IEEE International Symposium on Intelligent Signal Processing and Communication Systems 2017
NOVEMBER 6-9, 2017, XIAMEN, CHINA

Nov. 9 NA-L2 9:00-12:00

Fibonacci Sequence Weighted SAR ADC as Golden Section Search

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群馬大学
GUNMA UNIVERSITY

OUT LINE

- Introduction
- SAR ADC & Redundancy Design
- Golden Section Search & Fibonacci Search
- Fibonacci Sequence Weighted SAR ADC
- SAR ADC Based on Fibonacci Search
- Proof & Simulation
- Conclusion

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Background

Automotive electronics are in spotlight

High-speed, Reliable

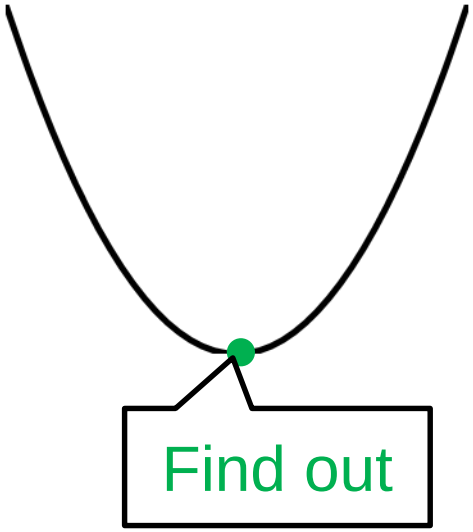
"SAR ADC" in microcontroller is needed.

In my laboratory

We have discovered various properties of SAR ADC using Fibonacci sequence.

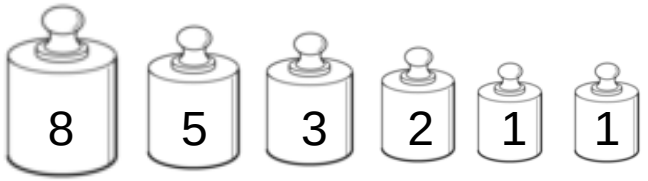
New Discovery

SAR ADC based on
golden section search
using unimodal function



equivalent

Fibonacci sequence weighted SAR ADC



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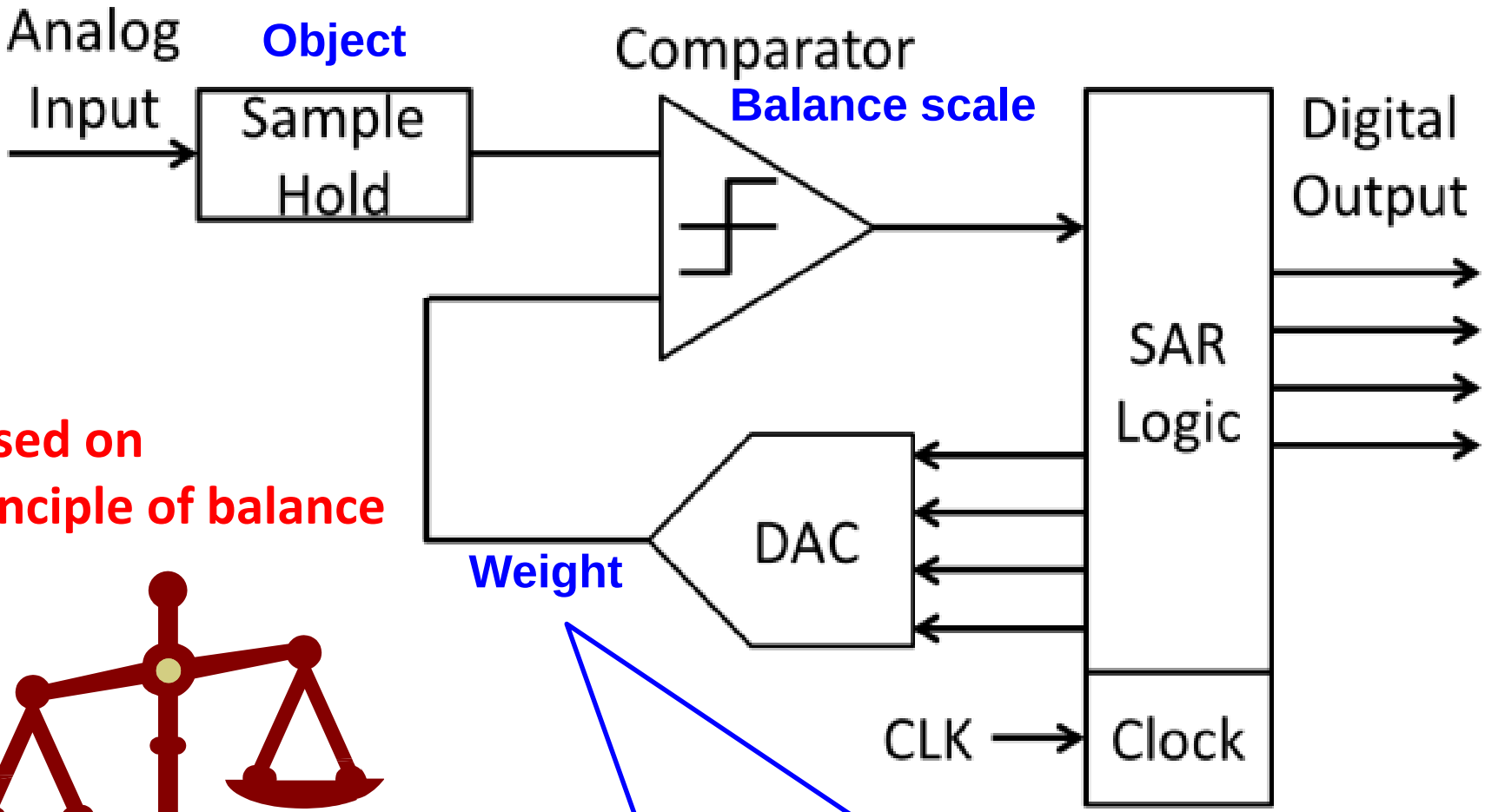
SAR ADC Configuration



Based on principle of balance

Generally use binary weight

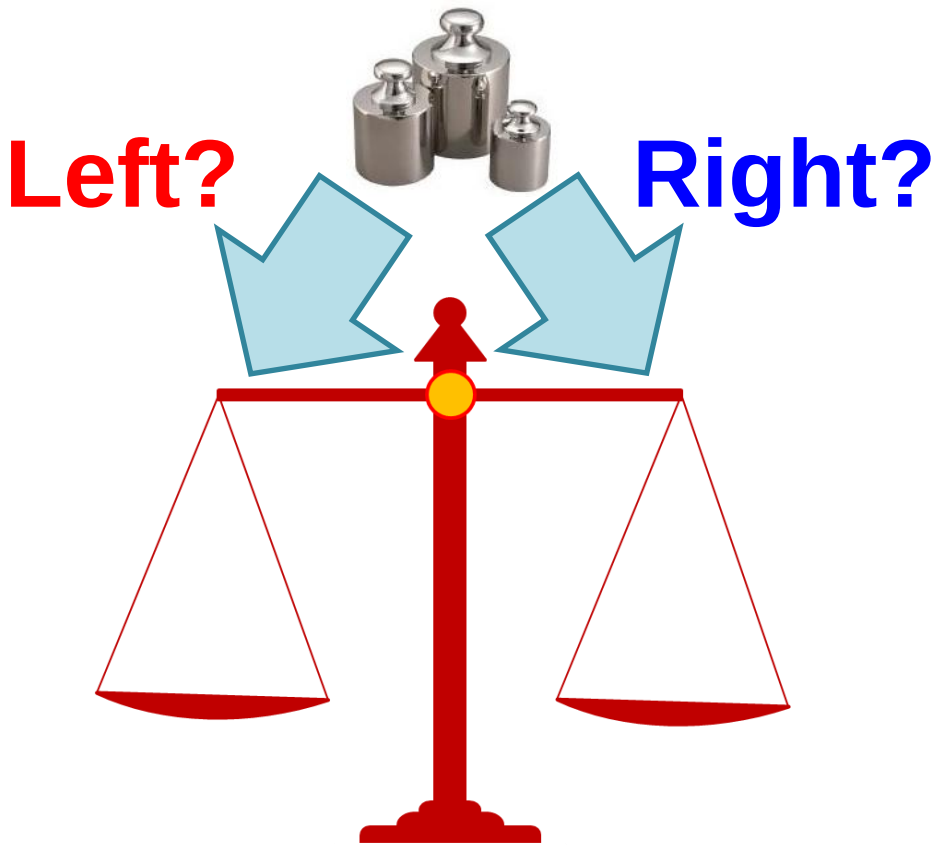
(1 , 2 , 4 , 8 , 16 , 32, 64 …)

Binary Search SAR ADC Operation

5bit-5step SAR ADC

- Analog Input：7.3 [V]
- Binary weight :

16, 8, 4, 2, 1



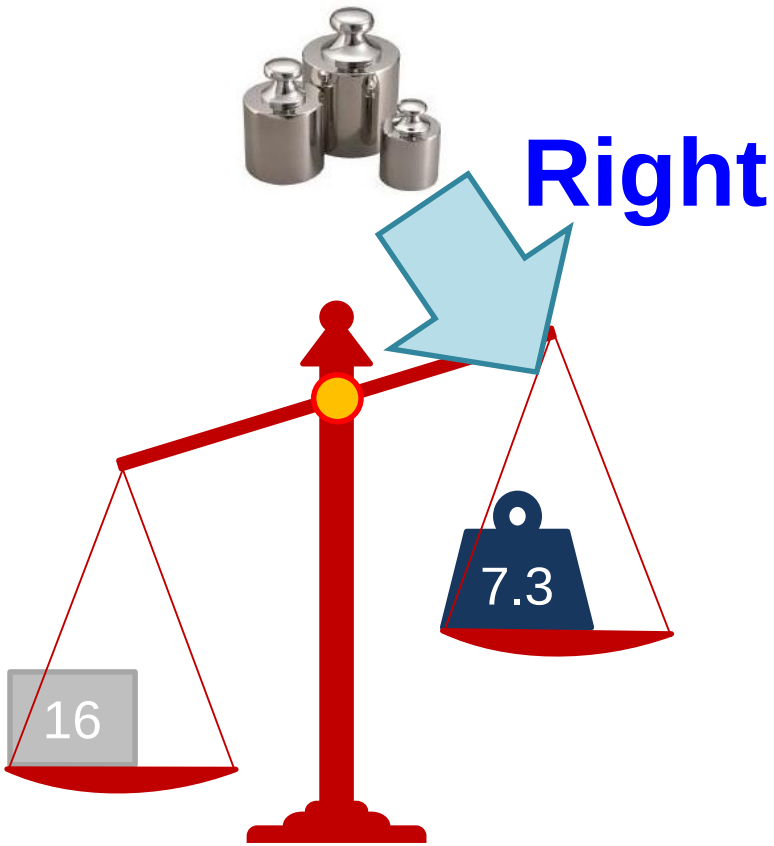
Step		1st	2nd	3rd	4th	5th	output
Weight p(k)		16	8	4	2	1	
Level	31						31
	30						30
	29						29
	28						28
	27						27
	26						26
	25						25
	24						24
	23						23
	22						22
	21						21
	20						20
	19						19
	18						18
	17						17
	16						16
	15						15
	14						14
	13						13
	12						12
	11						11
	10						10
	9						9
	8						8
	7						7
	6						6
	5						5
	4						4
	3						3
	2						2
	1						1
	0						0

Binary Search SAR ADC Operation

5bit-5step SAR ADC

- Analog Input：7.3 [V]
- Binary weight :

8, 4, 2, 1



Step		1st	2nd	3rd	4th	5th	output
Weight p(k)		16	8	4	2	1	
Level	31						31
	30						30
	29						29
	28						28
	27						27
	26						26
	25						25
	24						24
	23						23
	22						22
	21						21
	20						20
	19						19
	18						18
	17						17
	16						16
	15						15
	14						14
	13						13
	12						12
	11						11
	10						10
	9						9
	8						8
	7						7
	6						6
	5						5
	4						4
	3						3
	2						2
	1						1
	0						0

Down!

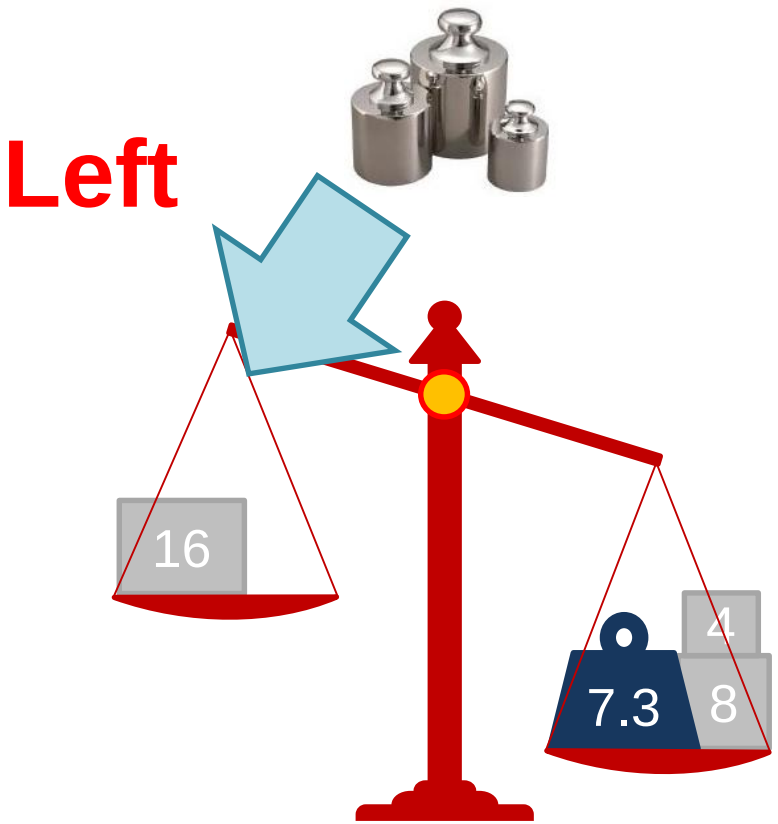
0

Binary Search SAR ADC Operation

5bit-5step SAR ADC

- Analog Input：7.3 [V]
- Binary weight :

2, 1



Step		1st	2nd	3rd	4th	5th	output
Weight p(k)		16	8	4	2	1	
Level	31						31
	30						30
	29						29
	28						28
	27						27
	26						26
	25						25
	24						24
	23						23
	22						22
	21						21
	20						20
	19						19
	18						18
	17						17
	16						16
	15						15
	14						14
	13						13
	12						12
	11						11
	10			UP!			10
	9						9
	8						8
	7						7
	6						6
	5						5
	4						4
	3	0	0	1			3
	2						2
	1						1
	0						0

Binary Search SAR ADC Operation

5bit-5step SAR ADC

- Analog Input：7.3 [V]
- Binary weight :

$$7.3 \Rightarrow 00111 \Rightarrow 7$$

$$16 - 8 - 4 + 2 + 1 + 0.5 - 0.5 = 7$$

Step	1st	2nd	3rd	4th	5th	output
Weight p(k)	16	8	4	2	1	
Level 31						31
30						30
29						29
28						28
27						27
26						26
25						25
24						24
23						23
22						22
21						21
20						20
19						19
18						18
17						17
16						16
15						15
14						14
13						13
12						12
11						11
10						10
9						9
8						8
7						7
6						6
5						5
4						4
3	0	0	1	1	1	3
2						2
1						1
0						0

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SAR ADC Redundancy Design

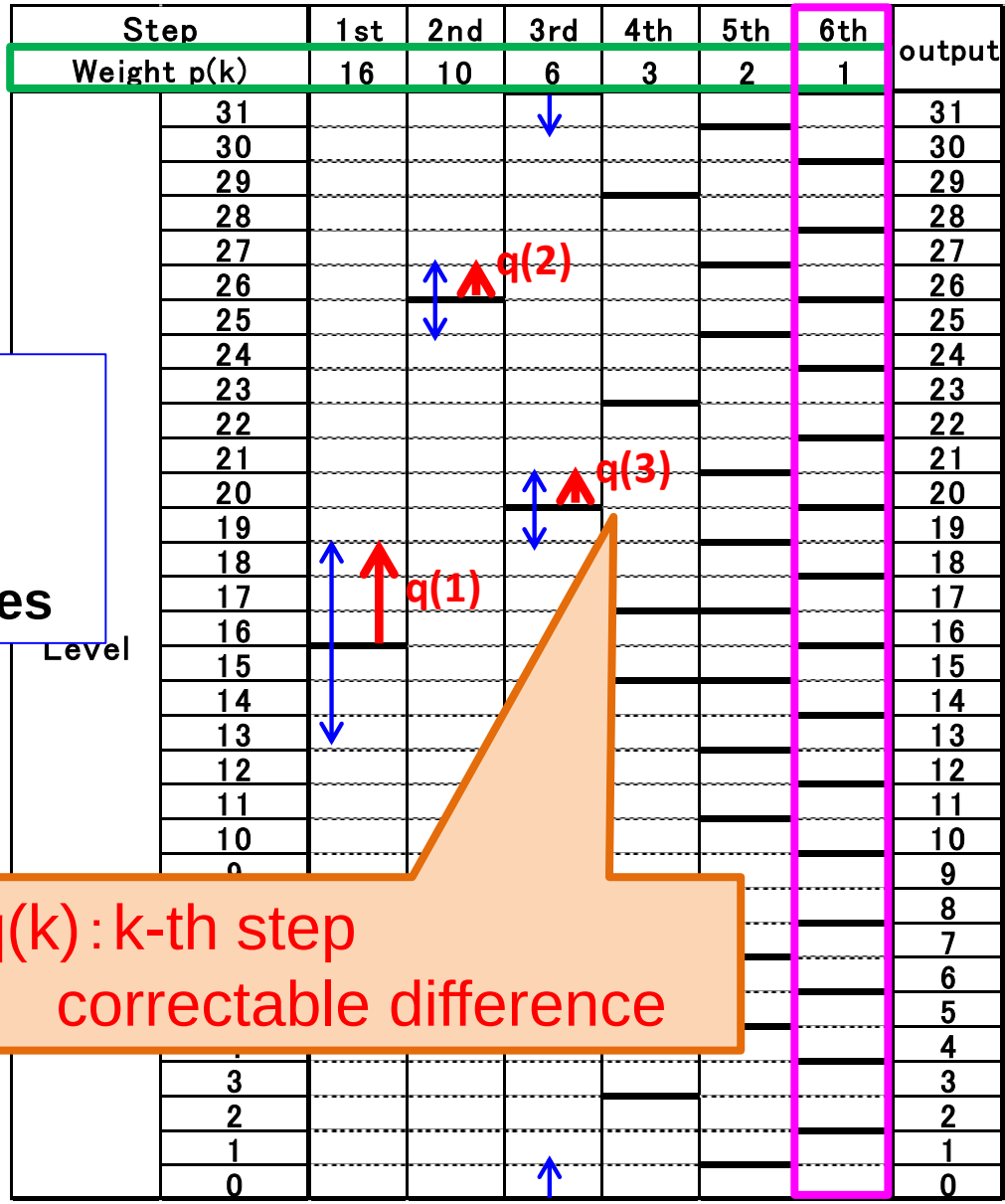
Redundancy

➡ Surplus, Extra

Using time redundancy

- **Extra comparison steps**
- **Change reference to Non-binary voltages**

Enable digital error correction!



Redundancy Design Operation(No Error)

4bit-5step SAR ADC

- Analog input : 6.3
- Redundant weight :

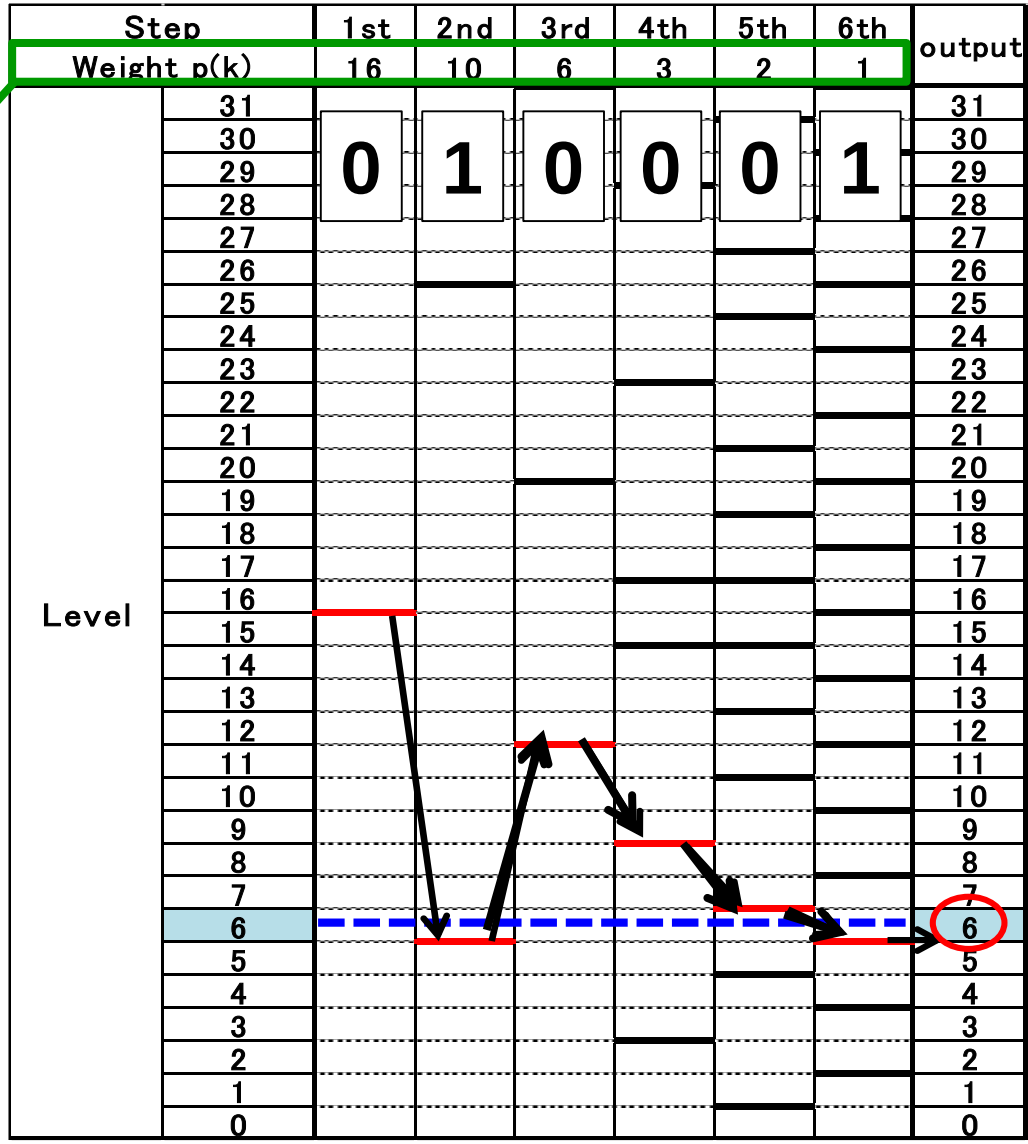
  16, 10, 6, 3, 2, 1

Correctable expression

6.3⇒010001⇒6

$$16 - 10 + 6 - 3 - 2 - 1 + 0.5 - 0.5$$

=6



Redundancy Design Operation(One Error)

4bit-5step SAR ADC

- Analog input：6.3
- Redundant weight :

  16, 10, 6, 3, 2, 1

Correctable expression

6.3⇒010001⇒6

Another expression

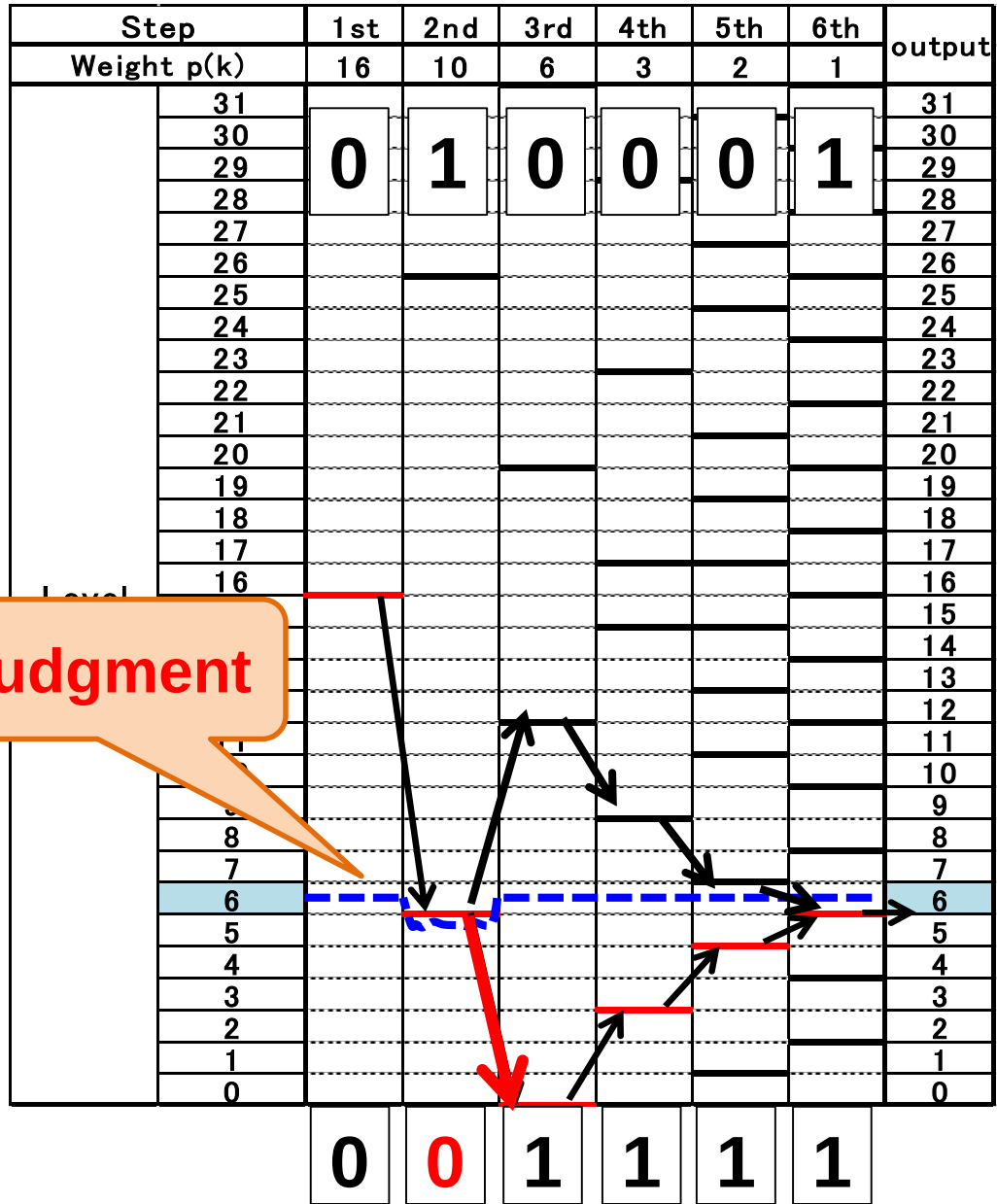
6.3⇒001111⇒6

$16 - 10 - 6 + 3 + 2 + 1 + 0.5 - 0.5$

$= 6$

Error correction

High-Reliability



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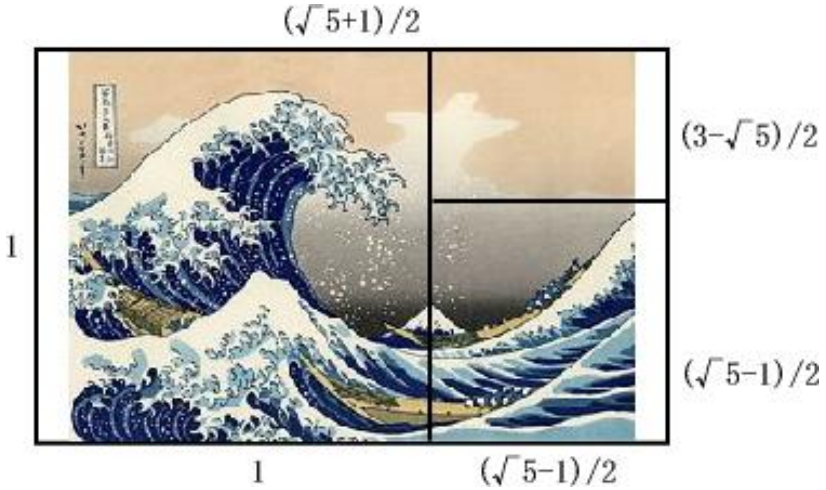
Golden Ratio

- Ratio satisfying $a : b = b : (a + b)$
- $\phi = \frac{b}{a} = \frac{1+\sqrt{5}}{2} = 1.618033988749895\cdots$
- The most beautiful ratio



Fibonacci Sequence

Definition (n=0,1,2,3…)

$$F_0 = 0$$
$$F_1 = 1$$
$$F_{n+2} = F_n + F_{n+1}$$

Fibonacci number

$$0, 1, 1, \mathbf{2}, 3, 5, \mathbf{8}, 13, 21, 34, \mathbf{55} \ldots$$

Property

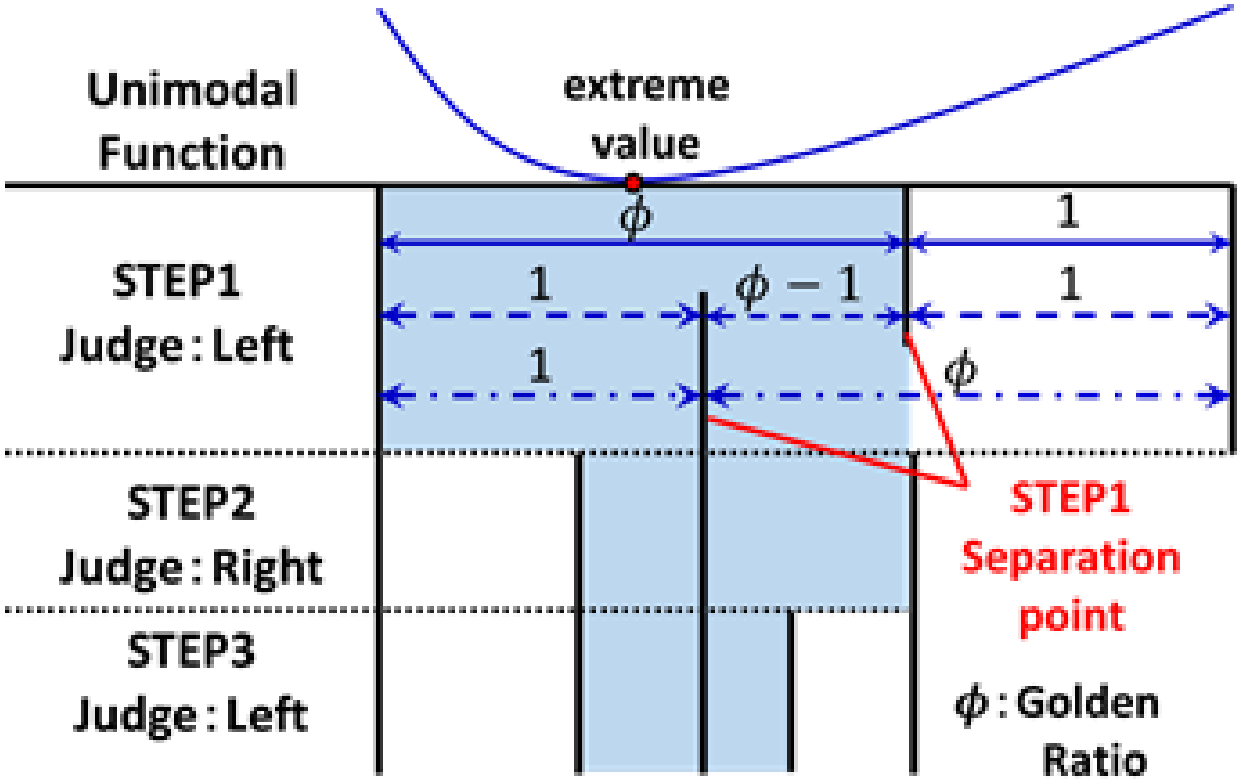
The closest terms ratio converges to *"Golden Ratio"* !

$$\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = 1.6180339887 \ldots = \phi$$

Golden Section Search

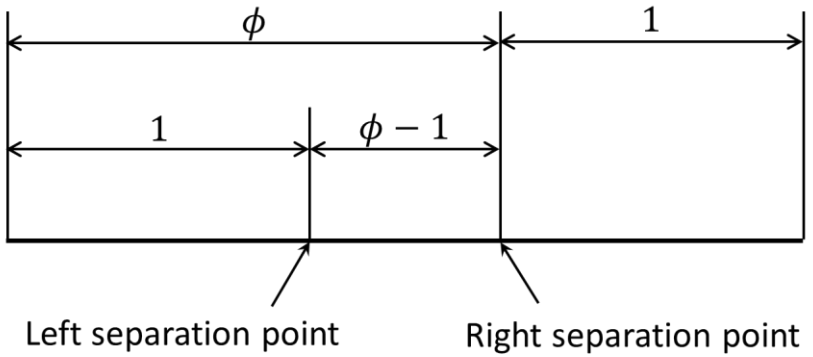
Finding of effectively extreme value of unimodal function

Division ratio = Golden ratio



Fibonacci Search

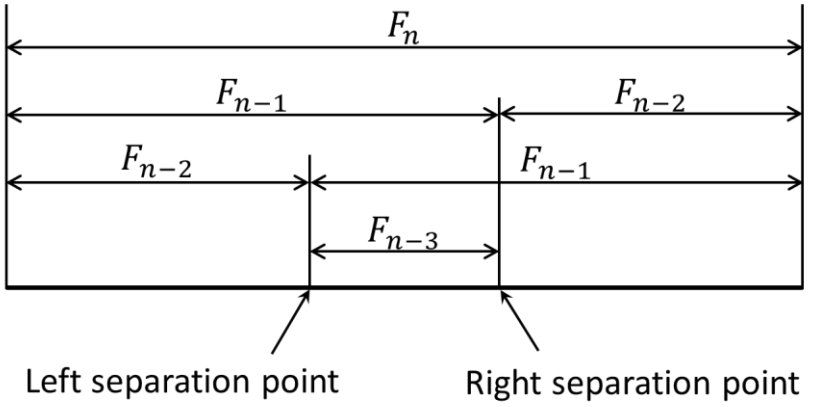
Golden Section Search
(ϕ:Golden ratio)



ADC treats only integers ⇒ **Difficult to realize**

Performing golden division with only integers

Fibonacci search
(F_x:Fibonacci number)



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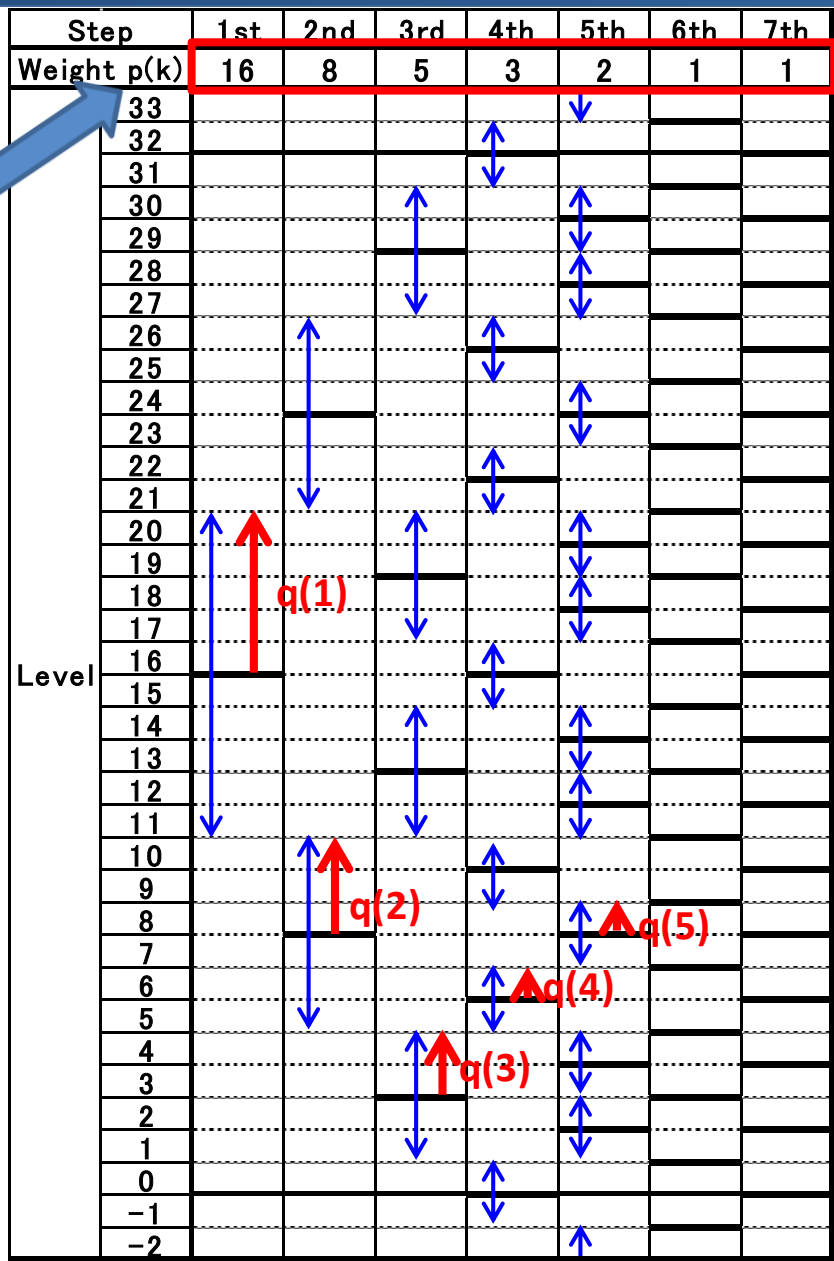
Fibonacci Sequence Weighted SAR ADC

Fibonacci sequence SAR ADC

Weight is Fibonacci sequence



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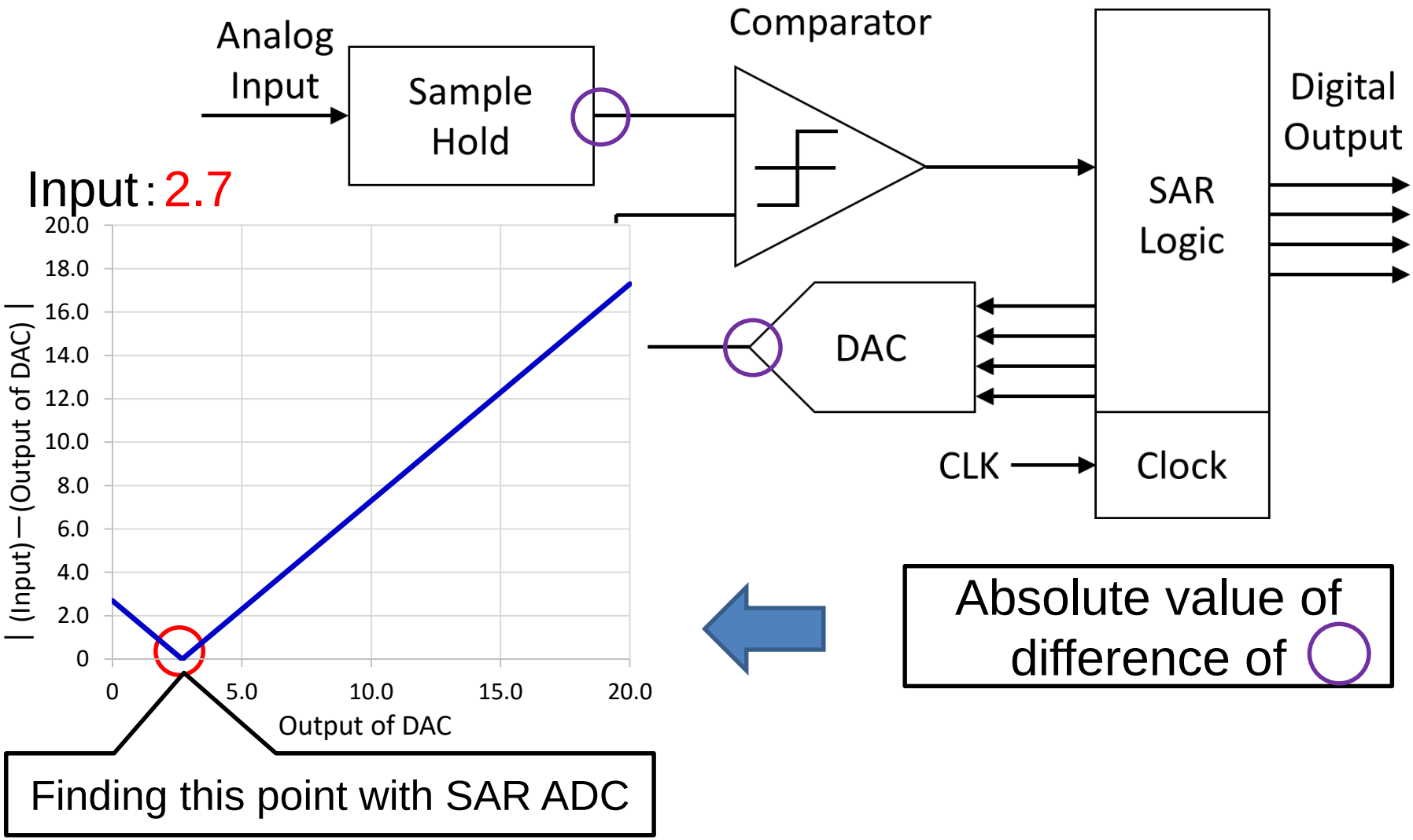
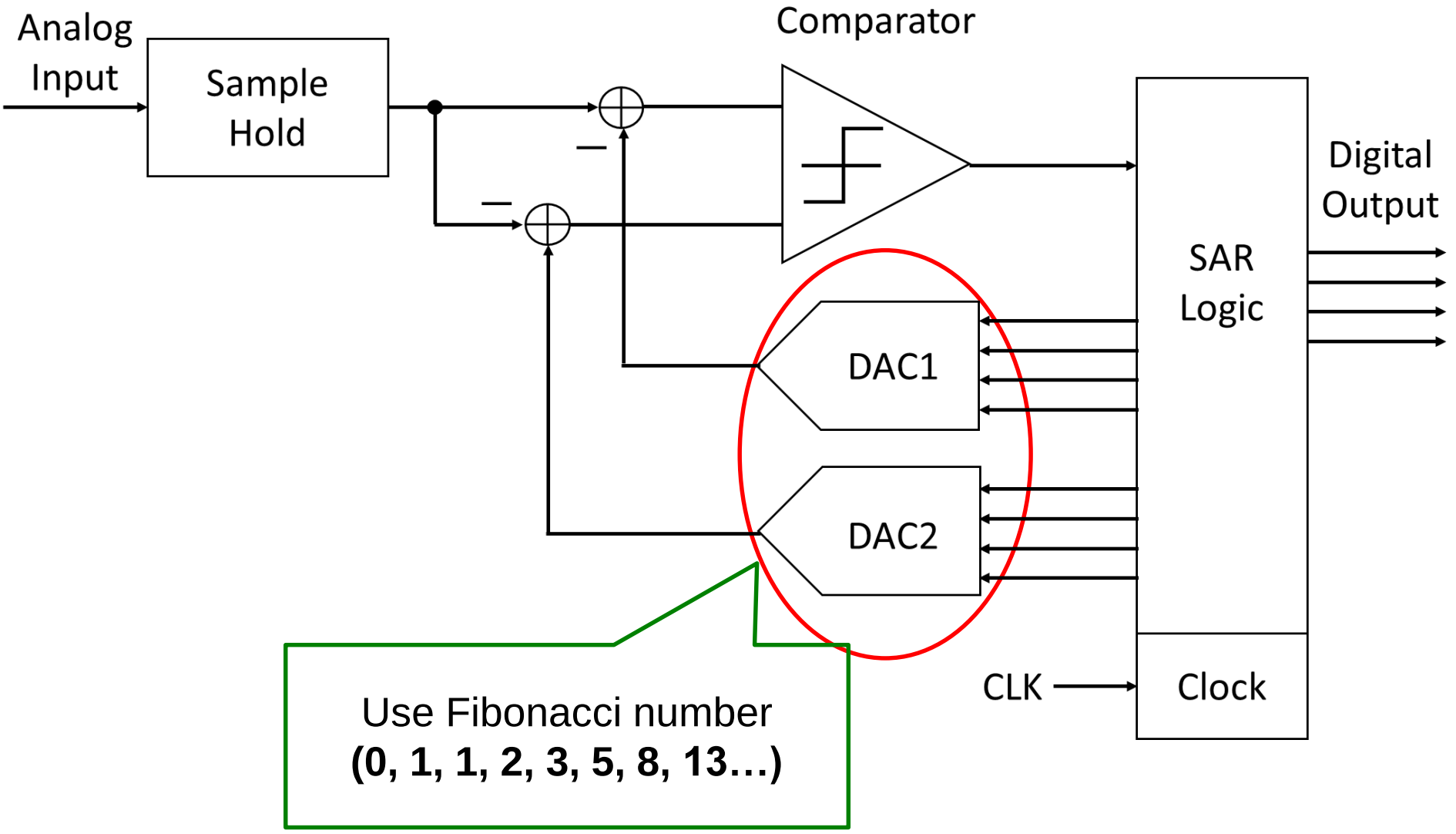
SAR ADC Based on Fibonacci Search

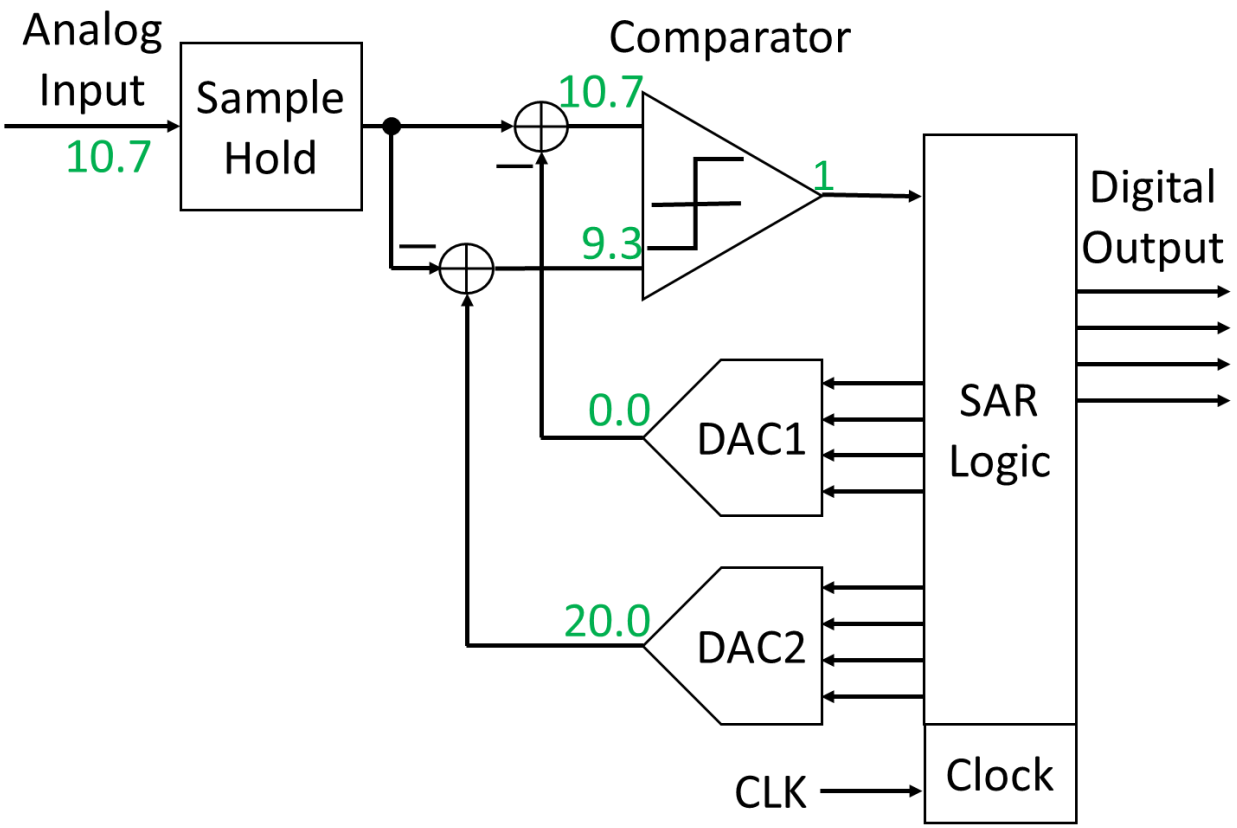


Unimodal function with local minimum → Use Fibonacci search

Block Diagram of SAR ADC Based on Fibonacci Search



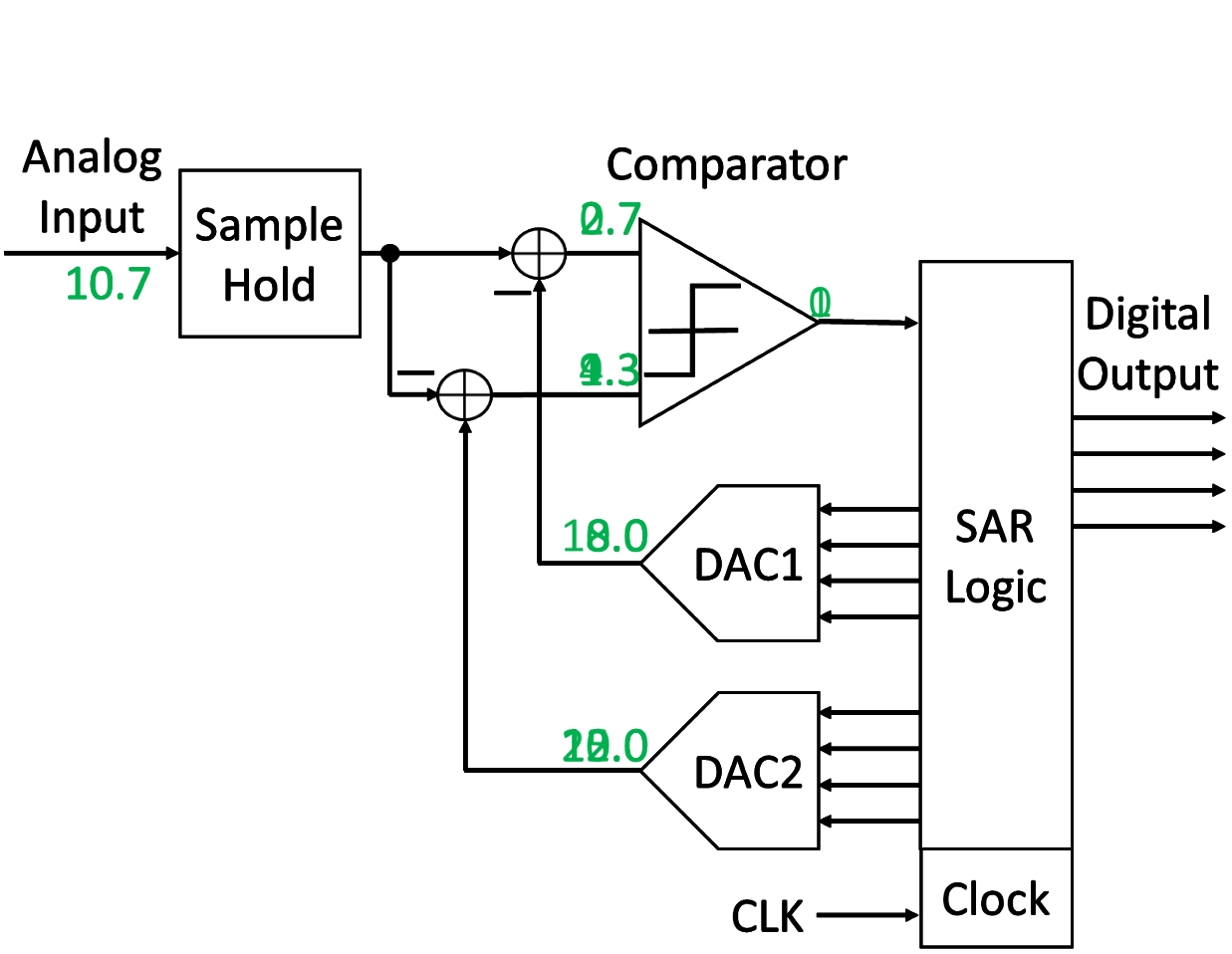
Operation of SAR ADC Based on Fibonacci Search



step	1st	2nd	3rd	4th	5th	output
20						20
19						19
18						18
17						17
16	9.3					16
15						15
14						14
13						13
12						12
11						11
10						10
9						9
8						8
7						7
6						6
5						5
4	10.7					4
3						3
2						2
1	1					1
0						0
−1						−1

Level

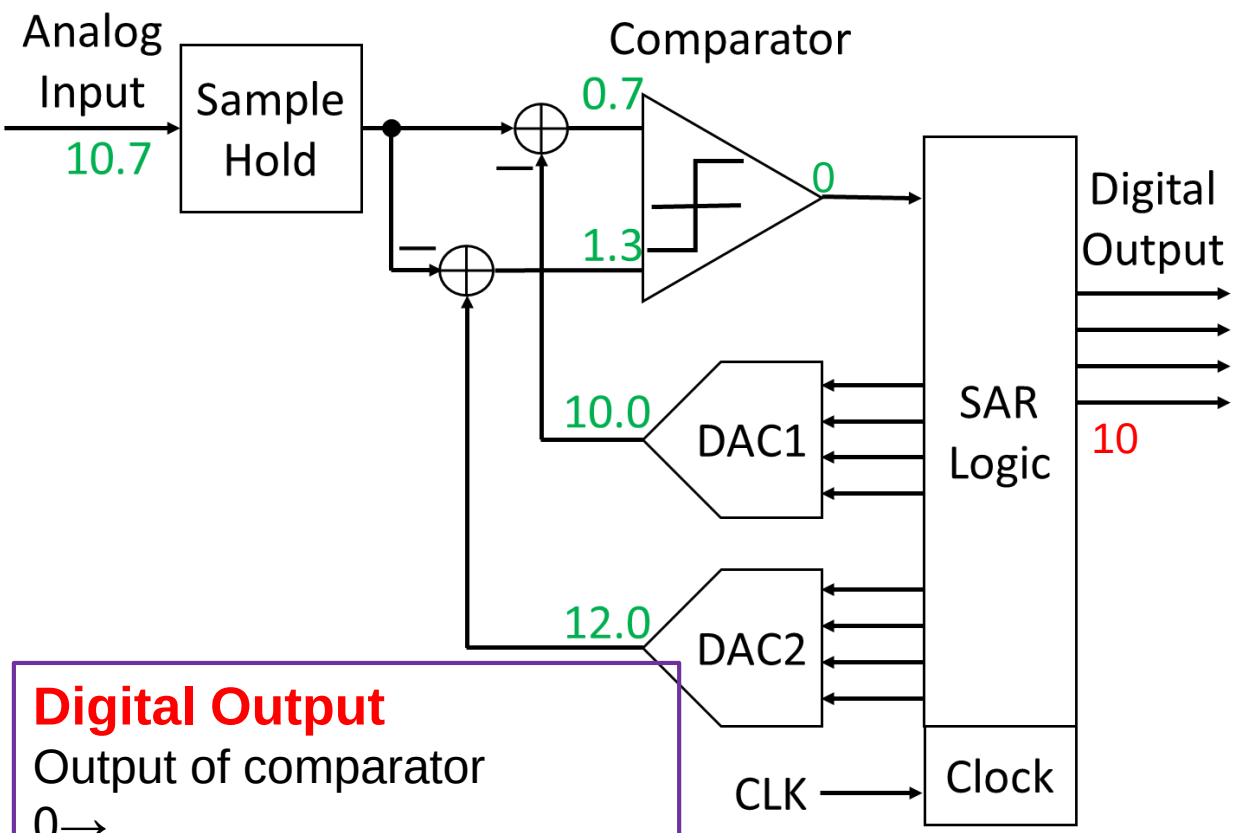
Operation of SAR ADC Based on Fibonacci Search



step	1st	2nd	3rd	4th	5th	output
20						20
19						19
18						18
17						17
16						16
15						15
14						14
13						13
12						12
11						11
10						10
9						9
8						8
7						7
6						6
5						5
4						4
3						3
2						2
1	1	0	0	1	0	1
0						0
-1						-1

Level

Operation of SAR ADC Based on Golden Section Search



step		1st	2nd	3rd	4th	5th	output
Level	20						20
	19						19
	18						18
	17						17
	16						16
	15						15
	14						14
	13						13
	12						12
	11						11
	10						10
	9						9
	8						8
	7						7
	6						6
	5						5
	4						4
	3						3
	2						2
	1	1	0	0	1	0	1
	0						0
	-1						-1

Digital Output

Output of comparator

0→

(Digital input of DAC1)

1→

(Digital input of ADAC1)+1

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Revised SAR ADC Based on Fibonacci Search

Comparison method before change

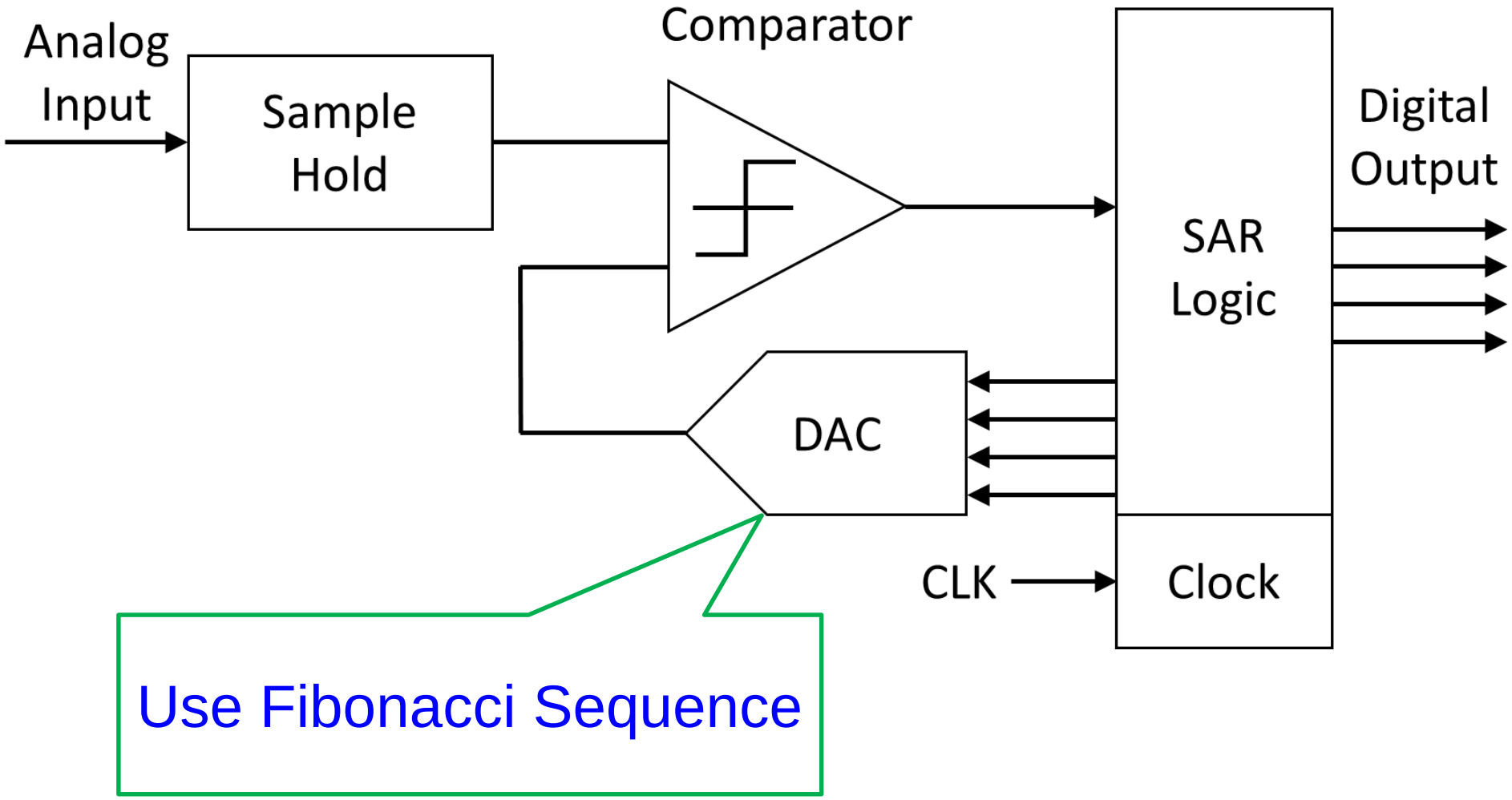
$$V_{in} - DAC1(\geq or \leq)DAC2 - V_{in}$$

$$2V_{in}(\geq or \leq)DAC1 + DAC2$$

$$V_{in}(\geq or \leq)\frac{1}{2}(DAC1 + DAC2)$$

Equivalent comparison method

Block Diagram of Revised SAR ADC Based on Fibonacci Search



Operation & DAC Revised SAR ADC Based on Fibonacci Search

Operation

1st step
Half the total current(20I)
$S_{L1}\sim S_{L6} \rightarrow ON$

Judgment:1

2nd step
Half the total current + 8I(28I)
$S_{L1}\sim S_{L6} + S_{R6} \rightarrow ON$

1

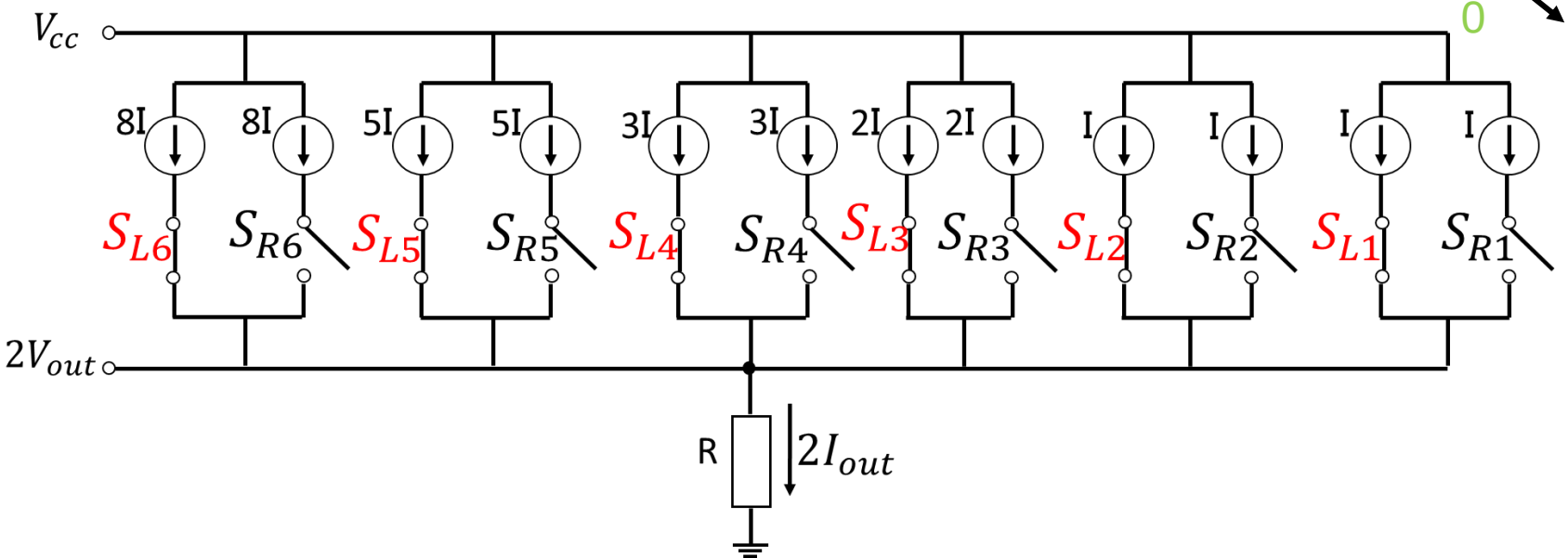
Judgment:0

2nd step
Half the total current - 8I(12I)
$S_{L1}\sim S_{L5} \rightarrow ON$

0

1

0



Revised SAR ADC Based on Fibonacci Search

Revised SAR ADC Based on Golden Section Search

Input 10.7

Output 10

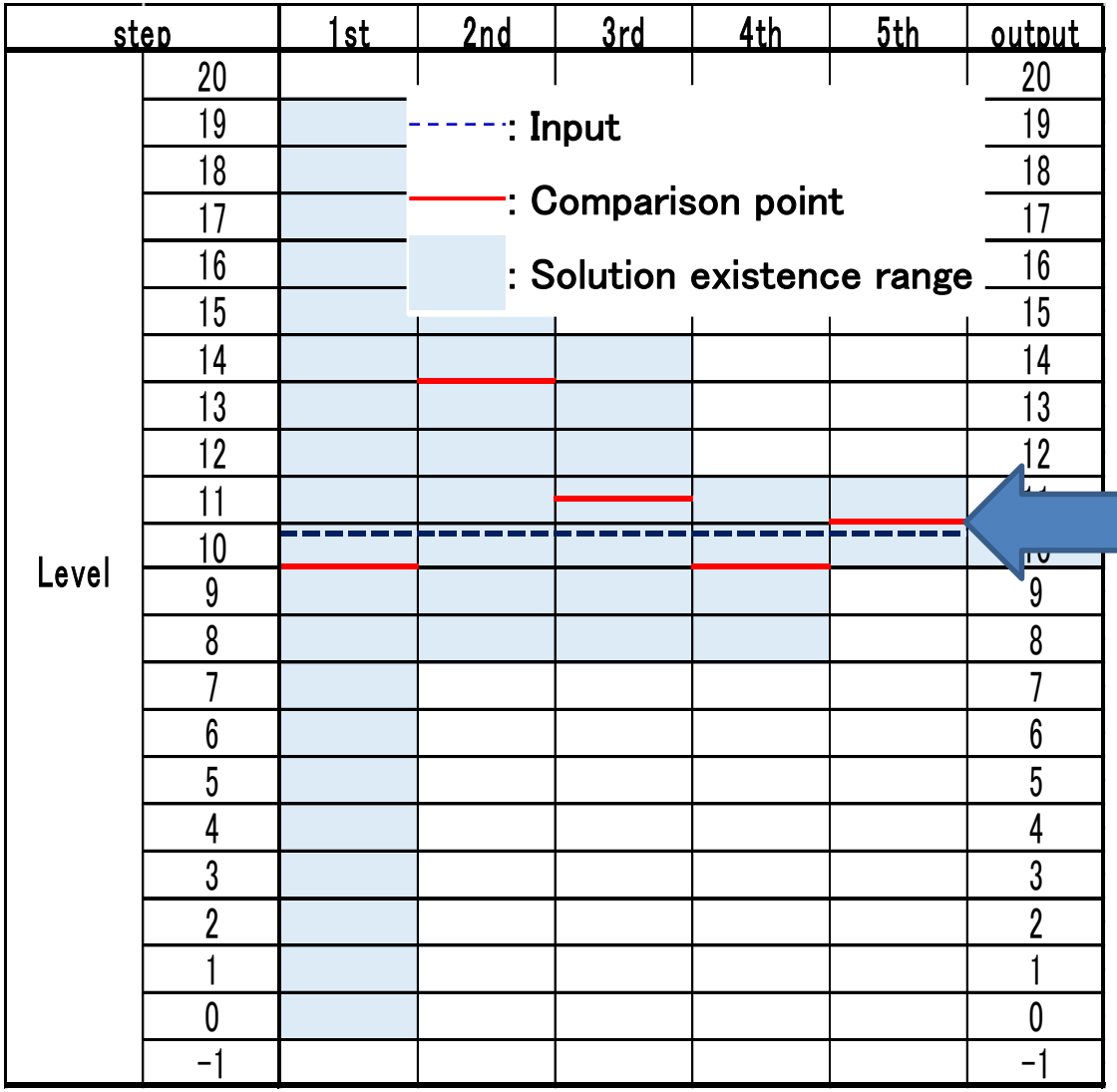
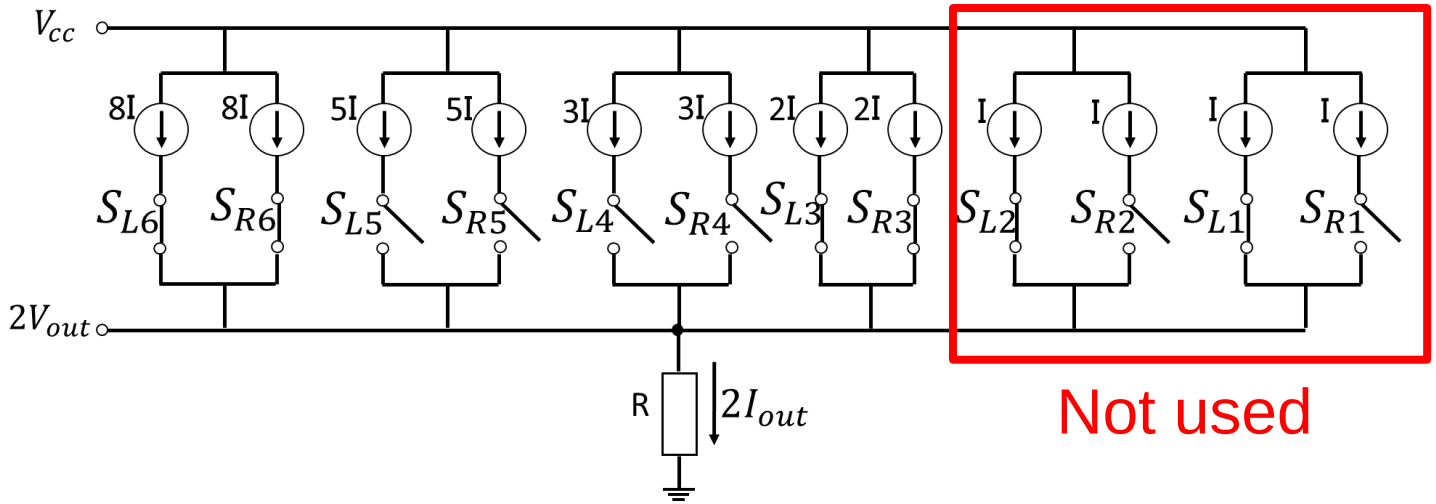


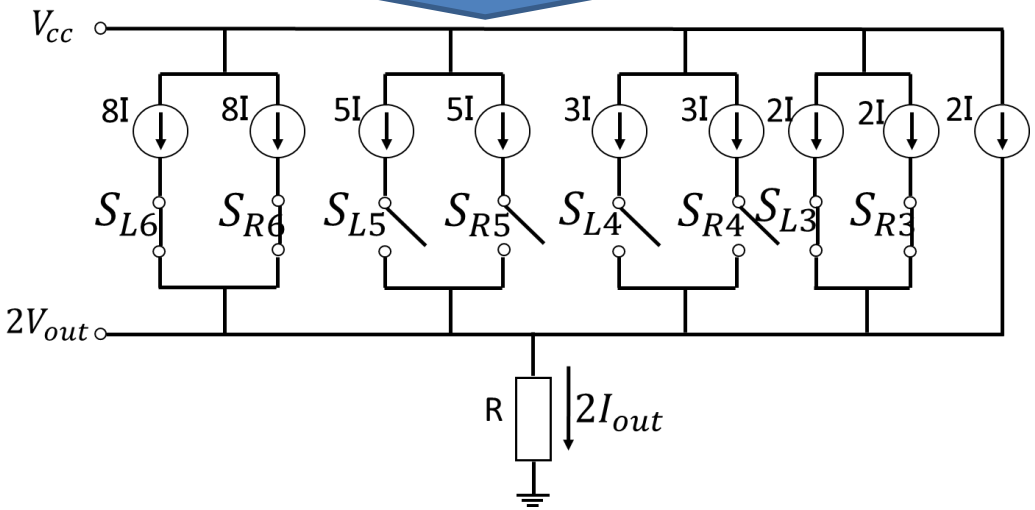
Figure of DAC at this time

Final Step of Revised SAR ADC Based on Fibonacci Search

DAC of final step



Further revised



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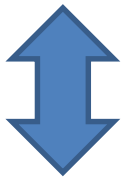
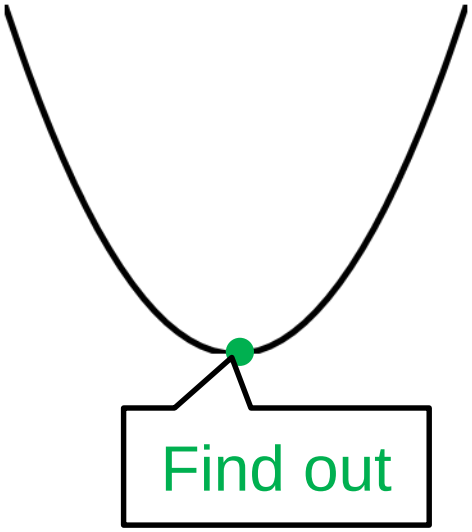
Proof Contents

SAR ADC based on
golden section search
using unimodal function



equivalent



Fibonacci sequence weighted SAR ADC

Prove above

Proof by Mathematical Expression

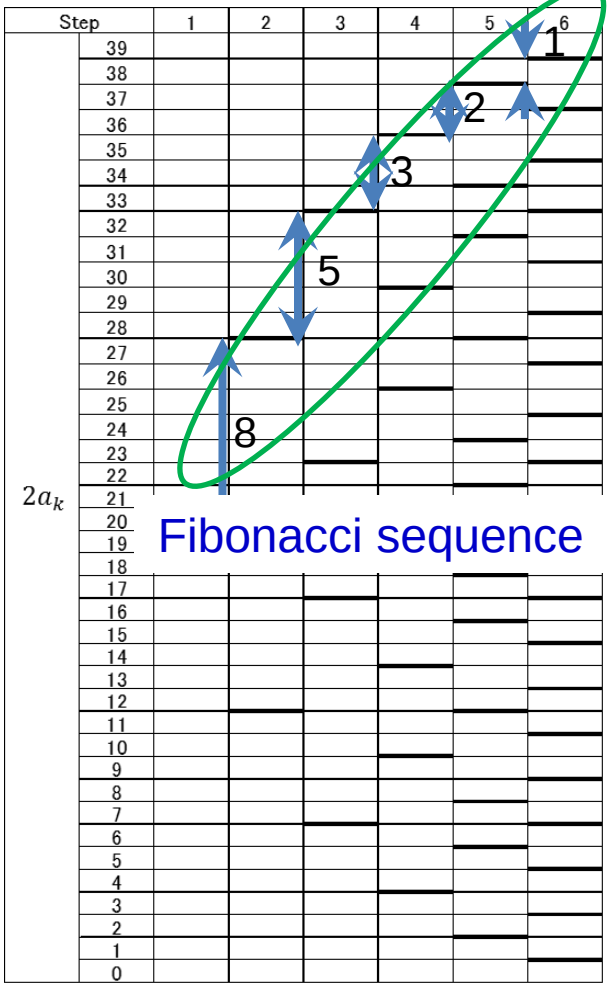
Fibonacci sequence

$$F_0 = 0$$
$$F_1 = 1$$
$$F_{n+2} = F_n + F_{n+1}$$

Comparison voltage

$$2a_k = (F_n + F_{n-1} + \cdots + F_1) + (\pm F_n \pm F_{n-1} \pm \cdots \pm F_1)$$

Comparison voltage when n=6



Difference of comparison voltages is Fibonacci sequence

Matches Fibonacci sequence weighted SAR ADC

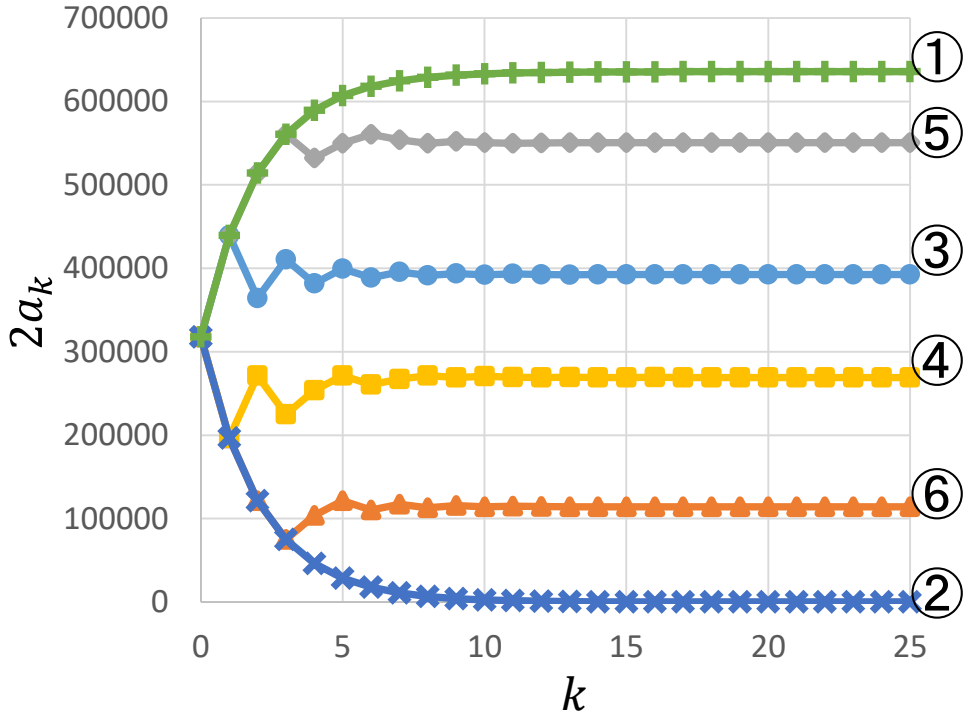
Simulation

Simulation condition

$$①V_{in} = 0,\ ②\ V_{in} = 317810,\ ③V_{in} = 196231.78,$$
$$④V_{in} = 57096.156,\ ⑤V_{in} = 275302.91,\ ⑥V_{in} = 134576.64$$

Convergence prediction

① 0, ② 635620, ③ 392462, ④ 114192, ⑤ 550604, ⑥ 269152



All converged to intended value

※ $n = 26$

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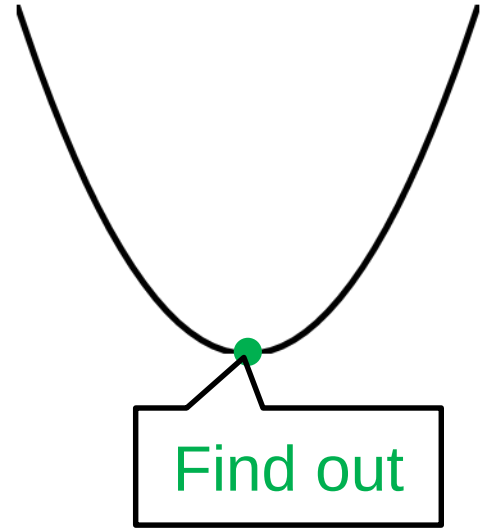
Conclusion

New Theorem

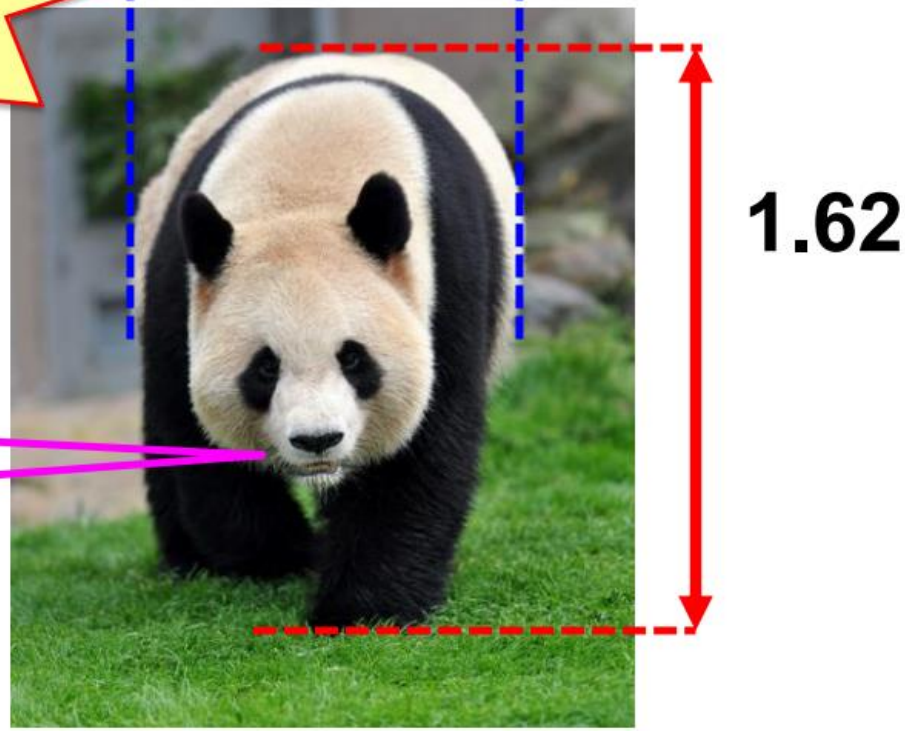


Equivalency

- SAR ADC using golden section search
- Fibonacci sequence weighted SAR ADC

Thank you for listening

Golden Ratio



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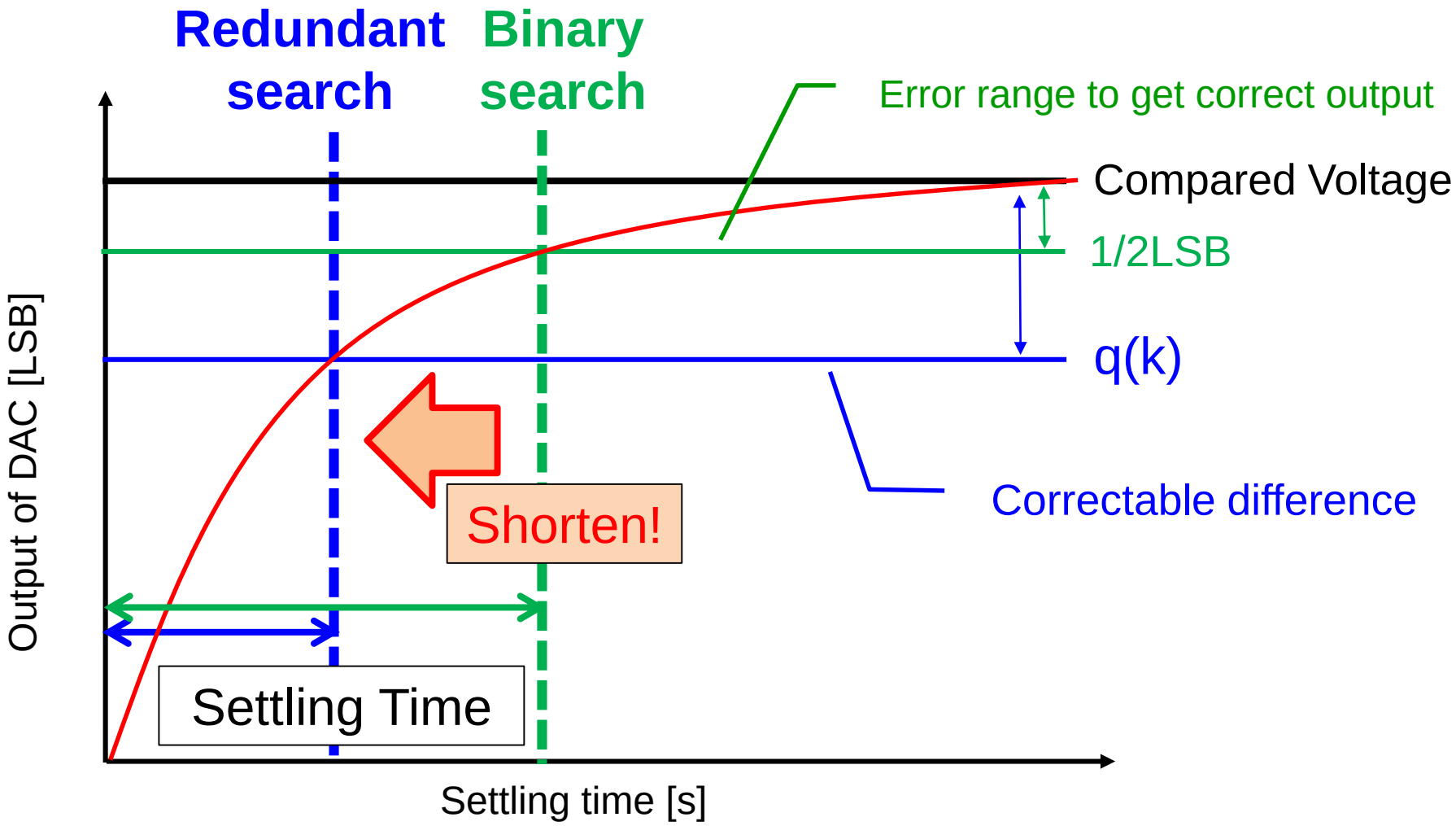
appendix

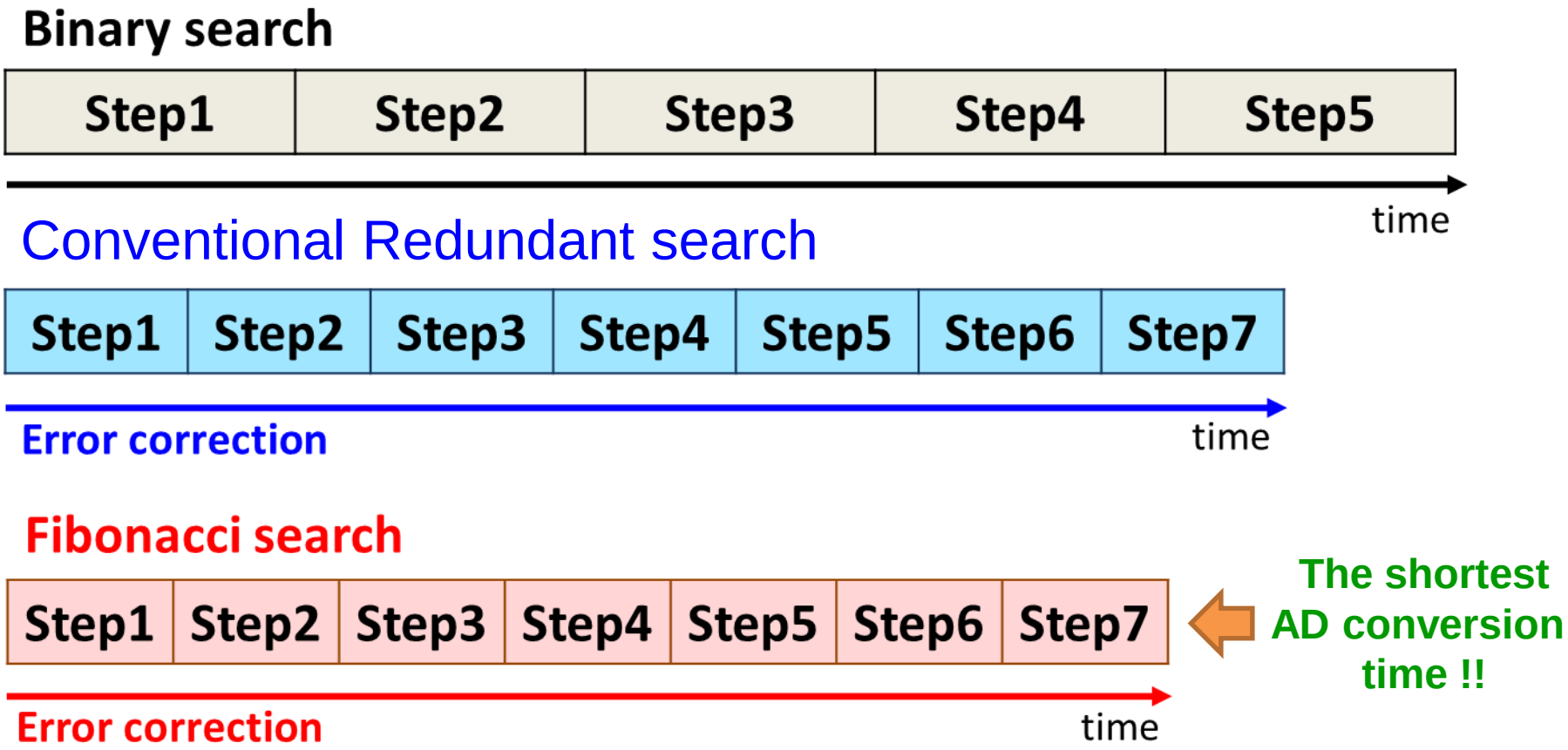
DAC Settling Time at Every Step

Shorten Conversion time

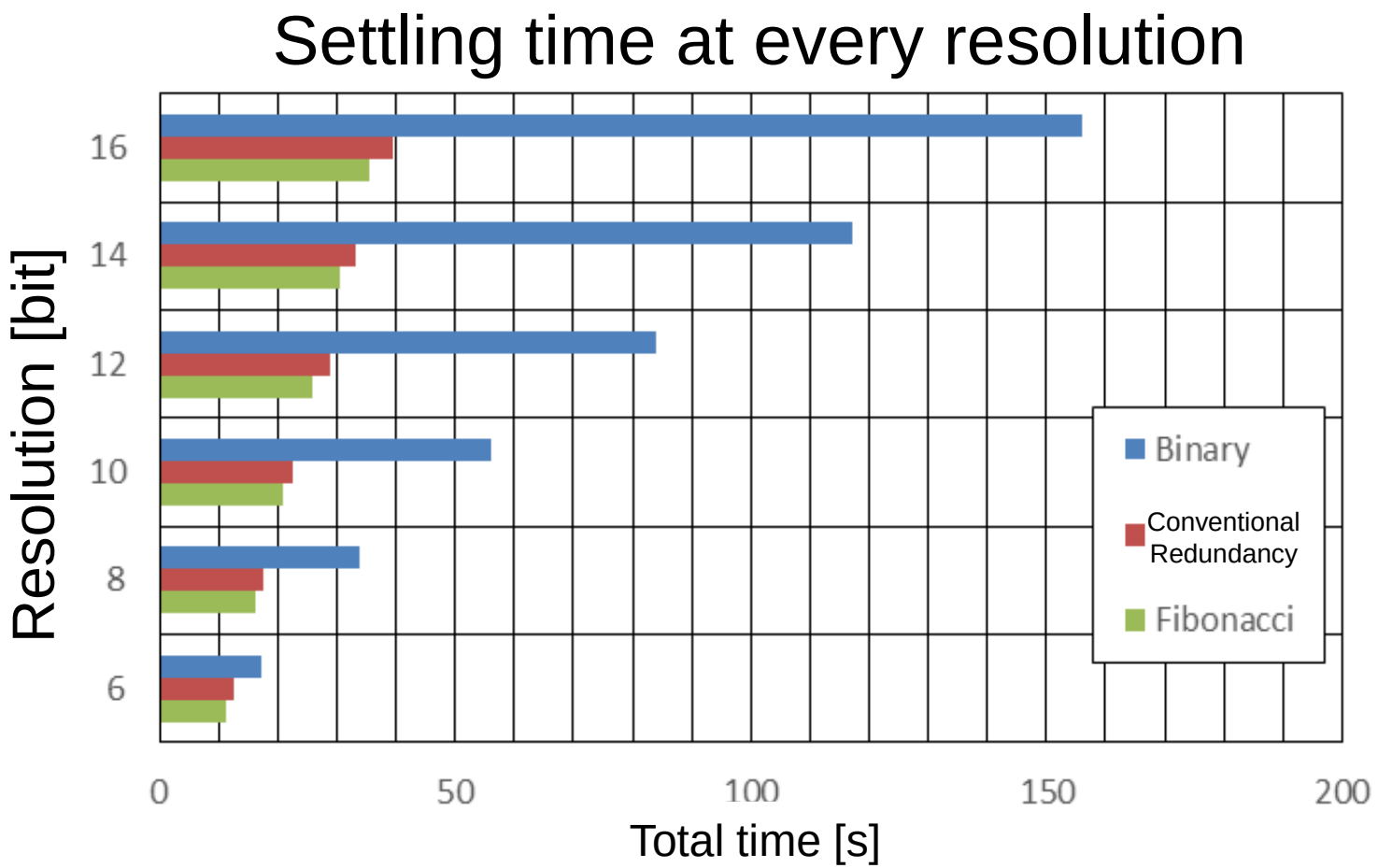


Reduction of Settling Time

5bit SAR ADC



Comparison of Incomplete Settling Time



At fixed clock,

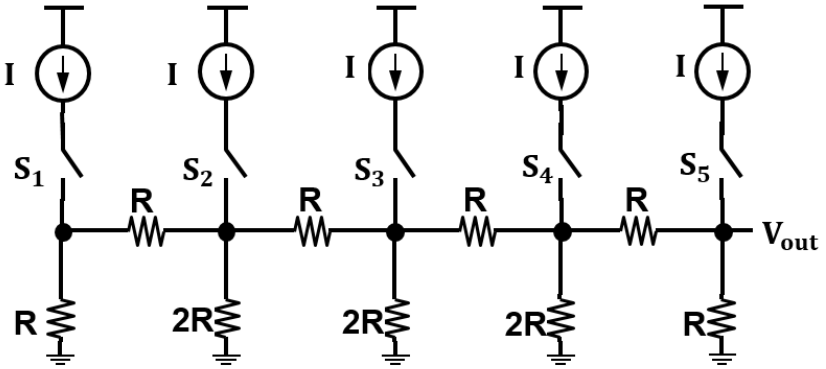
Fibonacci ⇨ the shortest AD conversion time !!

Conventional and Proposal DAC

Conventional

R-2R resistor ladder

⇒Generate **binary** voltage



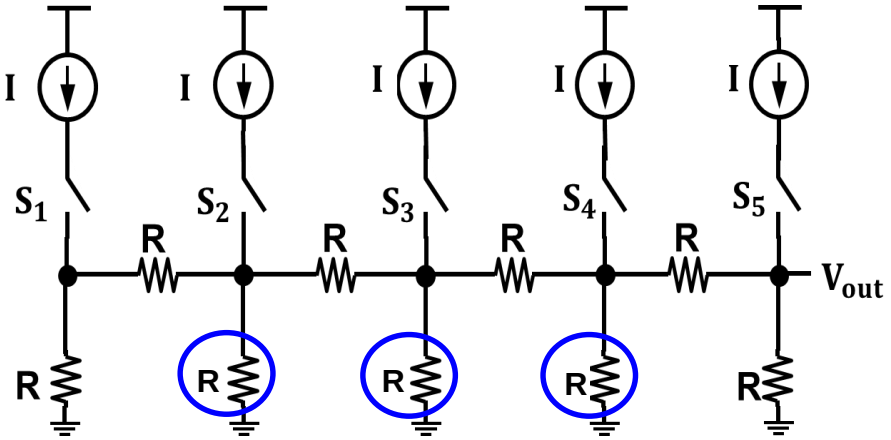
R-2R resistor ladder

Change all resistors to R

Proposal

R-R resistor ladder

⇒Generate **Fibonacci** voltage



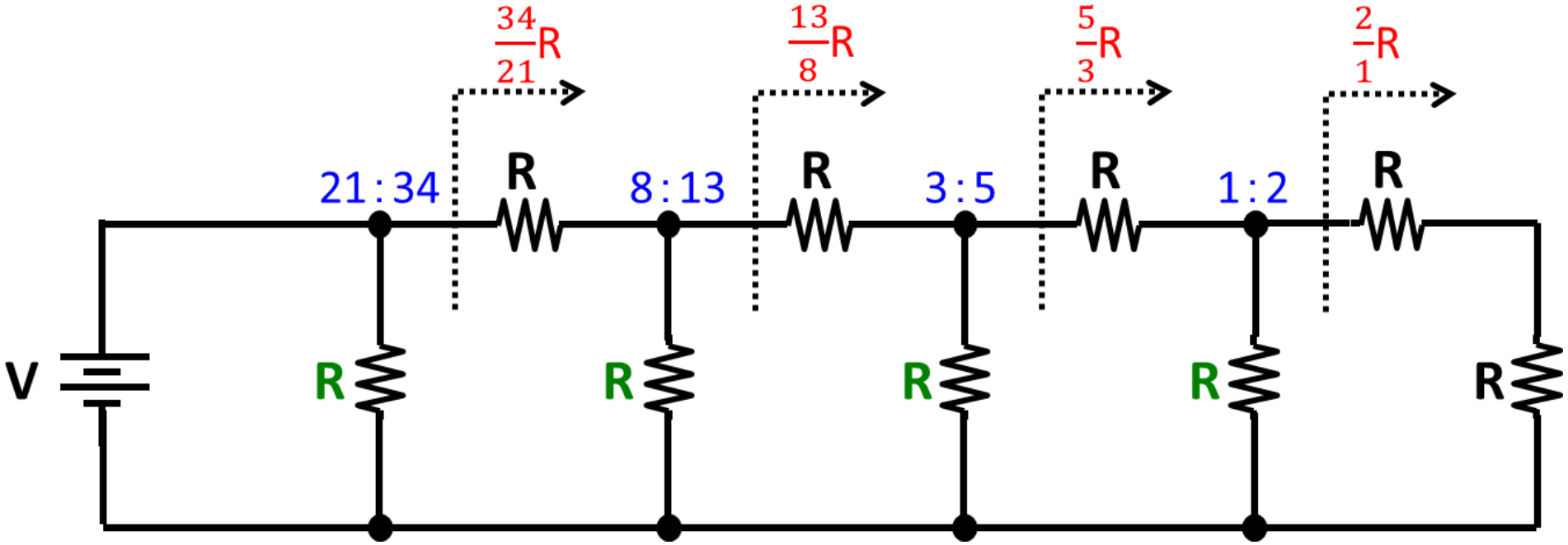
R-R resistor ladder

Realize Fibonacci DAC
by using simple circuit !

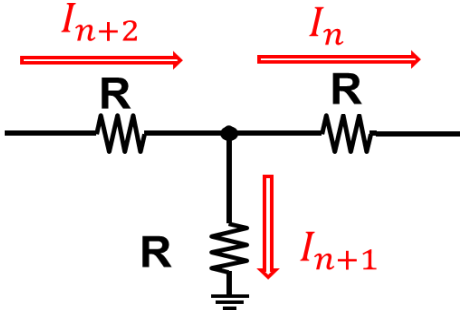
Principle of Fibonacci Voltage

New property

Divides current into **Fibonacci ratio** in each node



Principle



$$I_{n+2} = I_{n+1} + I_n$$

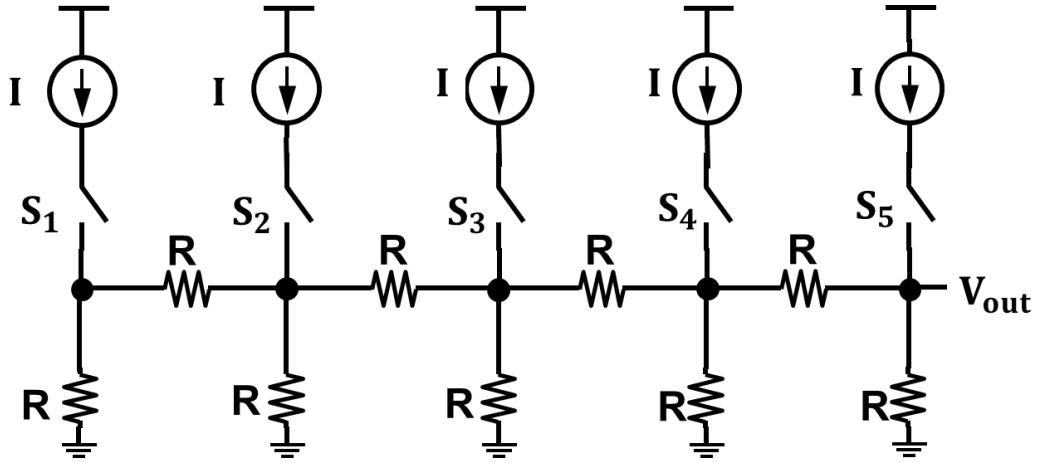
$$\Updownarrow$$

$$F_{n+2} = F_{n+1} + F_n$$

Proposal of R//R Fibonacci DAC

R-R resistor ladder

Generate
Fibonacci voltage
of **odd** term

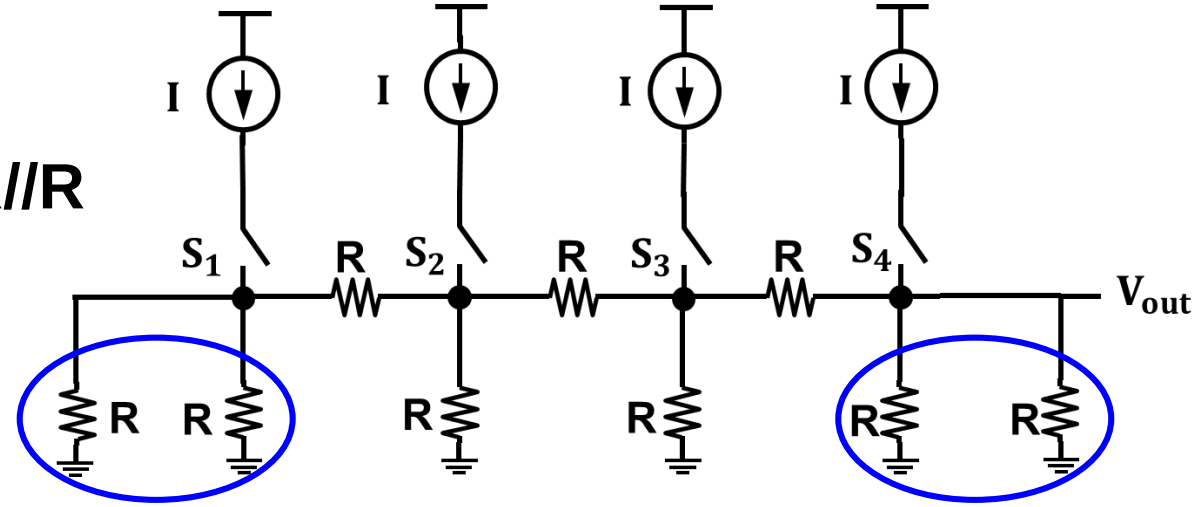


Change terminal resistors to
parallel resistors

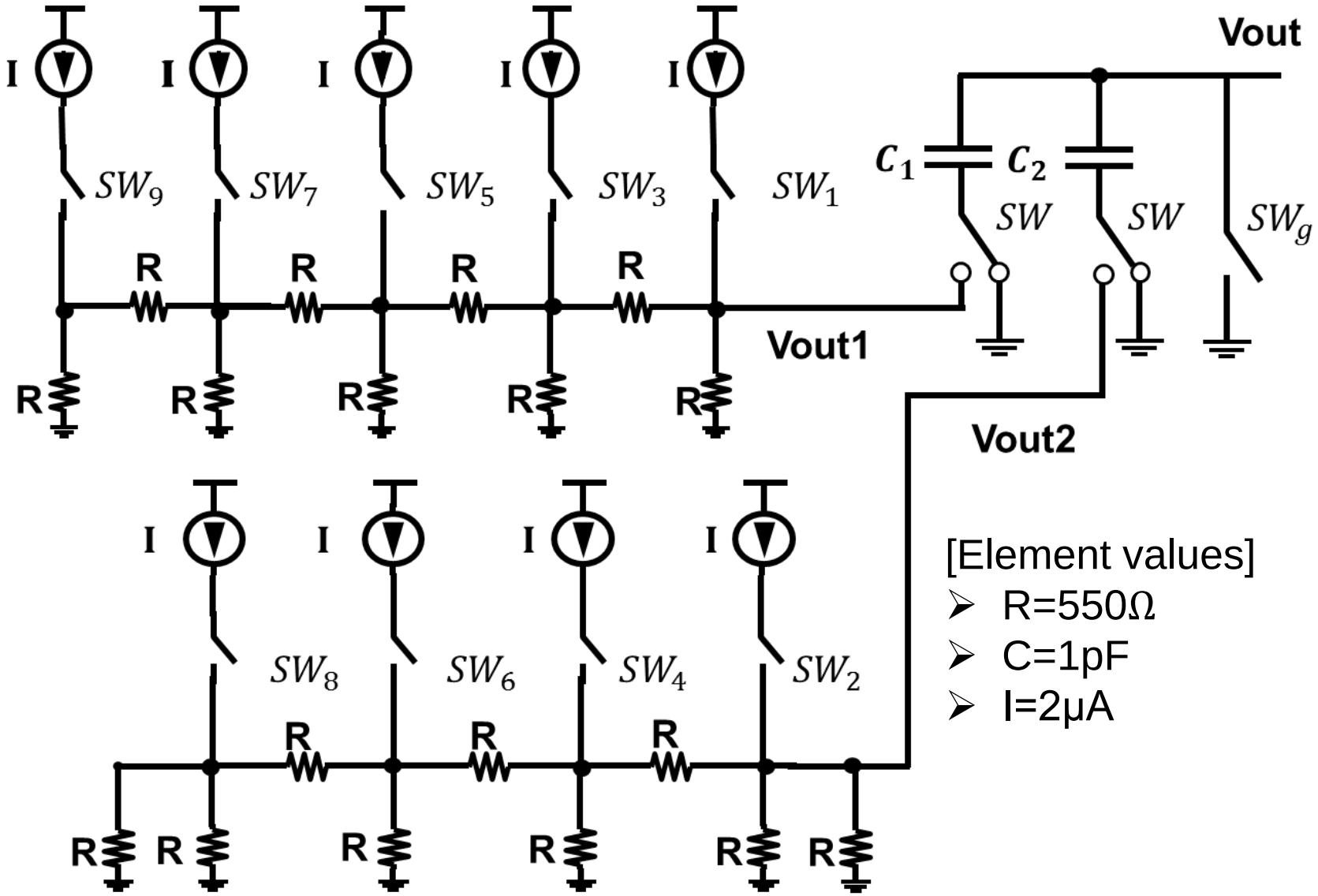
Proposal

R-R resistor ladder
with terminations of R//R

Generate
Fibonacci voltage
of **even** term



Fibonacci DAC simulation



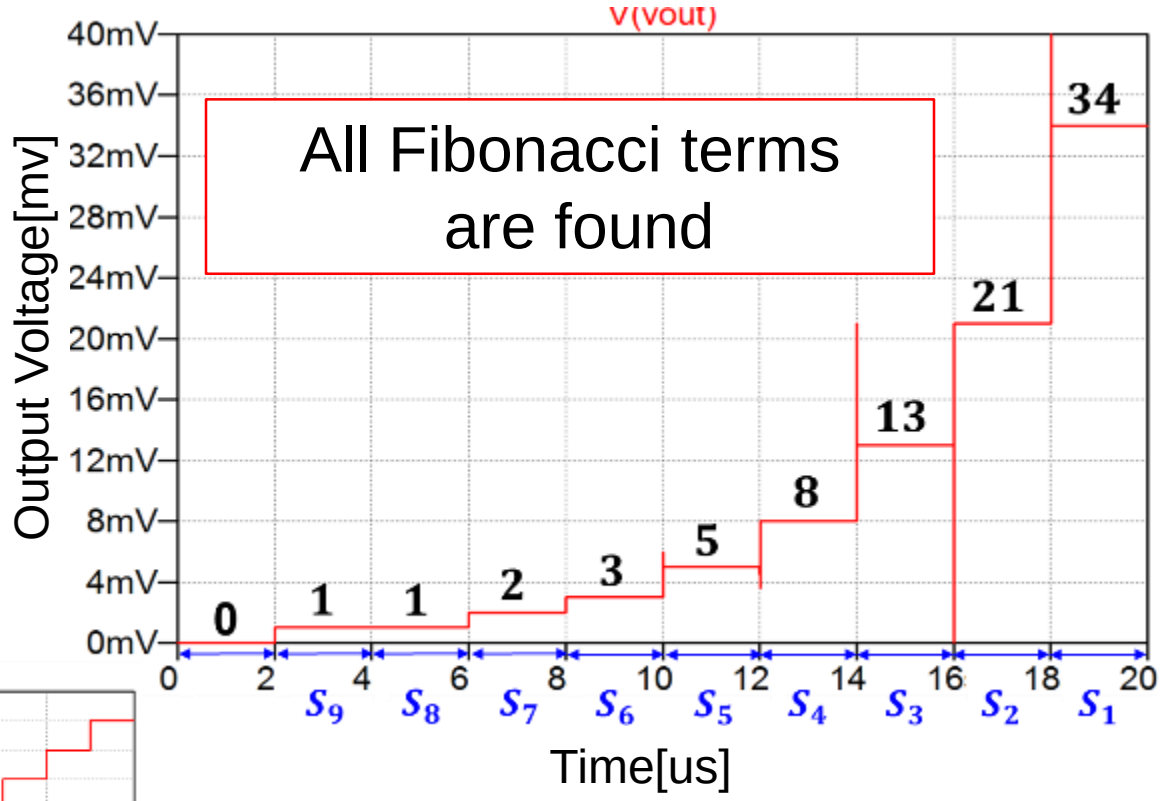
[Element values]

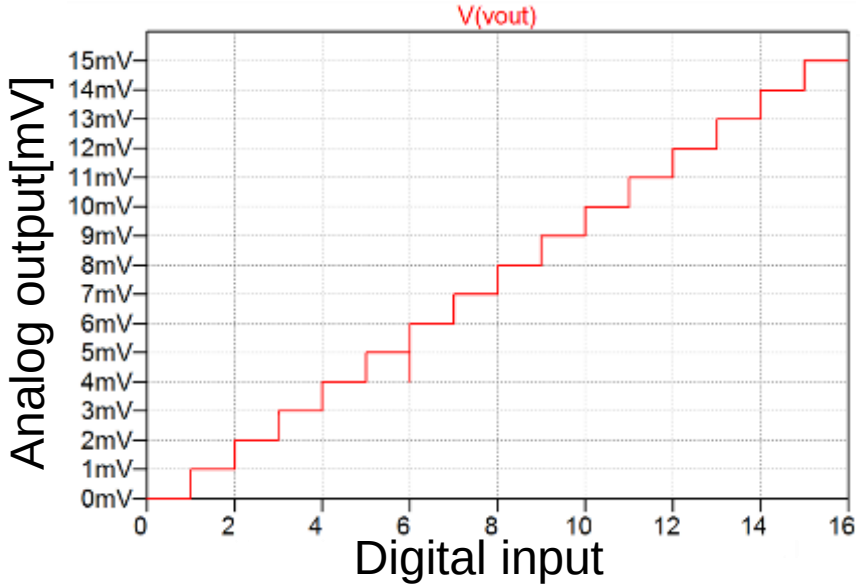
- R=550Ω
- C=1pF
- I=2μA

Simulation Result

Operation simulation

Each switch corresponds to a Fibonacci term





A-D conversion simulation

Combination of current sources realizes DAC function

Fibonacci DAC is realized