

# DWA Algorithm for Band-Pass $\Delta\Sigma$ DAC with Ternary Unit Cells

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# Outline

- ◆ Research Background
- ◆  $\Delta\Sigma$  DA Converter
  - DWA\* Algorithm (\* Data-Weighted Averaging)
- ◆ Simulation Verification
  - Binary, Ternary DWA Overview
  - $\Delta\Sigma$  DA Converter : HP type
  - $\Delta\Sigma$  DA Converter : BP type
- ◆ Conclusion

# Outline

## ◆ Research Background

## ◆ $\Delta\Sigma$ DA Converter

- DWA\* Algorithm (\* Data-Weighted Averaging)

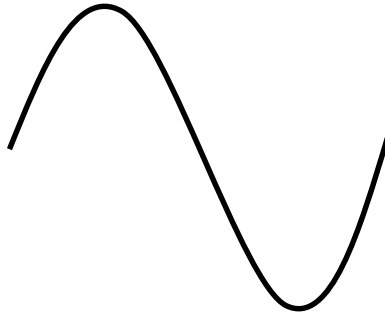
## ◆ Simulation Verification

- Binary, Ternary DWA Overview
- $\Delta\Sigma$  DA Converter : HP type
- $\Delta\Sigma$  DA Converter : BP type

## ◆ Conclusion

# Research Background

**Analog** signal



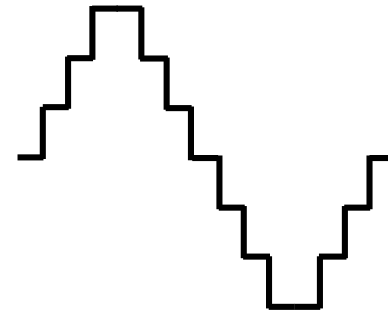
Continuous signal

Physical quantity existing in nature

ADC

DAC

**Digital** signal

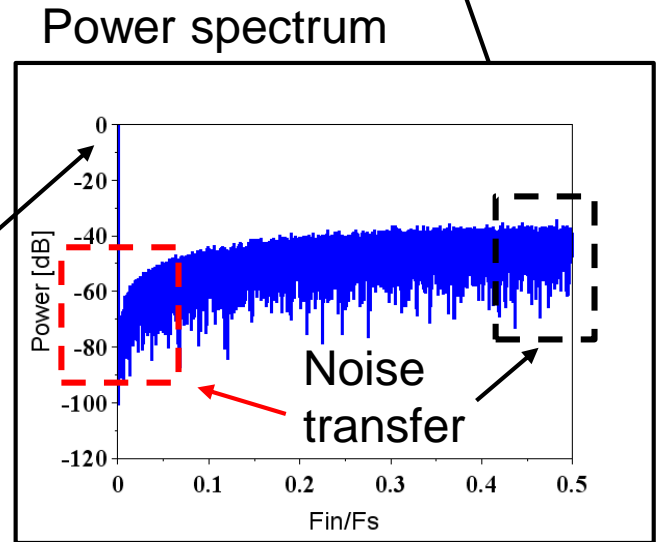
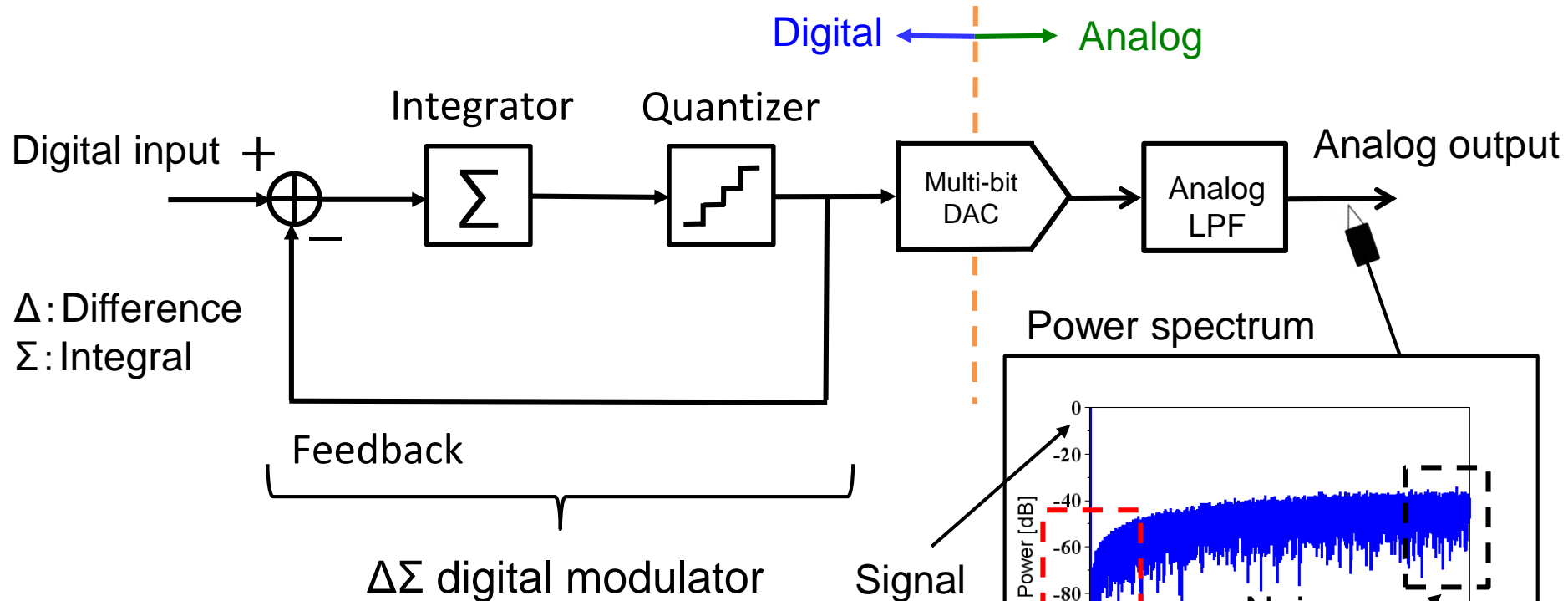


Discrete signal

Binary number

- $\Delta\Sigma$  Digital-to-Analog Converter ( $\Delta\Sigma$ DAC) → **Required**
  - Mostly digital circuit
  - High-resolution , High-linearity
  - DC signal , low frequency signal generation

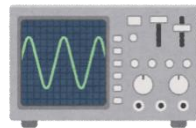
# $\Delta\Sigma$ Digital to Analog Converter (Low Pass)



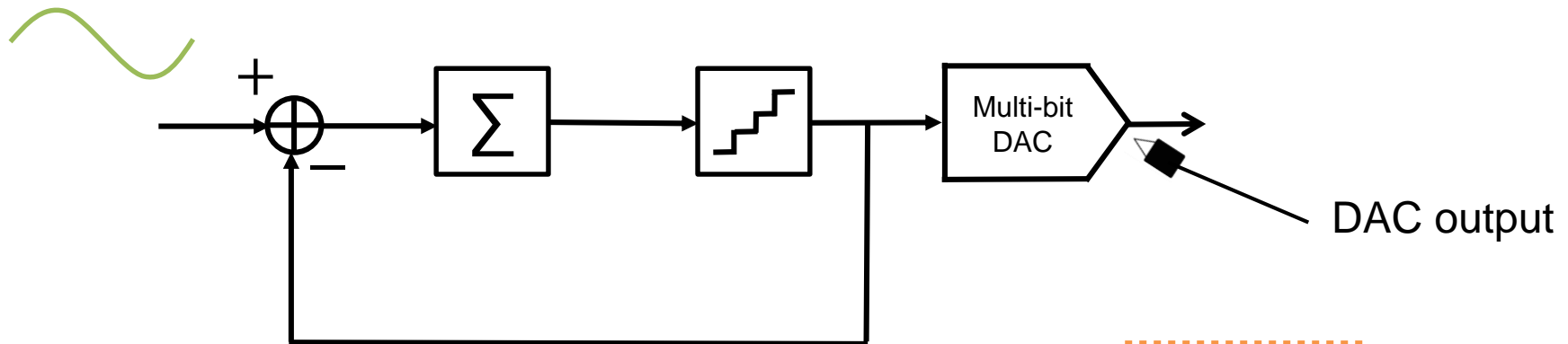
Noise  
 ↓  
 decrease at low frequency

## <Usage>

- Electric measurement
- Audio system

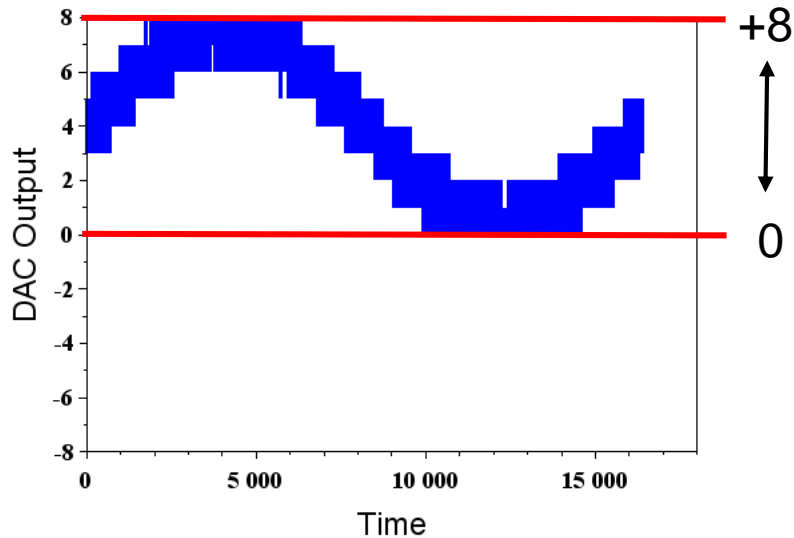


# Introduction to Ternary

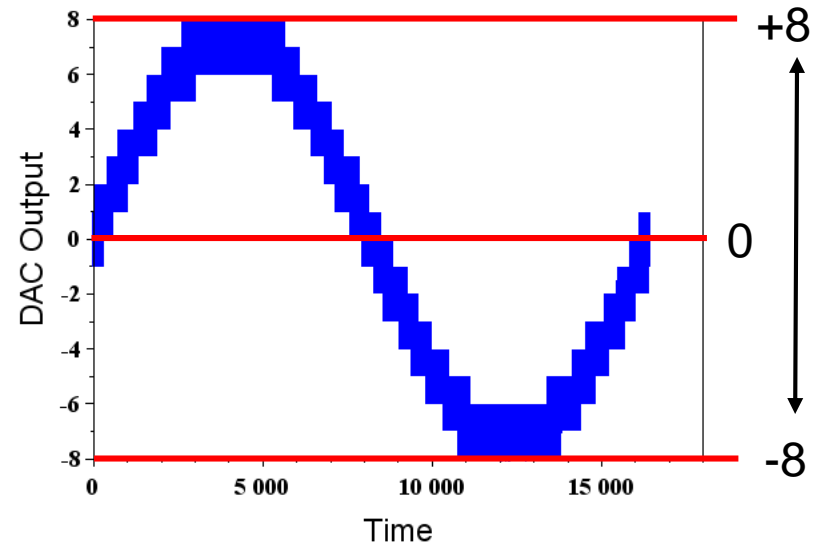


Forces

**Binary**  $\Rightarrow$  + and 0 value



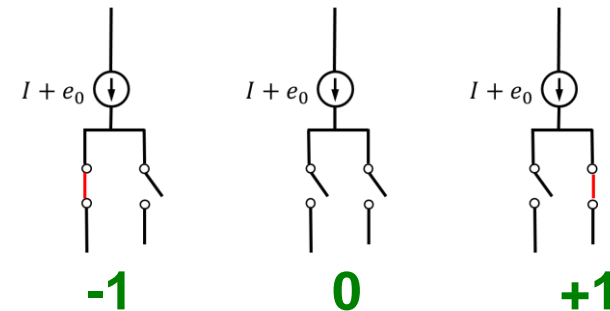
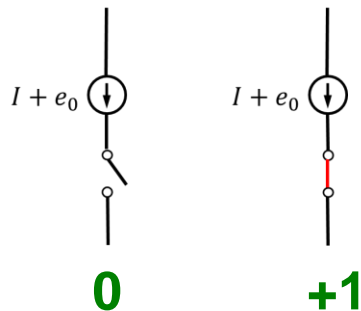
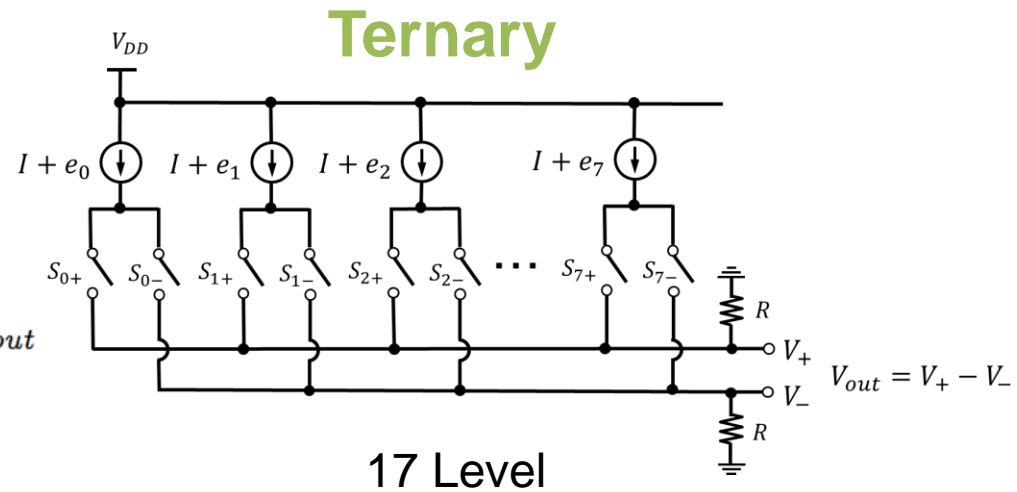
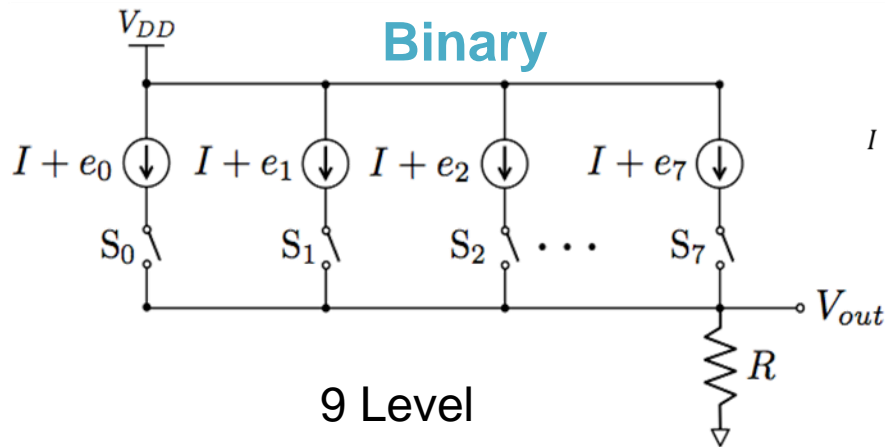
**Ternary**  $\Rightarrow$  +, - and 0 value



# Reasons for Ternary Usage

## ● Comparison

8 current sources



Binary unit cell → 1-bit

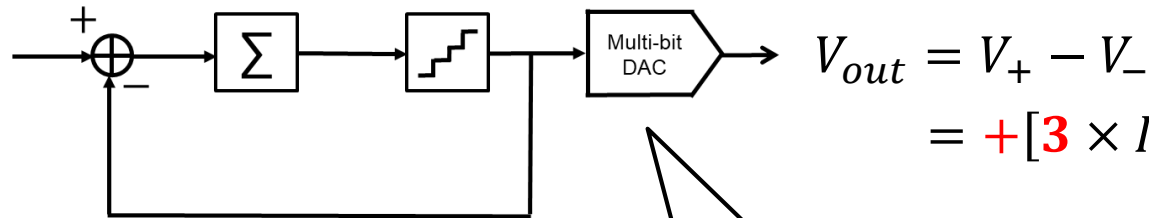
Ternary unit cell → 1.5-bit

Ternary →

Higher resolution for given current sources  
Smaller number of current sources for given resolution

# Multi-bit DAC Operation of Ternary (1/3)

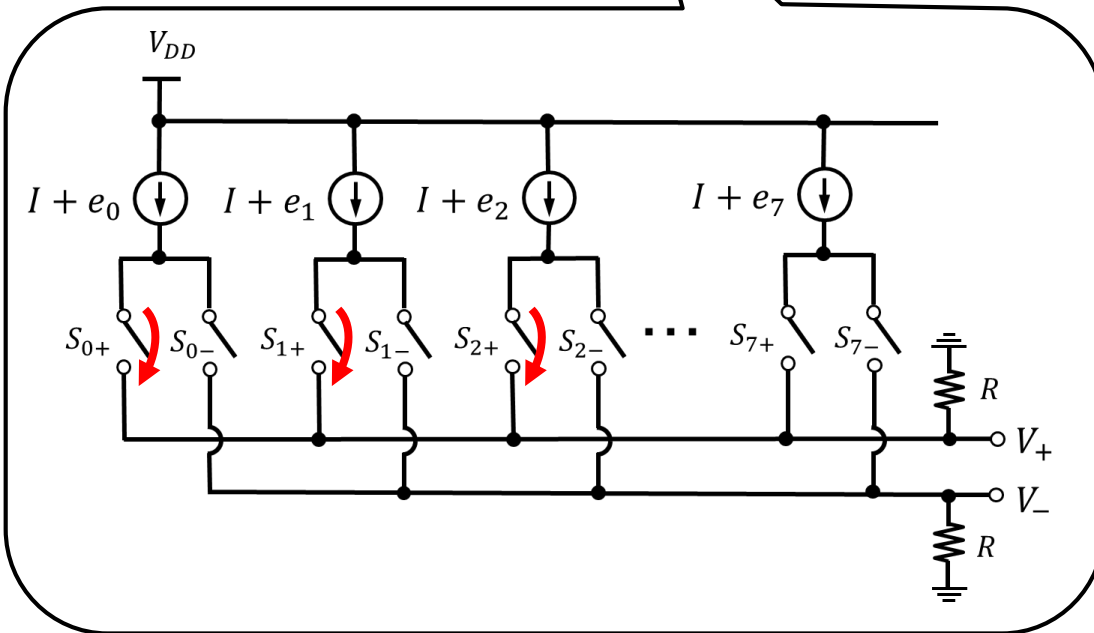
DAC input = **+3**



$$V_{out} = V_+ - V_-$$

$$= +[3 \times I + e_0 + e_1 + e_2] R$$

**Positive voltage**

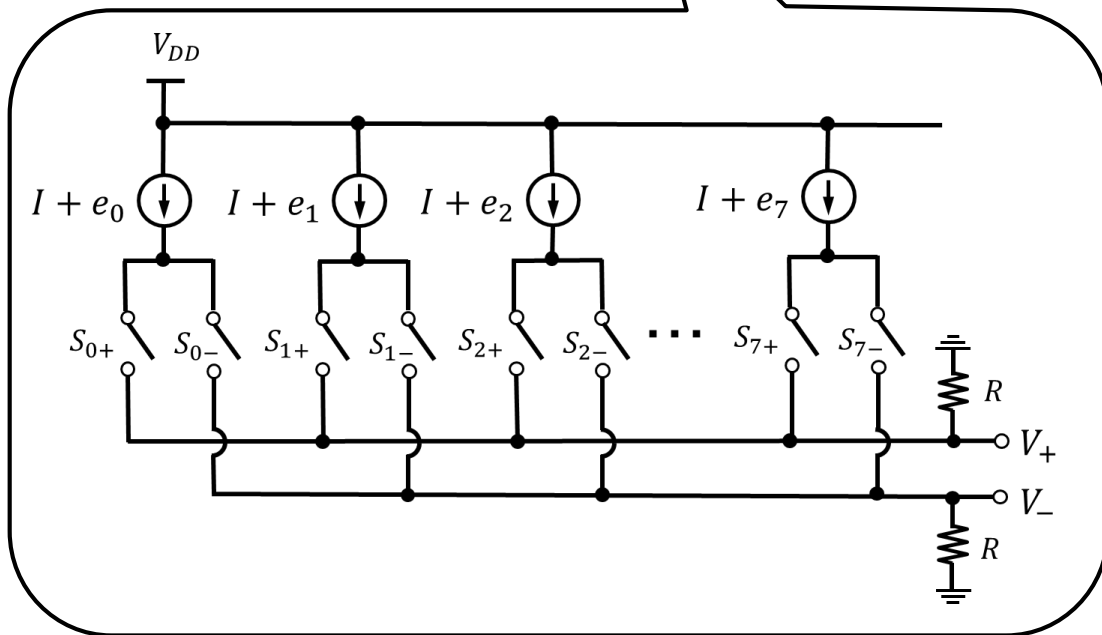
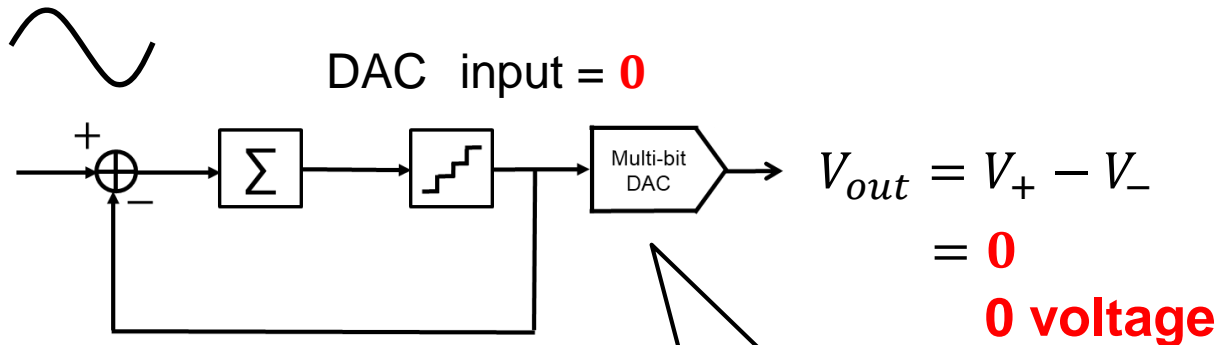


Digital	$V_{out}$
<b>+3</b>	<b><math>+[3I + e_0 + e_1 + e_2] R</math></b>
0	
-2	

current  $I_k = I + e_i$   
 $e_i$  : Variation in current cell



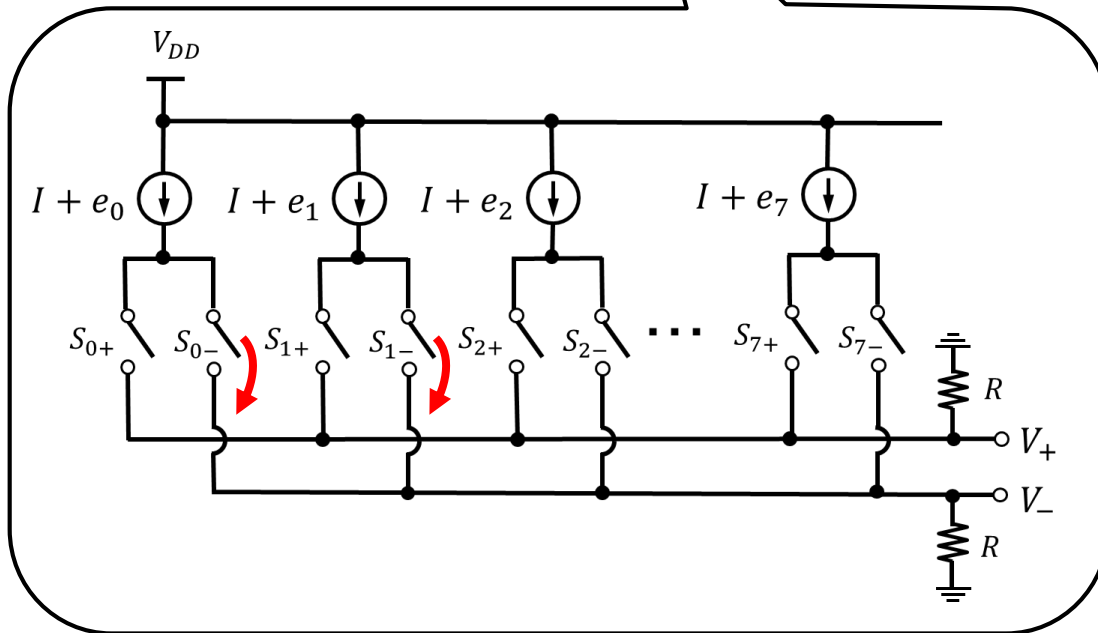
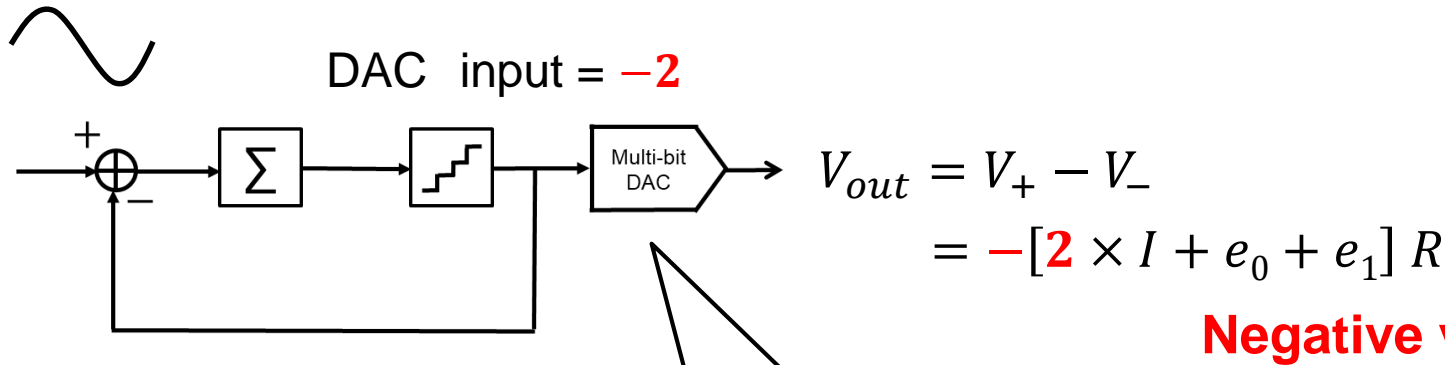
# Multi-bit DAC Operation of Ternary (2/3)



Digital	$V_{out}$
+3	$+ [3I + e_0 + e_1 + e_2] R$
<b>0</b>	<b>0</b>
-2	

$$\left[ \begin{array}{l} \text{current } I_k = I + e_i \\ e_i : \text{Variation in current cell} \end{array} \right]$$

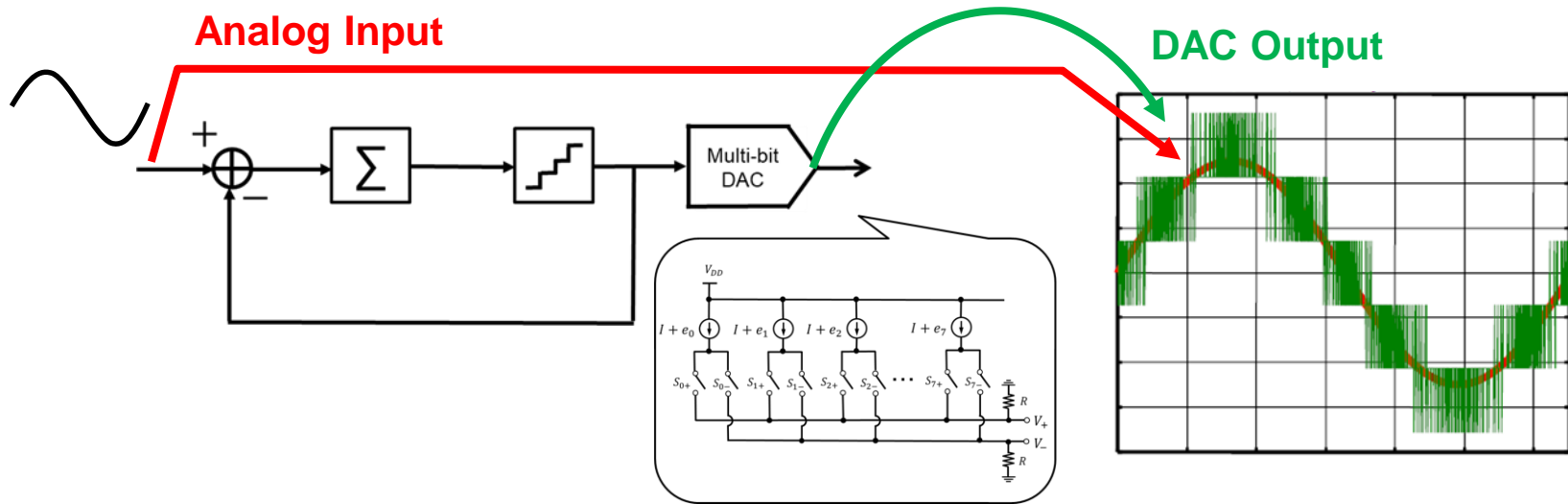
# Multi-bit DAC Operation of Ternary (3/3)



Digital	$V_{out}$
+3	$+[3I + e_0 + e_1 + e_2] R$
0	0
<b>-2</b>	<b><math>-[2 \times I + e_0 + e_1] R</math></b>

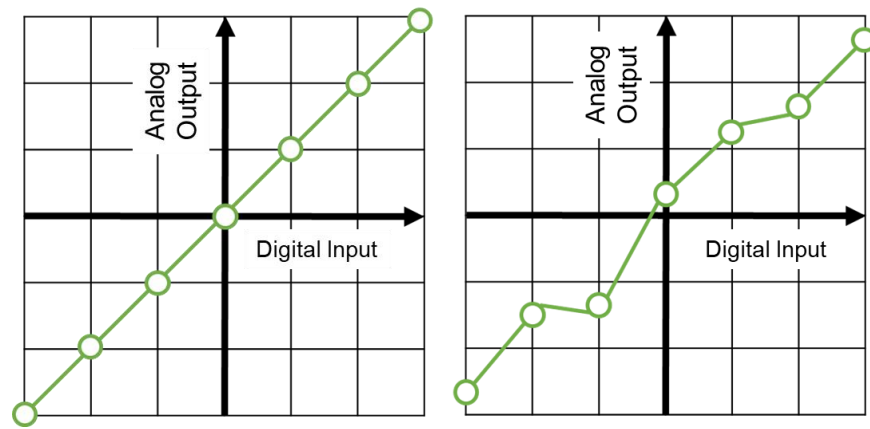
$$\left[ \begin{array}{l} \text{current } I_k = I + e_i \\ e_i : \text{Variation in current cell} \end{array} \right]$$

# Nonlinearity Problem of Multi-bit $\Delta\Sigma$ DAC



Ideal DAC

Real DAC



○ Multi-bit Output

## Merit

- Quantization error  $\Rightarrow$  Decrease
- Performance of following analog filter  $\Rightarrow$  Ease

## Nonlinearity Problem

- Characteristics mismatches among multiple unit cell  $\Rightarrow$  **Nonlinearity problem**

# Multi-bit DAC of Ternary (1/2)

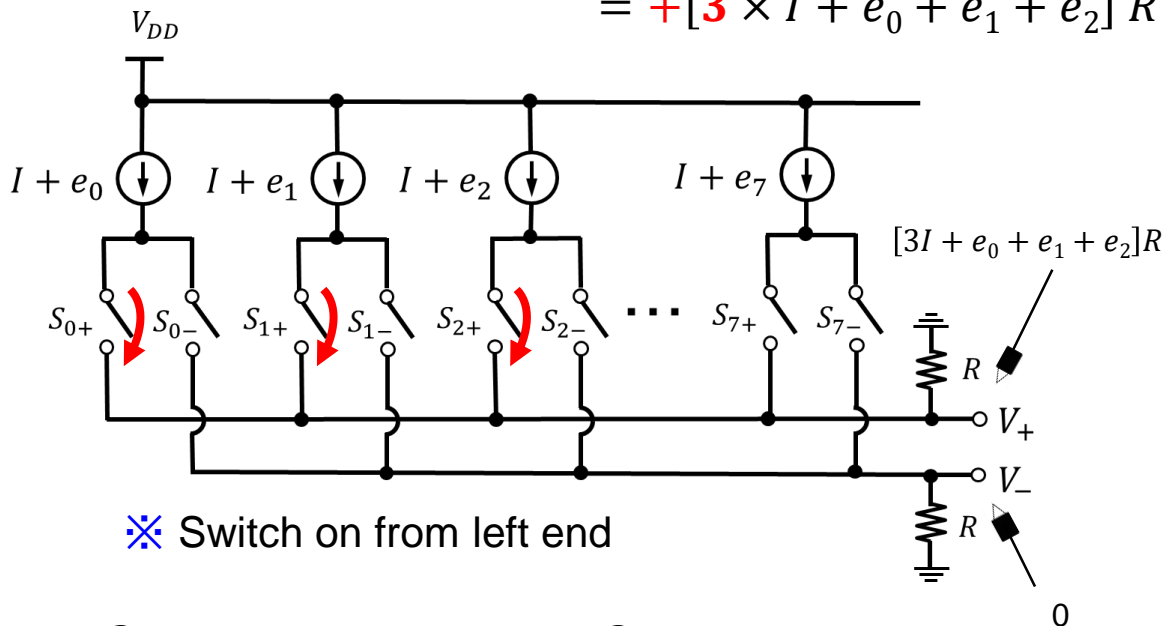
## ◆ Segment type DAC of ternary

$$\text{DAC input} = +3 \quad V_{out} = V_+ - V_-$$

$$= +[3 \times I + e_0 + e_1 + e_2] R$$



Positive Voltage  
Negative Voltage

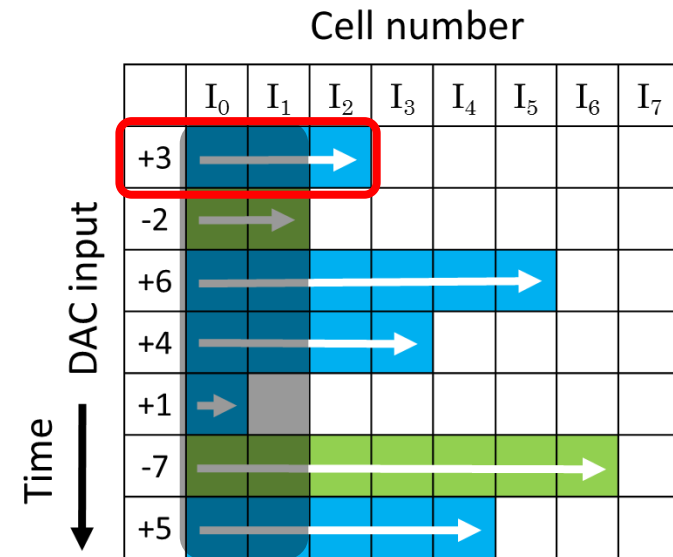


Current source in DAC

✓ Ideal  $\Rightarrow$  All equal

✓ Real  $\Rightarrow$  Process variation on manufacturing

$$\left[ \text{current } I_k = I + e_i \right]$$



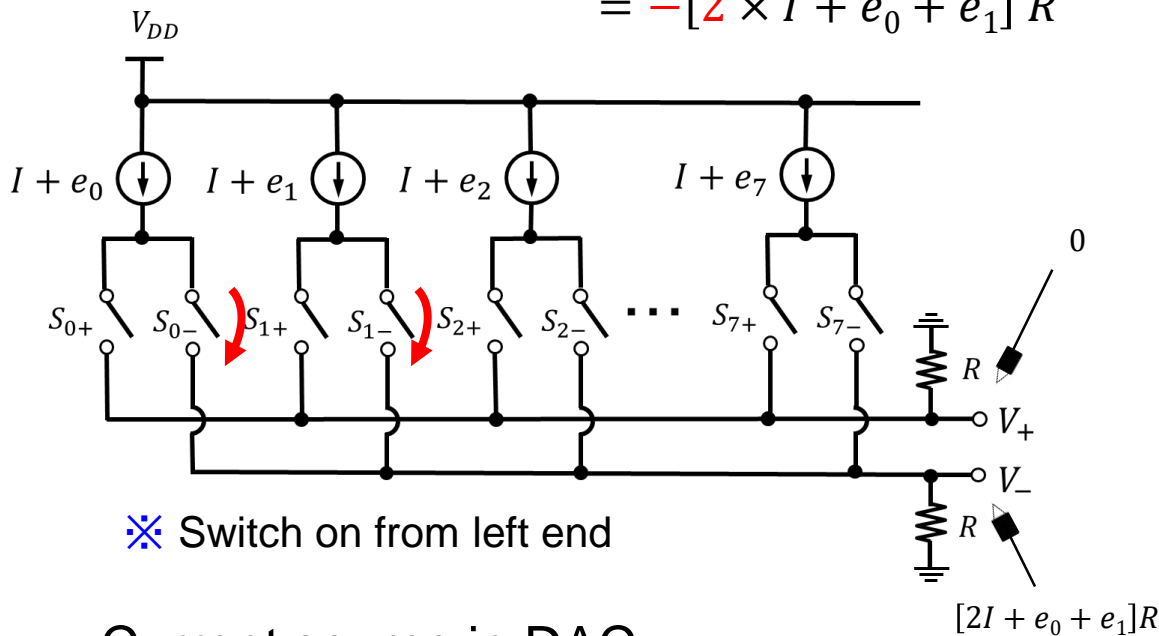
Unit cell mismatch  
 $\Rightarrow$  Accumulation

$e_i$

# Multi-bit DAC of Ternary (2/2)

## ◆ Segment type DAC of ternary

$$\begin{aligned} \text{DAC input} = -2 \quad V_{out} &= V_+ - V_- \\ &= -[2 \times I + e_0 + e_1] R \end{aligned}$$



Current source in DAC

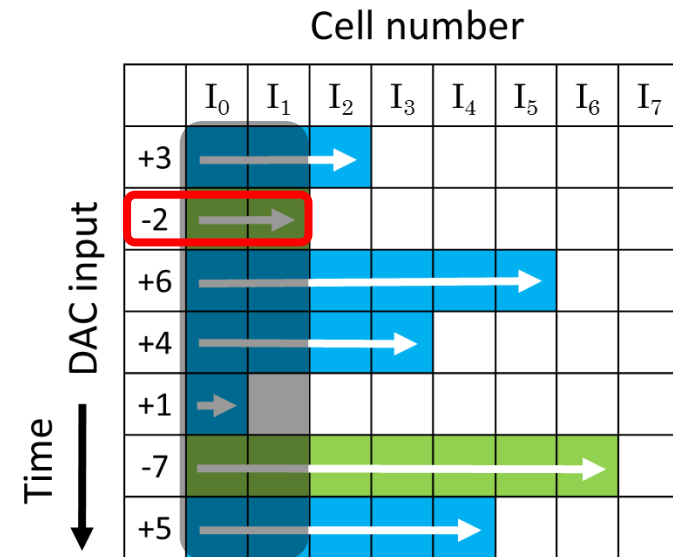
✓ Ideal  $\Rightarrow$  All equal

✓ Real  $\Rightarrow$  Process variation on manufacturing

$$\left[ \text{current } I_k = I + e_i \right]$$



Positive Voltage  
Negative Voltage



Unit cell mismatch  
 $\Rightarrow$  Accumulation

$e_i$

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# Unit Cell Current Average $I$

Unit cell current average :  $I = \frac{1}{8}(I_0 + I_1 + I_2 + I_3 + I_4 + I_5 + I_6 + I_7)$

$$\begin{array}{l}
 I_0 = I + e_0 \\
 I_1 = I + e_1 \\
 I_2 = I + e_2 \\
 I_3 = I + e_3 \\
 I_4 = I + e_4 \\
 I_5 = I + e_5 \\
 I_6 = I + e_6 \\
 I_7 = I + e_7
 \end{array}$$

total sum

$$\begin{aligned}
 &(I_0 + I_1 + I_2 + I_3 + I_4 + I_5 + I_6 + I_7) \\
 &= 4I + (e_0 + e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7)
 \end{aligned}$$

$I =$

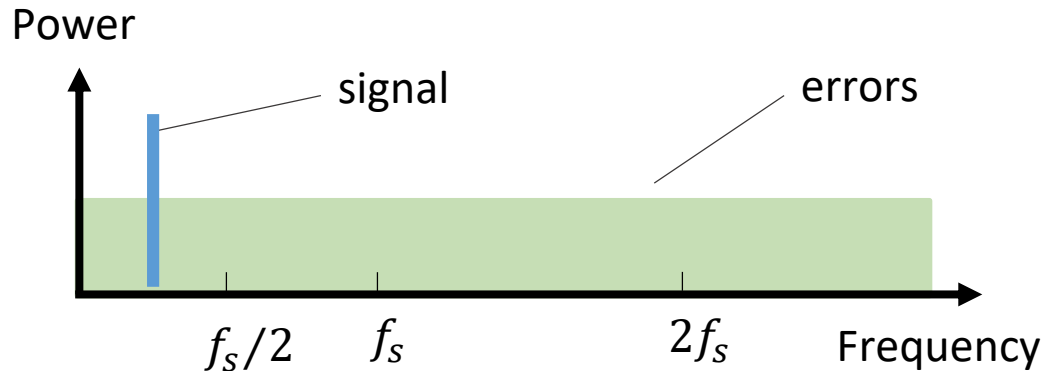
$$\begin{aligned}
 &\frac{1}{4}(I_0 + I_1 + I_2 + I_3 + I_4 + I_5 + I_6 + I_7) \\
 &- \frac{1}{4}(e_0 + e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7)
 \end{aligned}$$

0

mismatch :  $e_k$

# DWA Techniques

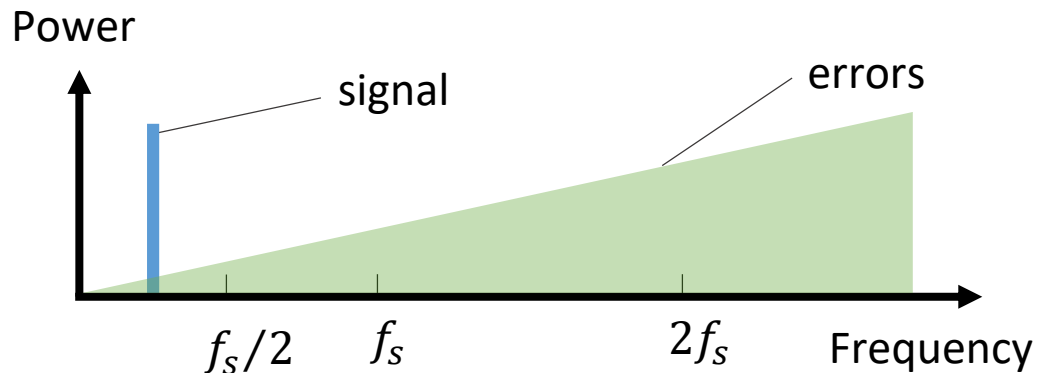
w/o DWA



- Errors are flat in spectrum



w/ DWA LP type



- Spectrum of errors are low-pass shaped.



# Multi-bit DAC of Ternary and **DWA I** (1/2)

◆ DWA\* type DAC of ternary (\*Data-Weighted Average)

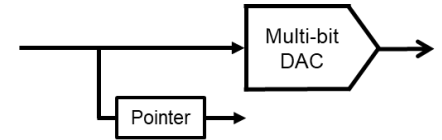
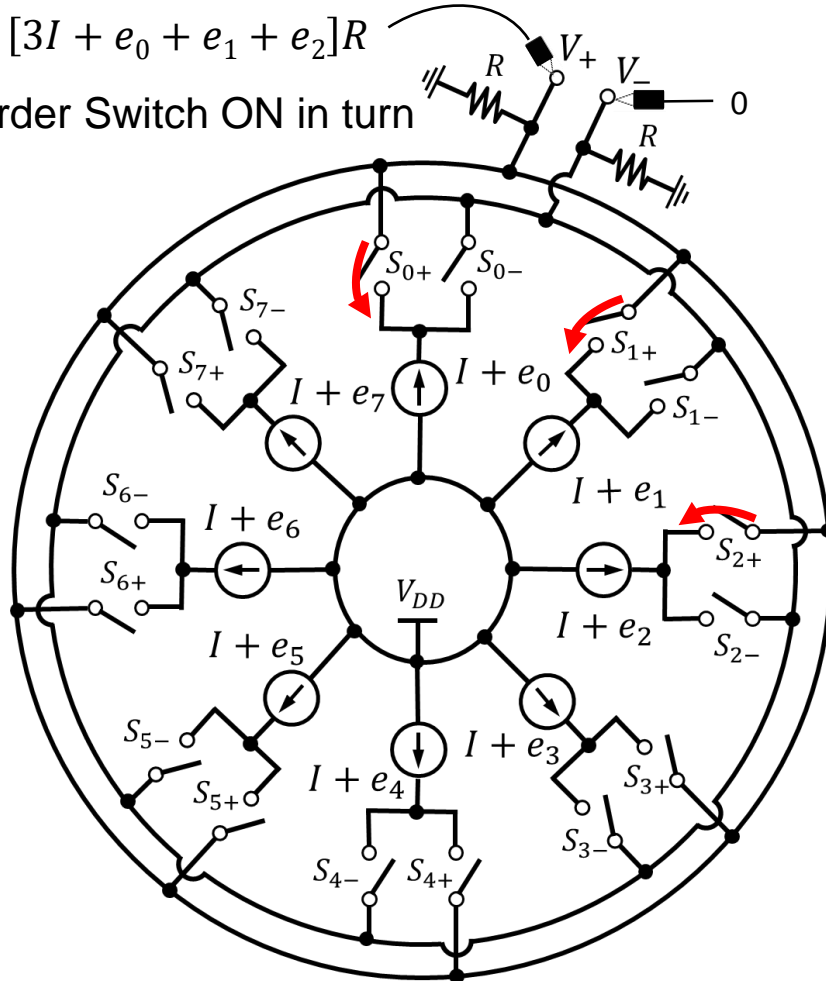
DAC input = **+3**

$$V_{out} = V_+ - V_-$$

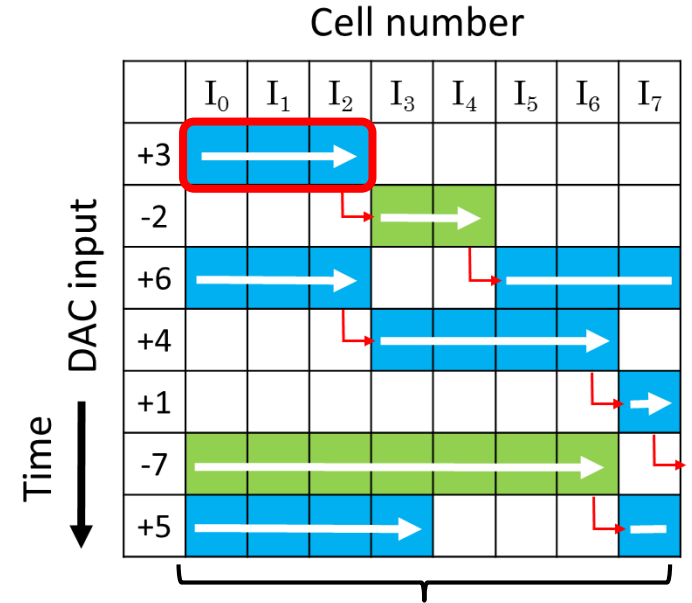
$$= +[3 \times I + e_0 + e_1 + e_2] R$$

$$[3I + e_0 + e_1 + e_2]R$$

※ In order Switch ON in turn



Positive Voltage  
negative Voltage



Distribute unit cell mismatch  
⇒ Averaging  
➡ Improve linearity

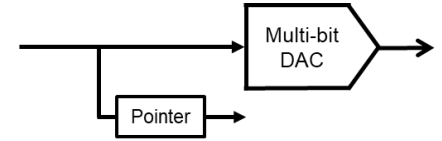
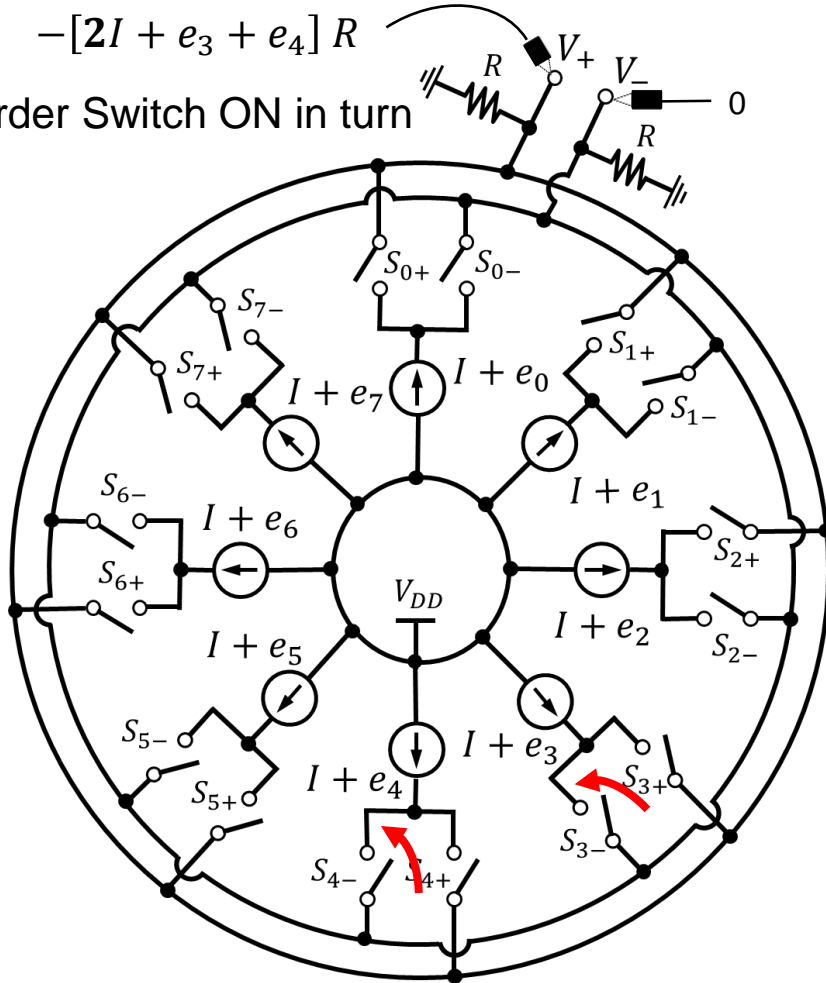
# Multi-bit DAC of Ternary and DWA I (2/2)

◆ DWA\* type DAC of ternary (\*Data-Weighted Average)

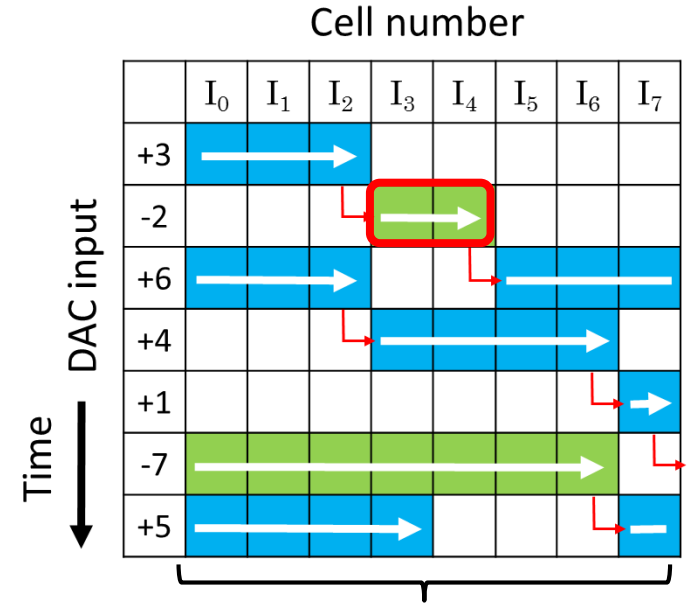
DAC input = **-2**       $V_{out} = V_+ - V_-$   
 $= -[2 \times I + e_3 + e_4] R$

$-[2I + e_3 + e_4] R$

※ In order Switch ON in turn



— Positive Voltage  
 — negative Voltage



Distribute unit cell mismatch  
 ⇒ Averaging  
 → Improve linearity

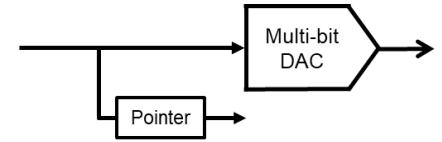
# Multi-bit DAC of Ternary and DWA II (1/3)

◆ DWA\* type DAC of ternary (\*Data-Weighted Average)

DAC input = **+3**

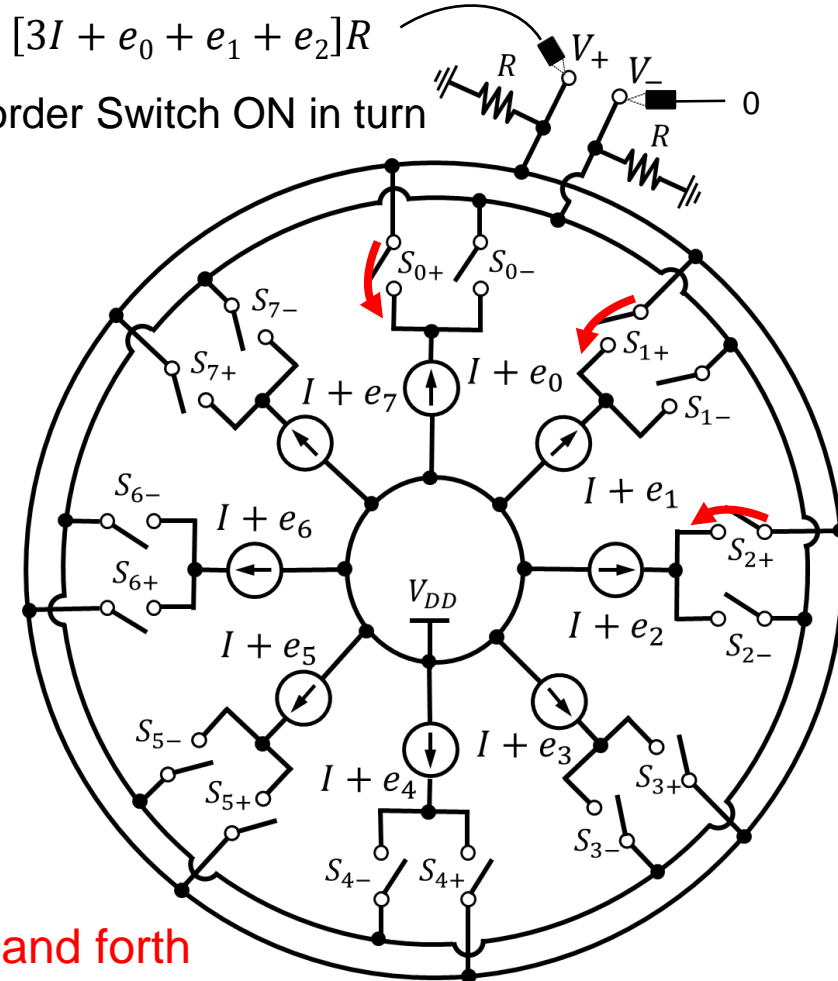
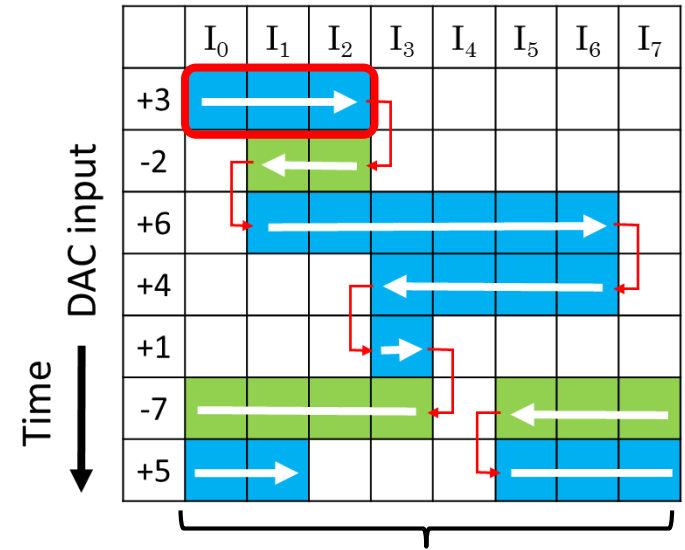
$$V_{out} = V_+ - V_-$$

$$= +[3 \times I + e_0 + e_1 + e_2] R$$



Positive Voltage (blue bar)  
negative Voltage (green bar)

Cell number



✦ In order Switch ON in turn

Back and forth

Distribute unit cell mismatch  
⇒ Averaging  
➡ Improve linearity

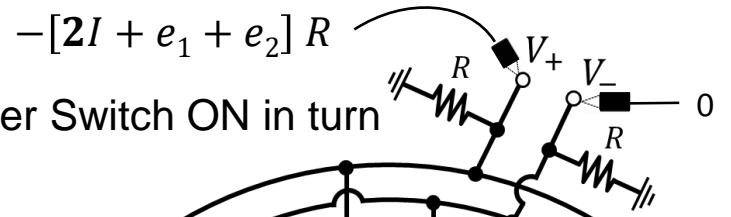
# Multi-bit DAC of Ternary and DWA II (2/3)

◆ DWA\* type DAC of ternary (\*Data-Weighted Average)

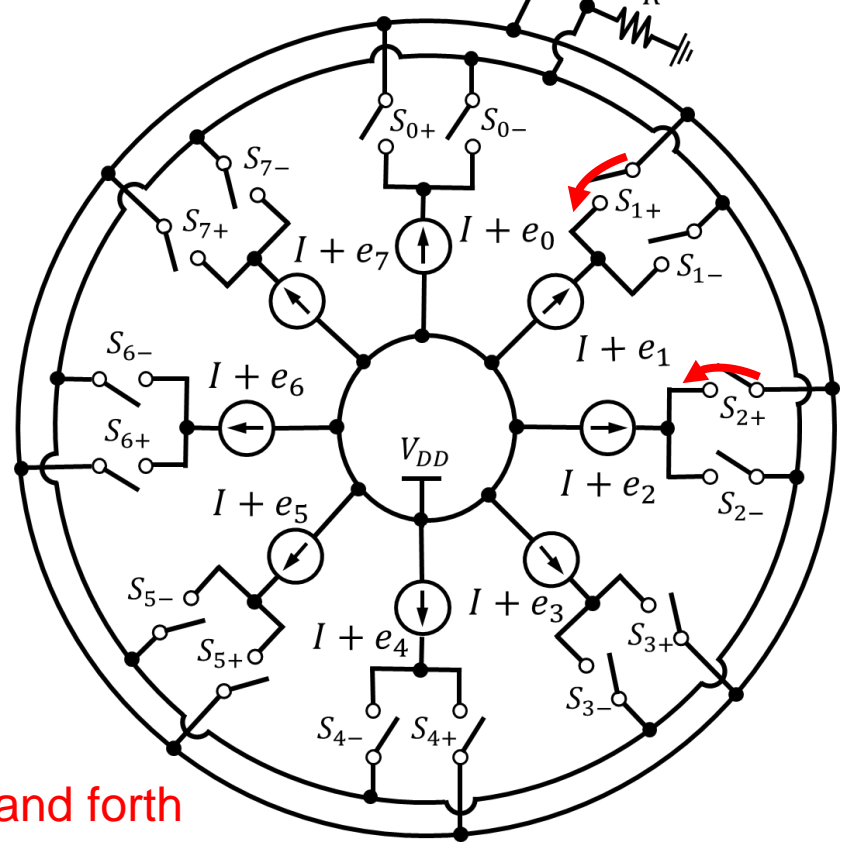
DAC input = **-2**

$$V_{out} = V_+ - V_-$$

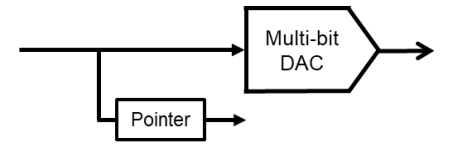
$$= -[2 \times I + e_1 + e_2] R$$



⊗ In order Switch ON in turn

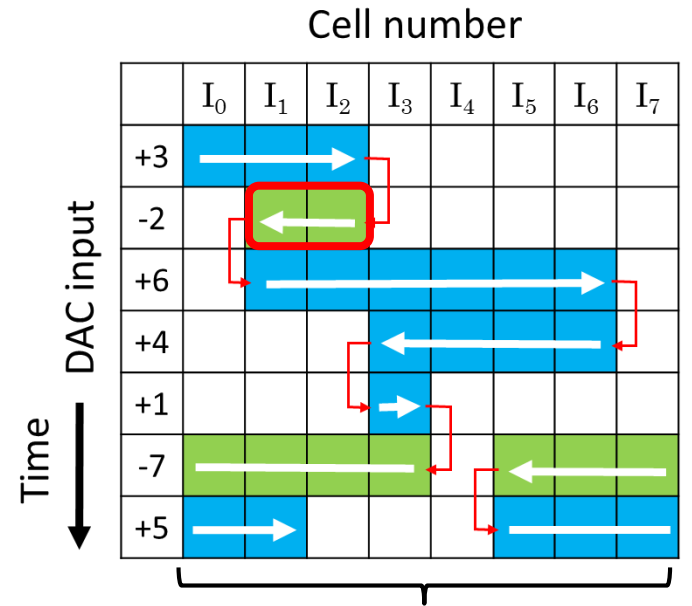


Back and forth



Positive Voltage (blue bar)

negative Voltage (green bar)



Distribute unit cell mismatch  
 ⇒ Averaging  
 → Improve linearity

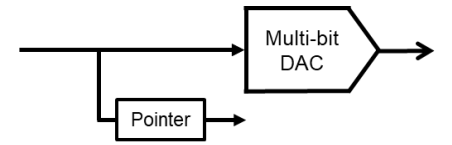
# Multi-bit DAC of Ternary and DWA II (3/3)

◆ DWA\* type DAC of ternary (\*Data-Weighted Average)

DAC input = +6

$$V_{out} = V_+ - V_-$$

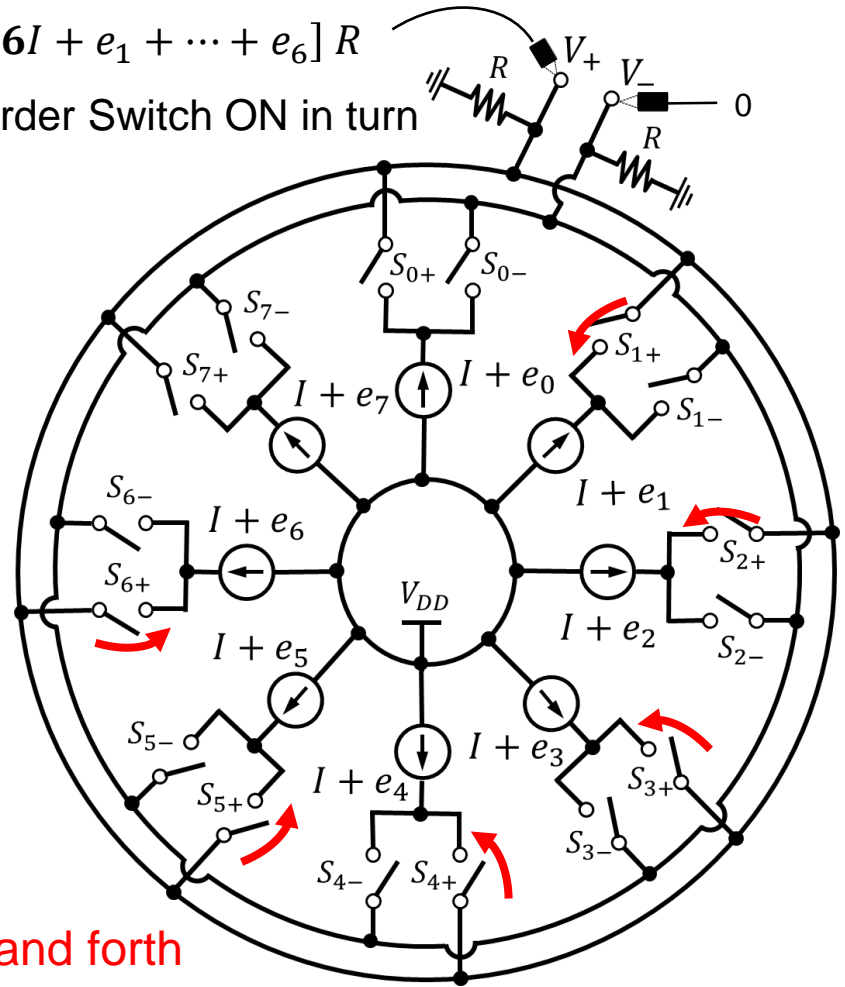
$$= +[6 \times I + e_1 + \dots + e_6] R$$



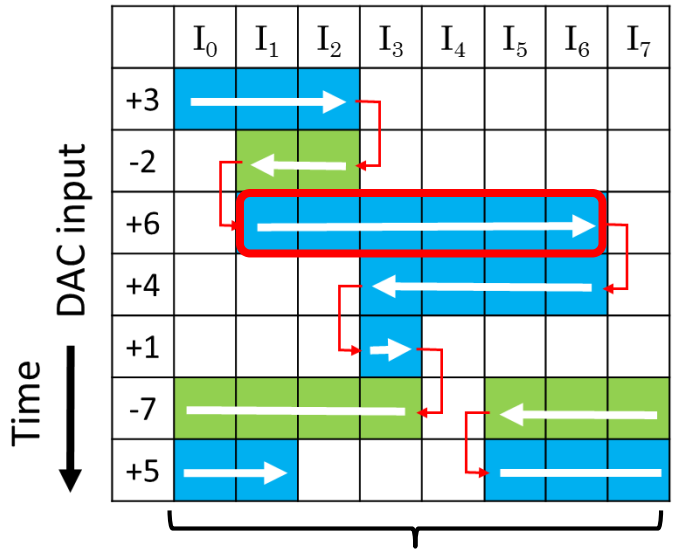
Positive Voltage (blue bar)  
 negative Voltage (green bar)

$$[6I + e_1 + \dots + e_6] R$$

⊗ In order Switch ON in turn



Cell number

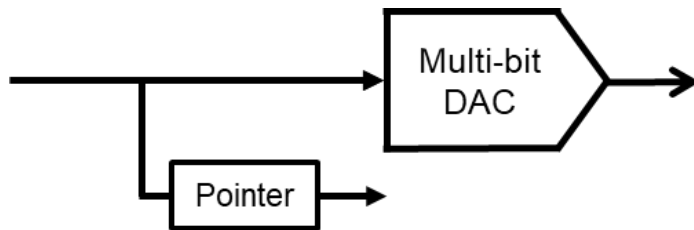


Distribute unit cell mismatch  
 ⇒ Averaging  
 → Improve linearity

Back and forth

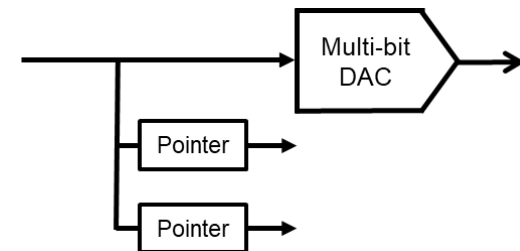
# DWA type I (Pointer)

## 1 Pointer



	I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>	I <sub>4</sub>	I <sub>5</sub>	I <sub>6</sub>	I <sub>7</sub>	
+3	→								
-2			↙	→					
+6	→				↙	→			
+4			↙	→					
+1							↙	→	
-7	→							↙	
+5	→							↙	→

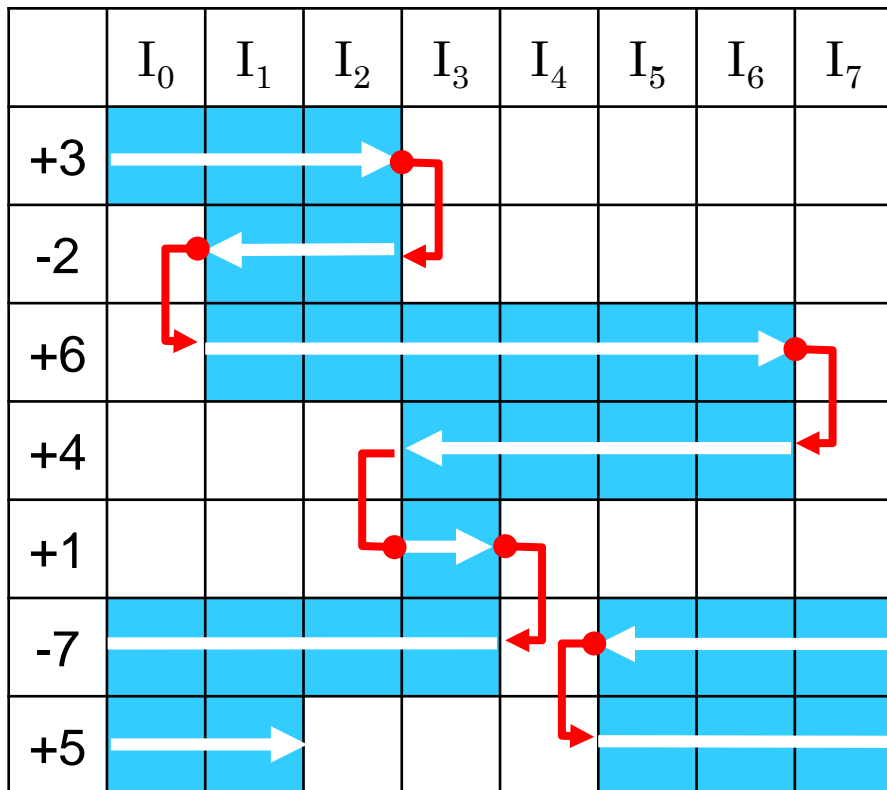
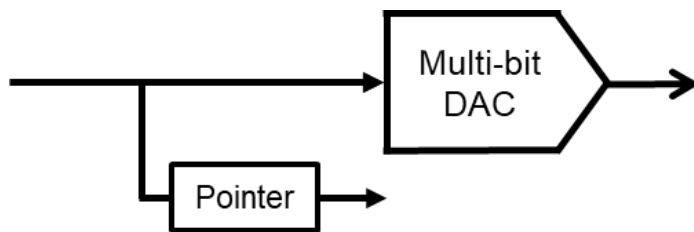
## 2 Pointers



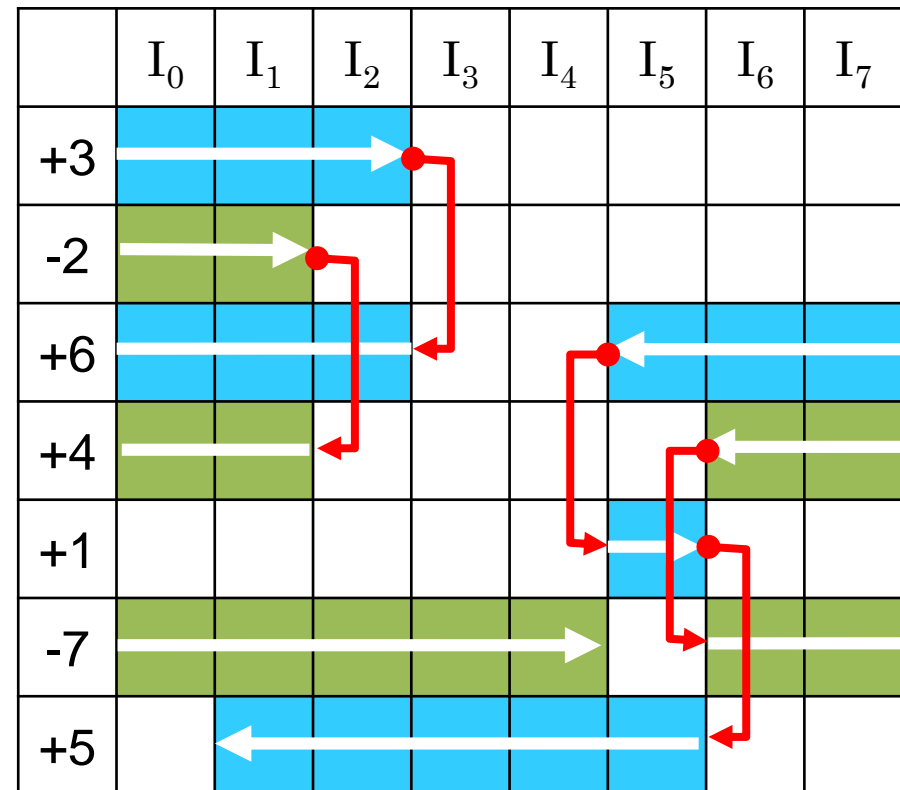
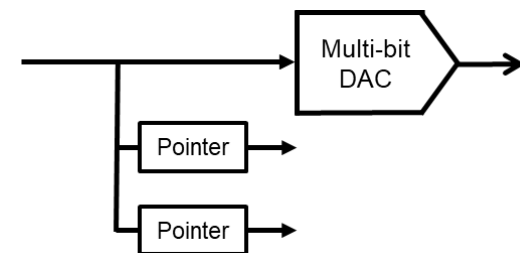
	I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>	I <sub>4</sub>	I <sub>5</sub>	I <sub>6</sub>	I <sub>7</sub>
+3	→							
-2	→		↙					
+6	→	↙	↙	→				
+4	↙	↙	→					
+1	↙	→					↙	
-7	→		↙	→			↙	→
+5		↙	→					

# DWA type II (Pointer)

## 1 Pointer



## 2 Pointers



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## ◆ $\Delta\Sigma$ DA Converter

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## ◆ Simulation verification

- Binary, Ternary DWA Overview
- $\Delta\Sigma$  DA Converter : HP type
- $\Delta\Sigma$  DA Converter : BP type

## ◆ Conclusion



# Binary, Ternary DWA Overview

Signal Band	Value	Number (N) of Signal Bands	DWA type
LP	Binary	1	I
	Ternary	1	I
HP	Binary	1	II
	Ternary	1	I
BP	Binary	2	II
	Binary	4	II
	Ternary	2	I
	Ternary	4	I

New Findings

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## ◆ Simulation verification

- Binary, Ternary DWA Overview

- $\Delta\Sigma$  DA Converter : HP type

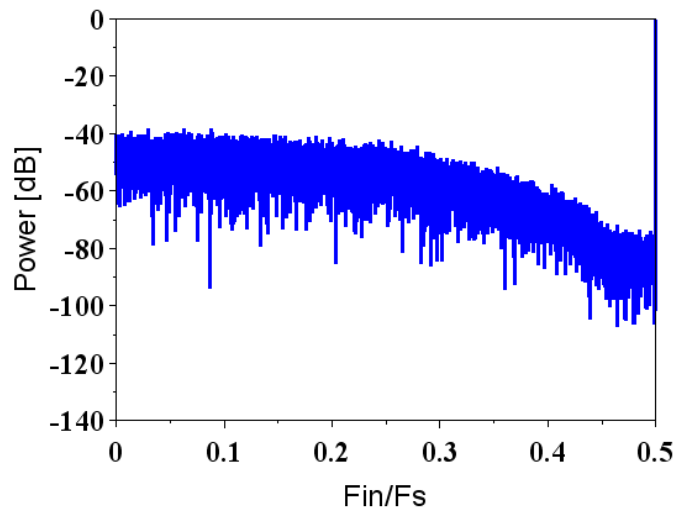
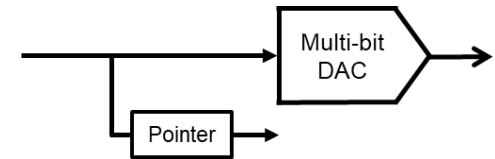
- $\Delta\Sigma$  DA Converter : BP type

## ◆ Conclusion

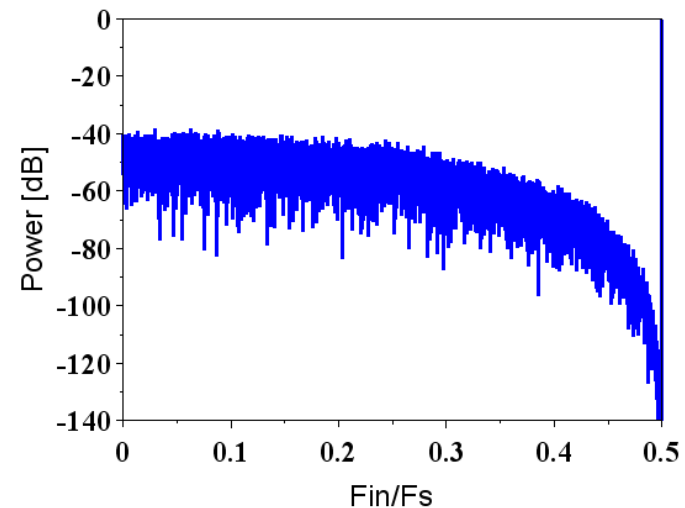
# High-Pass $\Delta\Sigma$ DAC (Binary)

- High-pass (HP)  $\Delta\Sigma$  DAC (N=1)

Segmented DAC with **binary** unit cells



< DWA type I >

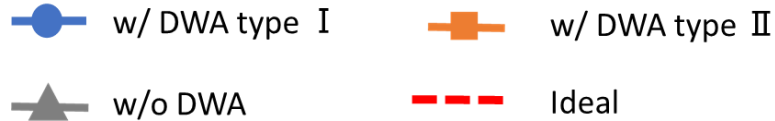
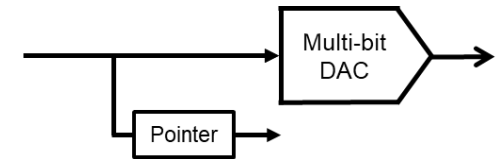


< DWA type II >

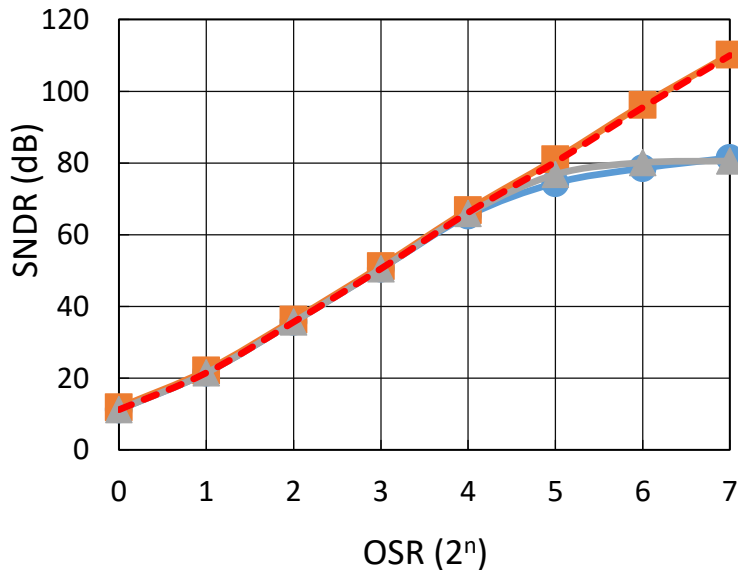
# High-Pass $\Delta\Sigma$ DAC (Binary)

## ● High-pass (HP) $\Delta\Sigma$ DAC (N=1)

Segmented DAC with **binary** unit cells

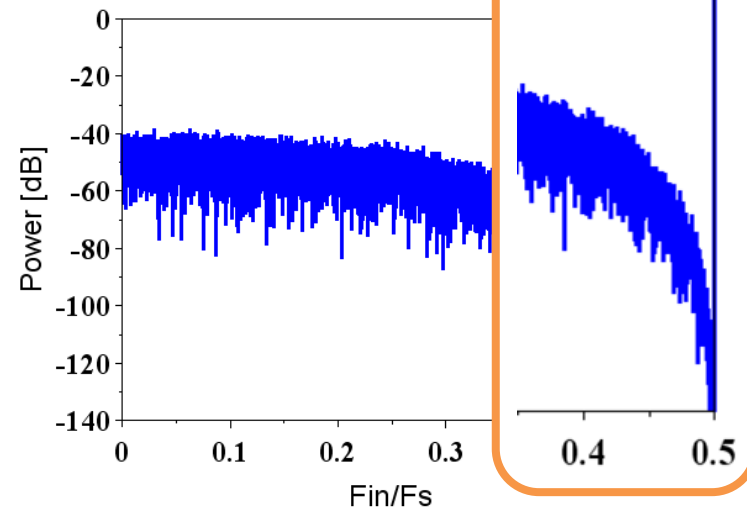


$\sigma = 0.1\%$



## Good Algorithm

- noise-shaping
- Reduced noise

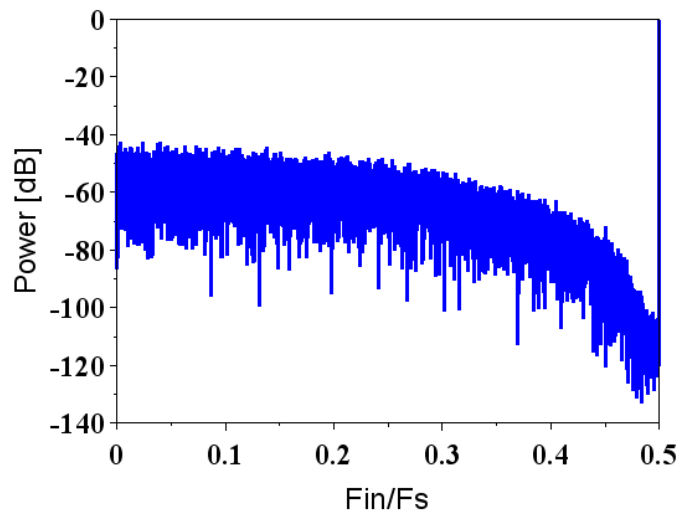
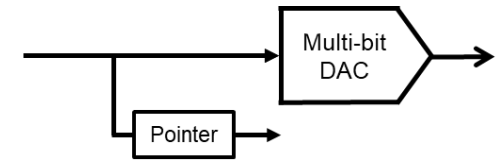


< DWA type II >

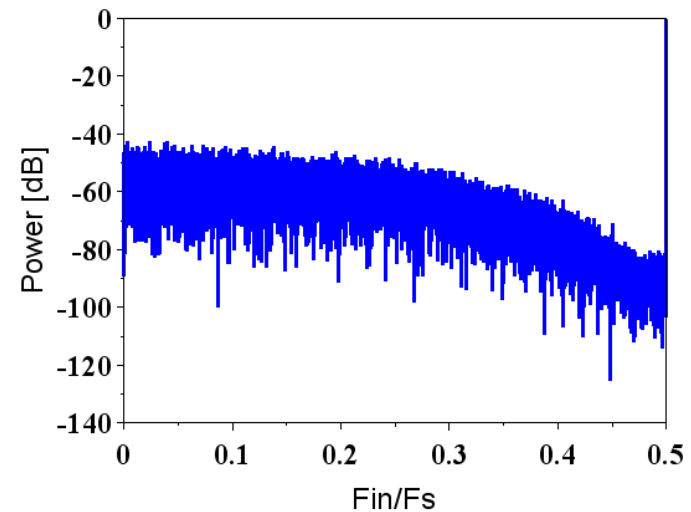
# High-Pass $\Delta\Sigma$ DAC (Ternary)

- High-pass (HP)  $\Delta\Sigma$  DAC (N=1)

Segmented DAC with **ternary** unit cells



< DWA type I >

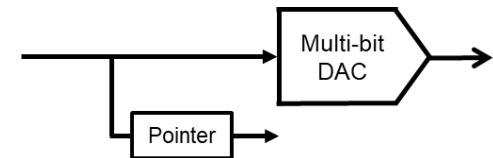


< DWA type II >

# High-Pass $\Delta\Sigma$ DAC (Ternary)

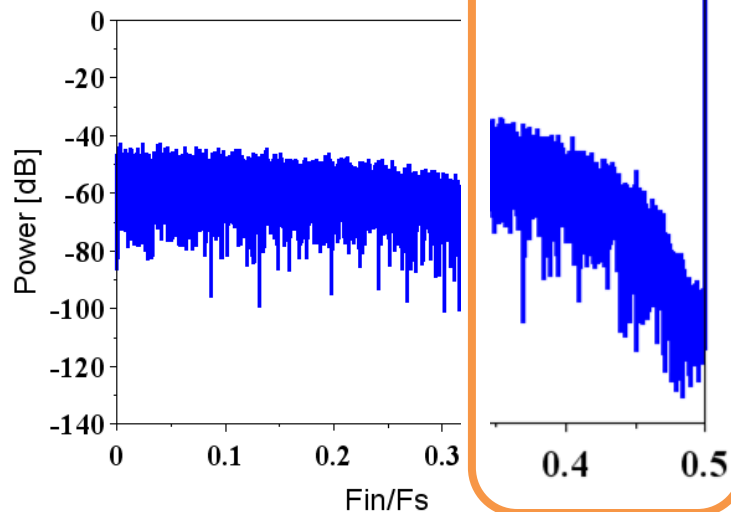
## ● High-pass (HP) $\Delta\Sigma$ DAC (N=1)

Segmented DAC with **ternary** unit cells

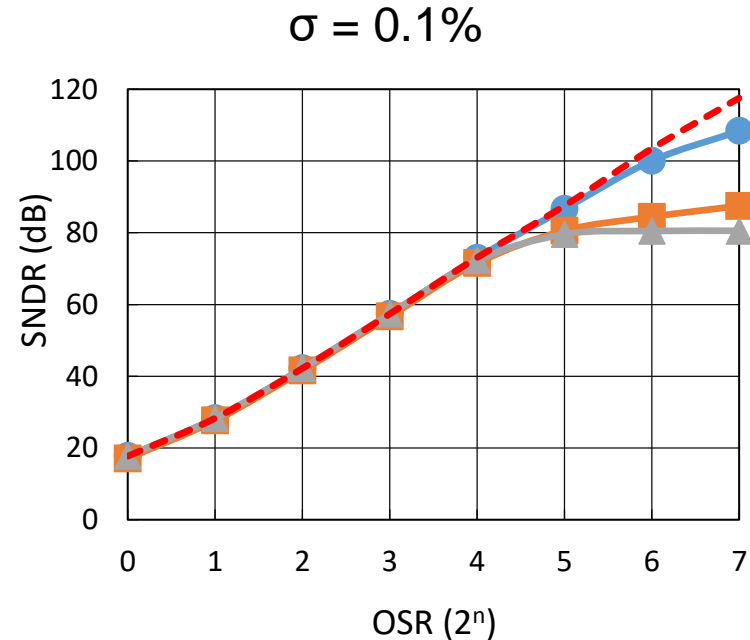


### Good Algorithm

- noise-shaping
- Reduced noise



< DWA type I >



# Outline

## ◆ Research Background

## ◆ $\Delta\Sigma$ DA Converter

- DWA\* Algorithm (\* Data-Weighted Averaging)

## ◆ Simulation verification

- Binary, Ternary DWA Overview

- $\Delta\Sigma$  DA Converter : HP type

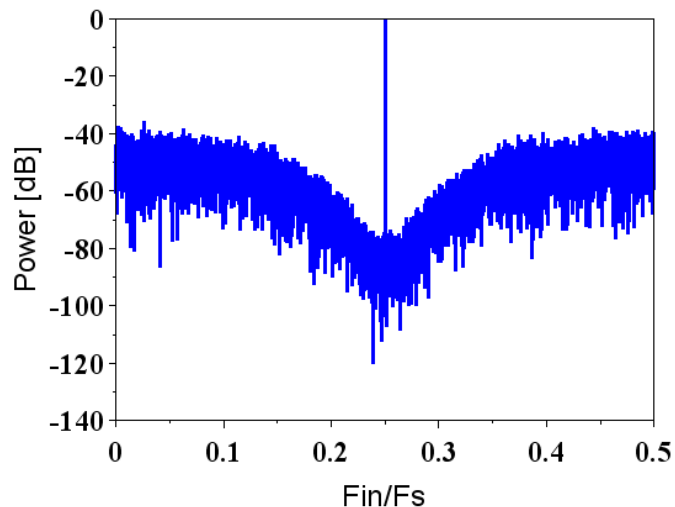
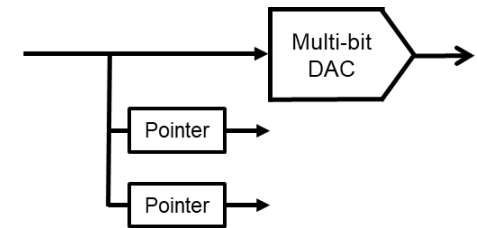
- $\Delta\Sigma$  DA Converter : BP type

## ◆ Conclusion

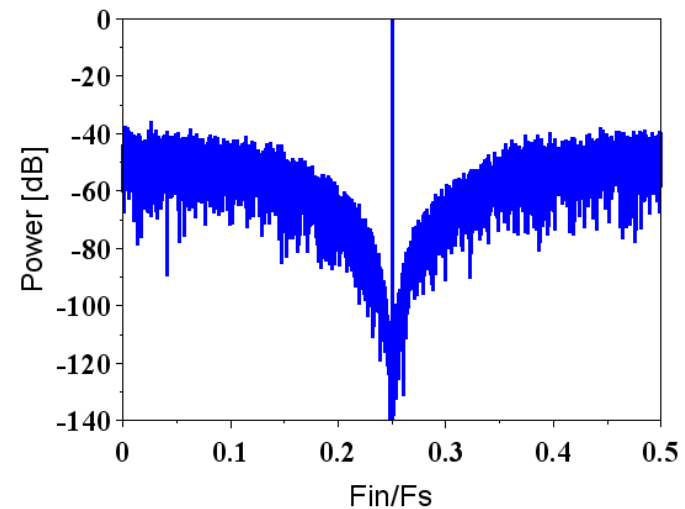
# Band-Pass $\Delta\Sigma$ DAC (Binary)

## ● Band-pass (BP) $\Delta\Sigma$ DAC (N=2)

segmented DAC with **binary** unit cells



< DWA type I >



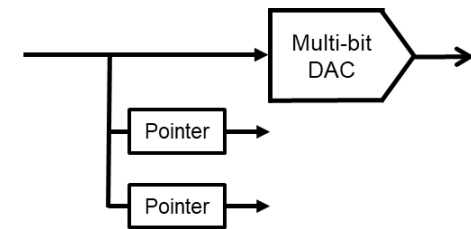
< DWA type II >



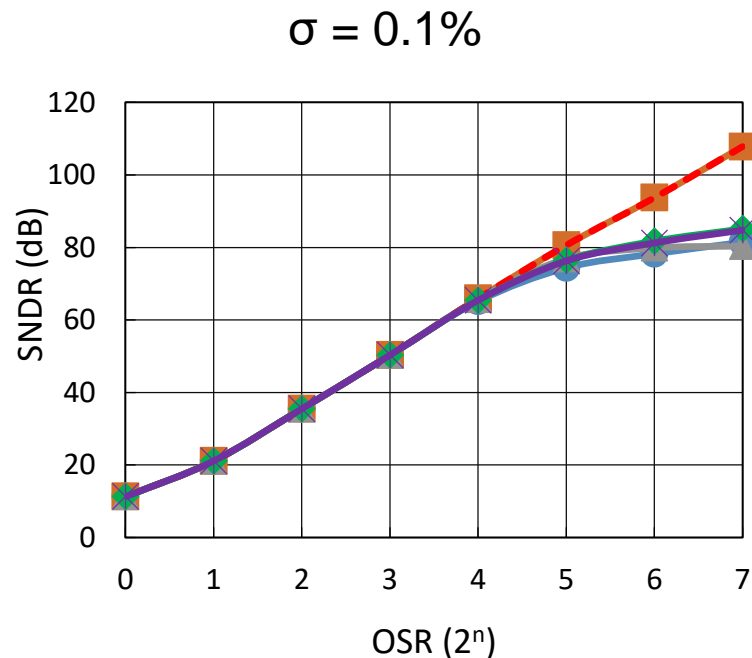
# Band-Pass $\Delta\Sigma$ DAC (Binary)

## ● Band-pass (BP) $\Delta\Sigma$ DAC (N=2)

Segmented DAC with **binary** unit cells

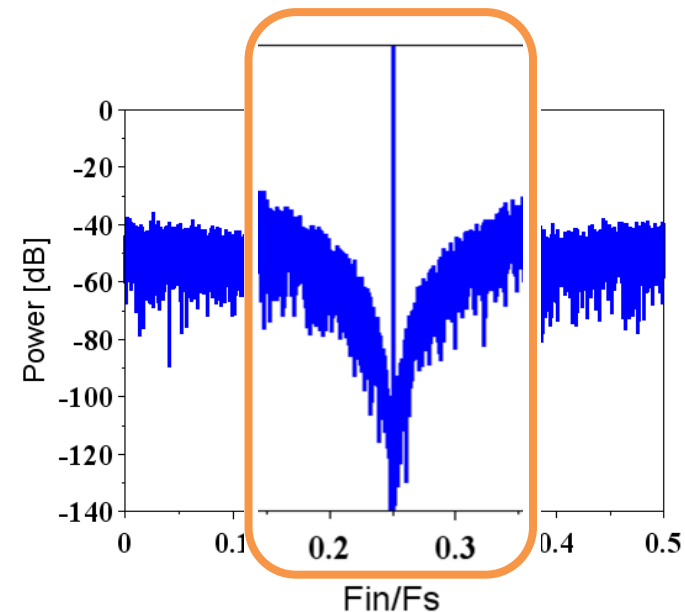


- w/ DWA type I (2 Pointer)    ■ w/ DWA type II (2 Pointer)
- ◆ w/ DWA type I (1 Pointer)    ✖ w/ DWA type II (1 Pointer)
- ▲ w/o DWA    - - - Ideal



## Good Algorithm

- noise-shaping

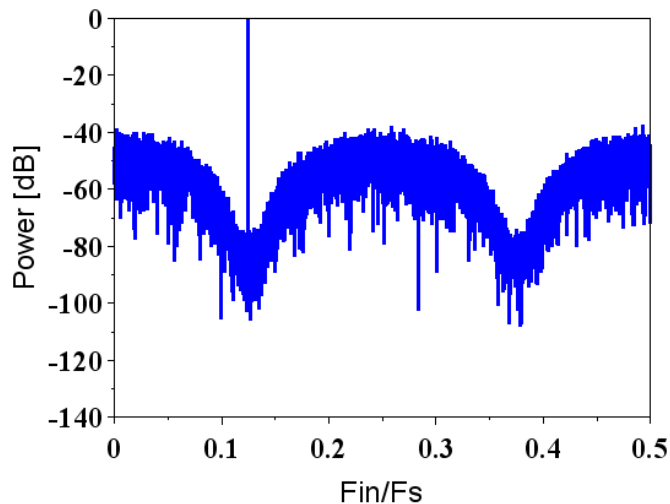
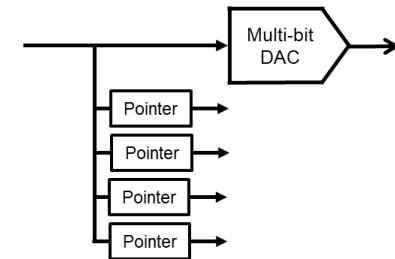


< DWA type II >

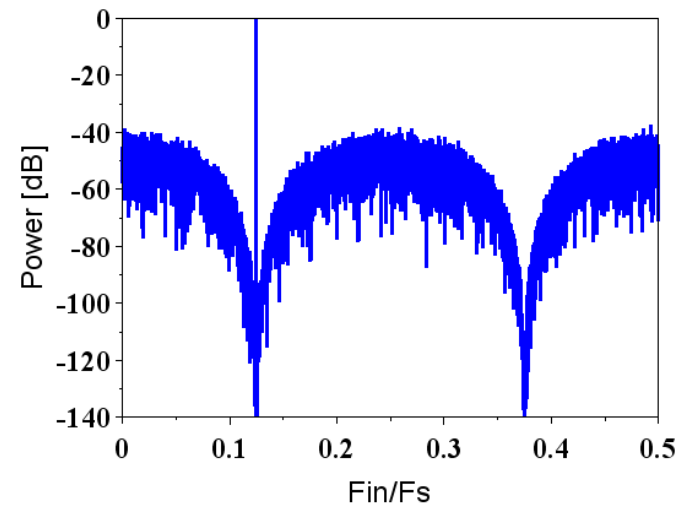
# Band-Pass $\Delta\Sigma$ DAC (Binary)

- Band-pass (BP)  $\Delta\Sigma$  DAC (N=4)

Segmented DAC with **binary** unit cells



< DWA type I >

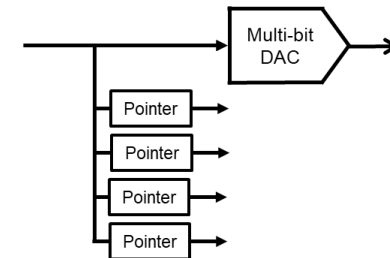


< DWA type II >

# Band-Pass $\Delta\Sigma$ DAC (Binary)

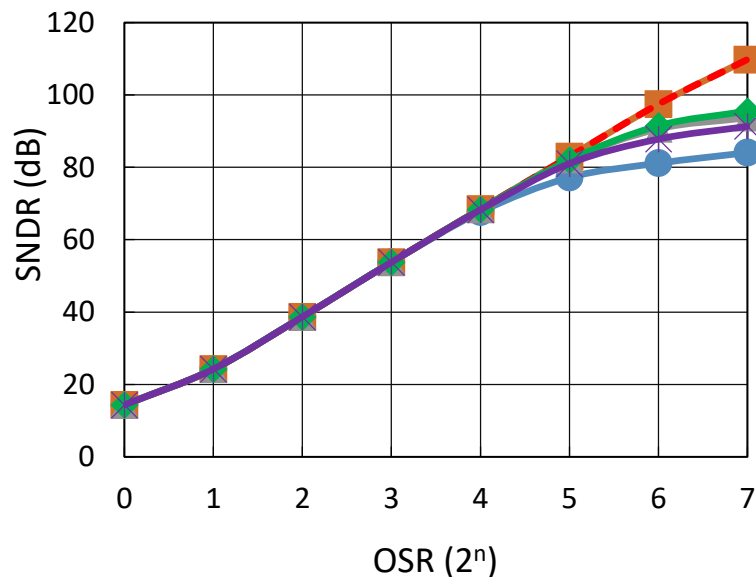
## ● Band-pass (BP) $\Delta\Sigma$ DAC (N=4)

Segmented DAC with **binary** unit cells



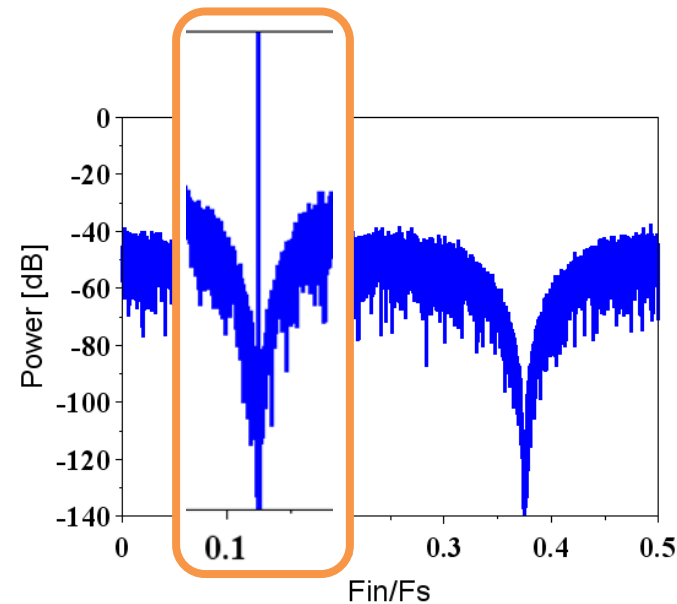
- w/ DWA type I (4 Pointer)    ■ w/ DWA type II (4 Pointer)
- ◆ w/ DWA type I (1 Pointer)    ✖ w/ DWA type II (1 Pointer)
- ▲ w/o DWA                            - - - Ideal

$\sigma = 0.1\%$



**Good Algorithm**

▪ noise-shaping

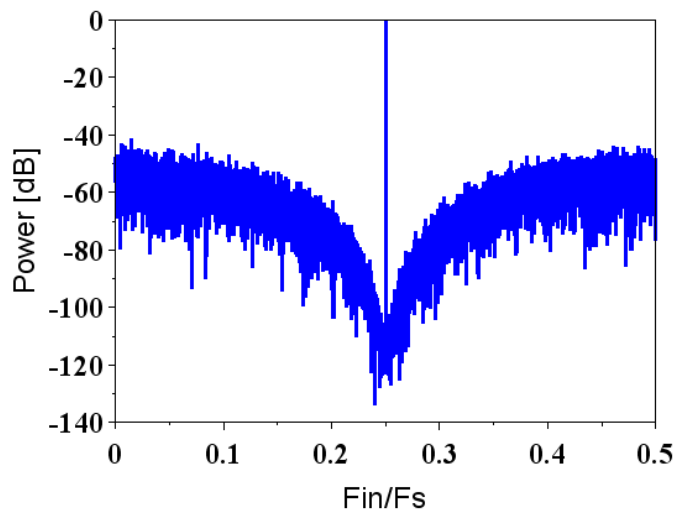
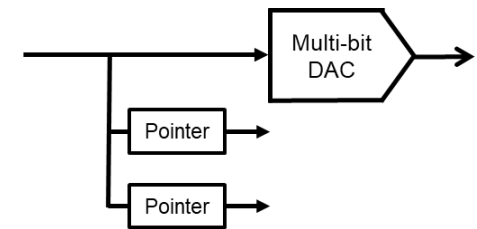


< DWA type II >

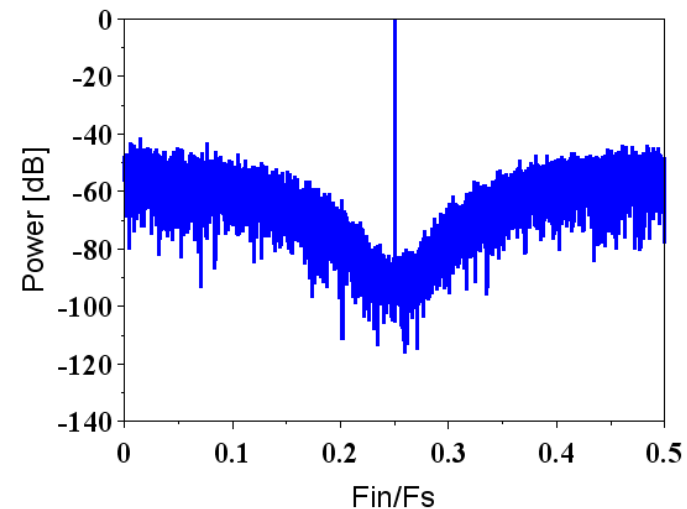
# Band-Pass $\Delta\Sigma$ DAC (Ternary)

- Band-pass (BP)  $\Delta\Sigma$  DAC (N=2)

Segmented DAC with **ternary** unit cells



< DWA type I >

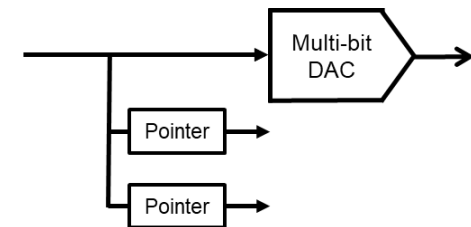


< DWA type II >

# Band-Pass $\Delta\Sigma$ DAC (Ternary)

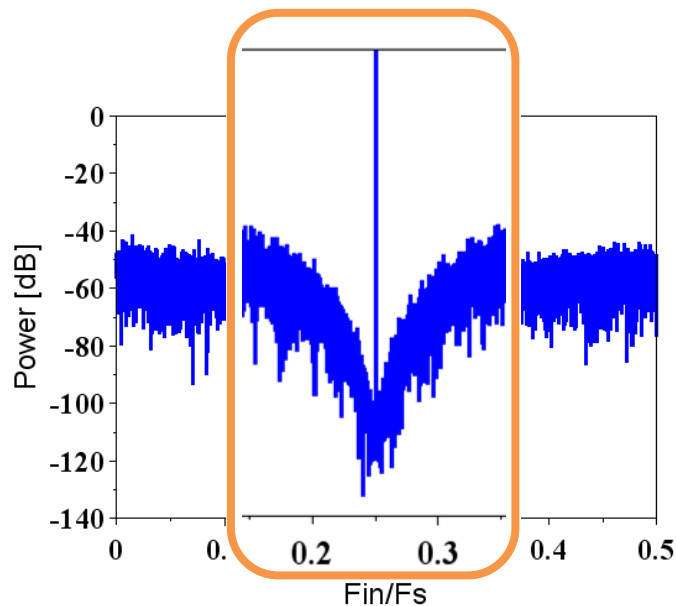
## ● Band-pass (BP) $\Delta\Sigma$ DAC (N=2)

Segmented DAC with **ternary** unit cells

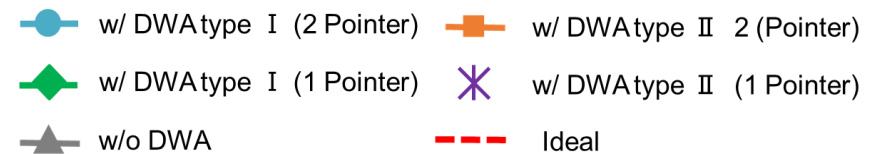


### Good Algorithm

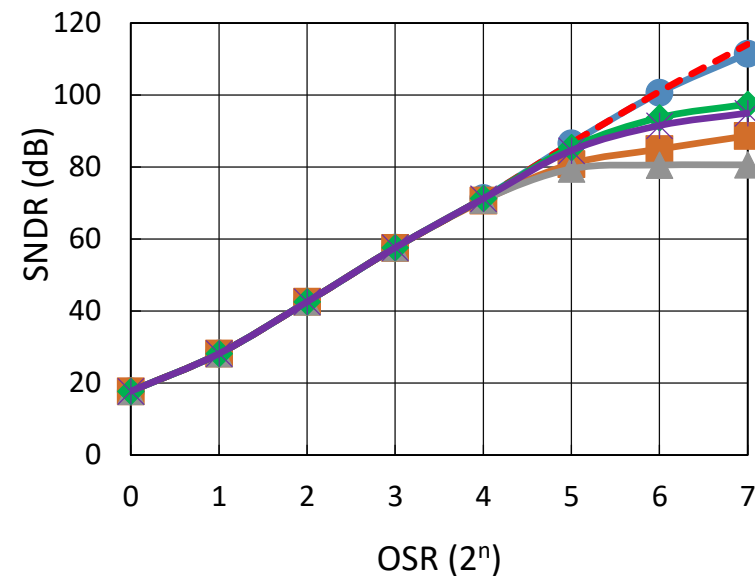
▪ noise-shaping



< DWA type I >



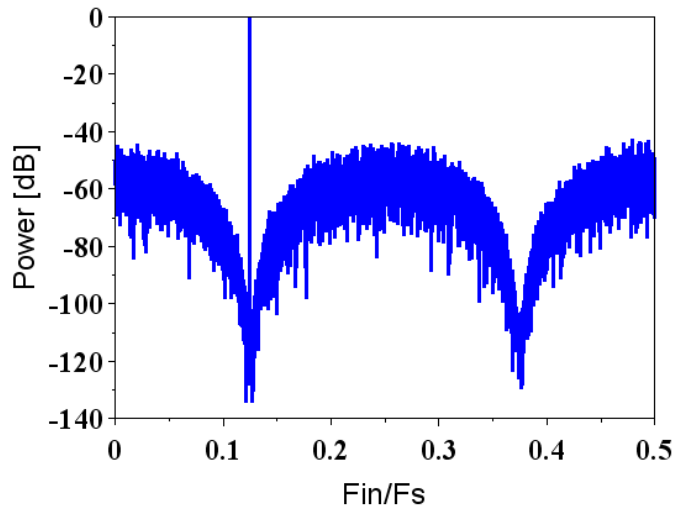
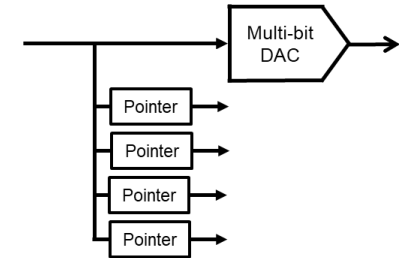
$\sigma = 0.1\%$



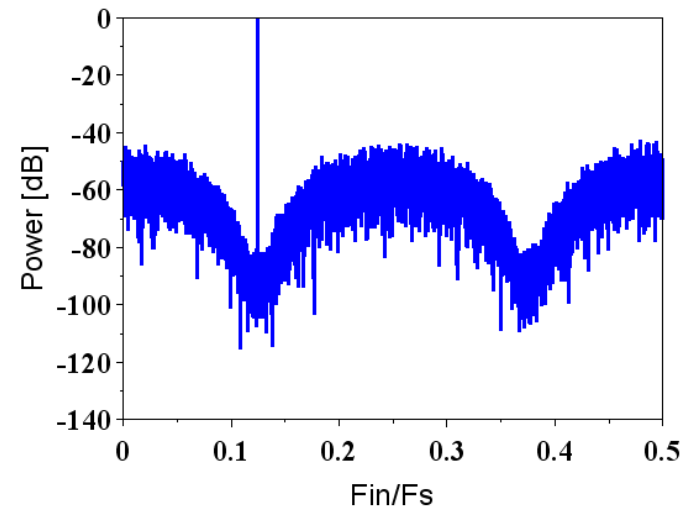
# Band-Pass $\Delta\Sigma$ DAC (Ternary)

- Band-pass (BP)  $\Delta\Sigma$  DAC (N=4)

Segmented DAC with **ternary** unit cells



< DWA type I >

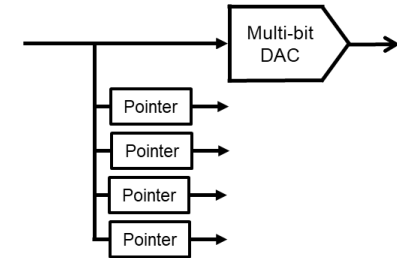


< DWA type II >

# Band-Pass $\Delta\Sigma$ DAC (Ternary)

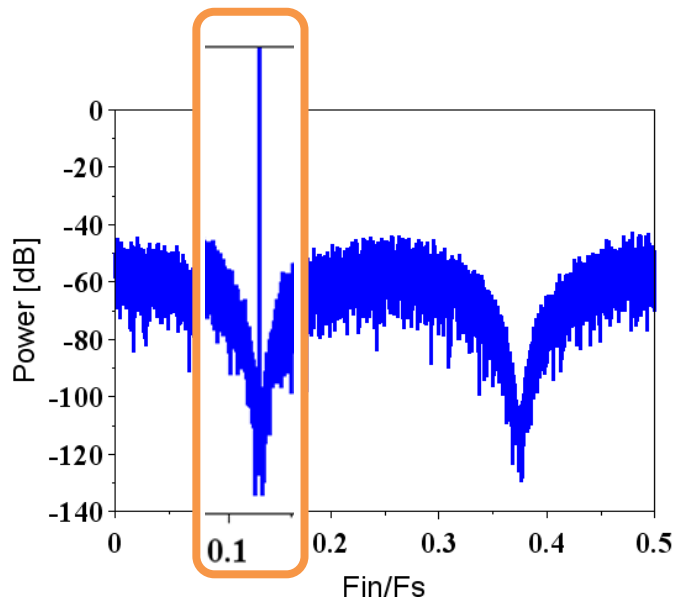
## ● Band-pass (BP) $\Delta\Sigma$ DAC (N=4)

Segmented DAC with **ternary** unit cells



### Good Algorithm

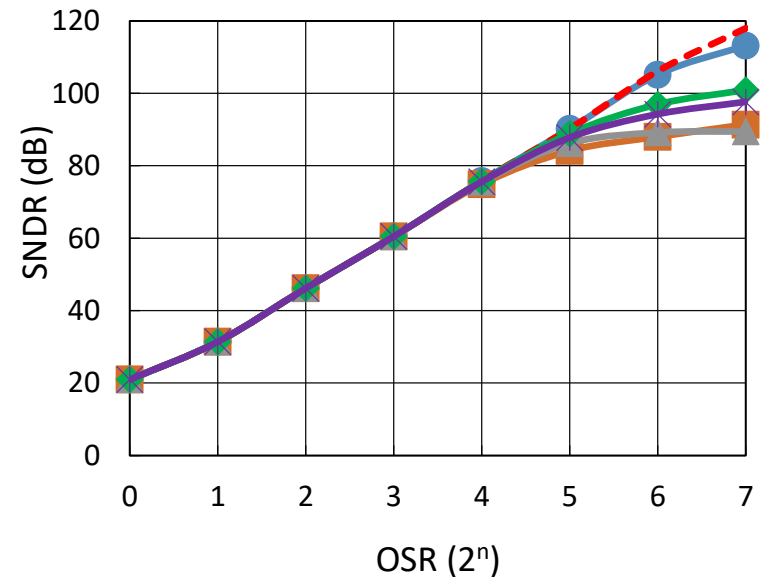
▪ noise-shaping



< DWA type I >

- w/ DWA type I (4 Pointer)
- w/ DWA type II (4 Pointer)
- ◆ w/ DWA type I (1 Pointer)
- ✱ w/ DWA type II (1 Pointer)
- ▲ w/o DWA
- Ideal

$\sigma = 0.1\%$



# Outline

## ◆ Research Background

## ◆ $\Delta\Sigma$ DA Converter

- DWA\* Algorithm (\* Data-Weighted Averaging)

## ◆ Simulation verification

- Binary, Ternary DWA Overview
- $\Delta\Sigma$  DA Converter : HP type
- $\Delta\Sigma$  DA Converter : BP type

## ◆ Conclusion



# Conclusion

- HP, BP multi-bit  $\Delta\Sigma$  DACs

- In case HP, BP  $\Delta\Sigma$  DACs with **ternary** unit cells, **DWA type I** with pointers alternately used is effective.



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Thank you for attention

# Appendix

# Look Up Table (LUT)

LUT: Save data in advance



Data corresponding to input  
output

example

Cat's age	Equivalent human age
1	17
2	23
3	28
4	32
5	36
6	40
7	44
8	48



Cats one year old



Human being 17 years old

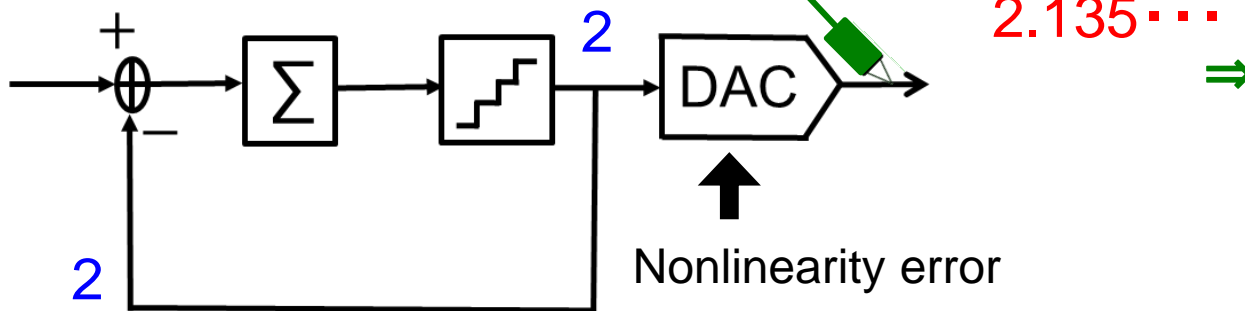
App.

# Self-Calibration Algorithm

Preparation

⇒ Feed back value measured with high precision  $\Delta\Sigma$  ADC

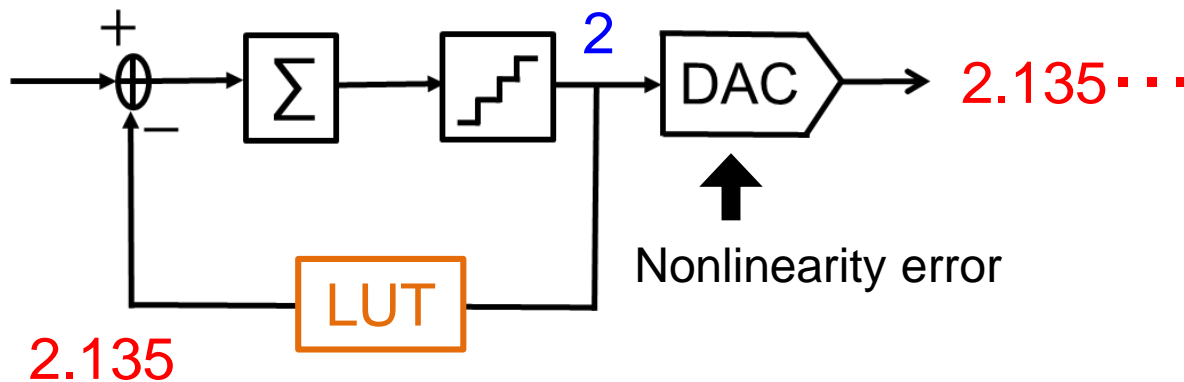
Save to LUT



LUT

Input	Output
0	
1	
2	2.135
3	
⋮	

Implementation of self-calibration algorithm



LUT

Input	Output
0	0.000
1	1.241
2	2.135
3	2.926
⋮	