

Frequency Estimation Sampling Circuit Using Analog Hilbert Filter and Residue Number System

Yudai Abe, Shogo Katayama, Congbing Li,
Anna Kuwana, Haruo Kobayashi

Division of Electronics and Informatics
Gunma University



OUTLINE

1. Research Background and Goal
2. Chinese Remainder Theorem
3. Proposed Waveform Sampling Circuit
4. Simulation Verification
5. Summary and Challenge

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Research Background

Next Generation Communication System "5G"



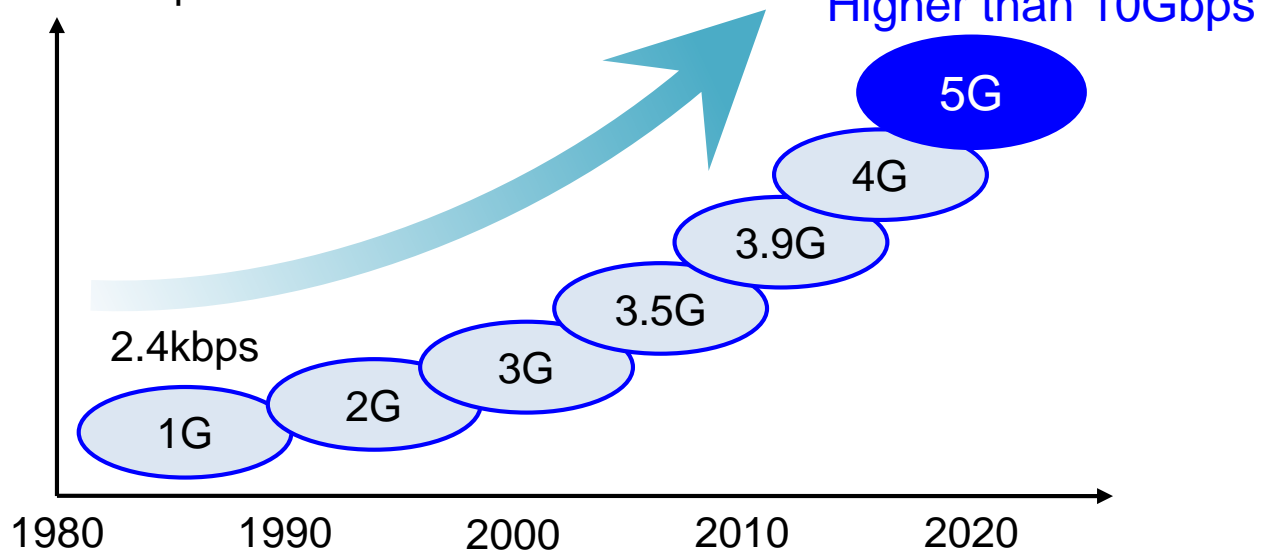
High frequencies
in communication systems



Electronic components
for high frequency bands



Communication speed



Our Research Goal

Estimate **high-frequency input signal**
with **multiple low-frequency** clock sampling circuits

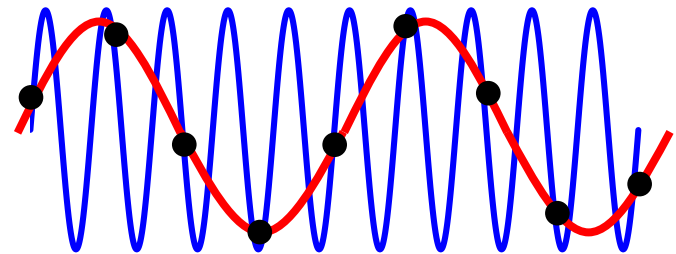
High-frequency sampling circuit is difficult to realize

Our Approach :

Sampling high frequency signal with multiple low frequency clocks



Use **Aliasing** proactively



Analog Hilbert filter and **residue number system**

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Chinese Remainder Theorem



Sun Tzu

Chinese arithmetic book 'Sun Tzu calculation'

孫子算經

“When dividing by 3, its residue is 2,
dividing by 5, its residue is 3,
dividing by 7, its residue is 2.
What is the original number ?”

Answer 23

Generalization



Chinese Remainder Theorem



Sun Tzu calculation

How to use the Chinese remainder theorem

He used to quickly find out how many soldiers there are.



Sun Tzu

“Divide into 3 people are there?”



How to use the Chinese remainder theorem

He used to quickly find out how many soldiers there are.



Sun Tzu

“Divide into 5 people.”

Remainder : 2



...



How to use the Chinese remainder theorem

He used to quickly find out how many soldiers there are.



Sun Tzu

“Divide into 7 people.”

Remainder : 3



How to use the Chinese remainder theorem

He used to quickly find out how many soldiers there are.



Sun Tzu

“Divide into 27 people.” in all.”



Remainder : 2

Example of Residue Number System

$$23 \% 3 = 2, \quad 23 \% 5 = 3, \quad 23 \% 7 = 2$$

- Natural numbers
3, 5, 7 (*relatively prime*)
 $N = 3 \times 5 \times 7 = 105$
- k ($0 \leq k \leq N-1 (=104)$)

a : Remainder of k dividing by 3 $a = \text{mod}3(k)$
 b : Remainder of k dividing by 5 $b = \text{mod}5(k)$
 c : Remainder of k dividing by 7 $c = \text{mod}7(k)$

$k \longleftrightarrow (a, b, c)$

one to one

Chinese remainder theorem

a	b	c	k
0	0	1	15
1	1	2	16
2	2	3	17
0	3	4	18
1	4	5	19
2	0	6	20
0	1	0	21
1	2	1	22
2	3	2	23
0	4	3	24
1	0	4	25
2	1	5	26
0	2	6	27
1	3	0	28
2	4	1	29

Residue number system

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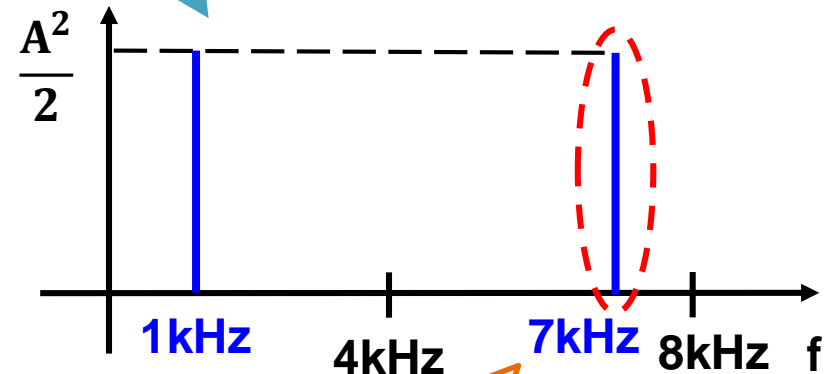
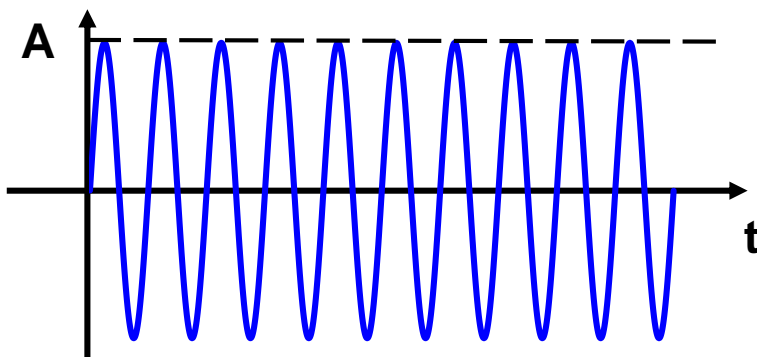
Aliasing Phenomenon

Sampling frequency : 8 kHz

FFT

Spectrums are folded within the sampling frequency band (**sampling theorem**)

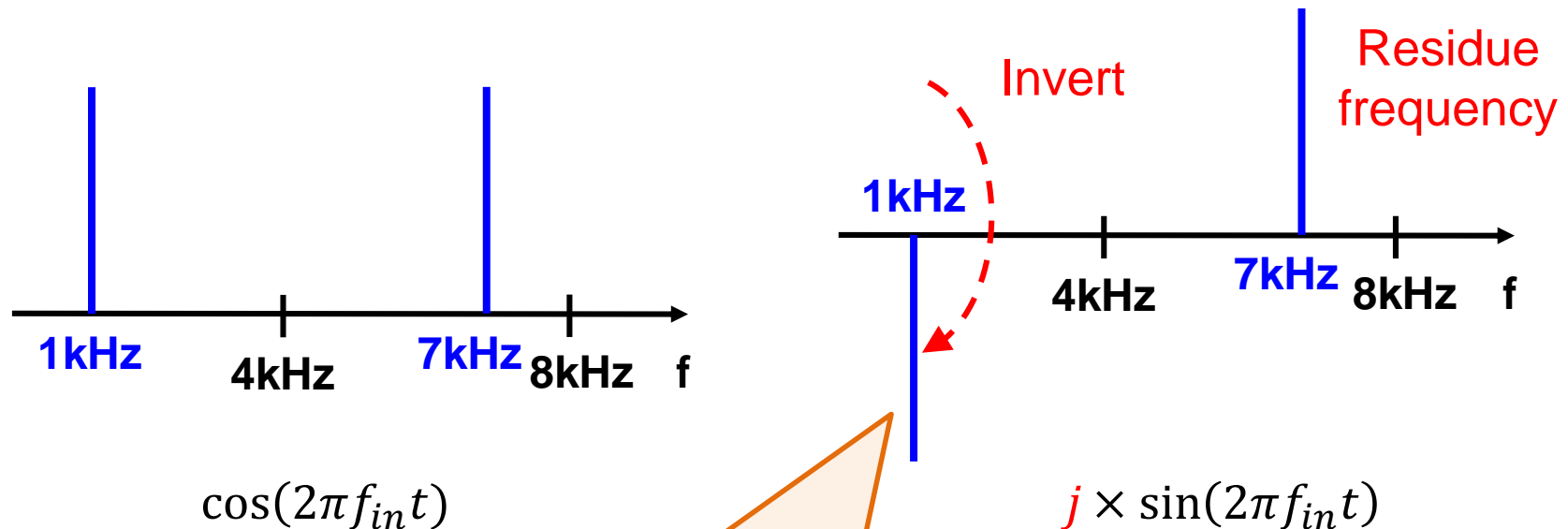
Waveform frequency : 31kHz



Residue frequency
(7 is the remainder of 31 divided by 8)

Complex FFT of $j \times \sin(2\pi f_{in} t)$

Complex FFT
 Input frequency : 31 kHz
 Sampling frequency : 8 kHz



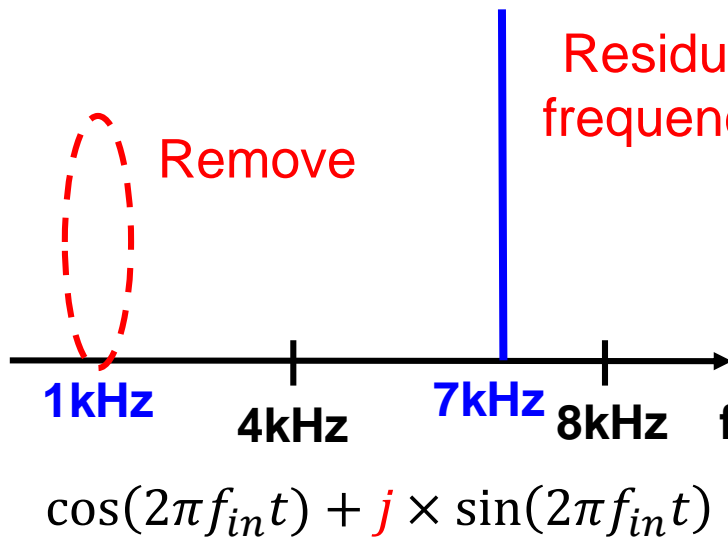
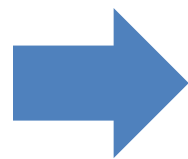
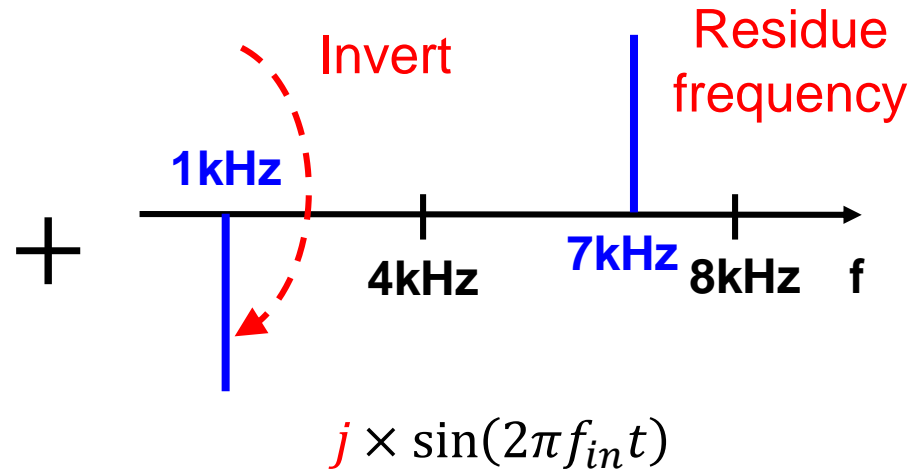
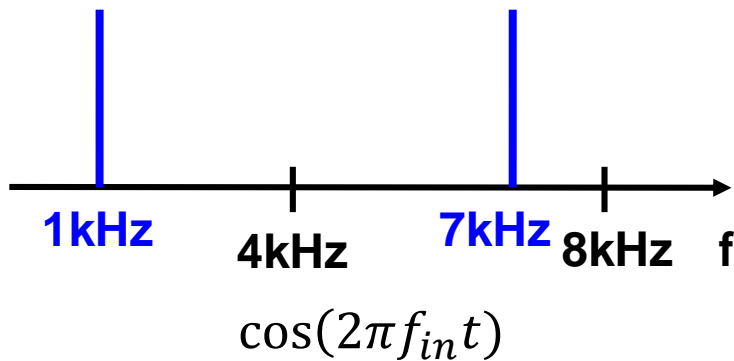
Inverted spectrum
 anti-symmetric at Nyquist frequency

Complex FFT of $\cos(2\pi f_{in}t) + j \times \sin(2\pi f_{in}t)$

Complex FFT

Input frequency : 31 kHz

Sampling frequency : 8 kHz



Extract spectrum
of the residual frequency

How Generate $j \times \sin(2\pi f_{in} t)$

Use Analog Hilbert filter

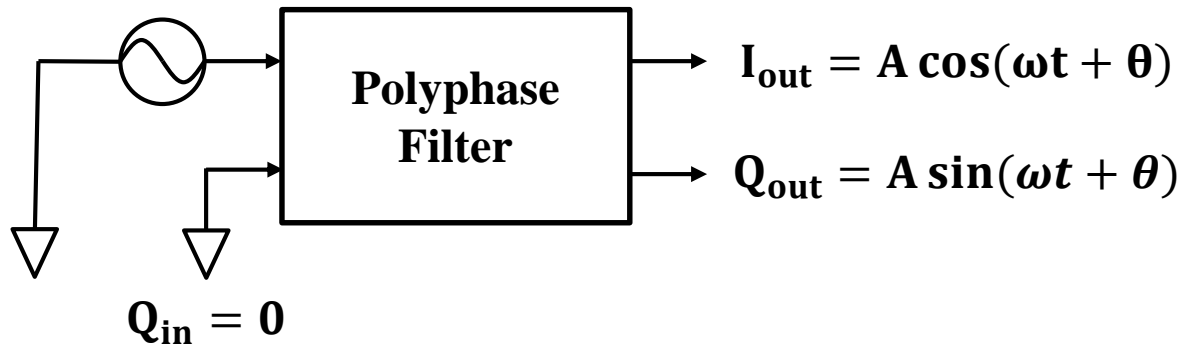


RC polyphase filter

David Hilbert
(German mathematician)
1862-1943

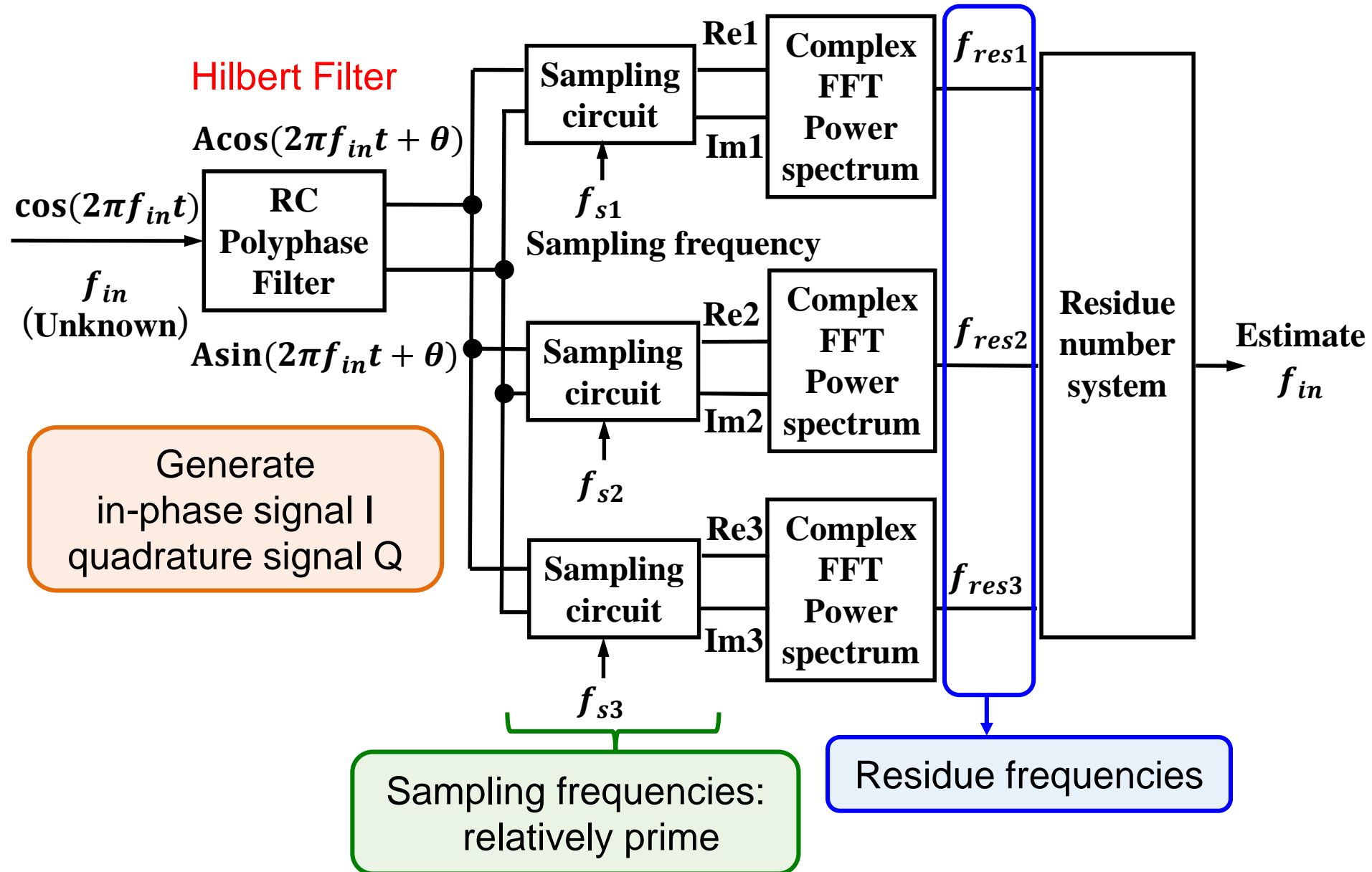


$$I_{in} = \cos(\omega t)$$



Generate **in-phase** and **quadrature** waves
from a single cosine wave

Proposed Sampling Circuit



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Simulation Settings

Complex FFT

- Input frequency : 12 GHz
- Frequency resolution : 1 kHz
- Sampling frequency : 229 kHz, 233 kHz, 239 kHz
(Relatively prime)
- Range of measurement : 0~2080622 kHz
(Note: $229 \times 233 \times 239 = 2080623$)

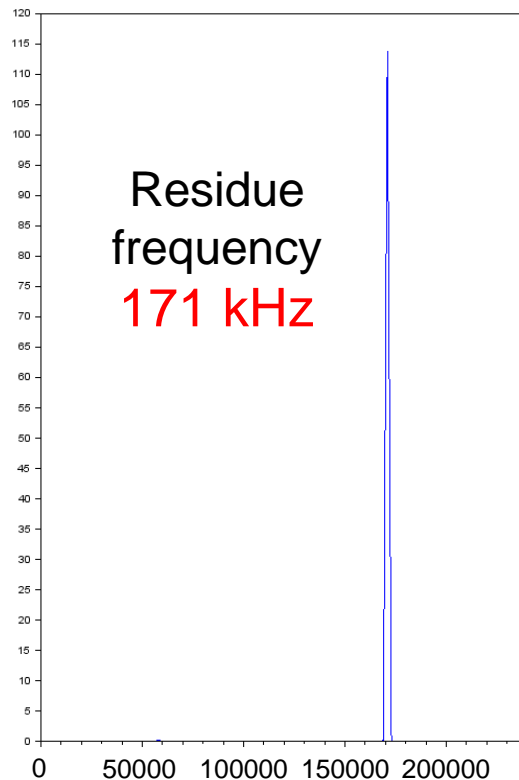
Measurement at 20 GHz
using sampling frequencies of \cong 200 kHz

Simulation Results

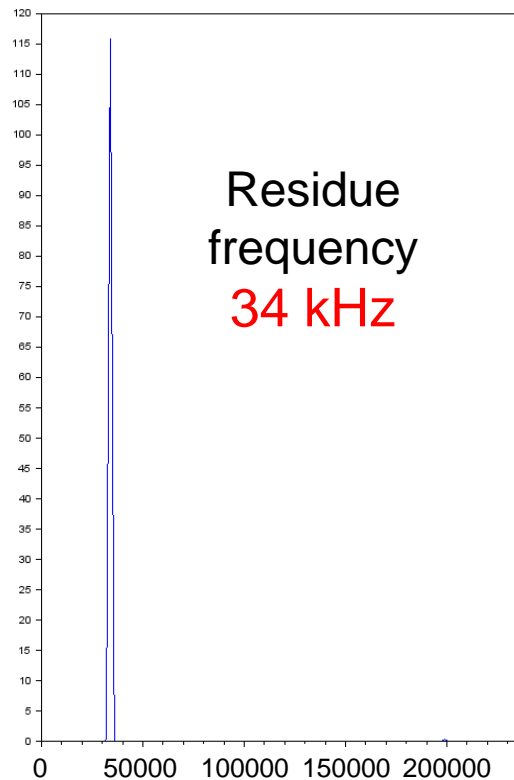
Complex FFT : $\cos(2\pi f_{in}t) + j \times \sin(2\pi f_{in}t)$

- Input frequency : 12 GHz
- Frequency resolution : 1 kHz
- Sampling frequency : 229 kHz 233 kHz 239 kHz

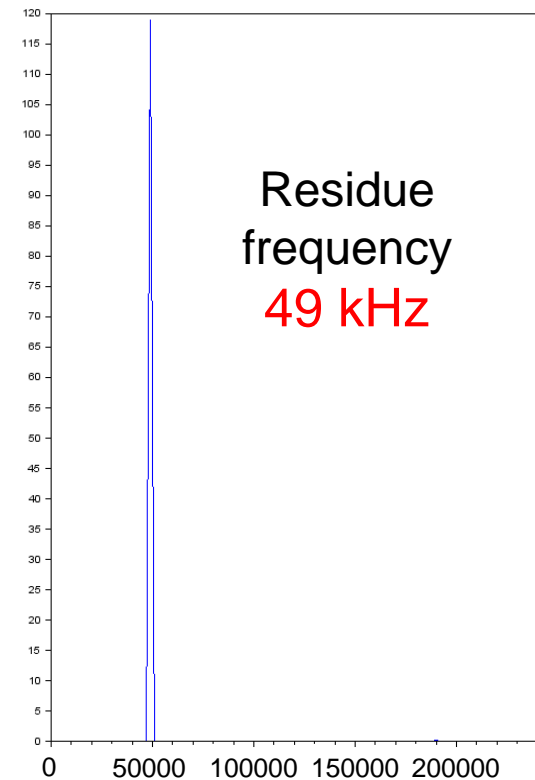
229 kHz Sampling



233 kHz Sampling



239 kHz Sampling



Frequency Estimation by Residue Number System

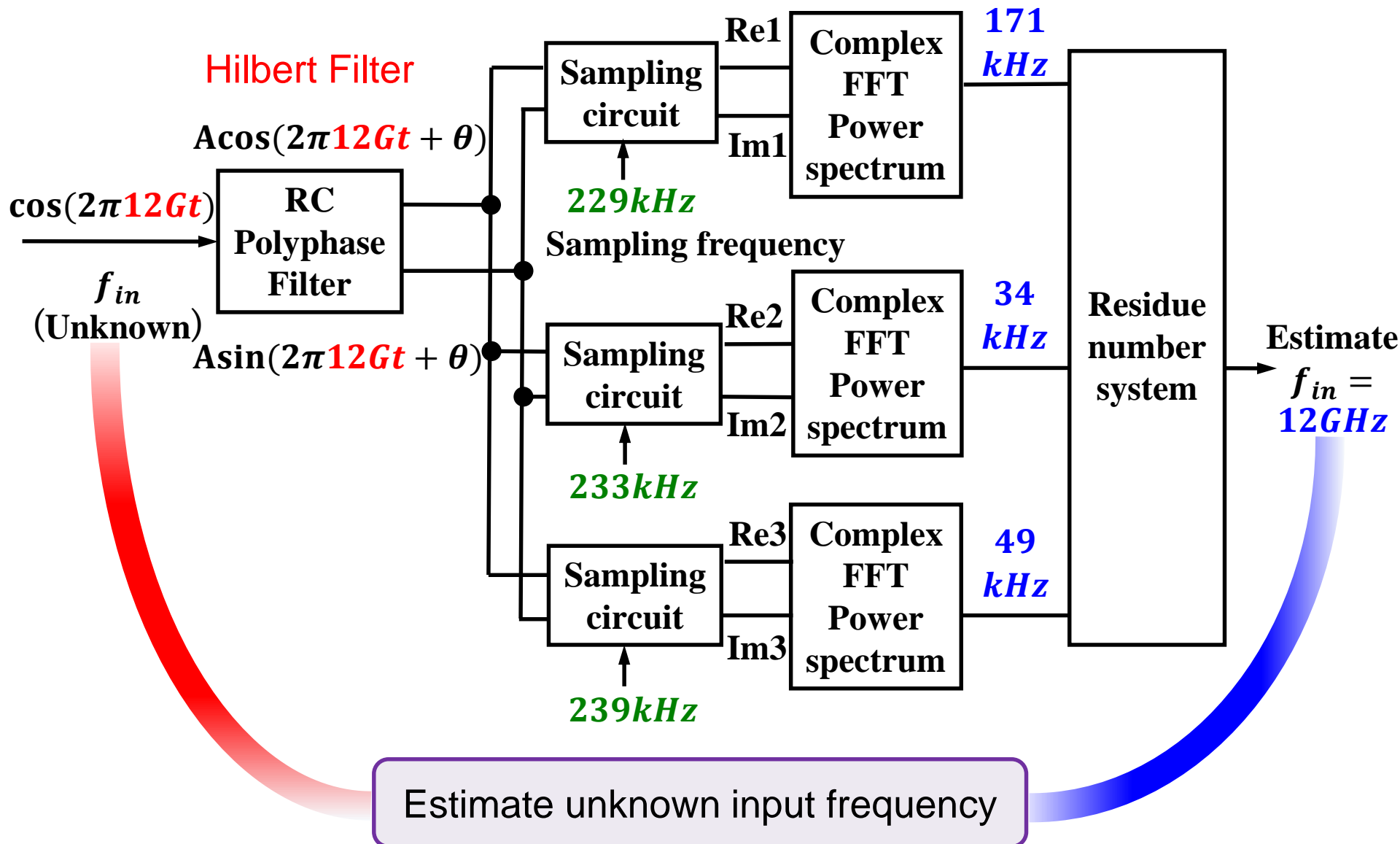
Residue frequencies
171 kHz, 34 kHz, 49 kHz

Input frequency estimation
using residue frequencies
and residue number system

Estimate input frequency **12GHz**

a [kHz]	b [kHz]	c [kHz]	k [kHz]
0	0	0	0
1	1	1	1
2	2	2	2
⋮	⋮	⋮	⋮
169	46	47	11999998
170	47	48	11999999
171	34	49	12000000
172	35	50	12000001
173	36	51	12000002
⋮	⋮	⋮	⋮
226	230	231	12752320
227	231	237	12752321
228	232	238	12752322

Simulation Result Overview



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Summary

- Proposed a method to estimate high-frequency signal using multiple low-frequency sampling circuits.
- Confirmed its operation by theory and simulation.
- Measurable range is wide:
proportional to multiplication of multiple sampling frequencies.

Challenge

- Estimated input frequency is discrete
 - ➡ Consider estimation with fine frequency resolution

Thank you for your attention