

Linearity Improvement Algorithms of Multi-bit $\Delta\Sigma$ DA Converter - Combination of Unit Cell Re-ordering and DWA -

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Outline

- ◆ Research Background
- ◆ $\Delta\Sigma$ DA converter
 - DWA* Algorithm (* Data-Weighted Averaging)
 - Unit Cell Re-ordering
- ◆ Simulation verification
 - 2nd - order $\Delta\Sigma$ DA converter : LP type
 - 2nd - order $\Delta\Sigma$ DA converter : HP type
- ◆ Conclusion

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- Unit Cell Re-ordering

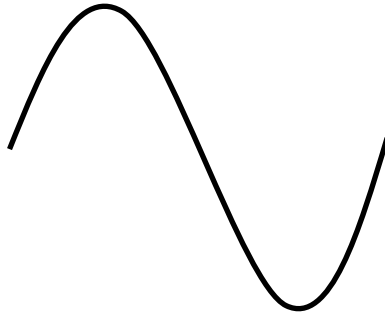
◆ Simulation verification

- 2nd - order $\Delta\Sigma$ DA converter : LP type
- 2nd - order $\Delta\Sigma$ DA converter : HP type

◆ Conclusion

Research Background

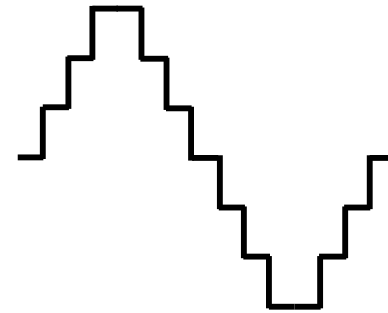
Analog signal



Continuous signal

Physical quantity existing in nature

Digital signal



Discrete signal

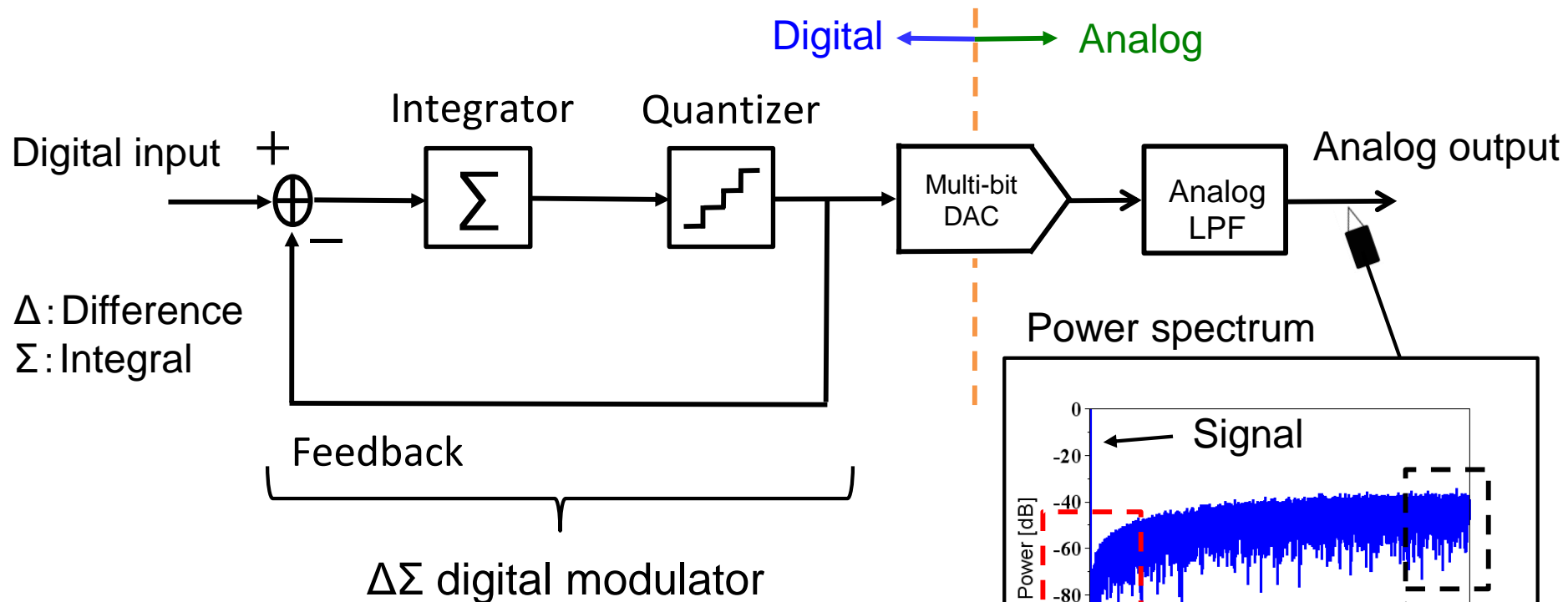
Binary number

ADC

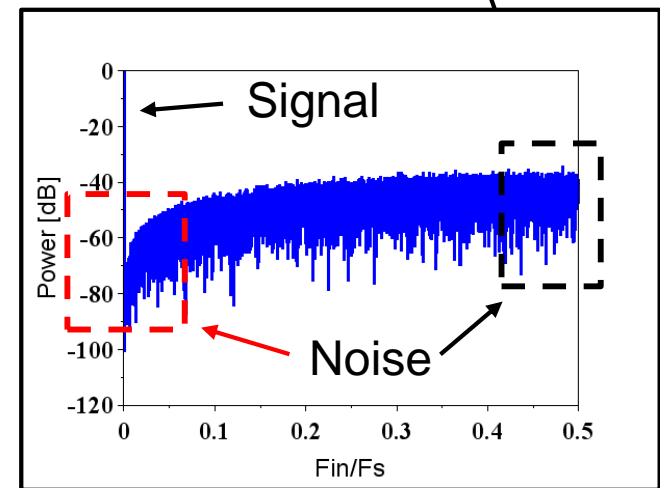
DAC

- $\Delta\Sigma$ Digital-to-Analog Converter ($\Delta\Sigma$ DAC) → **Required**
 - Mostly digital circuit
 - High-resolution , High-linearity
 - DC signal , low frequency signal generation

$\Delta\Sigma$ DA Converter (LP model)



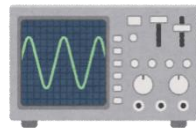
Power spectrum



Noise is **decreased at low frequency**
 increased at high frequency

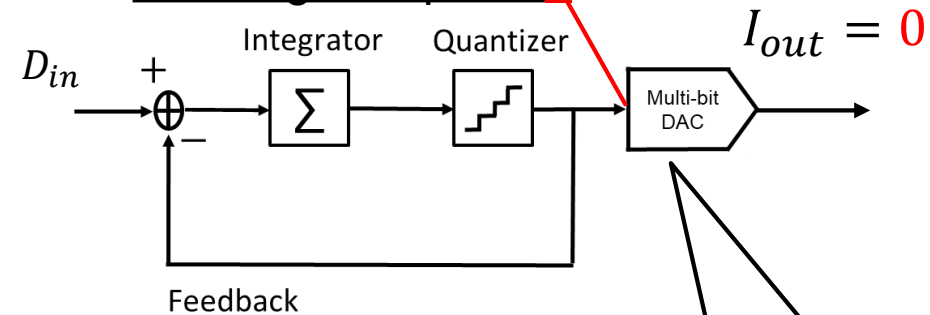
<Usage>

- Electric measurement
- Audio system

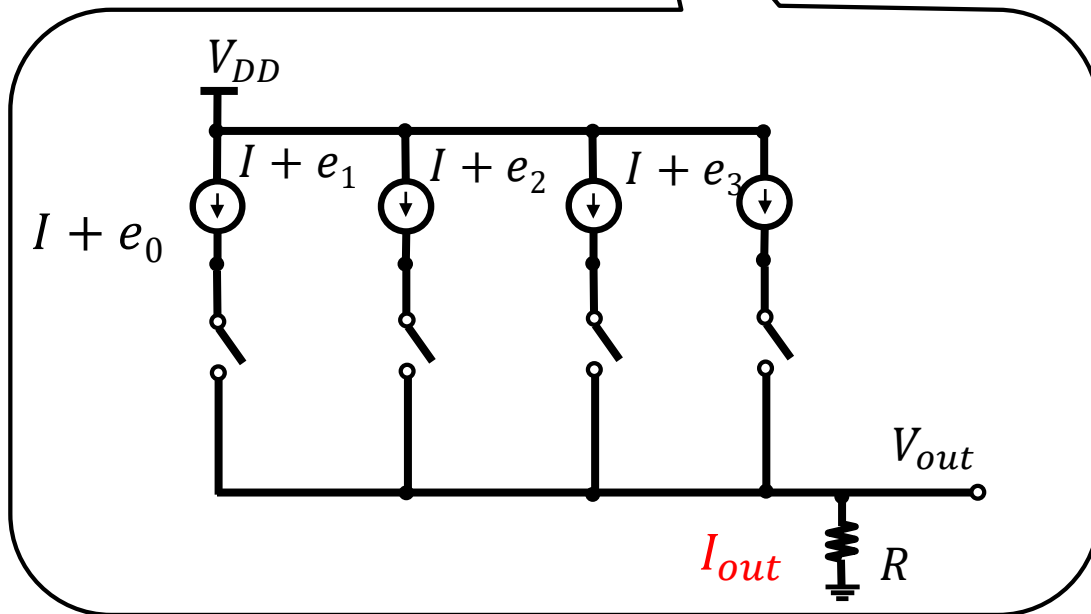


Multi-bit DAC Operation (1/3)

DAC digital input = 0

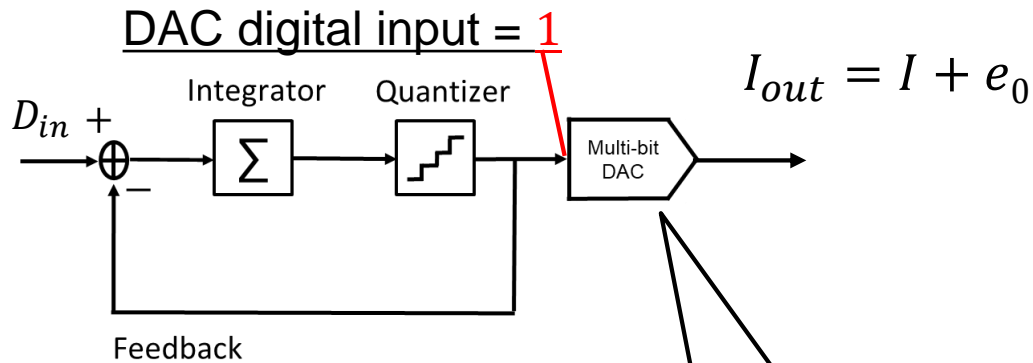


$$\left[\begin{array}{l} \text{Current } I_k = I + e_k \\ e_k : \text{Current source mismatches} \end{array} \right]$$

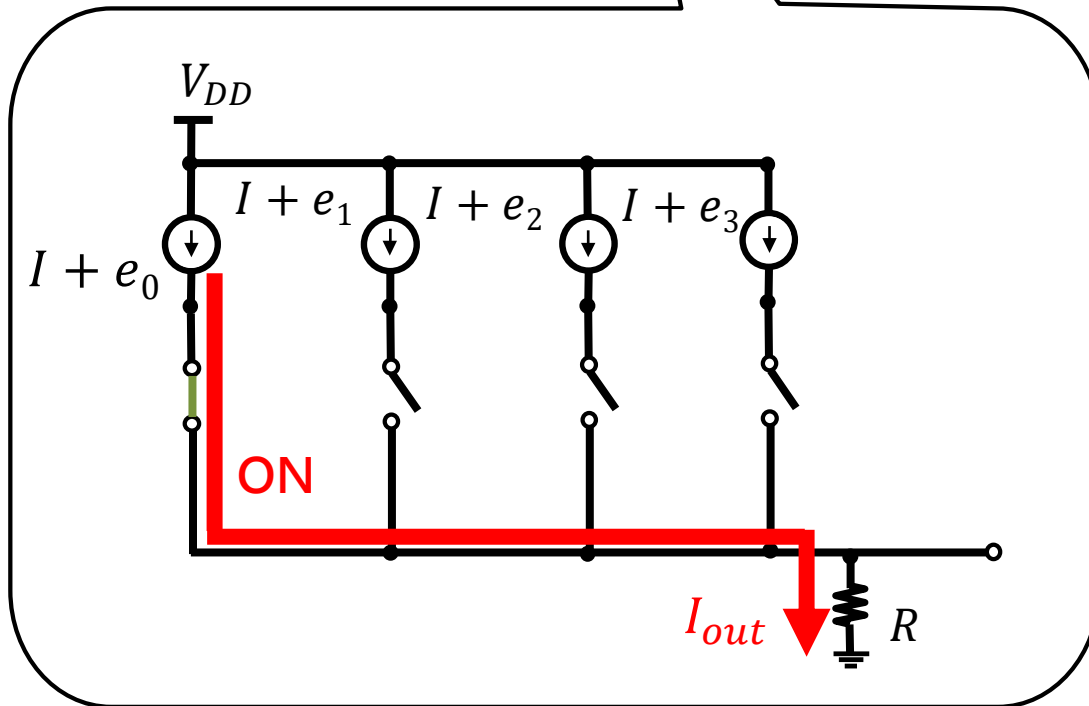


DAC Digital Input	I_{out}
0	0
1	
2	

Multi-bit DAC Operation (2/3)



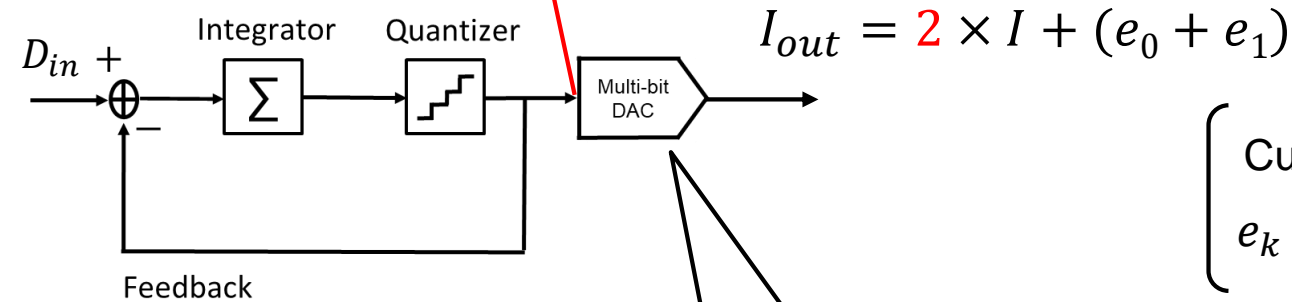
Current $I_k = I + e_k$
 e_k : Current source mismatches



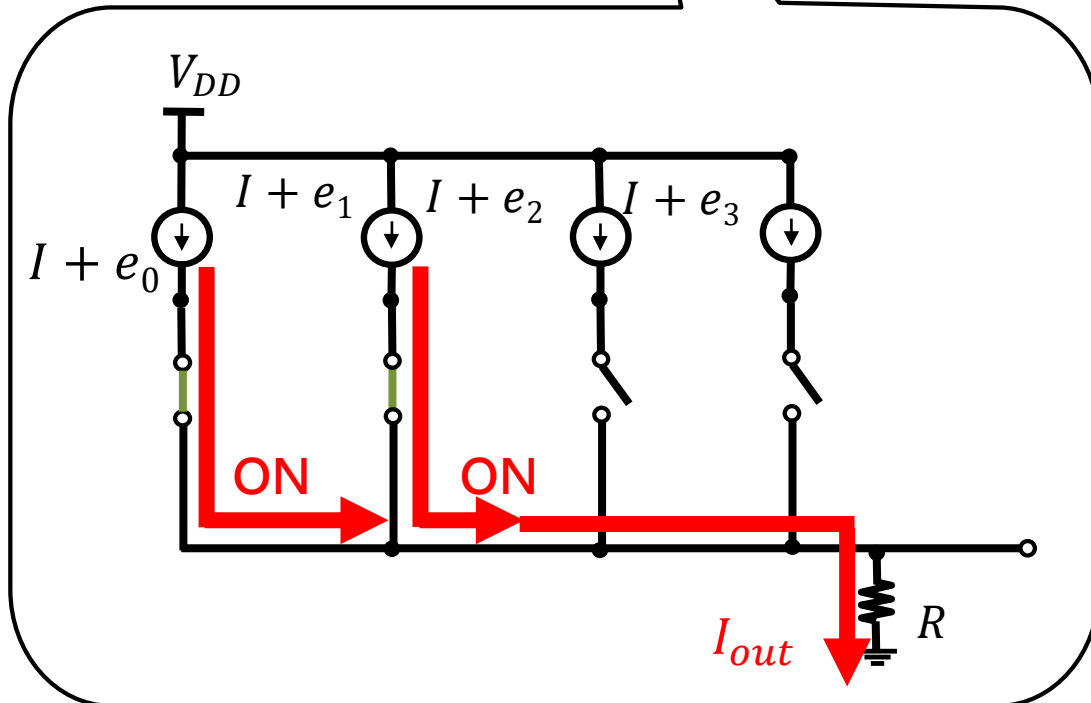
DAC Digital Input	I_{out}
0	0
1	$I + e_0$
2	

Multi-bit DAC Operation (2/3)

DAC digital input = 2



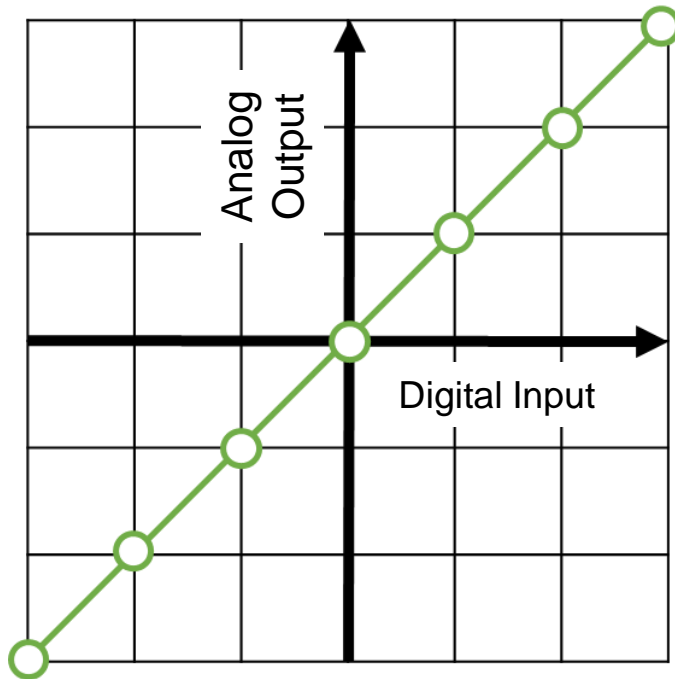
Current $I_k = I + e_k$
 e_k : Current source mismatches



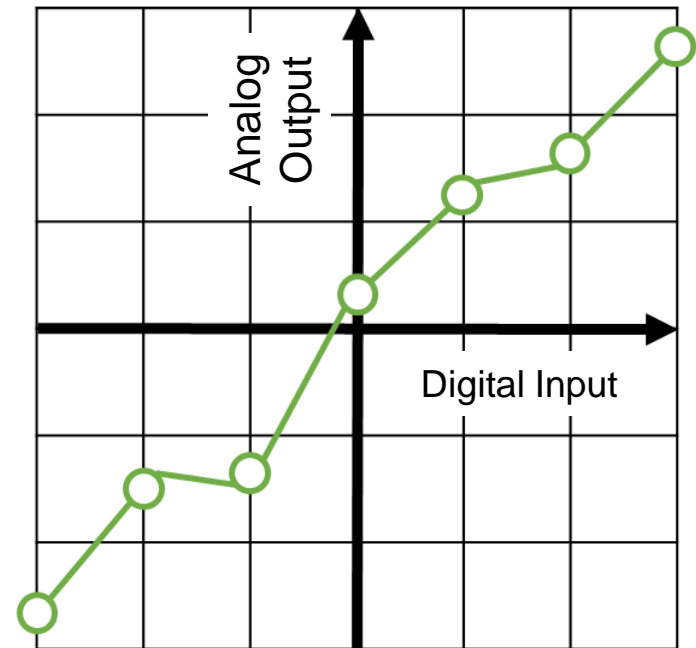
DAC input Digital	I_{out}
0	0
1	$I + e_0$
2	$2 \times I + (e_0 + e_1)$

Nonlinearity Problem of Multi-bit $\Delta\Sigma$ DAC

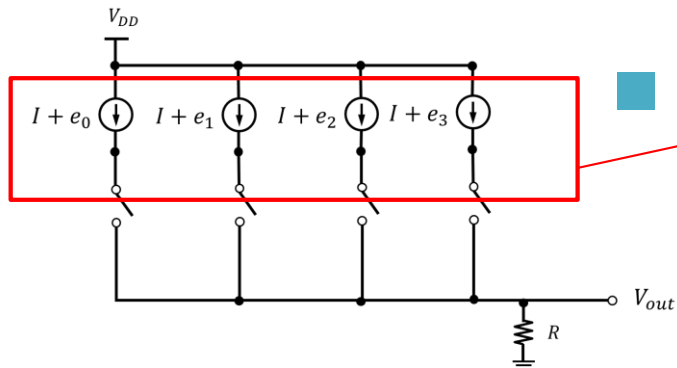
Ideal DAC



Real DAC



○ Multi-bit Output



■ **Currents** are different due to process variation inside IC chip

➤ $\Delta\Sigma$ DAC Nonlinearity Problem



Outline

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◆ **$\Delta\Sigma$ DA converter**

➤ **DWA*Algorithm (* Data-Weighted Averaging)**

➤ Unit Cell Re-ordering

◆ Simulation verification

➤ 2nd - order $\Delta\Sigma$ DA converter : LP type

➤ 2nd - order $\Delta\Sigma$ DA converter : HP type

◆ Conclusion

Unit Cell Current Average I

Unit cell current average : $I = \frac{1}{4}(I_0 + I_1 + I_2 + I_3)$

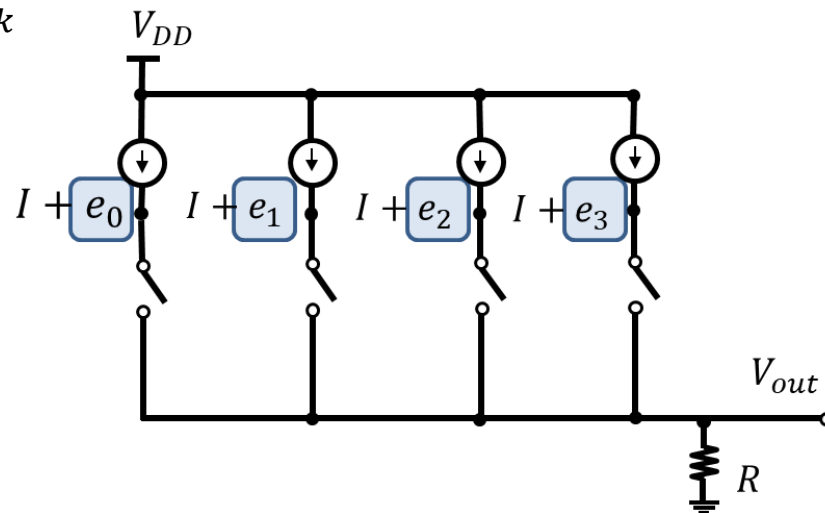
$$\begin{aligned} I_0 &= I + e_0 \\ I_1 &= I + e_1 \\ I_2 &= I + e_2 \\ I_3 &= I + e_3 \end{aligned}$$

total sum

$$(I_0 + I_1 + I_2 + I_3) = 4I + (e_0 + e_1 + e_2 + e_3)$$

$$I = \frac{1}{4}(I_0 + I_1 + I_2 + I_3) - \frac{1}{4}(e_0 + e_1 + e_2 + e_3)$$

mismatch : e_k



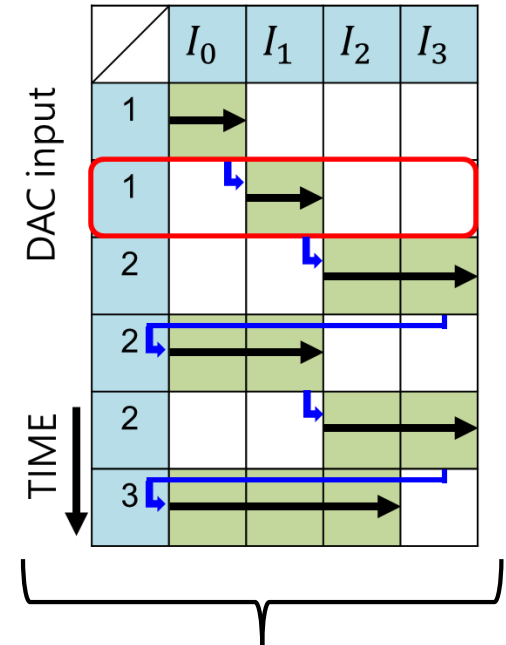
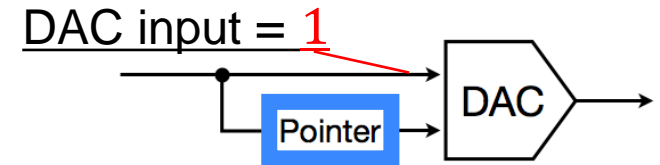
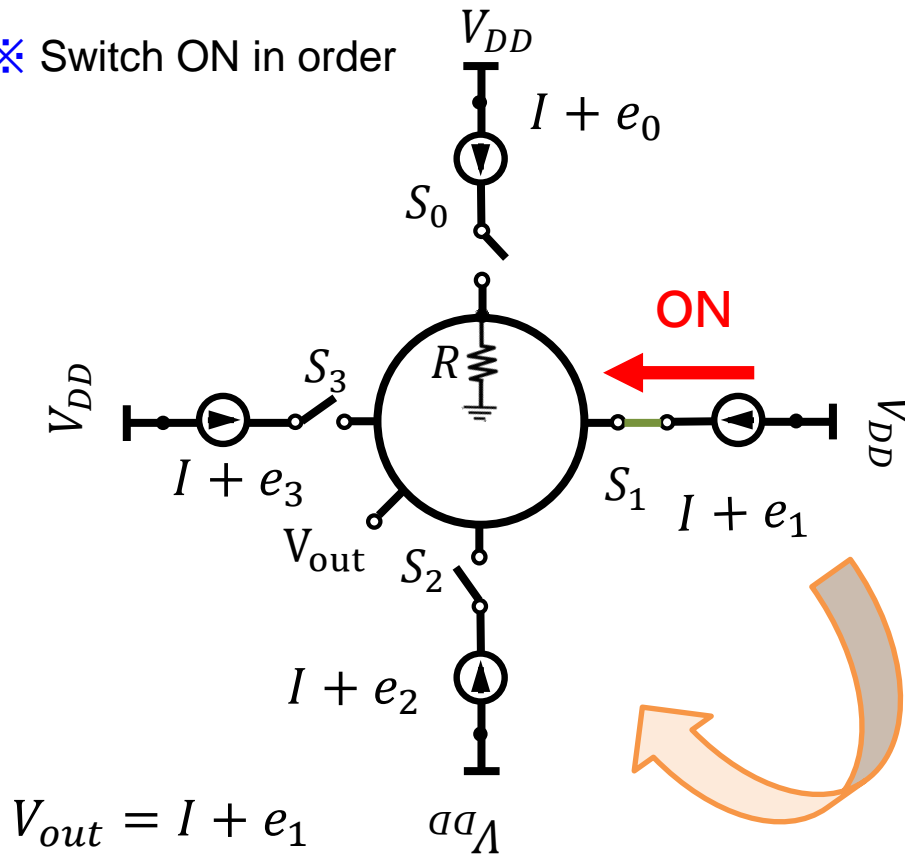
0

Multi-bit DAC + DWA I (1/2)

◆ DWA* DAC (*Data-Weighted Average)

$$I_{out} = I + e_1$$

※ Switch ON in order



Disperse mismatch of particular cells
⇒ Time-averaged

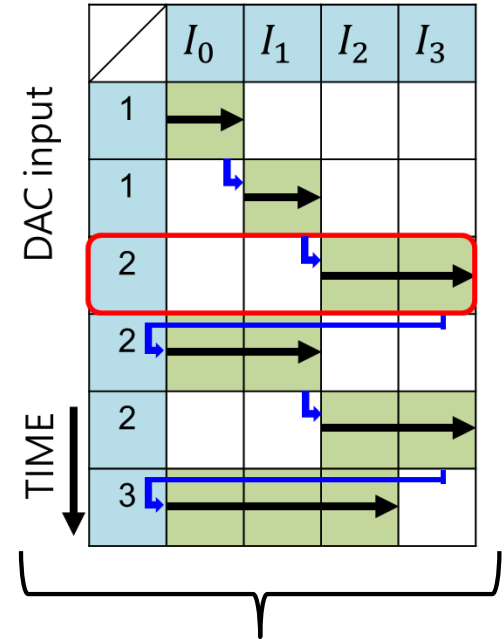
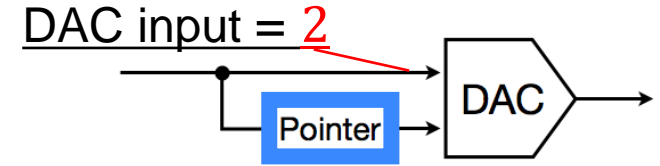
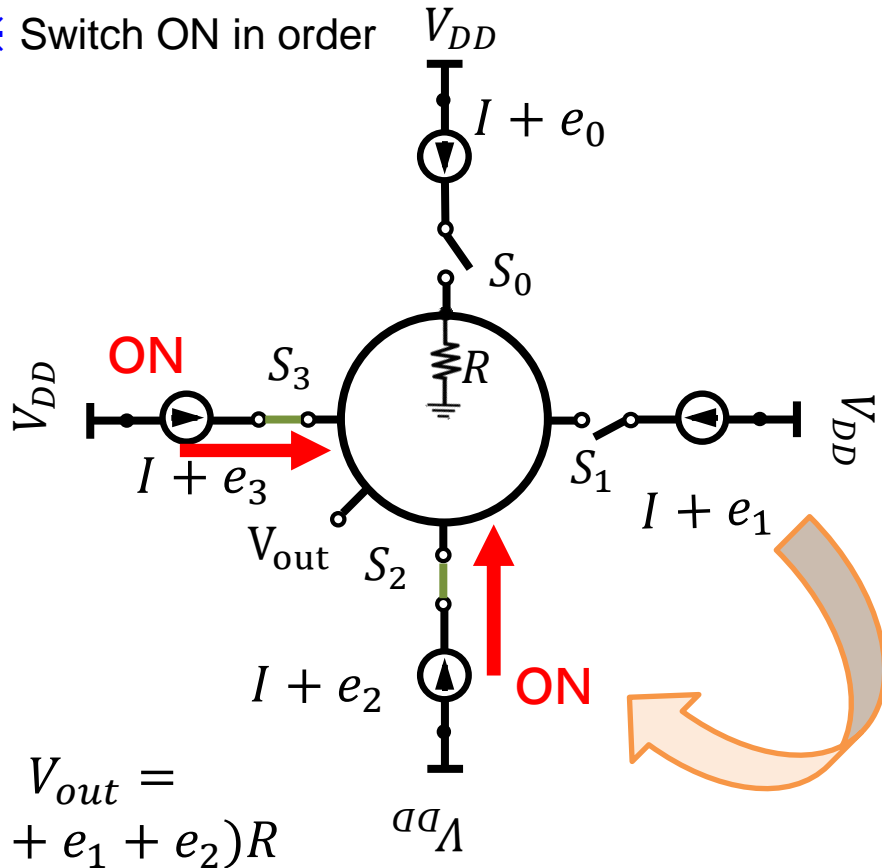
➔ Linearity improvement 😊

Multi-bit DAC + DWA I (2/2)

◆ DWA* DAC (*Data-Weighted Average)

$$I_{out} = 2 \times I + (e_1 + e_2)$$

✘ Switch ON in order



Disperse mismatch of particular cells
 \Rightarrow Time-averaged

➡ Linearity improvement 😊

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➤ Unit Cell Re-ordering

◆ Simulation verification

➤ 2nd - order $\Delta\Sigma$ DA converter : LP type

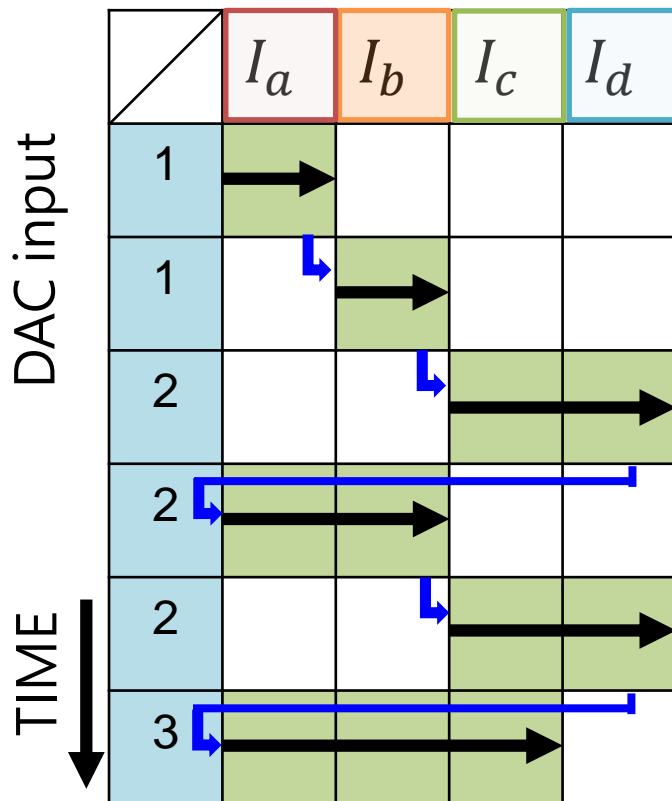
➤ 2nd - order $\Delta\Sigma$ DA converter : HP type

◆ Conclusion

Unit Cell Re-ordering

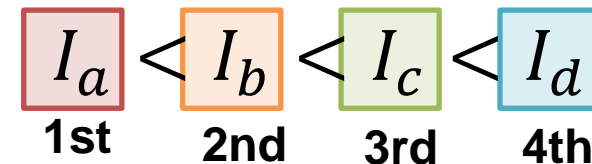
Multi-bit $\Delta\Sigma$ DAC

Measure magnitude relation of current cells using comparator
Perform rearrangement in software



The smallest

The largest



Combination of four unit cells

There are 24 ways to sort at 5 levels

Analog output : $0, I, 2I, 3I, 4I$

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◆ **Simulation verification**

- **2nd order $\Delta\Sigma$ DA converter : LP type**
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DAC Performance

SNDR

Signal-to-noise and distortion ratio

$$SNDR = \frac{P_{signal}}{P_{noise} + P_{distortion}}$$

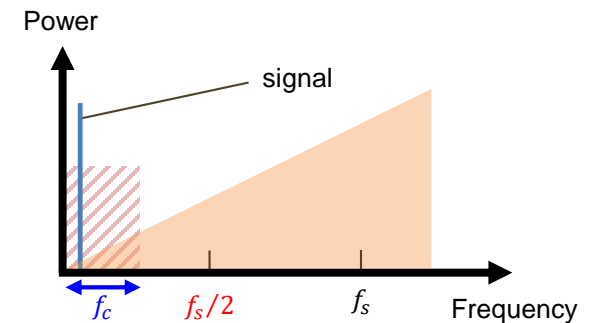
P_{signal} : signal power P_{noise} : noise power $P_{distortion}$: distortion power

OSR

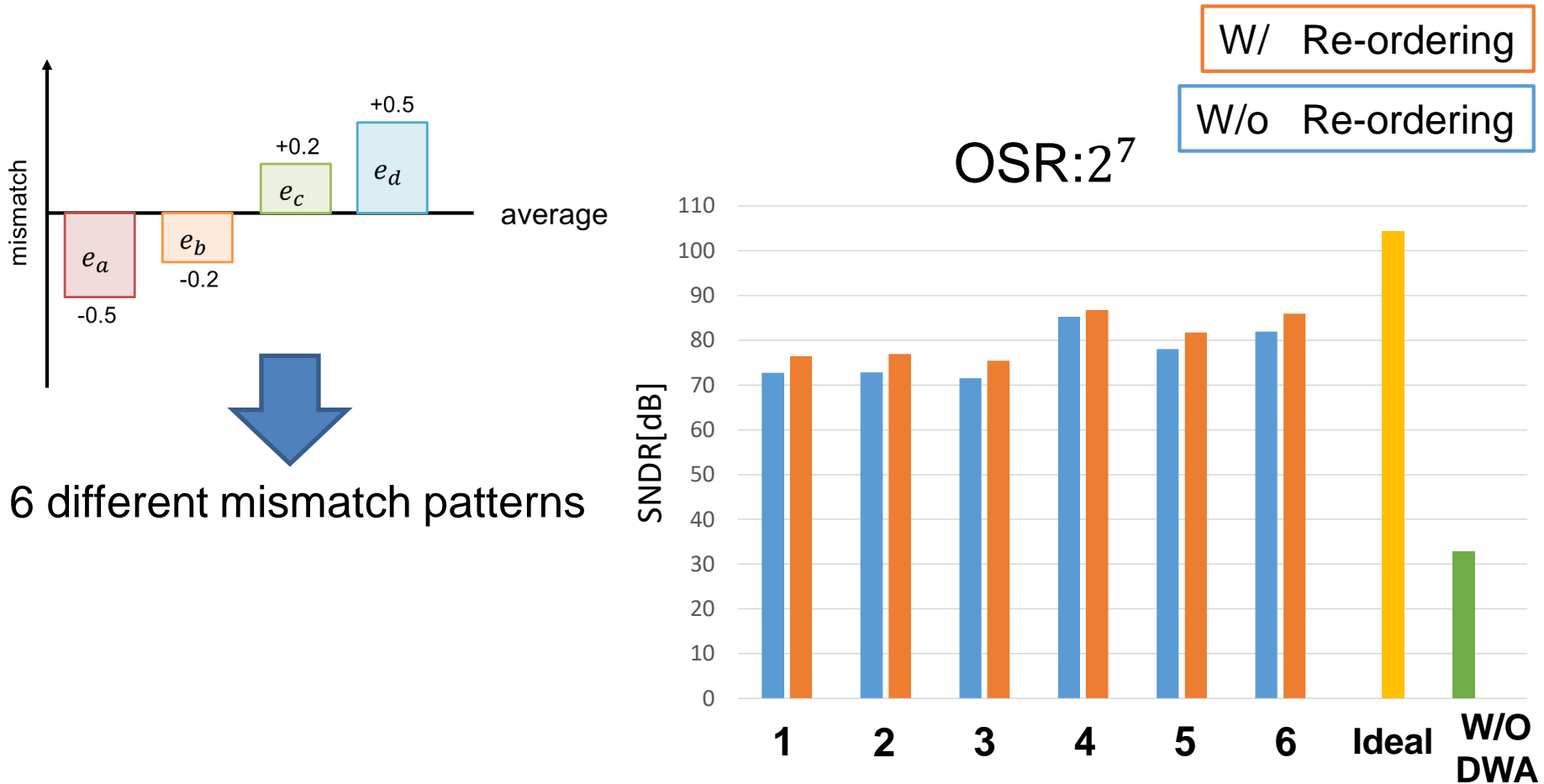
Over Sampling Ratio

$$OSR = \frac{f_s}{2f_c}$$

f_s : sampling frequency f_c : signal band



2nd-order $\Delta\Sigma$ DA Converter : LP type (1/3)



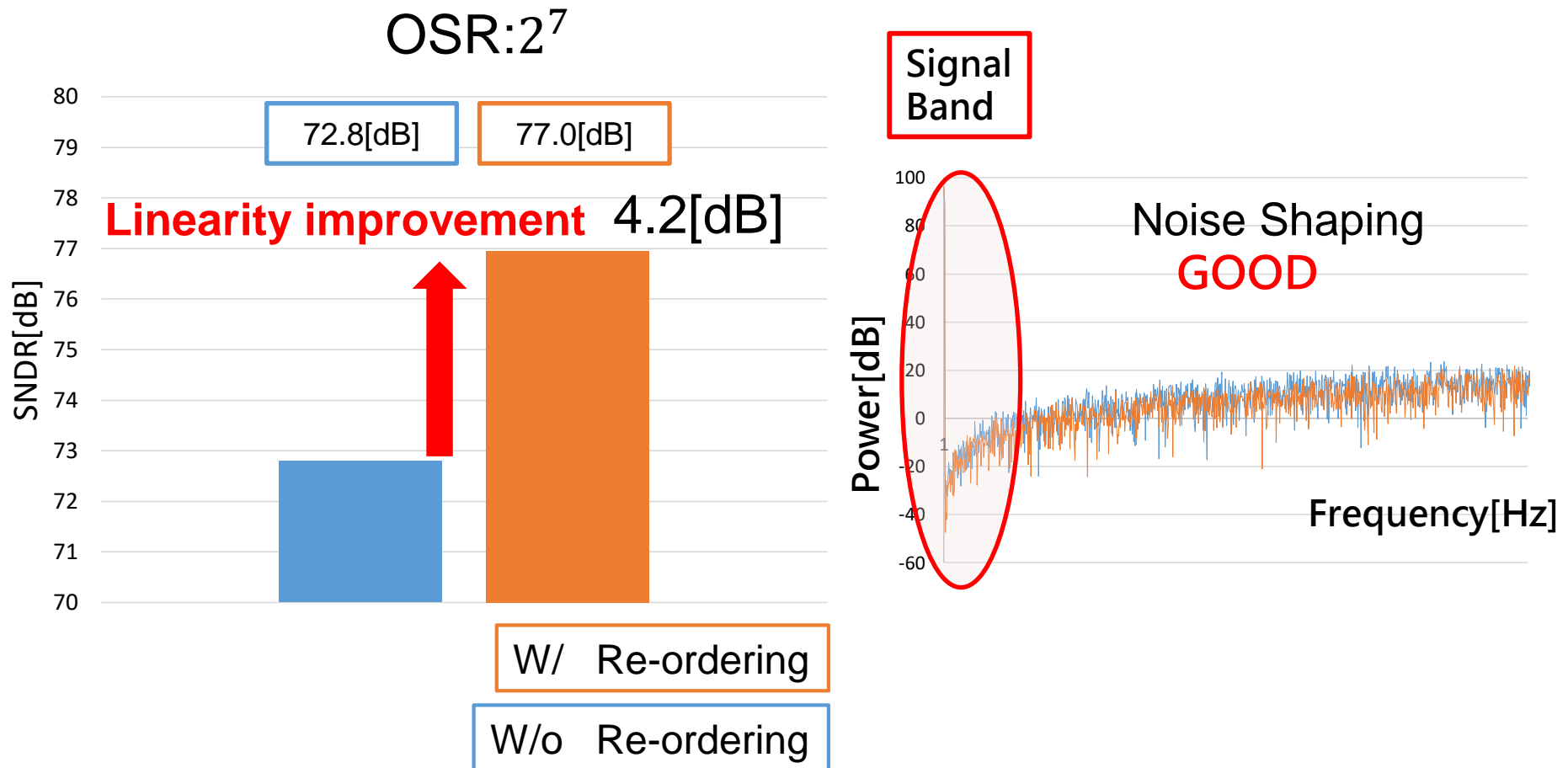
When 1st & 2nd , 2nd & 1st , 3rd & 4th , or 4th and 3rd
in order of combination is included



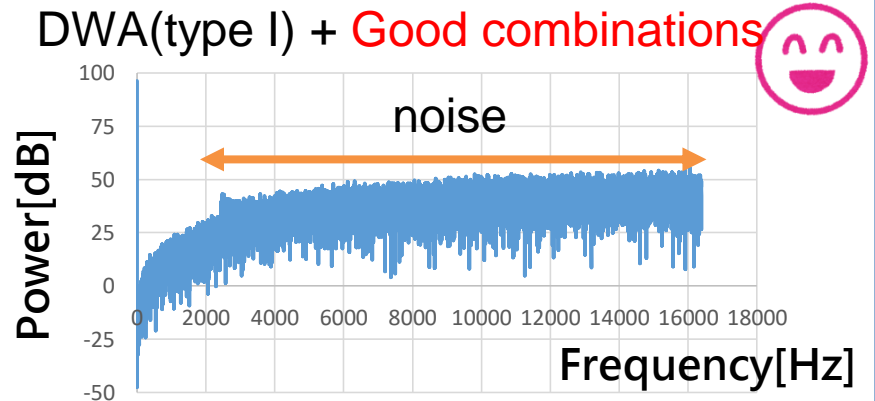
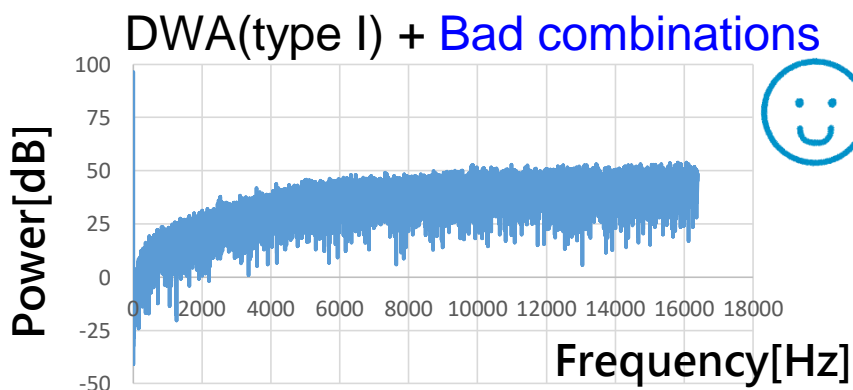
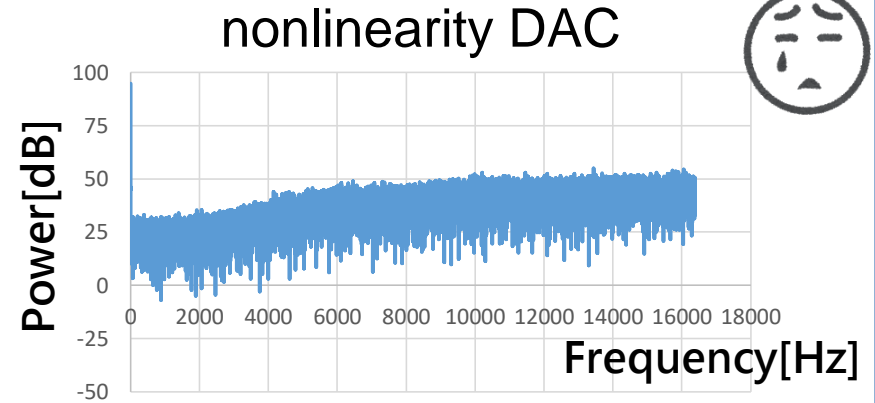
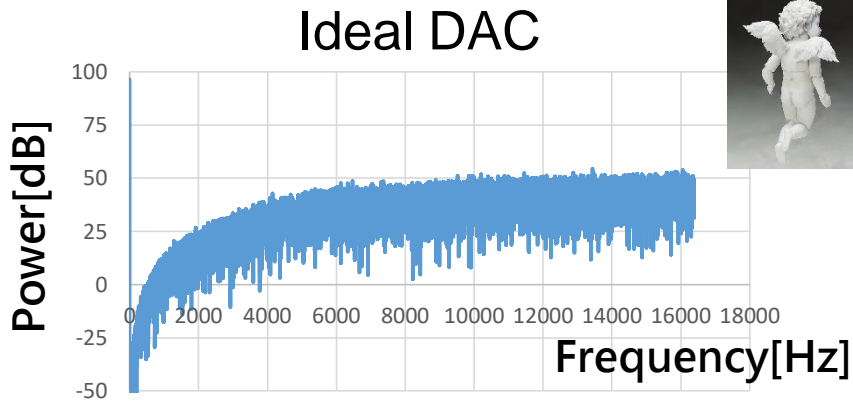
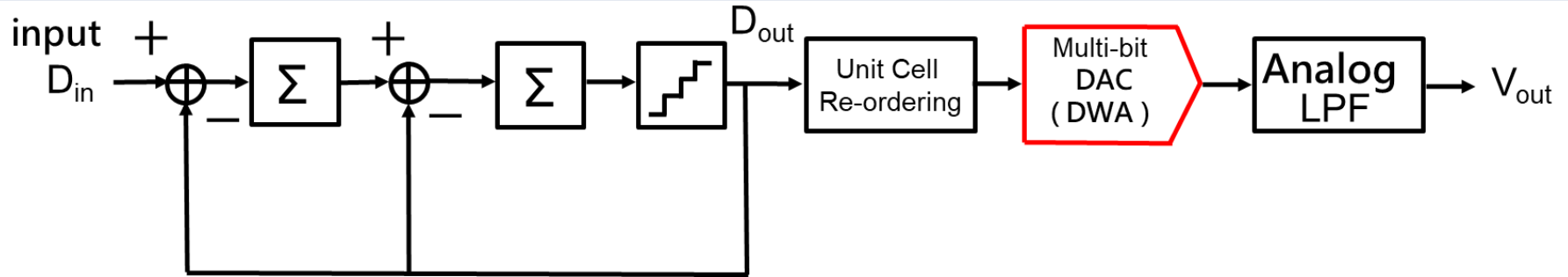
Good SNDR

2nd-order $\Delta\Sigma$ DA Converter : LP type (2/3)

Deviation from the average current among unit current cells
Pattern 2



2nd-order $\Delta\Sigma$ DA Converter : LP type (3/3)



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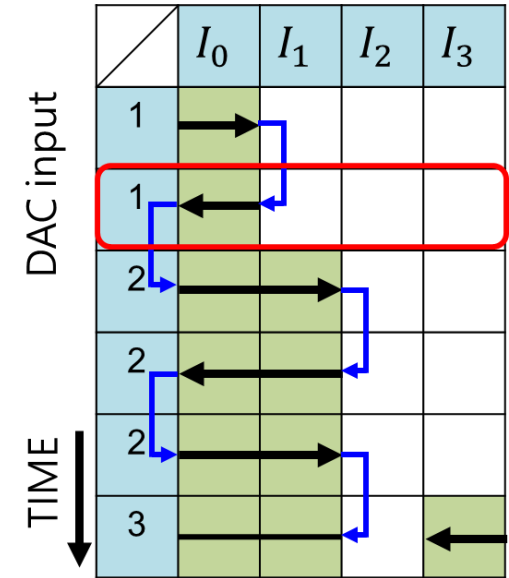
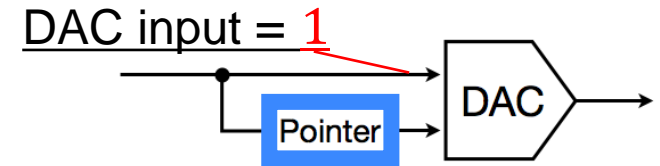
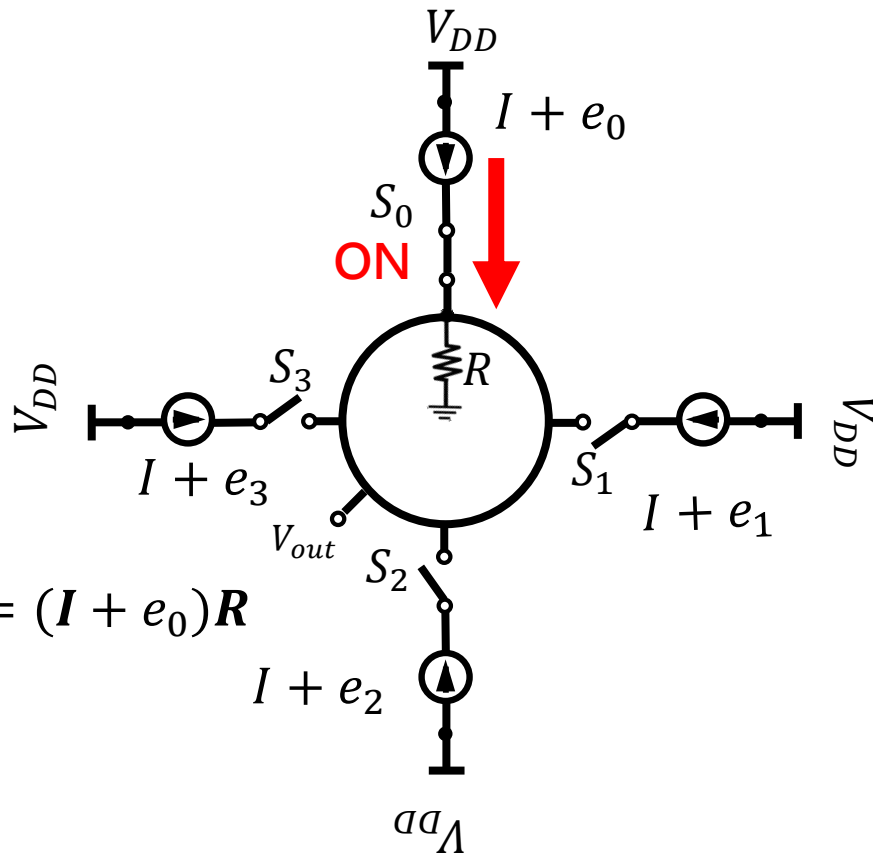
Multi-bit DAC + DWA II (1/2)

◆ DWA* II DAC (*Data-Weighted Average)

DAC input = 1

$$I_{out} = I + e_0$$

※ Switch ON in order



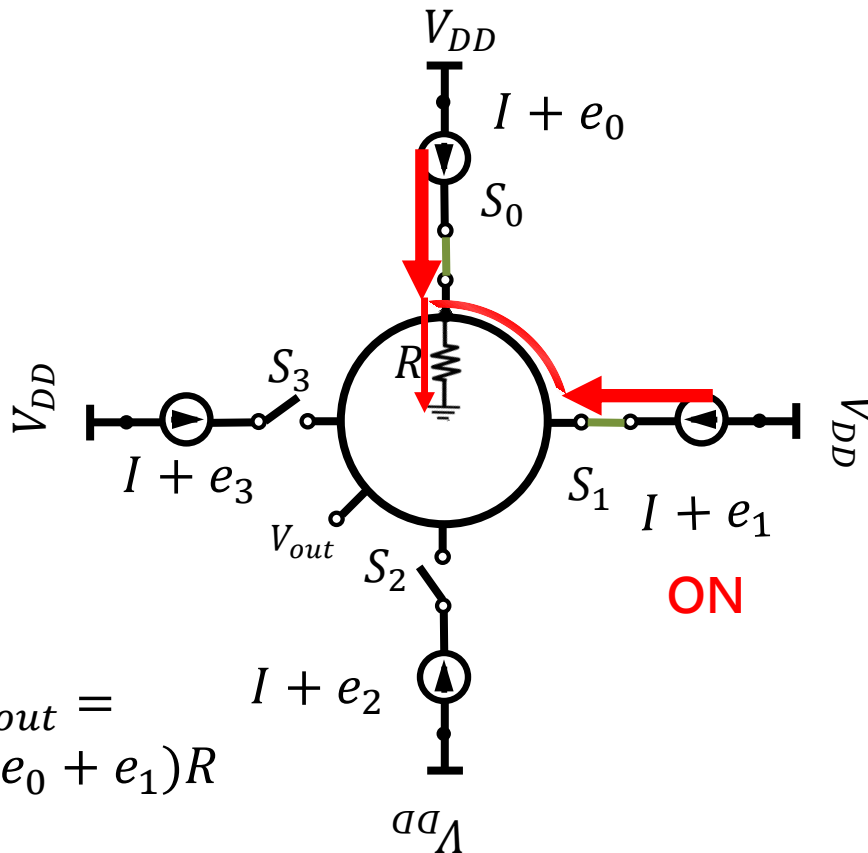
Back and forth

Multi-bit DAC + DWA II (2/2)

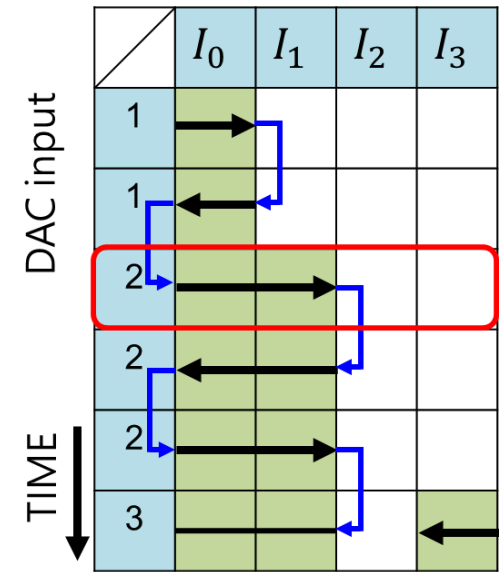
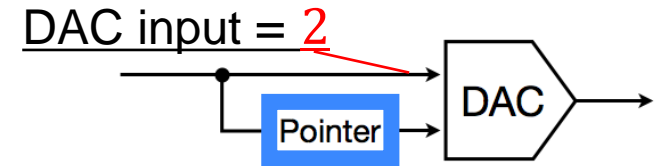
◆ DWA* II DAC (*Data-Weighted Average)

DAC input = 2 $I_{out} = 2 \times I + (e_0 + e_1)$

※ Switch ON in order

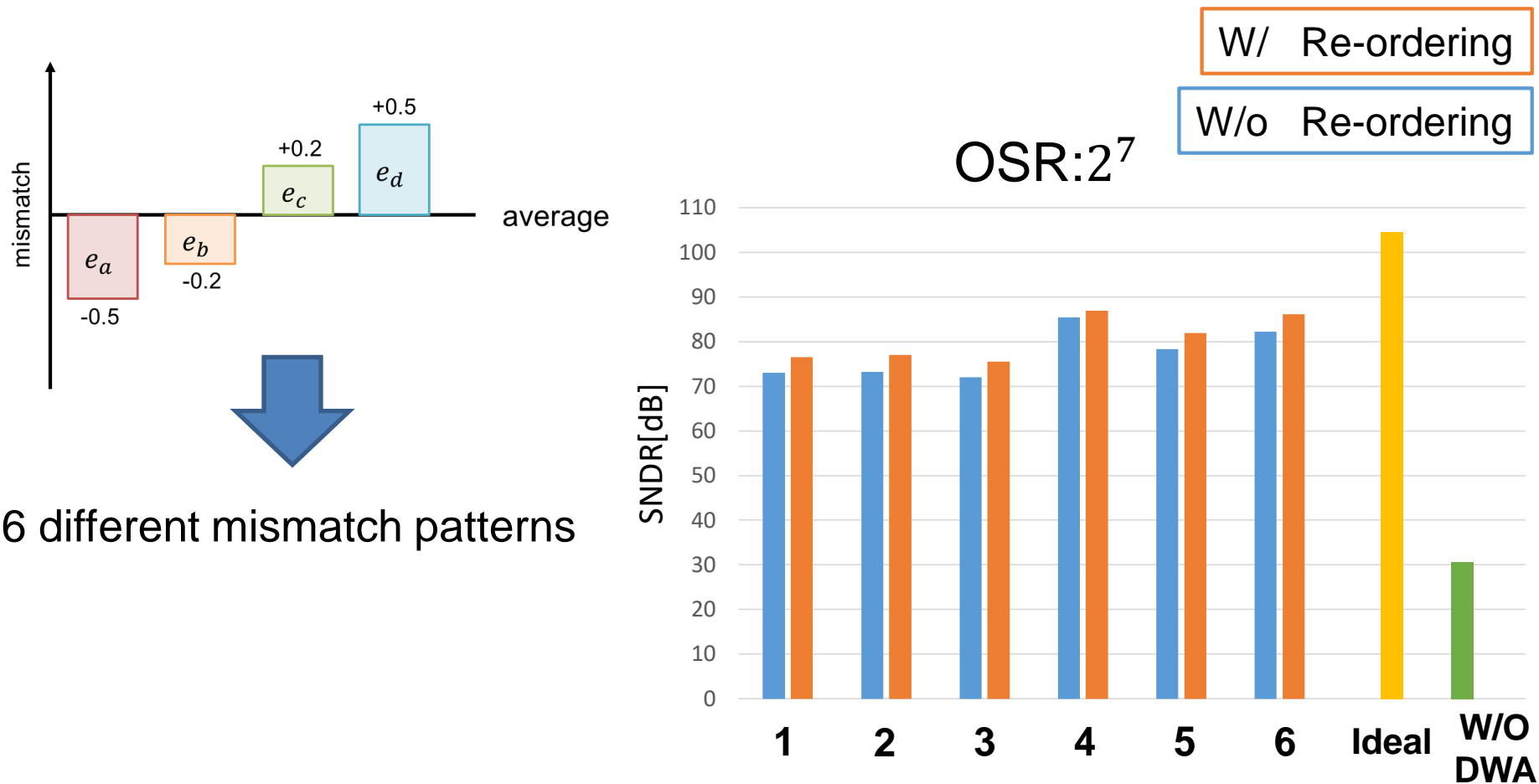


$$V_{out} = (2I + e_0 + e_1)R$$



Back and forth

2nd-order $\Delta\Sigma$ DA Converter : HP type (1/3)



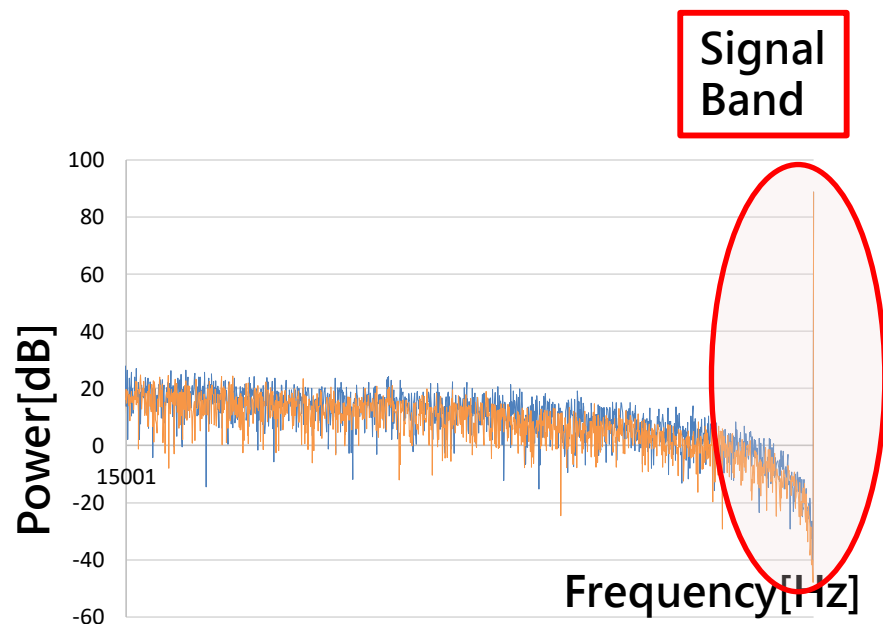
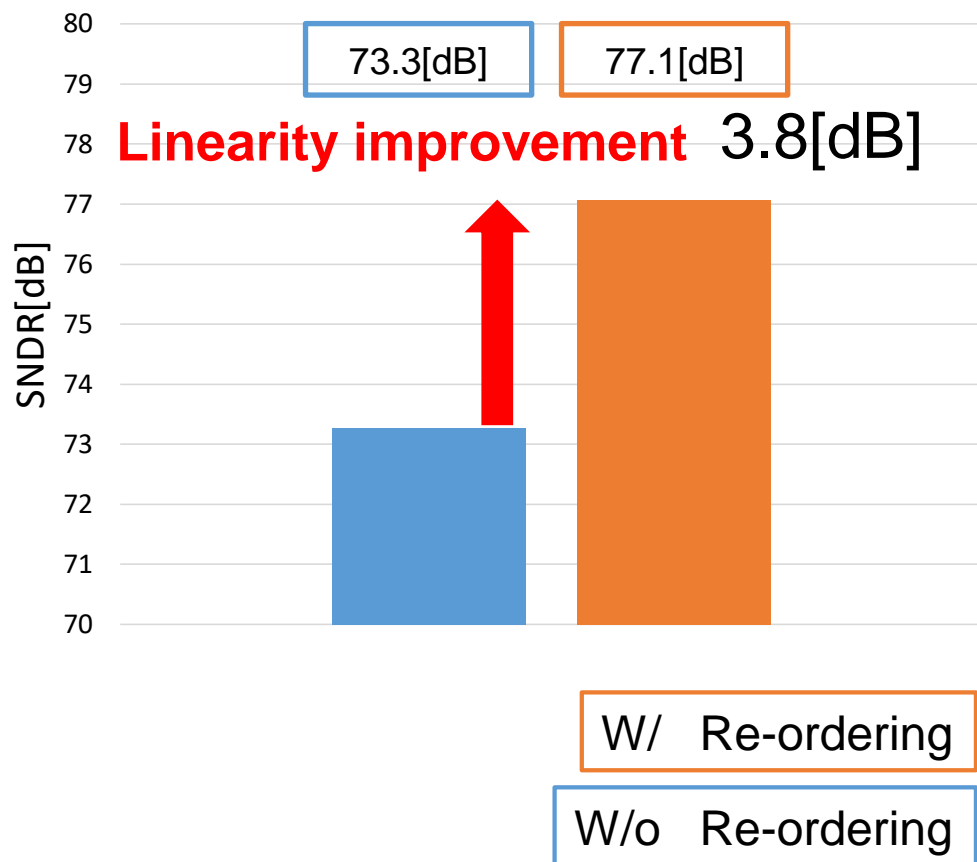
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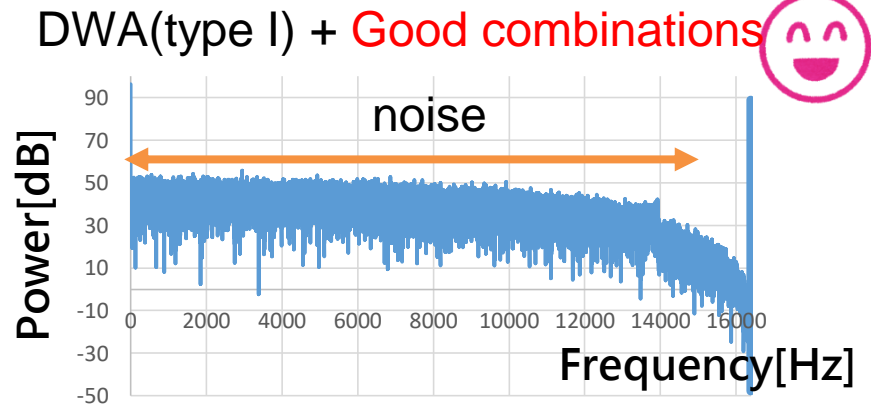
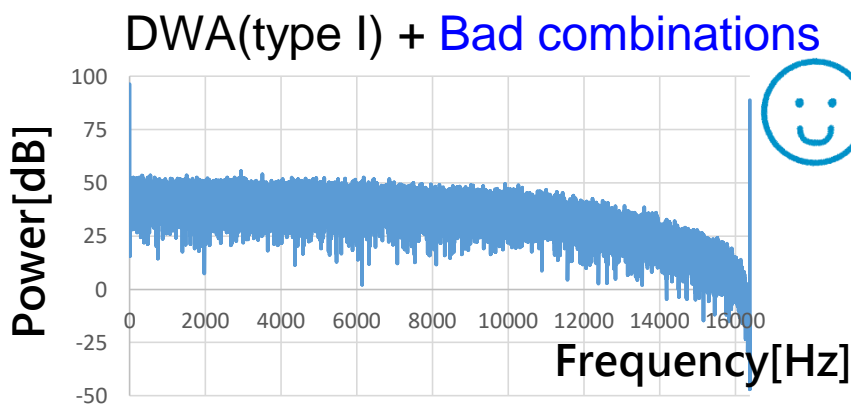
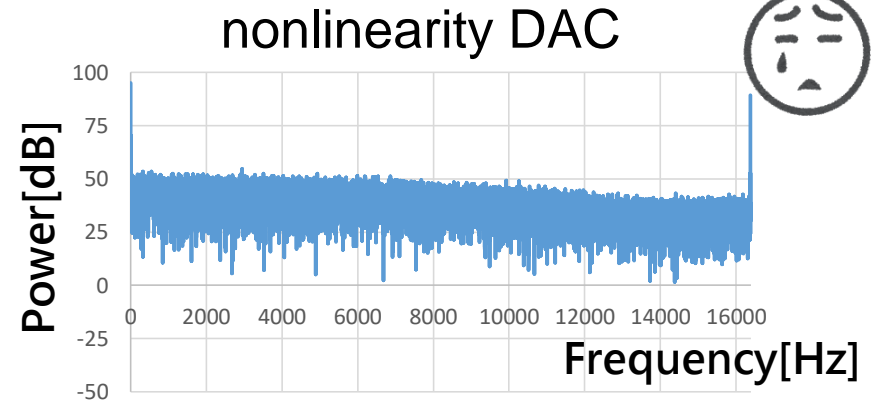
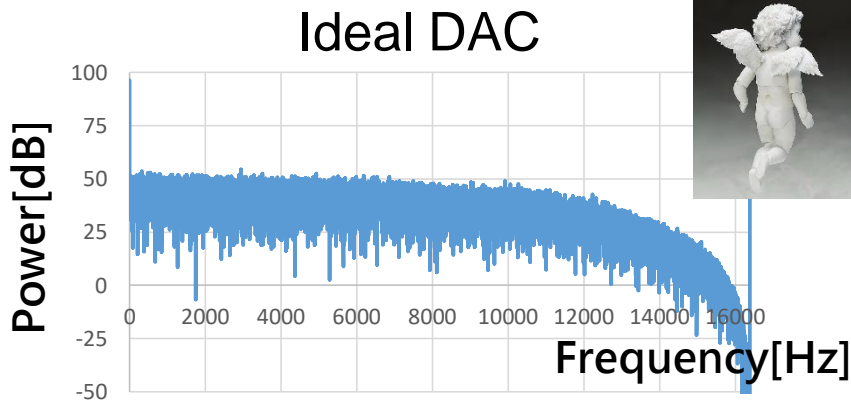
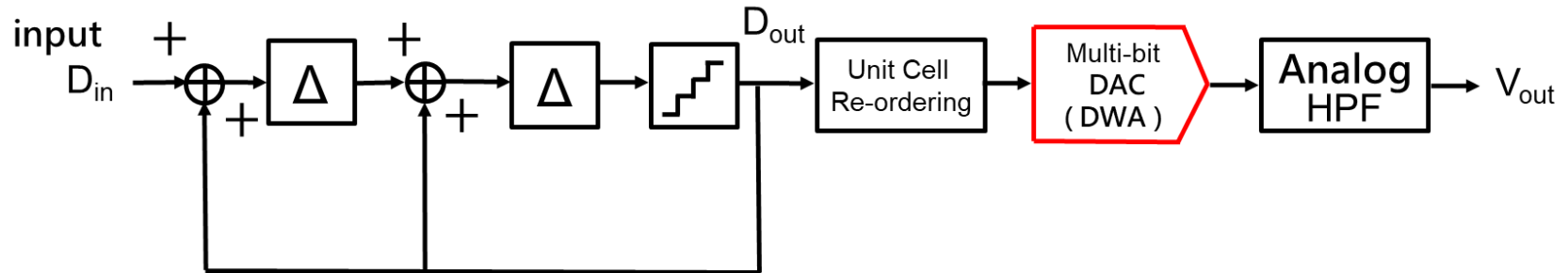
Good SNDR

2nd-order $\Delta\Sigma$ DA Converter : HP type (2/3)

Deviation from the average current among unit current cells
Pattern 2



2nd-order $\Delta\Sigma$ DA Converter : HP type (3/3)



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Conclusion

- Multi-bit $\Delta\Sigma$ DAC

Conventional:

Not using anything or using only DWA



Proposed:

Using **both unit cell re-ordering and DWA**



finding

Significant linearity improvement

