1006

Design and Analysis of for Operational Amplifier Stability Based on Control Theory

JianLong Wang, Gopal Adhikari, Nobukazu Tsukiji, Mayu Hirano, Haruo Kobayashi, Keita Kurihara,
Akihito Nagahama, Ippei Noda, Kohji Yoshii
Division of Electronics and Informatics, Gunma University, *1-5-1* Tenjin-cho, Kiryu *365-8515, Japan*
Ricoh Electronic Devices Co., Ltd., 13-1 Himemuro-cho, Ikeda, Osaka 536-8501 Japan
* Email: t15808004@gunma-u.ac.jp

Operational Amplifier Stability Research

Research Objective

Our proposal

For

Analysis and design of operational amplifier stability

Use

Routh-Hurwitz stability criterion

We can obtain

Explicit stability condition for circuit parameters
(which can NOT be obtained only with Bode plot).

We can verify

Equivalence between Nyquist and Routh-Hurwitz stability criteria

- ■ Electeonic Circuit Design Field
  - Bode plot (>90% frequently used)
  - Nyquist plot
- ■ Control Theory Field
  - Bode plot
  - Nyquist plot
  - Nicholas plot
  - Routh-Hurwitz stability criterion

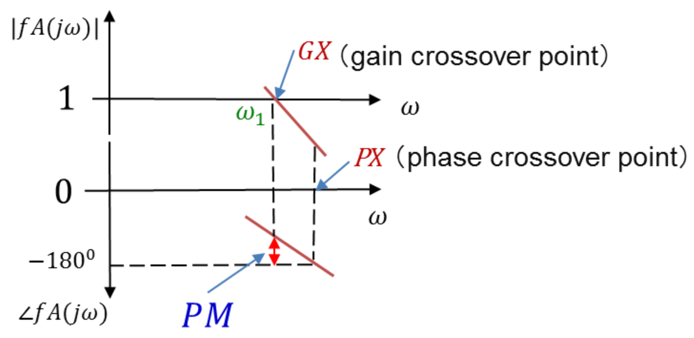
Very popular in control theory field
but rarely seen in electronic circuit books/papers

Stability Criteria

Nyquist stability Criteria

GX precedes PX ⇒ Feedback system is stable

Greater spacing between GX and PX ⇒ More stable



ω_1: gain crossover frequency

Phase margin : PM $= 180^0 + \angle fA(\omega = \omega_1)$

Bode plot does NOT show explicit stability conditions of circuit parameters.

Routh-Hurwitz stability Criteria

Characteristic equation:

$$D(s) = \alpha_n s^n + \alpha_{n-1} s^{n-1} + \cdots + \alpha_1 s + \alpha_0 = 0$$

Sufficient and necessary condition:

(i) $\alpha_i > 0$ for $i = 0,1,\ldots,n$

&

(ii) All values of Routh table's first columns are positive.

Routh table

s^n	α_n	α_{n-2}	α_{n-4}	α_{n-6}	⋯
s^{n-1}	α_{n-1}	α_{n-3}	α_{n-5}	α_{n-7}	⋯
s^{n-2}	$\beta_1 = \frac{\alpha_{n-1}\alpha_{n-2} - \alpha_n\alpha_{n-3}}{\alpha_{n-1}}$	$\beta_2 = \frac{\alpha_{n-1}\alpha_{n-4} - \alpha_n\alpha_{n-5}}{\alpha_{n-1}}$	β_3	β_4	⋯
s^{n-3}	$\gamma_1 = \frac{\beta_1\alpha_{n-3} - \alpha_{n-1}\beta_2}{\beta_1}$	$\gamma_2 = \frac{\beta_1\alpha_{n-5} - \alpha_{n-1}\beta_3}{\beta_1}$	γ_3	γ_4	⋯
⋮	⋮	⋮	⋮	⋮	⋮
s^0	α_0				

Equivalence at Mathematical Foundations

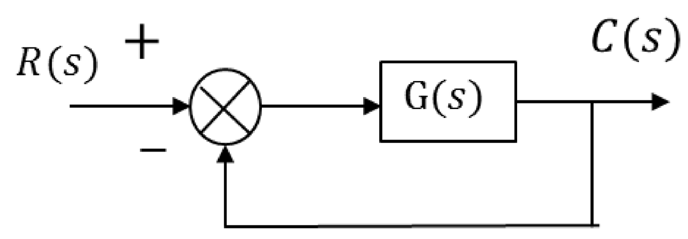
Two Examples

Ex.1 $G(s) = \dfrac{K(1+b_1 s)}{1 + a_1 s + a_2 s^2}$ One Zero, Two Poles

Ex.2 $G(s) = \dfrac{K(1+b_1 s)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$ One Zero, Three Poles

Stability Condition

Based on Routh-Hurwitz Criterion



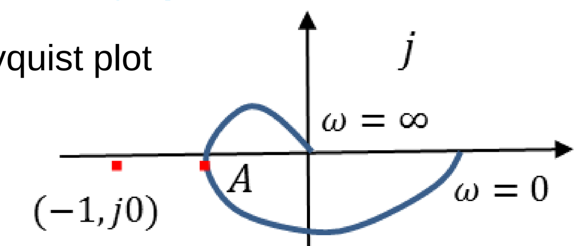
Block diagram of feedback system

Ex.1:
- $K > -\dfrac{a_1}{b_1}$ At condition: $b_1 > 0$
- $K < -\dfrac{a_1}{b_1}$ At condition: $b_1 < 0$

Ex.2:
- $K > \dfrac{a_3 - a_1 a_2}{a_2 b - a_3}$ At condition: $a_2 b - a_3 > 0$
- $K < \dfrac{a_3 - a_1 a_2}{a_2 b - a_3}$ At condition: $a_2 b - a_3 < 0$

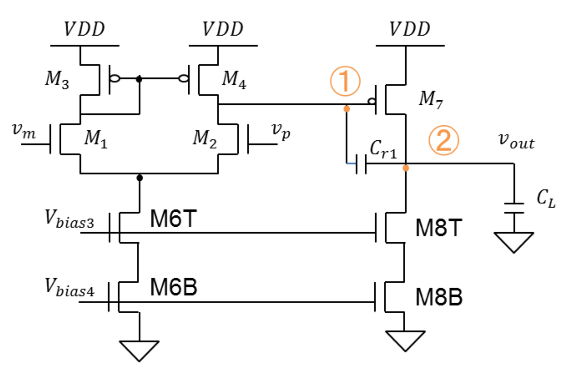
Based on Nyquist Criterion

Sketch chart of Nyquist plot

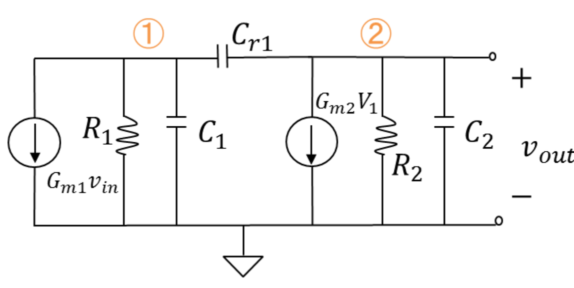


Ex.1:
- $-\dfrac{a_1}{b_1} < K < \dfrac{a_1}{b_1}$ At condition: $a_1 b_1 > 0$
- $\dfrac{a_1}{b_1} < K < -\dfrac{a_1}{b_1}$ At condition: $a_1 b_1 < 0$

Ex.2:
- $\dfrac{a_3 - a_1 a_2}{a_2 b - a_3} < K < \dfrac{a_3 - a_1 a_2}{a_3 - a_2 b}$ At condition: $(a_3 - a_1 a_2)(a_3 - a_2 b) > 0$
- $\dfrac{a_3 - a_1 a_2}{a_3 - a_2 b} < K < \dfrac{a_3 - a_1 a_2}{a_2 b - a_3}$ At condition: $(a_3 - a_1 a_2)(a_3 - a_2 b) < 0$

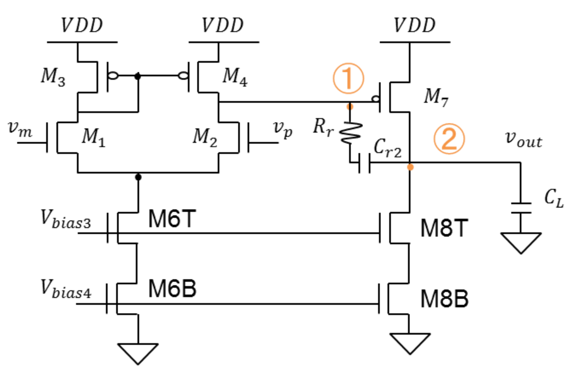


Transistor level circuit

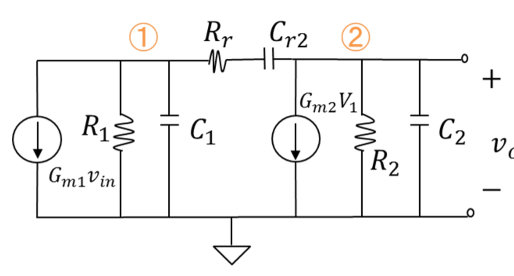


Small-signal model

Ex.1 Two-stage amplifier with C compensation



Transistor level circuit



Small-signal model

Ex.2 Two-stage amplifier with C,R compensation

Simulation Verification

Example I

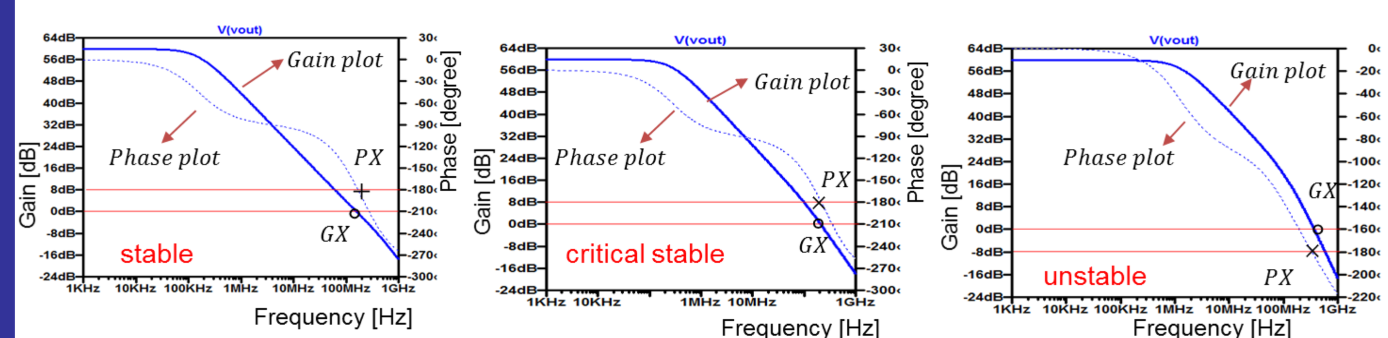
Closed-loop transfer function:

$$H(s) = \frac{A_0(1+b_1 s)}{1 + fA_0 + (a_1 + fA_0 b_1)s + a_2 s^2}$$

Explicit stability condition of parameters:

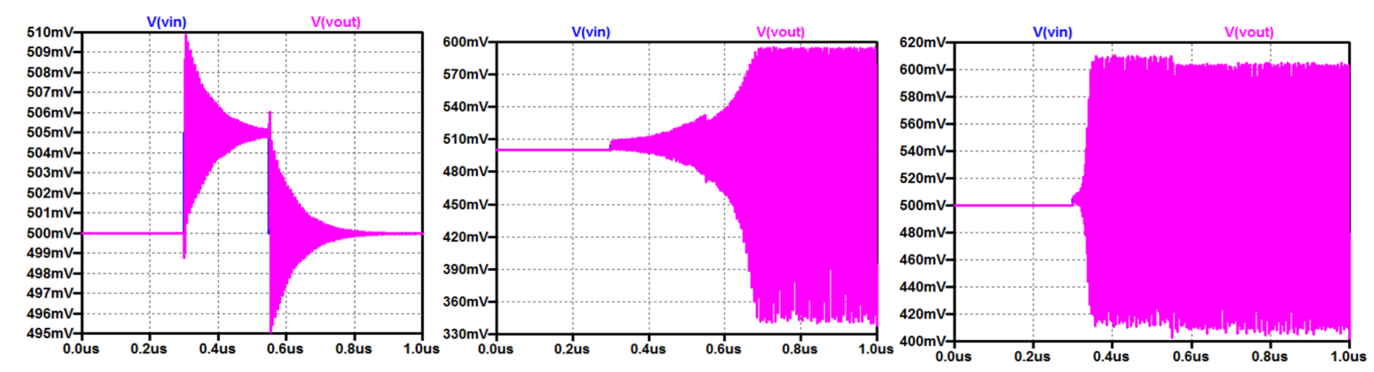
$$a_1 + fA_0 b_1 = R_1C_1 + R_2C_2 + (R_1 + R_2)C_r + (G_{m2} - fG_{m1})R_1R_2C_r > 0$$

⇒ C_r >79.57fF



Case (1) C_r = 150fF Case (2) C_r =79.57fF Case (3) C_r = 10fF

Consistency of Bode Plots and R-H Results



Case (1) C_r = 150fF Case (2) C_r =79.57fF Case (3) C_r = 10fF

Consistency of Transient Analysis and R-H Results

Example II

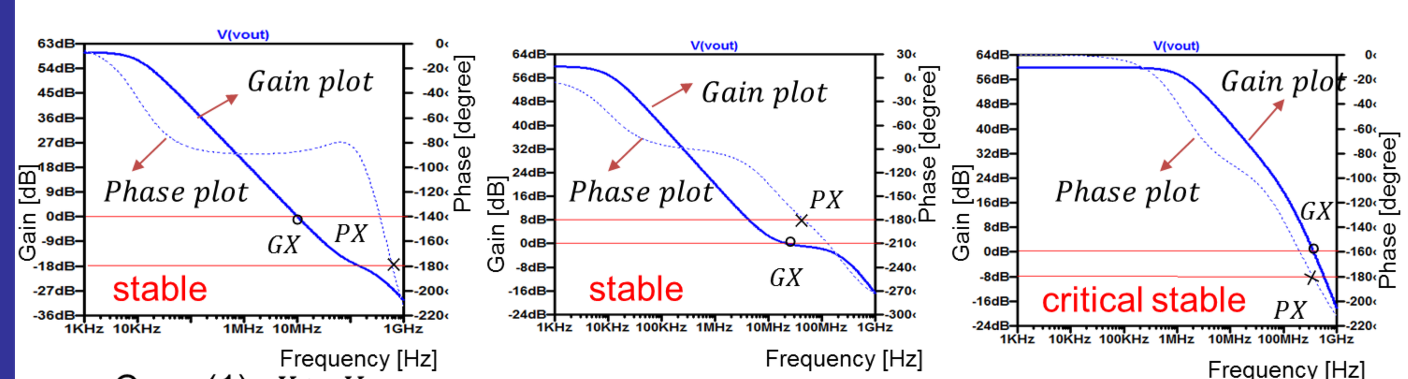
Closed-loop transfer function:

$$H(s) = \frac{A_0(1+b_1 s)}{1 + fA_0 + (a_1 + fA_0 b_1)s + a_2 s^2 + a_3 s^3}$$

Explicit stability condition of parameters:

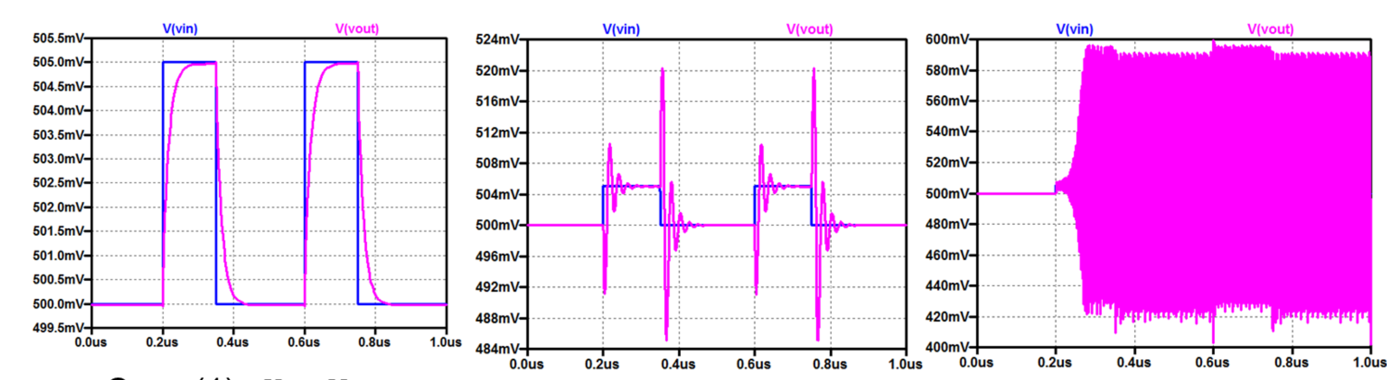
$$\frac{(a_1 + fA_0 d_1)a_2 - a_3(1 + fA_0)}{a_2} > 0$$

$$X = R_1C_1 + R_2C_2 + (R_1 + R_2 + R_r)C_r + (G_{m2} - fG_{m1} + fG_{m1}G_{m2}R_r)R_1R_2C_r > \frac{R_1R_2C_1C_2R_rC_r(1 + G_{m1}G_{m2}R_1R_2)}{R_1R_2(C_2C_r + C_1C_2 + C_1C_r) + R_rC_r(R_1C_1 + R_2C_2)} = Y$$



Case (1) $X > Y$ Case (2) $X > Y$ Case (3) $X \approx Y$

Consistency of Bode Plots and R-H Results



Case (1) $X > Y$ Case (2) $X > Y$ Case (3) $X \approx Y$

Consistency of Transient Analysis and R-H Results

Summary

Discussion

1. Depict small signal equivalent circuit
2. Derive open-loop transfer function
3. Derive closed-loop transfer function & obtain characteristics equation
4. Apply R-H stability criterion & obtain explicit stability condition
5. Use this stability condition for circuit parameters

Conclusion

- ● Equivalence between Nyquist and R-H stability criteria
- ● Equivalency of mathematical foundations
- ● R-H method, explicit circuit parameter conditions
- ● Consistency with Bode plot method, LTspice simulation

R-H method can be used with conventional Bode plot method.