

中国人の剰余定理(剰余系)

兵士数を数えるのに使用

群馬大学 阿部優大、小林春夫

Chinese Remainder Theorem



Sun Tzu

Chinese arithmetic book 'Sun Tzu calculation'

孫子算經

“When dividing by 3, its residue is 2,
dividing by 5, its residue is 3,
dividing by 7, its residue is 2.
What is the original number ?”

Answer 23

Generalization



Chinese Remainder Theorem



Sun Tzu calculation

How to use Chinese remainder theorem

He used to quickly find out how many soldiers there are.



Sun Tzu

“Divide into 3.”

Remainder : 2



...



How to use Chinese remainder theorem

He used to quickly find out how many soldiers there are.



Sun Tzu

“Divide into 5.”

Remainder : 3



How to use Chinese remainder theorem

He used to quickly find out how many soldiers there are.



“Divide into 7.”

Sun Tzu



How to use Chinese remainder theorem

He used to quickly find out how many soldiers there are.



Sun Tzu

“There are 23 people in all according to Chinese remainder theorem”



Example of Residue Number System

$$23 \% 3 = 2, \quad 23 \% 5 = 3, \quad 23 \% 7 = 2$$

- Natural numbers
3, 5, 7 (*relatively prime*)
 $N = 3 \times 5 \times 7 = 105$
- k ($0 \leq k \leq N-1 (=104)$)

a : Remainder of k dividing by 3 $a = \text{mod}3(k)$
 b : Remainder of k dividing by 5 $b = \text{mod}5(k)$
 c : Remainder of k dividing by 7 $c = \text{mod}7(k)$

$k \longleftrightarrow (a, b, c)$

one to one

Chinese remainder theorem

a	b	c	k
0	0	1	15
1	1	2	16
2	2	3	17
0	3	4	18
1	4	5	19
2	0	6	20
0	1	0	21
1	2	1	22
2	3	2	23
0	4	3	24
1	0	4	25
2	1	5	26
0	2	6	27
1	3	0	28
2	4	1	29

Residue number system

解説

孫子算経は兵法家の孫子(孫武)より
ずいぶん前の書で、直接は関係ないようである。

<https://kobaweb.ei.st.gunma-u.ac.jp/news/pdf/2014/2014-07-30joyo.pdf>

が、ここでは孫子が兵士の数を素早く数える
という話にした。