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Oct. 30, 2019 (Wed)

Invited

Analog / Mixed-Signal / RF Circuits for Complex Signal Processing

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Outline

- Motivation for Complex Signal Processing Research
- RC Polyphase Filter: Transfer Function
- RC Polyphase Filter: Flat Passband Gain Algorithm
- RC Polyphase Filter and Hilbert Filter
- Active Complex Bandpass Filters
- Complex Bandpass ΔΣ AD Modulator
- Complex Multi-Bandpass ΔΣ DA Modulator
- Conclusion

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- Conclusion

Why My Research for Complex Signal Processing ?

About 15 years ago

at IEEE International Solid-State Circuits Conference

San Francisco, CA

The most prestigious conference in IC design

Katholieke Universiteit Leuven (KU Leuven), Belgium

World top research group in analog IC design

presentation

Some simple circuit
with curious characteristics
(RC polyphase filter)

However,
I could not understand its principle

Complex Signal

There is NO physical complex signal.
It is only defined mathematically.

2 real signals: I, Q

I: In-Phase, Q: Quadrature-Phase

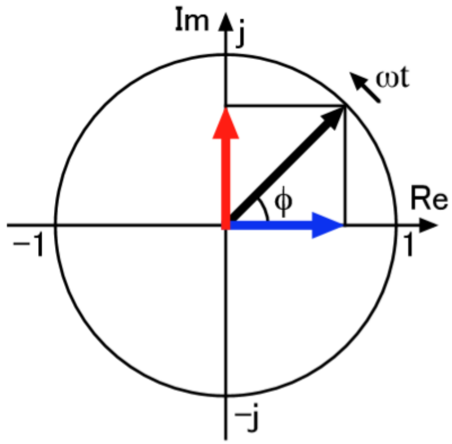
$V_{signal} = I + jQ$ Complex Signal

$V_{image} = I - jQ$ Image

$\Updownarrow$

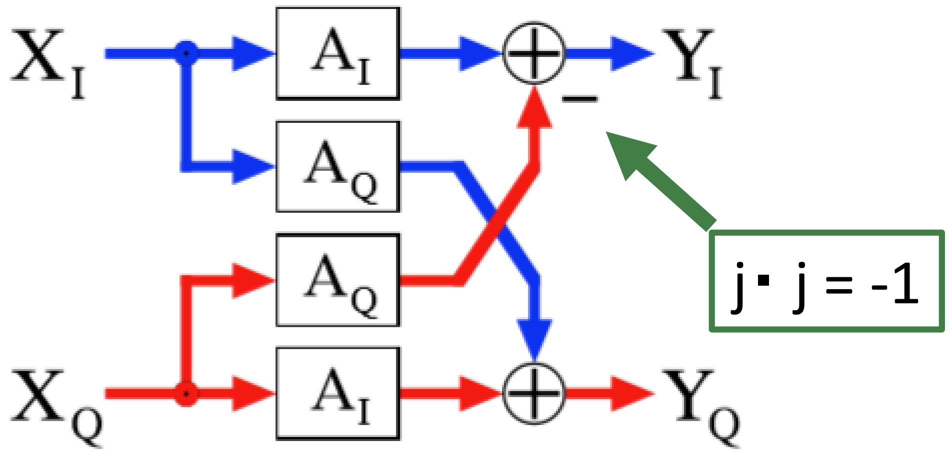
$I = [V_{signal} + V_{image}]/2$

$Q = [V_{signal} - V_{image}]/(2j)$



Gauss plane

Basic Complex Signal Processing Block



$$\dot{Y} = \dot{A} \cdot \dot{X}$$

$$\begin{aligned} Y_I + jY_Q &= (A_I + jA_Q) \cdot (X_I + jX_Q) \\ &= (A_I \cdot X_I - A_Q \cdot X_Q) \\ &\quad + j \cdot (A_I \cdot X_Q + A_Q \cdot X_I) \end{aligned}$$

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- Complex Multi-Bandpass ΔΣ DA Modulator

H. Kobayashi, J. Kang, T. Kitahara, S. Takigami, H. Sakamura, "Explicit Transfer Function of RC Polyphase Filter for Wireless Transceiver Analog Front-End'', IEEE Asia-Pacific Conference on ASICs, Taipei, Taiwan (Aug. 2002).

Goal of First Research

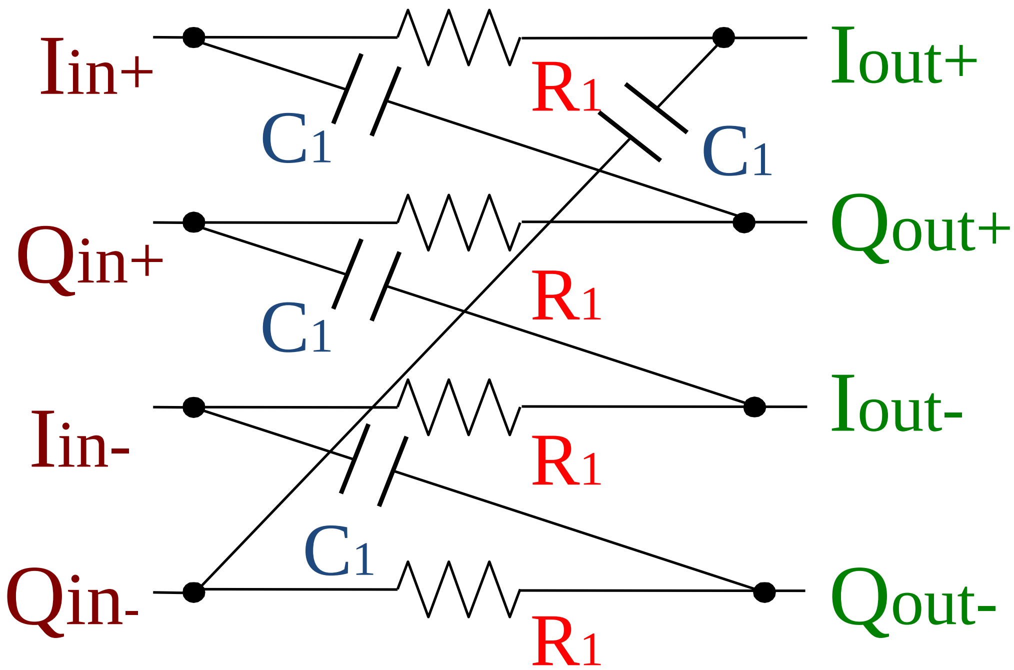
- To establish systematic design and analysis methods of RC polyphase filters.

- As its first step,
  to derive explicit transfer functions of
  the 1st-, 2nd- and 3rd-order
  RC polyphase filters.

Features of RC Polyphase Filter

- Its input and output are complex signal.
- Passive RC analog filter
- One of key components in wireless transceiver analog front-end
  - I, Q signal generation
  - Image rejection
- Its explicit transfer function was NOT derived yet at that time.

First-order RC Polyphase Filter



I: In-Phase, Q: Quadrature-Phase

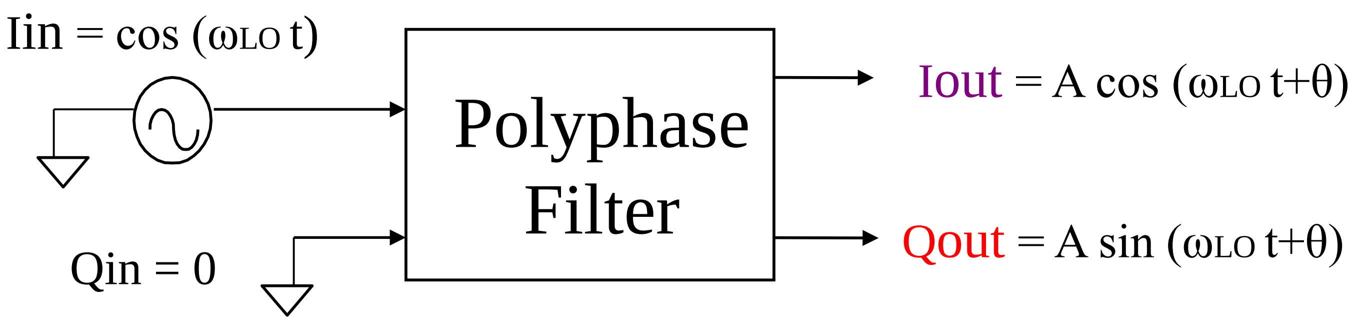
Differential Complex Input: $Vin = Iin + j\,Qin$

Differential Complex Output: $Vout = Iout + j\,Qout$

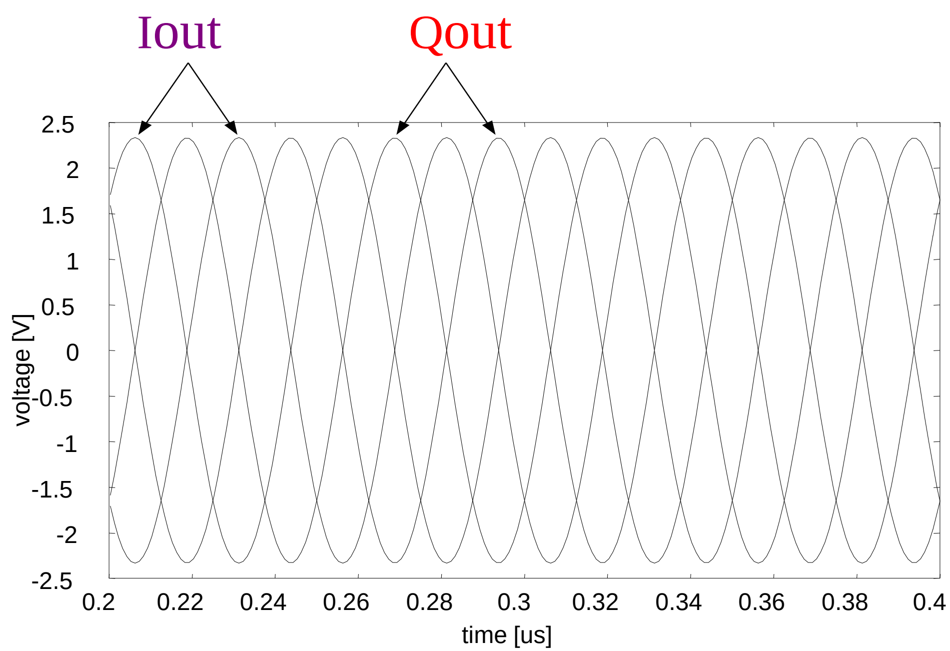
I, Q Signal Generation

Single cosine

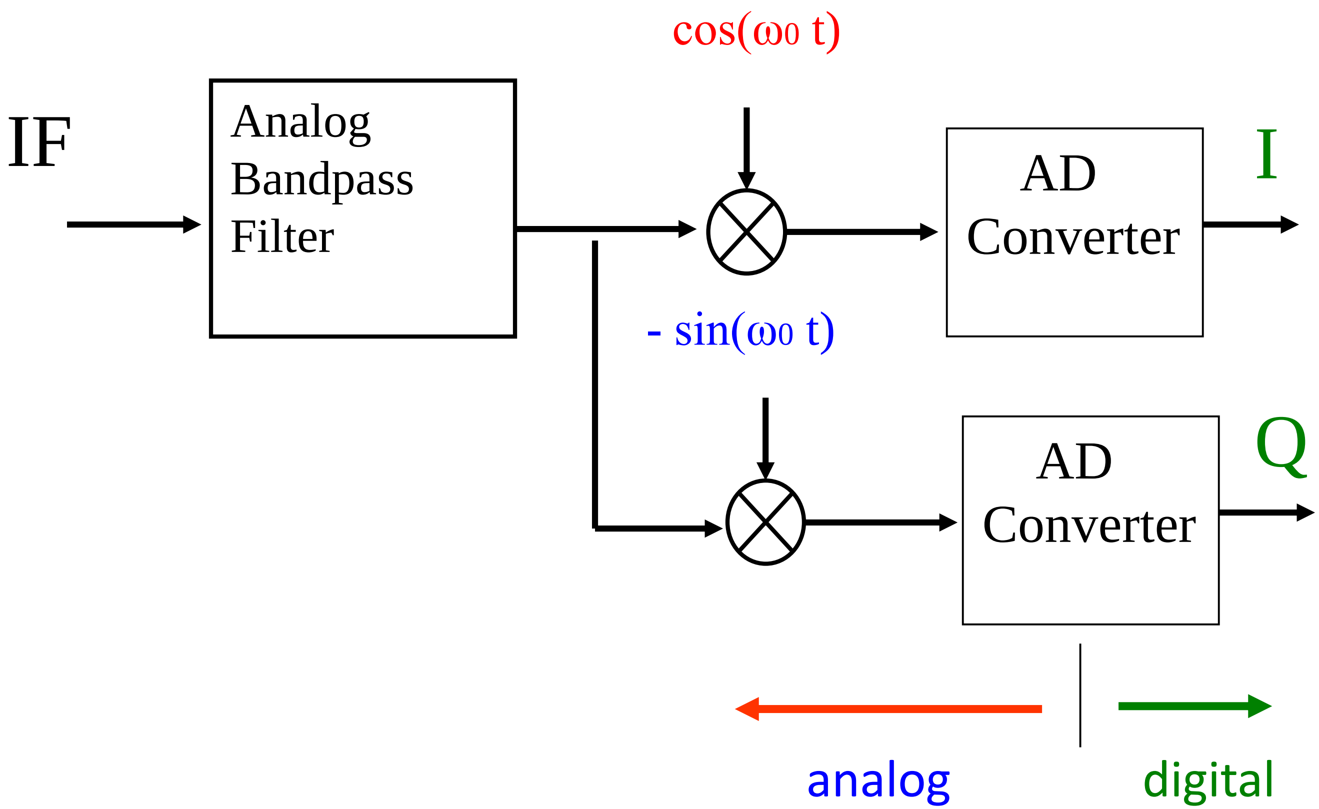
Cosine, Sine signals



$$\omega_{LO} = \frac{1}{R_1 C_1}$$



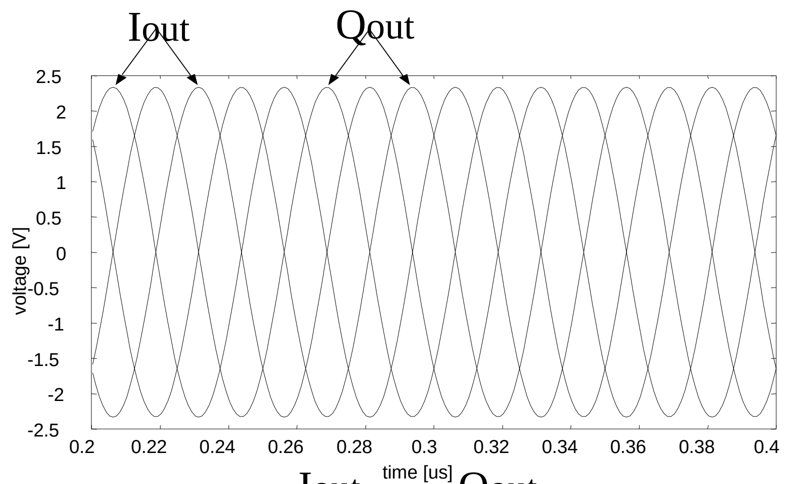
Cosine, Sine Signals in Receiver



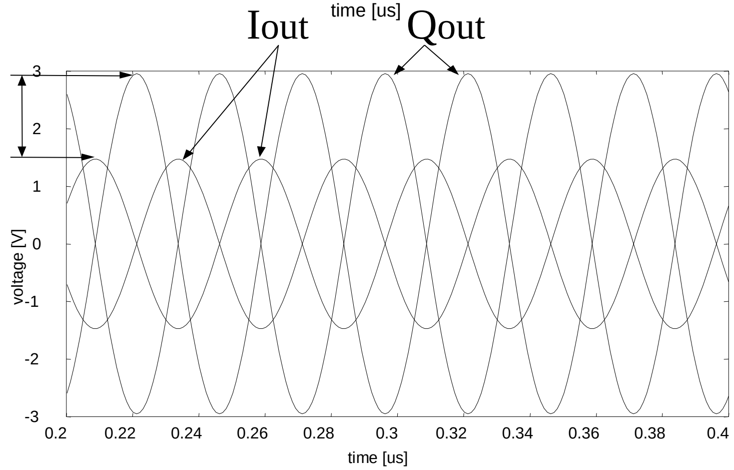
They are used for down conversion

Problem when $\omega_{LO} \neq 1/R_1C_1$

$$\boxed{\omega_{LO} = \frac{1}{R_1C_1}}$$



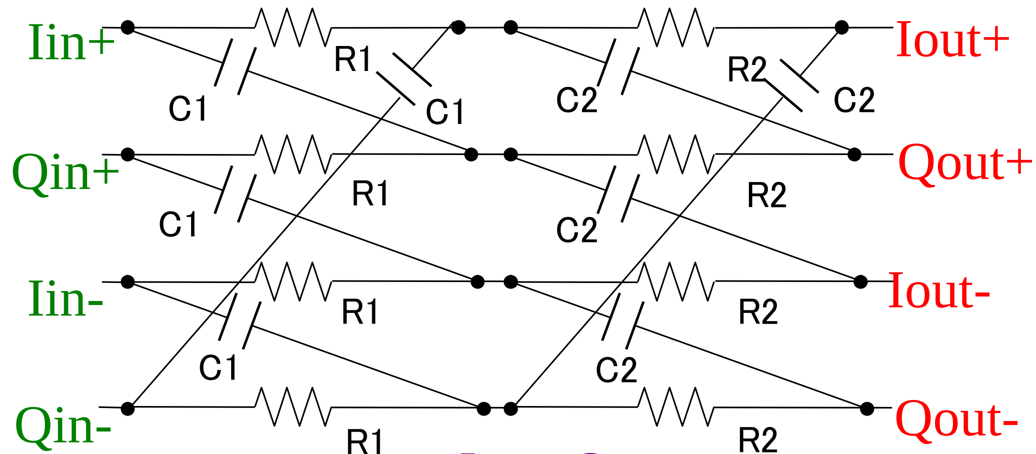
$$\boxed{\omega_{LO} = \frac{2}{R_1C_1}}$$

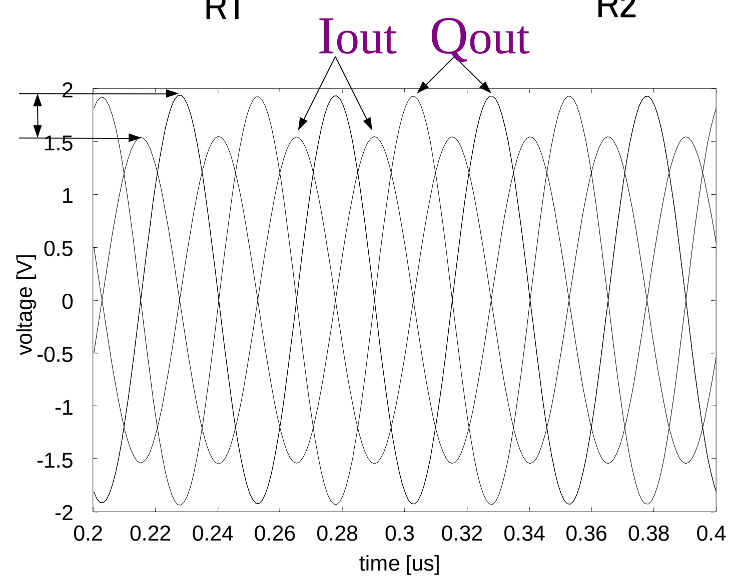


2nd-order RC Polyphase Filter

The problem of large difference between I_{out}, Q_{out} amplitudes can be alleviated.

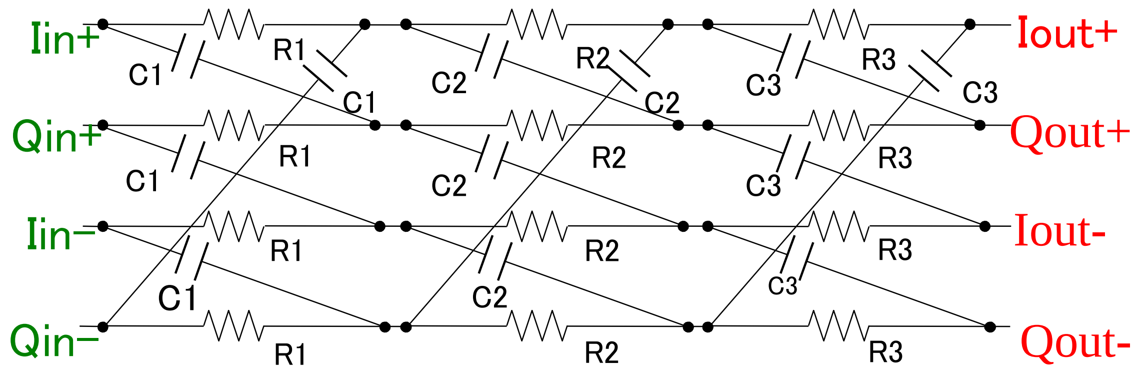
$$\boxed{\omega_{LO} = \frac{2}{R_1 C_1}}$$



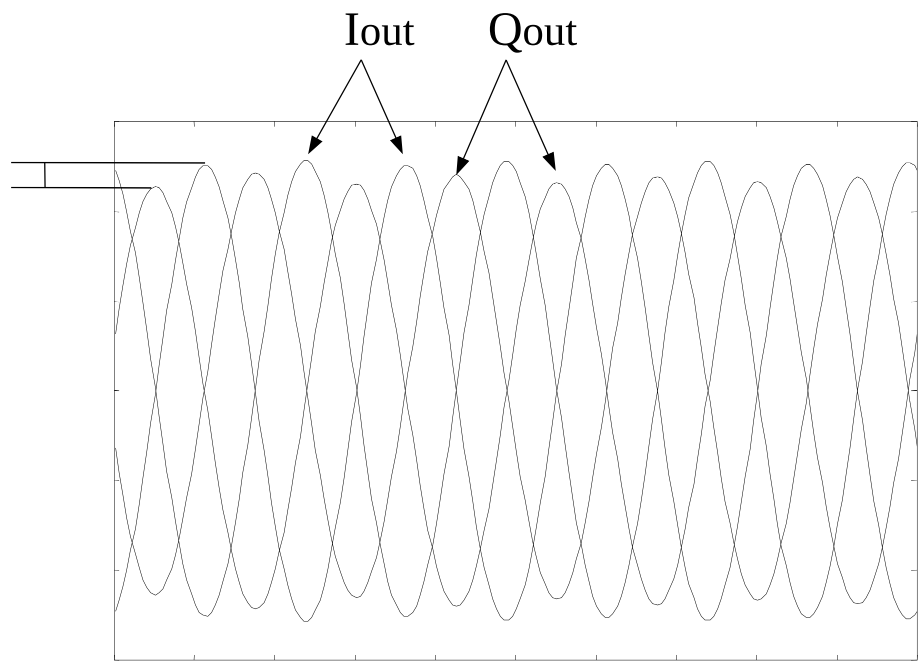


3rd-order RC Polyphase Filter

The amplitude difference problem is further alleviated.

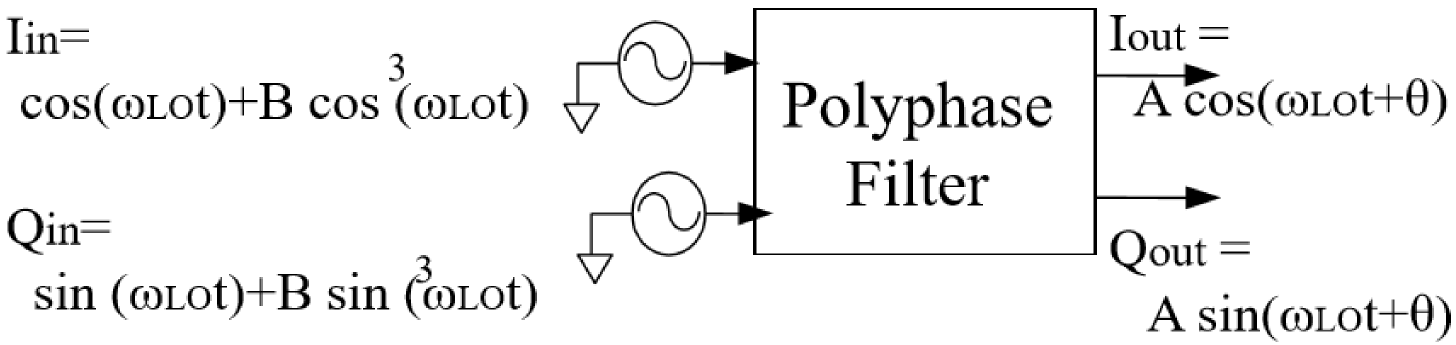


$$\boxed{\omega_{LO} = \frac{2}{R_1 C_1}}$$



Pure I, Q Signal Generation

3rd-order harmonics rejection



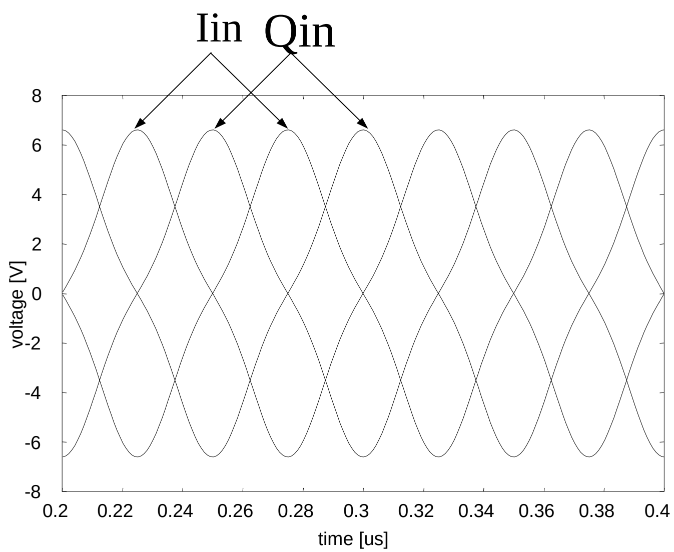
With
3rd-order harmonics.

Without
3rd-order harmonics.

Simulation of 3rd-order Harmonics Rejection

$$I_{in}(t) = \cos(\omega_{LO}t) + a\cos^3(\omega_{LO}t)$$

$$Q_{in}(t) = \sin(\omega_{LO}t) + a\sin^3(\omega_{LO}t)$$



$$\boxed{3\omega_{LO} = \frac{1}{R_1 C_1}}$$

$$I_{out}(t) = A\cos(\omega_{LO}t + \theta)$$

$$Q_{out}(t) = A\sin(\omega_{LO}t + \theta)$$

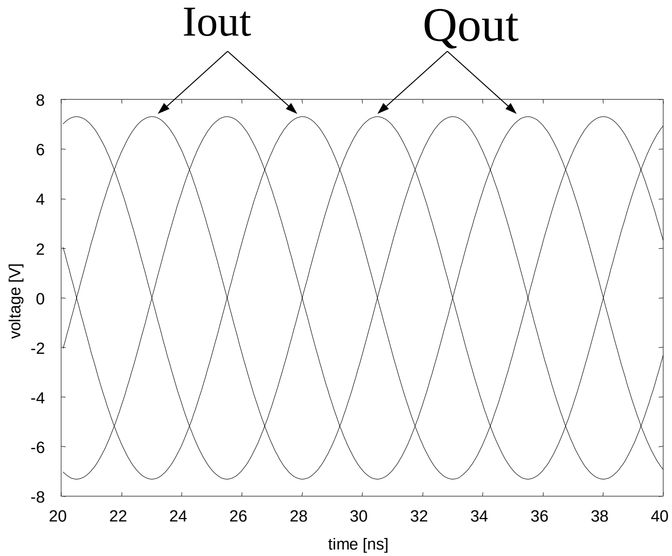
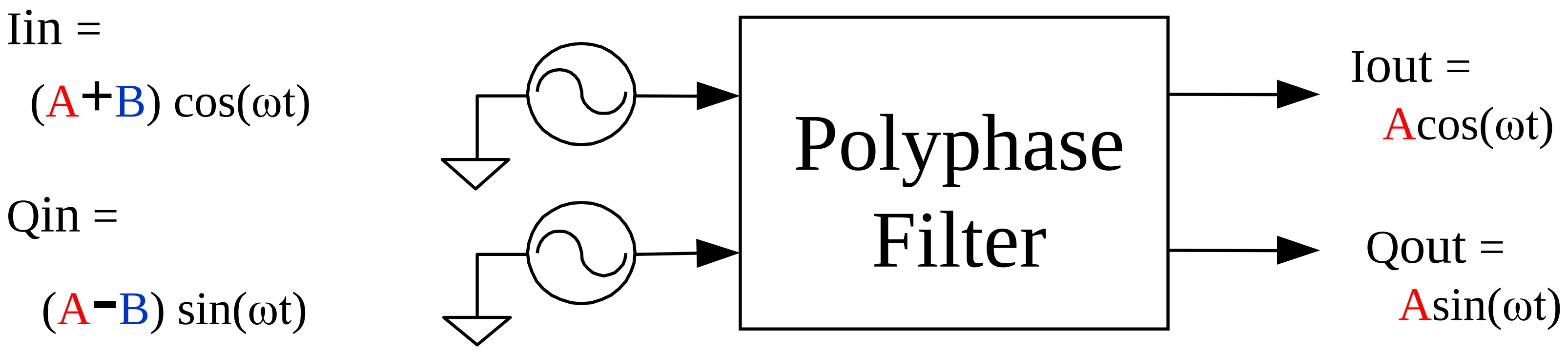


Image Rejection Filter



$$Ae^{j\omega t} + Be^{-j\omega t} \Rightarrow Ae^{j\omega t}$$

signal image

Complex Transfer Function

- Complex Signal Theory

- Complex input
- Complex output

$$V_{in}(j\omega) = I_{in} + j \cdot Q_{in}$$

$$V_{out}(j\omega) = I_{out} + j \cdot Q_{out}$$

- Complex
  Transfer Function

$$G(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$$

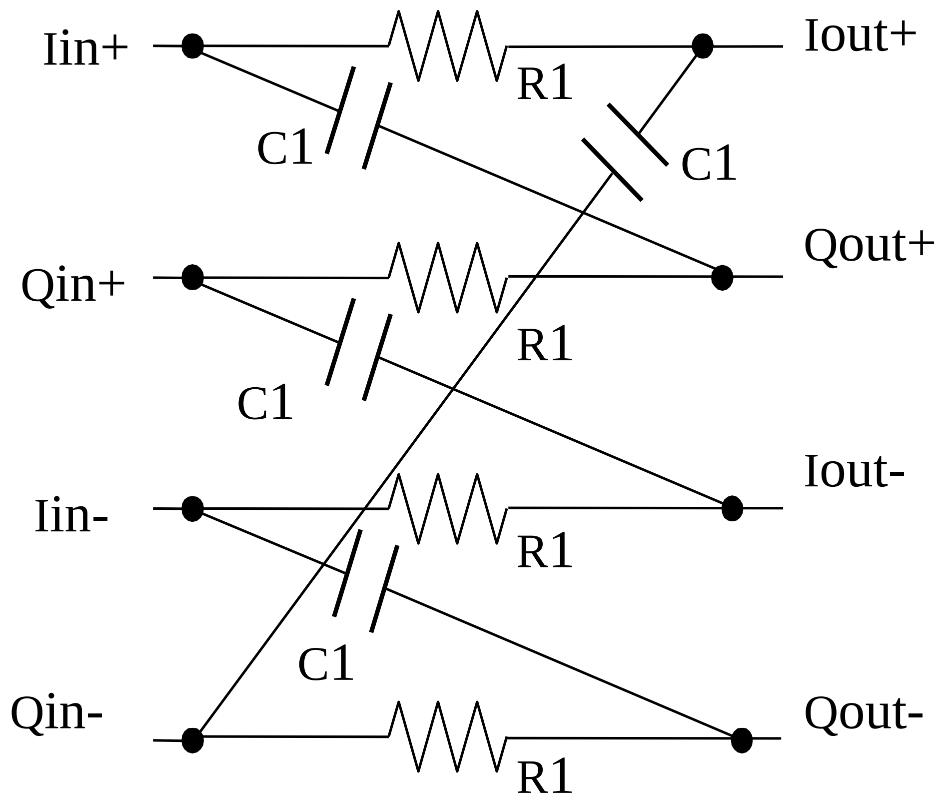
Signals in RC Polyphase Filter

Differential signal

$$I_{in}(t) = I_{in+}(t) - I_{in-}(t)$$
$$Q_{in}(t) = Q_{in+}(t) - Q_{in-}(t)$$
$$I_{out}(t) = I_{out+}(t) - I_{out-}(t)$$
$$Q_{out}(t) = Q_{out+}(t) - Q_{out-}(t)$$

Complex signal

$$V_{in}(t) = I_{in}(t) + jQ_{in}(t)$$
$$V_{out}(t) = I_{out}(t) + jQ_{out}(t)$$



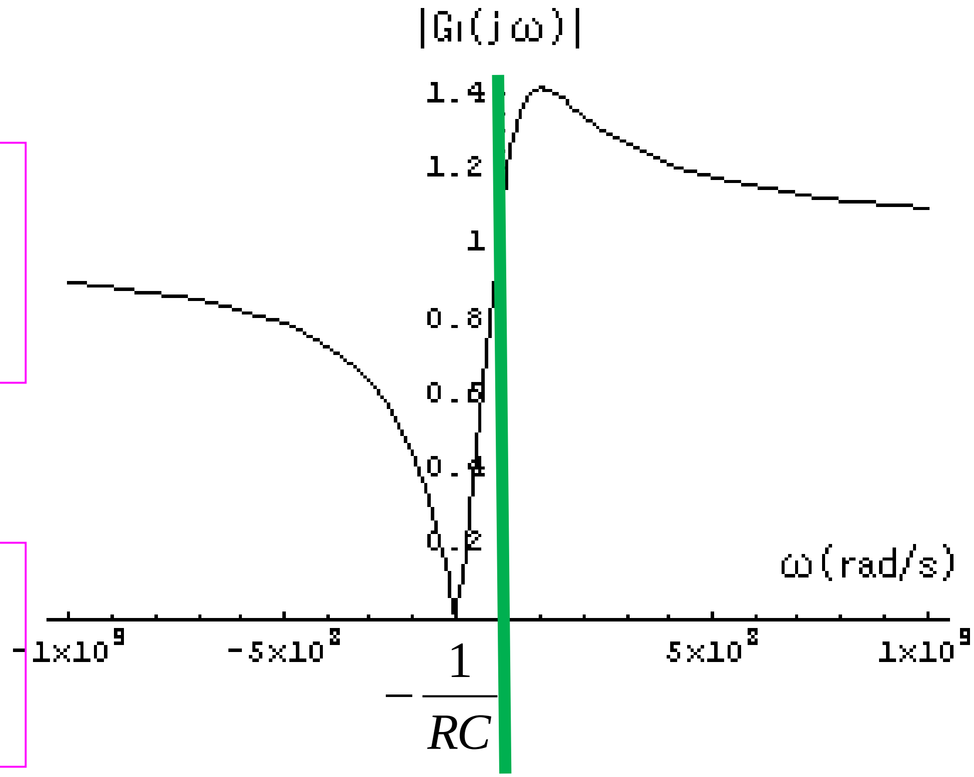
Transfer Function of RC Polyphase Filter

- Transfer Function

$$G_1(j\omega) = \frac{1+\omega RC}{1+j.\omega RC}$$

- Gain

$$\left|G_1(j\omega)\right| = \frac{\left|1+\omega RC\right|}{\sqrt{1+(\omega RC)^2}}$$



Asymmetric

Explanation of I, Q Signal Generation by $G_1(j\omega)$

$$Q_{in}(t) \equiv 0, \qquad I_{in}(t) = \cos(\omega t)$$

$$V_{in}(t) = I_{in}(t) + j\ Q_{in}(t) = \cos(\omega t) = \frac{1}{2}[e^{j\omega t} + e^{-j\omega t}]$$

$$V_{out}(t) = \frac{1}{2}[|G_1(j\omega)|e^{j(\omega t + \angle G_1(j\omega))} + |G_1(-j\omega)|e^{j(-\omega t + \angle G_1(-j\omega))}]$$

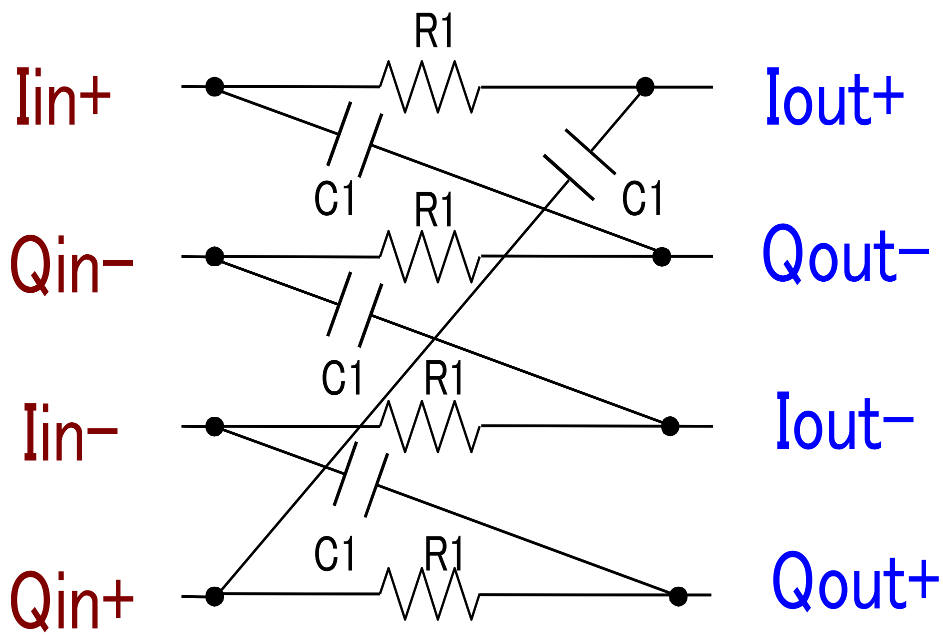
$$= \frac{\sqrt{2}}{2}\cos\left(\omega t - \frac{\pi}{4}\right) + \frac{j\sqrt{2}}{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

$$|G_1(-j\omega)|e^{j(-\omega t + \angle G_1(-j\omega))}] = 0$$

Here

$$|G_1(j\omega)|_{\omega=\frac{1}{RC}} = 0\,, \quad |G_1(j\omega)|_{\omega=\frac{1}{RC}} = \sqrt{2}\,, \quad \angle G_1(j\omega) = -\frac{\pi}{4}$$

Component Mismatch Case



$$R_{1Q+} := R_1 + \Delta R_{1Q+} \qquad R_{1Q-} := R_1 + \Delta R_{1Q-}$$

$$R_{1I+} := R_1 + \Delta R_{1I+} \qquad R_{1I-} := R_1 + \Delta R_{1I-}$$

$$C_{1Q+} := C_1 + \Delta C_{1Q+} \qquad C_{1Q-} := C_1 + \Delta C_{1Q-}$$

$$C_{1I+} := C_1 + \Delta C_{1I+} \qquad C_{1I-} := C_1 + \Delta C_{1I-}$$

$\Delta R_{1Q+}, \Delta R_{1Q-}, \Delta R_{1I+}, \Delta R_{1I-}$: Resistor variation

$\Delta C_{1Q+}, \Delta C_{1Q-}, \Delta C_{1I+}, \Delta C_{1I-}$: Capacitor variation

⇩

I, Q paths mismatch

Component Mismatch Effect

Mismatch components

$$V_{out} = \frac{1+\omega RC}{1+j\omega RC} V_{in} - \frac{(1+j)\omega RC}{2(1+j\omega RC)^2} \Delta X \overline{V_{in}}$$

Input Signal

Input Image Signal

$$\boxed{V_{in} = I_{in} + jQ_{in}} \quad \boxed{\overline{V}_{in} = I_{in} - jQ_{in}}$$

Derived by Y. Niki, Gunma University

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Y. Niki, S. Sasaki, N. Yamaguchi, J. Kang, T. Kitahara, H. Kobayashi "Flat Passband Gain Design Algorithm for 2nd-order RC Polyphase Filter," IEEE 11th International Conference on ASIC, Chengdu, China (Nov. 2015)

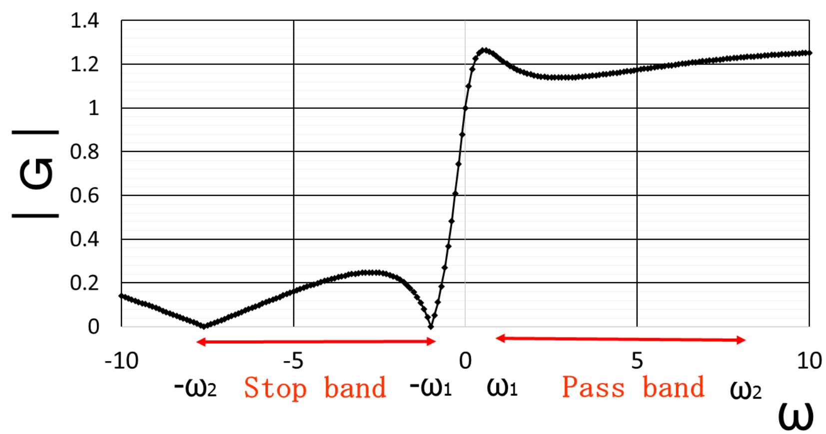
Transfer Function of 2nd-order RC Polyphase Filter

Transfer Function

$$G_2(j\omega) = \frac{(1+\omega R_1 C_1)(1+\omega R_2 C_2)}{1-\omega^2 R_1 C_1 R_2 C_2 + j\omega(C_1 R_1 + C_2 R_2 + 2R_1 C_2)}$$

Derivation is very complicated, so we used "Mathematica."

Gain |G2(jω)| characteristics



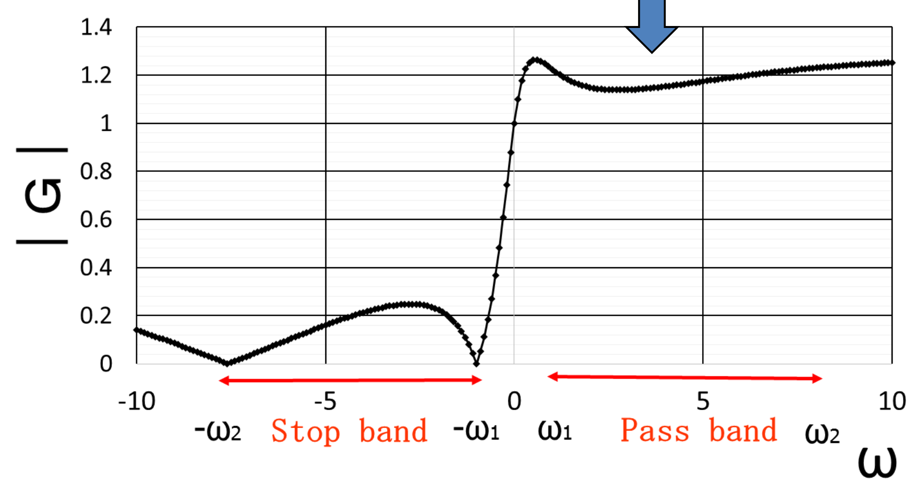
Need for Flat Passband Gain Algorithm

Transfer Function

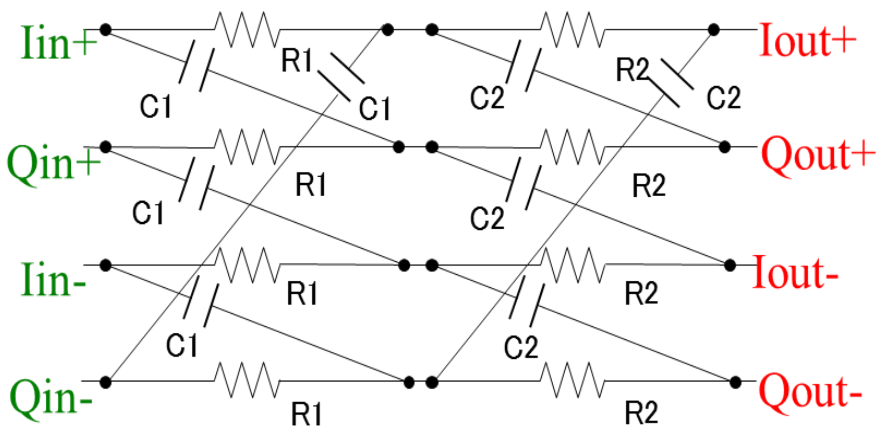
$$G_2(j\omega) = \frac{(1+\omega R_1 C_1)(1+\omega R_2 C_2)}{1-\omega^2 R_1 C_1 R_2 C_2 + j\omega(C_1 R_1 + C_2 R_2 + 2R_1 C_2)}$$

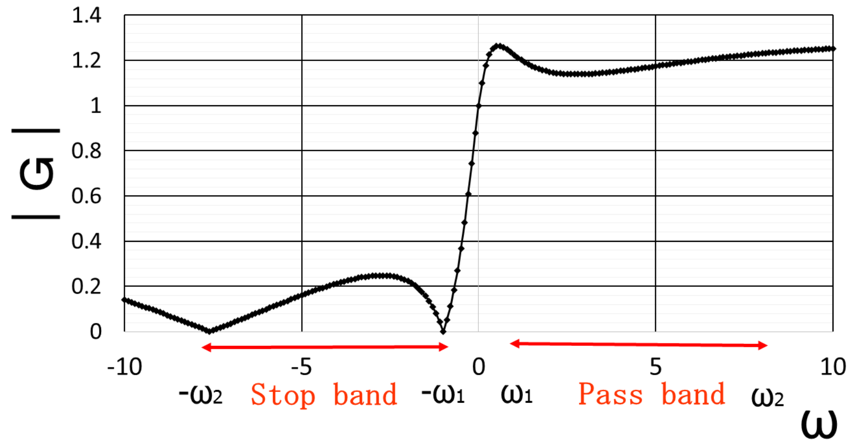
We need flat passband gain

Gain |G2(jω)| characteristics



Four Design Parameters



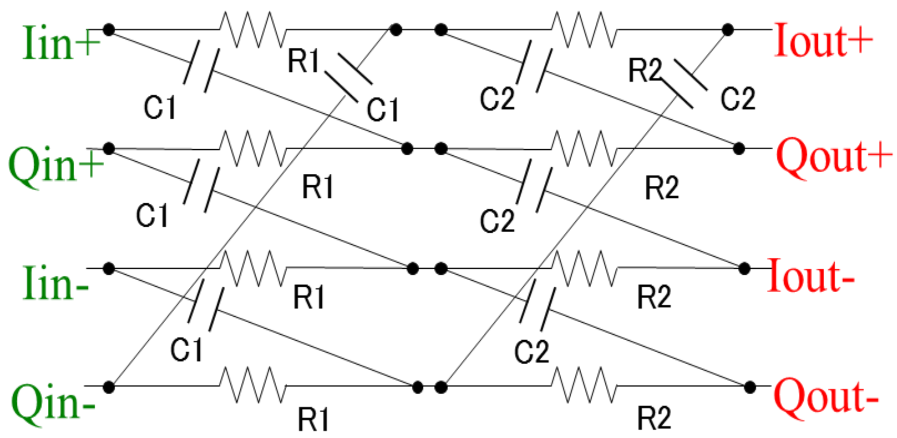


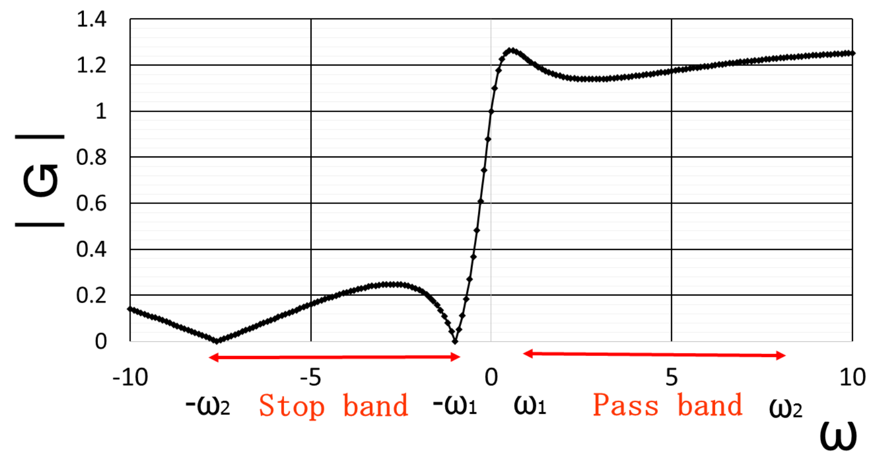
4 parameters : R_1, R_2, C_1, C_2

$$\omega_1 = \frac{1}{R_1 C_1}, \omega_2 = \frac{1}{R_2 C_2}, X = \frac{1}{R_2 C_1}, Y = \frac{1}{R_1 C_2}$$

4 constraints

Two Constraints from Filter Spec.

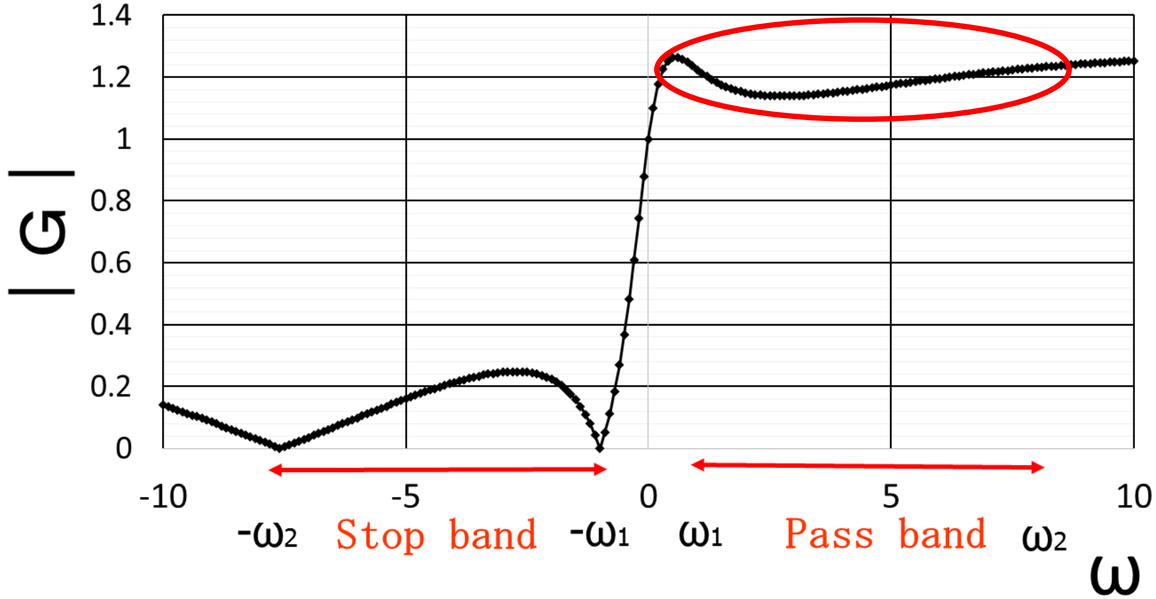




● 2 zeros : $-\omega_1 = \dfrac{-1}{R_1 C_1}$, $-\omega_2 = \dfrac{-1}{R_2 C_2}$

are given from the filter specification.

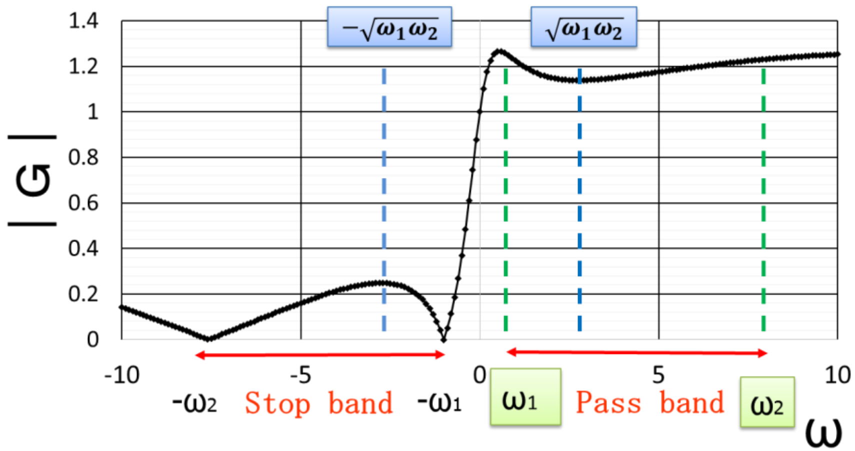
Proposed Algorithm Uses Third Constraint



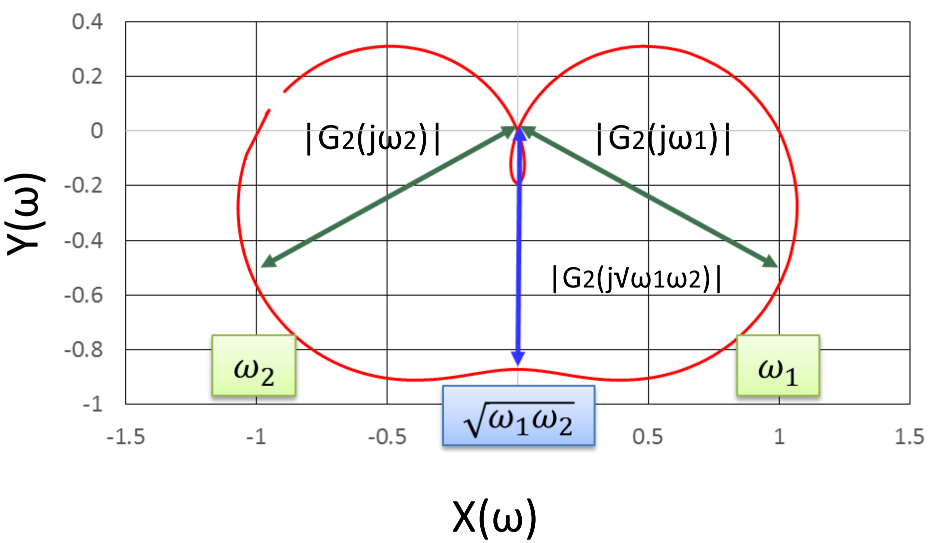
- We use the third constraint $X = \dfrac{1}{R_2 C_1}$ for passpand gain flattening.
- The fourth constraint is left for ease of IC realization.

Nyquist Chart of G2(jω)

Gain characteristics $|G_2(j\omega)|$



Nyquist chart of $G_2(j\omega)=X(\omega)+j\,Y(\omega)$

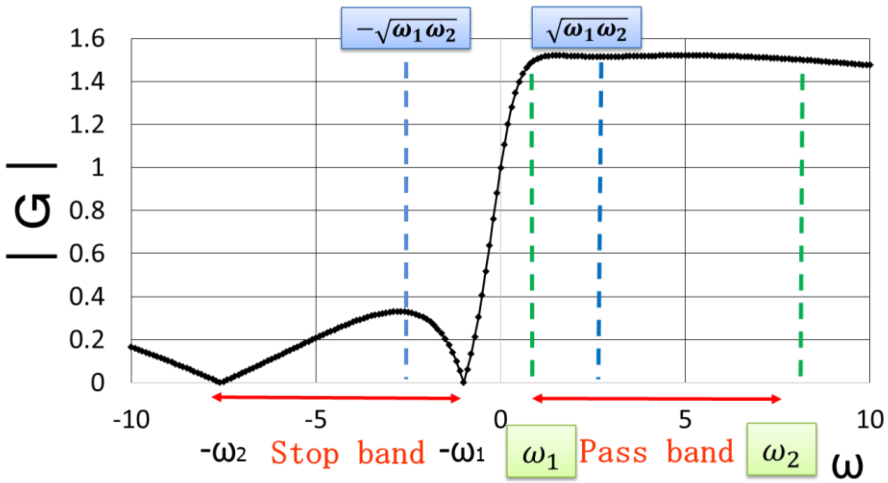


$$|G_2(j\omega_1)|=|G_2(j\omega_2)|$$

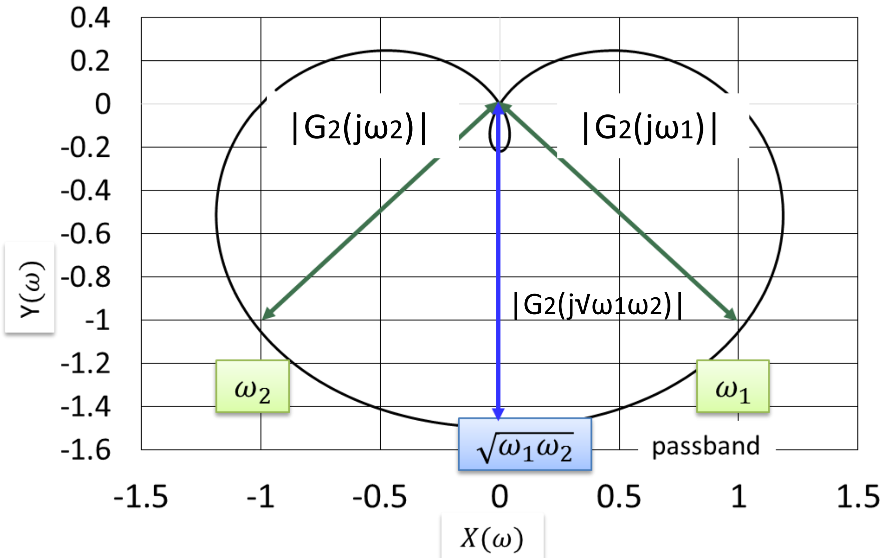
But in general $\quad |G_2(j\omega_1)|=|G_2(j\omega_2)|\neq|G_2(j\sqrt{\omega_1\omega_2})|$

Our Idea for Flat Passband Gain Algorithm

Gain characteristics $|G_2(j\omega)|$



Nyquist chart of $G_2(j\omega)=X(\omega)+j\,Y(\omega)$



If we make $|G_2(j\omega_1)| = |G_2(j\omega_2)| = |G_2(j\sqrt{\omega_1\omega_2})|$,
Passband gain becomes flat from ω_1 to ω_2.

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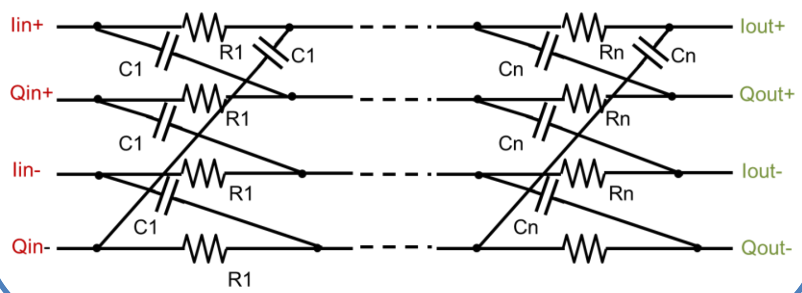
Y. Tamura, R. Sekiyama, K. Asami, H. Kobayashi, "RC Polyphase Filter As Complex Analog Hilbert Filter", IEEE 13th International Conference on Solid-State and Integrated Circuit Technology, Hangzhou, China (Oct. 2016)

Research Objective

RC Polyphase Filter

Analog

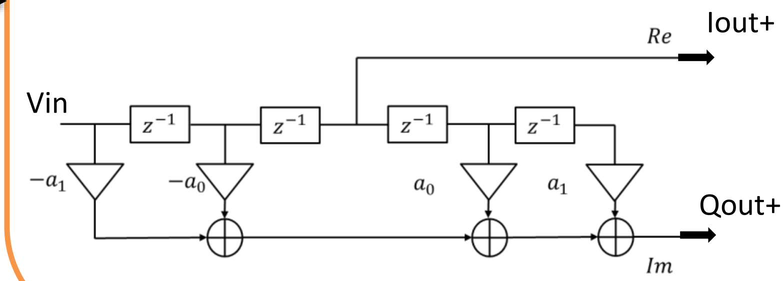
Complex (I, Q) input



Hilbert Filter

Digital

Real part (Vin) input



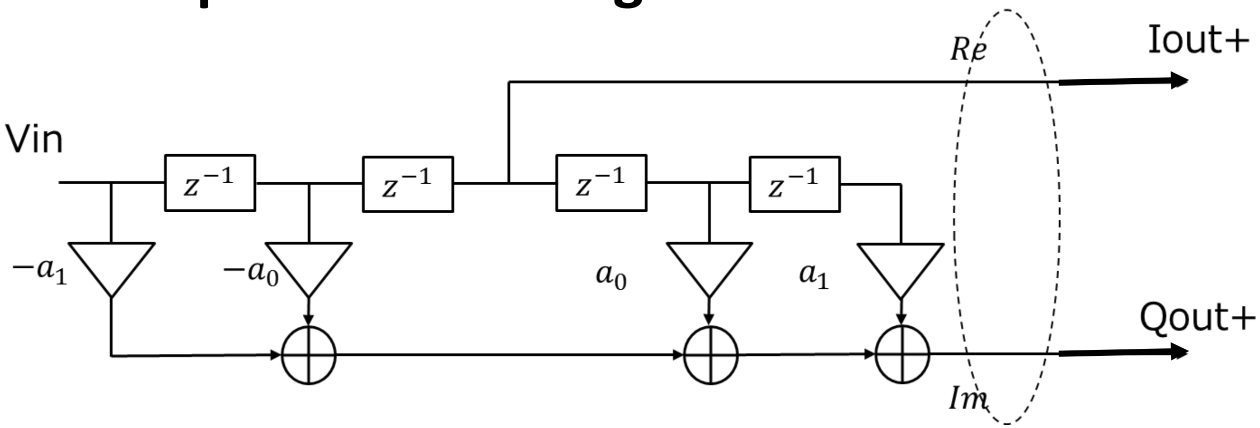
Analyze RC polyphase filter

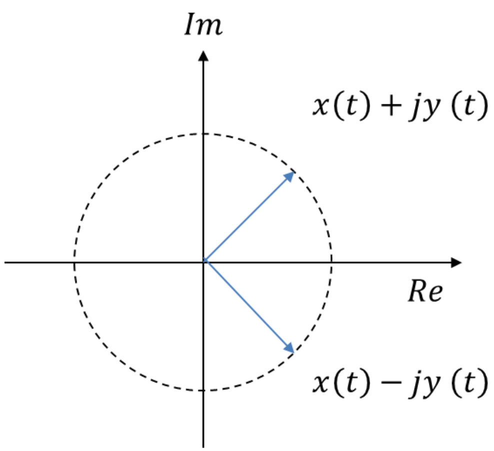
We found that relevance between

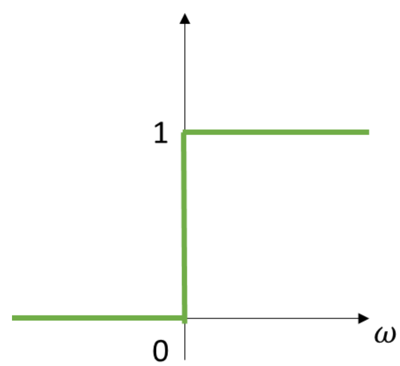
RC polyphase filter and Hilbert filter

Hilbert Filter

- Characteristics
  - Hilbert transform
  - 1 input and 2 outputs
  - **It is often implemented in digital filter**

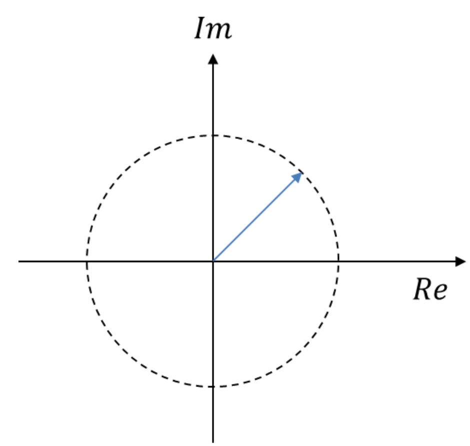






Gain

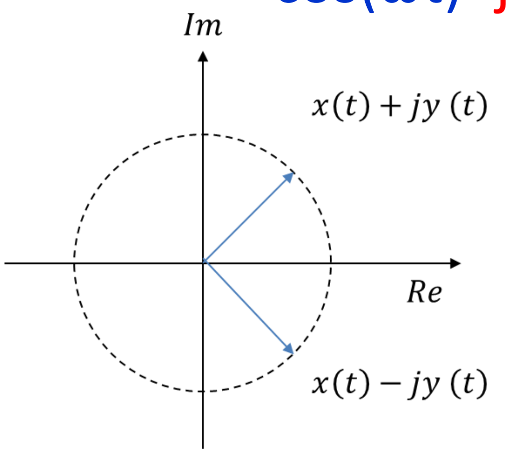
Phase



Cosine, Sine Generation with Hilbert Filter

ω component

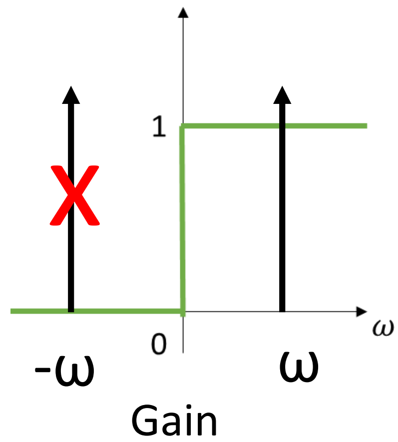
cos(ωt)+jsin(ωt)



cos(ωt)-jsin(ωt)

-ω component

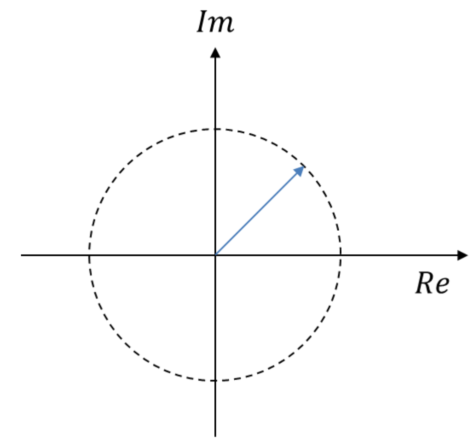
2 cos(ωt)



Hilbert filter

ω component

cos(ωt)+jsin(ωt)



cos(ωt)

sin(ωt)

Hilbert Transform

Complex signal from real signal $x\,(t)$

$$x(t) \rightarrow x(t) + jy\,(t)$$

Hilbert transform

$$y(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}d\tau = x(t) * \frac{1}{\pi t}$$

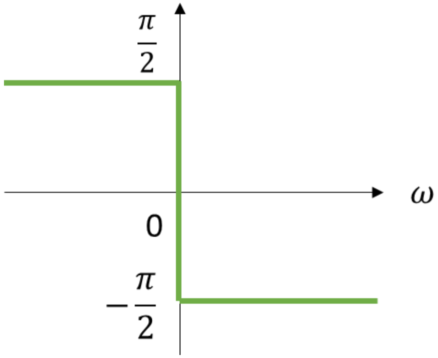
David Hilbert
1862-1943

Impulse response Fourier Transform

$$h(t) = \frac{1}{\pi t} \underset{Fourier}{\Longleftrightarrow} \boldsymbol{H(\omega)} = \begin{cases} \boldsymbol{-j} & \boldsymbol{(\omega > 0)} \\ \boldsymbol{j} & \boldsymbol{(\omega < 0)} \end{cases}$$

Frequency characteristic $H\,$(ω)

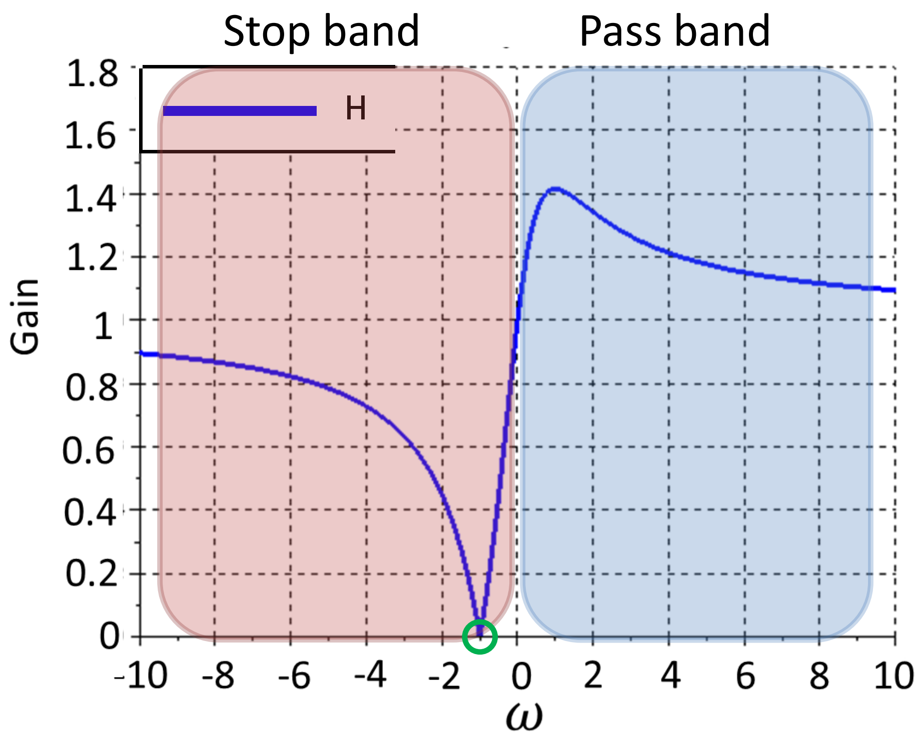
$$\boldsymbol{Y(\omega) = H(\omega)X(\omega)} = \begin{cases} \boldsymbol{-jX(\omega)} & \boldsymbol{(\omega > 0)} \\ \boldsymbol{jX(\omega)} & \boldsymbol{(\omega < 0)} \end{cases}$$

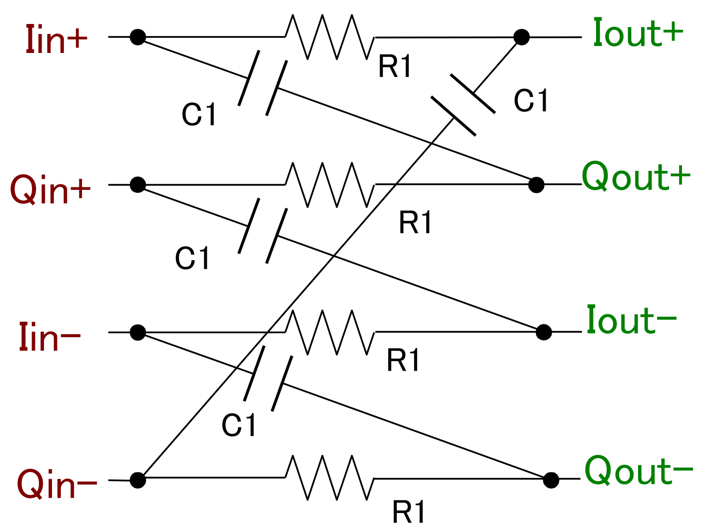


Phase

1st order RC Polyphase Filter : Analysis

$$H_1(j\omega) = \frac{1 + \omega R_1 C_1}{1 + j\omega R_1 C_1}$$: Transfer function





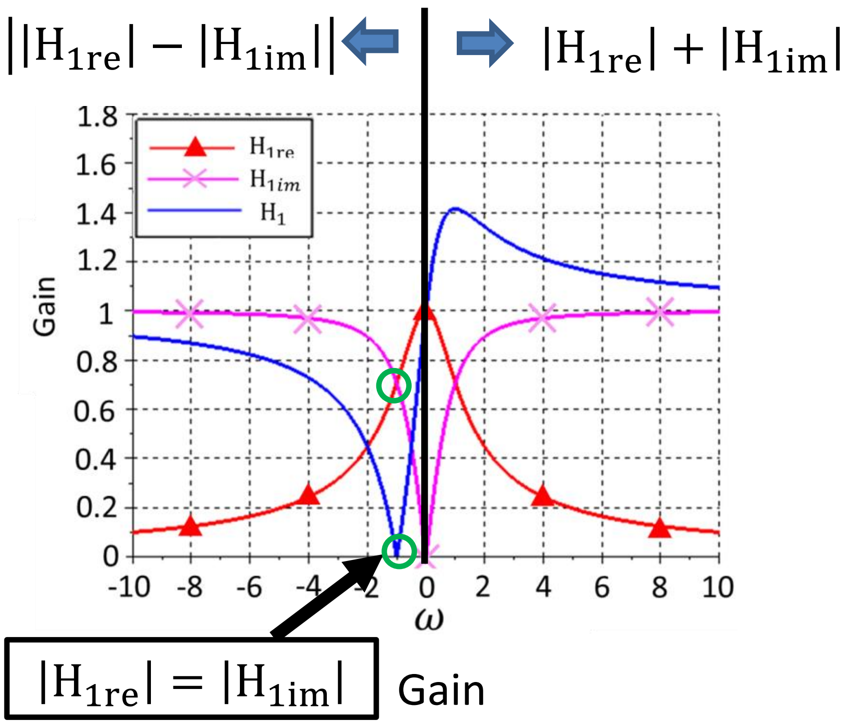
Zero: $\omega_k = \frac{1}{R_k C_k}$

1^{st} order RC Polyphase Filter : Gain and Phase

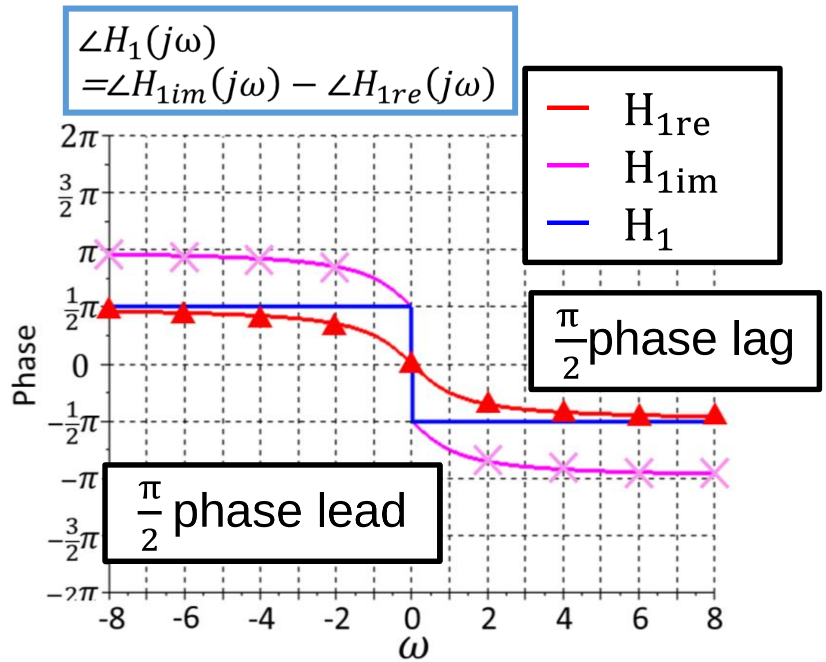
$$H_1(j\omega) = H_{1re}(j\omega) + jH_{1im}(j\omega)$$

$$H_{1re}(j\omega) = \frac{H_1(j\omega) + H_1^*(-j\omega)}{2} = \frac{1}{1 + j\omega R_1 C_1}$$

$$H_{1im}(j\omega) = \frac{H_1(j\omega) - H_1^*(-j\omega)}{2} = -j\frac{\omega R_1 C_1}{1 + j\omega R_1 C_1}$$



Gain



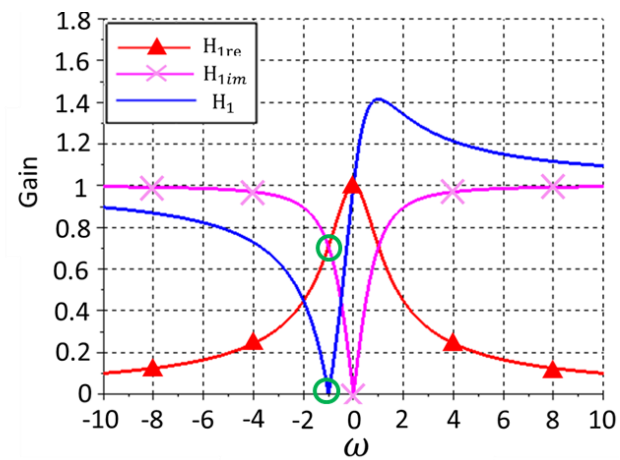
Phase

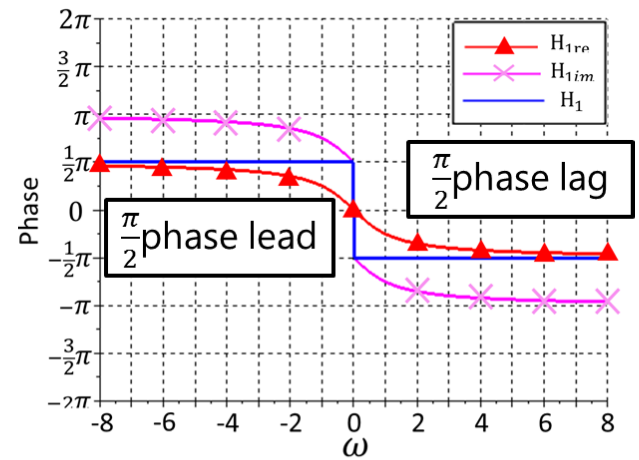
1st order case Analysis Results

Gain : Hilbert filter only at zero

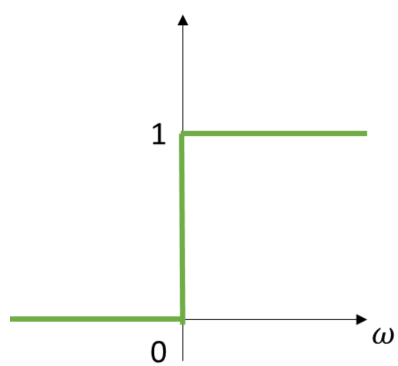
Phase : Completely Hilbert filter

RC Polyphase Filter

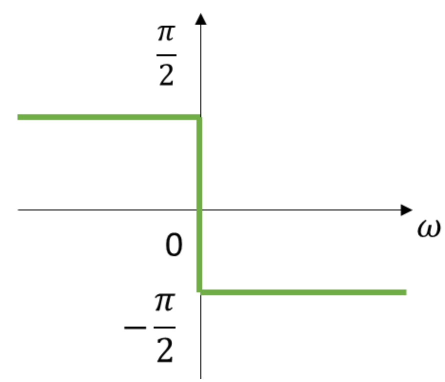




Hilbert filter

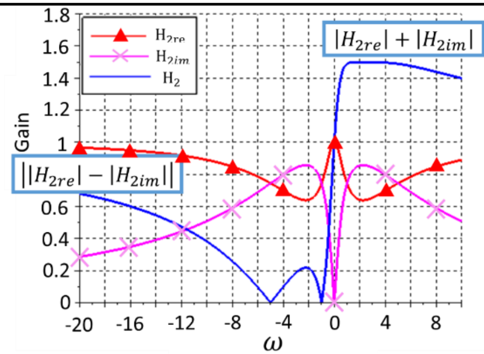
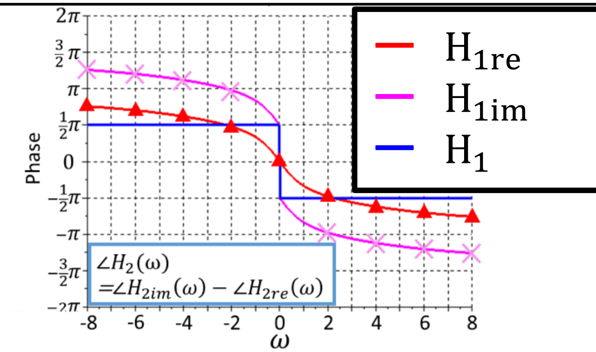
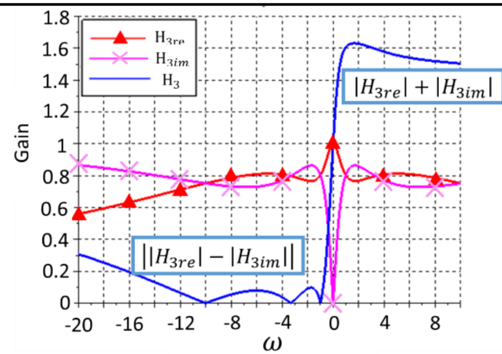
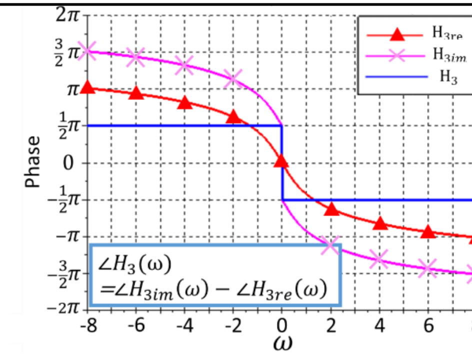
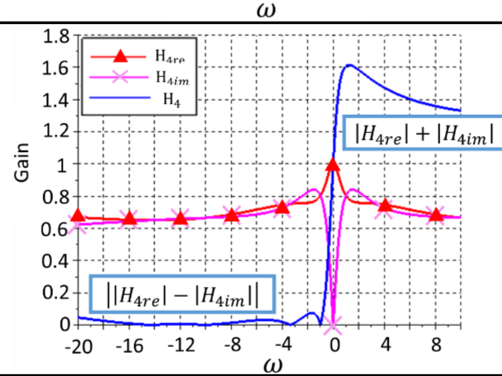
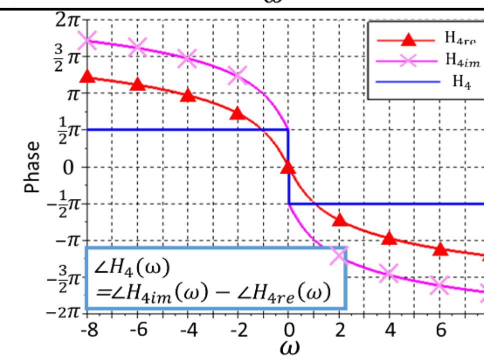


Gain



Phase

Results：2^{nd} to 4^{th} RC Polyphase Filter

	Gain	Phase
2^{nd}		
3^{rd}		
4^{th}		

Analysis Results and Consideration

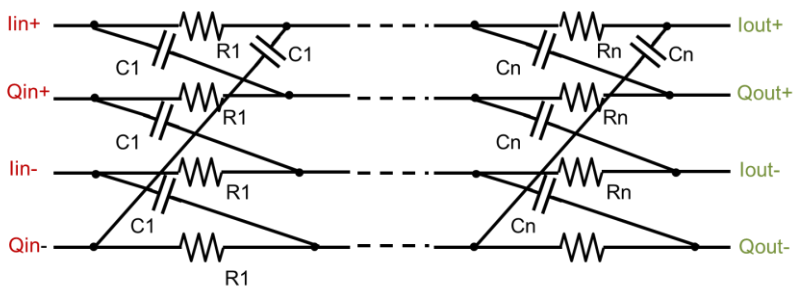
1st to 4th order RC Polyphase Filter Analysis results

Gain : Hilbert filter only at zeros

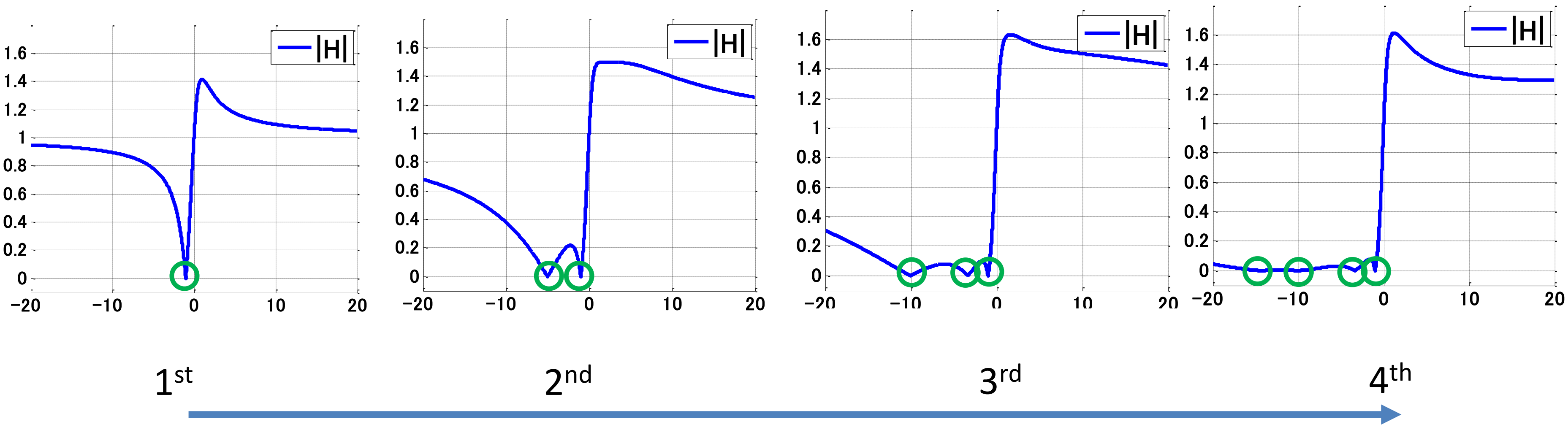
Phase : Completely Hilbert filter

Prove for general n-th order case

(n = 1, 2, 3, 4, 5, ...)



Order and Gain



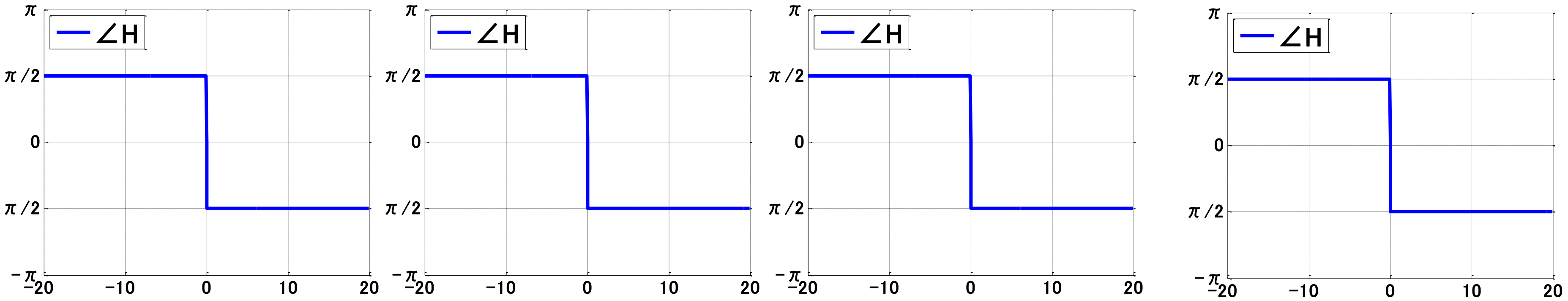
The higher orders,

the number of zeros increases;

$|Hre|$ and $|Him|$ becomes close in wide range

Close to ideal Hilbert transform

Order and Phase



1st 2nd 3rd 4th

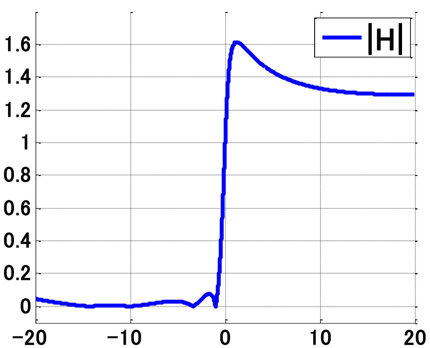
Phase characteristic is not changed

There is always 90 phase difference

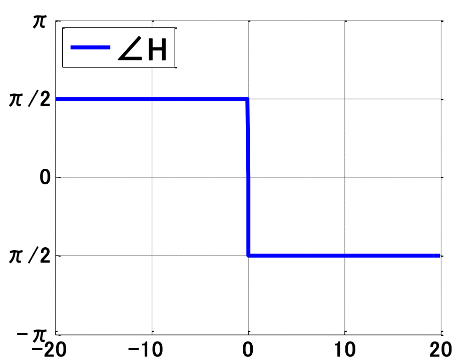
Fulfill Hilbert transform in full range

Summary of RCPF and Hilbert Filter

RC Polyphase Filter

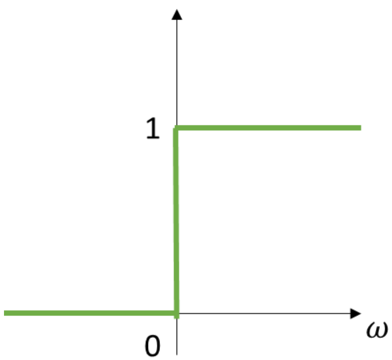


Gain

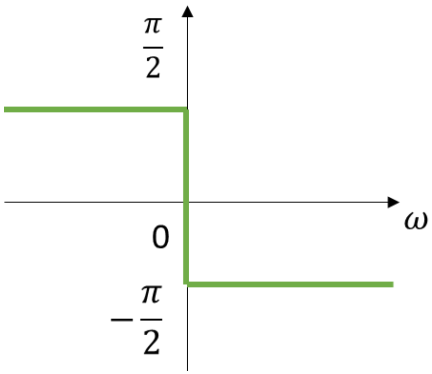


Phase

Hilbert Filter



Gain



Phase

RC polyphase filter is approximation of ideal Hilbert filter for complex input signal

Outline

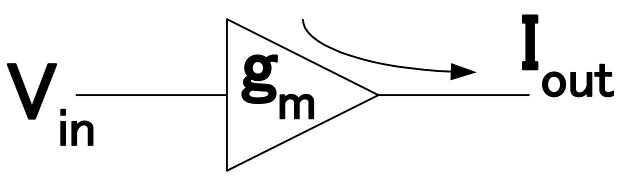
- Motivation for Complex Signal Processing Research
- RC Polyphase Filter: Transfer Function
- RC Polyphase Filter: Flat Passband Gain Algorithm
- RC Polyphase Filter and Hilbert Filter
- <u>Active Complex Bandpass Filters</u>
- Complex Bandpass ΔΣ AD Modulator
- Complex Multi-Bandpass ΔΣ DA Modulator

A. Hatta, N. Kushita, M. T.Tran, K. Asami, A. Kuwana, H. Kobayashi, "Relationship between Active Complex Bandpass Filter and Hilbert Filter" 5th Taiwan and Japan Conference on Circuits and Systems. Nikko, Japan (Aug. 2019)

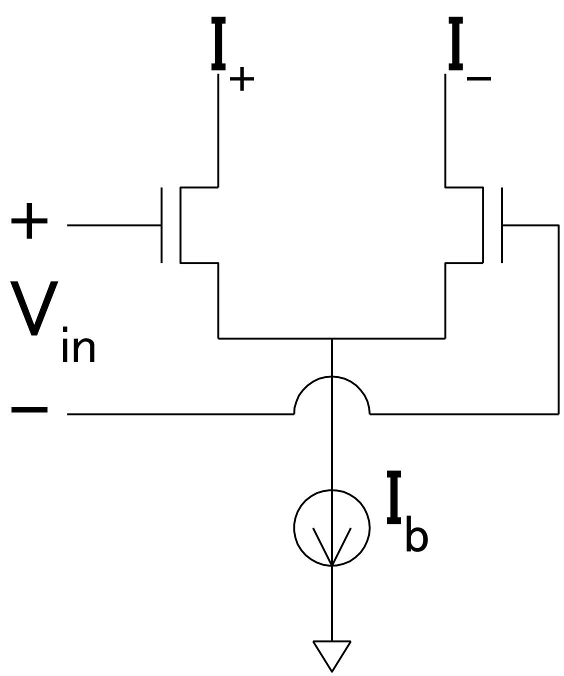
Gm : Transconductance

Input voltage: V_{in}

Output current : I_{out} $\qquad I_{out}=g_m V_{in}$

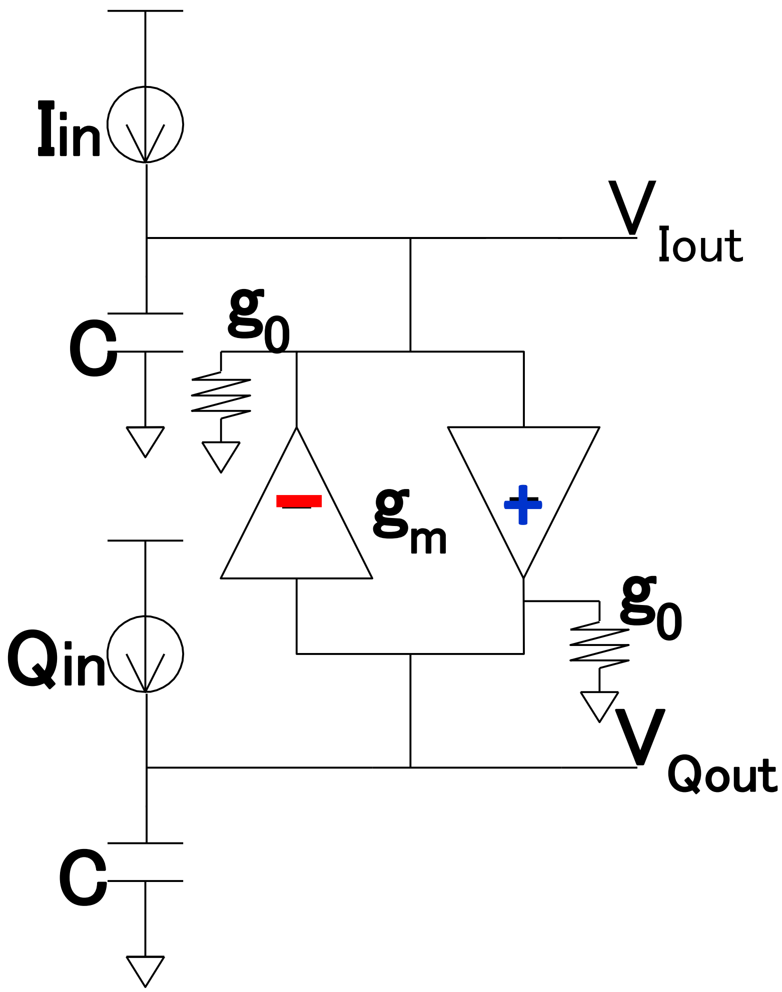


dimension of gm ⇨ $\frac{1}{R}$



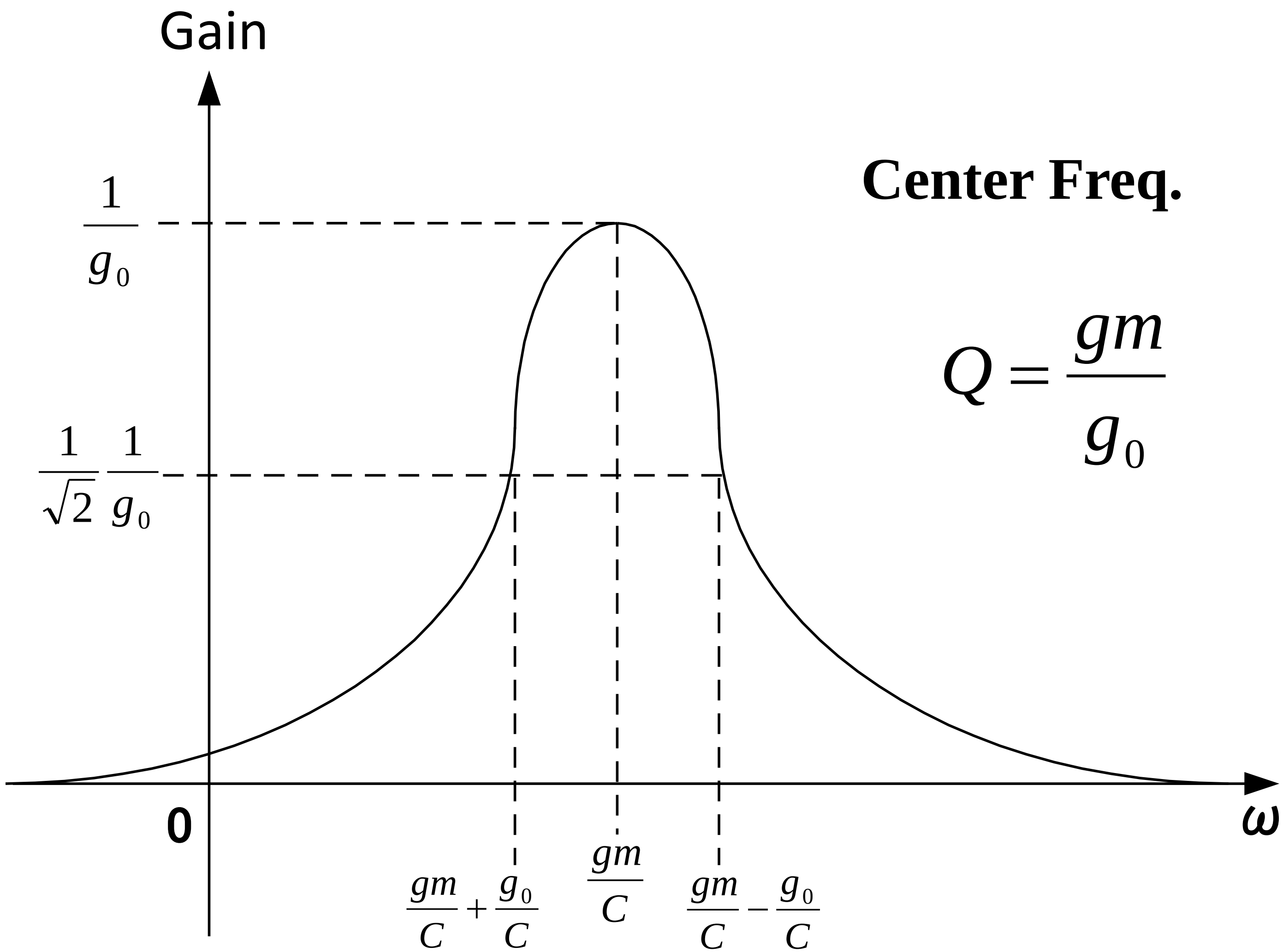
$$I_{out}=I_+-I_-$$
$$=g_m V_{in}$$

Complex Bandpass Gm-C Filter



$$\frac{V_{Iout} + jV_{Qout}}{I_{in} + jQ_{in}} = \frac{g_0 + sc - jg_m}{g_0^2 + g_m^2 + s^2C^2 + 2g_0 s\ C}$$

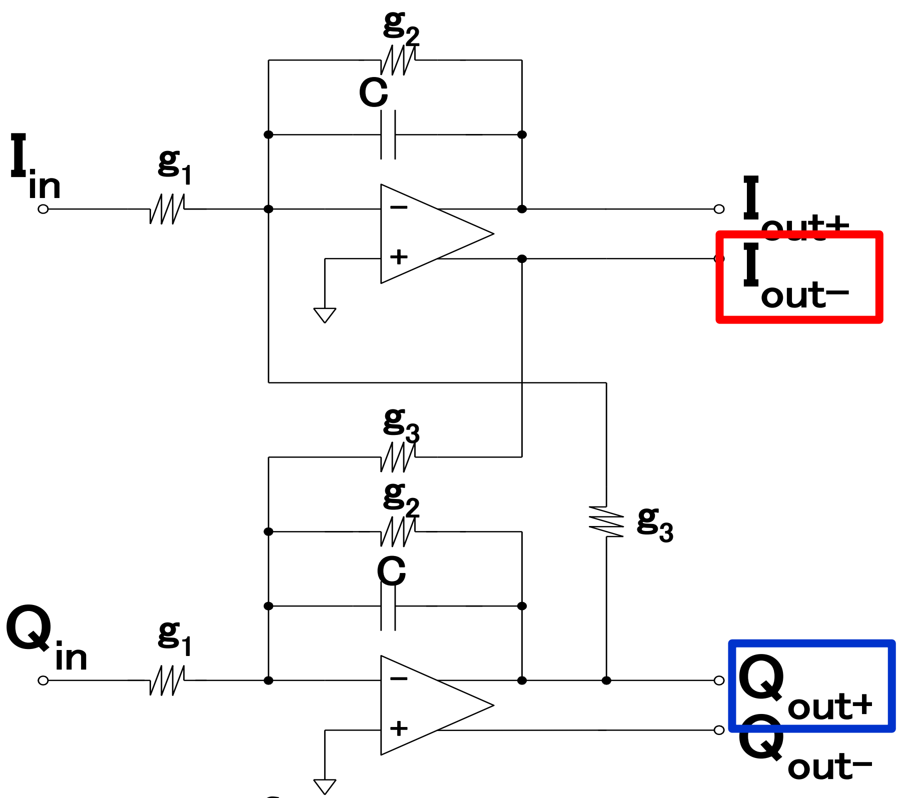
Gain of Complex Bandpass Gm-C Filter



Center Freq. $\frac{gm}{C}$

$$Q=\frac{gm}{g_0}$$

Complex Bandpass Active RC Filter



$$H(j\omega)=\frac{-g_1}{g_2+j(-g_3+\omega C)}$$

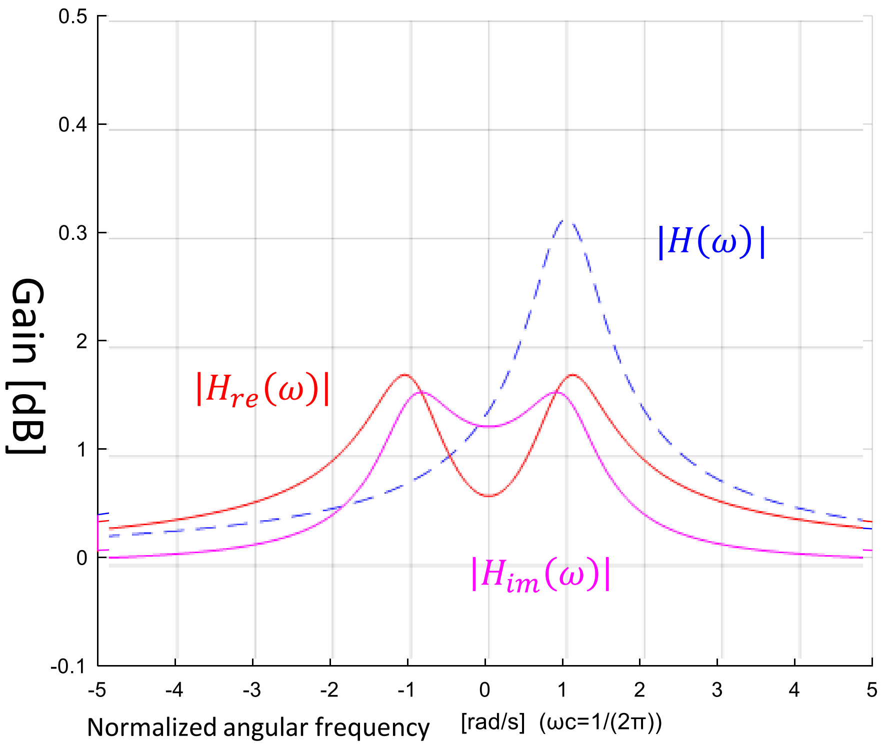
Center freq. $\omega_0=\dfrac{g_3}{C}$ $\qquad Q=\dfrac{g_3}{2g_2}$ $\qquad$ Gain $|H(j\omega)|=\dfrac{g_1}{g_2}$

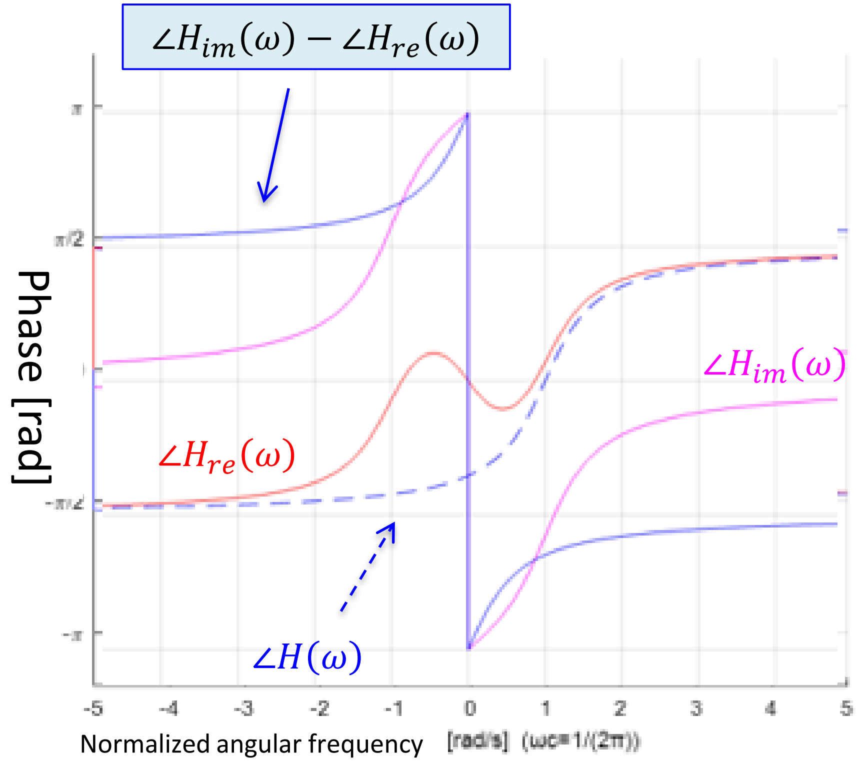
Transfer functions of complex bandpass Gm-C and active RC filters are the same.

Our Investigation Results

Gain : Poor Hilbert filter characteristics for both pass and stop bands

Phase : Hilbert filter only at large |ω|





Poor Hilbert filter characteristics of active complex bandpass filters

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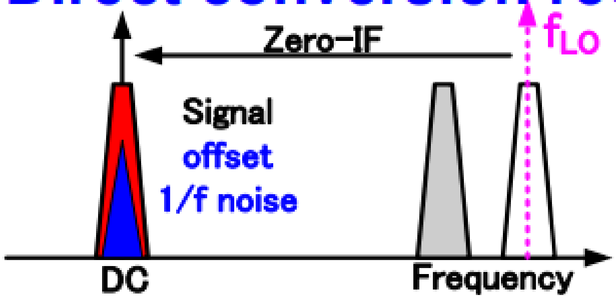
- Complex Multi-Bandpass ΔΣ DA Modulator

H. San, Y. Jingu, H. Wada, H. Hagiwara, A. Hayakawa, J. Kudoh, K. Yahagi,T. Matsuura, H. Nakane, H. Kobayashi, M. Hotta, T. Tsukada,K.Mashiko, A. Wada, "A Multibit Complex Bandpass Delta Sigma AD Modulator with I, Q Dynamic Matching and DWA Algorithm'', IEEE Asian Solid-State Circuits Conference, Hangzhou, China (Nov. 2006).

Receiver Architecture Comparison

Direct conversion receiver



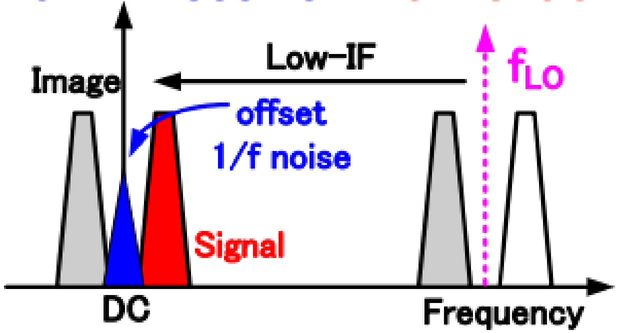
RF → Baseband

Zero-IF

⇒ No mage

Problem of DC offset, flicker noise

Low-IF receiver **Conventional**



RF → Low-IF

No problem of DC offset, flicker noise.

Image as well as signal are AD converted ⇒ Power is wasted

Quadrature-IF

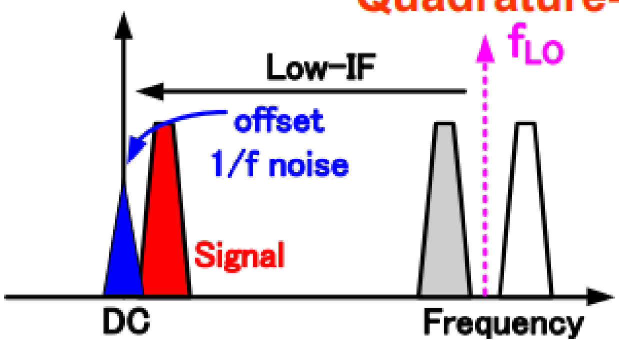
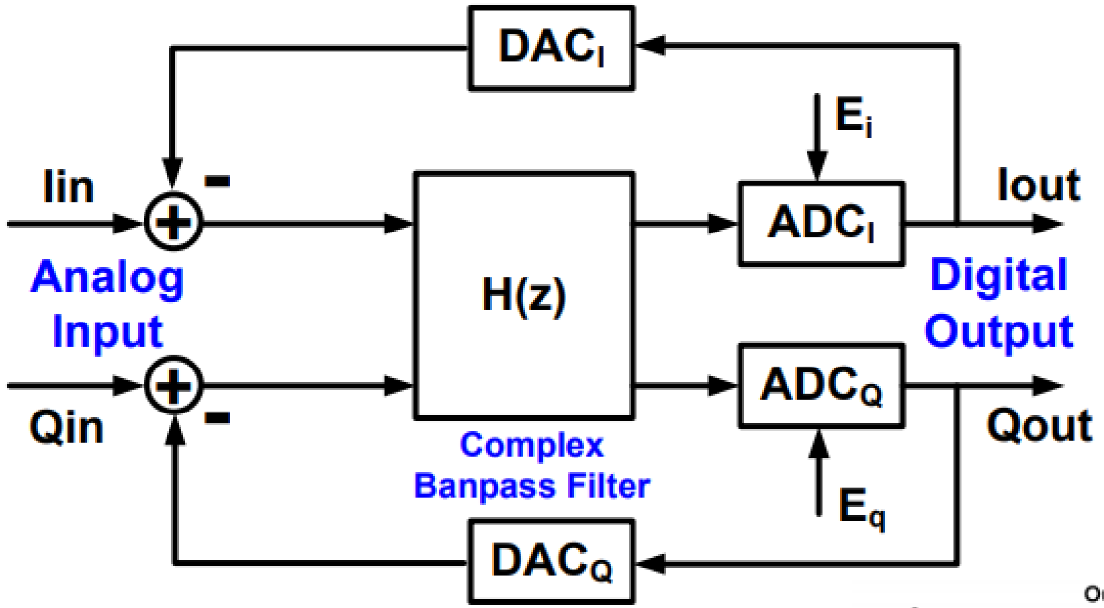
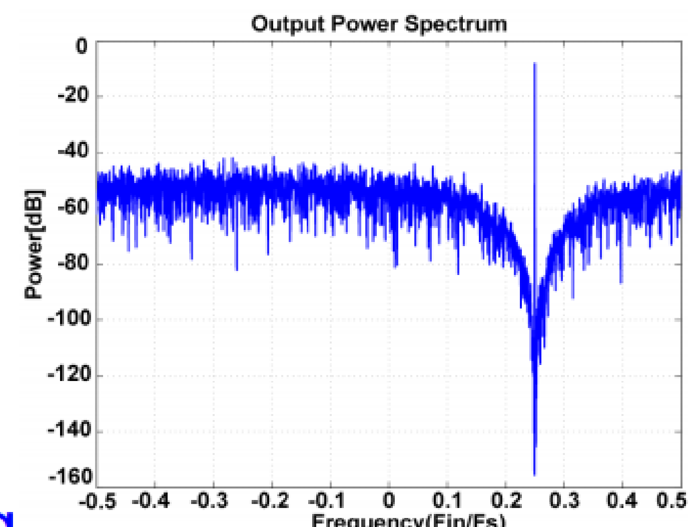


Image is not AD converted.

Complex Bandpass Delta-Sigma AD Modulator

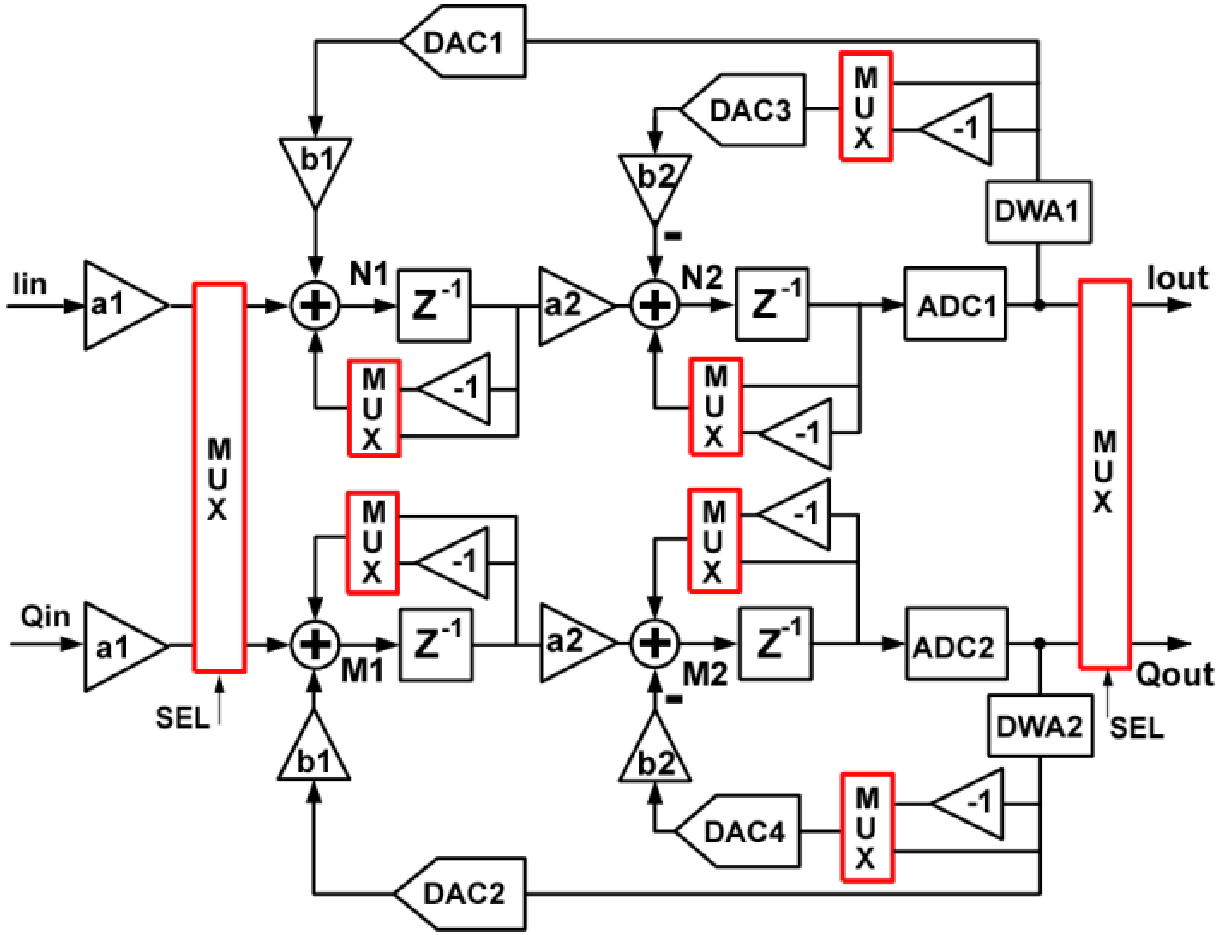


$$I_{out} + jQ_{out} = \frac{H}{1+H}(I_{in} + jQ_{in}) + \frac{1}{1+H}(E_i + jE_q)$$



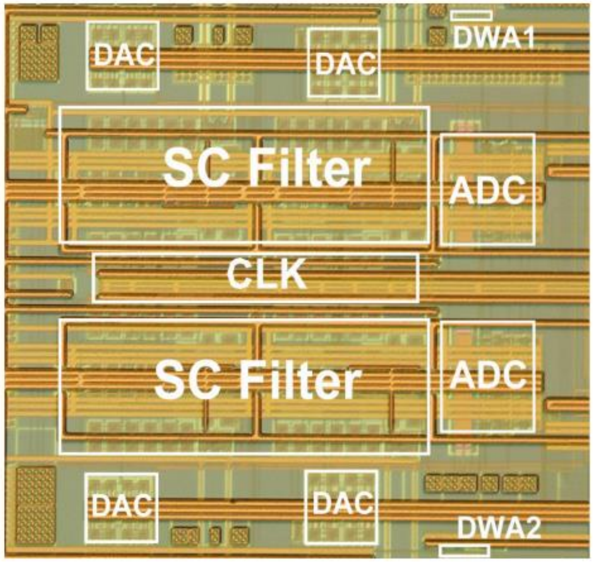
Complex bandpass noise-shaping

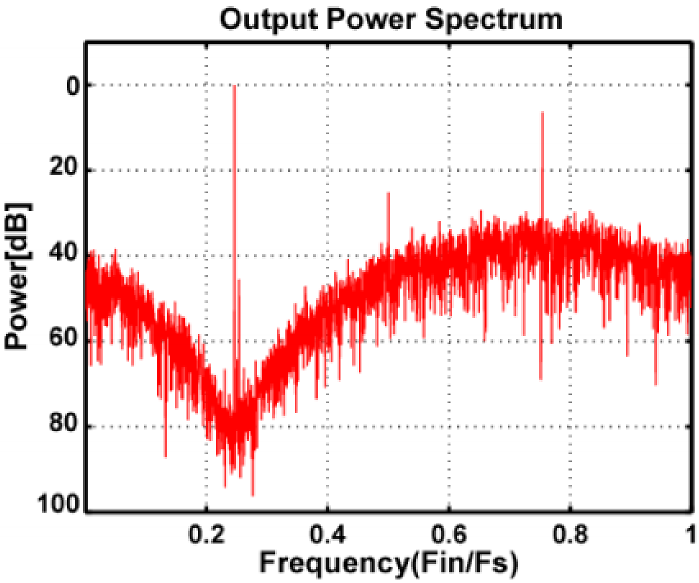
Proposed Complex Bandpass ΔΣ AD Modulator Configuration



- I, Q paths mismatch reduction
- Complex bandpass DWA algorithm for multi-bit DACs

Chip Implementation & Measurement





Technology	0.18-μm CMOS 1P6M
Supply voltage	2.8V
Sampling Frequency	20MHz
SNDR	64.5dB @ BW=78kHz
Power consumption	28.4mw
Active area	1.4mm*1.3mm

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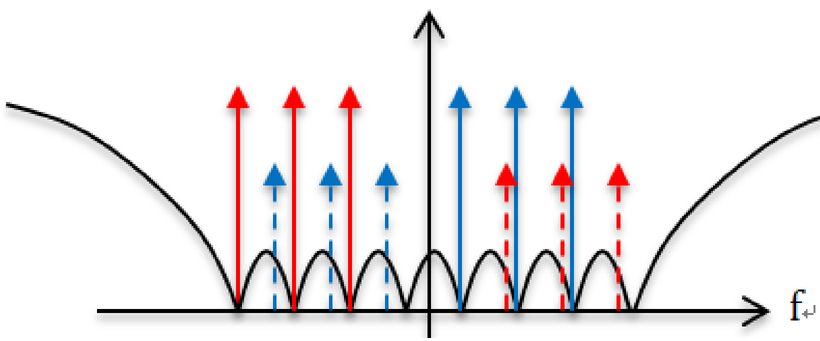
M. Murakami, H. Kobayashi, S. l N. B. Mohyar, O. Kobayashi, T. Miki, J. Kojima, "I-Q Signal Generation Techniques for Communication IC Testing and ATE Systems", IEEE International Test Conference, Fort Worth, TX (Nov. 2016).

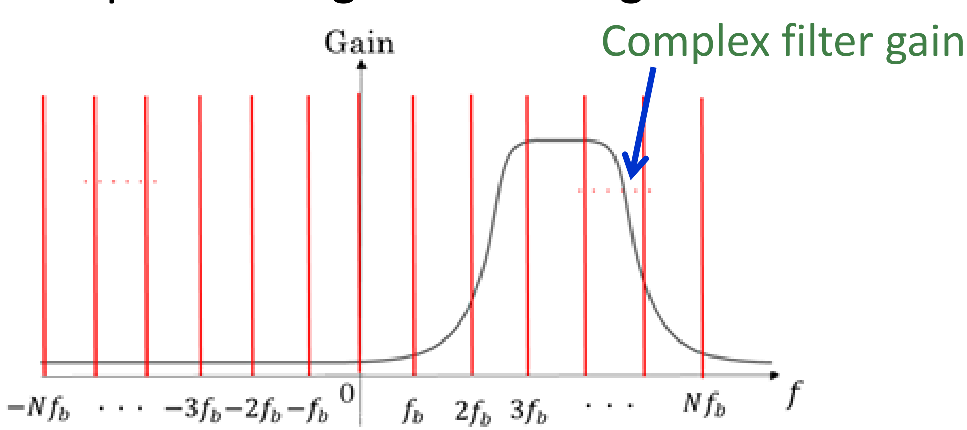
IC Testing with Complex Multi-tone Signal

(I) Image Rejection Ratio Testing of Communication ICs

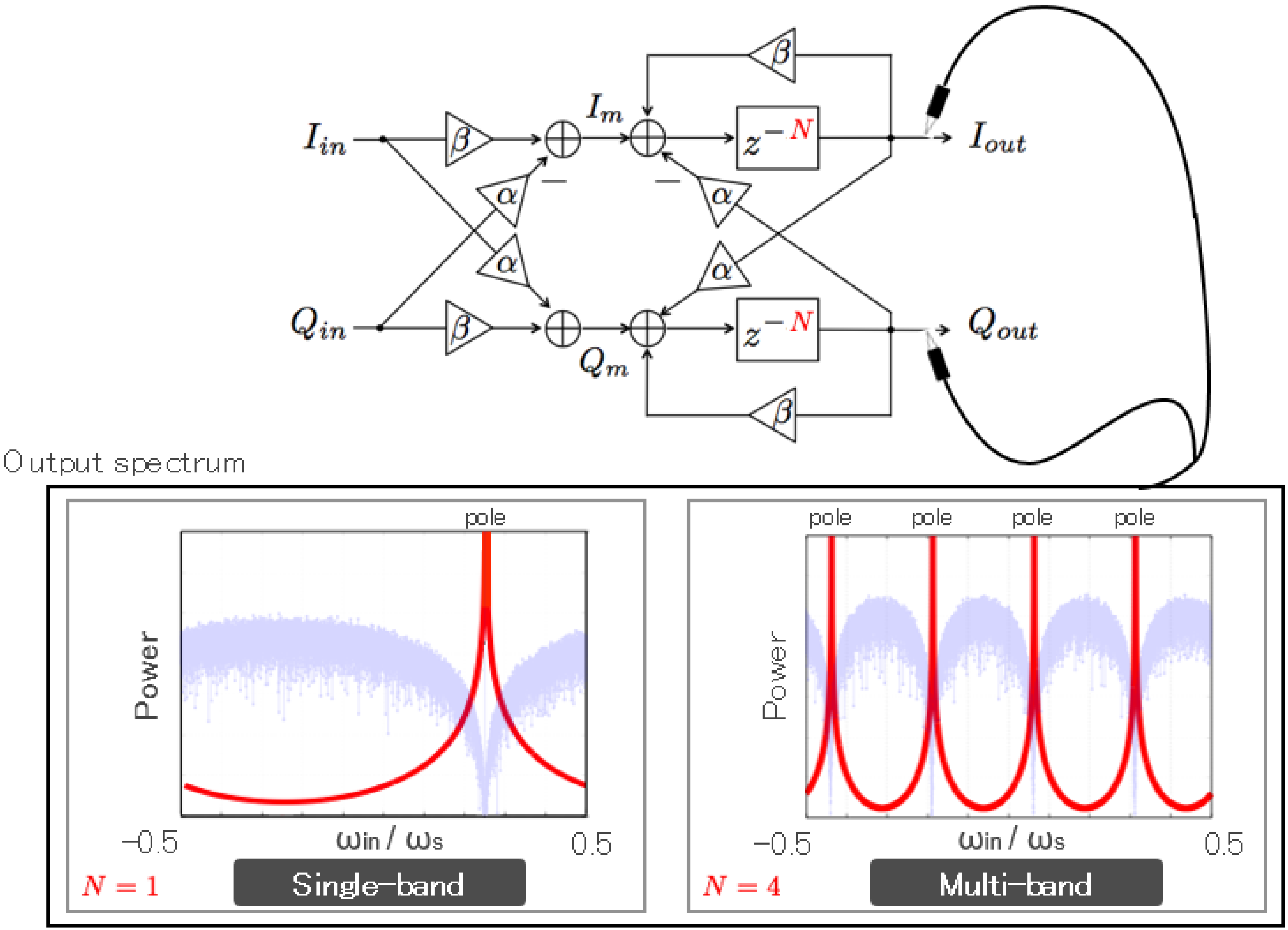


Negative freq. (input) ➡ Positive freq. (output)

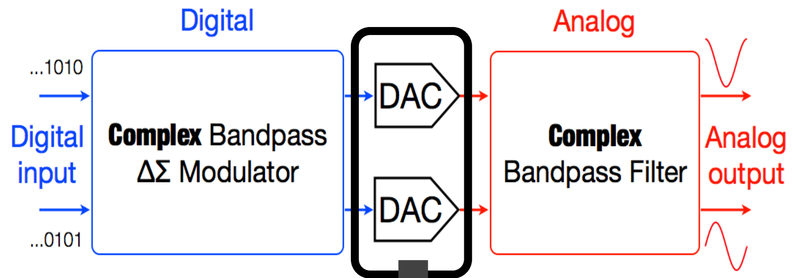
(II) Complex Analog Filter Testing

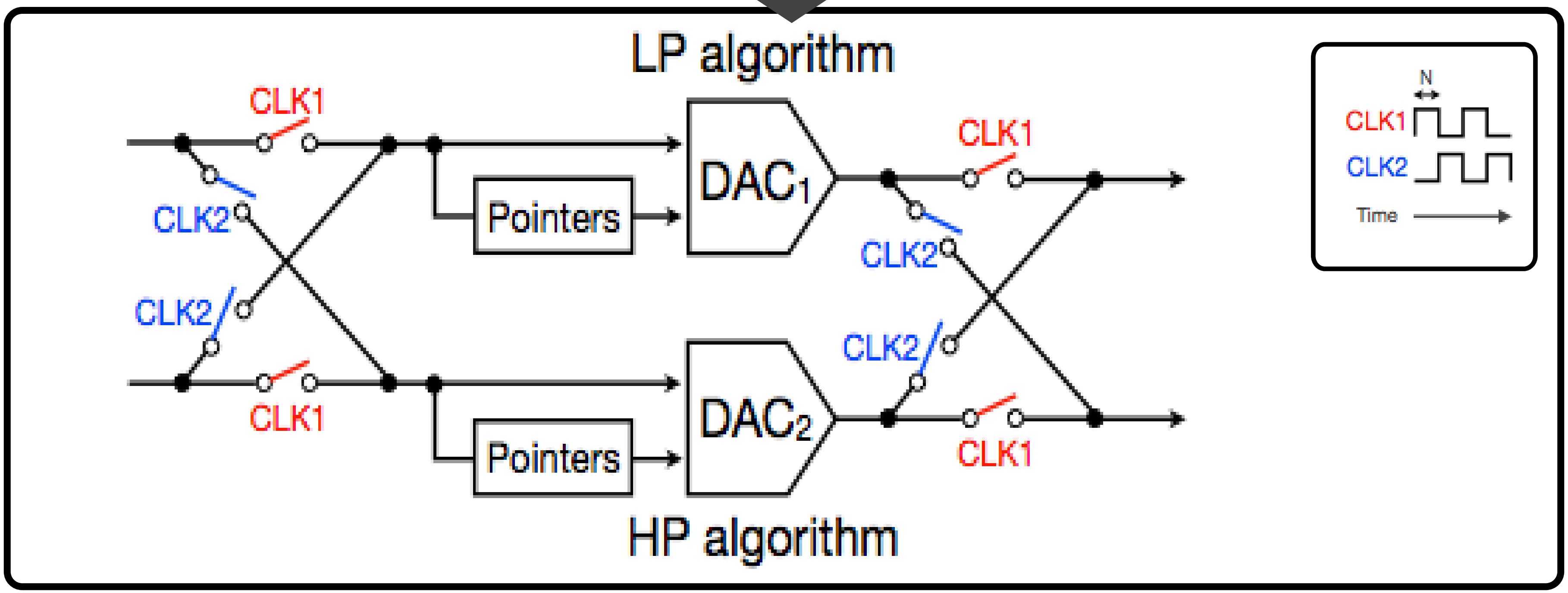


Complex Resonator



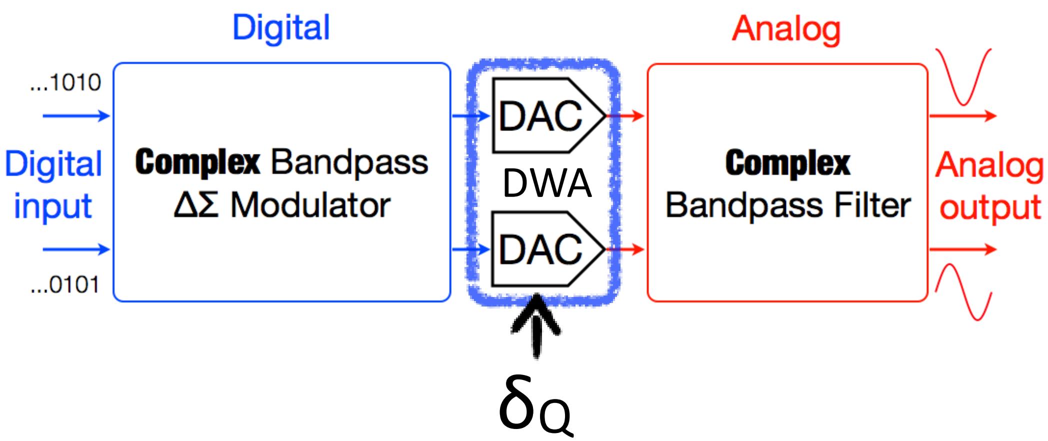
Complex N-Band DWA Algorithm

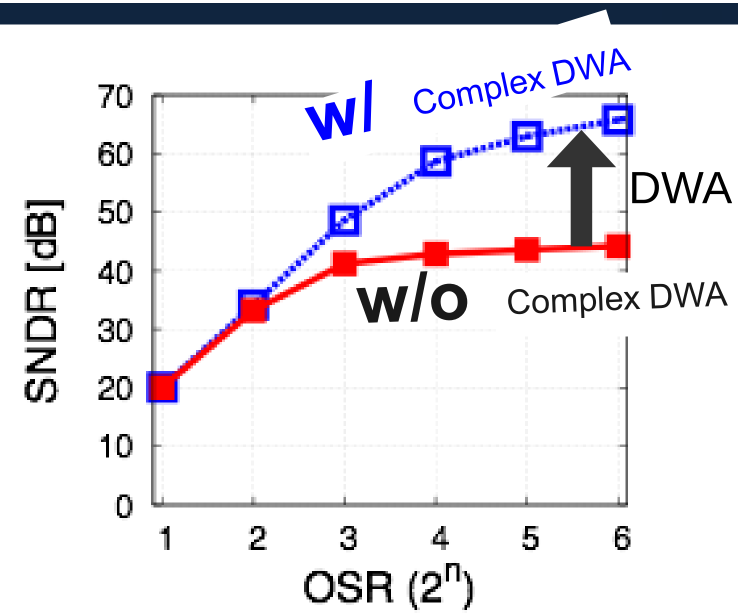


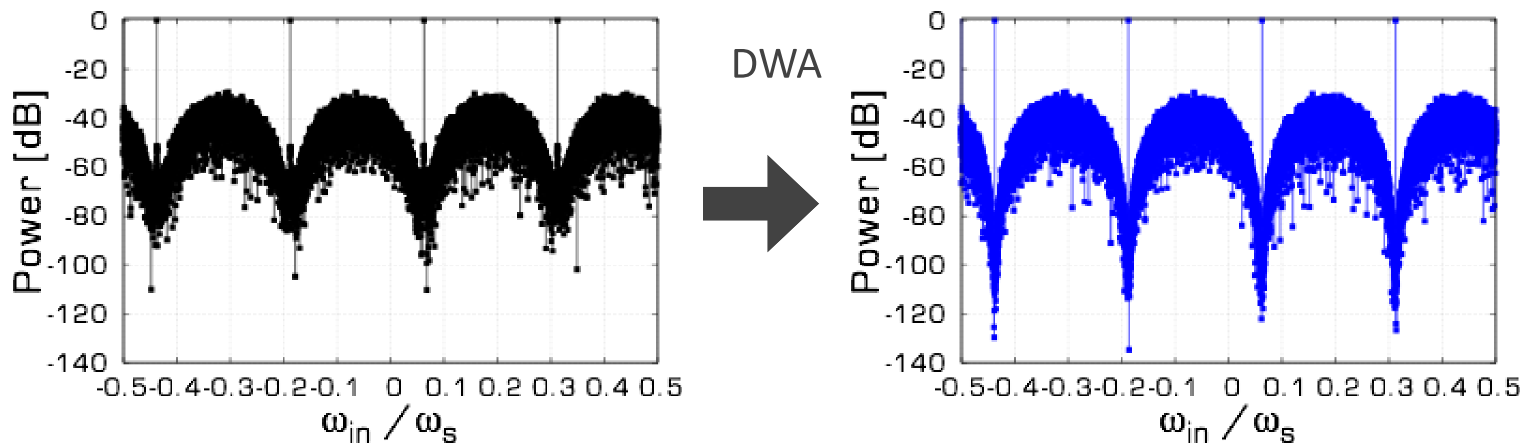


- ✦ Attach pointers
- ✦ Exchange upper-path and lower-path every N clock

Multi-tone Signal Generator







Notches filled with noise → Steep Notches

This work was done by Mr. Masahiro Murakami.

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- Conclusion

Conclusion

Complex filter is simple, but very interesting

Even somewhat mysterious !

To understand its principle, we use its complex transfer function and Hilbert transfer form.

These are useful for filter design as well as analysis

Our Recent Research Results

A2-3: 16: 45
Analysis and Evaluation Method of RC Polyphase Filter
K. Asami, N. Kushita, A. Hatta, M. T. Tran,
Y. Tamura, A. Kuwana, H. Kobayashi

A2-4: 16: 57
Flat Pass-Band Method with Two RC Band-Stop Filters
for 4-Stage Passive RC Polyphase Filter in Low-IF Receiver Systems
M. T. Tran, N. Kushita, A. Kuwana, H. Kobayashi

A2-5: 17: 09
Frequency Estimation Sampling Circuit
Using Analog Hilbert Filter and Residue Number System
Y. Abe, S. Katayama, C. Li, A. Kuwana, H. Kobayashi

Thank you for listening

謝謝

諸葛孔明　八卦 陣　may be complex.

But complex signal processing is NOT complex.

Appendix

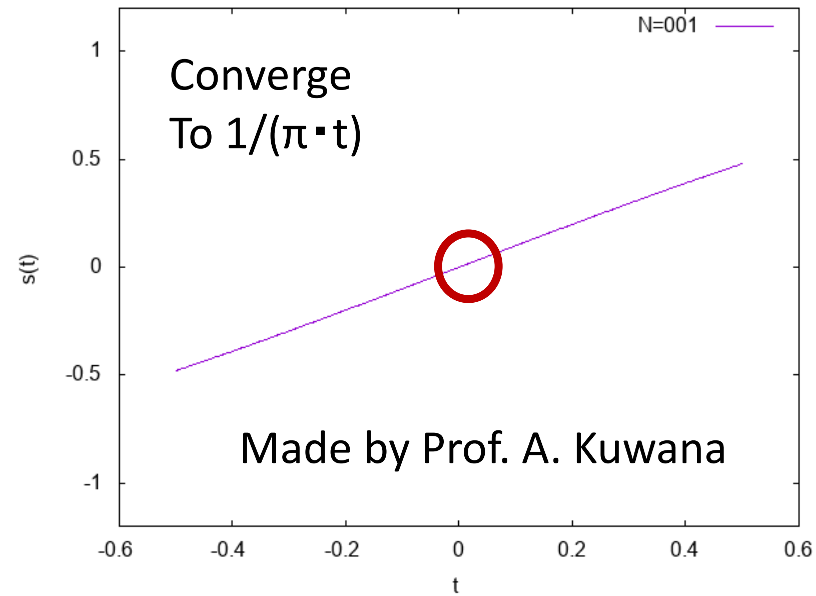
Hilbert Filter Impulse Response

$$s(t) = \frac{1}{N}\sum_{n=1}^{N} \sin(n\omega_0 t)$$

$= 1/(\pi \cdot t)$

$$\boxed{s(0)=0} \Rightarrow \boxed{\frac{1}{0} = 0}$$

ω_0=1.0,   N=700 animation



Imaginary part of impulse response for Hilbert filter

$$\int_0^\infty \cos(2\pi f t)\, df = \frac{1}{2}\delta(t) \longrightarrow \oplus \longrightarrow \int_0^\infty e^{2\pi f t} df = \frac{1}{2}\left(\delta(t) + \frac{j}{\pi t}\right)$$

$$\int_0^\infty \sin(2\pi f t)\, df = \frac{1}{2}\frac{1}{\pi t} \longrightarrow \otimes \longrightarrow \oplus$$

$j \longrightarrow \otimes$

By Product : Division by 0

$$s(t) = \frac{1}{N}\sum_{n=1}^{N} \sin(n\omega_0 t)$$ = 1/(π・t)

$$\boxed{\frac{1}{0} = 0}$$

Prof. Saburo Saito

DIVISION BY ZERO CALCULUS
(Draft)

SABUROU SAITOH

March 10, 2019

Note that the identity

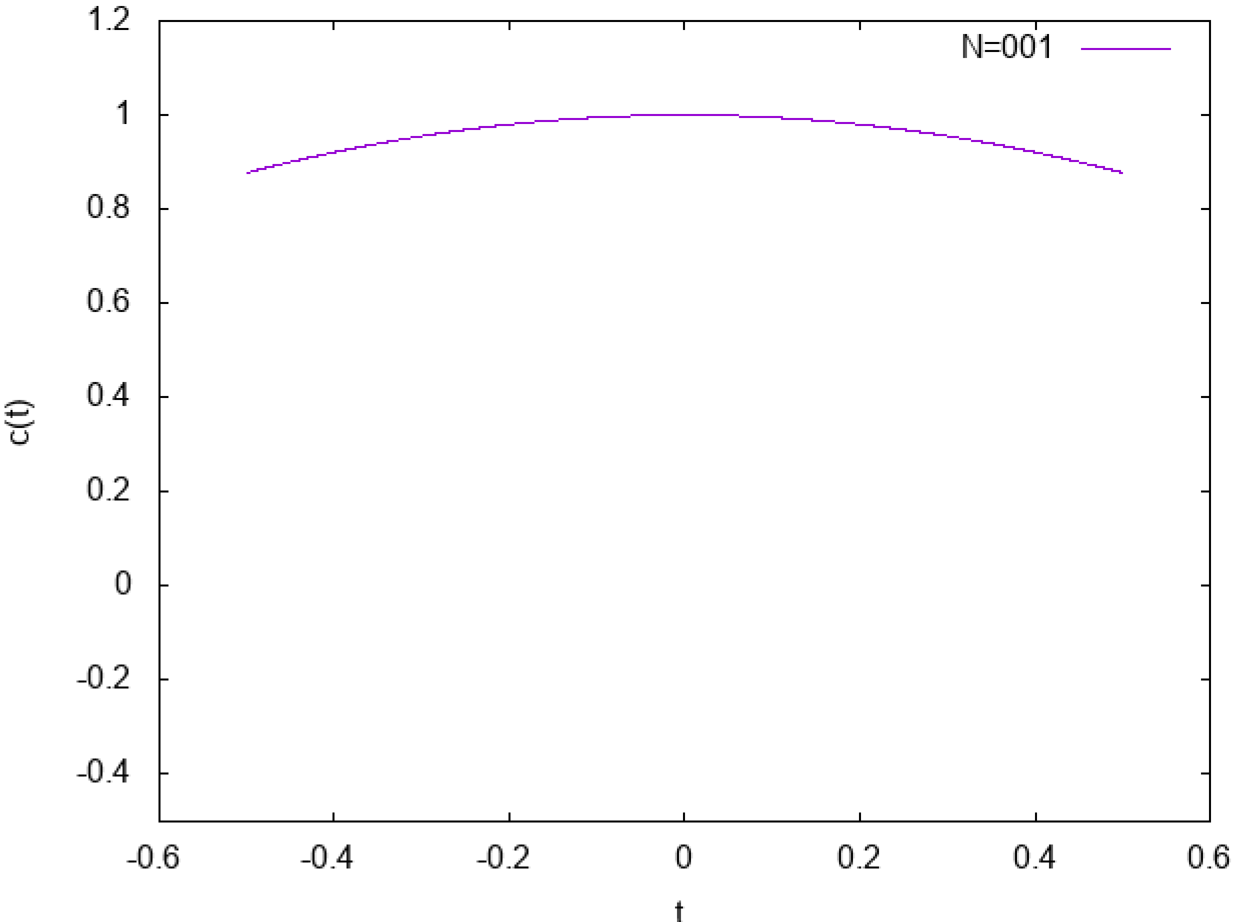
$$\int_0^\infty \sin(2\pi t\xi)d\xi = \frac{1}{2\pi}\frac{1}{t},$$

so, for $t = 0$, the both should be zero (H. Kobayashi: 2019.3.9.10:49).

Delta Function and Cosine Waves

$$c(t) = \frac{1}{N}\sum_{n=1}^{N} \cos(n\omega_0 t) \Rightarrow \delta(t)$$

ω_0=1.0, N=2000



Made by Prof. A. Kuwana