TP-L5: Adaptive, Non-linear and Multidimensional Signal Processing Ⅰ
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Equivalence Between Nyquist and Routh-Hurwitz Stability Criteria for Operational Amplifier Design

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Contents

- Research Objective & Background
- Stability Criteria
  - Nyquist Criterion
  - Routh-Hurwitz Criterion
- Equivalence at Mathematical Foundations
- Simulation Verification
- Discussion & Conclusion

Contents

Research Background (Stability Theory)

- **Electronic Circuit Design Field**
  - Bode plot (>90% frequently used)
  - Nyquist plot

- **Control Theory Field**
  - Bode plot
  - Nyquist plot
  - Nicholas plot
  - Routh-Hurwitz stability criterion

    ⇨ Very popular in control theory field
    but rarely seen in electronic circuit books/papers
  - Lyapunov function method

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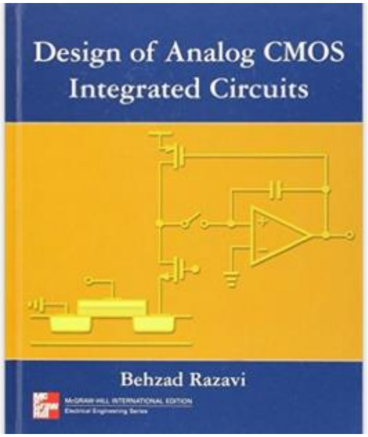
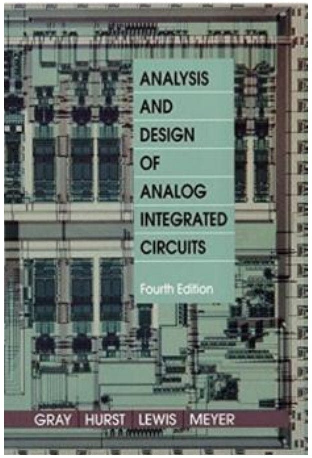
Electronic Circuit Text Book

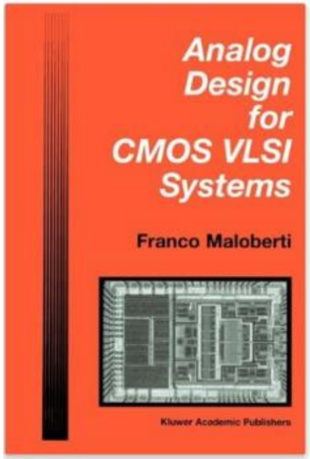
We were NOT able to find out any electronic circuit text book which describes Routh-Hurwitz method for operational amplifier stability analysis and design !



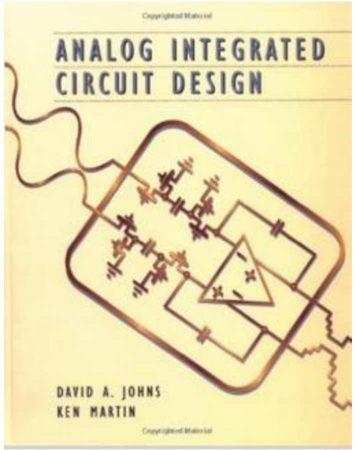
Razavi



Gray



Maloberti

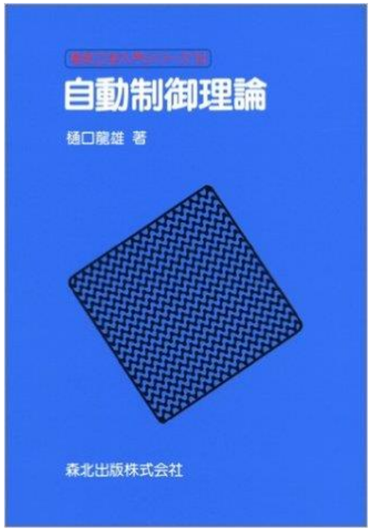


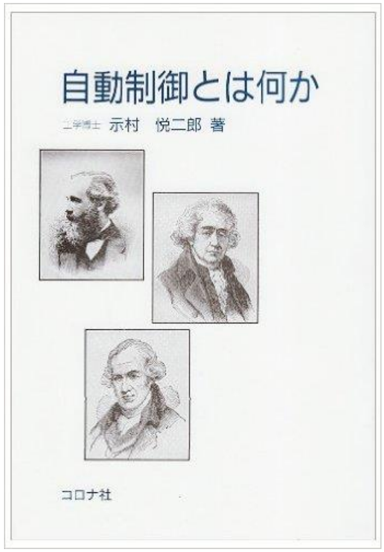
Martin

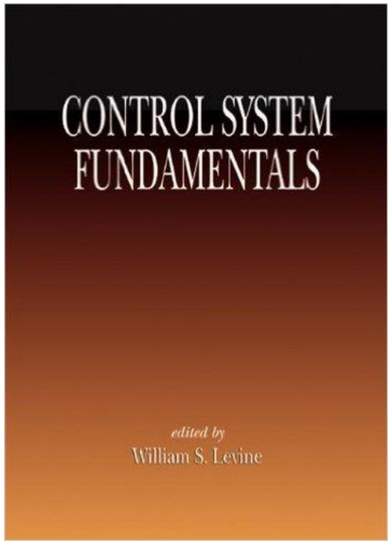
None of the above describes Routh-Hurwitz.
Only Bode plot is used.

Control Theory Text Book

Most of control theory text books
describe Routh-Hurwitz method
for system stability analysis and design !







Research Objective

Our proposal

For

Analysis and design of operational amplifier stability

Use

Routh-Hurwitz stability criterion

↓

We can obtain

Explicit stability condition for circuit parameters

(which can NOT be obtained only with Bode plot).

We can verify

Equivalence between Nyquist and Routh-Hurwitz Stability Criteria

Contents

- Research Objective & Background
- Stability Criteria
  - Nyquist Criterion — Nyquist plot / Bode plot
  - Routh-Hurwitz Criterion
- Equivalence at Mathematical Foundations
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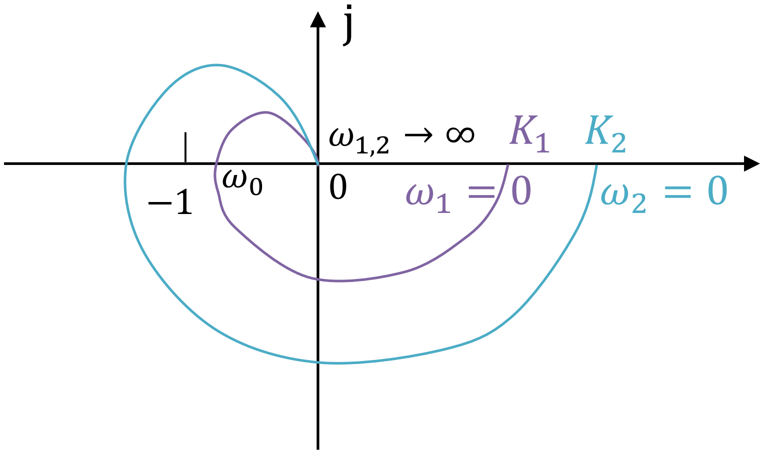
- Discussion & Conclusion

Nyquist plot

- Open-loop frequency characteristic

  ➡ Closed-loop stability



Nyquist plot of open-loop system

- Necessary and sufficient condition :

  When $\omega = 0 \to \infty$, $N = P - Z$

N : number, Nyquist plot anti-clockwise encircle point (-1,j0).

P: number, positive roots of open-loop characteristic equation.

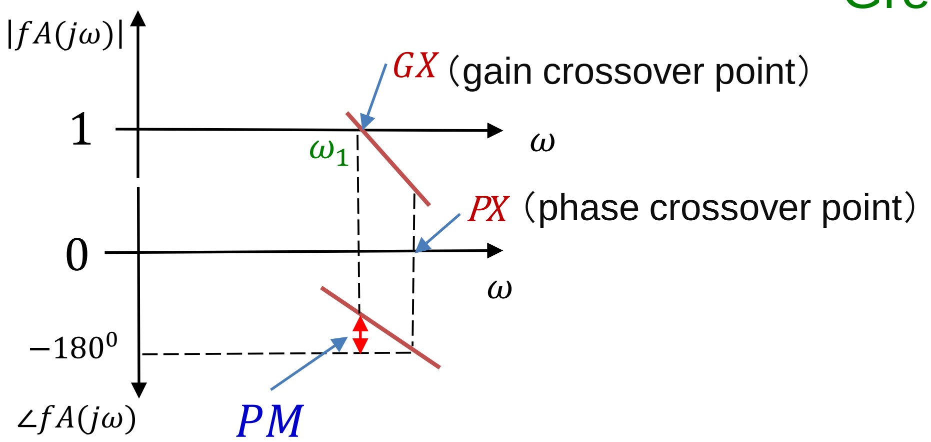
Z: number, positive roots of closed-loop characteristic equation.

- If the open-loop system is stable(P=0), the Nyquist plot mustn't encircle the point (-1,j0).

$$\angle G_{open}(j\omega_0) = -\pi, \left|G_{open}(j\omega_0)\right| < 1$$

Phase Margin from Bode Plot

GX precedes PX ⇒ Feedback system is stable



Greater spacing between GX and PX

⇓

More stable

ω_1: gain crossover frequency

Phase margin : $\mathrm{PM} = 180^0 + \angle fA(\omega = \omega_1)$

Bode plot is useful,
but it does NOT show explicit stability conditions of circuit parameters.

Contents

Routh Stability Criterion

Characteristic equation:

$$D(s) = \alpha_n s^n + \alpha_{n-1} s^{n-1} + \cdots + \alpha_1 s + \alpha_0 = 0$$

Routh table

s^n	α_n	α_{n-2}	α_{n-4}	α_{n-6}	$\cdots$
s^{n-1}	α_{n-1}	α_{n-3}	α_{n-5}	α_{n-7}	$\cdots$
s^{n-2}	$\beta_1 = \dfrac{\alpha_{n-1}\alpha_{n-2} - \alpha_n\alpha_{n-3}}{\alpha_{n-1}}$	$\beta_2 = \dfrac{\alpha_{n-1}\alpha_{n-4} - \alpha_n\alpha_{n-5}}{\alpha_{n-1}}$	β_3	β_4	$\cdots$
s^{n-3}	$\gamma_1 = \dfrac{\beta_1\alpha_{n-3} - \alpha_{n-1}\beta_2}{\beta_1}$	$\gamma_2 = \dfrac{\beta_1\alpha_{n-5} - \alpha_{n-1}\beta_3}{\beta_1}$	γ_3	γ_4	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s^0	α_0				

Sufficient and necessary condition:

(i) $\alpha_i > 0$ for $i = 0,1,\ldots,n$

&

(ii) All values of Routh table's first columns are positive.

Mathematical test

Determine whether given polynomial has all roots in the left-half plane.

Contents

Four Examples

Ex.1 $$G(s) = \frac{K}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ Zero Zero Point, Three Pole Points

Ex.2 $$G(s) = \frac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2}$$ One Zero Point, Two Pole Points

Ex.3 $$G(s) = \frac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ One Zero Point, Three Pole Points

Ex.4 $$G(s) = \frac{K(1 + b_1 s + b_2 s^2)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ Two Zero Points, Three Pole Points

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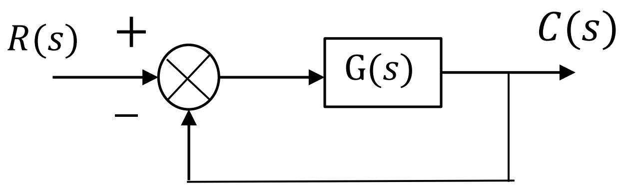
Based on Routh-Hurwitz Criterion

Example 1

Open-loop transfer function:

$$G(s) = \frac{K}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$



Block diagram of feedback system

Closed-loop transfer function:

$$H(s) = \frac{G(s)}{1 + G(s)} = \frac{K}{1 + K + a_1 s + a_2 s^2 + a_3 s^3}$$

Routh table

S^3	a_3	a_1
S^2	a_2	$1 + K$
S^1	$\frac{a_1 a_2 - a_3(1 + K)}{a_2}$	
S^0	$1 + K$	

Based on Routh-Hurwitz criterion:

$$a_3 > 0, a_2 > 0, 1 + K > 0,$$

$$\frac{a_1 a_2 - a_3(1 + K)}{a_2} > 0$$

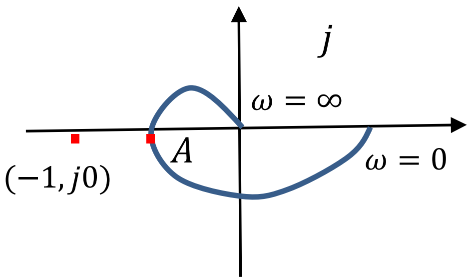
$$\Rightarrow \quad a_1 a_2 - a_3 > K a_3$$

Based on Nyquist Criterion

Frequency domain:

$$G(j\omega) = \frac{K}{1 - a_2\omega^2 + j(a_1\omega - a_3\omega^3)} = \frac{K[(1 - a_2\omega^2) - j(a_1\omega - a_3\omega^3)]}{(1 - a_2\omega^2)^2 + (a_1\omega - a_3\omega^3)^2}$$

Special frequency expressions



sketch chart of Nyquist plot

$$\angle G(j\omega) = -\pi$$

$$\Rightarrow (a_1\omega - a_3\omega^3) = 0$$

$$\Rightarrow \omega^2 = \frac{a_1}{a_3}$$ At point A

$$\Rightarrow |G(j\omega)| = \left|\frac{K\sqrt{(1 - a_2\omega^2)^2 + (a_1\omega - a_3\omega^3)^2}}{(1 - a_2\omega^2)^2 + (a_1\omega - a_3\omega^3)^2}\right| = \frac{K}{\left|1 - a_2\frac{a_1}{a_3}\right|}$$

Stability condition:

$$|G(j\omega)| < 1 \Rightarrow \begin{cases} a_1a_2 - a_3 < Ka_3 < a_3 - a_1a_2 \\ a_3 - a_1a_2 < Ka_3 < a_1a_2 - a_3 \end{cases}$$

At condition: $a_3 - a_1a_2 > 0$

At condition: $a_3 - a_1a_2 < 0$

Four Examples

Ex.1 $G(s) = \dfrac{K}{1 + a_1 s + a_2 s^2 + a_3 s^3}$ Zero Zero Point, Three Pole Points

Ex.2 $G(s) = \dfrac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2}$ One Zero Point, Two Pole Points

Ex.3 $G(s) = \dfrac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$ One Zero Point, Three Pole Points

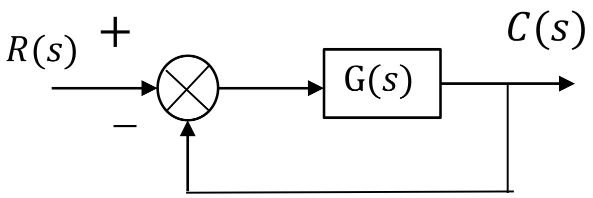
Ex.4 $G(s) = \dfrac{K(1 + b_1 s + b_2 s^2)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$ Two Zero Points, Three Pole Points

Based on Routh-Hurwitz Criterion

Example 2

Open-loop transfer function:

$$G(s) = \frac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2}$$



Closed-loop transfer function:

$$H(s) = \frac{G(s)}{1 + G(s)} = \frac{K + Kb_1 s}{1 + K + (a_1 + Kb_1)s + a_2 s^2}$$

Routh table

S^2	a_2	$1 + K$
S^1	$a_1 + Kb_1$	0
S^0	$1 + K$	

Based on Routh-Hurwitz criterion:

$$a_2 > 0, \ 1 + K > 0$$

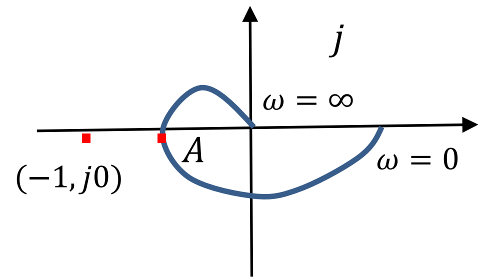
$$a_1 + Kb_1 > 0$$

$$\Rightarrow \begin{cases} K > -\dfrac{a_1}{b_1} & \text{At condition: } b_1 > 0 \\ K < -\dfrac{a_1}{b_1} & \text{At condition: } b_1 < 0 \end{cases}$$

Based on Nyquist Criterion

Frequency domain:

$$G(j\omega) = \frac{K(1+b_1(j\omega))}{1+a_1(j\omega)+a_2(j\omega)^2} = \frac{K\,(1-a_2\omega^2\ +b_1a_1\omega^2)+jK(b_1\omega-a_1\omega-a_2b_1\omega^3)}{(1-a_2\omega^2)^2+a_1^2\omega^2}$$



sketch chart of Nyquist plot

Special frequency expressions

$$\angle G(j\omega) = -\pi$$

$$\Rightarrow \quad b_1\omega - a_1\omega - a_2b_1\omega^3 = 0$$

$$\Rightarrow \quad \omega^2 = \frac{1}{a_2}\left(1-\frac{a_1}{b_1}\right)$$ At point A

$$\Rightarrow \quad |G(j\omega)| = \left|\frac{K1-a_2\omega^2\ +b_1a_1\omega^2}{(1-a_2\omega^2)^2+a_1^2\omega^2}\right| = \frac{K\left|\frac{a_1}{b_1}+\frac{a_1}{a_2}(b_1-a_1)\right|}{\left|\left(\frac{a_1}{b_1}\right)^2+\frac{a_1}{a_2}\frac{a_1}{b_1}\ (b_1-a_1)\right|} = K\left|\frac{b_1}{a_1}\right|$$

Stability condition:

$$|G(j\omega)| < 1 \quad \Rightarrow \quad \begin{cases} -\frac{a_1}{b_1} < K < \frac{a_1}{b_1} & \text{At condition: } a_1b_1 > 0 \\ \frac{a_1}{b_1} < K < -\frac{a_1}{b_1} & \text{At condition: } a_1b_1 < 0 \end{cases}$$

Four Examples

Ex.1 $$G(s) = \frac{K}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ Zero Zero Point, Three Pole Points

Ex.2 $$G(s) = \frac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2}$$ One Zero Point, Two Pole Points

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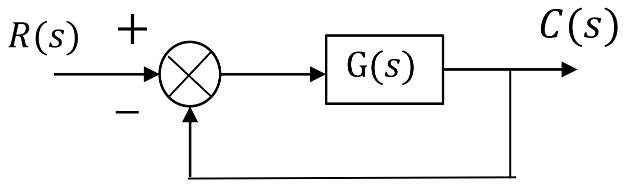
Ex.4 $$G(s) = \frac{K(1 + b_1 s + b_2 s^2)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ Two Zero Points, Three Pole Points

Based on Routh-Hurwitz Criterion

Example 3



Open-loop transfer function:

$$G(s) = \frac{K(1+bs)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$

Closed-loop transfer function:

$$H(s) = \frac{G(s)}{1+G(s)} = \frac{K + Kbs}{1 + K + (a_1 + Kb)s + a_2 s^2 + a_3 s^3}$$

Routh table

S^3	a_3	$a_1 + Kb$
S^2	a_2	$1 + K$
S^1	$\frac{a_2(a_1+Kb) - a_3(1+K)}{a_2}$	
S^0	$1 + K$	

Based on Routh-Hurwitz criterion:

$a_3 > 0$ $a_2 > 0$

$1 + K > 0$

$$\frac{a_2(a_1+Kb) - a_3(1+K)}{a_2} > 0 \Rightarrow \begin{cases} K > \frac{a_3 - a_1 a_2}{a_2 b - a_3} \\ K < \frac{a_3 - a_1 a_2}{a_2 b - a_3} \end{cases}$$

At condition: $a_2 b - a_3 > 0$

At condition: $a_2 b - a_3 < 0$

Based on Nyquist Criterion

Frequency domain:

$$G(j\omega)=\frac{K(1+bj\omega)}{1-a_2\omega^2+j(a_1\omega-a_3\omega^3)}=\frac{K[(1-a_2\omega^2+a_1b\omega^2-a_3b\omega^4)+j(b\omega-a_2b\omega^3-a_1\omega+a_3\omega^3)]}{(1-a_2\omega^2)^2+(a_1\omega-a_3\omega^3)^2}$$

Special frequency expressions

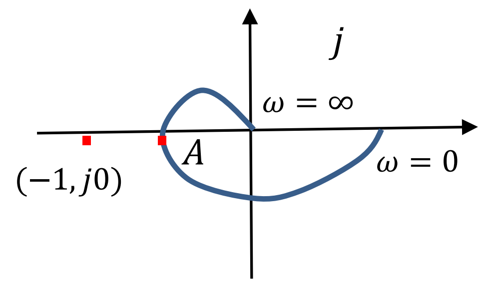
$$\angle G(j\omega)=-\pi$$

$$\Rightarrow\quad b\omega-a_2b\omega^3-a_1\omega+a_3\omega^3=0$$

$$\Rightarrow\quad \omega^2=\frac{a_1-b}{a_3-a_2b}$$

At point A

$$\Rightarrow\quad |G(j\omega)|=\left|\frac{K(1-a_2\omega^2+a_1b\omega^2-a_3b\omega^4)}{(1-a_2\omega^2)^2+(a_1\omega-a_3\omega^3)^2}\right|=K\left|\frac{a_3-a_2b}{a_3-a_1a_2}\right|$$



sketch chart of Nyquist plot

Stability condition:

$$|G(j\omega)|<1 \Rightarrow \begin{cases}\dfrac{a_3-a_1a_2}{a_2b-a_3}<K<\dfrac{a_3-a_1a_2}{a_3-a_2b} & \text{At condition: } (a_3-a_1a_2)(a_3-a_2b)>0\\[2ex] \dfrac{a_3-a_1a_2}{a_3-a_2b}<K<\dfrac{a_3-a_1a_2}{a_2b-a_3} & \text{At condition: } (a_3-a_1a_2)(a_3-a_2b)<0\end{cases}$$

Four Examples

Ex.1 $$G(s) = \frac{K}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ Zero Zero Point, Three Pole Points

Ex.2 $$G(s) = \frac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2}$$ One Zero Point, Two Pole Points

Ex.3 $$G(s) = \frac{K(1 + b_1 s)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ One Zero Point, Three Pole Points

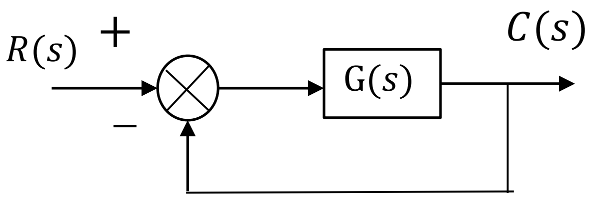
Ex.4 $$G(s) = \frac{K(1 + b_1 s + b_2 s^2)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$ Two Zero Points, Three Pole Points

Based on Routh-Hurwitz Criterion

Example 4

Open-loop transfer function:

$$G(s) = \frac{K(1 + b_1 s + b_2 s^2)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$



Closed-loop transfer function:

$$H(s) = \frac{G(s)}{1 + G(s)} = \frac{K + Kb_1 s + Kb_2 s^2}{1 + K + (a_1 + Kb_1)s + (a_2 + Kb_2)s^2 + a_3 s^3}$$

Routh table

S^3	a_3	$a_1 + Kb_1$
S^2	$a_2 + Kb_2$	$1 + K$
S^1	$\frac{(a_2 + Kb_2)(a_1 + Kb_1) - a_3(1 + K)}{(a_2 + Kb_2)}$	
S^0	$1 + K$	

Based on Routh-Hurwitz criterion:

$$a_3 > 0,\ a_2 + Kb_2 > 0,\ 1 + K > 0$$

$$\frac{(a_2 + Kb_2)(a_1 + Kb_1) - a_3(1 + K)}{(a_2 + Kb_2)} > 0$$

Based on Routh-Hurwitz Criterion

$$(a_2 + Kb_2)(a_1 + Kb_1) - a_3(1 + K) > 0 \quad (1)$$

Set one function:

$$f(K) = (a_2 + Kb_2)(a_1 + Kb_1) - a_3(1 + K)$$
$$= K^2 b_1 b_2 + K a_1 b_2 + K a_2 b_1 - K a_3 + a_1 a_2 - a_3$$

- Domain of definition $K \in (0, +\infty)$
- Initial value: $f(0) = a_1 a_2 - a_3$
- Derived function: $f'(K) = 2K b_1 b_2 + a_1 b_2 + a_2 b_1 - a_3$

if

$$f'(K) = 2K b_1 b_2 + a_1 b_2 + a_2 b_1 - a_3 > 0$$
$$f(0) = a_1 a_2 - a_3 \geq 0$$

$\Rightarrow$ $f(K) > 0$ $\Rightarrow$ (1) will be established

Stability condition is becoming:

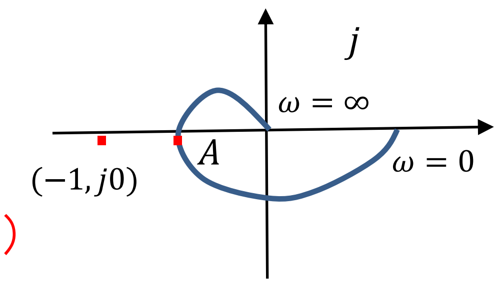
$2K b_1 b_2 > -a_1 b_2 - a_2 b_1 + a_3$ at condition: $a_1 a_2 - a_3 \geq 0$

Based on Nyquist Criterion

Frequency domain:

$$G(j\omega) = \frac{K + Kb_1(j\omega) + Kb_2(j\omega)^2}{1 - a_2\omega^2 + j(a_1\omega - a_3\omega^3)}$$

$$= \frac{K\,(1 - a_2\omega^2 - b_2\omega^2 + a_2b_2\omega^4 + a_1b_1\omega^2 - a_3b_1\omega^4) + jK(a_3\omega^3 - a_1\omega + a_1b_2\omega^3 - a_3b_2\omega^5 + b_1\omega - a_2b_1\omega^3)}{(1 - a_2\omega^2)^2 + (a_1\omega - a_3\omega^3)^2}$$



sketch chart of Nyquist plot

Special frequency expressions

$$\angle G(j\omega) = -\pi$$

⇒ $$a_3\omega^3 - a_1\omega + a_1b_2\omega^3 - a_3b_2\omega^5 + b_1\omega - a_2b_1\omega^3 = 0 \quad (2)$$

⇒ $$1 - a_2\omega^2 = \frac{(1 - b_2\omega^2)(a_1 - a_3\omega^2)}{b_1}$$ At point A

⇒ $$|G(j\omega)| = \frac{K|1 - a_2\omega^2 - b_2\omega^2 + a_2b_2\omega^4 + a_1b_1\omega^2 - a_3b_1\omega^4|}{|(1 - a_2\omega^2)^2 + (a_1\omega - a_3\omega^3)^2|}$$

$$= \cdots = \frac{Kb_1}{(a_1 - a_3\omega^2)}$$

(2) ⇒ $$a_3b_2\omega^4 + (a_2b_1 - a_1b_2 - a_3)\omega^2 + a_1 - b_1 = 0$$

⇒ $$\omega^2 = \frac{a_3 + a_1b_2 - a_2b_1 \pm \sqrt{(a_2b_1 - a_1b_2 - a_3)^2 - 4a_3b_2(a_1 - b_1)}}{2a_3b_2} \approx \frac{a_3 + a_1b_2 - a_2b_1}{2a_3b_2}$$

Based on Nyquist Criterion

Nyquist criterion

$$|G(j\omega)| = \frac{K|b_1|}{|a_1 - a_3\omega^2|} = \frac{K|b_1|}{\left|a_1 - a_3\frac{a_3 + a_1b_2 - a_2b_1}{2a_3b_2}\right|} = \frac{K|2b_1b_2|}{|a_1b_2 + a_2b_1 - a_3|} < 1$$

Stability condition:

$$a_3 - a_1b_2 - a_2b_1 < 2Kb_1b_2 < a_1b_2 + a_2b_1 - a_3$$ At condition: $a_1b_2 + a_2b_1 - a_3 > 0$

$$a_1b_2 + a_2b_1 - a_3 < 2Kb_1b_2 < a_3 - a_1b_2 - a_2b_1$$ At condition: $a_1b_2 + a_2b_1 - a_3 < 0$

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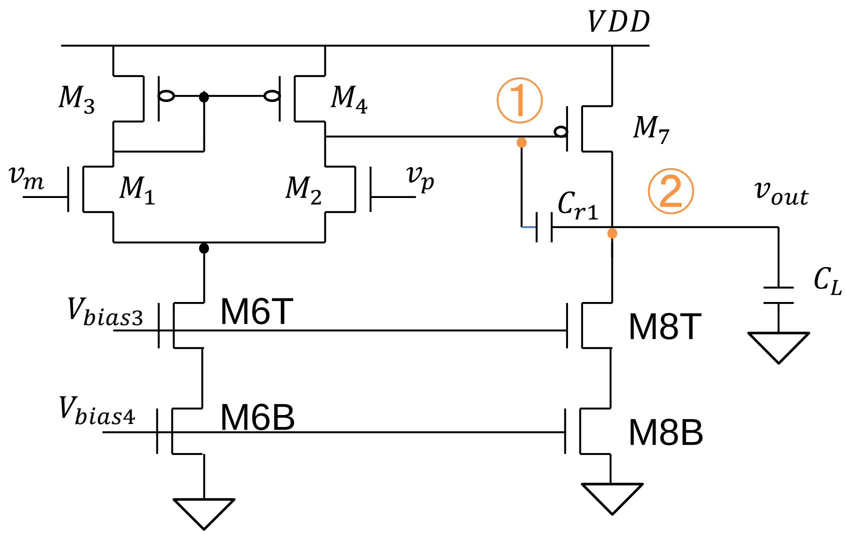
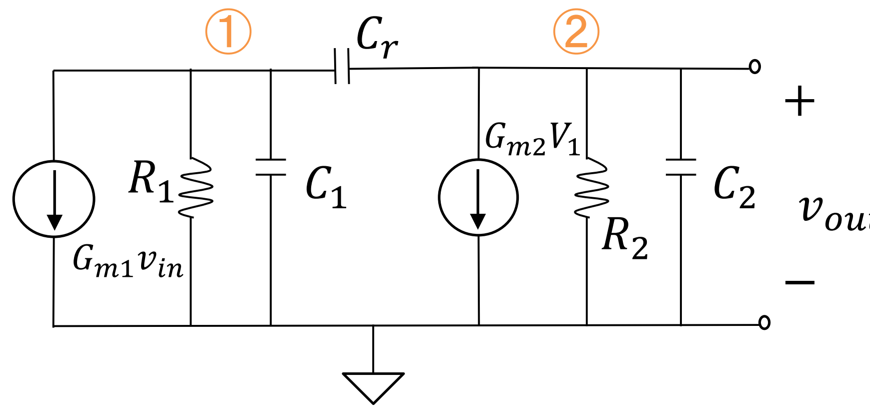
- Equivalence at Mathematical Foundations
- Simulation Verification
  - Ex.1: Two-stage amplifier with C compensation
  - Ex.2: Two-stage amplifier with C, R compensation
- Discussion & Conclusion

Amplifier Circuit and Small Signal Model



Transistor level circuit



Small-signal model

Open-loop transfer function from small signal model

$$A(s) = \frac{v_{out}(s)}{v_{in}(s)} = A_0 \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}$$

$$b_1 = -\frac{C_r}{G_{m2}} \qquad A_0 = G_{m1} G_{m2} R_1 R_2$$

$$a_1 = R_1 C_1 + R_2 C_2 + (R_1 + R_2 + R_1 G_{m2} R_2) C_r \qquad a_2 = R_1 R_2 (C_1 C_2 + C_1 C_r + C_2 C_r)$$

Routh-Hurwitz method

Closed-loop transfer function:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{A(s)}{1+fA(s)} = \frac{A_0(1+b_1 s)}{1+fA_0+(a_1+fA_0b_1)s+a_2s^2}$$

Short-channel CMOS parameters:

$$R_1 = r_{on}||r_{op} = 111k\Omega$$

$$R_2 = r_{op}||R_{ocasn} \approx r_{op} = 333k\Omega$$

$$G_{m1} = g_{mn} = 150\,uA/V$$

$$G_{m2} = g_{mp} = 150\,uA/V$$

$$C_1 = C_{dg4} + C_{dg2} + C_{gs7} = 13.6fF$$

$$C_2 = C_L + C_{gd8} \approx C_L + 1.56fF$$
$$= 101.56fF \quad (C_L = 100fF)$$

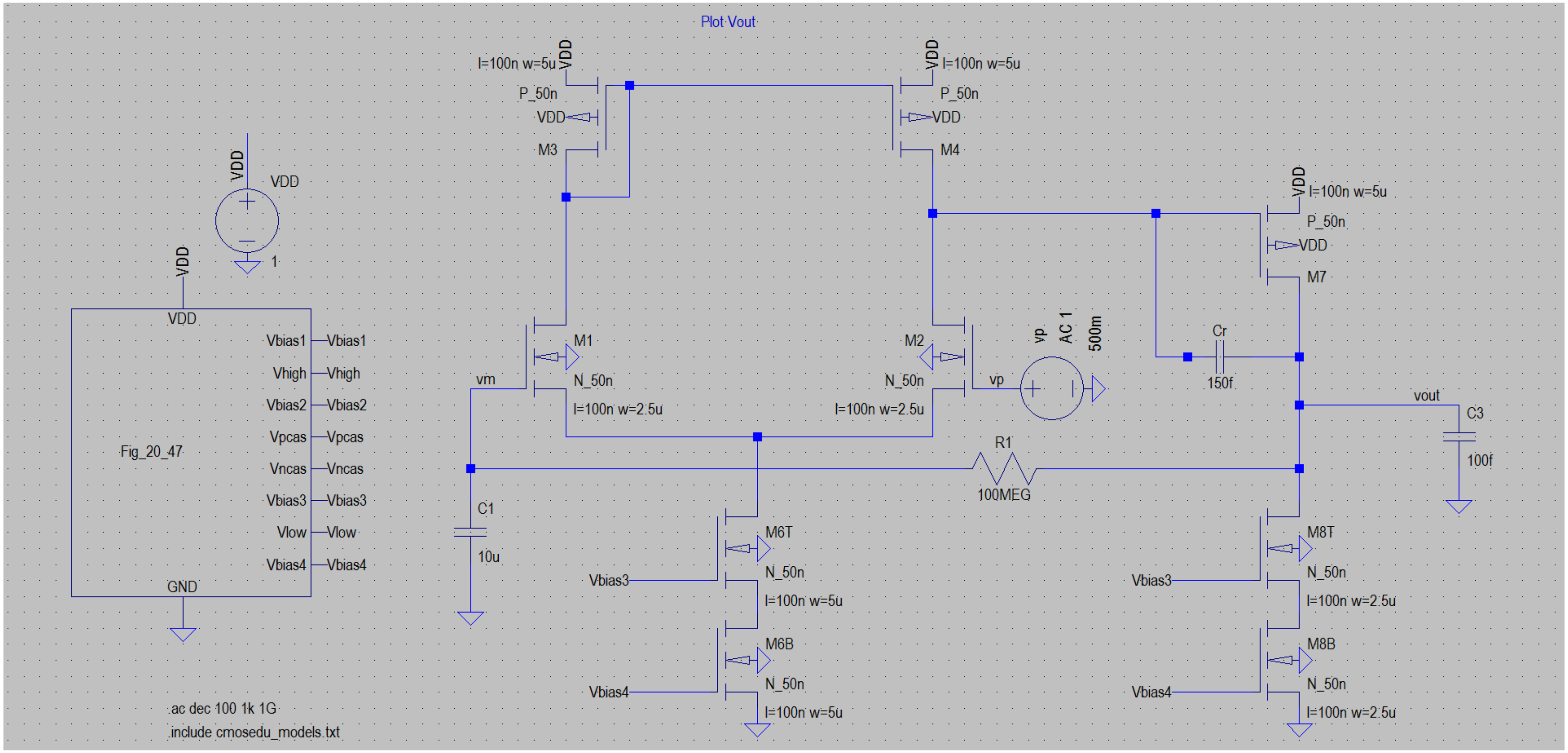
Explicit stability condition of parameters:

$$a_1 + fA_0b_1$$
$$= R_1C_1 + R_2C_2 + (R_1+R_2)C_r + (G_{m2} - fG_{m1})R_1R_2C_r > 0$$

⇨ $C_r >$ 79.57fF

Based on the different values of the transistor parameters,
We can obtain appropriate C_r.

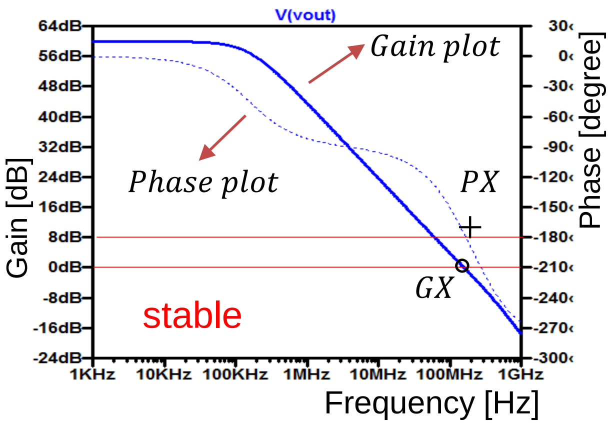
AC analysis



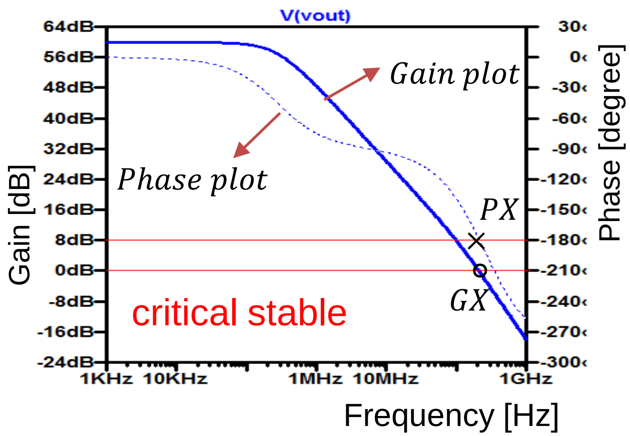
Ltspice simulation circuit

Consistency of Bode Plots and R-H Results

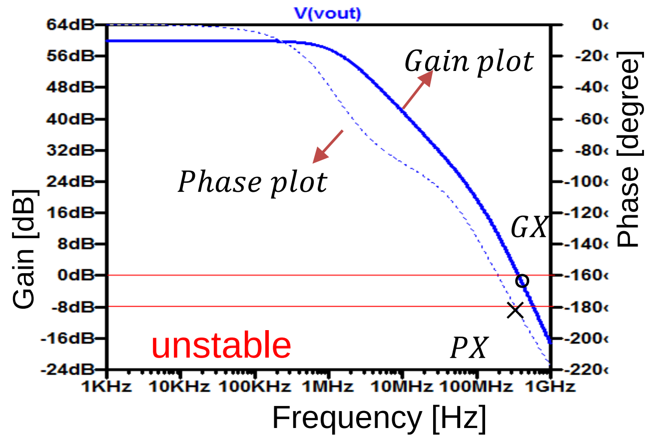
case	C_r	SPICE
(1)	150f F	stable
(2)	79.57f F	critical stable
(3)	10f F	unstable



Case (1) $C_r = 150\text{f F}$

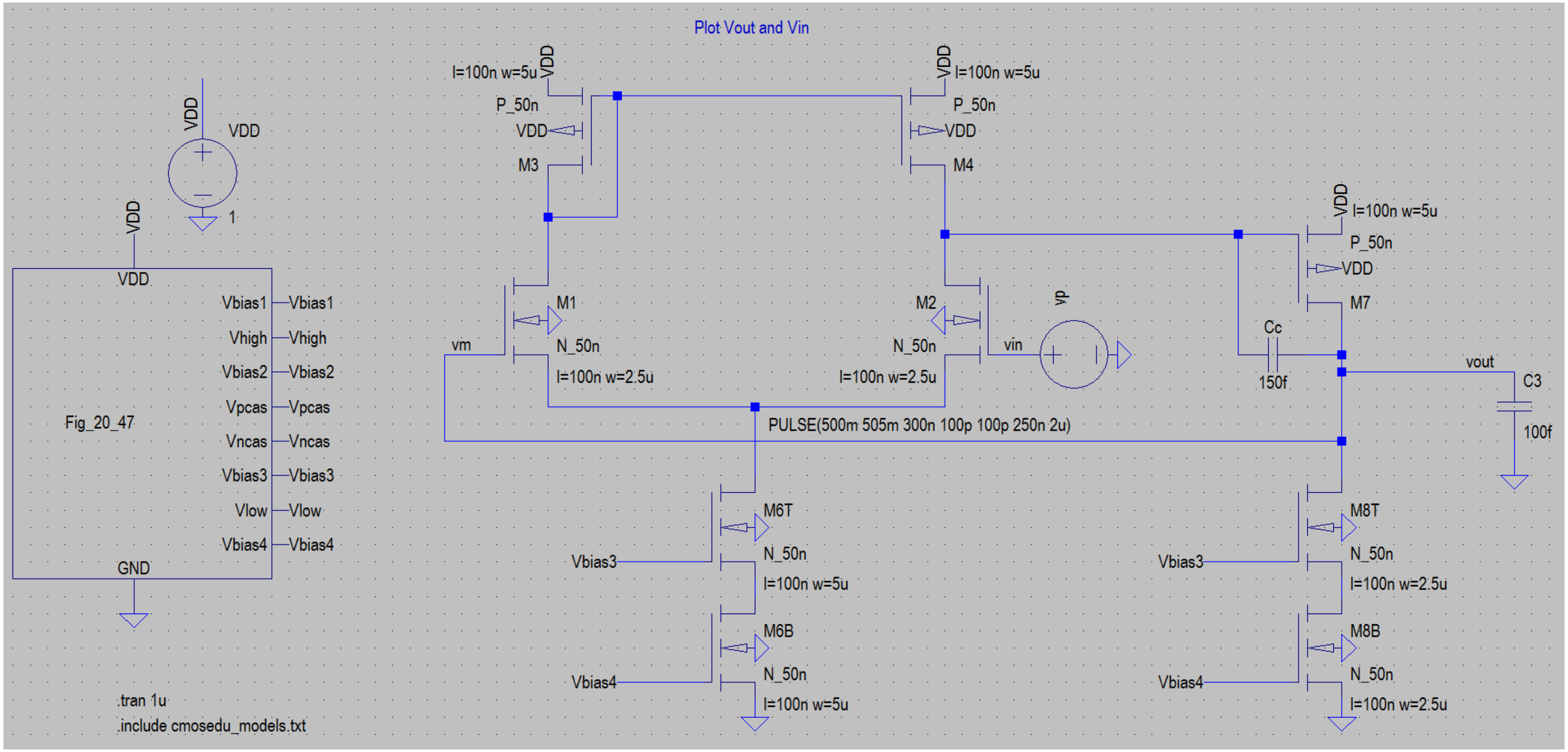


Case (2) $C_r = 79.57\text{f F}$



Case (3) $C_r = 10\text{f F}$

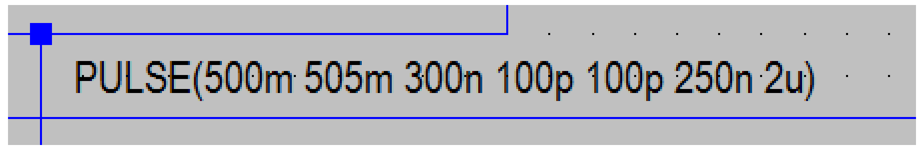
Transient Analysis

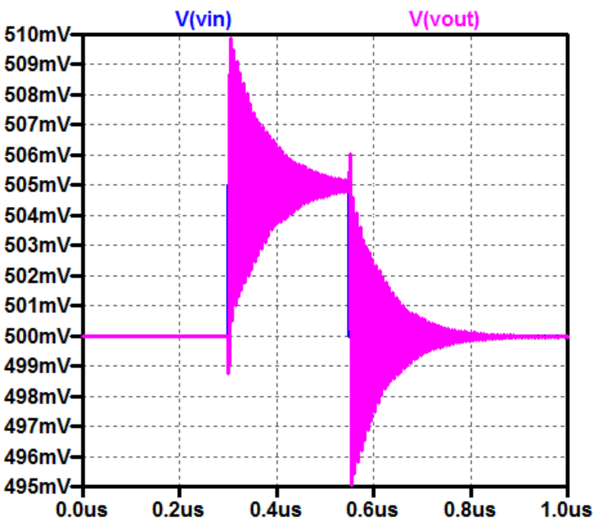


Ltspice simulation circuit

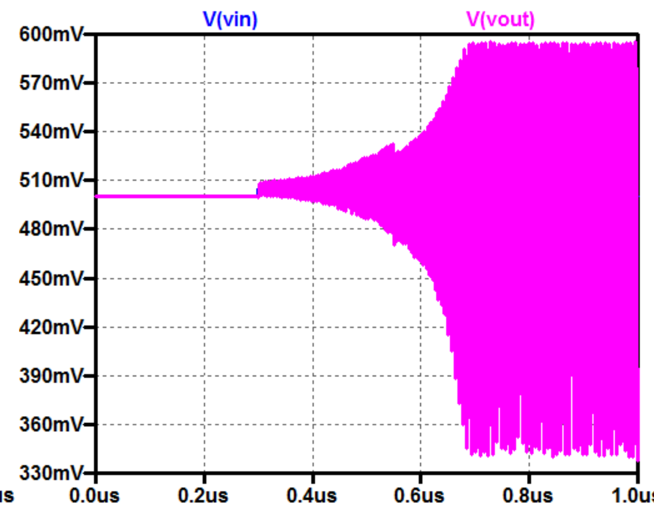
Consistency of Transient Analysis and R-H Results

Pulse response

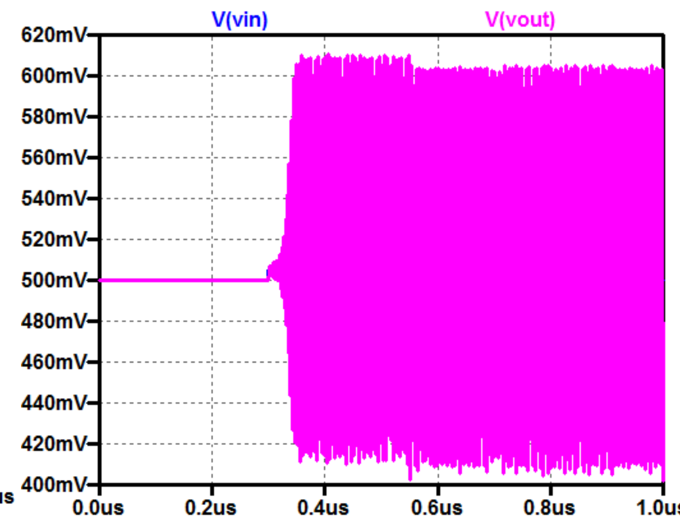




Case (1) $C_r = 150\text{fF}$

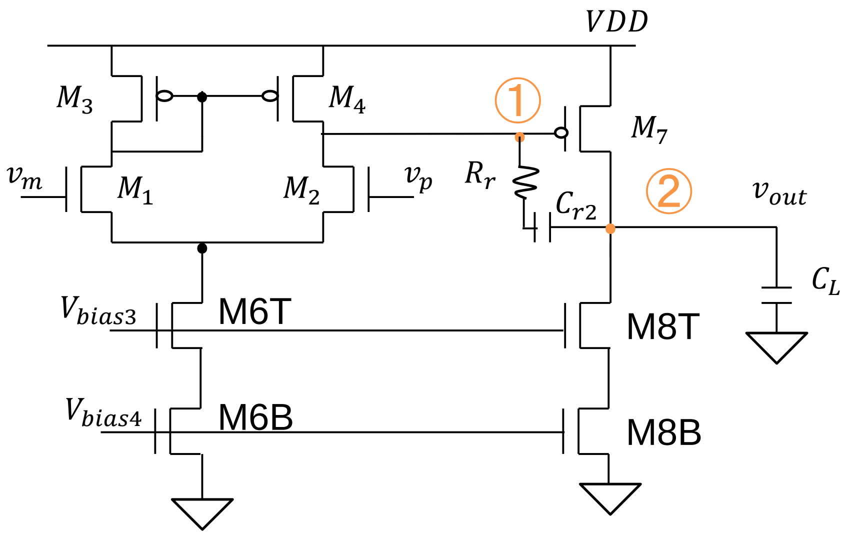


Case (2) C_r =79.57fF

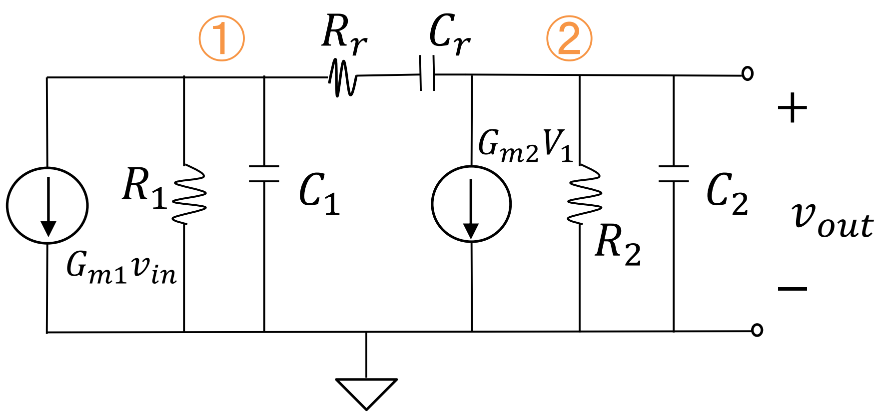


Case (3) $C_r = 10\text{fF}$

Amplifier Circuit and Small Signal Model



Transistor level circuit



Small-signal model

Open-loop transfer function:

$$A(s) = \frac{v_{out}(s)}{v_{in}(s)} = A_0 \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$

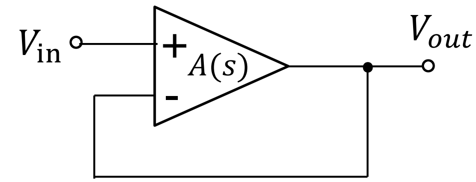
$A_0 = G_{m1} G_{m2} R_1 R_2$ $\quad b_1 = -\left(\frac{1}{G_{m2}} - R_r\right) C_r$ $\quad a_1 = R_1 C_1 + R_2 C_2 + (R_1 + R_2 + R_r + R_1 R_2 G_{m2}) C_r$

$a_2 = R_1 R_2 (C_1 C_2 + C_1 C_r + C_2 C_r) + R_r C_r (R_1 C_1 + R_2 C_2)$ $\quad a_3 = R_1 R_2 R_r C_1 C_2 C_r$

Routh-Hurwitz method

Closed-loop transfer function:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{A(s)}{1 + fA(s)} = \frac{A_0(1 + b_1 s)}{1 + fA_0 + (a_1 + fA_0 b_1)s + a_2 s^2 + a_3 s^3}$$



$f = 1$

Explicit stability condition of parameters:

$$\frac{(a_1 + fA_0 d_1)a_2 - a_3(1 + fA_0)}{a_2} > 0$$

$$R_1C_1 + R_2C_2 + (R_1 + R_2 + R_r)C_r + (G_{m2} - fG_{m1} + fG_{m1}G_{m2}R_r)R_1R_2C_r > \frac{R_1R_2C_1C_2R_rC_r(1 + G_{m1}G_{m2}R_1R_2)}{R_1R_2(C_2C_r + C_1C_2 + C_1C_r) + R_rC_r(R_1C_1 + R_2C_2)}$$

Short-channel CMOS parameters:

$R_1 = r_{on}||r_{op} = 111k\Omega$ $\quad G_{m1} = g_{mn} = 150\,uA/V$

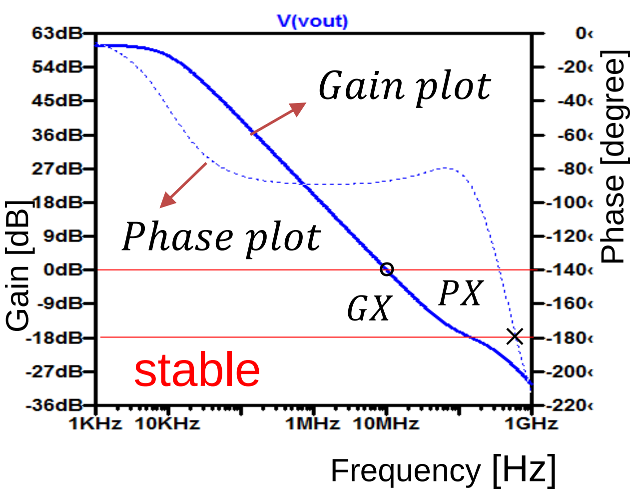
$R_2 = r_{op}||R_{ocasn} \approx r_{op} = 333k\Omega$ $\quad G_{m2} = g_{mp} = 150\,uA/V$

$C_1 = C_{dg4} + C_{dg2} + C_{gs7} = 13.6fF$ $\quad C_2 = C_L + C_{gd8} \approx C_L + 1.56fF = 101.56fF$

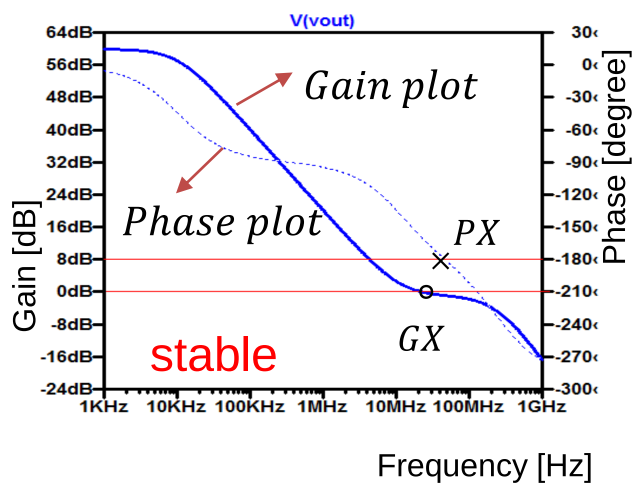
$$\underbrace{3.5 \times 10^{-8} + 3.7 \times 10^{10} C_r + R_r C_r + 831.7 R_r}_{X} > \underbrace{\frac{4.3 \times 10^{-8} R_r C_r}{5.1 \times 10^{-17} + 4.3 \times 10^{-3} C_r + 3.5 \times 10^{-8} R_r C_r}}_{Y}$$

Consistency of Bode Plots and R-H Results

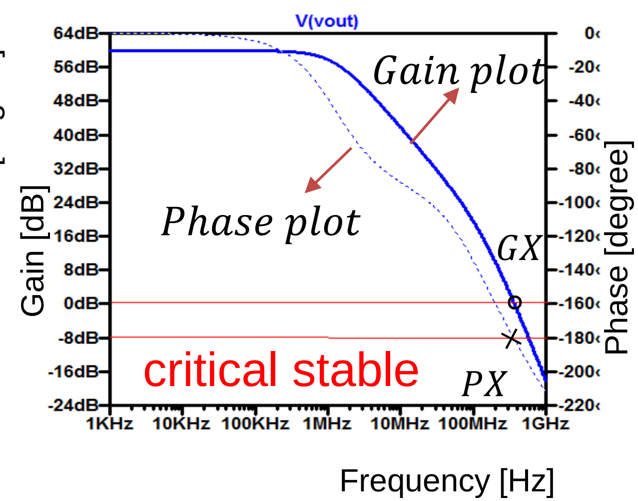
	parameter values				R-H criterion	Bode plot
case	R_r	C_r	X	Y		SPICE simulation
(1)	6.5k	2.4p	1.41×10^{-5}	6.13×10^{-8}	$X > Y$	stable
(2)	1	2.4p	1.10×10^{-6}	9.94×10^{-12}	$X > Y$	stable
(3)	7k	10f	9.8×10^{-8}	3.10×10^{-8}	$X \approx Y$	critical



Case (1) $X > Y$



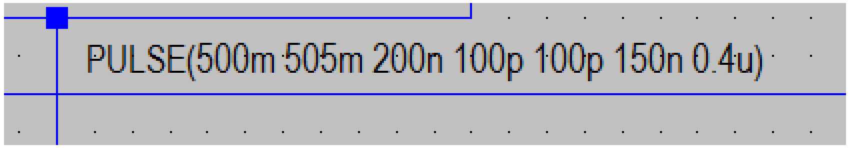
Case (2) $X > Y$



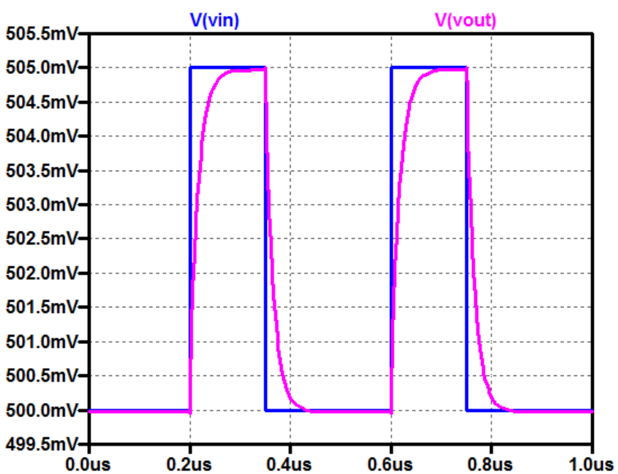
Case (3) $X \approx Y$

Consistency of Transient Analysis and R-H Results

Pulse response

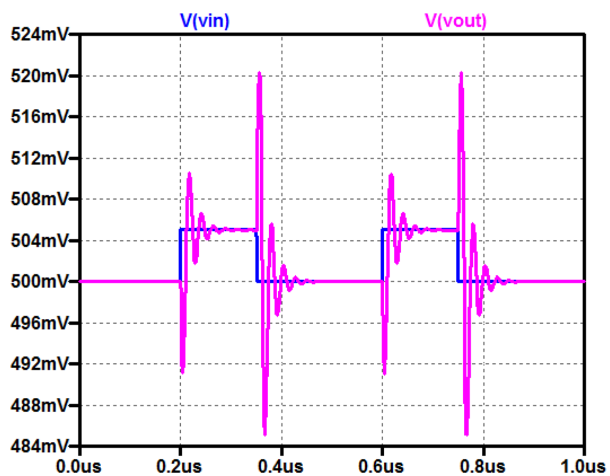


$X > Y$ Stable



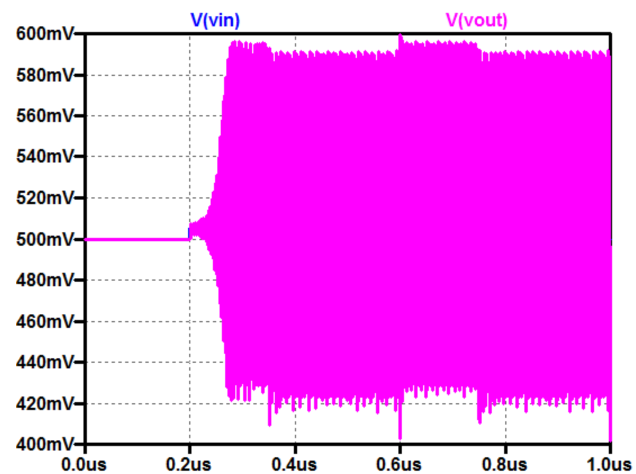
Case (1)

$X > Y$ Stable



Case (2)

$X \approx Y$ Critical



Case (3)

Contents

Discussion

Depict small signal equivalent circuit of amplifier

Derive open-loop transfer function

Derive closed-loop transfer function
& obtain characteristics equation

Apply R-H stability criterion
& obtain explicit stability condition

Especially effective for

Multi-stage opamp (high-order system)

Limitation

Explicit transfer function with polynomials of s has to be derived.

Conclusion

- Show the equivalence between Nyquist and R-H stability criteria for analysis and design of the opamp stability under some conditions.
- Equivalency of their mathematical foundations was shown.
- R-H method, explicit circuit parameter conditions can be obtained for feedback stability.
- Consistency with Bode plot method has been confirmed with SPICE simulation.

⇓

R-H method can be used
with conventional Bode plot method.

Dr. Yuji Gendai is acknowledged
for his stimulating comments

**Thank you
for your kind attention.**

Q&A

Q:

You said: R-H method very popular in control theory field,
but it is rarely seen in electronic circuit field.
Why do you propose using R-H method?
Did you compare the proposed method and the Bode plot method?

A:

Bode plot often be used, it can show stability information directly.
From Bode plot, we can only find out the stability, and
judge whether the system stable or unstable.
But we don’t know internal connection between circuit parametersand stability.
For example, which parameter values influence the stability, capacitor, resistor or transconductor of transistor.
I have done simulation by LTspice, and by comparing, we can see
that Bode plot results and R-H method results are consistent.