**Operational Amplifier Stability Research**

**Research Objective**

- **For** Analysis and design of operational amplifier stability
- **Use** Routh-Hurwitz stability criterion
  
We can obtain explicit stability condition for circuit parameters (which can NOT be obtained only with Bode plot)

**Stability Criteria**

- **Electronic Circuit Design Field**
  - Bode plot (>90% frequently used)
  - Nyquist plot

- **Control Theory Field**
  - Bode plot
  - Nyquist plot
  - Nichols plot
  - Routh-Hurwitz stability criteria

*Very popular in control theory field but rarely seen in electronic circuit books/papers*

### Equivalence at Mathematical Foundations

#### Two Examples

**Ex.1** \( G(s) = \frac{K(1+b_1 s)}{1+a_1 s+a_2 s^2} \) One Zero, Two Poles

**Ex.2** \( G(s) = \frac{K(1+b_1 s)}{1+a_1 s+a_2 s^2+a_3 s^3} \) One Zero, Three Poles

#### Stability Condition

**Based on Routh-Hurwitz Criterion**

\[
K > \frac{a_2}{b_2} \\
\frac{a_1}{a_2} > \frac{b_1}{b_2} \\
\frac{a_2}{a_3} > \frac{b_2}{b_3} \\
\frac{a_3}{a_4} > \frac{b_3}{b_4} \\
\frac{a_4}{a_5} > \frac{b_4}{b_5} \\
\frac{a_5}{a_6} > \frac{b_5}{b_6} \\
\]

At condition:
- \( b_1 > 0 \)
- \( a_1b_1 > a_2 > 0 \)
- \( a_2b_2 > a_3 > 0 \)
- \( a_3b_3 > a_4 > 0 \)
- \( a_4b_4 > a_5 > 0 \)
- \( a_5b_5 > a_6 > 0 \)

**Based on Nyquist Criterion**

**Sketch chart of Nyquist plot**

At condition:
- \( a_1b_1 > a_2 > 0 \)
- \( a_2b_2 > a_3 > 0 \)
- \( a_3b_3 > a_4 > 0 \)
- \( a_4b_4 > a_5 > 0 \)
- \( a_5b_5 > a_6 > 0 \)

**Simulation Verification**

**Equivalence with Bode method**

Closed-loop transfer function:

\[
H(s) = \frac{A_0(1+b_1 s)}{1+a s+b s^2} + a_3 s^2
\]

Explicit stability condition of parameters:

\[
a_1 + f A_0 b_1 = R_1 C_1 + R_2 C_2 + (R_1 + R_2) C_2 + (f C_12 - f C_22) R_1 R_2 C_2 > 0
\]

**Consistency of Bode Plots and R-H Results**

**Practicability**

For stable feedback system, necessary PM value: 45° or 60°

\[
C_1 = \frac{f C_11}{f C_13} \pm 10^{-4} \text{PM} \approx 2.095 \times 10^{-12} \text{PM} + 2.493 \times 10^{-12} \text{PM} 
\]

\[
P_M = 45^\circ \text{degree}, C_1 = 2.569 \times 10^{-6} \text{F} = 0.25694 \text{nF}
\]

**Conclusion**

- Equivalence between Nyquist and R-H stability criteria
- Equivalency of mathematical foundations
- R-H method, explicit circuit parameter conditions
- Consistency with Bode plot method, LTspice simulation

R-H method can be used with conventional Bode plot method.