P063

Analysis and Design of Operational Amplifier Stability Based on Control Theory

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Operational Amplifier Stability Research

Research Objective

Our proposal

For

Analysis and design of operational amplifier stability

Use

Routh-Hurwitz stability criterion

We can obtain

Explicit stability condition for circuit parameters (which can NOT be obtained only with Bode plot).

Relationship between R-H parameters and phase margin

- Electeonic Circuit Design Field
  - Bode plot (>90% frequently used)
  - Nyquist plot
- Control Theory Field
  - Bode plot
  - Nyquist plot
  - Nicholas plot
  - Routh-Hurwitz stability criterion

Very popular in control theory field but rarely seen in electronic circuit books/papers

Stability Criteria

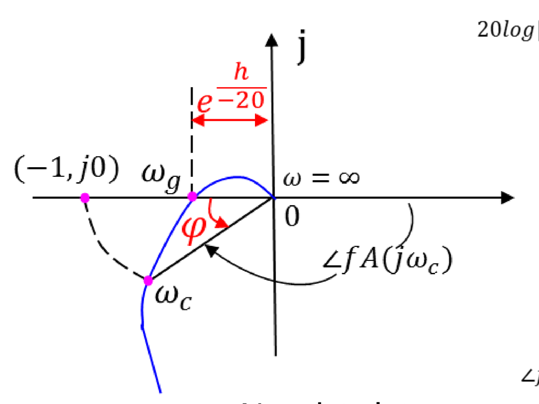
Signal Expression

Time domain → (Laplace transform) Frequency domain

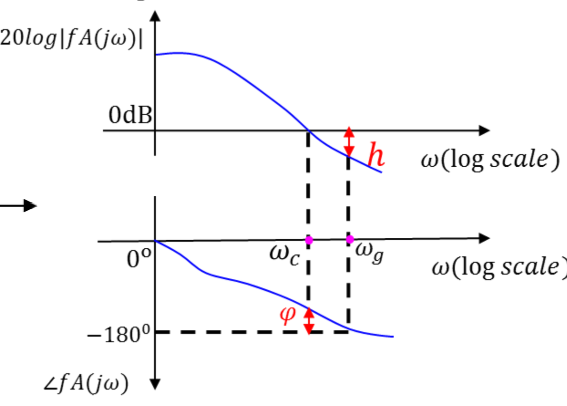
$A(t)$ → $A(jw)$ $A(s)$

Nyquist stability Criteria

h: Gain Margin φ: Phase Margin



Nyquist plot



Bode plot

Bode plot does NOT show explicit stability conditions of circuit parameters.

Routh-Hurwitz stability Criteria

$$\frac{A(s)}{1+A(s)} = \frac{N(s)}{D(s)} = \frac{b_n s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{\alpha_n s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0}$$

Routh table

S^n	α_n	α_{n-2}	α_{n-4}	α_{n-6}	$\cdots$
S^{n-1}	α_{n-1}	α_{n-3}	α_{n-5}	α_{n-7}	$\cdots$
S^{n-2}	$\beta_1 = \frac{\alpha_{n-1}\alpha_{n-2} - \alpha_n\alpha_{n-3}}{\alpha_{n-1}}$	$\beta_2 = \frac{\alpha_{n-1}\alpha_{n-4} - \alpha_n\alpha_{n-5}}{\alpha_{n-1}}$	β_3	β_4	$\cdots$
S^{n-3}	$\gamma_1 = \frac{\beta_1\alpha_{n-3} - \alpha_{n-1}\beta_2}{\beta_1}$	$\gamma_2 = \frac{\beta_1\alpha_{n-5} - \alpha_{n-1}\beta_3}{\beta_1}$	γ_3	γ_4	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S^0	α_0				

Equivalence at Mathematical Foundations

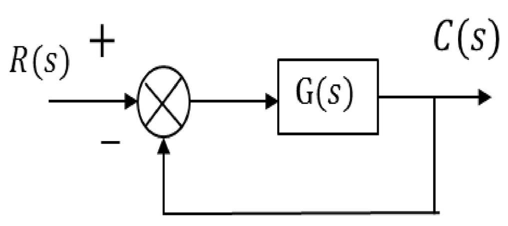
Two Examples

Ex.1 $G(s) = \frac{K(1+b_1 s)}{1+a_1 s + a_2 s^2}$ One Zero, Two Poles

Ex.2 $G(s) = \frac{K(1+b_1 s)}{1+a_1 s + a_2 s^2 + a_3 s^3}$ One Zero, Three Poles

Stability Condition

Based on Routh-Hurwitz Criterion



feedback system

S^2	a_2	$1+K$
S^1	$a_1 + Kb_1$	0
S^0	$1+K$	

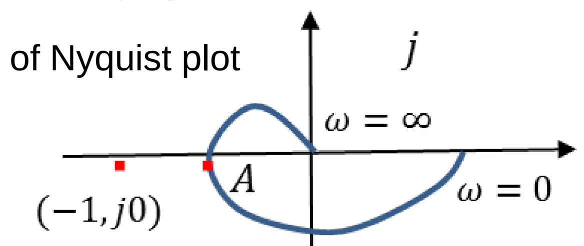
Routh sequence

Ex.1
- $K > -\frac{a_1}{b_1}$ At condition: $b_1 > 0$
- $K < -\frac{a_1}{b_1}$ At condition: $b_1 < 0$

Ex.2
- $K > \frac{a_3 - a_1 a_2}{a_2 b - a_3}$ At condition: $a_2 b - a_3 > 0$
- $K < \frac{a_3 - a_1 a_2}{a_2 b - a_3}$ At condition: $a_2 b - a_3 < 0$

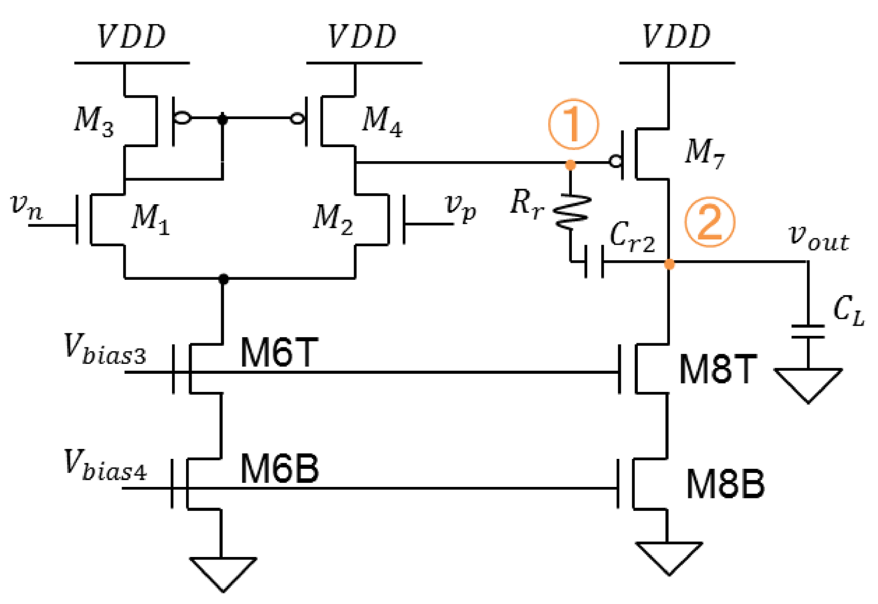
Based on Nyquist Criterion

Sketch chart of Nyquist plot

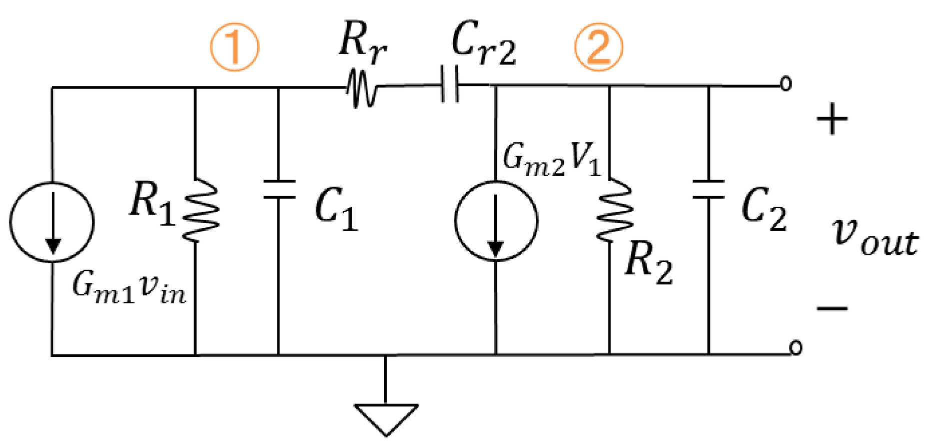


Ex.1
- $-\frac{a_1}{b_1} < K < \frac{a_1}{b_1}$ At condition: $a_1 b_1 > 0$
- $\frac{a_1}{b_1} < K < -\frac{a_1}{b_1}$ At condition: $a_1 b_1 < 0$

Ex.2
- $\frac{a_3 - a_1 a_2}{a_2 b - a_3} < K < \frac{a_3 - a_1 a_2}{a_3 - a_2 b}$ At condition: $(a_3 - a_1 a_2)(a_3 - a_2 b) > 0$
- $\frac{a_3 - a_1 a_2}{a_3 - a_2 b} < K < \frac{a_3 - a_1 a_2}{a_2 b - a_3}$ At condition: $(a_3 - a_1 a_2)(a_3 - a_2 b) < 0$



(a) Transistor level circuit



(b) Small-signal model

Ex.2 Two-stage amplifier with compensation network using a nulling resistor

Simulation Verification

Equivalence with Bode method

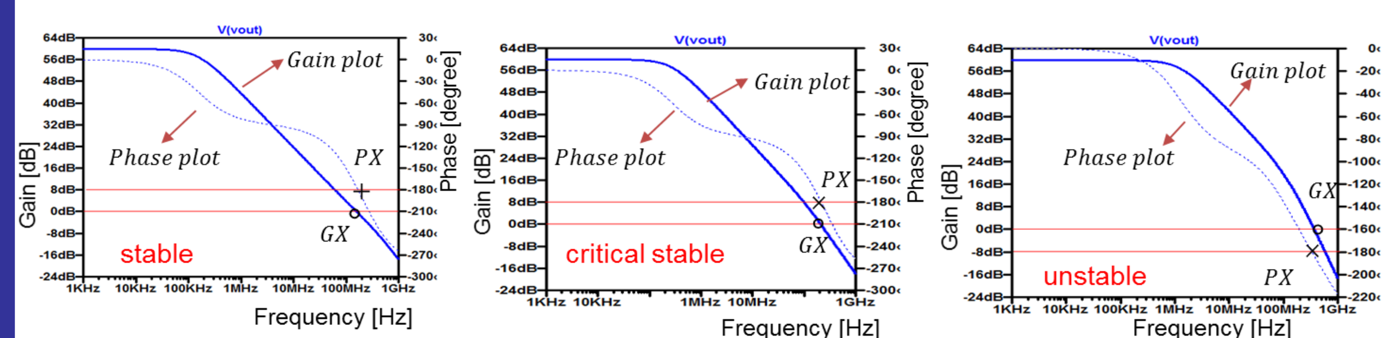
Closed-loop transfer function:

$$H(s) = \frac{A_0(1+b_1 s)}{1 + fA_0 + (a_1 + fA_0 b_1)s + a_2 s^2}$$

Explicit stability condition of parameters:

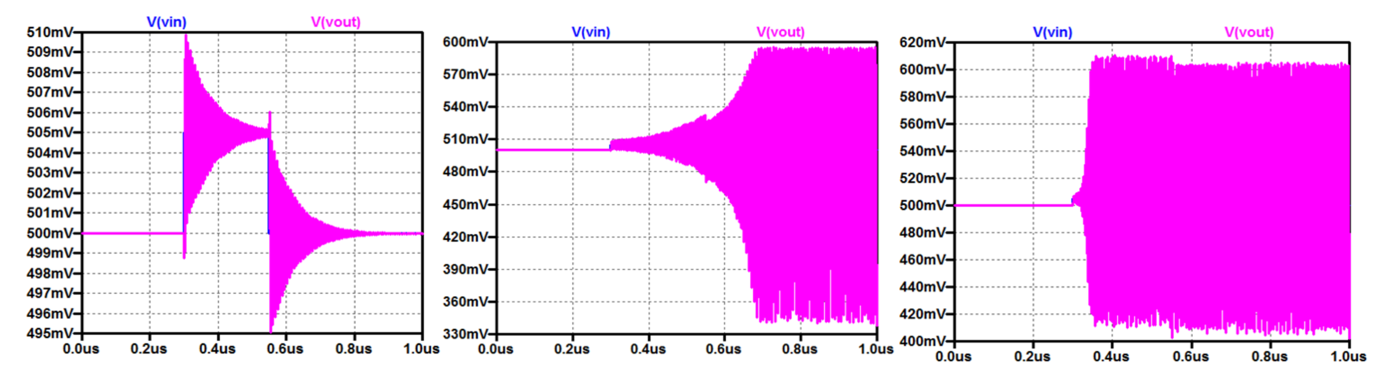
$$a_1 + fA_0 b_1 = R_1C_1 + R_2C_2 + (R_1+R_2)C_r + (G_{m2} - fG_{m1})R_1R_2C_r > 0$$

C_r >79.57fF



Case (1) C_r = 150fF Case (2) C_r =79.57fF Case (3) C_r = 10fF

Consistency of Bode Plots and R-H Results



Case (1) C_r = 150fF Case (2) C_r =79.57fF Case (3) C_r = 10fF

Consistency of Transient Analysis and R-H Results

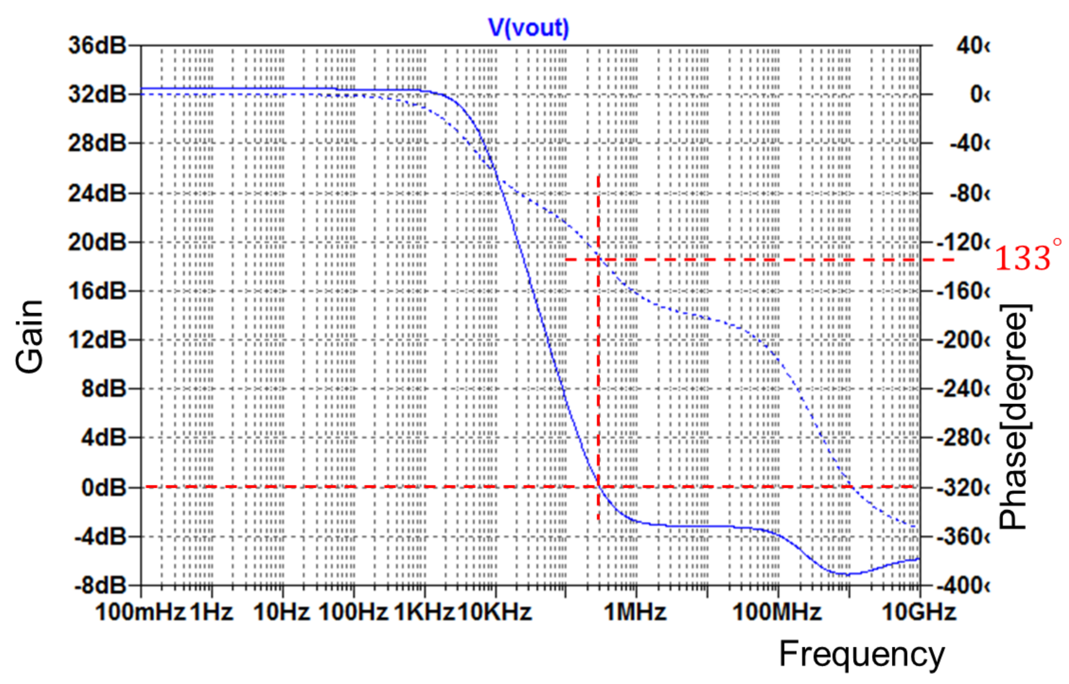
Practicability

For stable feedback system,

necessary PM value: 45^o or 60^o

$$C_{r1} = f_1(PM) = 6.343e^{-15}PM^3 - 2.091e^{-13}PM^2 + 2.493e^{-12}PM - 9.822e^{-12}$$

PM=45degree, $C_{r1} = 2.5694e^{-10}F = 0.25694\text{nF}$



phase: v(vout)=(-4.66844e-005dB,-133.013°) at 301437

$$Phase\ Margin = 180^\circ - 133^\circ = 47^\circ$$

experimental result is close to calculation result

Summary

Discussion

1. Depict small signal equivalent circuit
2. Derive open-loop transfer function
3. Derive closed-loop transfer function & obtain characteristics equation
4. Apply R-H stability criterion & obtain explicit stability condition
5. Use this stability condition for circuit parameters

Conclusion

- Equivalence between Nyquist and R-H stability criteria
- Equivalency of mathematical foundations
- R-H method, explicit circuit parameter conditions
- Consistency with Bode plot method, LTspice simulation

R-H method can be used with conventional Bode plot method.