

# A Second-order DWA Algorithm for Multi-bit $\Delta \Sigma$ ADC/DAC

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GUNMA UNIVERSITY

# OUTLINE

**Introduction & Objective**

**What is DWA ?**

**Multi-bit  $\Delta\Sigma$ ADC/DAC**

**Proposed 2<sup>nd</sup>-order DWA Algorithm**

**Circuit Realization of 2<sup>nd</sup>-order DWA Algorithm**

**Simulation Verification**

**Summary**

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## Introduction & Objective

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# Background

The  $\Delta\Sigma$  ADCs/DACs are used for IoT applications → Sensor Face Interface  
Narrow Band Internet of Things

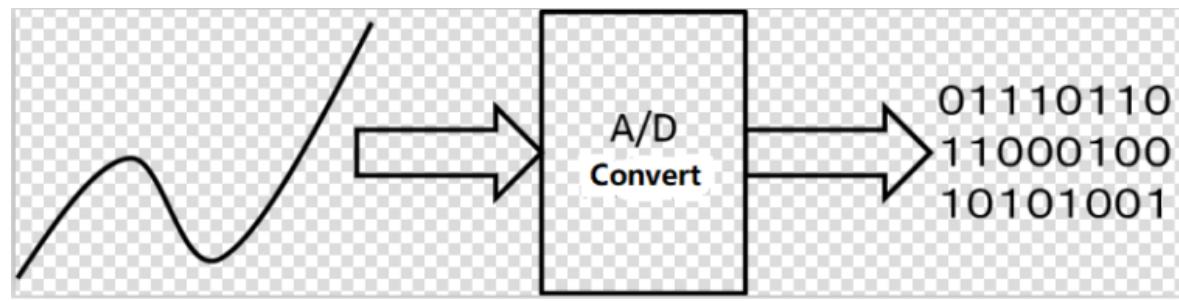
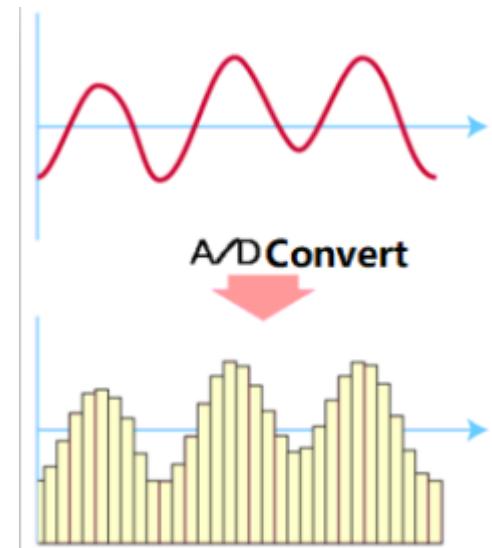


Mult bit configuration  
(a)small hardware  
(b)non-linearity



## DWA algorithm

- conventional:1st-order DWA
- our approach:2<sup>nd</sup>-order DWA



# What is DWA ?

ADC/DAC/TDC digital calibration techniques  
prevail in nano-CMOS era.

- Error Correction
  - No measurement of errors
  - Redundancy usage

DWA:  
Data Weighted Averaging

# DWA Techniques

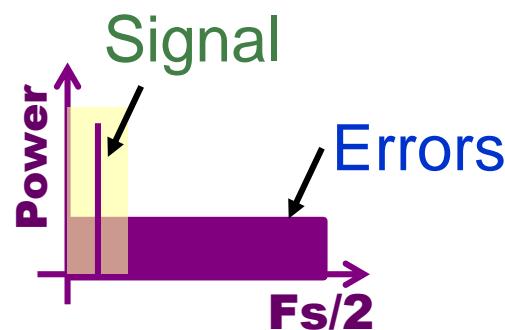
- Error Correction
  - No measurement of errors
  - Redundancy usage
    - Time averaging of errors
    - Spectrum shaping of errors

**DWA:**

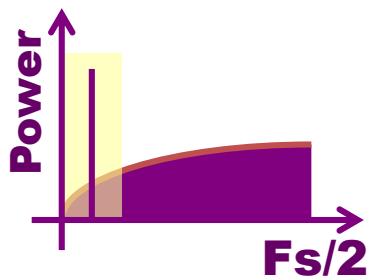
Data Weighted Averaging

**DEM:**

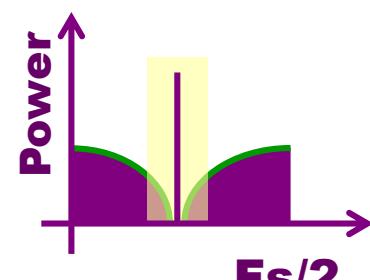
Dynamic Element Matching



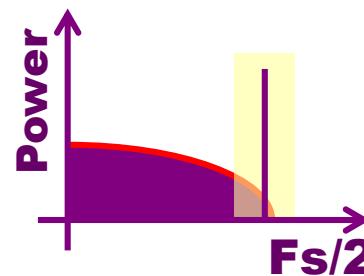
W/O DWA



LP DWA



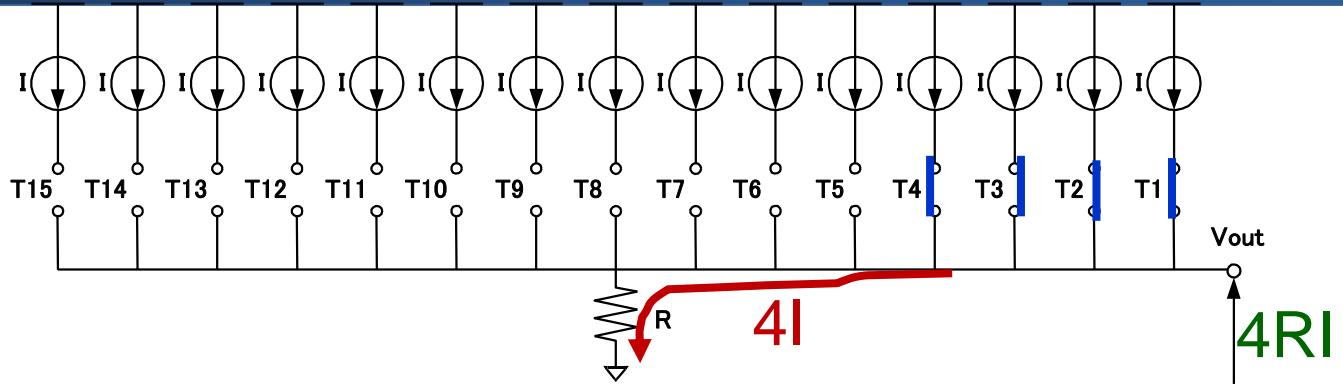
BP DWA



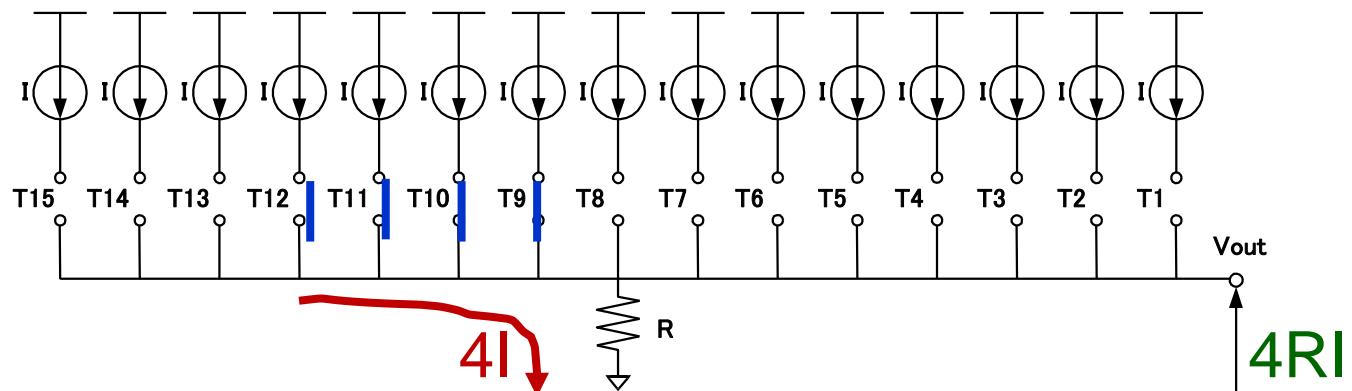
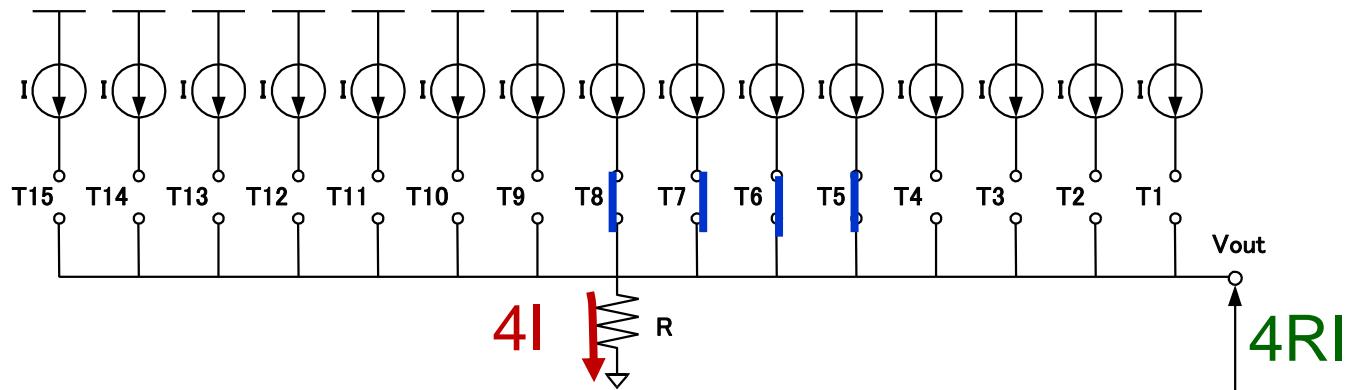
HP DWA

# Segment DAC with Redundancy

Digital  
input  
 $= 4$

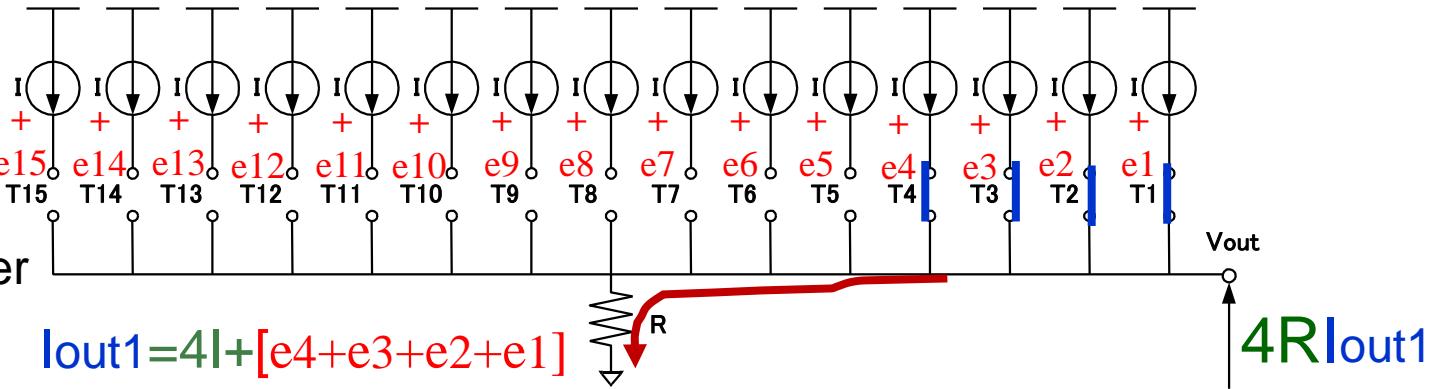


Multiple  
realization  
configurations



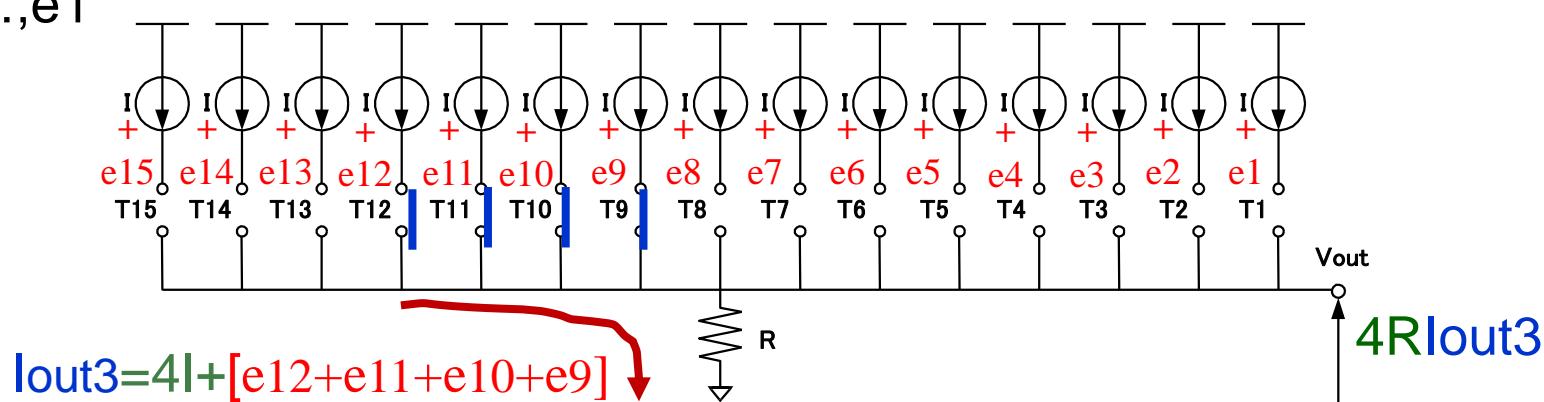
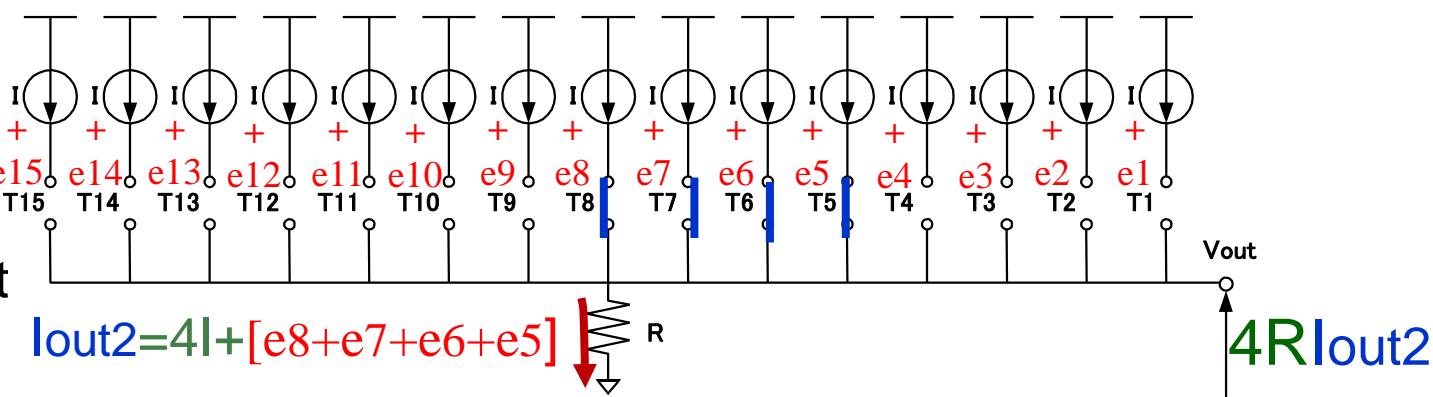
# Unit Cell Mismatches

Current  
mismatches  
 $e_{15}, e_{14}, \dots, e_1$   
spectrum shaping  
by cell selection order

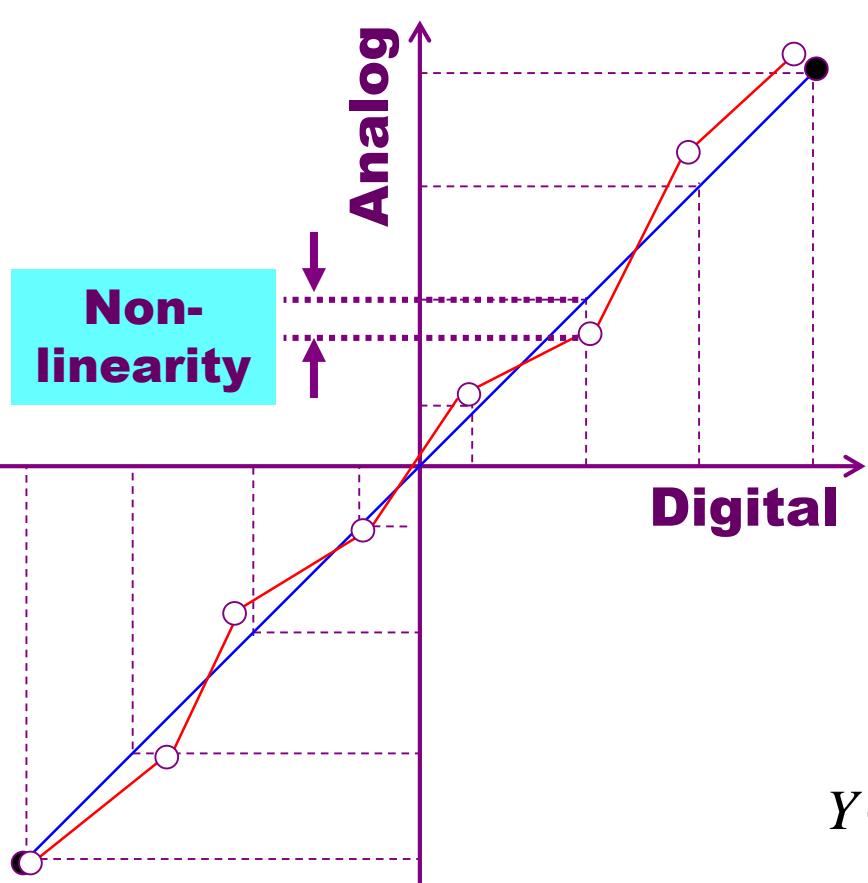


DWA algorithm

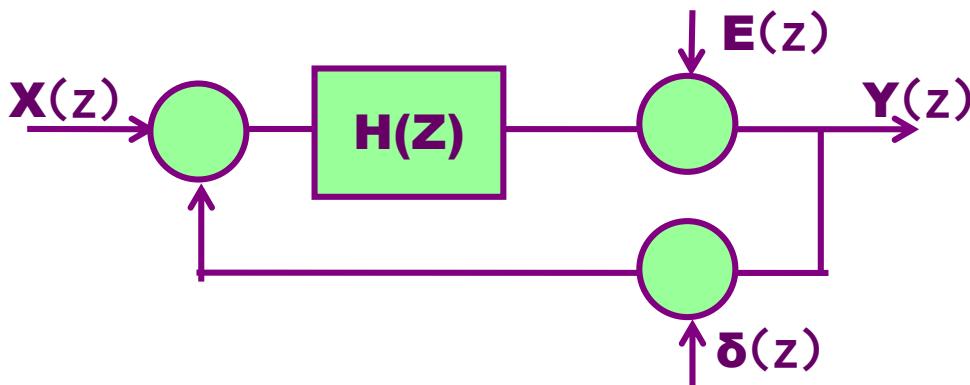
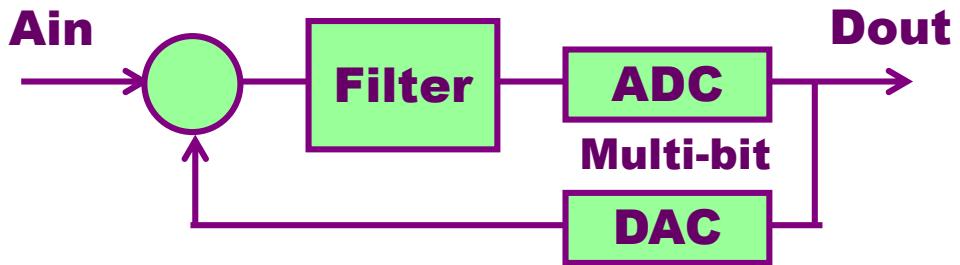
No measurement  
of  
 $e_{15}, e_{14}, \dots, e_1$



# Multi-bit DAC Nonlinearity



- :Single-bit Output
- :Multi-bit Output

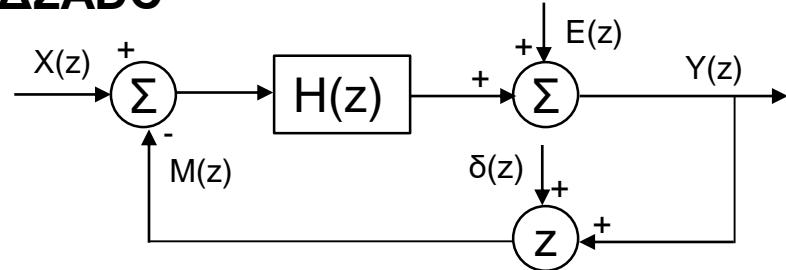


$$Y(z) = \frac{H(z)}{1 + H(z)} \{X(z) - \delta(z)\} + \frac{1}{1 + H(z)} E(z)$$

**δ(z) is NOT noise-shaped**

# Multi-bit $\Delta\Sigma$ ADC/DAC

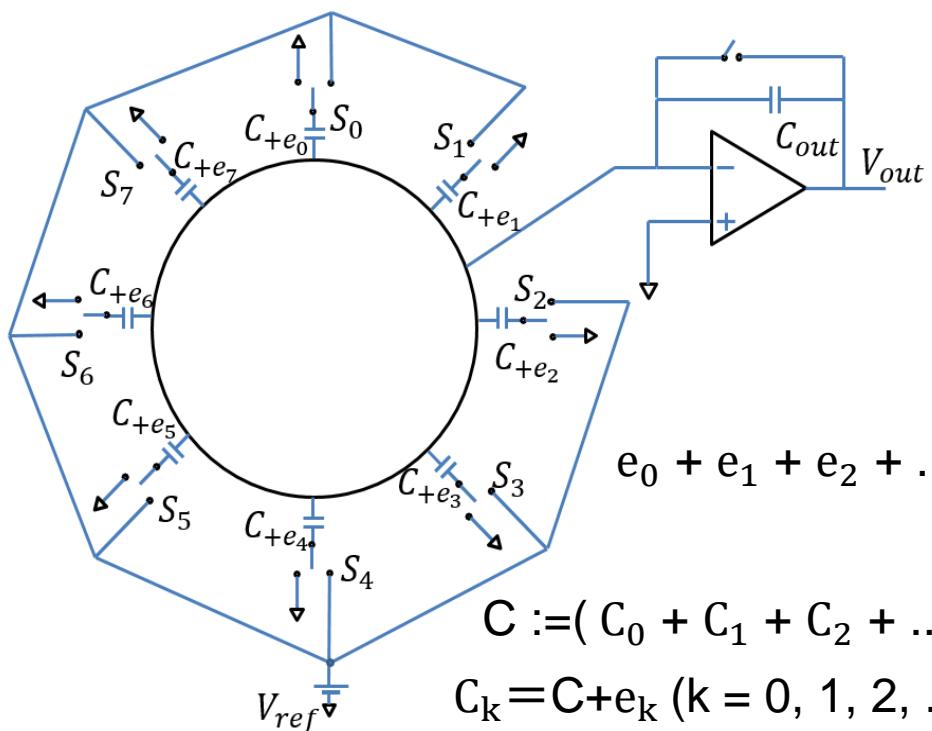
$\Delta\Sigma$ ADC



$$Y(z) = \frac{H(z)}{1+H(z)} [X(z) - \delta(z)] + \frac{1}{1+H(z)} E(z)$$

nonlinearity

a multi-bit(9bit) DAC with the switched capacitor



When a digital input is  $m$  and the DWA algorithm is not used, then the output voltage



$$V_{out} = -m \frac{C}{C_{out}} V_{ref} + \delta$$

$$e_0 + e_1 + e_2 + \dots + e_7 = 0$$

$$C := (C_0 + C_1 + C_2 + \dots + C_7)/8$$

$$C_k = C + e_k \quad (k = 0, 1, 2, \dots, 7).$$

Nonlinearity  $\delta$  of the DAC is expressed by

$$\delta = -\frac{e_0 + e_1 + e_2 + \dots + e_{m-1}}{C_{out}} V_{ref}$$

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**Proposed 2<sup>nd</sup>-order DWA Algorithm**

Circuit Realization of 2<sup>nd</sup>-order DWA Algorithm

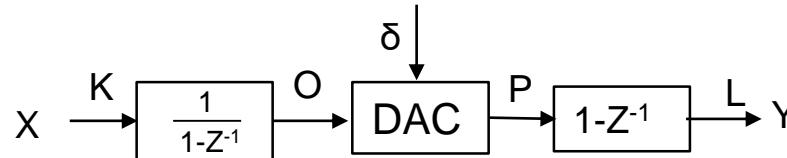
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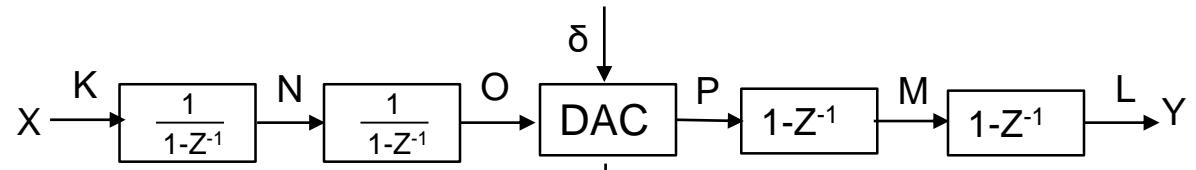
# Proposed 2<sup>nd</sup>-order DWA Algorithm

## 2<sup>nd</sup>-order Noise-Shaping of DAC Nonlinearity

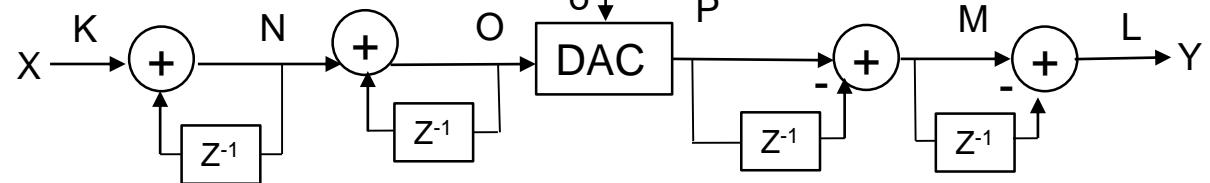
**(a) 1st-order DWA architecture**



**(b) 2<sup>nd</sup>-order DWA architecture**

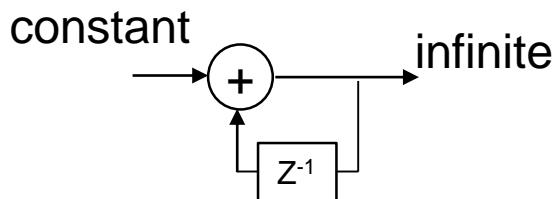


**(c) equivalent block diagram**



$$Y(z) = X(z) + (1 - z^{-1})^2 \delta(z)$$

**problem**



does not work well

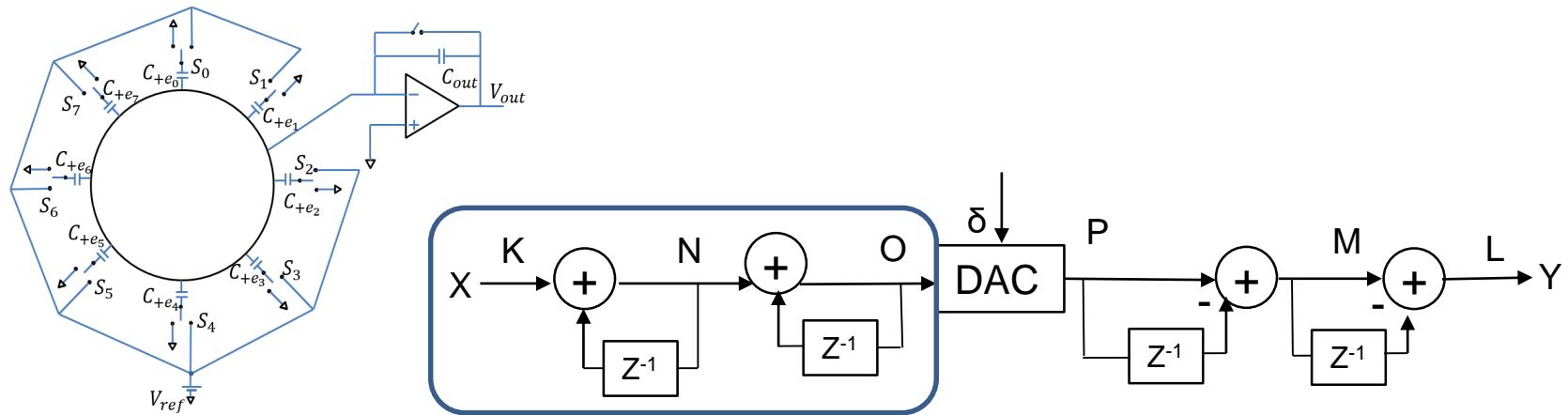


# Operation Proposed 2<sup>nd</sup>-order DWA Algorithm(1)

## Example

DAC input at time 0:  $D(0)=3$

Input	$C_0$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$
3	+	+	+					



# Operation Proposed 2<sup>nd</sup>-order DWA Algorithm(2.1)

## A. Two pointers Po+, Po-

Plus side:

Pointer:  $P_{0+}(n) = \text{mod}_8[D(n - 1) + \sum_{k=0}^{n-2} P_{0+}(k)]$

Signal start:  $S_+(n) = \text{mod}_8[P_{0+}(n) + 1]$

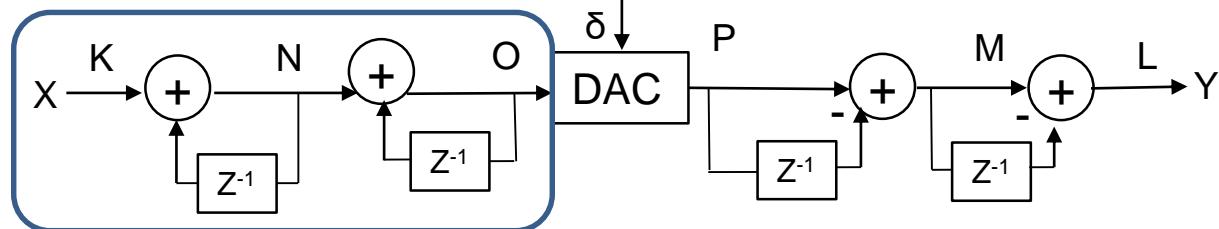
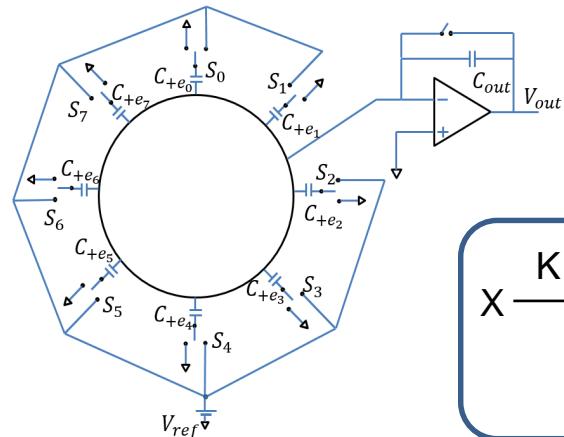
Number:  $A_+(n) = D(n) + \text{mod}_8[A_+(n - 1)]$

Minus side:

Pointer:  $P_{0-}(n) = P_{0+}(n - 1)$

Signal start:  $S_-(n) = \text{mod}_8[P_{0-}(n - 1) + 1]$

Number:  $A_-(n) = \text{mod}_8[A_-(n - 1)]$



## Example

DAC input at time 1:  $D(1)=4$

$P_{0+}(1)=3$

$S_+(1)=4$

$A_+(1)=7$

$P_{0-}(1)=0$

$S_-(1)=1$

$A_-(1)=3$

Input	$C_0$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$
4				+	+	+	+	+
	+	+						

$D(0)+D(1)=7$

# Operation Proposed 2<sup>nd</sup>-order DWA Algorithm(2.2)

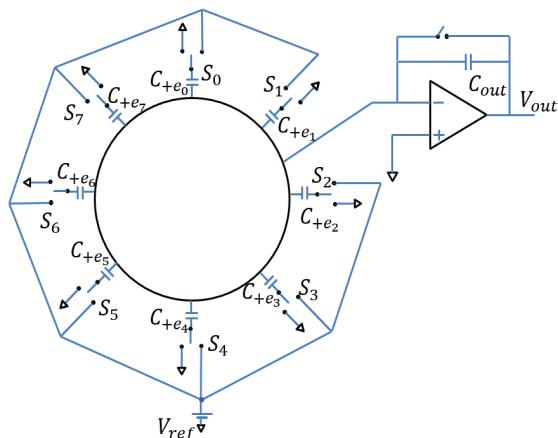
## B. Assignment of "Positive" and "Negative" to Cell

(i) Assign "positive : +"  
to the unit cells of

- $S_+(n)$
- $\text{mod}_8(S_+(n) + 1)$
- $\text{mod}_8(S_+(n) + 2)$
- ...
- $\text{mod}_8(S_+(n) + A_+ - 1)$

(ii) Assign " negative : -"  
to the unit cells of

- $S_-(n)$
- $\text{mod}_8(S_-(n) + 1)$ ,
- $\text{mod}_8(S_-(n) + 2)$ ,
- ....
- $\text{mod}_8(S_-(n) + A_- - 1)$

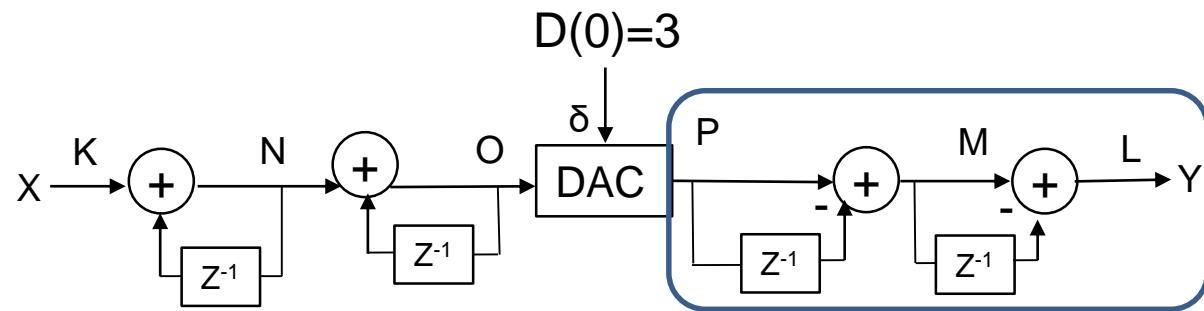


## Example

DAC input at time 1:  $D(1)=4$

$$\begin{array}{ll} P_{0+}(1)=3 & P_{0-}(1)=0 \\ S_+(1)=4 & S_-(1)=1 \\ A_+(1)=7 & A_-(1)=3 \end{array}$$

input	$C_0$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$
4				+	+	+	+	+
	+	+						
	-	-	-					



# Operation Proposed 2<sup>nd</sup>-order DWA Algorithm(2.3)

## C. Decision of Unit Cell Value: -1, 0, 1 or 2

(i) Condition for unit cell value of -1:

"+" is k+2 times assigned" and "–" is k+3 times assigned."

(ii) Condition for unit cell value of 0:

"+" is k+2 times assigned" and "–" is k+2 times assigned."

(iii) Condition for unit cell value of +1:

"+" is k+2 times assigned" and "–" is k+1 times assigned."

(iv) Condition for unit cell value of 2:

"+" is k+2 times assigned" and "–" is k times assigned."

## Example

DAC input at time 1:  $D(1)=4$

$$P_{0+}(1)=3$$

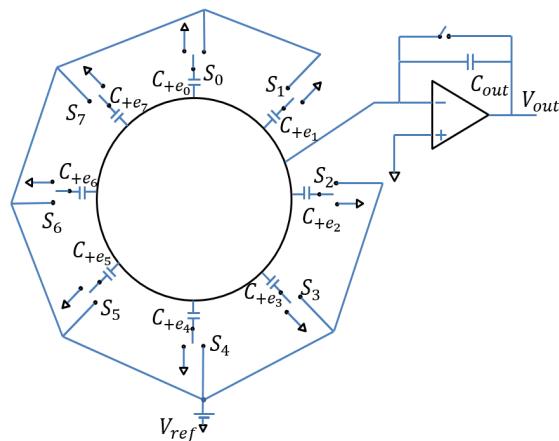
$$S_+(1)=4$$

$$A_+(1)=7$$

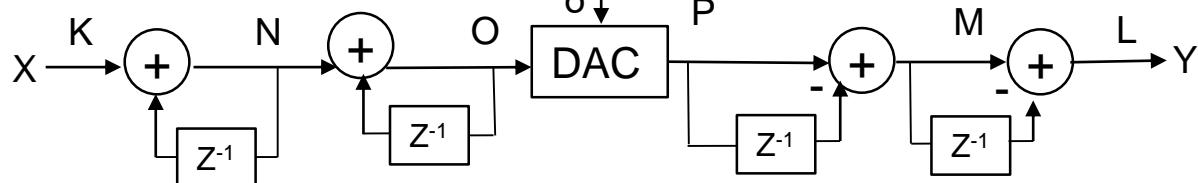
$$P_{0-}(1)=0$$

$$S_-(1)=1$$

$$A_-(1)=3$$



input	$C_0$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$
4				+	+	+	+	+
	+	+		-				
result			-		+	+	+	+



# Operation Proposed 2<sup>nd</sup>-order DWA Algorithm(3.1)

## A. Two pointers Po+, Po-

Plus side:

Pointer:  $P_{0+}(n) = \text{mod}_8[D(n - 1) + \sum_{k=0}^{n-2} P_{0+}(k)]$

Signal start:  $S_+(n) = \text{mod}_8[P_{0+}(n) + 1]$

Number:  $A_+(n) = D(n) + \text{mod}_8[A_+(n - 1)]$

Minus side:

Pointer:  $P_{0-}(n) = P_{0+}(n - 1)$

Signal start:  $S_-(n) = \text{mod}_8[P_{0-}(n - 1) + 1]$

Number:  $A_-(n) = \text{mod}_8[A_-(n - 1)]$

## Example

DAC input at time 2:  $D(2)=2$

$P_{0+}(2)=4$

$S_+(2)=5$

$A_+(2)=9$

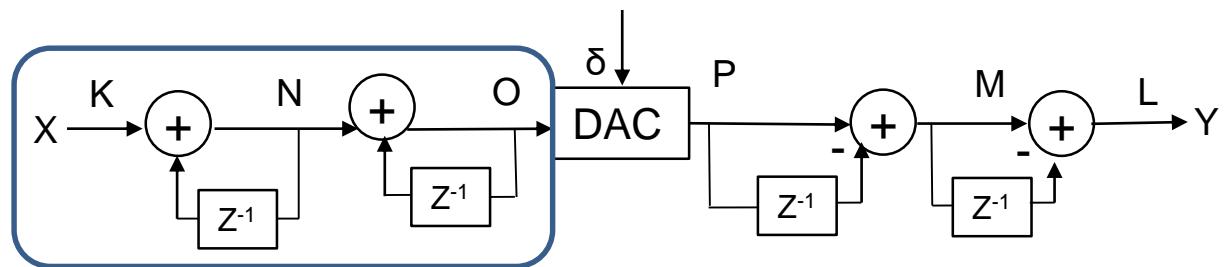
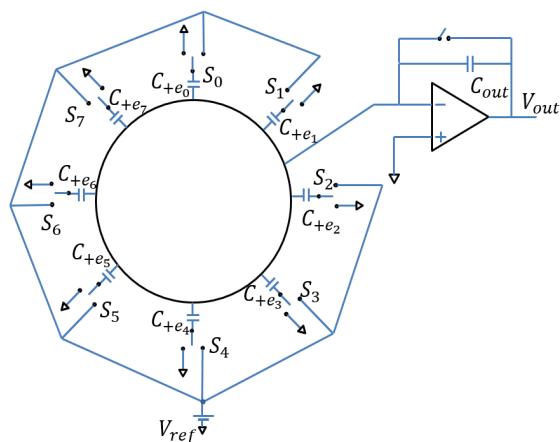
$P_{0-}(2)=3$

$S_-(2)=4$

$A_-(2)=7$

Input	$C_0$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$
2			+	+	+	+	+	+
	+	+	+					

$D(0)+D(1)+D(2)=9$



# Operation Proposed 2<sup>nd</sup>-order DWA Algorithm(3.2)

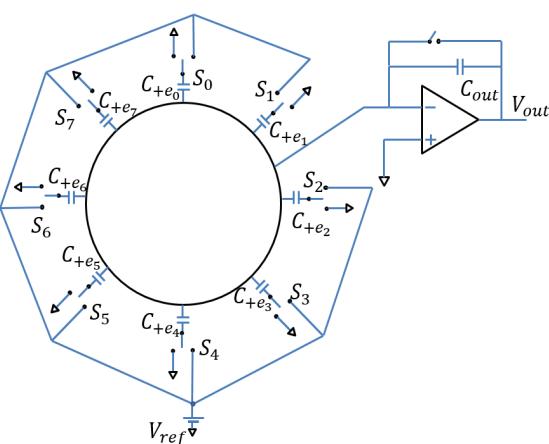
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(i) Assign "positive : +"  
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- $\text{mod}_8(S_+(n) + 1)$
- $\text{mod}_8(S_+(n) + 2)$
- ...
- $\text{mod}_8(S_+(n) + A_+ - 1)$

(ii) Assign " negative : -"  
to the unit cells of

- $S_-(n)$
- $\text{mod}_8(S_-(n) + 1),$
- $\text{mod}_8(S_-(n) + 2),$
- ....
- $\text{mod}_8(S_-(n) + A_- - 1)$



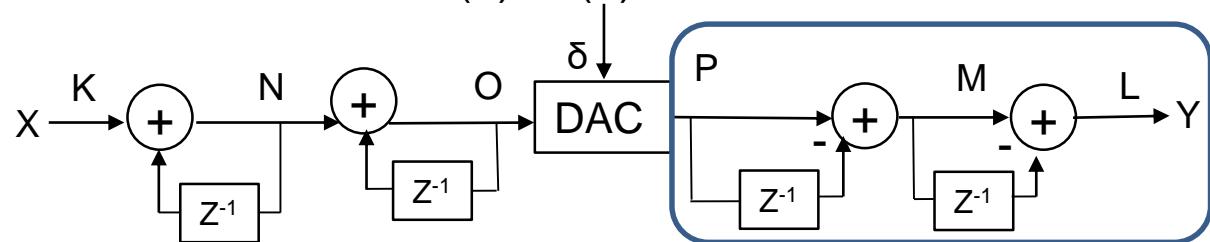
## Example

DAC input at time 2:  $D(2)=2$

$$\begin{array}{ll} P_{0+}(2)=4 & P_{0-}(2)=3 \\ S_+(2)=5 & S_-(2)=4 \\ A_+(2)=9 & A_-(2)=7 \end{array}$$

Input	$C_0$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$
2			+	+	+	+	+	+
	+	+	+					
					-	-	-	-
	-	-						

$$D(0)+D(1)=7$$



# Operation Proposed 2<sup>nd</sup>-order DWA Algorithm(3.3)

## C. Decision of Unit Cell Value: -1, 0, 1 or 2

(i) Condition for unit cell value of -1:

"+ is k+2 times assigned" and "- is k+3 times assigned."

(ii) Condition for unit cell value of 0:

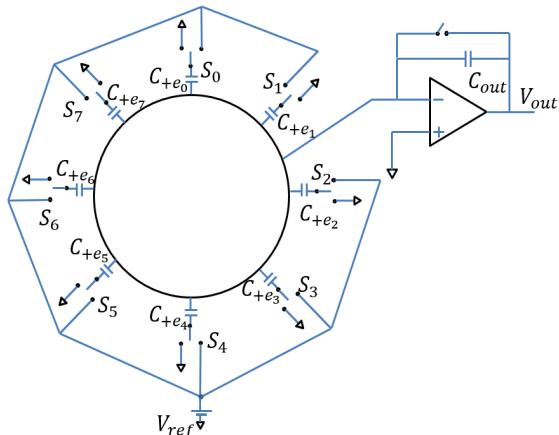
"+ is k+2 times assigned" and "- is k+2 times assigned."

(iii) Condition for unit cell value of +1:

"+ is k+2 times assigned" and "- is k+1 times assigned."

(iv) Condition for unit cell value of 2:

"+ is k+2 times assigned" and "- is k times assigned."



## Example

DAC input at time 2:  $D(2)=2$

$$P_{0+}(2)=4$$

$$S_+(2)=5$$

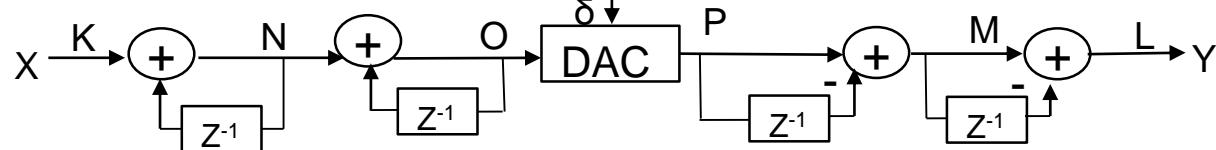
$$A_+(2)=1$$

$$P_{0-}(2)=3$$

$$S_-(2)=4$$

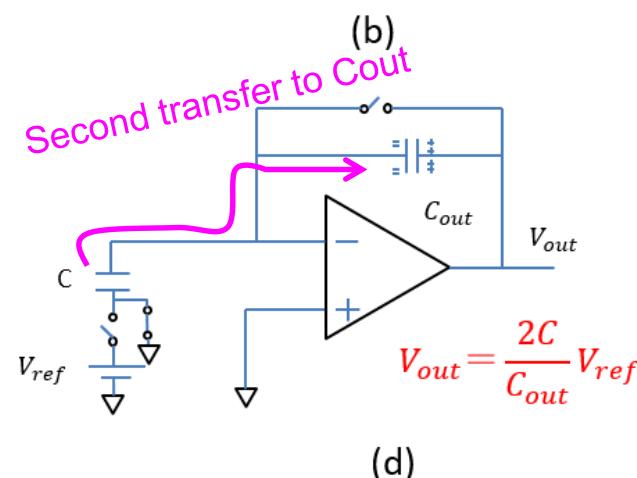
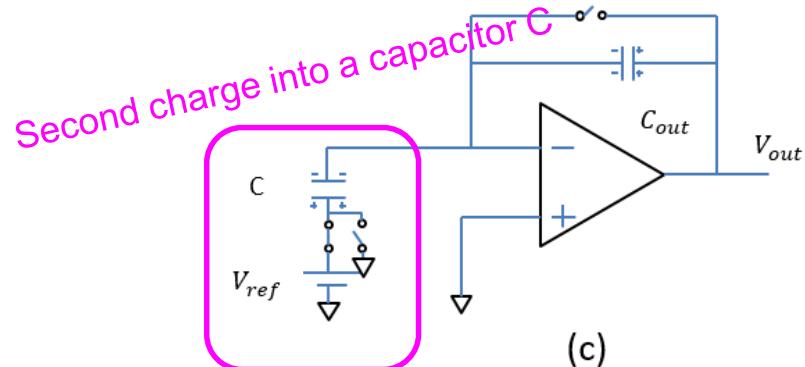
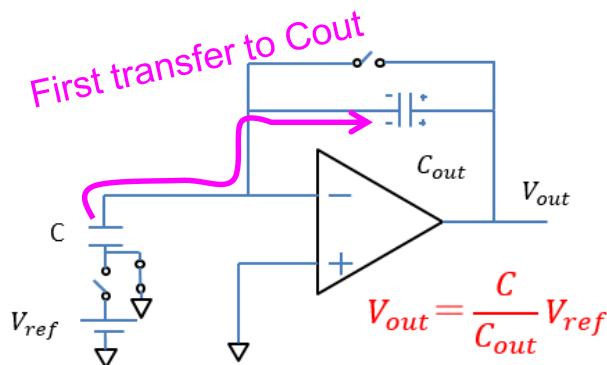
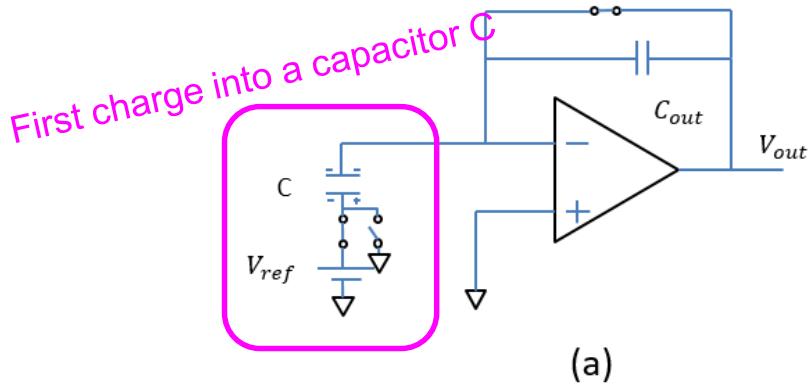
$$A_-(2)=7$$

Input	$C_0$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$
2			+	+	+	+	+	+
	+	+	+	-	-	-	-	-
	-	-						
result			++					

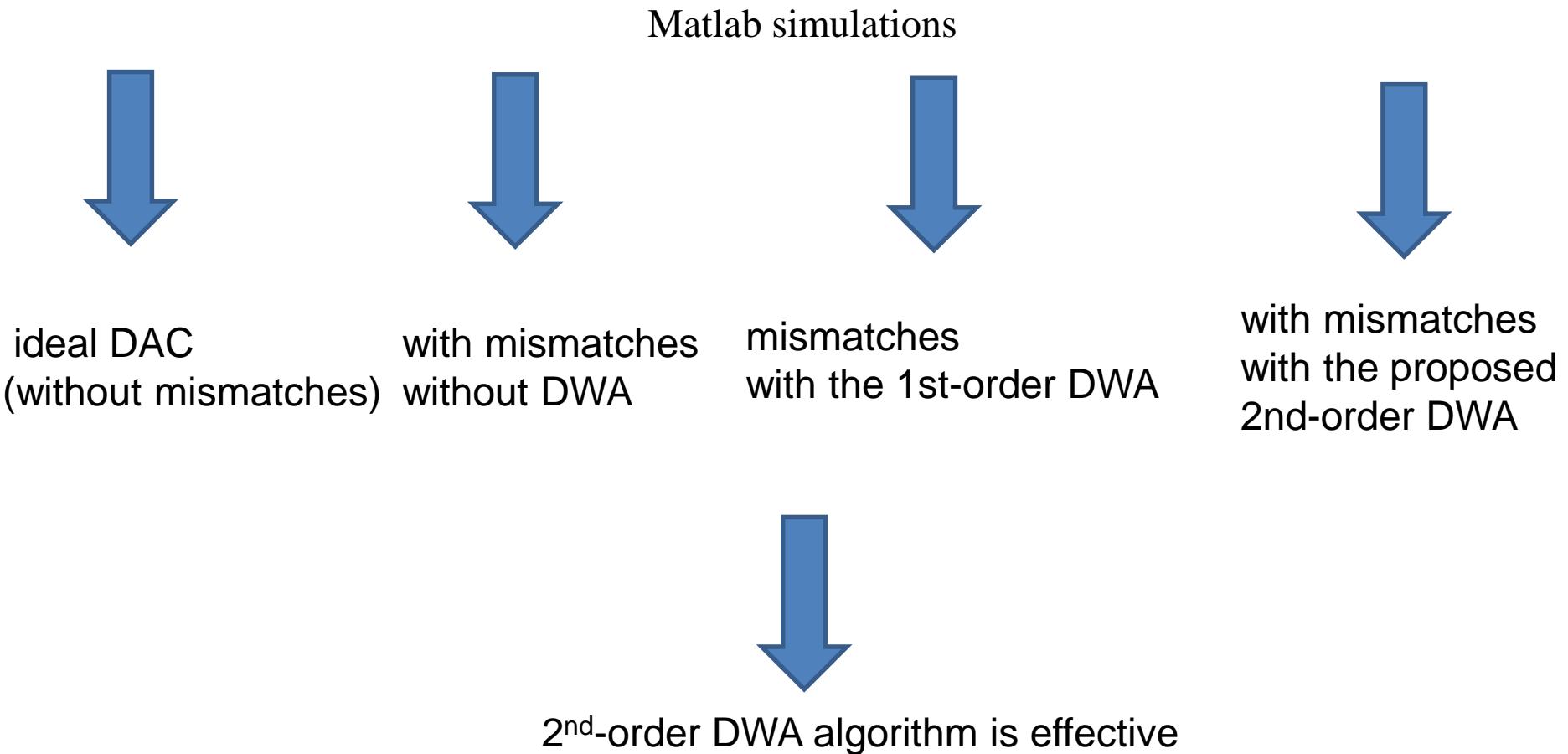


# Circuit Realization of 2<sup>nd</sup>-order DWA Algorithm

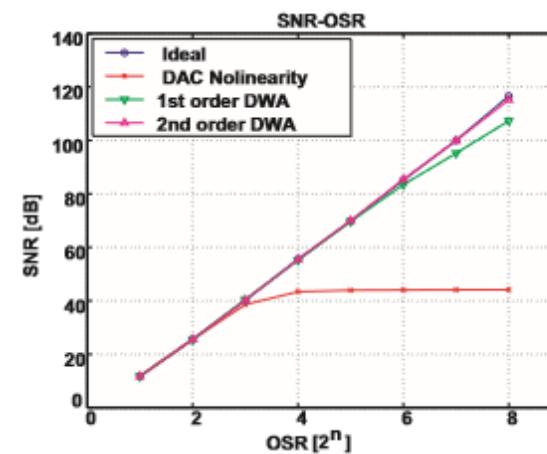
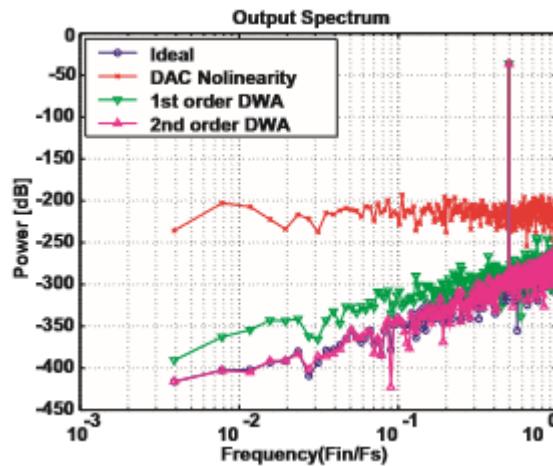
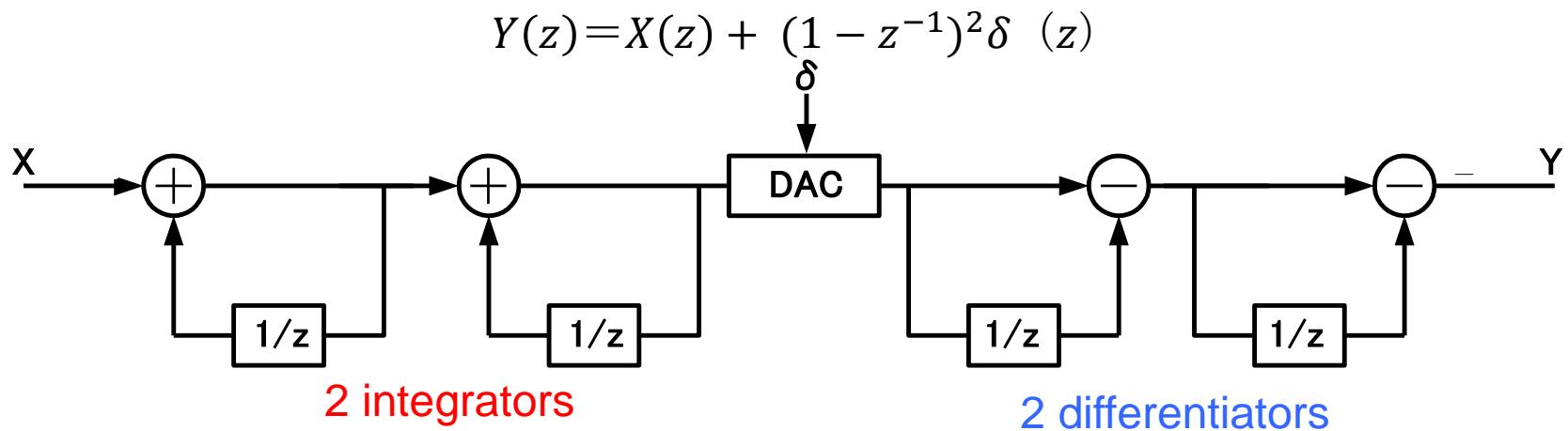
DWA algorithm needs twice unit cell output and minus unit cell output realized with 2 clock operation



# Simulation Verification



# 2<sup>nd</sup>-order DWA



- 2<sup>nd</sup>-order DWA is more effective
- But its circuit/operation become complicated

# Summary

- a second-order DWA algorithm
- reduce mismatches with 2nd-order noise-shaping
- requires a two clock operation with switched capacitor circuit
- make the circuit operation slow, but easy to implement

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**thank you for your listening**