Low-Distortion One-Tone and Two-Tone Signal Generation
Using AWG Over Full Nyquist Region

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Abstract - This paper describes algorithms, simulation and experiment verifications of a novel harmonic distortion suppression technique for an arbitrary waveform generator (AWG). It can automatically suppress the harmonics and the image components over a full Nyquist region of the AWG by preprocessing the sinusoidal waveform digital data using "phase switching method". We show this technique can apply to cancel the harmonics from mid-frequency regions. Also we show two-tone signal generation with IMD suppression. With these methods, distortion components close to the signal are suppressed simply by changing DSP program or waveform memory contents —AWG nonlinearity identification is not required—and spurious components, generated far from the signal band, are relatively easy to remove using an analog filter.

Keyword: Analog/Mixed-Signal IC Testing, Low Distortion Signal Generation, Arbitrary Waveform Generator, Sine Wave, Two-Tone Signal

1. Introduction

LSI production testing is becoming important in the semiconductor industry, because its testing cost is increasing while its silicon cost per transistor is decreasing [1]. Especially low-cost and high-quality testing of analog/mixed-signal circuits are important.

An Arbitrary Waveform Generator (AWG) consists of a DSP (or waveform memory) and a DAC. We can use the AWG to generate arbitrary analog waveforms simply by changing the DSP program, and Automatic Test Equipment (ATE) often uses AWGs. However, due to AWG nonlinearities, sinusoidal signals generated by AWGs include harmonics that degrade the accuracy of analog/mixed-signal circuit testing when the generated signals are used for the testing input.

This paper presents methods for generating a low-distortion one-tone and two-tone signals with harmonics, images and IMD suppression from low, mid frequency regions. Table I clarifies the position of this paper in this field.

As shown in the Fig.1, the regions where harmonics appear near the fundamental wave can be divided into three: “low-frequency”, “mid-frequency” and “high-frequency” regions.

Fig.1 Regions where harmonics appear near the fundamental wave

Note that here mid-frequency means “the frequency region around fs/4”, and high-frequency means “up to approximately the Nyquist frequency (fs/2) of the DAC in the AWG, where fs is a sampling frequency of the DAC”. Also HD stands for harmonic distortion, and IMD stands for intermodulation distortion.

The reference [2] shows a low-distortion one-tone signal generation algorithm with the AWG nonlinearity compensation, but it requires AWG nonlinearity identification. On the other hand, we have investigated the phase switching algorithm for one-tone and two-tone signal generation which does not require AWG nonlinearity identification. For the low-frequency signal generation, the references [3] [4] [5] show its algorithms, experimental results at laboratory level and ATE application results, respectively. The reference [6][7] shows the two-tone case. For the high-frequency signal generation, the reference [8] shows only HD3 image cancellation algorithm of one-tone signal. The paper [9] shows high-frequency single-tone and two-tone signals generation with image and IMD suppression. This paper
shows mid-frequency single-tone and two-tone signals generation with image and IMD suppression.

Then with this paper and the references [3-9], low-distortion one-tone and two-tone signal generation using AWG over full Nyquist region becomes possible.

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2. Theory of Phase-Switching Method and Low-Frequency Signal Generation

The AWG generates an analog signal through a DAC whose digital input is provided from DSP. Hence the nonlinearity of the DAC causes harmonic distortion, and then we propose methods to cancel the DAC nonlinearity effects with the DSP program change as pre-distortion.

2.1 Single-Tone Signal Generation

The direct sinusoidal signal generation method with AWG uses the following, where $D_{in}$ is a digital input signal to the DAC from DSP inside the AWG.

$$D_{in} = A \sin(2\pi f_{in} n T_s)$$

(1)

For the low-distortion signal generation with the AWG [3-5], our phase switching method uses the following:

$$D_{in} = \begin{cases} X_0 = A \sin(2\pi f_{in} n T_s + \varphi_0) & \text{even} \\ X_1 = A \sin(2\pi f_{in} n T_s - \varphi_1) & \text{odd} \end{cases}$$

(2)

For low-frequency generation

$$\varphi = \varphi_0 - \varphi_1 = (2m - 1)\pi / N$$

(3)

Fig. 2 Phase-switching method

Here $m = 0, 1, 2, ..., n$ is an integer, while $T_s$ is a sampling period. The DSP output signal $D_{in}$ consists of $X_0$ and $X_1$, and they are interleaved (Fig.2).

Now we consider the case that the AWG has 3rd order distortions and we consider to cancel their effects to generate a low-frequency sine signal. Consider the case that the AWG has 3rd order distortion:

$$\begin{align*}
Y(n T_s) &= a_1 D_{in} + a_3 D_{in}^3 \\
&= a_1 X_0 + a_3 X_0^3 \\
&= a_1 X_1 + a_3 X_1^3
\end{align*}$$

(4)

Here, considering the 3rd order harmonics (HD3), $X_0$ and $X_1$ are as shown in equations (5) and (6).

$$a_1 X_0 + a_3 X_0^3 = \frac{4a_1 A + 3a_3 A^3}{4} \sin(2\pi f_{in} n T_s + \varphi_0) - \frac{a_3 A^3}{4} \sin(2\pi \cdot 3 f_{in} n T_s + 3\varphi_0)$$

(5)

$$a_1 X_1 + a_3 X_1^3 = \frac{4a_1 A + 3a_3 A^3}{4} \sin(2\pi f_{in} n T_s + \varphi_1) - \frac{a_3 A^3}{4} \sin(2\pi \cdot 3 f_{in} n T_s + 3\varphi_1)$$

(6)

The interleave operation can be expressed by the following equation:

$$\begin{align*}
Y(n T_s) &= \frac{1}{2} \left( 1 + \cos \left( 2\pi \frac{f_s}{2} n T_s \right) \right) (a_1 X_0 + a_3 X_0^3) \\
&+ \frac{1}{2} \left( 1 - \cos \left( 2\pi \frac{f_s}{2} n T_s \right) \right) (a_1 X_1 + a_3 X_1^3) \\
&= \frac{a_1 A + 3a_3 A^3}{8} (e^{j\varphi_0} + e^{j\varphi_1}) \sin(2\pi f_{in} n T_s) \\
&- \frac{a_3 A^3}{8} (e^{j3\varphi_0} + e^{j3\varphi_1}) \sin(2\pi \cdot 3 f_{in} n T_s) \\
&- \frac{a_1 A + 3a_3 A^3}{8} (e^{j\varphi_0} - e^{j\varphi_1}) \sin \left( 2\pi \frac{f_s}{2} - 2 f_{in} n T_s \right) \\
&+ \frac{a_3 A^3}{8} (e^{j3\varphi_0} - e^{j3\varphi_1}) \sin \left( 2\pi \frac{f_s}{2} - 3 f_{in} n T_s \right)
\end{align*}$$

(7)

To cancel 3rd order harmonics, $\varphi_0$ and $\varphi_1$ have to satisfy the following condition:

$$e^{j3\varphi_0} + e^{j3\varphi_1} = 0$$

(8)

$$\varphi_0 - \varphi_1 = \frac{2m - 1}{3} \pi \quad (m = 1, 2, 3 \ldots)$$

(9)

In brief, it takes the following values:

$$\varphi_0 = \frac{\pi}{6}, \quad \varphi_1 = -\frac{\pi}{6}$$

(10)

If the phase difference is determined by the above method, it is possible to create a waveform that removes only arbitrary harmonic components.

Also for the simulation conditions, we set

$$f_{in}/f_s = 3/200, \quad A = 1, \quad a_1 = 1, \quad a_3 = -0.01$$

Figs 3, 4 show simulation results for the direct and phase switching methods respectively, and we see in Fig 3 that HD3 component is removed by the phase switching. Spurious appears on the high frequency side, but it is removed with an analog filter. Rather than directly filtering the third harmonic, setting order and design of the filter is simplified.

![AWG](AWG.png)

Fig. 2 Phase-switching method

![Signal vs. HD3](SignalvsHD3.png)

Fig. 3. $Y(n T_s)$ spectrum with the direct low-frequency signal generation method with AWG 3rd-order distortions. (using Eq.1).
Fig. 4 $Y(nT_s)$ spectrum with the phase-switching low-frequency signal generation with AWG 3rd-order distortion (using Eq. 2).

2.2 Multiple Harmonics suppression

Now we consider the case that the AWG has 3rd and 5th order distortions (Eq. 11) and we consider to cancel their effects to generate a low-distortion sine signal:

$$Y(nT_s) = a_1D_{in} + a_3D_{in}^3 + a_5D_{in}^5$$  \hspace{1cm} (11)

Then let

$$D_{in} = \begin{cases} X_0 = A \sin(2\pi f_{out} nT_s - \varphi_a - \varphi_b) & n = 4k \\ X_1 = A \sin(2\pi f_{out} nT_s + \varphi_a - \varphi_b) & n = 4k + 1 \\ X_2 = A \sin(2\pi f_{out} nT_s - \varphi_a + \varphi_b) & n = 4k + 2 \\ X_3 = A \sin(2\pi f_{out} nT_s + \varphi_a + \varphi_b) & n = 4k + 3 \end{cases}$$ \hspace{1cm} (12)

$$\varphi_a = \frac{\pi}{6}, \quad \varphi_b = \frac{\pi}{10}$$

Also for the simulation conditions, we set

$$f_{in}/f_s = 1/200$$

$$A = 1, \quad a_1 = 1, \quad a_3 = -0.01, \quad a_5 = -0.001$$

Fig. 5 shows the power spectrum with the direct method. Fig. 6 shows the one with the proposed method with Eq. 12; we see that there are no harmonics around the fundamental signal.

Fig. 5 $Y(nT_s)$ spectrum with the direct low-frequency signal generation method with AWG 3rd and 5th-order distortions (using Eq. 1).

Fig. 6 $Y(nT_s)$ spectrum with the phase-switching low-frequency signal generation with AWG 3rd and 5th-order distortion (using Eq. 12).

2.3 Two-Tone Signal Generation

Two-tone signal testing is frequently used for testing of communication application circuits. When the 3rd-order nonlinearity is dominant in the AWG and $f_{out1}, f_{out2}$ are used, IMD3 components $(2f_{out1}-f_{out2}, 2f_{out2}-f_{out1})$ are serious because they are close to the signals $(f_{out1}, f_{out2})$ and are difficult to remove with an analog filter. Then we consider to apply the phase switching algorithm. Suppose that the AWG has 3rd-order distortion. Suppose the following:

$$Y(nT_s) = a_1D_{in} + a_3D_{in}^3$$ \hspace{1cm} (13)

For the direct method, we use

$$D_{in} = A \sin(2\pi f_{out} nT_s) + B \sin(2\pi f_{out} nT_s)$$ \hspace{1cm} (14)

Simulation conditions are as follows:

$$f_{out1}/f_s = 3/200, \quad f_{out2}/f_s = 4/200$$

$$A = 1, \quad B = 1, \quad a_1 = 1, \quad a_3 = -0.01$$

Then the output spectrum is shown in Fig. 7.

Fig. 7 $Y(nT_s)$ spectrum with the direct low-frequency two-tone signal generation method with AWG 3rd-order distortion (using Eqs. 13).

We proposed to use Eqs. (15), (16) for a low-frequency two-tone signal without IMD3 [3, 6]:

$$D_{in} = \begin{cases} X_0 = A \sin(2\pi f_{out} nT_s + \varphi_o) + B \sin(2\pi f_{out} nT_s - \varphi_o) & n: \text{even} \\ X_1 = A \sin(2\pi f_{out} nT_s - \varphi_o) + B \sin(2\pi f_{out} nT_s + \varphi_o) & n: \text{odd} \end{cases}$$ \hspace{1cm} (15)

$$\varphi_o = \frac{\pi}{6}$$ \hspace{1cm} (16)

Fig. 8 shows simulation results.
3. Mid-Frequency Signal Generation
3.1 Single-Tone Signal Generation
Let us consider the mid-frequency signal generation case. We propose the following:

\[ D_{in} = \begin{cases} 
X_0 = A \sin(2\pi f_{in} n T_s + \varphi_0) & n = 4k - 3, \ 4k - 2 \\
X_1 = A \sin(2\pi f_{in} n T_s + \varphi_1) & n = 4k - 1, \ 4k \\
\varphi_0 - \varphi_1 = (2m - 1)\pi/N 
\end{cases} \tag{17} \]

For the simulation conditions, we set

\[ f_{in}/f_s = 49/200 \]

\[ A = 1, \ a_1 = 1, \ a_3 = -0.01 \]

With the proposed algorithm, only the 3rd order harmonics can be cancelled.

\[ f_{in}/f_s = 50/200 \]

\[ A = 1, \ a_1 = 1, \ a_3 = -0.01 \]

The theoretical amplitude values of the fundamental frequency are as follows:

- Direct method

\[ 20 \log_{10} \left( \frac{4a_1A + 3a_3A^3}{4} \right) = -6.108\text{dB} \]

- Phase-switching method

\[ 20 \log_{10} \left( \frac{4a_1A + 3a_3A^3}{4} \right) = -6.086\text{dB} \]

Output power spectra are shown in Figs. 11 and 12.

3.2 Multiple Harmonics Suppression
We propose to use Eqs. 10, 11 to generate a mid-frequency signal.

\[ X_n = A \sin(2\pi f_{in} n T_s + \varphi_n) \]

\[ D_{in} = \begin{cases} 
X_1 = A \sin(2\pi f_{in} n T_s + \varphi_1 + \varphi_a) & n = 8k - 5, \ 8k - 4 \\
X_2 = A \sin(2\pi f_{in} n T_s + \varphi_2 + \varphi_b) & n = 8k - 3, \ 8k - 2 \\
X_3 = A \sin(2\pi f_{in} n T_s + \varphi_3 + \varphi_c) & n = 8k - 1, \ 8k \\
\varphi_a = \frac{\pi}{6}, \ \varphi_b = \frac{\pi}{10} 
\end{cases} \tag{18} \]

Fig. 13 shows the power spectrum with the direct method. Fig. 14 shows the one with the proposed method with Eqs. 18, 19; we see that there are no harmonics and spurious around the signal.
3.3 Two-Tone Signal Generation

For mid-frequency two-tone signal generation without IMD3, the condition for $f_1, f_2$ shown in Fig.15 should be satisfied.

\[ f_1 = \frac{f_s}{4} - f_k, \quad f_2 = \frac{f_s}{4} + f_k \]

Fig.15. Condition of mid-frequency two-tone signal frequency configuration.

We propose to use Eqs. 17, 18:

\[
\begin{align*}
D_n &= A \sin(2\pi f_{in} n T_s + \varphi_0) + B \sin(2\pi f_{in} n T_s - \varphi_0) \quad n = 4k - 3, 4k - 2 \\
D_0 &= A \sin(2\pi f_{in} n T_s - \varphi_0) + B \sin(2\pi f_{in} n T_s + \varphi_0) \quad n = 4k - 1, 4k
\end{align*}
\]

Simulation condition is as follows:

\[ f_s/f_{in} = 49/200, \quad f_1/f_s = 51/200 \quad (f_k = 1) \]

The output power spectrum is shown in Fig. 17, and we see that the IMD3 components are cancelled. On the other hand, Fig. 16 shows the one with the direct method using Eq.14 where IMD3 components appear. Although spurious appears on the low frequency side and the high frequency side, it is removed by using BPF.

4. Experimental Verification

We have performed experiments at the laboratory level, to confirm the effectiveness of our methods. We have generated waveforms by the proposed algorithms with an AWG (Tektronix AFG3102), and measured their spectrum with a spectrum analyzer (Advantest R3267).

Fig.18 shows the AWG output spectrum measurement result of the single tone signal by the direct mid-frequency signal generation method, and Fig.19 shows the phase switching method. The experimental conditions are shown below.

\[ f_s = 4 \text{MHz}, \quad f_{in} = 980 \text{kHz} \]

In Fig.19, it is found that the 3rd harmonic is suppressed by -26 dB as compared with before application.

Fig.20 shows the measurement result of the AWG output spectrum of the two-tone signal by the direct mid-frequency signal generation method, and Fig. 21 shows the measurement result of the phase switching method. The experimental conditions are shown below.

\[ f_s = 4 \text{MHz}, \quad f_{in} = 980 \text{kHz}, \quad f_2 = 1020 \text{kHz} \]

Also in Fig. 21, suppression of -15 dB is seen for IMD3 component compared with before application.

Although it was not completely suppressed up to the noise floor in either single tone or two tone methods, suppression of about 20 dB was observed. The cause of this imperfect suppression is considered to be due to the distortion of the spectrum analyzer and the finite
(not-zero) rise / fall time of the AWG generation signal.

Fig.18. Spectrum with the direct mid-frequency signal generation method with AWG 3rd-order distortion.

Fig.19. Spectrum with the phase-switching mid-frequency signal generation with AWG 3rd-order distortion

Fig.20. Spectrum with the direct mid-frequency two-tone signal generation method with AWG 3rd-order distortion

Fig.21. Spectrum with the phase-switching mid-frequency two-tone signal generation method with 3rd order distortions.

5. Conclusion
We have described low-distortion one-tone and two-tone generation algorithms with an AWG using the phase switching technique over full Nyquist range. Simulation and experiment results show their effectiveness.

References