Invited

Analog / Mixed-Signal / RF Circuits for Complex Signal Processing

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Gunma University
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Outline

- Motivation for Complex Signal Processing Research
- RC Polyphase Filter: Transfer Function
- RC Polyphase Filter: Flat Passband Gain Algorithm
- RC Polyphase Filter and Hilbert Filter
- Active Complex Bandpass Filters
- Complex Bandpass ΔΣ AD Modulator
- Complex Multi-Bandpass ΔΣ DA Modulator
- Conclusion
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- Conclusion
Why My Research for Complex Signal Processing?

About 15 years ago

at IEEE International Solid-State Circuits Conference
San Francisco, CA

The most prestigious conference in IC design

Katholieke Universiteit Leuven (KU Leuven), Belgium

World top research group in analog IC design

presentation

Some simple circuit
with curious characteristics
(RC polyphase filter)

However,
I could not understand its principle
There is NO physical complex signal. It is only defined mathematically.

2 real signals: I, Q

I: In-Phase, Q: Quadrature-Phase

\[ V_{signal} = I + jQ \quad \text{Complex Signal} \]
\[ V_{image} = I - jQ \quad \text{Image} \]

\[ I = \frac{V_{signal} + V_{image}}{2} \]
\[ Q = \frac{V_{signal} - V_{image}}{2j} \]
Basic Complex Signal Processing Block

\[ j \cdot j = -1 \]

\[ Y = \hat{A} \cdot \hat{X} \]

\[ Y_I + jY_Q = (A_I + jA_Q) \cdot (X_I + jX_Q) \]

\[ = (A_I \cdot X_I - A_Q \cdot X_Q) + j \cdot (A_I \cdot X_Q + A_Q \cdot X_I) \]
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Goal of First Research

• To establish systematic design and analysis methods of RC polyphase filters.

• As its first step, to derive explicit transfer functions of the 1st-, 2nd- and 3rd-order RC polyphase filters.
Features of RC Polyphase Filter

• Its input and output are complex signal.
• Passive RC analog filter
• One of key components in wireless transceiver analog front-end
  - I, Q signal generation
  - Image rejection
• Its explicit transfer function was NOT derived yet at that time.
First-order RC Polyphase Filter

I:\ In-Phase,  Q:\ Quadrature-Phase

Differential Complex Input: \( V_{\text{in}} = I_{\text{in}} + jQ_{\text{in}} \)

Differential Complex Output: \( V_{\text{out}} = I_{\text{out}} + jQ_{\text{out}} \)
**I, Q Signal Generation**

**Single cosine**

\[ I_{in} = \cos (\omega_{LO} t) \]

\[ Q_{in} = 0 \]

\[ \omega_{LO} = \frac{1}{R_1 C_1} \]

**Cosine, Sine signals**

\[ I_{out} = A \cos (\omega_{LO} t + \theta) \]

\[ Q_{out} = A \sin (\omega_{LO} t + \theta) \]

Polyphase Filter

![Graph showing I_{out} and Q_{out} vs time](image-url)
Cosine, Sine Signals in Receiver

They are used for down conversion
Problem when $\omega_{LO} \neq 1/R_1C_1$

\[
\omega_{LO} = \frac{1}{R_1C_1}
\]

\[
\omega_{LO} = \frac{2}{R_1C_1}
\]
The problem of large difference between $I_{\text{out}}$, $Q_{\text{out}}$ amplitudes can be alleviated.

\[ \omega_{LO} = \frac{2}{R_1C_1} \]
The amplitude difference problem is further alleviated.

$$\omega_{LO} = \frac{2}{R_1 C_1}$$
Pure I, Q Signal Generation

3rd-order harmonics rejection

\[
\begin{align*}
I_{in} &= \cos(\omega_{lot}) + B \cos^3(\omega_{lot}) \\
Q_{in} &= \sin(\omega_{lot}) + B \sin^3(\omega_{lot})
\end{align*}
\]

With 3rd-order harmonics.

Without 3rd-order harmonics.
Simulation of 3rd-order Harmonics Rejection

\[ I_{in}(t) = \cos(\omega_{LO}t) + a \cos^3(\omega_{LO}t) \]

\[ Q_{in}(t) = \sin(\omega_{LO}t) + a \sin^3(\omega_{LO}t) \]

\[ 3\omega_{LO} = \frac{1}{R_1 C_1} \]

\[ I_{out}(t) = A \cos(\omega_{LO}t + \theta) \]

\[ Q_{out}(t) = A \sin(\omega_{LO}t + \theta) \]
Image Rejection Filter

\[ I_{\text{in}} = (A + B) \cos(\omega t) \]
\[ Q_{\text{in}} = (A - B) \sin(\omega t) \]

\[ I_{\text{out}} = A \cos(\omega t) \]
\[ Q_{\text{out}} = A \sin(\omega t) \]

\[ Ae^{j\omega t} + Be^{-j\omega t} \]

\[ Ae^{j\omega t} \]

signal  image
Complex Transfer Function

- Complex Signal Theory
- Complex input
- Complex output
- Complex Transfer Function

\[
V_{in} (j\omega) = I_{in} + j \cdot Q_{in}
\]

\[
V_{out} (j\omega) = I_{out} + j \cdot Q_{out}
\]

\[
G(j\omega) = \frac{V_{out} (j\omega)}{V_{in} (j\omega)}
\]
Signals in RC Polyphase Filter

Differential signal

\[ I_{in}(t) = I_{in+}(t) - I_{in-}(t) \]
\[ Q_{in}(t) = Q_{in+}(t) - Q_{in-}(t) \]
\[ I_{out}(t) = I_{out+}(t) - I_{out-}(t) \]
\[ Q_{out}(t) = Q_{out+}(t) - Q_{out-}(t) \]

Complex signal

\[ V_{in}(t) = I_{in}(t) + jQ_{in}(t) \]
\[ V_{out}(t) = I_{out}(t) + jQ_{out}(t) \]
• **Transfer Function**

\[ G_1(j\omega) = \frac{1 + \omega RC}{1 + j\omega RC} \]

• **Gain**

\[ |G_1(j\omega)| = \frac{|1 + \omega RC|}{\sqrt{1 + (\omega RC)^2}} \]

Asymmetric
Explanation of I, Q Signal Generation by $G_1(j\omega)$

$$Q_{in}(t) \equiv 0, \quad I_{in}(t) = \cos(\omega t)$$

$$V_{in}(t) = I_{in}(t) + j\ Q_{in}(t) = \cos(\omega t) = \frac{1}{2} [e^{j\omega t} + e^{-j\omega t}]$$

$$V_{out}(t) = \frac{1}{2} [ |G_1(j\omega)| e^{j(\omega t + \angle G_1(j\omega))} + |G_1(-j\omega)| e^{j(-\omega t + \angle G_1(-j\omega))} ]$$

$$= \frac{\sqrt{2}}{2} \cos \left( \omega t - \frac{\pi}{4} \right) + \frac{j\sqrt{2}}{2} \sin \left( \omega t - \frac{\pi}{4} \right)$$

Here:

$$|G_1(j\omega)|_{\omega=\frac{1}{RC}} = 0, \quad |G_1(j\omega)|_{\omega=\frac{1}{RC}} = \sqrt{2}, \quad \angle G_1(j\omega) = -\frac{\pi}{4}$$
Component Mismatch Case

\[ R_{1Q^+} := R_1 + \Delta R_{1Q^+} \quad R_{1Q^-} := R_1 + \Delta R_{1Q^-} \]

\[ R_{1I^+} := R_1 + \Delta R_{1I^+} \quad R_{1I^-} := R_1 + \Delta R_{1I^-} \]

\[ C_{1Q^+} := C_1 + \Delta C_{1Q^+} \quad C_{1Q^-} := C_1 + \Delta C_{1Q^-} \]

\[ C_{1I^+} := C_1 + \Delta C_{1I^+} \quad C_{1I^-} := C_1 + \Delta C_{1I^-} \]

\[ \Delta R_{1Q^+}, \Delta R_{1Q^-}, \Delta R_{1I^+}, \Delta R_{1I^-} : \text{Resistor variation} \]

\[ \Delta C_{1Q^+}, \Delta C_{1Q^-}, \Delta C_{1I^+}, \Delta C_{1I^-} : \text{Capacitor variation} \]

I, Q paths mismatch
\[ V_{out} = \frac{1 + \omega RC}{1 + j \omega RC} V_{in} - \frac{(1 + j) \omega RC}{2(1 + j \omega RC)^2} \Delta X \bar{V}_{in} \]

\[ V_{in} = I_{in} + j Q_{in} \]

\[ \bar{V}_{in} = I_{in} - j Q_{in} \]

Derived by Y. Niki, Gunma University
Motivation for Complex Signal Processing Research

RC Polyphase Filter: Transfer Function

RC Polyphase Filter: Flat Passband Gain Algorithm

RC Polyphase Filter and Hilbert Filter

Active Complex Bandpass Filters

Complex Bandpass ΔΣ AD Modulator

Complex Multi-Bandpass ΔΣ DA Modulator

Y. Niki, S. Sasaki, N. Yamaguchi, J. Kang, T. Kitahara, H. Kobayashi
“Flat Passband Gain Design Algorithm for 2nd-order RC Polyphase Filter,”
IEEE 11th International Conference on ASIC, Chengdu, China (Nov. 2015)
Transfer Function of 2\textsuperscript{nd}-order RC Polyphase Filter

**Transfer Function**

\[ G_2(j\omega) = \frac{(1 + \omega R_1 C_1)(1 + \omega R_2 C_2)}{1 - \omega^2 R_1 C_1 R_2 C_2 + j\omega(C_1 R_1 + C_2 R_2 + 2R_1 C_2)} \]

Derivation is very complicated, so we used "Mathematica."

Gain \(|G_2(j\omega)|\) characteristics

![Graph showing gain characteristics]
Need for Flat Passband Gain Algorithm

Transfer Function

\[ G_2(j\omega) = \frac{(1 + \omega R_1 C_1)(1 + \omega R_2 C_2)}{1 - \omega^2 R_1 C_1 R_2 C_2 + j\omega(C_1 R_1 + C_2 R_2 + 2R_1 C_2)} \]

We need flat passband gain

Gain \(|G_2(j\omega)|\) characteristics
Four Design Parameters

4 parameters: \( R_1, R_2, C_1, C_2 \)

\[
\begin{align*}
\omega_1 &= \frac{1}{R_1 C_1}, \\
\omega_2 &= \frac{1}{R_2 C_2}, \\
X &= \frac{1}{R_2 C_1}, \\
Y &= \frac{1}{R_1 C_2}
\end{align*}
\]

4 constraints
Two Constraints from Filter Spec.

- 2 zeros: $\omega_1 = \frac{-1}{R_1 C_1}$, $\omega_2 = \frac{-1}{R_2 C_2}$

are given from the filter specification.
We use the third constraint \( X = \frac{1}{R_2C_1} \) for passband gain flattening.

The fourth constraint is left for ease of IC realization.
Nyquist Chart of $G_2(j\omega)$

Gain characteristics $|G_2(j\omega)|$

Nyquist chart of $G_2(j\omega) = X(\omega) + j Y(\omega)$

$|G_2(j\omega_1)| = |G_2(j\omega_2)|$

But in general

$|G_2(j\omega_1)| = |G_2(j\omega_2)| = |G_2(j\sqrt{\omega_1\omega_2})|$
If we make $|G_2(j\omega_1)| = |G_2(j\omega_2)| = |G_2(j\sqrt{\omega_1\omega_2})|$, Passband gain becomes flat from $\omega_1$ to $\omega_2$. 

\[ |G_2(j\omega)| = |X(\omega) + jY(\omega)| \]
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- **RC Polyphase Filter and Hilbert Filter**
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Research Objective

Analyze RC polyphase filter

We found that relevance between RC polyphase filter and Hilbert filter
Hilbert Filter

■ Characteristics
  • Hilbert transform
  • 1 input and 2 outputs
  • It is often implemented in digital filter
Cosine, Sine Generation with Hilbert Filter

\[ \cos(\omega t) + j\sin(\omega t) \]

\[ \cos(\omega t) - j\sin(\omega t) \]

\[ 2\cos(\omega t) \]

\[ \omega \text{ component} \]

Hilbert filter

\[ \text{Gain} \]

\[ \omega \text{ component} \]

\[ \cos(\omega t) + j\sin(\omega t) \]
Hilbert Transform

Complex signal from real signal $x(t)$

$$x(t) \rightarrow x(t) + jy(t)$$

Hilbert transform

$$y(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau = x(t) \ast \frac{1}{\pi t}$$

Impulse response Fourier Transform

$$h(t) = \frac{1}{\pi t} \quad \text{Fourier}$$

Frequency characteristic $H(\omega)$

$$Y(\omega) = H(\omega)X(\omega) = \begin{cases} -jX(\omega) & (\omega > 0) \\ jX(\omega) & (\omega < 0) \end{cases}$$
$H_1(j\omega) = \frac{1 + \omega R_1 C_1}{1 + j\omega R_1 C_1}$ : Transfer function

Zero: $\omega_k = \frac{1}{R_k C_k}$
$H_1(j\omega) = H_{1re}(j\omega) + jH_{1im}(j\omega)$

$H_{1re}(j\omega) = \frac{H_1(j\omega) + H_1^*(-j\omega)}{2} = \frac{1}{1 + j\omega R_1 C_1}$

$H_{1im}(j\omega) = \frac{H_1(j\omega) - H_1^*(-j\omega)}{2} = -j \frac{\omega R_1 C_1}{1 + j\omega R_1 C_1}$

$|H_{1re}| - |H_{1im}| \rightarrow |H_{1re}| + |H_{1im}|$

Gain

$|H_{1re}| = |H_{1im}|$

Phase

$\angle H_1(j\omega) = \angle H_{1im}(j\omega) - \angle H_{1re}(j\omega)$

1st order RC Polyphase Filter : Gain and Phase
1st order case Analysis Results

**Gain:** Hilbert filter only at zero

**Phase:** Completely Hilbert filter

---

**RC Polyphase Filter**

**Hilbert filter**

---

Gain: Hilbert filter only at zero

Phase: Completely Hilbert filter
Results: 2\textsuperscript{nd} to 4\textsuperscript{th} RC Polyphase Filter

<table>
<thead>
<tr>
<th></th>
<th>Gain</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2\textsuperscript{nd}</strong></td>
<td><img src="image" alt="Gain Graph" /></td>
<td><img src="image" alt="Phase Graph" /></td>
</tr>
<tr>
<td><strong>3\textsuperscript{rd}</strong></td>
<td><img src="image" alt="Gain Graph" /></td>
<td><img src="image" alt="Phase Graph" /></td>
</tr>
<tr>
<td><strong>4\textsuperscript{th}</strong></td>
<td><img src="image" alt="Gain Graph" /></td>
<td><img src="image" alt="Phase Graph" /></td>
</tr>
</tbody>
</table>
1\textsuperscript{st} to 4\textsuperscript{th} order RC Polyphase Filter Analysis results

Gain: Hilbert filter only at zeros
Phase: Completely Hilbert filter

Prove for general n-th order case
\((n = 1, 2, 3, 4, 5, \ldots)\)
Order and Gain

The higher orders, the number of zeros increases; $|Hre|$ and $|Him|$ becomes close in wide range.

Close to ideal Hilbert transform
Order and Phase

Phase characteristic is not changed
There is always 90 phase difference
Fulfill Hilbert transform in full range
RC polyphase filter is approximation of ideal Hilbert filter for complex input signal
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Gm : Transconductance

Input voltage: $V_{in}$

Output current: $I_{out}$

$I_{out} = g_m V_{in}$

\[ I_{out} = I_+ - I_- = g_m V_{in} \]

Dimension of $g_m$ \( \frac{1}{R} \)

\[ V_{in} \rightarrow g_m \rightarrow I_{out} \]

\[ I_{in} \rightarrow I_+ \]

\[ I_{in} \rightarrow I_- \]

\[ I_b \]
Complex Bandpass Gm-C Filter

\[
\begin{align*}
V_{\text{Iout}} + jV_{\text{Qout}} &= \frac{I_{\text{in}} + jQ_{\text{in}}}{g_0 + sC - jg_m} \\
&= \frac{g_0^2 + g_m^2 + s^2C^2 + 2g_0sC}{g_0^2 + g_m^2 + s^2C^2 + 2g_0sC}
\end{align*}
\]
Gain of Complex Bandpass Gm-C Filter

Center Freq. \( \frac{g_m}{C} \)

\[ Q = \frac{g_m}{g_0} \]
Complex Bandpass Active RC Filter

Transfer functions of complex bandpass Gm-C and active RC filters are the same.

$$H(j\omega) = \frac{-g_1}{g_2 + j(-g_3 + \omega C)}$$

Center freq. $$\omega_0 = \frac{g_3}{C}$$  

$$Q = \frac{g_3}{2g_2}$$  

Gain $$|H(j\omega)| = \frac{g_1}{g_2}$$
Our Investigation Results

Gain: Poor Hilbert filter characteristics for both pass and stop bands
Phase: Hilbert filter only at large $|\omega|$
Motivation for Complex Signal Processing Research

RC Polyphase Filter: Transfer Function

RC Polyphase Filter: Flat Passband Gain Algorithm

RC Polyphase Filter and Hilbert Filter

Active Complex Bandpass Filters

Complex Bandpass $\Delta \Sigma$ AD Modulator

Complex Multi-Bandpass $\Delta \Sigma$ DA Modulator

"A Multibit Complex Bandpass Delta Sigma AD Modulator with I, Q Dynamic Matching and DWA Algorithm", IEEE Asian Solid-State Circuits Conference, Hangzhou, China (Nov. 2006).
Receiver Architecture Comparison

**Direct conversion receiver**

- RF → Baseband
- Zero-IF
  - ⇒ No image
- Problem of DC offset, flicker noise

**Low-IF receiver**

- RF → Low-IF
  - No problem of DC offset, flicker noise.
  - Image as well as signal are AD converted ⇒ Power is wasted

**Quadrature-IF**

- Image is not AD converted.
Complex Bandpass Delta-Sigma AD Modulator

\[ I_{\text{out}} + jQ_{\text{out}} = \frac{H}{1+H} (I_{\text{in}} + jQ_{\text{in}}) + \frac{1}{1+H} (E_i + jE_q) \]

Complex bandpass noise-shaping
Proposed Complex Bandpass $\Delta\Sigma$ AD Modulator Configuration

- I, Q paths mismatch reduction
- Complex bandpass DWA algorithm for multi-bit DACs
Chip Implementation & Measurement

![Chip Image]

**Output Power Spectrum**

<table>
<thead>
<tr>
<th>Technology</th>
<th>0.18-µm CMOS 1P6M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply voltage</td>
<td>2.8V</td>
</tr>
<tr>
<td>Sampling Frequency</td>
<td>20MHz</td>
</tr>
<tr>
<td>SNDR</td>
<td>64.5dB @ BW=78kHz</td>
</tr>
<tr>
<td>Power consumption</td>
<td>28.4mw</td>
</tr>
<tr>
<td>Active area</td>
<td>1.4mm*1.3mm</td>
</tr>
</tbody>
</table>
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IC Testing with Complex Multi-tone Signal

(I) Image Rejection Ratio Testing of Communication ICs

![Image of frequency response with I, Q imbalance]

- Negative freq. (input)
- Positive freq. (output)

(II) Complex Analog Filter Testing

![Diagram of frequency response with complex filter gain]
Complex Resonator

Output spectrum

- $N = 1$: Single-band
- $N = 4$: Multi-band
Attach pointers

Exchange upper-path and lower-path every N clock
Multi-tone Signal Generator

This work was done by Mr. Masahiro Murakami.
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Conclusion
Conclusion

Complex filter is simple, but very interesting

Even somewhat mysterious!

To understand its principle, we use its complex transfer function and Hilbert transfer form.

These are useful for filter design as well as analysis.
A2-3: 16: 45
Analysis and Evaluation Method of RC Polyphase Filter
K. Asami, N. Kushita, A. Hatta, M. T. Tran,
Y. Tamura, A. Kuwana, H. Kobayashi

A2-4: 16: 57
Flat Pass-Band Method with Two RC Band-Stop Filters for 4-Stage Passive RC Polyphase Filter in Low-IF Receiver Systems
M. T. Tran, N. Kushita, A. Kuwana, H. Kobayashi

A2-5: 17: 09
Frequency Estimation Sampling Circuit Using Analog Hilbert Filter and Residue Number System
Y. Abe, S. Katayama, C. Li, A. Kuwana, H. Kobayashi
Thank you for listening
謝謝

諸葛孔明 八卦陣 may be complex.

But complex signal processing is NOT complex.
Appendix
Hilbert Filter Impulse Response

\[ s(t) = \frac{1}{N} \sum_{n=1}^{N} \sin(n \omega_0 t) \]

\[ = \frac{1}{(\pi \cdot t)} \]

\( s(0) = 0 \) \( \Rightarrow \frac{1}{0} = 0 \)

\( \omega_0 = 1.0, \ N = 700 \) animation

Converge To \( \frac{1}{(\pi \cdot t)} \)

Imaginary part of impulse response for Hilbert filter

\[ \int_0^\infty \cos(2\pi ft) \, df = \frac{1}{2} \delta(t) \]

\[ \int_0^\infty e^{2\pi ft} \, df = \frac{1}{2} \left( \delta(t) + \frac{j}{\pi t} \right) \]

\[ \int_0^\infty \sin(2\pi ft) \, df = \frac{1}{2} \frac{1}{\pi t} \]
By Product: Division by 0

\[ s(t) = \frac{1}{N} \sum_{n=1}^{N} \sin(n\omega_0 t) = 1/(\pi \cdot t) \]

\[ \frac{1}{0} = 0 \]

DIVISION BY ZERO CALCULUS
(Draft)

SABUROU SAITOH
March 10, 2019

Note that the identity

\[ \int_{0}^{\infty} \sin(2\pi t \xi) d\xi = \frac{1}{2\pi} \frac{1}{t}, \]

so, for \( t = 0 \), the both should be zero (H. Kobayashi: 2019.3.9.10:49).
Delta Function and Cosine Waves

\[ c(t) = \frac{1}{N} \sum_{n=1}^{N} \cos(n\omega_0 t) \rightarrow \delta(t) \]

\[ \omega_0 = 1.0, \quad N = 2000 \]