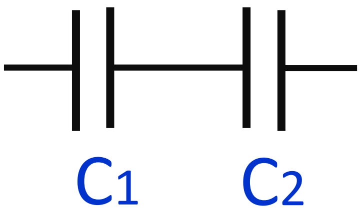
Momentum conservation law
⇔ Charge conservation law

Difference

Any connection of m1 & m2 > m1, m2



Series connection of C1 & C2 < C1, C2

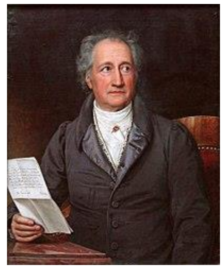


Bridge Through Plank Constant

"Let there be light !"

Old testament

"Mehr Licht !"



by J. W. von Goethe

Uncertainty Relationship

Analogy to Principle

$$\Delta\omega \, \Delta\tau \geq 1/2$$

$$(h/(2\pi)) \, \Delta\omega \, \Delta\tau \geq h/(4\pi)$$

Energy in the light: $E = (h/(2\pi)) \, \omega$

$$\Delta E \, \Delta\tau \geq h/(4\pi)$$

Uncertainty Principle

Measurement and Simulation

Measurement : *Active*, *Passive*

Active: Stimulus → Device

Response → Measured

Device state → Disturbed.

Passive: No stimulus

Device state → Not disturbed.

Uncertainty principle

⇨ all measurements disturb device state.

Circuit simulation ⇨ No disturbance.

Contents

Now, Close to Ultimate Limitation

h/(4π) = 5.2 x 10^(-35)

In case

C=0.01fF, V=0.1V, Δt= 1ps

$\Delta E = (1/2)\, C\, V^2$

⇩

ΔE Δt = 5.0 x 10^(-32)

With subtle change of conditions, both become comparable.

Contents

Conclusion

Our strong belief:

Analog electronic circuit

- Its design tradeoff as well as FOM

↓ Explained with

Analogy to uncertainty principle/relationship.

- Uncertainty principle and relationship

↓

Its ultimate performance limitation

Final Statement

Current status of circuit design and analysis area

⇩

Only individual techniques have been developed.

⇩

We need to establish a unified theory
for circuit design and analysis area.

Thank you for listening