Flat Pass-Band Method with Two RC Band-Stop Filters for 4-Stage Passive RC Polyphase Filter in Low-IF Receiver Systems

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Outline

1. Research Background
   - Applications of RC Poly-phase Network
   - Analog Complex Signal Processing Concepts
2. Analysis of 4-Stage Passive RC Network
   - Frequency Responses
   - Image Rejection Ratio
3. Proposed Model of 4-Stage Passive RC Network
   - Flat pass-band gain with two RC band-stop filters
   - Experimental Results
4. Conclusions

1. Research Background

Research Objective & Design Approach

Objective

Design of Image Rejection Filter for Blue-tooth Receiver:

- ○ Low image noise
- ○ Simple Model of RC Network

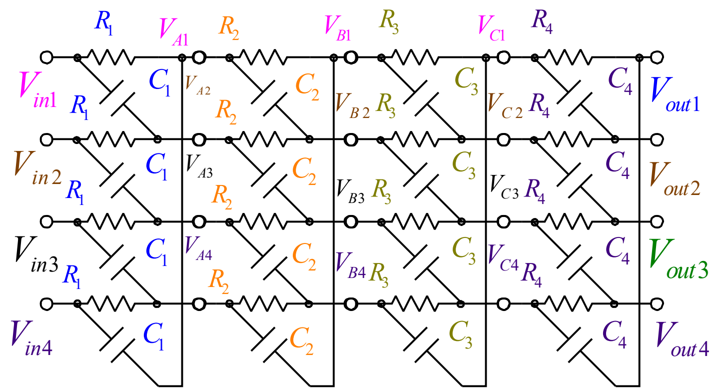
Approach

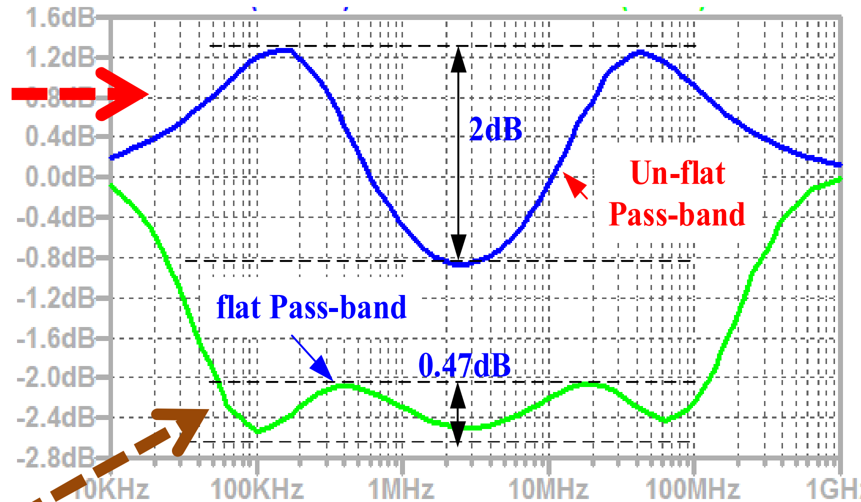
- ○ Derivation of transfer function of 4-stage passive rc network based on widened superposition principle
- ○ Mathematical analysis of transfer function of 4-stage passive RC poly-phase filter
- ○ Flat pass-band gain with two RC band-stop filters

1. Research Background

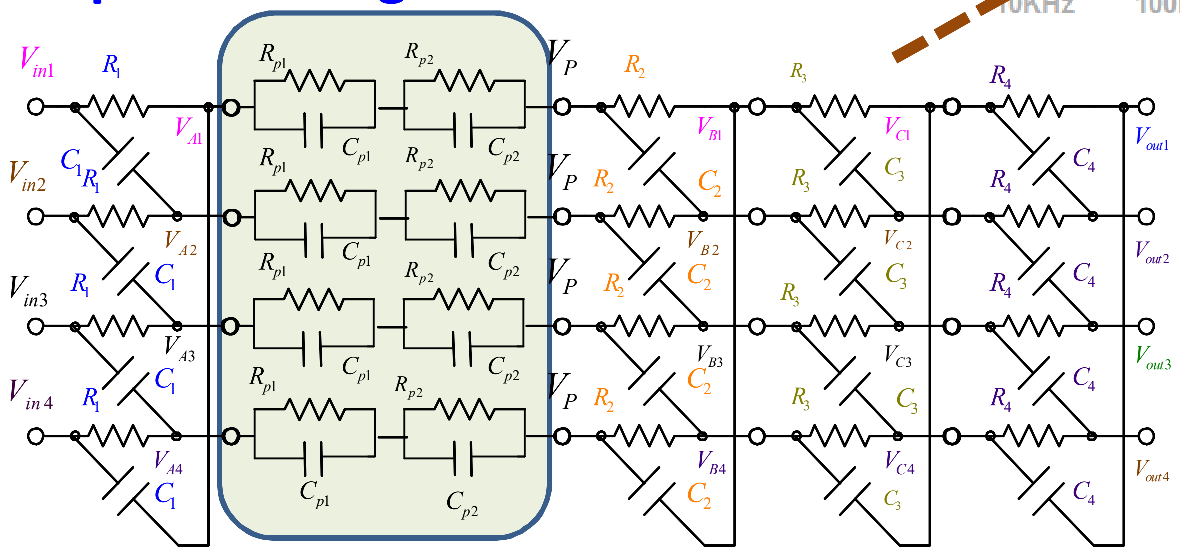
Design Achievements of This Work

Conventional Design





Proposed Design



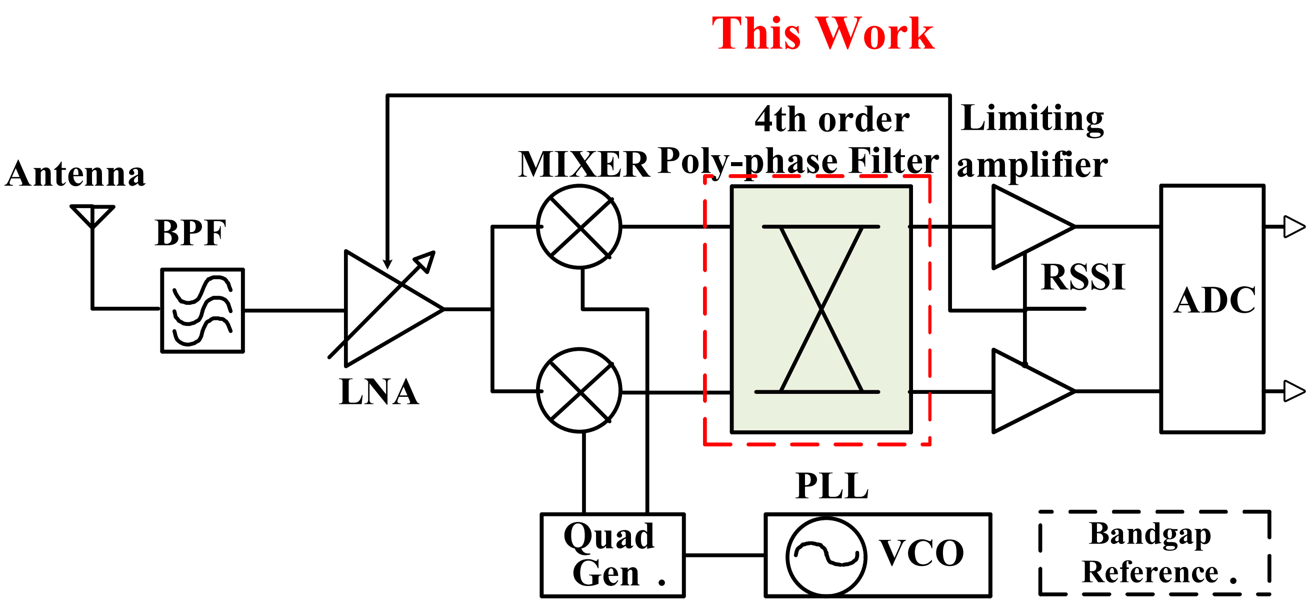
Two RC band-stop filters

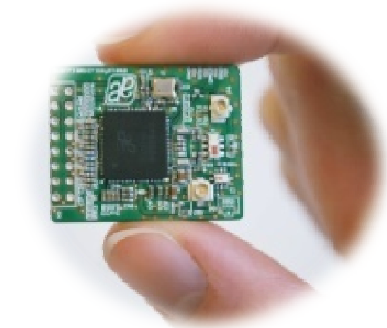
Pass-band Gain Ripple Reduction

from 2dB → 0.47dB

1. Research Background

Low-IF Receiver System Architecture





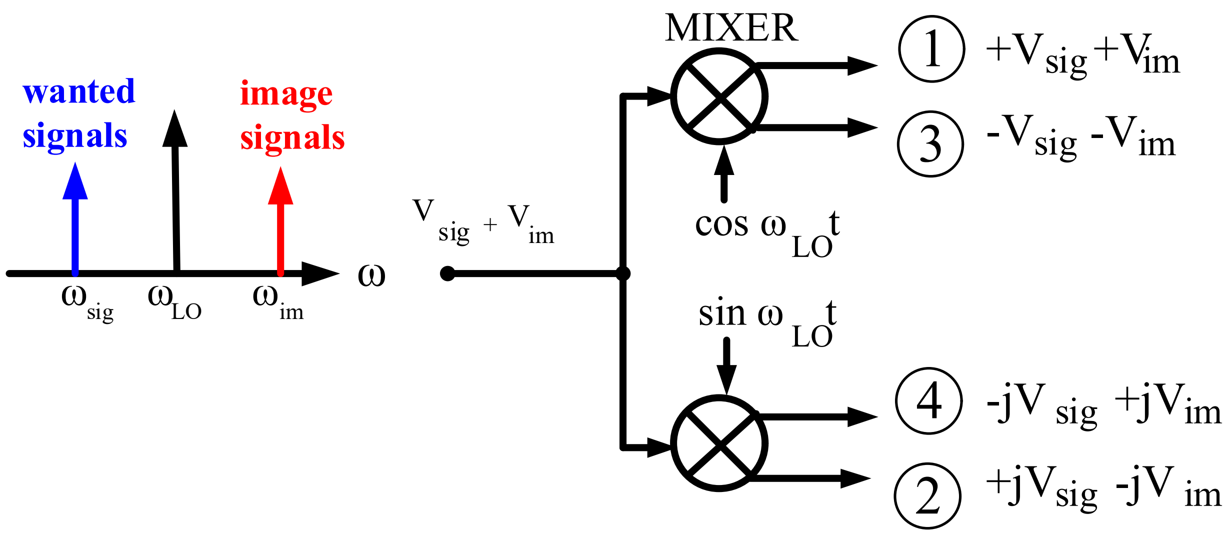
Applications:
ZigBee, Bluetooth, TV Set Top Box …

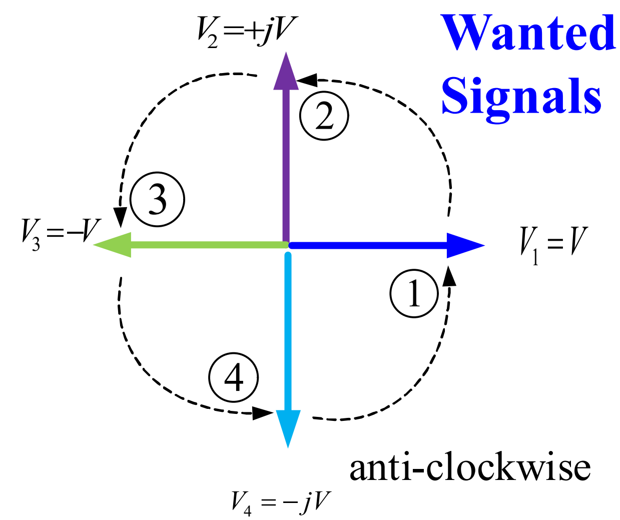
Merits
- Low-cost
- Small-size
- High-integration

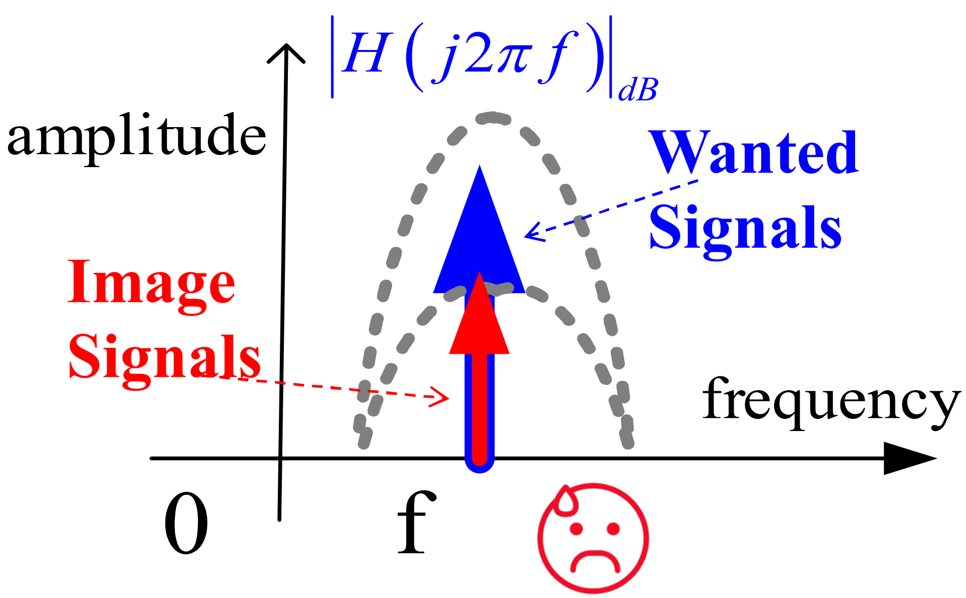
Demerits
- Image Noises
- Power Loss
- Noise Figure

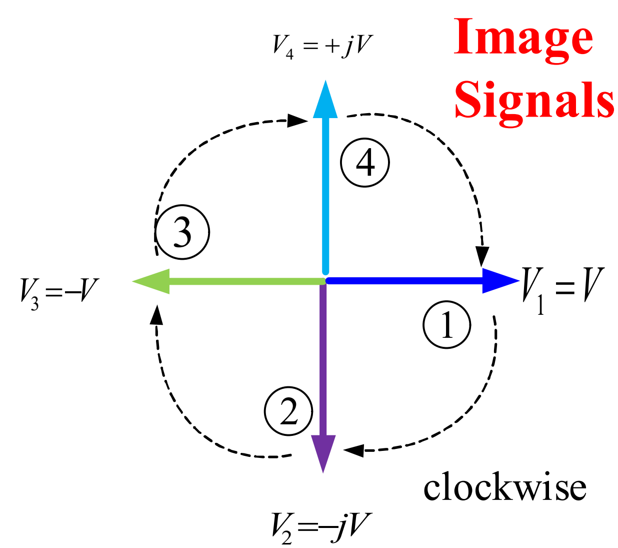
1. Research Background

Frequency Plane of Low-IF Signals



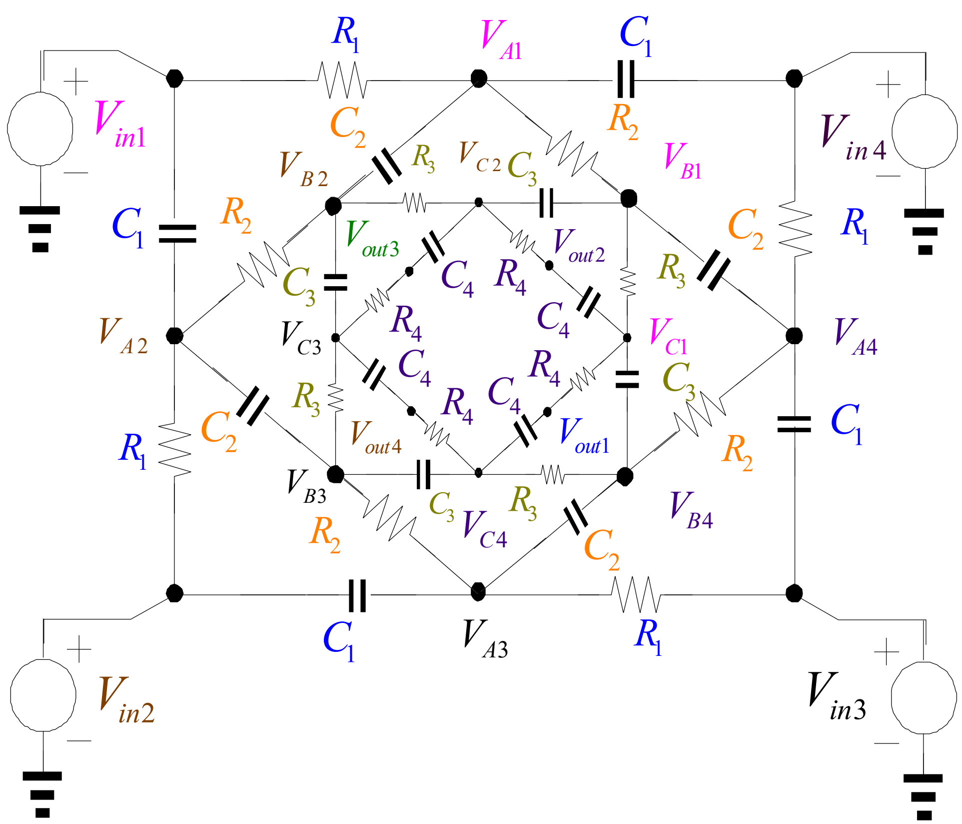






2. Analysis of 4-Stage Passive RC Network

Design of 4-Stage Passive RC Network



Parameter	PPF
Input IF frequency	5MHz
Input IF Power	-60dBm
NF	10dB
Gain	0dB
Current Consumption	<5mA
Supply Voltage	1.8V
Image Rejection Ratio IMRR	< -30 dB

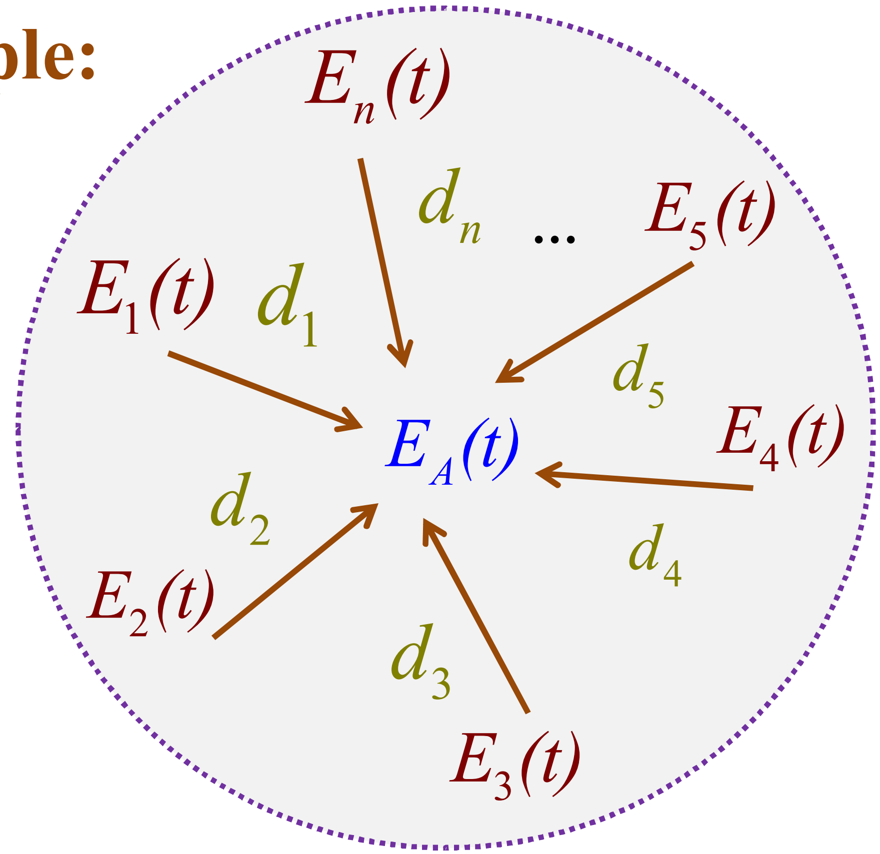
2. Analysis of 4-Stage Passive RC Network

Definition of Widened Superposition Principle

A superposition of energy at *one place* is proportional with their *input sources* and *resistance distances of transmission spaces*.

Widened Superposition Principle:

$$E_A(t)\sum_{i=1}^{n}\frac{1}{d_i}=\sum_{i=1}^{n}\frac{E_i(t)}{d_i}$$



$E_A(t)$: *energy at one place*

$E_i(t)$: *input sources*

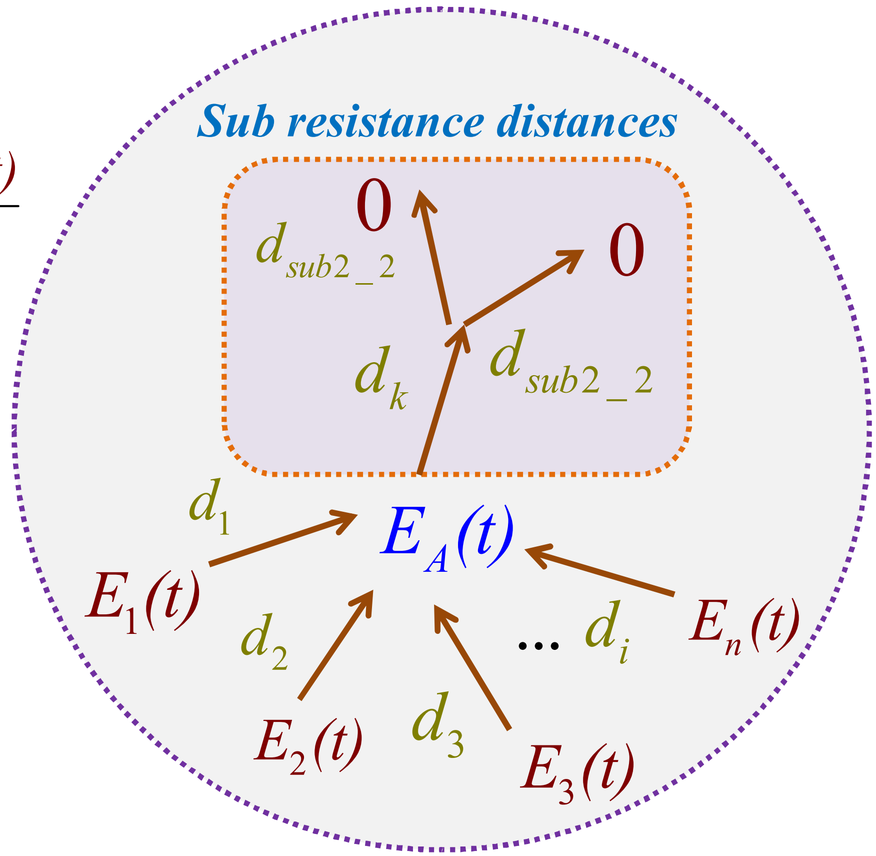
$d_i(t)$: *resistance distances*

2. Analysis of 4-Stage Passive RC Network

Widened Superposition principle with sub branches

If a resistance distance is *not direct* or *divided into small branches*, *an energy* at every small place will be calculated by the *direct resistance distances* of every branch.

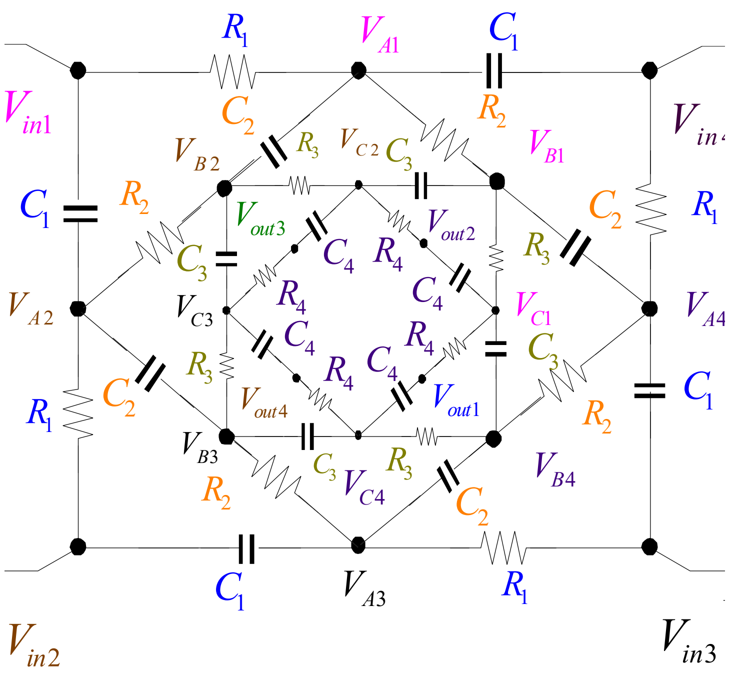
$$E_A(t)\left(\sum_{i=1}^{n}\frac{1}{d_i}+\frac{1}{d_k+\dfrac{1}{\dfrac{1}{d_{k\_sub1}}+\dfrac{1}{d_{k\_sub2}}+\dots}}\right)=\sum_{i=1}^{n}\frac{E_i(t)}{d_i}$$



$E_A(t)$: *energy at one place*
$E_i(t)$: *input sources*
$d_i(t)$: *resistance distances*
$d_k(t)$: *sub resistance distances*

2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages on 1st Loop



Widened Superposition Principle

$$\begin{cases} V_{A1}\,Y_{total\_A} = \dfrac{V_{in1}}{R_1} + \dfrac{V_{in4}}{Z_{C1}} \\ V_{A2}\,Y_{total\_A} = \dfrac{V_{in2}}{R_1} + \dfrac{V_{in1}}{Z_{C1}} \\ V_{A3}\,Y_{total\_A} = \dfrac{V_{in3}}{R_1} + \dfrac{V_{in2}}{Z_{C1}} \\ V_{A4}\,Y_{total\_A} = \dfrac{V_{in4}}{R_1} + \dfrac{V_{in3}}{Z_{C1}} \end{cases}$$

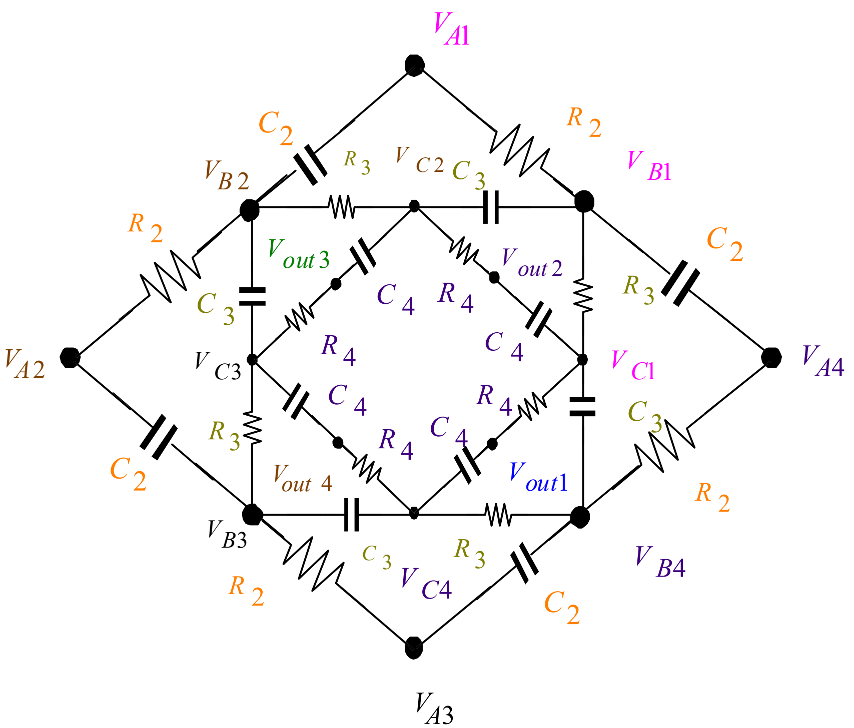
Here:

$$Y_{total\_A} = \frac{1}{R_1} + \frac{1}{Z_{C1}} + \frac{1}{R_2 + \dfrac{1}{\dfrac{1}{Z_{C2}} + \dfrac{1}{R_3 + \dfrac{1}{\dfrac{1}{Z_{C3}} + \dfrac{2}{Z_{C4}+R_4}}} + \dfrac{1}{Z_{C3} + \dfrac{1}{\dfrac{1}{R_3} + \dfrac{2}{Z_{C4}+R_4}}}}} + \frac{1}{Z_{C2} + \dfrac{1}{\dfrac{1}{R_2} + \dfrac{1}{R_3 + \dfrac{1}{\dfrac{1}{Z_{C3}} + \dfrac{2}{Z_{C4}+R_4}}} + \dfrac{1}{Z_{C3} + \dfrac{1}{\dfrac{1}{R_3} + \dfrac{2}{Z_{C4}+R_4}}}}}$$

$$= \frac{\left[2R_1Z_{C1} + (R_1+Z_{C1})(R_2+Z_{C2})\right]\left[2R_3Z_{C3} + (R_3+Z_{C3})(R_4+Z_{C4})\right] + 2(R_3+Z_{C3}+R_4+Z_{C4})\left[(R_1+Z_{C1})R_2Z_{C2} + R_1Z_{C1}(R_2+Z_{C2})\right]}{R_1Z_{C1}\left\{(R_2+Z_{C2})\left[2R_3Z_{C3} + (R_3+Z_{C3})(R_4+Z_{C4})\right] + 2R_2Z_{C2}(R_3+Z_{C3}+R_4+Z_{C4})\right\}}$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages on 2nd Loop



Widened Superposition Principle

$$
\begin{cases}
V_{B1} Y_{total\_B} = \dfrac{V_{A1}}{R_2} + \dfrac{V_{A4}}{Z_{C2}} \\
V_{B2} Y_{total\_B} = \dfrac{V_{A2}}{R_2} + \dfrac{V_{A1}}{Z_{C2}} \\
V_{B3} Y_{total\_B} = \dfrac{V_{A3}}{R_2} + \dfrac{V_{A2}}{Z_{C2}} \\
V_{B4} Y_{total\_B} = \dfrac{V_{A4}}{R_2} + \dfrac{V_{A3}}{Z_{C2}}
\end{cases}
$$

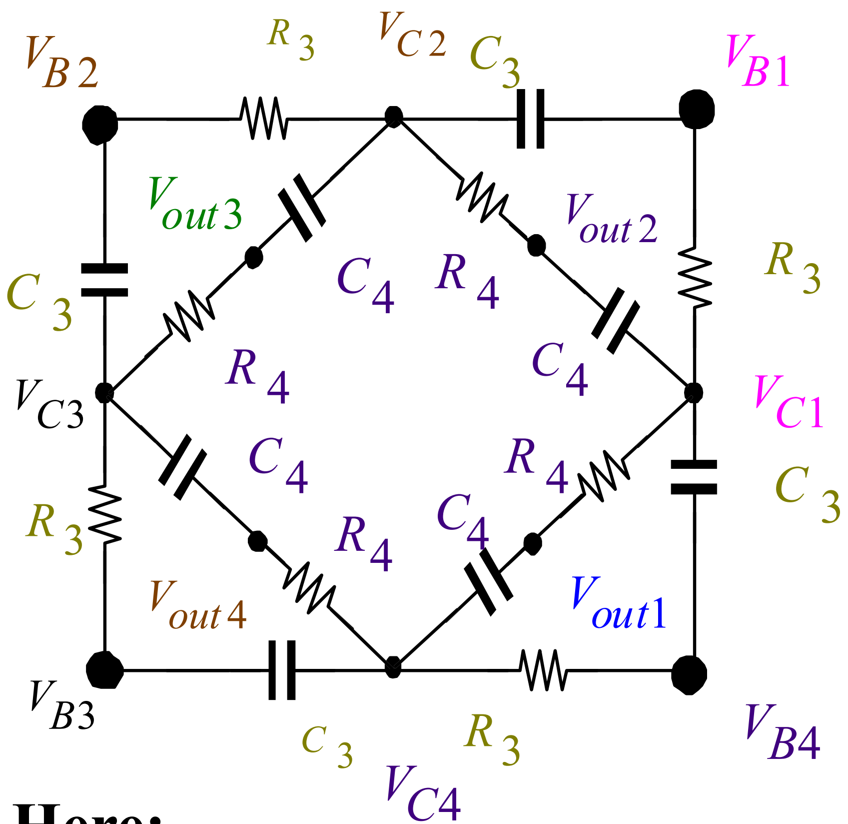
Here:

$$
Y_{total\_B} = \frac{1}{R_2} + \frac{1}{Z_{C2}} + \frac{1}{R_3 + \dfrac{1}{\dfrac{1}{Z_{C3}} + \dfrac{2}{Z_{C4} + R_4}}} + \frac{1}{Z_{C3} + \dfrac{1}{\dfrac{1}{R_3} + \dfrac{2}{Z_{C4} + R_4}}}
$$

$$
= \frac{(R_2 + Z_{C2})\left[2R_3 Z_{C3} + (R_3 + Z_{C3})(R_4 + Z_{C4})\right] + 2R_2 Z_{C2}(R_3 + Z_{C3} + R_4 + Z_{C4})}{R_2 Z_{C2}\left[2R_3 Z_{C3} + (R_3 + Z_{C3})(R_4 + Z_{C4})\right]}
$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages on 3rd Loop



Widened Superposition Principle

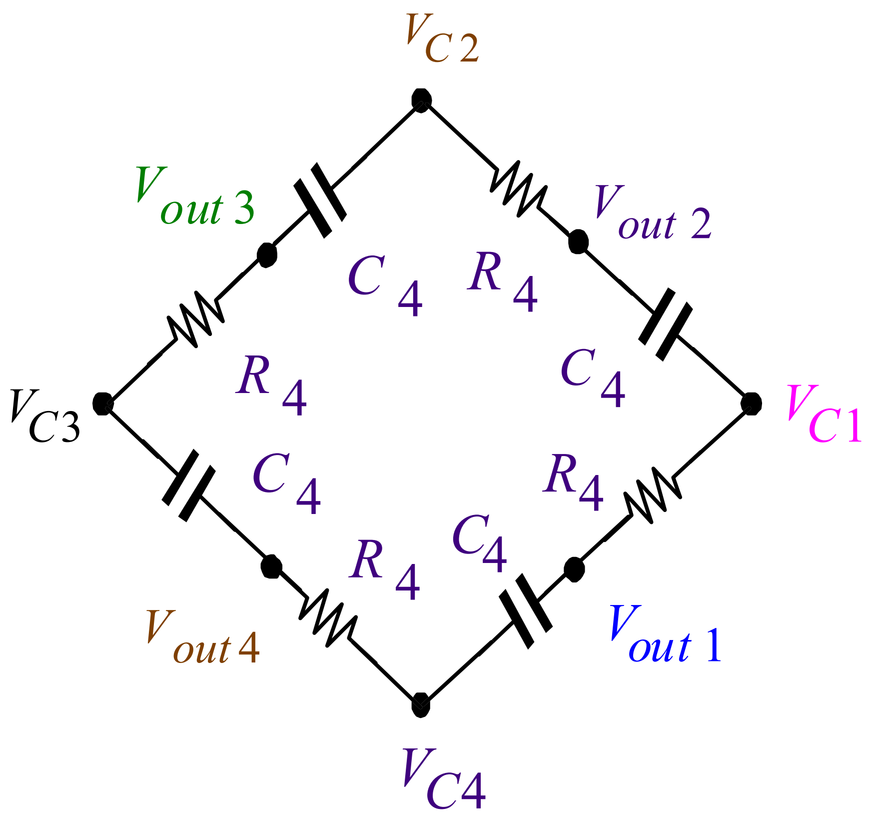
$$\begin{cases} V_{C1} Y_{total\_C} = \dfrac{V_{B1}}{R_3} + \dfrac{V_{B4}}{Z_{C3}} \\ V_{C2} Y_{total\_C} = \dfrac{V_{B2}}{R_3} + \dfrac{V_{B1}}{Z_{C3}} \\ V_{C3} Y_{total\_C} = \dfrac{V_{B3}}{R_3} + \dfrac{V_{B2}}{Z_{C3}} \\ V_{C4} Y_{total\_C} = \dfrac{V_{B4}}{R_3} + \dfrac{V_{B3}}{Z_{C3}} \end{cases}$$

Here:

$$Y_{total\_C} = \frac{1}{R_3} + \frac{1}{Z_{C3}} + \frac{2}{Z_{C4} + R_4} = \frac{\left(Z_{C3} + R_3\right)\left(Z_{C4} + R_4\right) + 2R_3 Z_{C3}}{R_3 Z_{C3}\left(Z_{C4} + R_4\right)}$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages on 4th Loop



Widened Superposition Principle

$$\begin{cases} V_{out1}\,Y_{total\_D} = \dfrac{V_{C1}}{R_4} + \dfrac{V_{C4}}{Z_{C4}} \\ V_{out2}\,Y_{total\_D} = \dfrac{V_{C2}}{R_4} + \dfrac{V_{C1}}{Z_{C4}} \\ V_{out3}\,Y_{total\_D} = \dfrac{V_{C3}}{R_4} + \dfrac{V_{C2}}{Z_{C4}} \\ V_{out4}\,Y_{total\_D} = \dfrac{V_{C4}}{R_4} + \dfrac{V_{C3}}{Z_{C4}} \end{cases}$$

Here:

$$Y_{total\_D} = \frac{1}{R_4} + \frac{1}{Z_{C4}} = \frac{R_4 + Z_{C4}}{R_4 Z_{C4}}$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages on 4th Loop

Voltages on 4th loop

$$
\begin{cases}
V_{out1} = \dfrac{V_{in1}\left(Z_{C4}\left(\begin{aligned}&Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]+\left(\frac{V_{in4}}{V_{in1}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]\right\}\\&+\left(\frac{V_{in4}}{V_{in1}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]+\left(\frac{V_{in3}}{V_{in4}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]\right\}\end{aligned}\right)+\left(\frac{V_{in4}}{V_{in1}}\right)R_4\left(\begin{aligned}&Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]+\left(\frac{V_{in3}}{V_{in4}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]\right\}\\&+\left(\frac{V_{in3}}{V_{in4}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]+\left(\frac{V_{in2}}{V_{in3}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]\right\}\end{aligned}\right)\right)}{\left[2R_1Z_{C1}+\left(Z_{C2}+R_2\right)\left(Z_{C1}+R_1\right)\right]\left[2R_3Z_{C3}+\left(Z_{C3}+R_3\right)\left(Z_{C4}+R_4\right)\right]+2\left(Z_{C3}+R_3+Z_{C4}+R_4\right)\left[\left(Z_{C1}+R_1\right)R_2Z_{C2}+\left(Z_{C2}+R_2\right)R_1Z_{C1}\right]} \\[2ex]
V_{out2} = \dfrac{V_{in2}\left(Z_{C4}\left(\begin{aligned}&Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]+\left(\frac{V_{in1}}{V_{in2}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]\right\}\\&+\left(\frac{V_{in1}}{V_{in2}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]+\left(\frac{V_{in4}}{V_{in1}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]\right\}\end{aligned}\right)+\left(\frac{V_{in1}}{V_{in2}}\right)R_4\left(\begin{aligned}&Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]+\left(\frac{V_{in4}}{V_{in1}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]\right\}\\&+\left(\frac{V_{in4}}{V_{in1}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]+\left(\frac{V_{in3}}{V_{in4}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]\right\}\end{aligned}\right)\right)}{\left[2R_1Z_{C1}+\left(Z_{C2}+R_2\right)\left(Z_{C1}+R_1\right)\right]\left[2R_3Z_{C3}+\left(Z_{C3}+R_3\right)\left(Z_{C4}+R_4\right)\right]+2\left(Z_{C3}+R_3+Z_{C4}+R_4\right)\left[\left(Z_{C1}+R_1\right)R_2Z_{C2}+\left(Z_{C2}+R_2\right)R_1Z_{C1}\right]} \\[2ex]
V_{out3} = \dfrac{V_{in3}\left(Z_{C4}\left(\begin{aligned}&Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]+\left(\frac{V_{in2}}{V_{in3}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]\right\}\\&+\left(\frac{V_{in2}}{V_{in3}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]+\left(\frac{V_{in1}}{V_{in2}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]\right\}\end{aligned}\right)+\left(\frac{V_{in2}}{V_{in3}}\right)R_4\left(\begin{aligned}&Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]+\left(\frac{V_{in1}}{V_{in2}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]\right\}\\&+\left(\frac{V_{in1}}{V_{in2}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]+\left(\frac{V_{in4}}{V_{in1}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]\right\}\end{aligned}\right)\right)}{\left[2R_1Z_{C1}+\left(Z_{C2}+R_2\right)\left(Z_{C1}+R_1\right)\right]\left[2R_3Z_{C3}+\left(Z_{C3}+R_3\right)\left(Z_{C4}+R_4\right)\right]+2\left(Z_{C3}+R_3+Z_{C4}+R_4\right)\left[\left(Z_{C1}+R_1\right)R_2Z_{C2}+\left(Z_{C2}+R_2\right)R_1Z_{C1}\right]} \\[2ex]
V_{out4} = \dfrac{V_{in4}\left(Z_{C4}\left(\begin{aligned}&Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]+\left(\frac{V_{in3}}{V_{in4}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]\right\}\\&+\left(\frac{V_{in3}}{V_{in4}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]+\left(\frac{V_{in2}}{V_{in3}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]\right\}\end{aligned}\right)+\left(\frac{V_{in3}}{V_{in4}}\right)R_4\left(\begin{aligned}&Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]+\left(\frac{V_{in2}}{V_{in3}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]\right\}\\&+\left(\frac{V_{in2}}{V_{in3}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]+\left(\frac{V_{in1}}{V_{in2}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]\right\}\end{aligned}\right)\right)}{\left[2R_1Z_{C1}+\left(Z_{C2}+R_2\right)\left(Z_{C1}+R_1\right)\right]\left[2R_3Z_{C3}+\left(Z_{C3}+R_3\right)\left(Z_{C4}+R_4\right)\right]+2\left(Z_{C3}+R_3+Z_{C4}+R_4\right)\left[\left(Z_{C1}+R_1\right)R_2Z_{C2}+\left(Z_{C2}+R_2\right)R_1Z_{C1}\right]}
\end{cases}
$$

2. Analysis of 4-Stage Passive RC Network

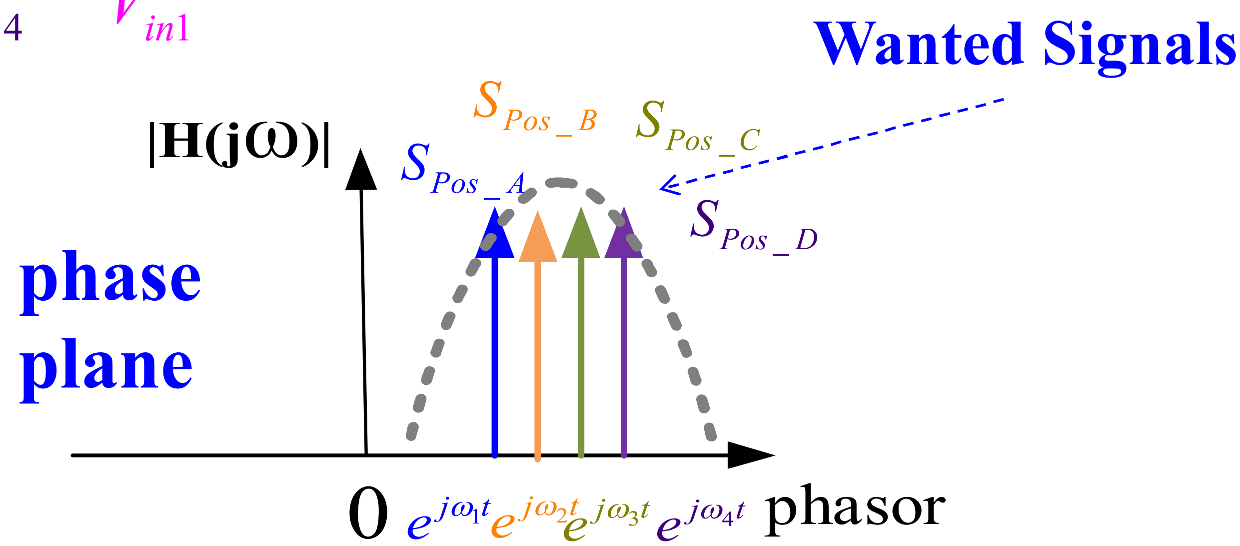
Model of Input Wanted Signals

$$S_{Pos\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\}$$

$$=\{1;j;-1;-j\}V_{pos}(t)=\left\{1;e^{j\frac{\pi}{2}};e^{j\pi};e^{j\frac{3\pi}{2}}\right\}\sum_{k=1}^{n}A_k\cos(\omega_k t+\theta_k)$$

Positive poly-phase signals

$$\frac{V_{in1}}{V_{in2}}=\frac{V_{in2}}{V_{in3}}=\frac{V_{in3}}{V_{in4}}=\frac{V_{in4}}{V_{in1}}=-j$$



2. Analysis of 4-Stage Passive RC Network

Derivation of Transfer Function (Wanted Signal)

Transfer Function

$$H = \frac{V_{out1}}{V_{in1}} = \frac{V_{out2}}{V_{in2}} = \frac{V_{out3}}{V_{in3}} = \frac{V_{out4}}{V_{in4}}$$

$$= \frac{\left(Z_{C4} - jR_4\right)\left(Z_{C3} - jR_3\right)\left(Z_{C2} - jR_2\right)\left(Z_{C1} - jR_1\right)}{\left\{\begin{array}{l}\left[2R_1Z_{C1} + \left(Z_{C2} + R_2\right)\left(Z_{C1} + R_1\right)\right]\left[2R_3Z_{C3} + \left(Z_{C3} + R_3\right)\left(Z_{C4} + R_4\right)\right] \\ +2\left(Z_{C3} + R_3 + Z_{C4} + R_4\right)\left[\left(Z_{C1} + R_1\right)R_2Z_{C2} + \left(Z_{C2} + R_2\right)R_1Z_{C1}\right]\end{array}\right\}}$$

$$Z_{C1} = \frac{1}{j2\pi fC_1}; Z_{C2} = \frac{1}{j2\pi fC_2}$$

$$Z_{C3} = \frac{1}{j2\pi fC_3}$$

$$Z_{C4} = \frac{1}{j2\pi fC_4}$$

$$f_1 = \frac{1}{2\pi R_1C_1}; f_2 = \frac{1}{2\pi R_2C_2}; f_{21} = \frac{1}{2\pi R_2C_1};$$

$$f_3 = \frac{1}{2\pi R_3C_3}; f_{31} = \frac{1}{2\pi R_3C_1}; f_{32} = \frac{1}{2\pi R_3C_2};$$

$$f_4 = \frac{1}{2\pi R_4C_4}; f_{41} = \frac{1}{2\pi R_4C_1}; f_{42} = \frac{1}{2\pi R_4C_2}; f_{43} = \frac{1}{2\pi R_4C_3};$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Transfer Function (Wanted Signal)

Transfer Function

$$H = \frac{\left\{\begin{aligned} & s^4 + js^3\left(f_4+f_3+f_2+f_1\right) - s^2\left[f_4\left(f_3+f_2+f_1\right)+f_3\left(f_2+f_1\right)+f_2f_1\right] \\ & -js\left\{f_4\left[f_3\left(f_2+f_1\right)+f_2f_1\right]+f_3f_2f_1\right\}+f_4f_3f_2f_1 \end{aligned}\right\}}{\left\{\begin{aligned} & s^4 + s^3\left(f_4+f_3+f_2+f_1+2\left[\left(f_{43}+f_{42}+f_{41}\right)+\left(f_{32}+f_{31}\right)+f_{21}\right]\right) \\ & +s^2\left[\begin{aligned} & f_4\left(f_3+f_2+f_1\right)+f_3\left(f_2+f_1\right)+f_2f_1 \\ & +4f_{43}f_{21}+2\left(\begin{aligned} & f_4\left(\left(f_{32}+f_{31}\right)+f_{21}\right)+f_3\left(\left(f_{42}+f_{41}\right)+f_{21}\right) \\ & +f_2\left(\left(f_{43}+f_{41}\right)+f_{31}\right)+f_1\left(\left(f_{43}+f_{42}\right)+f_{32}\right) \end{aligned}\right) \end{aligned}\right] \\ & +s\left\{f_4\left[f_3\left(f_2+f_1\right)+f_2f_1\right]+f_3f_2f_1+2\left(\begin{aligned} & f_4\left(f_3f_{21}+f_2f_{31}+f_1f_{32}\right) \\ & +f_3\left(f_2f_{41}+f_1f_{42}\right)+f_2f_1f_{43} \end{aligned}\right)\right\} \\ & +f_4f_3f_2f_1 \end{aligned}\right\}}$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Transfer Function (Wanted Signal)

$$a_1 = f_4 + f_3 + f_2 + f_1$$

$$a_2 = f_4\left(f_3 + f_2 + f_1\right) + f_3\left(f_2 + f_1\right) + f_2 f_1$$

$$a_3 = f_4\left[f_3\left(f_2 + f_1\right) + f_2 f_1\right] + f_3 f_2 f_1$$

$$a_4 = f_4 f_3 f_2 f_1$$

$$a_5 = a_1 + 2\left[\left(f_{43} + f_{42} + f_{41}\right) + \left(f_{32} + f_{31}\right) + f_{21}\right]$$

$$a_6 = a_2 + 4 f_{43} f_{21} + 2\begin{pmatrix} f_4\left(\left(f_{32} + f_{31}\right) + f_{21}\right) + f_3\left(\left(f_{42} + f_{41}\right) + f_{21}\right) \\ + f_2\left(\left(f_{43} + f_{41}\right) + f_{31}\right) + f_1\left(\left(f_{43} + f_{42}\right) + f_{32}\right) \end{pmatrix}$$

$$a_7 = a_3 + 2\left(f_4\left(f_3 f_{21} + f_2 f_{31} + f_1 f_{32}\right) + f_3\left(f_2 f_{41} + f_1 f_{42}\right) + f_2 f_1 f_{43}\right)$$

Transfer function

$$H_{Pos}\left(j2\pi f\right) = \frac{s^4 + j a_1 s^3 - a_2 s^2 - j a_3 s + a_4}{s^4 + a_5 s^3 + a_6 s^2 + a_7 s + a_4}$$

2. Analysis of 4-Stage Passive RC Network

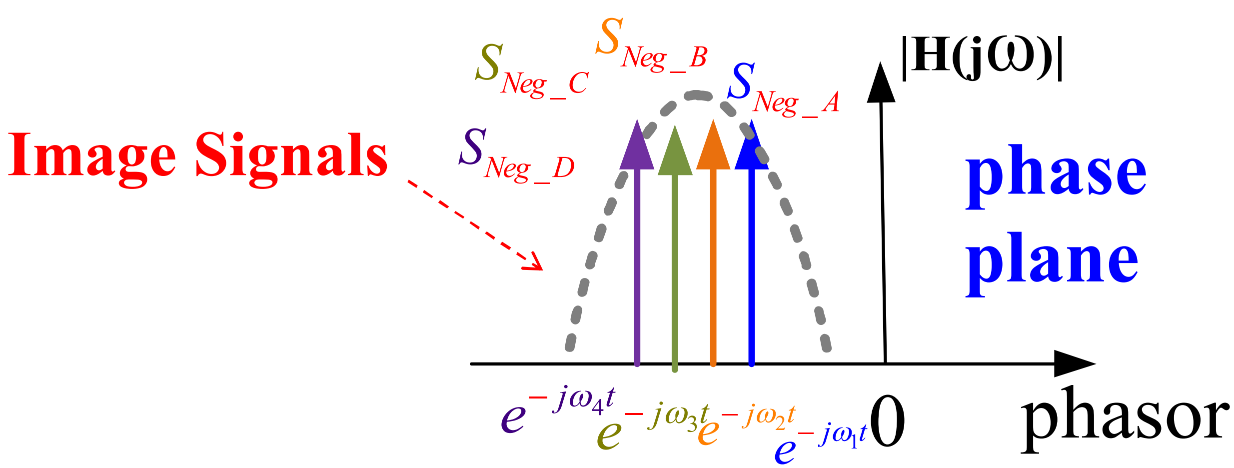
Model of Input Image Signals

$$S_{Neg\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\}$$

$$=\{V(t);-jV(t);-V(t);jV(t)\}=\left\{V(t);e^{-j\frac{\pi}{2}}V(t);e^{-j\pi}V(t);e^{-j\frac{3\pi}{2}}V(t)\right\}$$

Negative poly-phase signals

$$\frac{V_{in1}}{V_{in2}}=\frac{V_{in2}}{V_{in3}}=\frac{V_{in3}}{V_{in4}}=\frac{V_{in4}}{V_{in1}}=+j$$



2. Analysis of 4-Stage Passive RC Network

Derivation of Transfer Function (Image Signal)

Transfer Function

$$H = \frac{\left(Z_{C4} + jR_4\right)\left(Z_{C3} + jR_3\right)\left(Z_{C2} + jR_2\right)\left(Z_{C1} + jR_1\right)}{\left\{\begin{array}{l}\left[2R_1 Z_{C1} + \left(Z_{C2} + R_2\right)\left(Z_{C1} + R_1\right)\right]\left[2R_3 Z_{C3} + \left(Z_{C3} + R_3\right)\left(Z_{C4} + R_4\right)\right] \\ +2\left(Z_{C3} + R_3 + Z_{C4} + R_4\right)\left[\left(Z_{C1} + R_1\right)R_2 Z_{C2} + \left(Z_{C2} + R_2\right)R_1 Z_{C1}\right]\end{array}\right\}}$$

$$H = \frac{\left\{s^4 - js^3\left(f_4 + f_3 + f_2 + f_1\right) - s^2\left[f_4\left(f_3 + f_2 + f_1\right) + f_3\left(f_2 + f_1\right) + f_2 f_1\right] + js\left\{f_4\left[f_3\left(f_2 + f_1\right) + f_2 f_1\right] + f_3 f_2 f_1\right\} + f_4 f_3 f_2 f_1\right\}}{\left\{\begin{array}{l} s^4 + s^3\left(f_4 + f_3 + f_2 + f_1 + 2\left[\left(f_{43} + f_{42} + f_{41}\right) + \left(f_{32} + f_{31}\right) + f_{21}\right]\right) + s^2\left[\begin{array}{l} f_4\left(f_3 + f_2 + f_1\right) + f_3\left(f_2 + f_1\right) + f_2 f_1 \\ +4 f_{43} f_{21} + 2\left(\begin{array}{l} f_4\left(\left(f_{32} + f_{31}\right) + f_{21}\right) + f_3\left(\left(f_{42} + f_{41}\right) + f_{21}\right) \\ +f_2\left(\left(f_{43} + f_{41}\right) + f_{31}\right) + f_1\left(\left(f_{43} + f_{42}\right) + f_{32}\right)\end{array}\right)\end{array}\right] \\ +s\left\{f_4\left[f_3\left(f_2 + f_1\right) + f_2 f_1\right] + f_3 f_2 f_1 + 2\left(\begin{array}{l} f_4\left(f_3 f_{21} + f_2 f_{31} + f_1 f_{32}\right) \\ +f_3\left(f_2 f_{41} + f_1 f_{42}\right) + f_2 f_1 f_{43}\end{array}\right)\right\} + f_4 f_3 f_2 f_1\end{array}\right\}}$$

$$H_{Neg}\left(j2\pi f\right) = \frac{s^4 - ja_1 s^3 - a_2 s^2 + ja_3 s + a_4}{s^4 + a_5 s^3 + a_6 s^2 + a_7 s + a_4}$$

2. Analysis of 4-Stage Passive RC Network

Composed Transfer Function of 4-Stage Passive RC Network

Wanted Signals

$$H_{Pos}\left(j2\pi f\right)=\frac{s^4+ja_1s^3-a_2s^2-ja_3s+a_4}{s^4+a_5s^3+a_6s^2+a_7s+a_4};\forall f>0$$

Image Signals

$$H_{Neg}\left(j2\pi f\right)=\frac{s^4-ja_1s^3-a_2s^2+ja_3s+a_4}{s^4+a_5s^3+a_6s^2+a_7s+a_4};\forall f<0$$

Composed transfer function

$$\left|H\left(f\right)\right|=\frac{\left|f^4+a_1f^3+a_2f^2+a_3f+a_4\right|}{\sqrt{\left(f^4+a_4-a_6f^2\right)^2+\left(a_7f-a_5f^3\right)^2}};\forall f\in R$$

Image Rejection Ratio

$$IRR=\frac{H_{Pos}\left(j2\pi f\right)}{H_{Neg}\left(j2\pi f\right)}=\frac{\left(f_1+f\right)\left(f_2+f\right)\left(f_3+f\right)\left(f_4+f\right)}{\left(f_1-f\right)\left(f_2-f\right)\left(f_3-f\right)\left(f_4-f\right)}$$

2. Analysis of 4-Stage Passive RC Network

Analysis of Transfer Function (Positive Frequency)

$$\left|H(f)\right| = \frac{f^4 + a_1 f^3 + a_2 f^2 + a_3 f + a_4}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}}; \forall f > 0$$

$$\lim_{f\to 0}\left|H(f)\right| = \lim_{f\to 0}\frac{f^4 + a_1 f^3 + a_2 f^2 + a_3 f + a_4}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}} = 1$$

$$\lim_{f\to \infty}\left|H(f)\right| = \lim_{f\to \infty}\frac{f^4 + a_1 f^3 + a_2 f^2 + a_3 f + a_4}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}} = 1$$

$$\min\left(\left|H(f)\right|\right) as\ f = \sqrt[4]{f_4 f_3 f_2 f_1}$$

Applying Cauchy-Schwarz inequality theorem:

$$a, b > 0, a^2 + b^2 \geq 2ab;\ \min\left(a^2 + b^2\right) = 2ab\ as\ "a = b"$$

$$\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2 \geq 2\left|\left(f^4 + a_4 - a_6 f^2\right)\right|\left|\left(a_7 f - a_5 f^3\right)\right|$$

$$\left|\left(f^4 + a_4 - a_6 f^2\right)\right| = \left|\left(a_7 f - a_5 f^3\right)\right|; \Rightarrow \left[\begin{array}{l} f_{\min 1} = \ldots \\ f_{\min 2} = \ldots \end{array}\right. \Rightarrow \left\{\begin{array}{l} \max 1\left(\left|H(f)\right|\right) \\ \max 2\left(\left|H(f)\right|\right) \end{array}\right.$$

2. Analysis of 4-Stage Passive RC Network

Analysis of Transfer Function (Negative Frequency)

$$\left|H(f)\right| = \frac{f^4 - a_1 f^3 + a_2 f^2 - a_3 f + a_4}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}}; \forall f < 0$$

$$\lim_{f \to 0} \left|H(f)\right| = \lim_{f \to 0} \frac{f^4 - a_1 f^3 + a_2 f^2 - a_3 f + a_4}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}} = 1$$

$$\lim_{f \to -\infty} \left|H(f)\right| = \lim_{f \to -\infty} \frac{f^4 - a_1 f^3 + a_2 f^2 - a_3 f + a_4}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}} = 1$$

$$\min\left(\left|H(f)\right|_{dB}\right) = -\infty; as \; f = -f_1 \vee f = -f_2 \vee f = -f_3 \vee f = -f_4$$

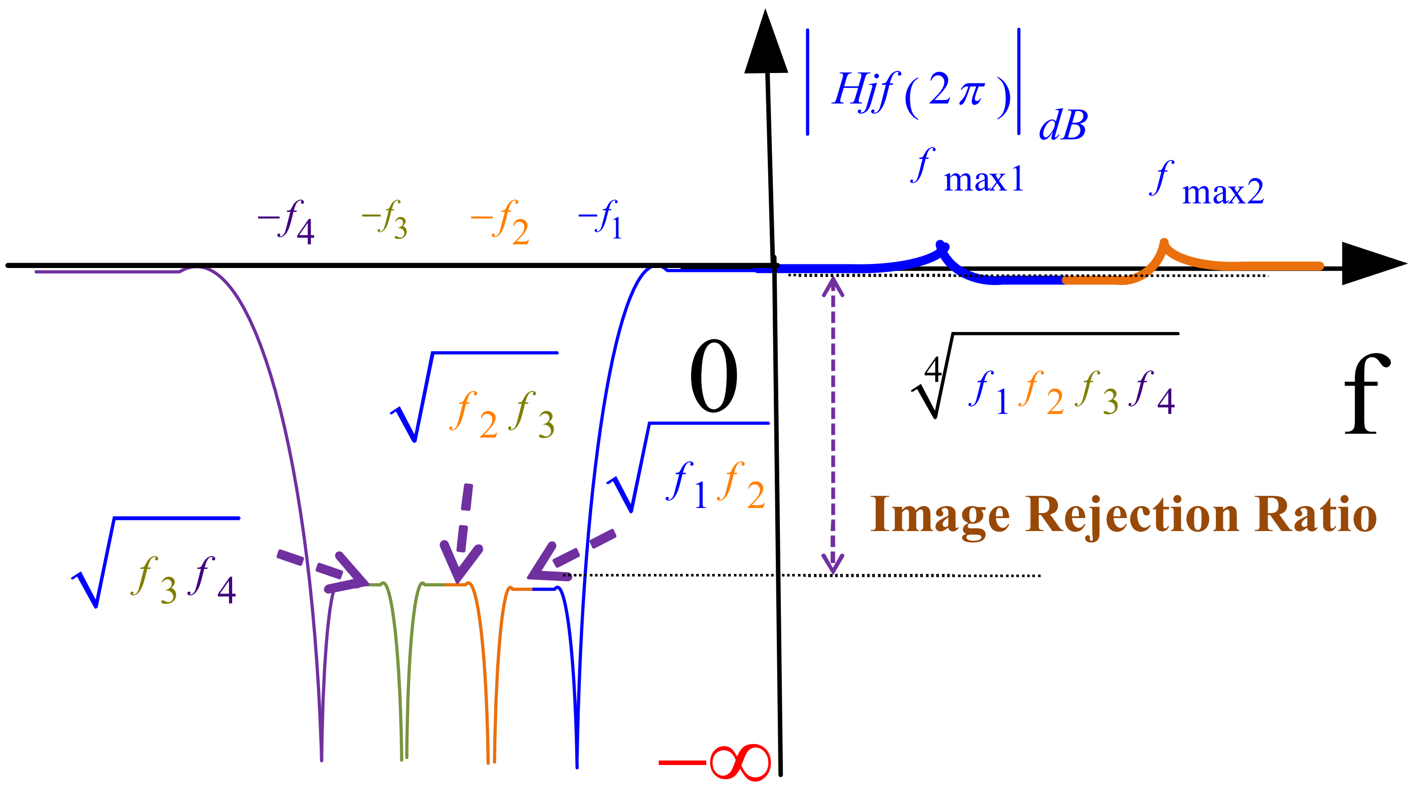
$$\max 1\left(\left|H(f)\right|\right) as \; f = \sqrt{f_1 f_2}; \max 2\left(\left|H(f)\right|\right) as \; f = \sqrt{f_2 f_3}; \max 3\left(\left|H(f)\right|\right) as \; f = \sqrt{f_3 f_4}$$

2. Analysis of 4-Stage Passive RC Network

Frequency Responses of 4-Stage Passive RC Network

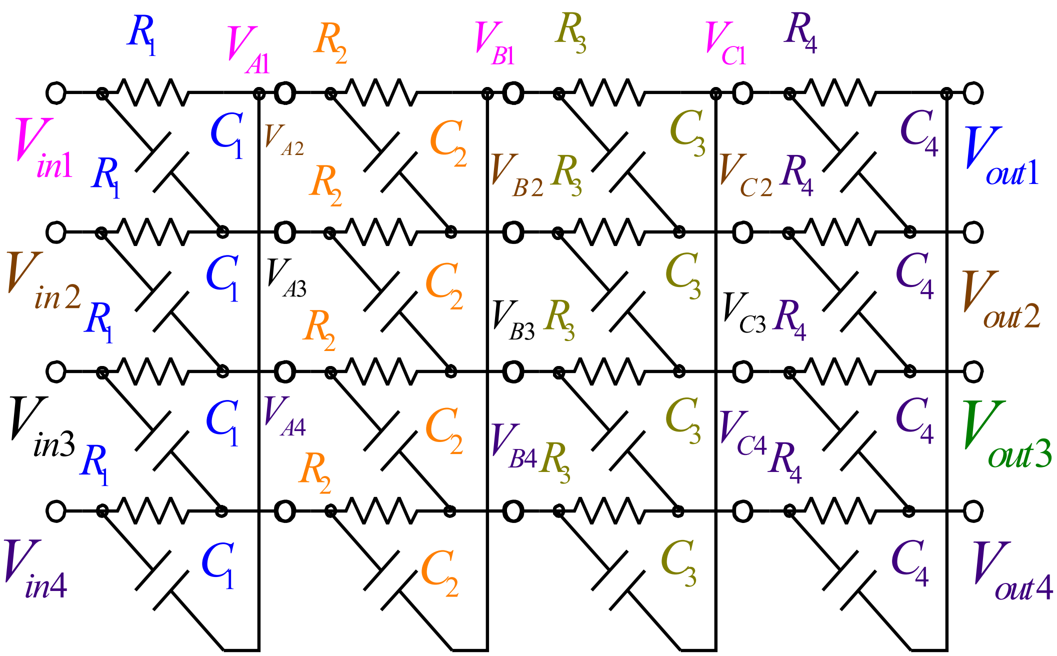
Transfer function $$|H(f)| = \frac{\left|f^4 + a_1 f^3 + a_2 f^2 + a_3 f + a_4\right|}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}}; \forall f \in R$$

Frequency Responses



3. Proposed Model of 4-Stage Passive RC Network

Model of 4-Stage Passive RC Network



$$R_1 = 1k\Omega; C_1 = 227pF;$$
$$R_2 = 1k\Omega; C_2 = 106pF;$$
$$R_3 = 1k\Omega; C_3 = 39.8pF;$$
$$R_4 = 1k\Omega; C_4 = 19.9pF;$$

$$f_1 = 0.7MHz;$$
$$f_2 = 1.5MHz; f_{21} = 0.7MHz;$$
$$f_3 = 4MHz; f_{31} = 0.7MHz; f_{32} = 1.5MHz;$$
$$f_4 = 8MHz; f_{41} = 0.7MHz; f_{42} = 1.5MHz; f_{43} = 4MHz;$$

3. Proposed Model of 4-Stage Passive RC Network

Simplified Model of 4-Stage Passive RC Network

$$a_1 = f_4 + f_3 + f_2 + f_1 = 1.42 * 10^7$$

$$a_2 = f_4\left(f_3 + f_2 + f_1\right) + f_3\left(f_2 + f_1\right) + f_2 f_1 = 5.95 * 10^{13}$$

$$a_3 = f_4\left[f_3\left(f_2 + f_1\right) + f_2 f_1\right] + f_3 f_2 f_1 = 8.3 * 10^{19}$$

$$a_4 = f_4 f_3 f_2 f_1 = 3.36 * 10^{25}$$

$$a_5 = a_1 + 2\left[\left(f_{43} + f_{42} + f_{41}\right) + \left(f_{32} + f_{31}\right) + f_{21}\right] = 3.24 * 10^7$$

$$a_6 = a_2 + 4 f_{43} f_{21} + 2\begin{pmatrix} f_4\left(\left(f_{32} + f_{31}\right) + f_{21}\right) + f_3\left(\left(f_{42} + f_{41}\right) + f_{21}\right) \\ + f_2\left(\left(f_{43} + f_{41}\right) + f_{31}\right) + f_1\left(\left(f_{43} + f_{42}\right) + f_{32}\right) \end{pmatrix} = 1.66 * 10^{14}$$

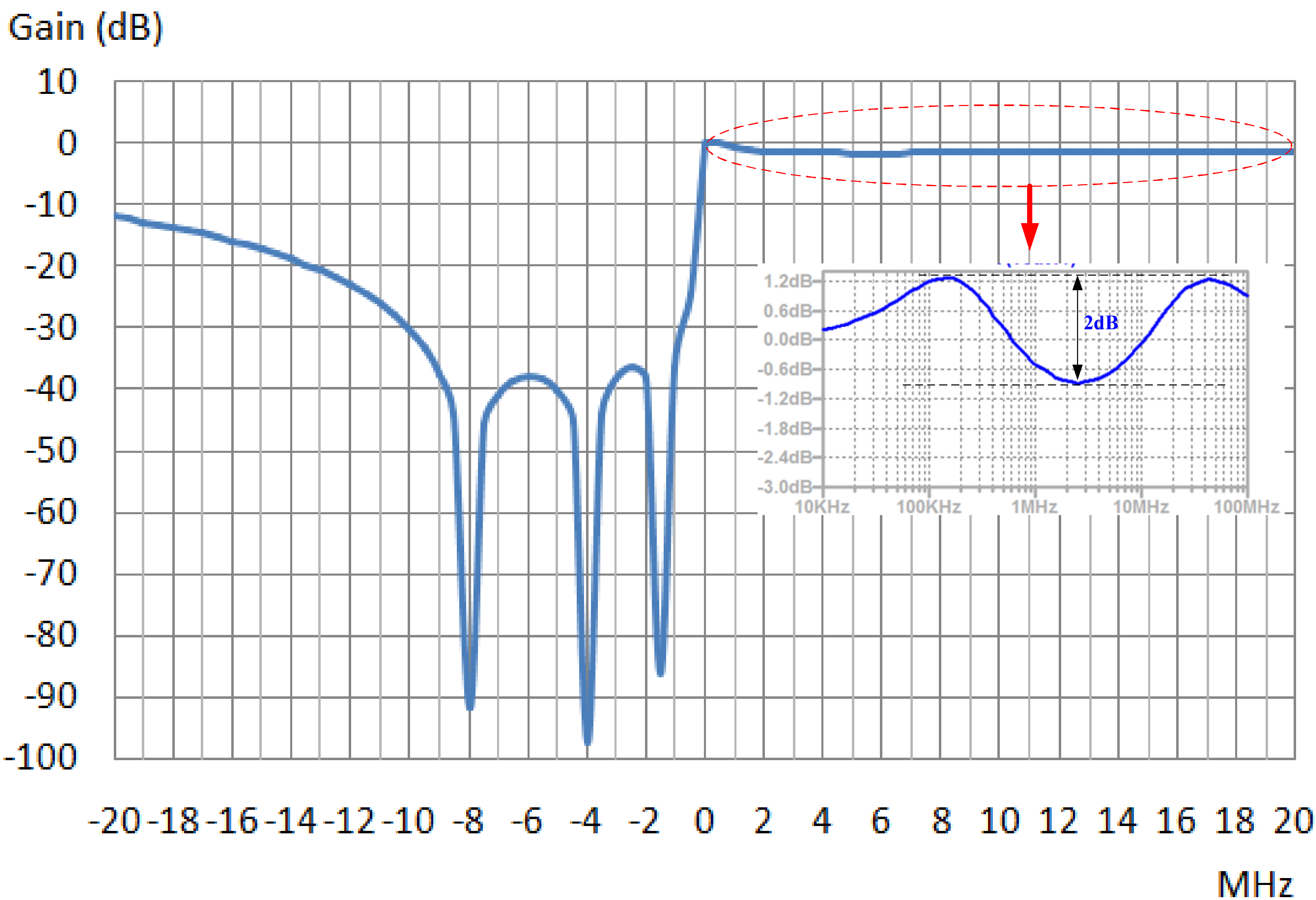
$$a_7 = a_3 + 2\left(f_4\left(f_3 f_{21} + f_2 f_{31} + f_1 f_{32}\right) + f_3\left(f_2 f_{41} + f_1 f_{42}\right) + f_2 f_1 f_{43}\right) = 1.87 * 10^{20}$$

Transfer function

$$\left|H(f)\right|_{dB} = \frac{\left|f^4 + 1.42 * 10^7 f^3 + 5.95 * 10^{13} f^2 + 8.3 * 10^{19} f + 3.36 * 10^{25}\right|}{\sqrt{\left(f^4 + 3.36 * 10^{25} - 1.66 * 10^{14} f^2\right)^2 + \left(1.87 * 10^{20} f - 3.24 * 10^7 f^3\right)^2}}$$

3. Proposed Model of 4-Stage Passive RC Network

Experimental Results of 4-Stage Passive RC Network



3. Proposed Model of 4-Stage Passive RC Network

Algorithm of Flat Pass-band Gain

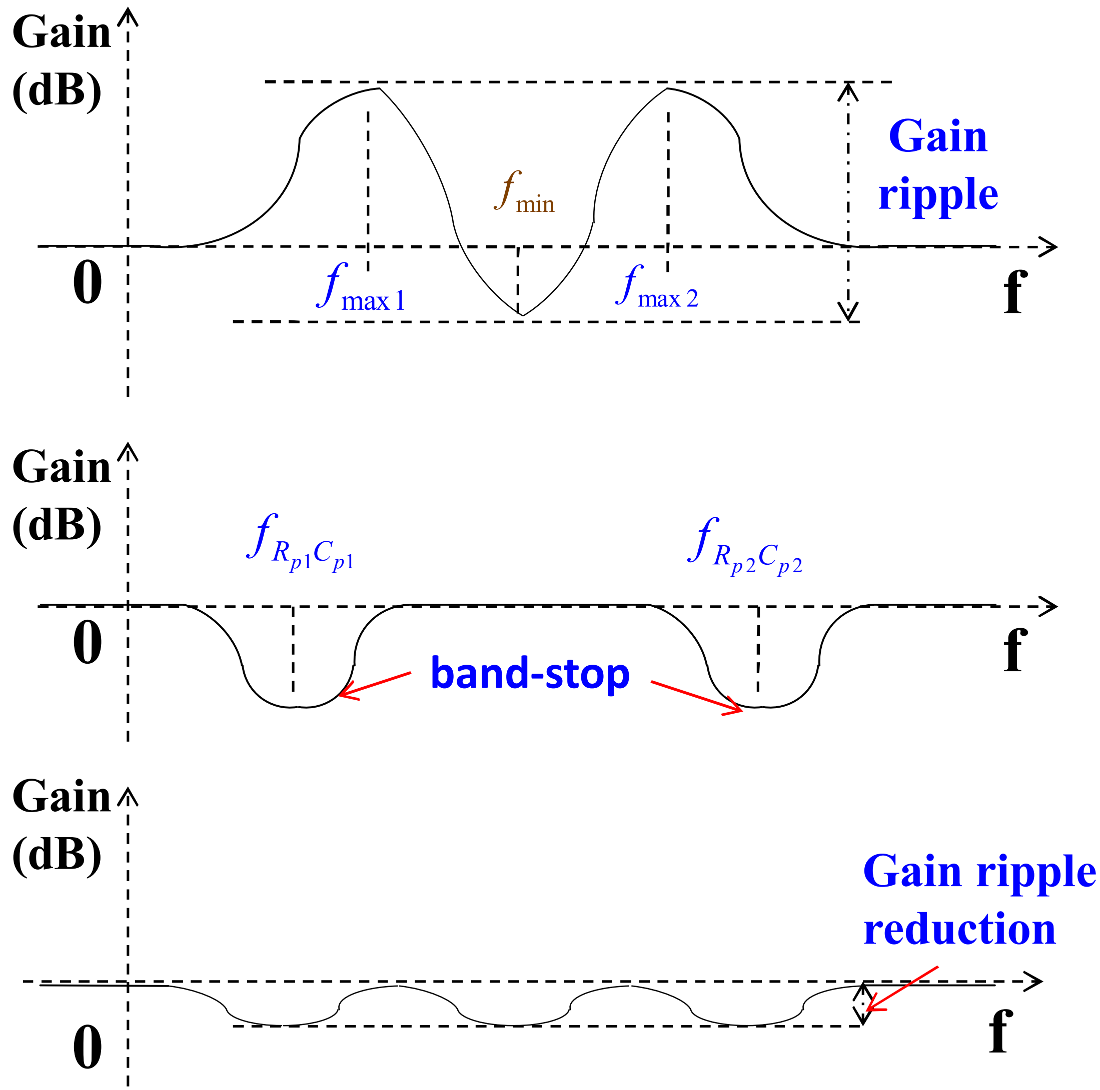
Pass-band Gain

+

Two RC band-stop filter

↓

Improvement of Pass-band Gain

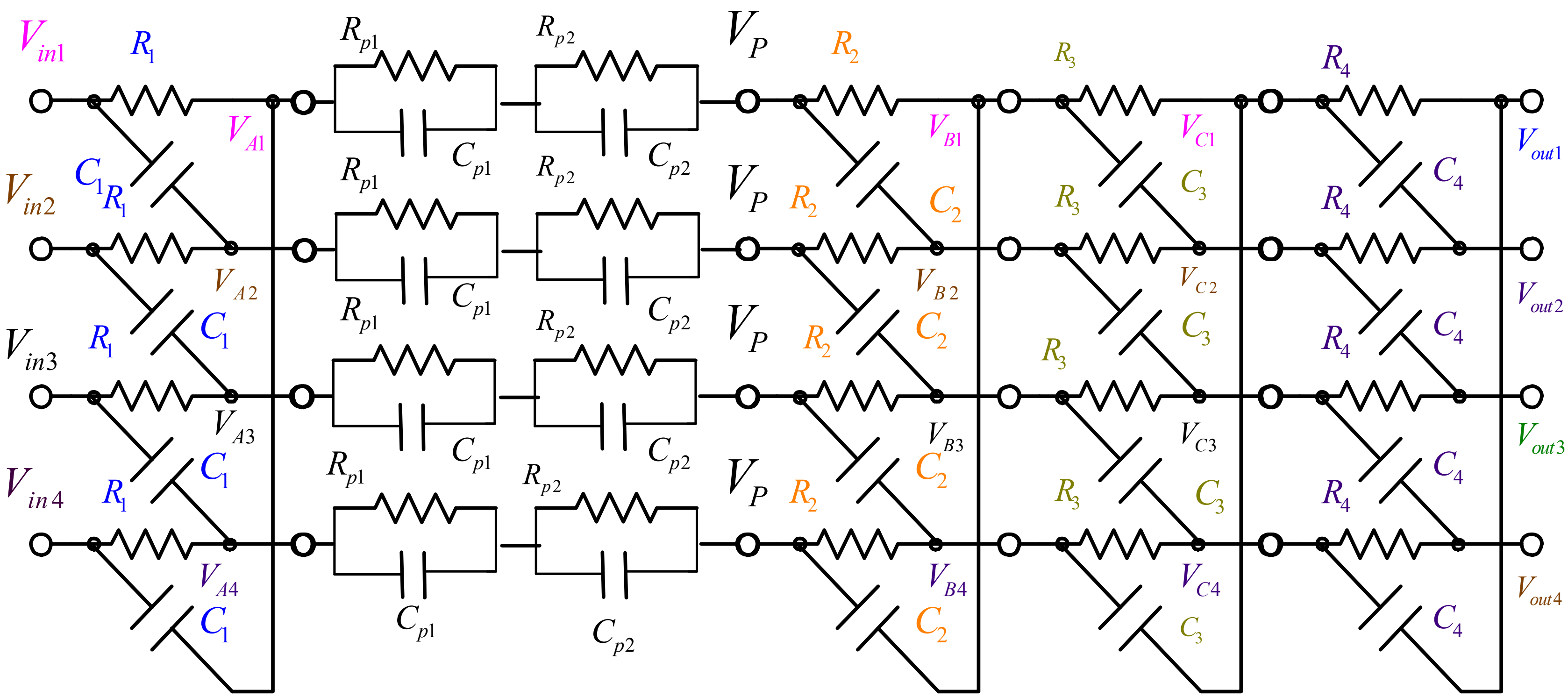


3. Proposed Model of 4-Stage Passive RC Network

Proposed Design of Two RC Band-stop Filters

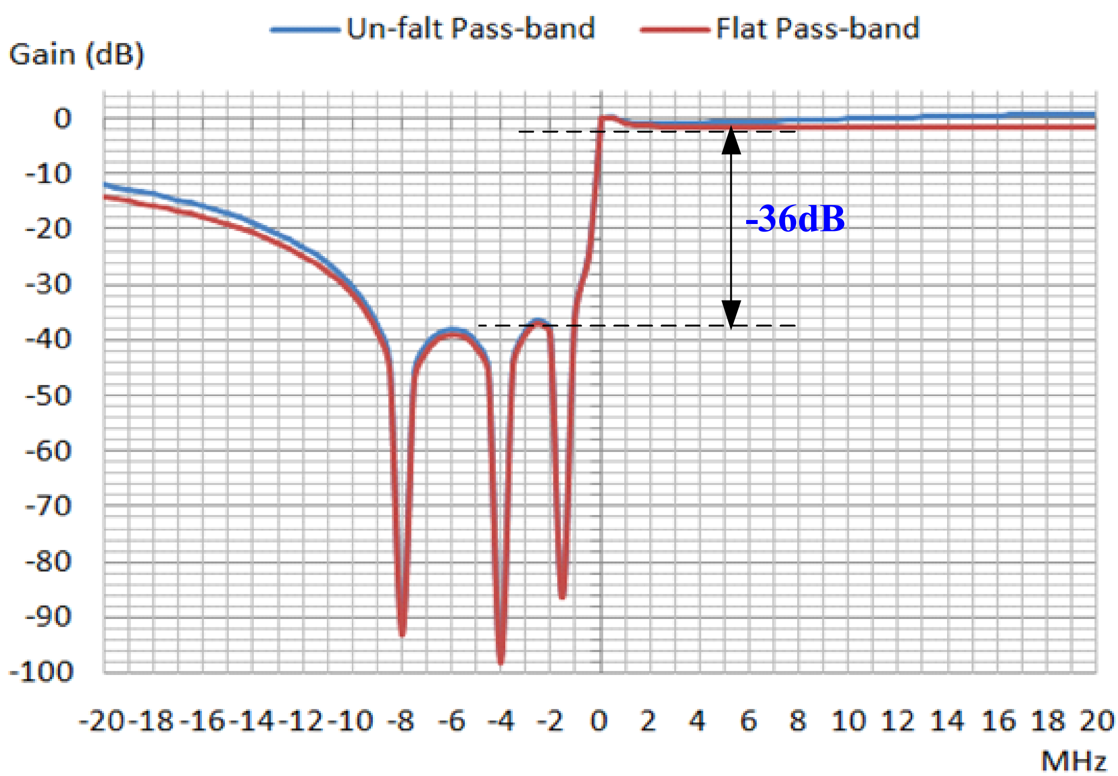
$$R_1 = 1k\Omega; C_1 = 227pF; R_{p1} = 6k\Omega; C_{p1} = 5nF; R_{p2} = 100\Omega; C_{p2} = 12pF;$$

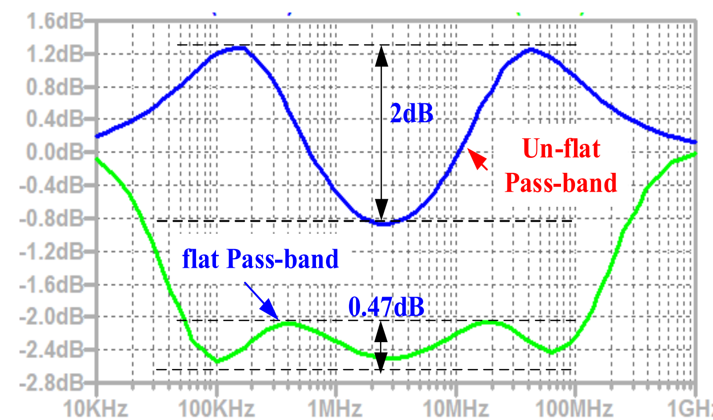
$$R_2 = 1k\Omega; C_2 = 106pF; R_3 = 1k\Omega; C_3 = 39.8pF; R_4 = 1k\Omega; C_4 = 19.9pF;$$



3. Proposed Model of 4-Stage Passive RC Network

Experimental Results of Proposed Polyphase Filter





Flat Pass-band Gain

Improvement of pass-band gain ripple

→from 2dB into 0.47dB

4. Conclusions

This Work:

- **Derivation of transfer function of poly-phase network based on superposition principle**
- **Mathematical analysis and model of 4-stage passive RC poly-phase filter**
- **Improvement of pass-band gain ripple from 2dB into 0.47dB with two RC band-stop filters**
- **Image rejection ratio: -36dB**

Future of Work

- **Analysis of IQ mismatches of poly-phase signals**
- **Analysis of Parasitic of RC components**

Thanks for your kind attention!

谢 谢