SC4-1 13:30-14:00
Oct. 27, 2017 (Fri)

Fundamental Design Tradeoff and Performance Limitation of Electronic Circuits Based on Uncertainty Relationships

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My First Research

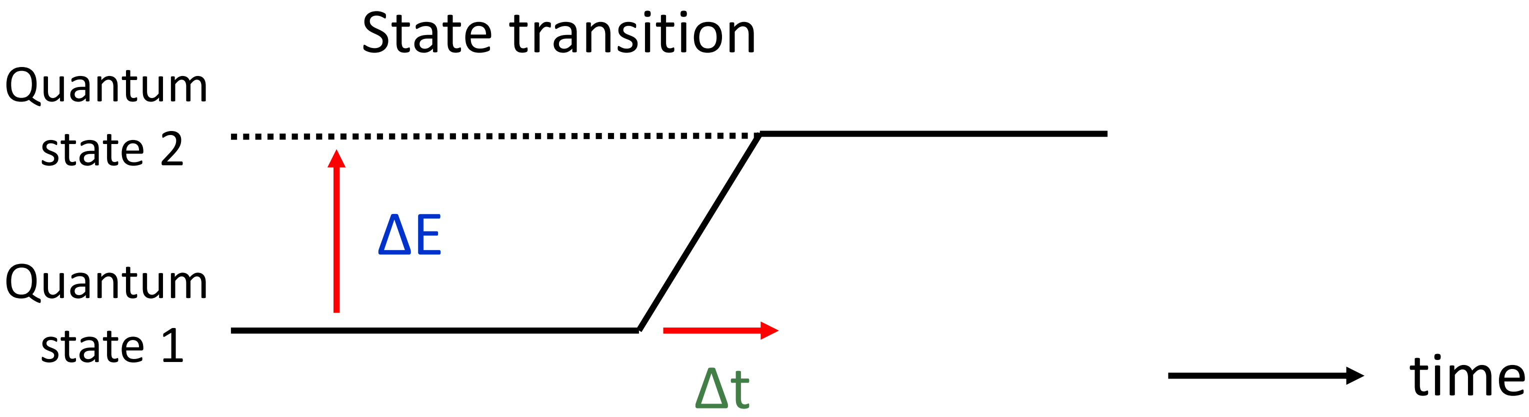
Computer with Superconductor (Josephson Device)

Under supervision of Prof. Ko Hara (原　宏)
at University of Tokyo
Physicist

Undergraduate (Bachelor) course, 4th year

[1] K. Hara, H. Kobayashi, S. Takagi, F. Shiota, "Simulation of a Multi-Josephson Switching Device", Japanese J. of Applied Physics (1980).

Research Motivation of This Paper



$\Delta E \, \Delta t \geq h/(4\pi)$ Uncertainty principle

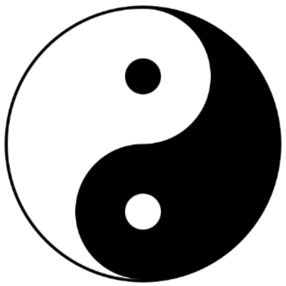
My strong impression :

Transition time Δt ⇒ Time uncertainty

Our Statement

Uncertainty relationships are everywhere
in electronic circuits

Our conjecture



陰陽思想
太極図

Ultimately, some would converge to
Heisenberg uncertainty principle
in quantum physics.

Contents

Contents

Research Objective

- Our Objective

In analog electronic circuits

- Clarify tradeoff among their performance indices
- Provide their fundamental limitation

- Our Approach

Based on

- Uncertainty principle in quantum mechanics
- Uncertainty relationship in signal processing

Contents

Uncertainty Principle in Quantum Mechanics

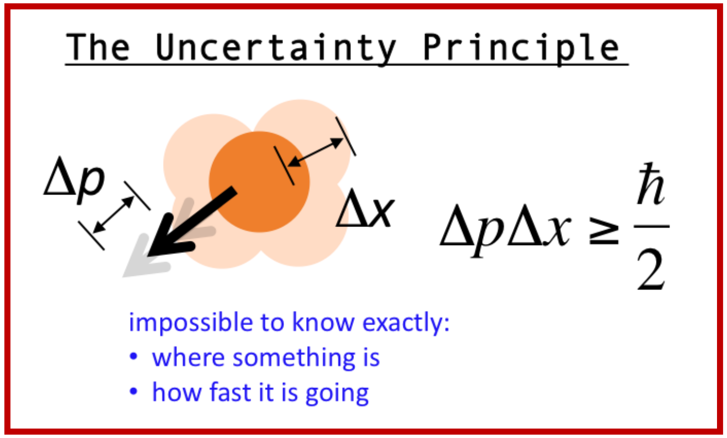
W. K. Heisenberg

$$\Delta t \, \Delta E \geq h/(4\pi)$$

t: time, *E*: energy

$$\Delta x \, \Delta p \geq h/(4\pi)$$

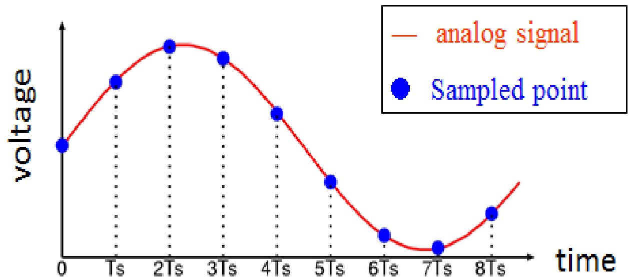
x: position, *p*: momentum.

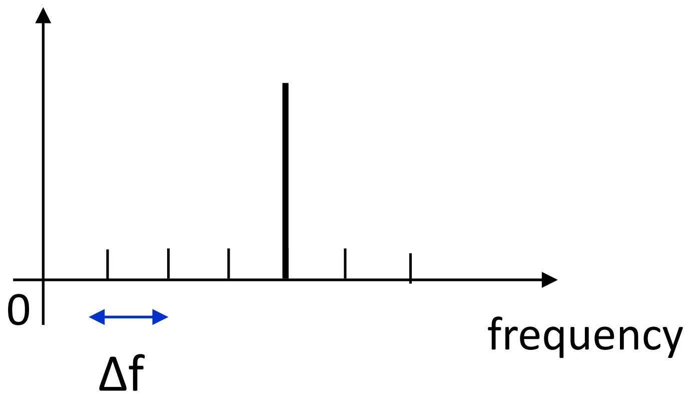


These cannot be proved ⇨ *principle.*

Uncertainty Relationship in Signal Processing (1)

- Discrete Fourier Transform (DFT)





Sampling frequency : fs
Sampling period: Ts (= 1/fs)

Number of DFT points :N

$\Delta f = fs/N = 1/(Ts\ N)$

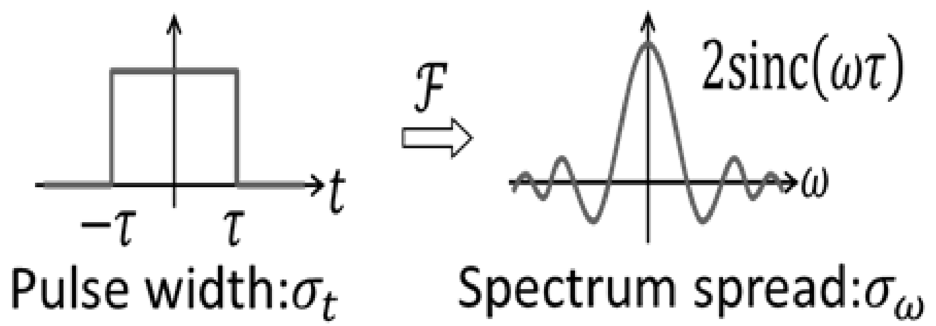
Time & frequency resolution

$$\Delta f\ Ts = 1/N$$

This can be proved mathematically ⇨ *Relationship*

Uncertainty Relationship in Signal Processing (2)

- Uncertainty Relationship
  between Time & Frequency of Continuous Waveform



$$\sigma_\tau \sigma_\omega \geq \frac{1}{2}$$

This can be proved mathematically ⇨ *Relationship*

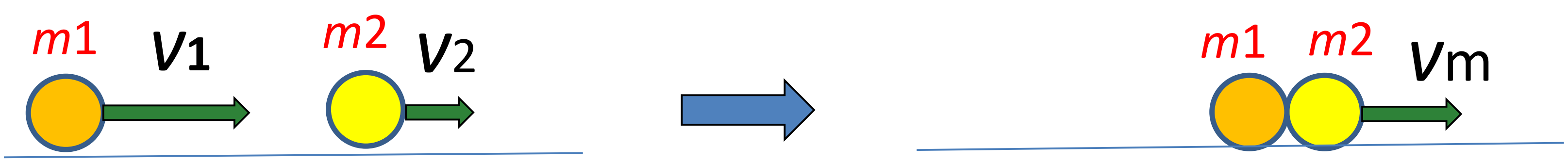
Contents

Importance of Invariant (1)

Invariant quantity ⇨
clarify phenomena & characteristics

Conservation Law in Physics :

- Energy conservation law
- Mass conservation law
- Momentum conservation law
- Charge conservation law



$p1 = m1\ v1, \quad p2 = m2\ v2$

$p1' = m1\ v_m, \quad p2' = m2\ v_m$

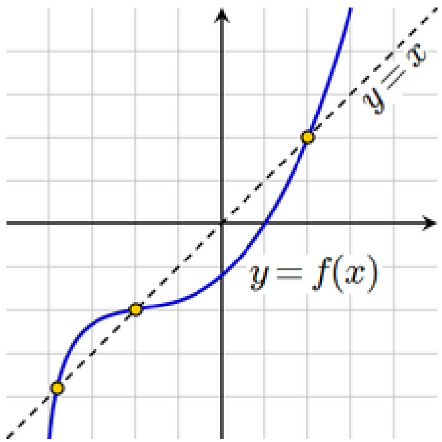
$p1 + p2 = p1' + p2'$

Importance of Invariant (2)

Invariant quantity ⇨
clarify phenomena & characteristics

Fixed-Point in Mathematics :

$$f(x) = x$$



Utility for Voyage

Compass


Polaris

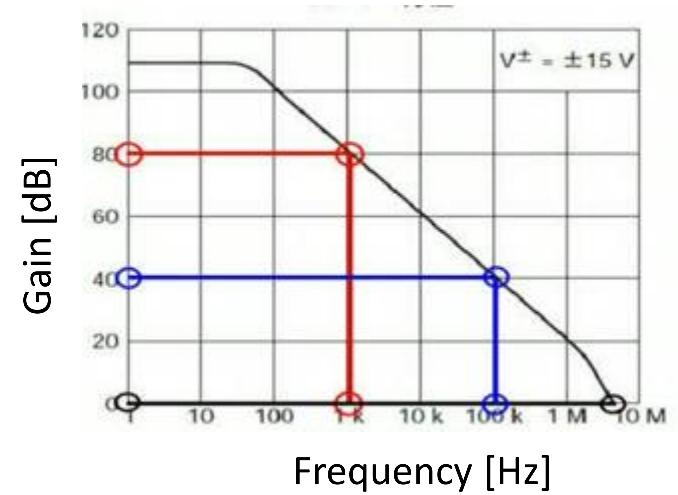
Contents

Gain, Signal Band and Power

- For a given amplifier

Gain・bandwidth = constant

Gain → large, bandwidth → narrow



- Amplifier Performance

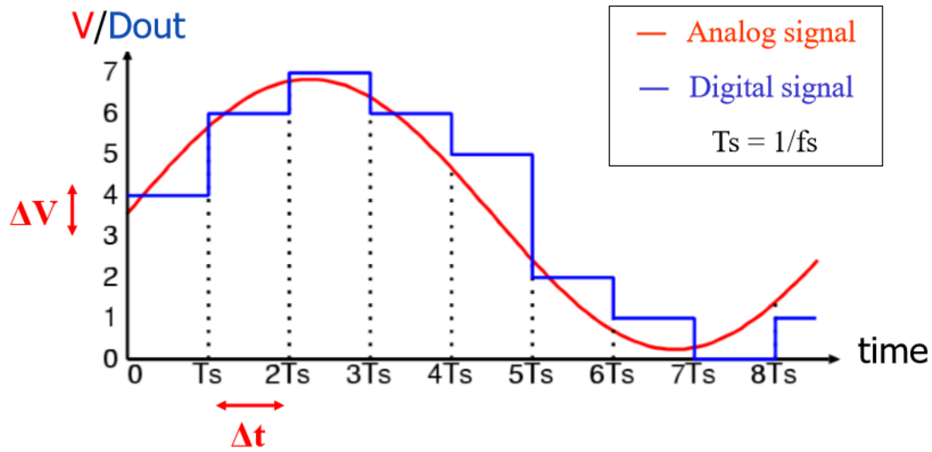
$$\text{FOM} = \frac{\text{Power}}{\text{Gain} \cdot \text{Bandwidth}}$$

Technology constant

→ Converge to uncertainty principle

conjecture

ADC Sampling Speed, Resolution and Power



Sampling period: Δt

Resolution: $V_{full} / \Delta V = 2^n$

Power: P

$$FOM = \Delta t \cdot \Delta V \cdot P / V_{full}$$
$$= \Delta t \cdot P / 2^n$$

FOM =

$$\frac{\text{Voltage Resolution} \cdot \text{Power}}{\text{Sampling Speed}}$$

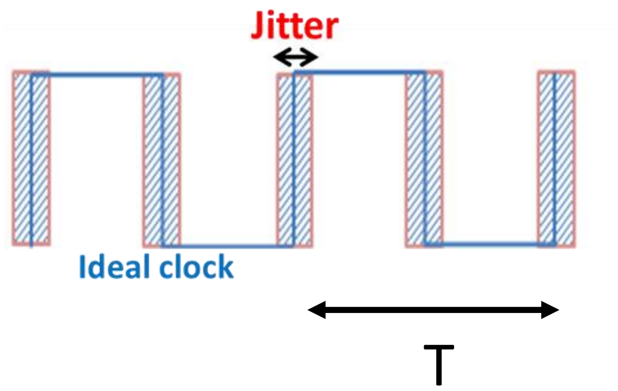
Technology constant

FOM → Smaller, ADC → Better

→ Converge to uncertainty principle

conjecture

Clock Jitter, Power



Clock jitter: Δt

Clock generator energy : E

power : P

Design tradeoff

$\Delta t \cdot E \geq K_1$ ↔ $(\Delta t / T) P \geq K_1$

Power → larger, Jitter → smaller

Noise, Capacitor

Analogy

p (momentum) ⇔ Q (charge)

v (velocity) ⇔ V (voltage)

m (mass) ⇔ C (capacitor)

Momentum conservation law

⇔ Charge conservation law

Uncertainty principle

$\Delta x \, \Delta p \geq K$ ⇔ $\Delta V \; f \, \Delta Q \geq K$

⇔ $C \; \Delta V^2 \; f \geq K$

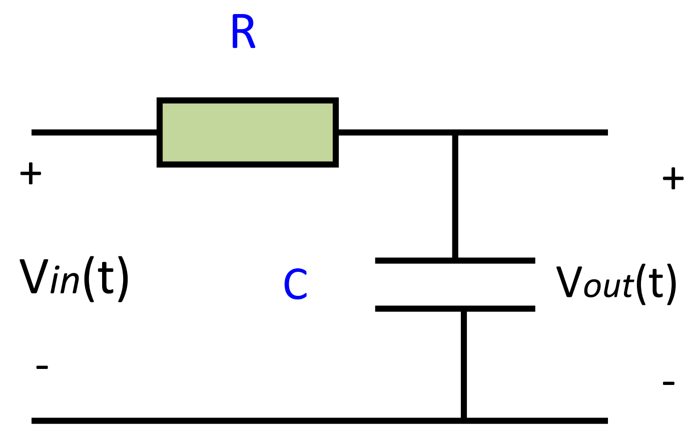
Noise bandwidth: f

Noise power $\Delta V^2 = kT/C$

C → large, Noise → small

Noise, Capacitor (2)

- For a given T=RC
  the same gain & phase characteristics
  for different (R_1, C_1), (R_2, C_2), …
  with $R_1 C_1 = R_2 C_2 = \ldots = T$



- For a given V_{out}

$E_c = (1/2)\, C\, V_{out}^2$

$V_{noise}^2 = kT/\, C$

Transfer function

$G(s) = 1/\,(1 + sRC\,)$

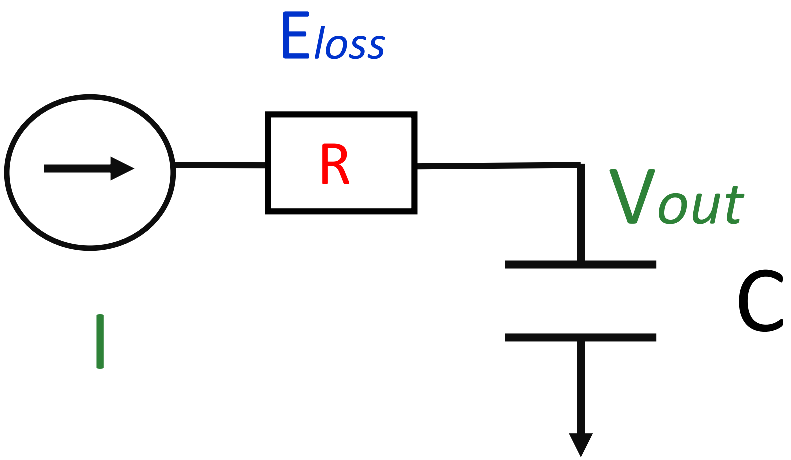
C → large, R → small
Same gain & phase characteristics
⇨ Low noise
Large energy

Capacitor Charge & Loss

$E_{loss} = (R \cdot I) \cdot I \cdot T$
$= R \cdot C \cdot V \cdot I$

$V_{out} = I \cdot T / C$

I : Charge Current
T: Charge Duration



$$E_{loss} \cdot T = R \cdot C \cdot V_{out}$$

Uncertainty relationship

For given R, C, V_{out}

I → small, T → long ⇨ E_{loss} → small

Analog Electronic Circuits

Performance tradeoffs are everywhere in circuits

$$\Delta a \, \Delta b \geq K$$

- In some cases, these can be proved.

  Uncertainty relationship

- In other cases, these can NOT be proved.

  For a given technology

$$\Delta a \, \Delta b = K$$

K: Technology constant

Technology → advance ⇨ K → smaller

Conjecture: this converges to uncertainty principle

Analog Circuit and Quantum Mechanics

Myth

- Real world signals → analog
- Computer world signals → digital.

Truth

- quantum mechanics →
  signals in nature → digital (discrete).

- Current → average of electrons' moves
- Electronic noises → their variation.

Conjecture

- Analog electronic circuit performance
  → Limited by quantum mechanics

Analogy

In Physics, analogy is just a coincidence,

NOT inevitable.

Analogy

p (momentum) ⇔ Q (charge)

v (velocity) ⇔ V (voltage)

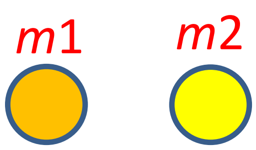
m (mass) ⇔ C (capacitor)

Momentum conservation law

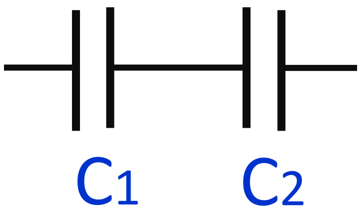
⇔ Charge conservation law

Difference

Any connection of m1 & m2 > m1, m2



Series connection of C1 & C2 < C1, C2



Bridge Through Plank Constant

"Let there be light !"

Old testament

"Mehr Licht !"

by J. W. von Goethe

Uncertainty Relationship
Analogy to Principle

$$\Delta\omega\ \Delta\tau \geq 1/2$$

$$(h/(2\pi))\ \Delta\omega\ \Delta\tau \geq h/(4\pi)$$

Energy in the light: $E = (h/(2\pi))\ \omega$

$$\Delta E\ \Delta\tau \geq h/(4\pi)$$

Uncertainty Principle

Measurement and Simulation

Measurement : *Active*, *Passive*

Active: Stimulus → Device

Response → Measured

Device state → Disturbed.

Passive: No stimulus

Device state → Not disturbed.

Uncertainty principle

⇨ all measurements disturb device state.

Circuit simulation ⇨ No disturbance.

Contents

- Research Objective
- Uncertainty Principle and Relationship

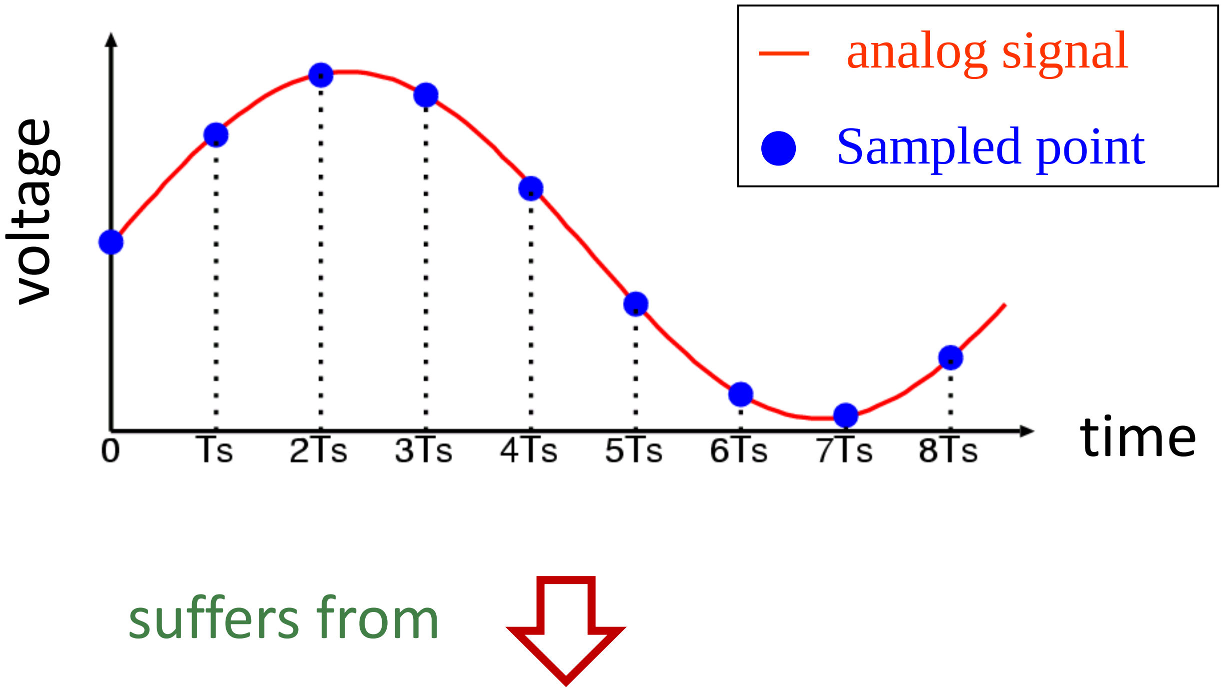
Example of
Uncertainty Relationship
In Signal Processing

nce Analogy
to Uncertainty Relationship and Invariant

- <u>Waveform Sampling Circuit</u>

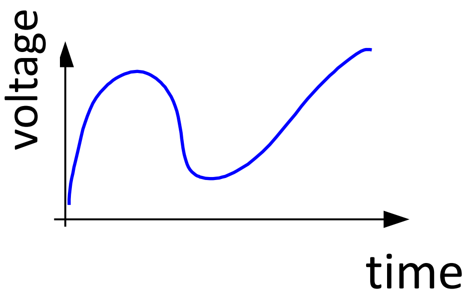
[2] M. Arai , H. Kobayashi , et. al.,"Finite Aperture Time Effects in Sampling Circuit," IEEE 11th International Conference on ASIC, Chengdu (Nov. 2015).

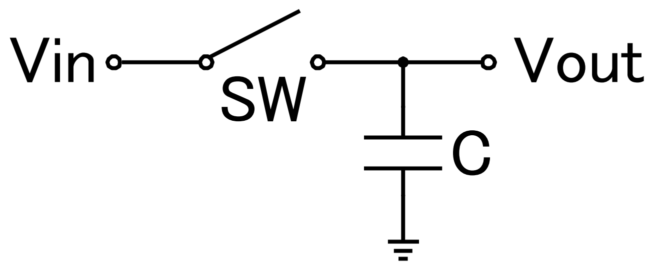
Waveform Sampling

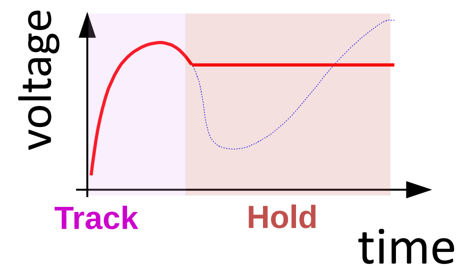


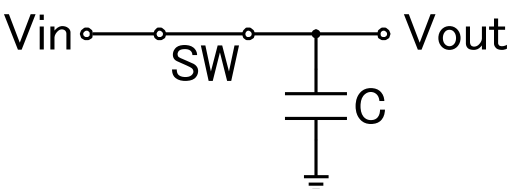
- Finite aperture time (non-zero turn-off time)
- Aperture jitter

Sampling Circuit



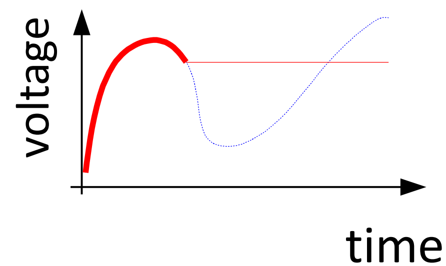


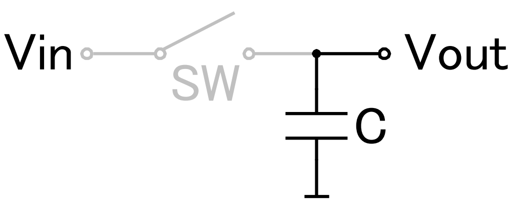




- SW: ON
  - Vout(t) = Vin(t)

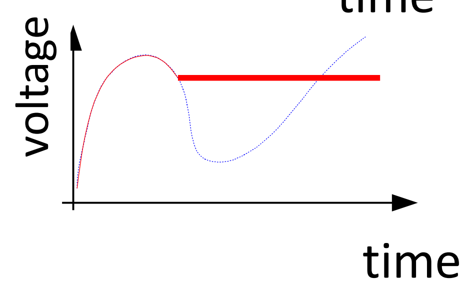
  Track mode



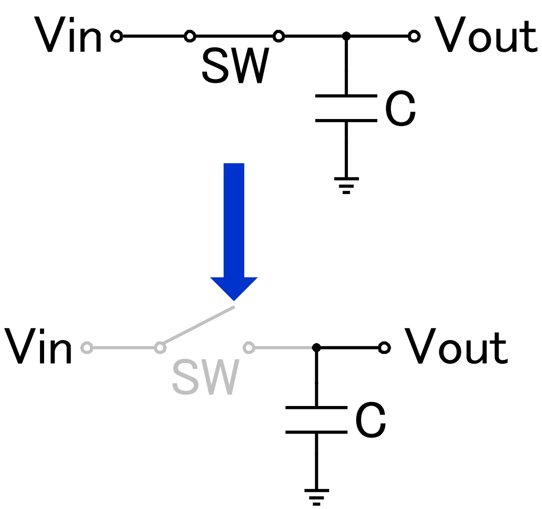


- SW: OFF
  - Vout(t) = Vin(t_{OFF})

  Hold mode



Finite Aperture Time

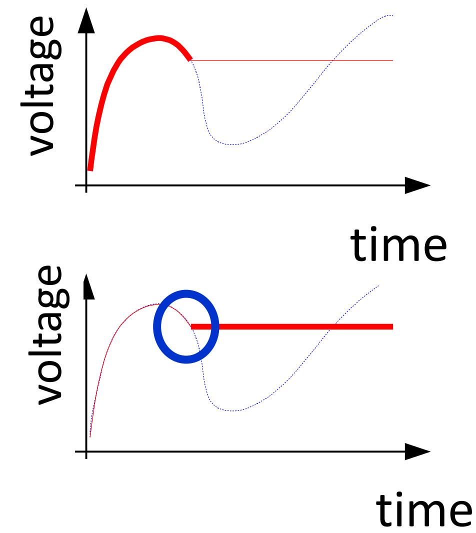


- <u>SW: ON</u>
  - Vout(t) = Vin(t)

  Track mode

- <u>SW: OFF</u>
  - Vout(t) = Vin(t_{OFF})

  Hold mode



Finite transition time from track to hold modes

Analogy with Camera Shutter Speed

Camera: Finite Shutter Speed

Moving Object

Blurred

Sampling Circuit:
Finite Aperture Time

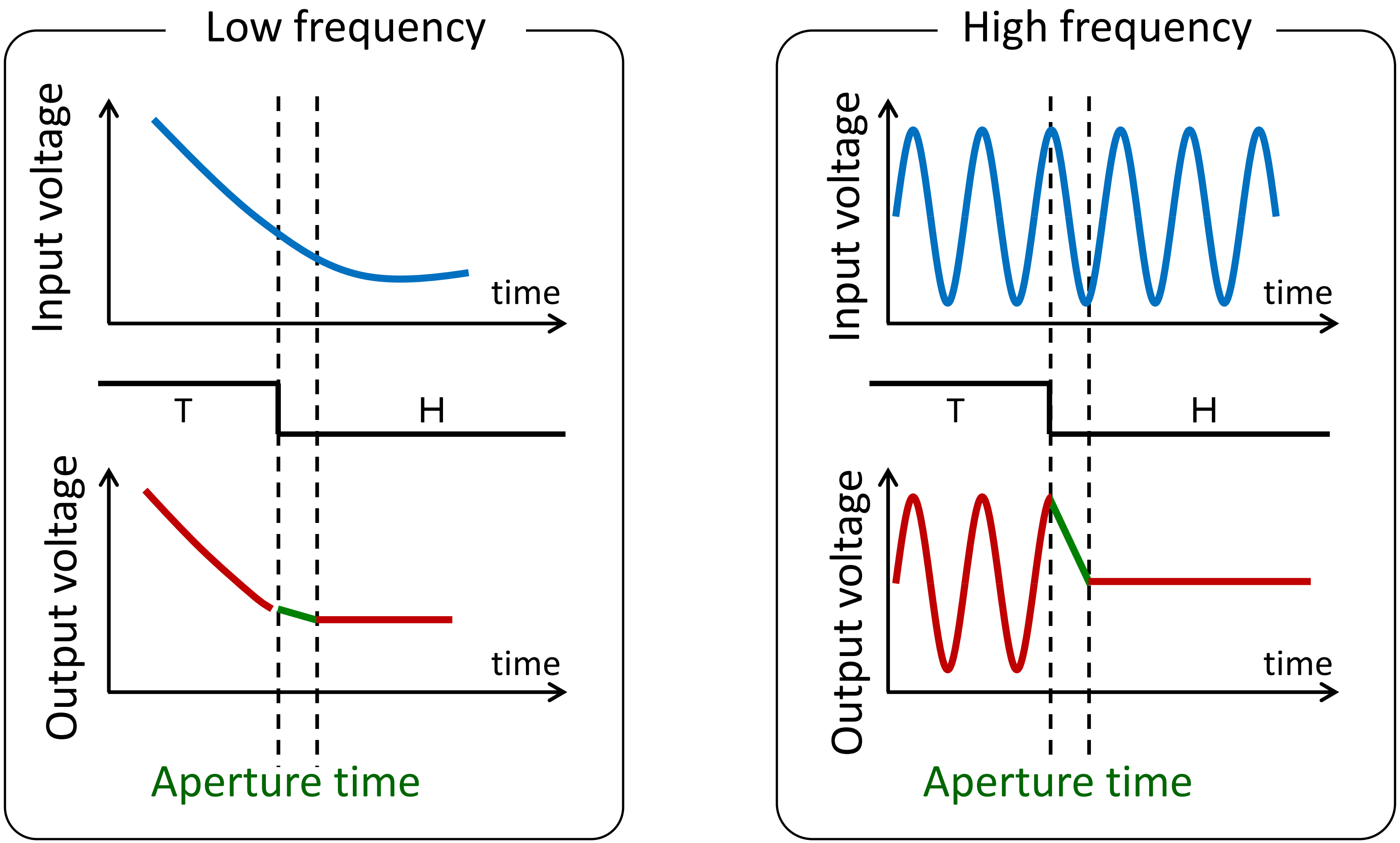
Input signal

High frequency

Acquired signal

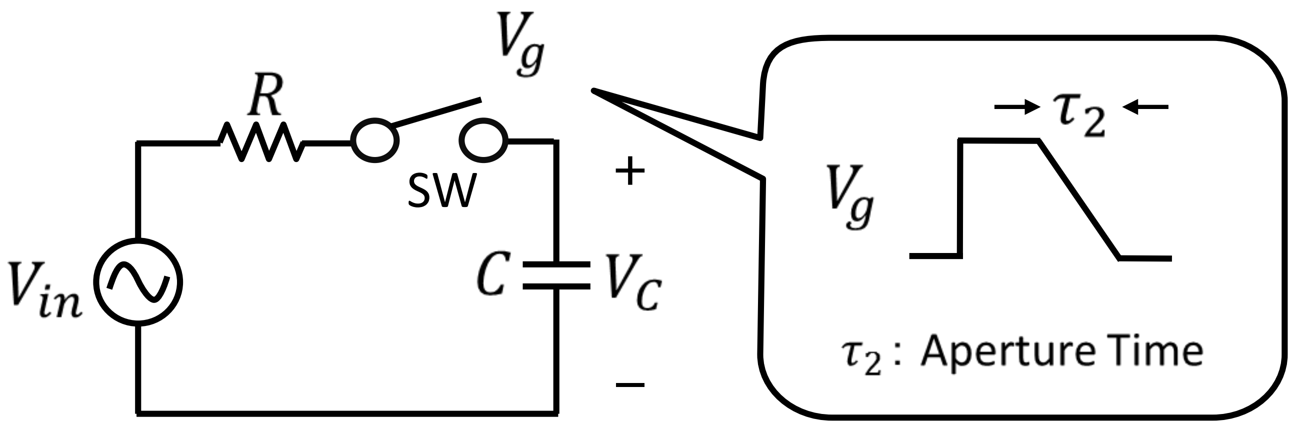
Low pass filtered

Signal Frequency and Aperture Time

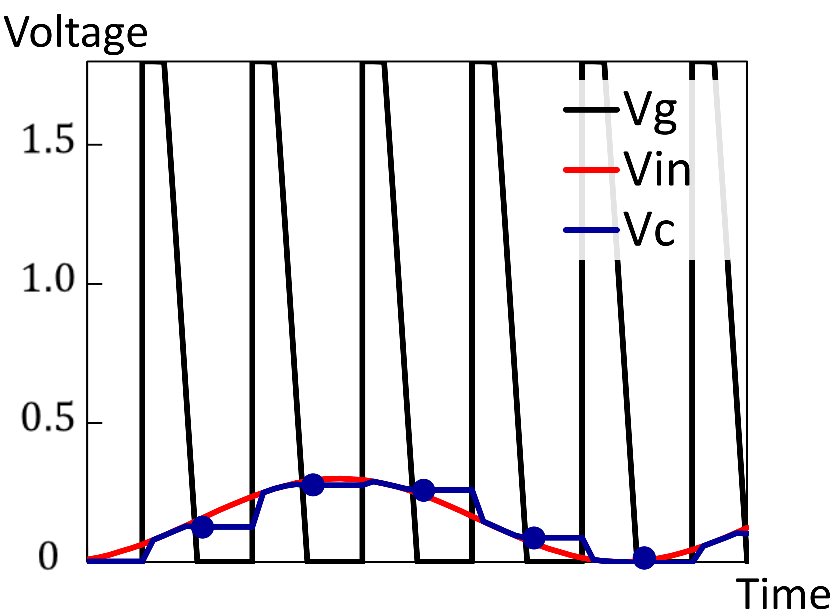


Higher frequency signal ⇒ More affected by finite aperture time

Transfer Function Derivation



Track Hold Circuit



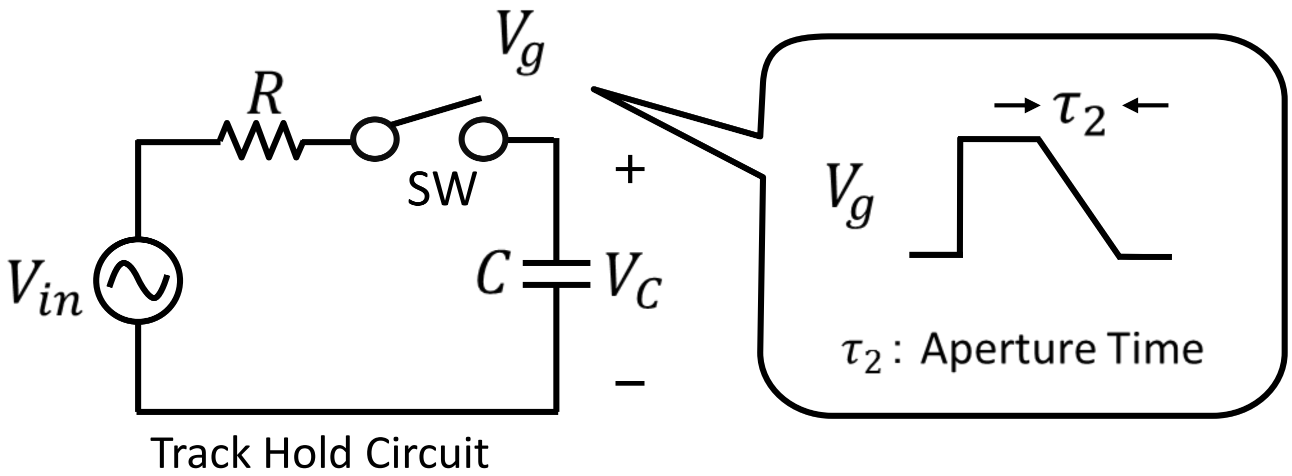
Obtain values of ●

Equivalent time sampling

Obtain gain, phase for each frequency

Frequency transfer function

Derived Transfer Function



$$\frac{V_C}{V_{in}} = \frac{sinc(\omega\tau_2)}{sinc(\omega\tau_2) + j\omega\tau_1}$$

$\tau_1 = R\,C$

Transfer function in case of finite aperture time

[3] A. Abidi, M. Arrai, K. Niitsu, H. Kobayashi, "Finite Aperture Time Effects in Sampling Circuits," 24th IEICE Workshop on Circuits and Systems, Awaji Island, Japan (Aug. 2011)

Consistency with Zero Aperture Time Case

$$\frac{V_C}{V_{in}} = \frac{sinc(\omega\tau_2)}{sinc(\omega\tau_2) + j\omega\tau_1}$$

$$(\tau_1 = RC \ , \ \tau_2 = \tau\ )$$

Transfer function in case of finite aperture time

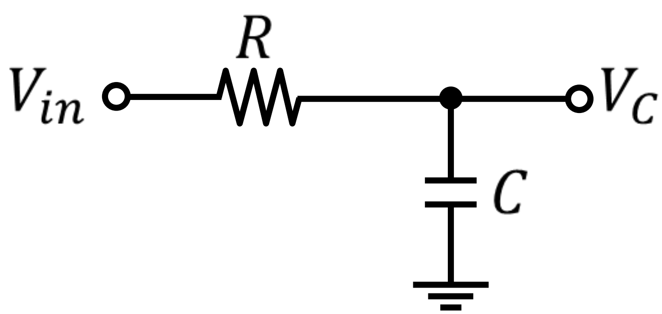
$\tau_2 \to 0$

$sinc(\omega\tau_2) \to 1$

$$\frac{V_C}{V_{in}} = \frac{1}{1 + j\omega\tau_1}$$

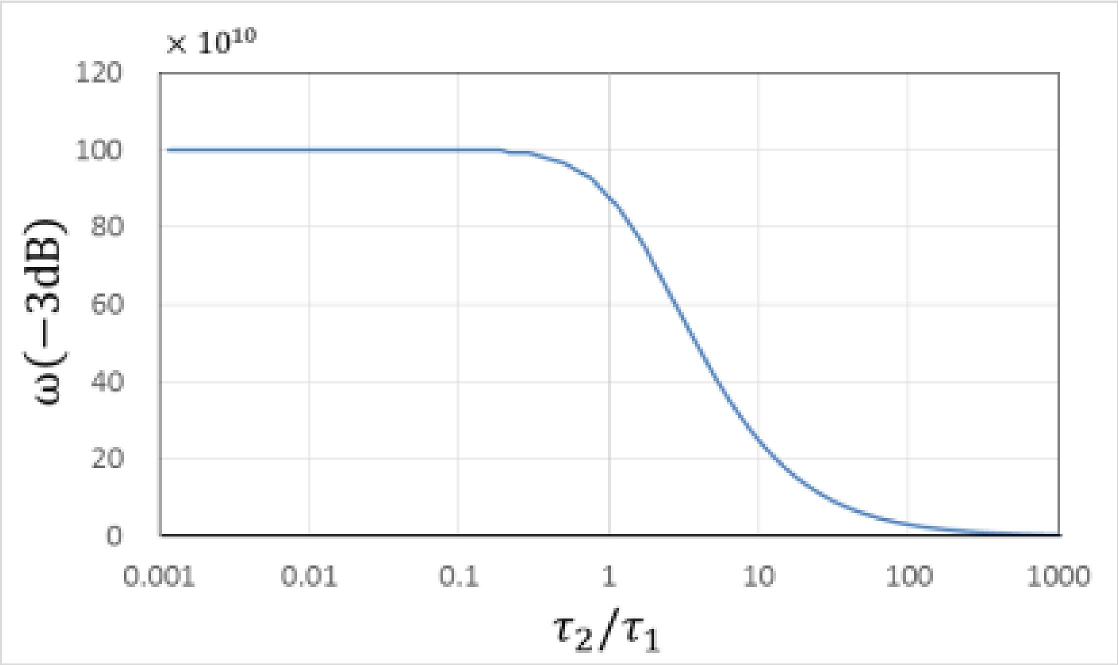
$$(\tau_1 = RC\ )$$

Transfer function in case of zero aperture time



τ_1, τ_2 Effects to Bandwidth

Numerical calculation from the derived transfer function



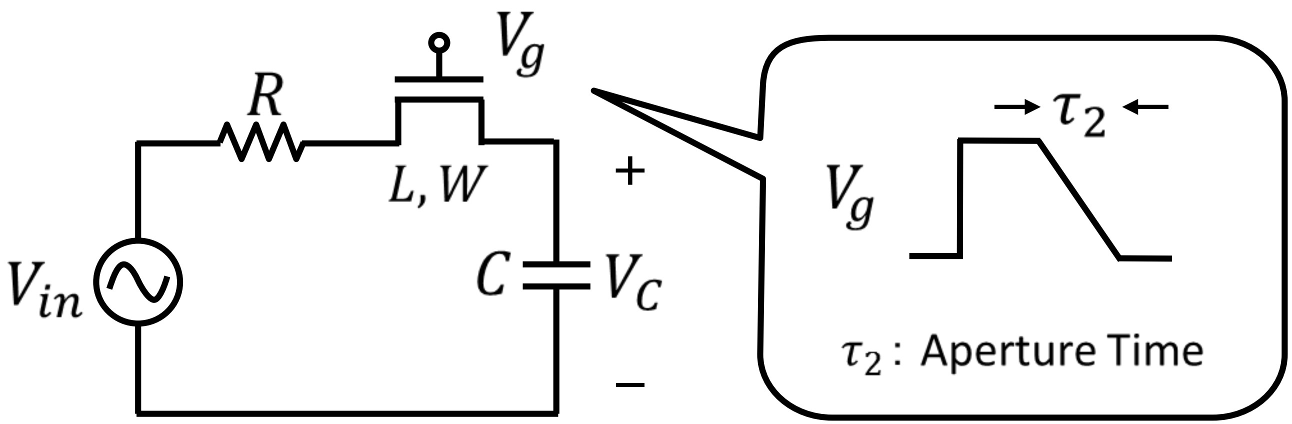
τ_1 (= R C) : fixed

τ_2 (aperture time) : varied

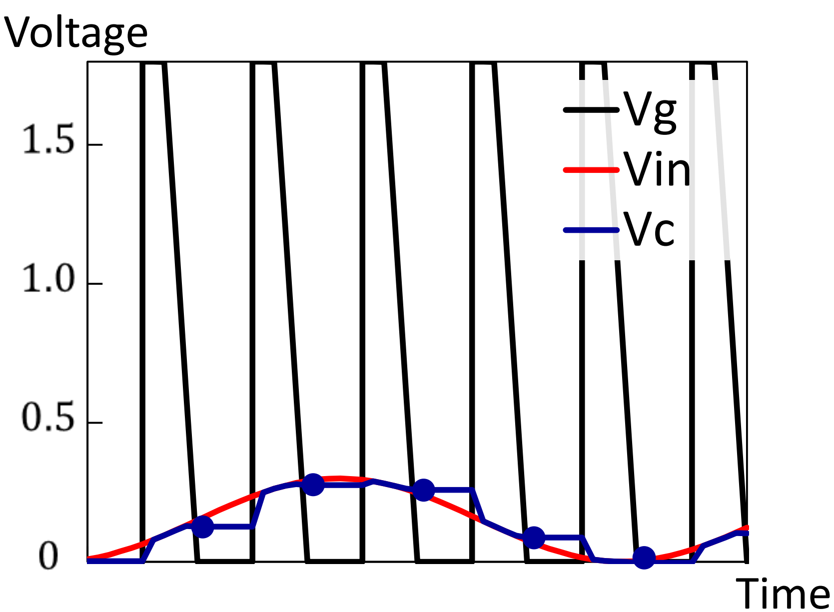
Bandwidth starts to decrease at $\tau_2 / \tau_1 = 1$

τ_1, τ_2 effects to bandwidth are comparable.

SPICE Simulation Verification



Track Hold Circuit



Obtain values of ●

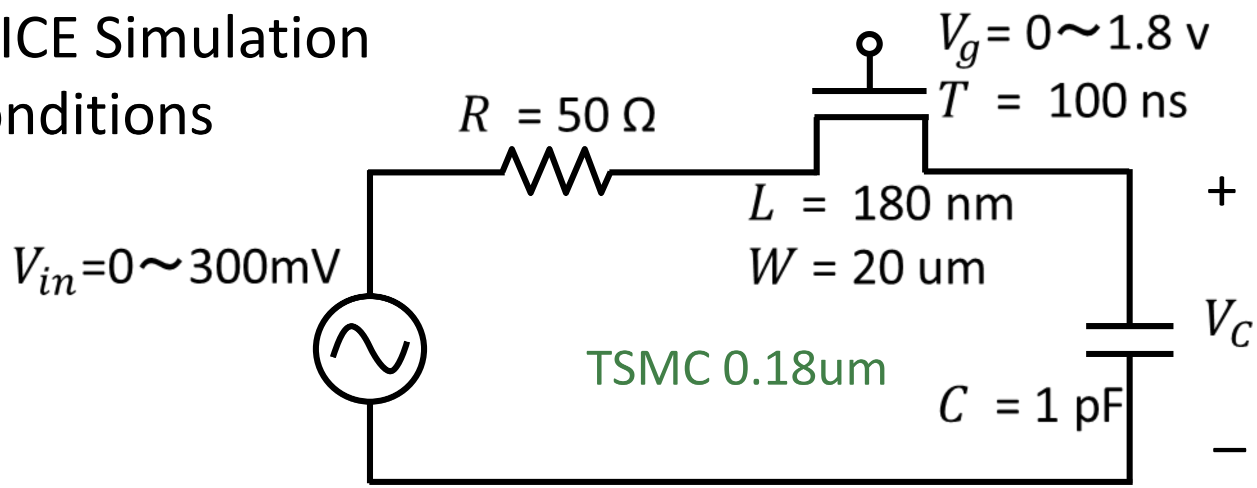
Equivalent time sampling

Obtain gain, phase for each frequency

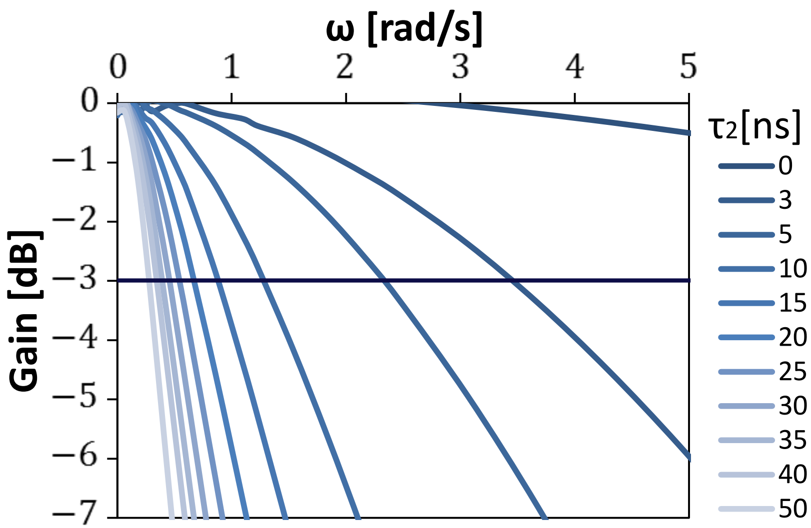
Frequency transfer function

Results

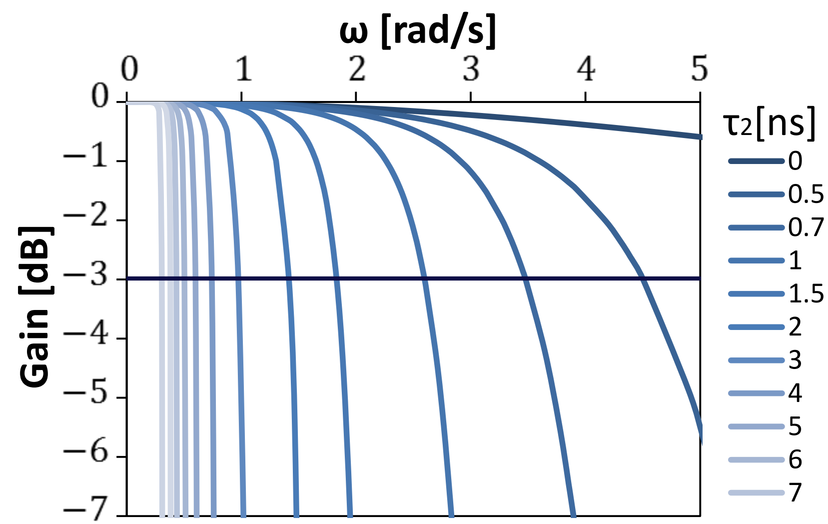
- SPICE Simulation Conditions



- Results

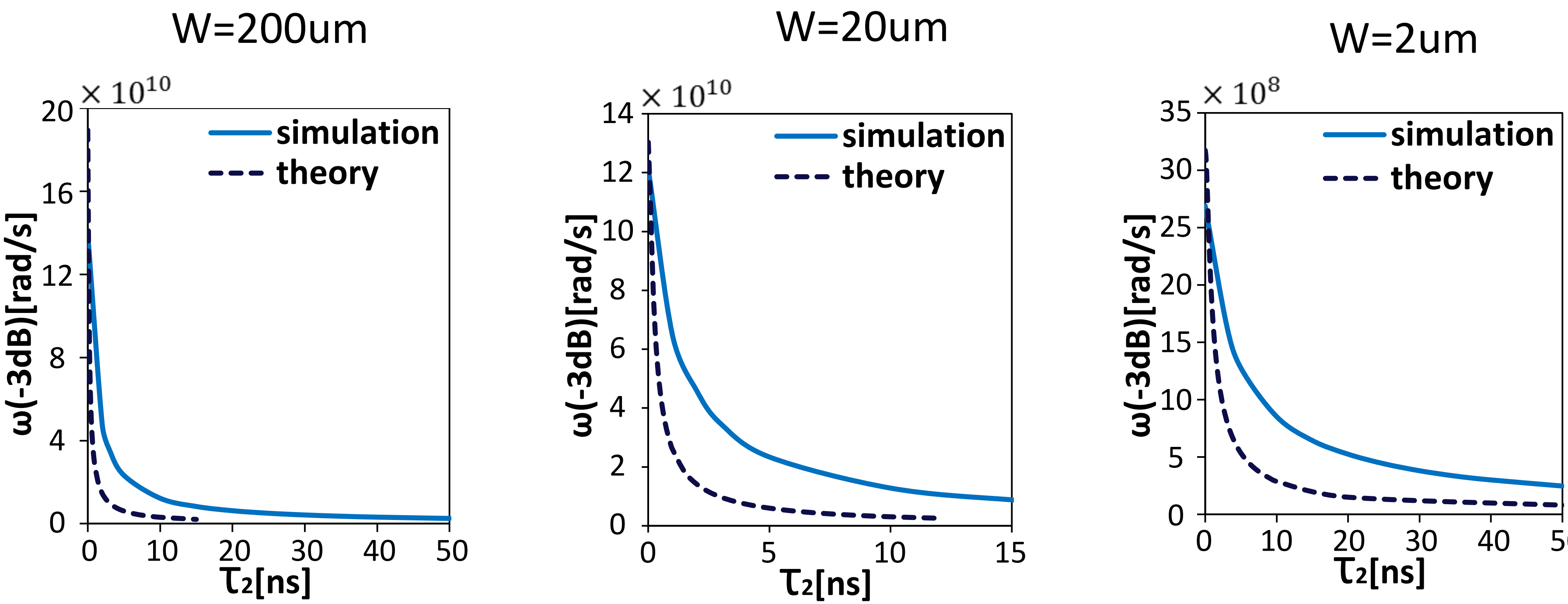


SPICE Simulation Results



Theory
(Derived Transfer Function)

Comparison of -3dB Bandwidth

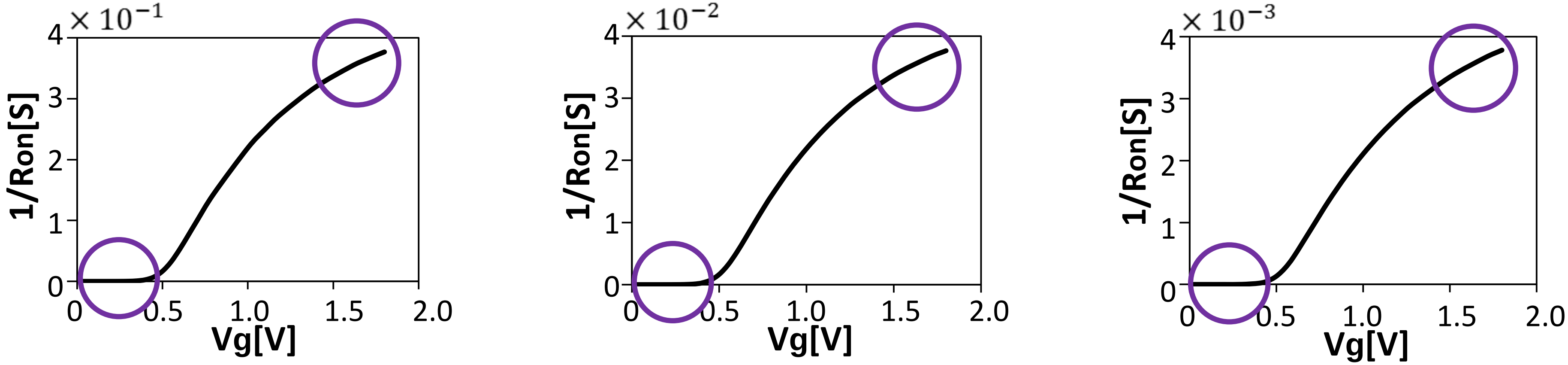


Simulation $\neq$ Theory

Large discrepancies !

NMOS ON-Conductance Nonlinearity

W=200um W=20um W=2um



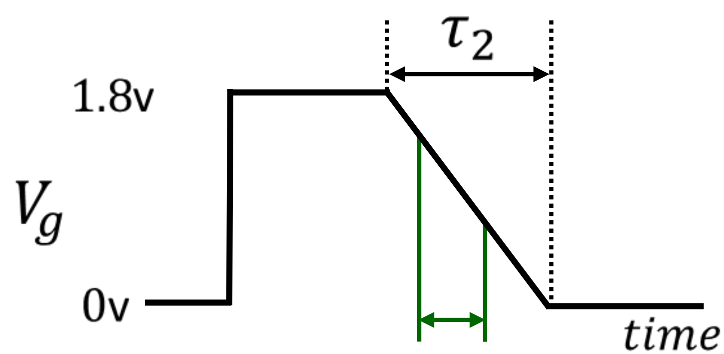
○ region

⇩

Strong nonlinearity of 1/Ron

⇩

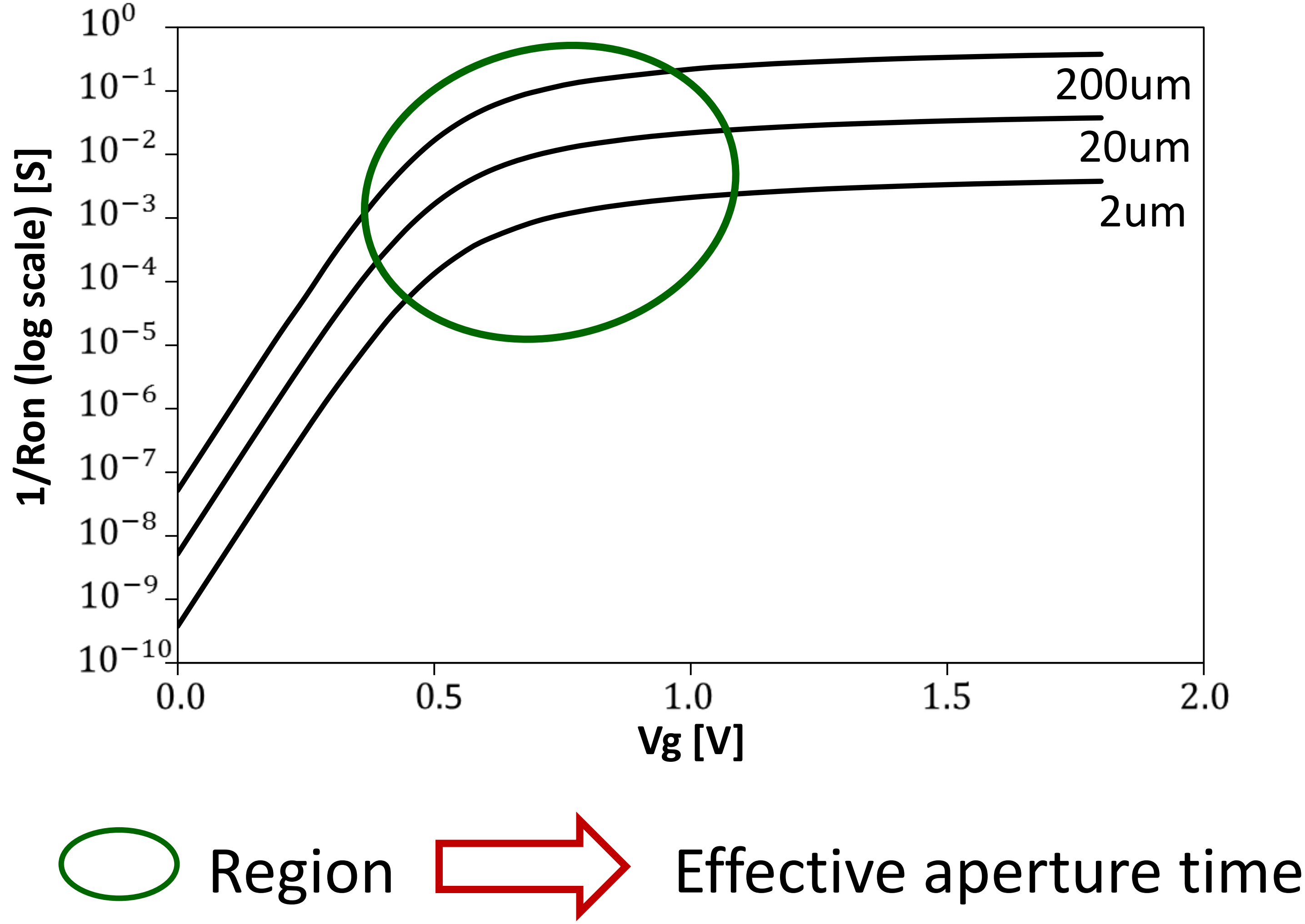
Define effective aperture time τ_{eff}



Effective aperture time

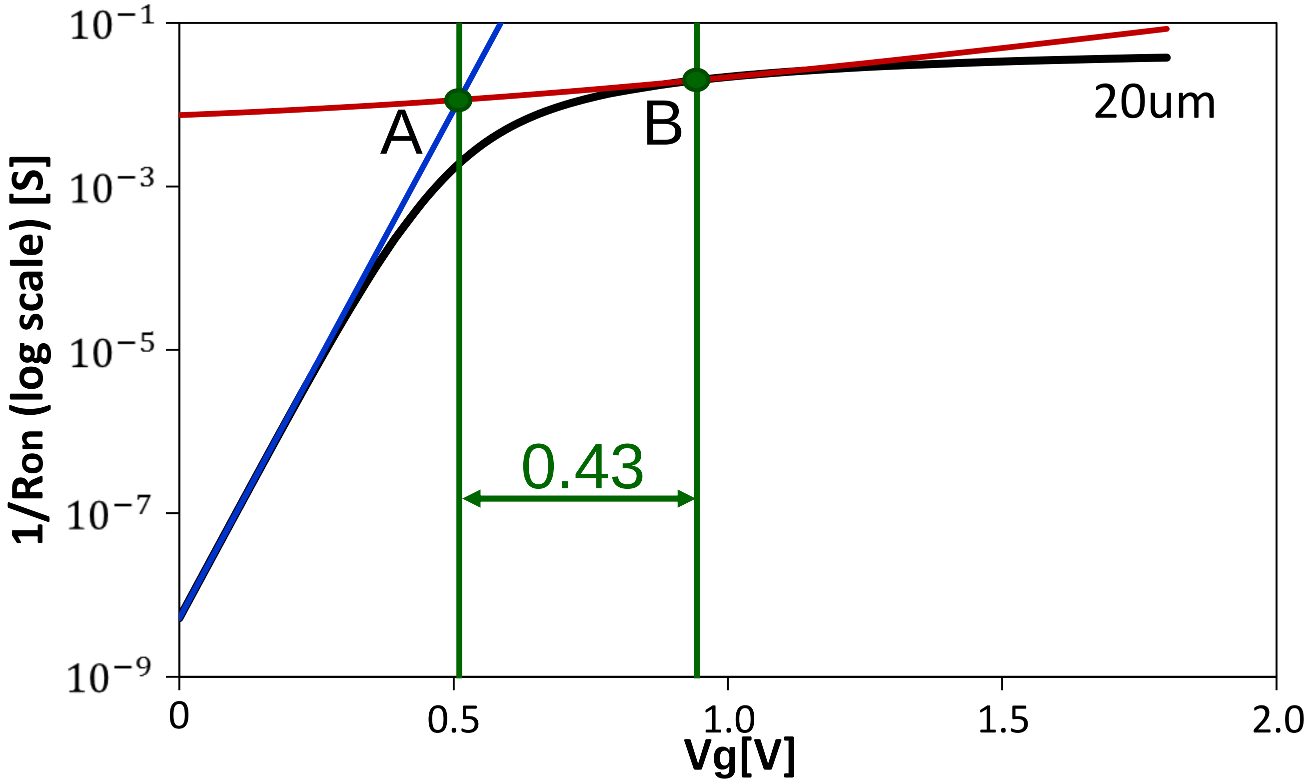
τ_{eff}

ON-Conductance and Effective Aperture Time



Empirical Effective Aperture Time Derivation

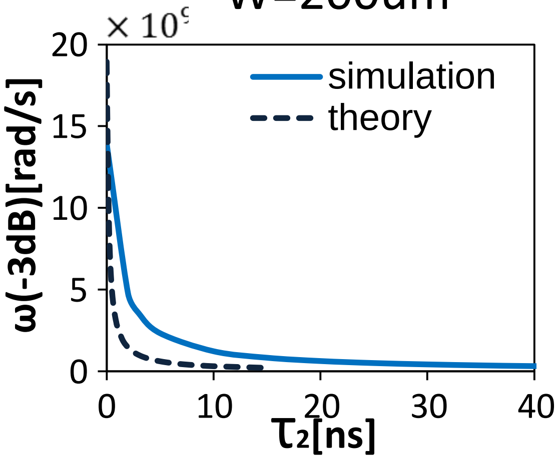
$$y = \left(y_{V_{th}} - 9 \times 10^{-8} \frac{W}{L} \cdot V_{th}\right) e^{\left(\frac{x}{V_{th}} - 1\right)} + 9 \times 10^{-8} \frac{W}{L} \cdot V_{th}$$



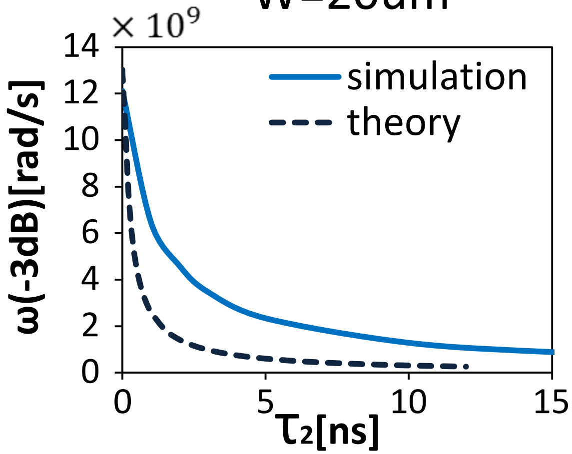
$\therefore$ Effective Aperture Time $\tau_{\text{eff}} = \frac{0.43}{1.8} \times \tau_2$

Discussion Again

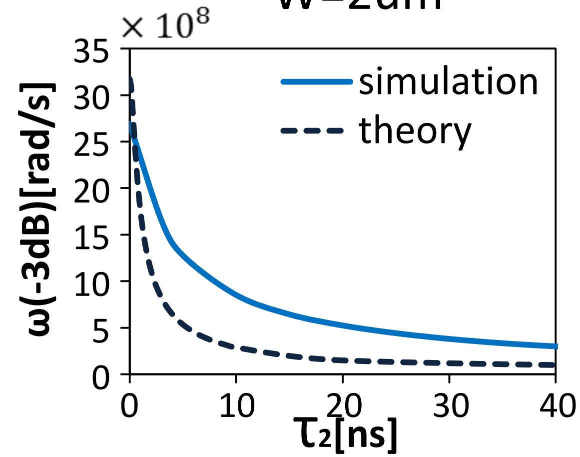
W=200um



W=20um



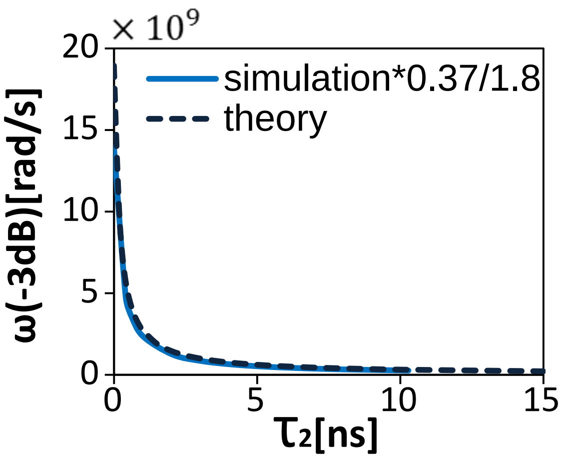
W=2um

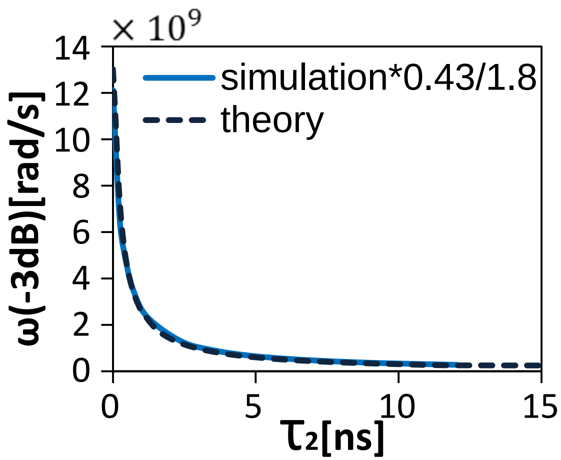


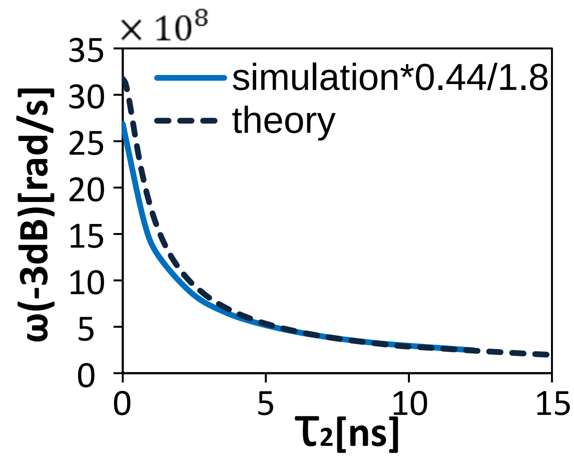
⇩ X 0.37/1.8

⇩ X 0.43/1.8

⇩ X 0.44/1.8



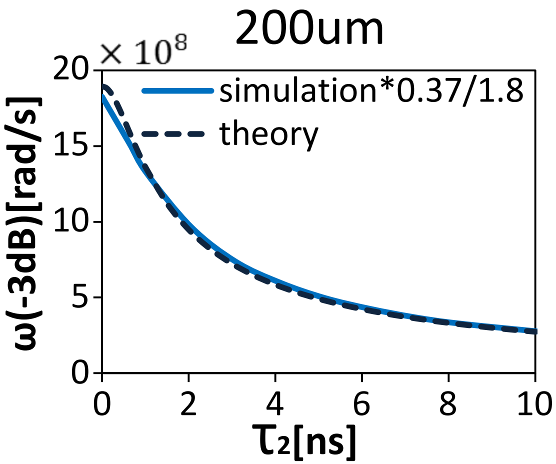


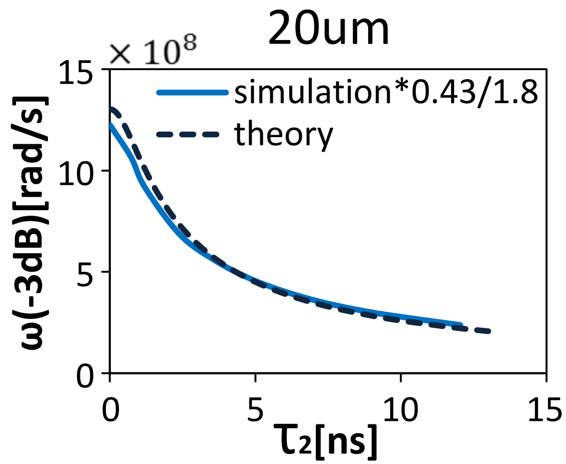


SPICE simulation results ≅ Theory

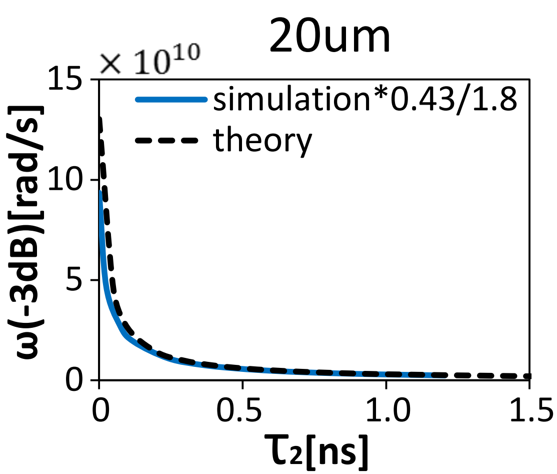
Various Values for RC, W

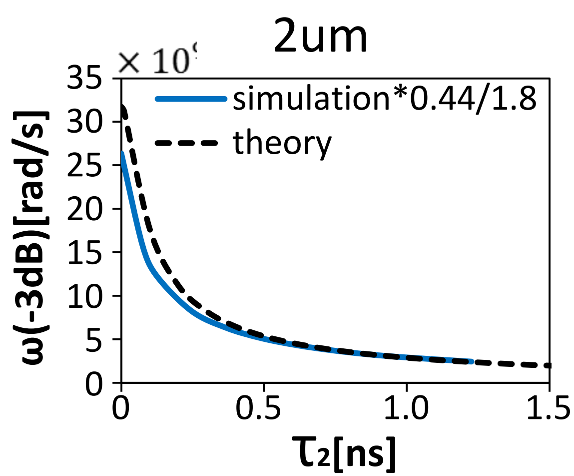
R=50Ω,   C=10pF





R=50Ω,   C=0.1pF

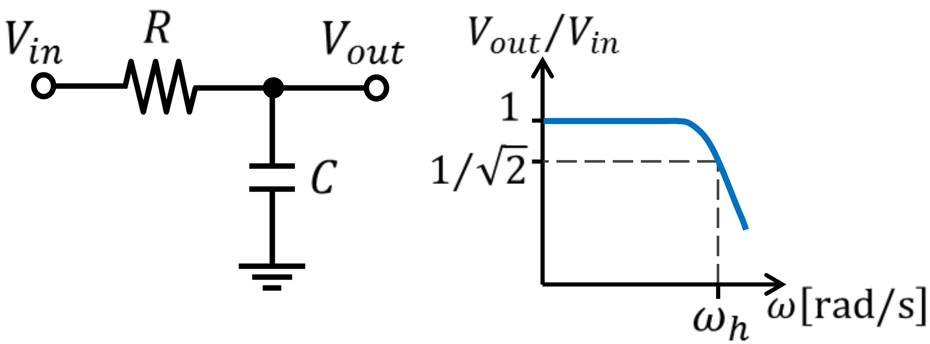




SPICE simulation results $\cong$ Theory

Trade-off of Time Constant and Bandwidth

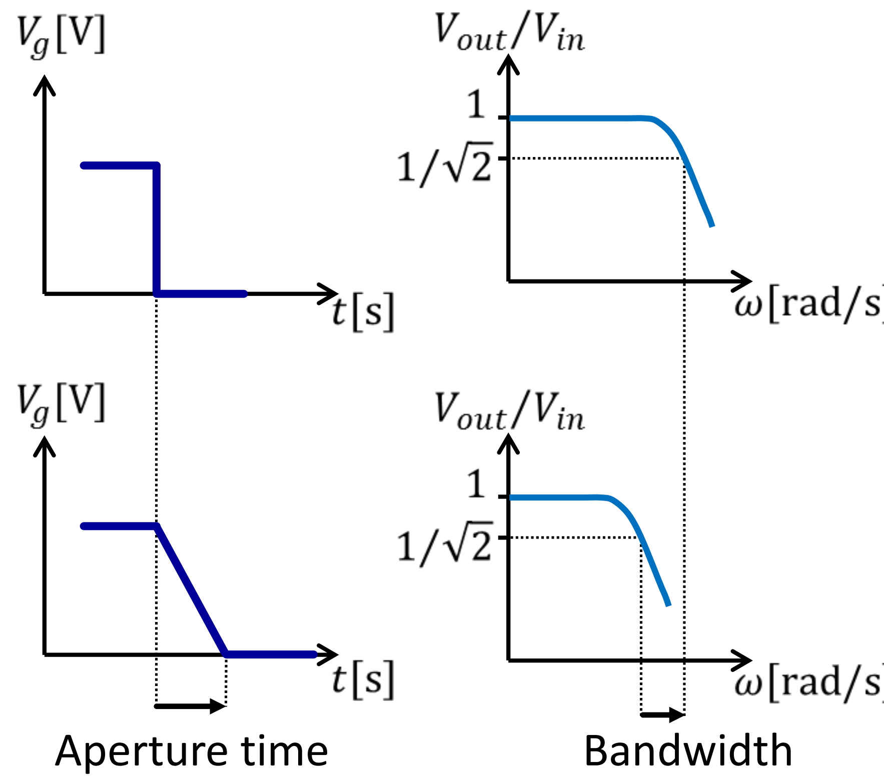
- RC time constant and bandwidth



$$\omega_h = \frac{1}{\tau_1}$$

$$\begin{cases} \tau_1 : \text{RC} \\ \omega_h : \text{bandwidth} \end{cases}$$

- Aperture time and bandwidth



Time		Band : ω_h
Short	⟶	Wide
Long	⟶	Narrow

Summary

- Derived explicit transfer function of sampling circuit with finite aperture time effect.
- Verified it with SPICE simulation
- Introduced concept of effective finite aperture time
- Showed uncertainty relationship between time constants and bandwidth in sampling circuit.

Contents

- Research Objective
- Uncertainty Principle and Relationship
- Invariant Quantity
- Electronic Circuit Performance Analogy to Uncertainty Relationship and Invariant
- Waveform Sampling Circuit
- Conclusion

Conclusion

Our strong belief:

Analog electronic circuit

- Its design tradeoff as well as FOM

  ↓ Explained with

  Analogy to uncertainty principle/relationship.

- Uncertainty principle and relationship

  ↓

  Its ultimate performance limitation

Final Statement

Current status of circuit design and analysis area

⇩

Only individual techniques have been developed.

大道以多岐亡羊，學者以多方喪生

（列子）

⇩

We need to establish a unified theory for circuit design and analysis area.

Thank you for listening

謝謝