Frequency Estimation Sampling Circuit
Using Analog Hilbert Filter
and Residue Number System

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1. Research Background and Goal
2. Chinese Remainder Theorem
3. Proposed Waveform Sampling Circuit
4. Simulation Verification
5. Summary and Challenge
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Next Generation Communication System “5G”

- High frequencies in communication systems
- Electronic components for high frequency bands

Communication speed:
- 1G: 2.4kbps
- 2G
- 3G
- 3.9G
- 3.5G
- 4G
- 5G: Higher than 10Gbps
Our Research Goal

Estimate high-frequency input signal with multiple low-frequency clock sampling circuits

High-frequency sampling circuit is difficult to realize

Our Approach:

Sampling high frequency signal with multiple low frequency clocks

Use **Aliasing** proactively

**Analog Hilbert filter and residue number system**
OUTLINE

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2. **Chinese Remainder Theorem**
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Chinese Remainder Theorem

Chinese arithmetic book ‘Sun Tzu calculation’

孙子算經

“When dividing by 3, its residue is 2, dividing by 5, its residue is 3, dividing by 7, its residue is 2. What is the original number?”

Answer 23

Generalization

Sun Tzu calculation

Chinese Remainder Theorem
How to use the Chinese remainder theorem

He used to quickly find out how many soldiers there are.

"Divide into 3 people," there?

Sun Tzu
How to use the Chinese remainder theorem

He used to quickly find out how many soldiers there are.

Sun Tzu

“Divide into 5 people.”

Remainder : 2
How to use the Chinese remainder theorem

He used to quickly find out how many soldiers there are.

Sun Tzu

“Divide into 7 people.”

Remainder : 3
How to use the Chinese remainder theorem

He used to quickly find out how many soldiers there are.

Sun Tzu

“Divide into 27 people.” In all.

Remainder : 2
Example of Residue Number System

- Natural numbers
  3, 5, 7 (relatively prime)
  N = 3 × 5 × 7 = 105
- k (0 ≤ k ≤ N-1 (=104))

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</table>

23 % 3 = 2, 23 % 5 = 3, 23 % 7 = 2

k ↔ (a, b, c)
one to one

Chinese remainder theorem

Residue number system
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Aliasing Phenomenon

Waveform frequency: 31 kHz

Sampling frequency: 8 kHz

FFT

Spectrums are folded within the sampling frequency band (sampling theorem)

Residue frequency (7 is the remainder of 31 divided by 8)

Waveform frequency: 31 kHz
Complex FFT of $j \times \sin(2\pi f_{in}t)$

Complex FFT
Input frequency : 31 kHz
Sampling frequency : 8 kHz

Invert
Residue frequency

$\cos(2\pi f_{in}t)$

$j \times \sin(2\pi f_{in}t)$

Inverted spectrum
anti-symmetric at Nyquist frequency
Complex FFT of \( \cos(2\pi f_{in}t) + j \times \sin(2\pi f_{in}t) \)

Complex FFT
Input frequency : 31 kHz
Sampling frequency : 8 kHz

Extract spectrum of the residual frequency

\( \cos(2\pi f_{in}t) \)
\( j \times \sin(2\pi f_{in}t) \)
How Generate $j \times \sin(2\pi f_{in} t)$

Use Analog Hilbert filter

RC polyphase filter

David Hilbert
(German mathematician)
1862-1943

$I_{in} = \cos(\omega t)$

$I_{out} = A \cos(\omega t + \theta)$

$Q_{out} = A \sin(\omega t + \theta)$

$Q_{in} = 0$

Generate in-phase and quadrature waves from a single cosine wave
Proposed Sampling Circuit

RC Polyphase Filter

\[ \cos(2\pi f_{in}t) \]

\[ \sin(2\pi f_{in}t) \]

Hilbert Filter

\[ A \cos(2\pi f_{in}t + \theta) \]

\[ A \sin(2\pi f_{in}t + \theta) \]

Generate in-phase signal I and quadrature signal Q

Sampling frequencies: relatively prime

Sampling frequencies:

- \( f_{s1} \)
- \( f_{s2} \)
- \( f_{s3} \)

Residue frequencies:

- \( f_{res1} \)
- \( f_{res2} \)
- \( f_{res3} \)

Estimate \( f_{in} \)
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Simulation Settings

Complex FFT

- Input frequency: 12 GHz
- Frequency resolution: 1 kHz
- Sampling frequency: 229 kHz, 233 kHz, 239 kHz
  (Relatively prime)
- Range of measurement: 0~2080622 kHz
  (Note: $229 \times 233 \times 239 = 2080623$)

Measurement at 20 GHz using sampling frequencies of $\approx 200$ kHz
Simulation Results

Complex FFT: \( \cos(2\pi f_{in}t) + j \times \sin(2\pi f_{in}t) \)

- Input frequency: 12 GHz
- Frequency resolution: 1 kHz
- Sampling frequency: 229 kHz, 233 kHz, 239 kHz

Residue frequency
- 229 kHz Sampling: 171 kHz
- 233 kHz Sampling: 34 kHz
- 239 kHz Sampling: 49 kHz

Input frequency: 12 GHz
Frequency resolution: 1 kHz
Sampling frequency: 229 kHz, 233 kHz, 239 kHz
Frequency Estimation by Residue Number System

Residue frequencies
171 kHz, 34 kHz, 49 kHz

Input frequency estimation using residue frequencies and residue number system

Estimate input frequency 12 GHz

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<th>c [kHz]</th>
<th>k [kHz]</th>
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Simulation Result Overview

Simulation of a Polyphase Filter

- **Input Signal**: \( f_{in} \) (Unknown)
- **Filter**: RC Polyphase Filter
- **Sampling Frequency**: 229 kHz
- **Estimated Frequency**: 229 kHz, 233 kHz, 239 kHz, 171 kHz, 34 kHz, 49 kHz
- **Output**: Estimate unknown input frequency \( f_{in} = 12 \text{GHz} \)
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Summary and Challenge

Summary

● Proposed a method to estimate high-frequency signal using multiple low-frequency sampling circuits.

● Confirmed its operation by theory and simulation.

● Measurable range is wide: proportional to multiplication of multiple sampling frequencies.

Challenge

● Estimated input frequency is discrete

Consider estimation with fine frequency resolution
Thank you for your attention