Overshoot Cancelation Based on Balanced Charge-Discharge Time Condition for Buck Converter in Mobile Applications

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Outline

1. Research Background
   • Applications of Switching Power Supply
   • Basic Switching Converter Architecture

2. Analysis of Step-down Switching Converter
   • Conventional Classical Technique
   • Superposition Principle

3. Proposed Design of Buck Converter
   • Overshoot Improvement with Parallel RLC Network
   • Experimental Results

4. Conclusions
1. Research Background

Research Objective & Approach

**Objective**
Development of switching power supply with
- Fast response & high efficiency
- Low EMI noise
- Small output ripple

**Approach**
- Analysis of buck converter system based on **classical technique** and **superposition principle**
- Overshoot reduction using **parallel RLC network**
1. Research Background
Design Achievements of This Work

Condition for overshoot cancelation

Balanced charge-discharge time

\[ |Z_L| = |Z_C| = 2R \Rightarrow \omega L = \frac{1}{\omega C} = 2R \]

Here, \( \omega = \frac{1}{\sqrt{LC}} = \frac{1}{2RC} \)

Imbalance of charge-discharge time

Overshoot improvement with parallel RLC

Overshoot improvement from 1Vpp into 0.1Vpp
1. Research Background
Basic Switching Converter Architecture

High Efficiency Switching
- Reduce energy consumption
- Extend battery operating time
- Minimize costs of systems

Merits
- Downsizing
- Light Weight
- High Efficiency

Demerits
- Output Ripple
- Switching noise
- Harmonic noise
1. Research Background
Switching Regulator

Independence of PWM Frequency

\[ V_{\text{out}} = \frac{T_{\text{ON}}}{(T_{\text{ON}} + T_{\text{OFF}})} V_{\text{in}} \]

Charge

Discharge

\[ V_{\text{Charge}}(t_i) = V_{\text{Discharge}}(t_{i-1}) \left(1 - e^{-\frac{-t}{\tau_{\text{ON}}}}\right) \]

\[ V_{\text{Discharge}}(t_i) = \frac{V_{\text{Charge}}(t_i)}{e^{-\frac{-t}{\tau_{\text{OFF}}}}} \]
1. Research Background
Harmonics of PWM Signals

$$S_{PWM}(t) = \frac{4}{\pi} \left( \sin(2\pi ft) + \frac{1}{3} \sin(3*2\pi ft) + \frac{1}{5} \sin(5*2\pi ft) + ... \right)$$

- **50% Duty Cycle**

$$S_{PWM}(t) = \frac{4}{\pi} \left( \sin(2\pi ft) + \frac{1}{3} \sin(3*2\pi ft) + \frac{1}{5} \sin(5*2\pi ft) + ... \right)$$

- **75% Duty Cycle**

$$S_{PWM}(t) = \frac{4}{\pi} \left( \sin(2\pi ft) + \frac{1}{2} \sin(2*2\pi ft) + \frac{1}{3} \sin(3*2\pi ft) + \frac{1}{4} \sin(4*2\pi ft) + \frac{1}{5} \sin(5*2\pi ft) + ... \right)$$

- **25% Duty Cycle**

$$S_{PWM}(t) = \frac{4}{\pi} \left( \sin(2\pi ft) - \frac{1}{2} \sin(2*2\pi ft) + \frac{1}{3} \sin(3*2\pi ft) - \frac{1}{4} \sin(4*2\pi ft) + \frac{1}{5} \sin(5*2\pi ft) + ... \right)$$
2. Analysis of Step-down Switching Converter

Conventional Classical Technique

**Inductor**

\[ i(t) = \frac{1}{L} \int_{-\infty}^{t_0} v(\tau)d\tau + \frac{1}{L} \int_{t_0}^{t} v(\tau)d\tau \]

\[ = i(t^-_0) + \frac{1}{L} \int_{t_0}^{t} v(\tau)d\tau \]

**Capacitor**

\[ v(t) = \frac{1}{C} \int_{-\infty}^{t_0} i(\tau)d\tau + \frac{1}{C} \int_{t_0}^{t} i(\tau)d\tau \]

\[ = v(t^-_0) + \frac{1}{C} \int_{t_0}^{t} i(\tau)d\tau \]

- **Advantages**
  - Converts differential equation into algebraic equation.
  - Rapidly provides stability & transient response.
- **Disadvantages**
  - Applicable only to Linear, Time-Invariant (LTI) systems
2. Analysis of Step-down Switching Converter

Linear Graph of Buck Converter (Switch ON)

\[
\begin{bmatrix}
\dot{i_L}(t) \\
\dot{v}_C(t)
\end{bmatrix} =
\begin{bmatrix}
0 & -1/L \\
1/C & -1/RC
\end{bmatrix}
\begin{bmatrix}
i_L(t) \\
v_C(t)
\end{bmatrix} +
\begin{bmatrix}
1/L \\
0
\end{bmatrix} v_i(t)
\]

\[
y(t) = [0 \quad 1] \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + [0] v_i(t)
\]

\[
\frac{1}{L} V_C(s) + sC\left(s + \frac{1}{RC}\right)V_C(s) = \frac{1}{L} V_i(s)
\]

\[
I_L(s) = C\left(s + \frac{1}{RC}\right)V_C(s)
\]
2. Analysis of Step-down Switching Converter

Analysis of Buck Converter (Switch ON)

\[ V_C(s) = \frac{1}{LC} \left( \frac{s}{s^2 + \frac{s}{RC} + \frac{1}{LC}} \right) V_i(s) \]

\[ s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0 \]

\[ s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \quad \text{and} \quad s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \]

\[ \omega_{2RC} = \frac{1}{2RC}; \quad \omega_{LC} = \frac{1}{\sqrt{LC}}; \]

\[ V_{\text{charge}}(t) = A_{ch1}e^{-\omega_{2RC}t + \sqrt{(\omega_{2RC})^2 - \omega_{LC}^2}t} + A_{ch2}e^{-\omega_{2RC}t - \sqrt{(\omega_{2RC})^2 - \omega_{LC}^2}t} \]
2. Analysis of Step-down Switching Converter

Linear Graph of Buck Converter (Switch OFF)

\[
\begin{bmatrix}
\cdot i_L(t) \\
\cdot v_C(t)
\end{bmatrix} = \begin{bmatrix}
0 & -\frac{1}{L} \\
-\frac{1}{C} & -\frac{1}{RC}
\end{bmatrix} \begin{bmatrix}
i_L(t) \\
v_C(t)
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix} v_i(t)
\]

\[
y(t) = [0 \ 1] \begin{bmatrix}
i_L(t) \\
v_C(t)
\end{bmatrix} + [0] v_i(t)
\]

\[
\begin{cases}
\frac{1}{L} V_C(s) - sC\left(s + \frac{1}{RC}\right) V_C(s) = 0 \\
I_L(s) = -C\left(s + \frac{1}{RC}\right) V_C(s)
\end{cases}
\]
2. Analysis of Step-down Switching Converter

Analysis of Buck Converter (Switch OFF)

\[
\left( s^2 - \frac{s}{RC} + \frac{1}{LC} \right) V_C(s) = 0
\]

\[
s^2 - \frac{1}{RC} s + \frac{1}{LC} = 0
\]

\[
s_{dis1} = \frac{1}{2RC} + \sqrt{\left( \frac{1}{2RC} \right)^2 - \frac{1}{LC}} \quad \lor \quad s_{dis2} = \frac{1}{2RC} - \sqrt{\left( \frac{1}{2RC} \right)^2 - \frac{1}{LC}}
\]

\[
\omega_{2RC} = \frac{1}{2RC}; \quad \omega_{LC} = \frac{1}{\sqrt{LC}};
\]

\[
V_{\text{discharge}}(t) = A_{dis1} e^{\left( \omega_{2RC} + \sqrt{\left( \omega_{2RC} \right)^2 - \omega_{LC}^2} \right) t} + A_{dis2} e^{\left( \omega_{2RC} + \sqrt{\left( \omega_{2RC} \right)^2 - \omega_{LC}^2} \right) t}
\]
2. Analysis of Step-down Switching Converter

Balanced Charge-Discharge Time Condition

\[ \overline{V_{out}} = \frac{1}{(T_{ON} + T_{OFF})} \left[ \int_{0}^{T_{ON}} A_{ch1} e^{-\omega_{2RC}t + \sqrt{(\omega_{2RC})^2 - \omega_{LC}^2} t} \, dt + \int_{T_{ON}}^{T_{OFF}} A_{ch2} e^{-\omega_{2RC}t - \sqrt{(\omega_{2RC})^2 - \omega_{LC}^2} t} \, dt \right] \]

\[ \omega_{2RC} = \omega_{LC} \iff \omega = \frac{1}{\sqrt{LC}} = \frac{1}{2RC} \]

\[ \omega L = \frac{1}{\omega C} = 2R \]

Balanced Charge-Discharge Time Condition

\[ |Z_L| = |Z_C| = 2R \]
2. Analysis of Step-down Switching Converter

Proposed Analysis Model of Buck Converter

Switch ON

Switch OFF

Model of Buck Converter
2. Analysis of Step-down Switching Converter

Definition of Widened Superposition Principle

A superposition of energy at one place is proportional with their input sources and resistance distances of transmission spaces.

Widened Superposition Principle:

\[ E_A(t) \sum_{i=1}^{n} \frac{1}{d_i} = \sum_{i=1}^{n} \frac{E_i(t)}{d_i} \]

\[ E_A(t) \] : energy at one place  
\[ E_i(t) \] : input sources  
\[ d_i(t) \] : resistance distances
2. Analysis of Step-down Switching Converter

Transfer Function of Proposed Analysis Model

\[ E_A(t) \sum_{i=1}^{n} \frac{I}{d_i} = \sum_{i=1}^{n} \frac{E_i(t)}{d_i} \]

Proposed analysis model

Superposition principle

Output Voltage

\[ V_o \left( \frac{1}{Z_L} + \frac{1}{Z_C} + \frac{1}{R} \right) = \frac{V_{in}}{Z_L} \]

\[ V_o = V_{in} \frac{RZ_C}{R(Z_L + Z_C) + Z_L Z_C} \]

Transfer Function

\[ H = \frac{V_o}{V_{in}} = \frac{RZ_C}{R(Z_L + Z_C) + Z_L Z_C} \]

\[ H (j\omega) = \frac{1}{LC} \frac{1}{(j\omega)^2 + j\omega \frac{1}{RC} + \frac{1}{LC}} \]
2. Analysis of Step-down Switching Converter

Condition for Max Power Propagation

Transfer Function

\[ H(j\omega) = \frac{1}{LC} \frac{1}{(j\omega)^2 + j\omega \frac{1}{RC} + \frac{1}{LC}} \]

\[ H(j2\pi f) = \frac{1}{(2\pi)^2 LC} \frac{1}{(j * f)^2 + j * f \frac{1}{2\pi RC} + \frac{1}{(2\pi)^2 LC}} \]

\[ |H(f)| = \frac{1}{(2\pi)^2 LC} \sqrt{\left( \frac{1}{(2\pi)^2 LC} - f^2 \right)^2 + \left( \frac{f}{2\pi RC} \right)^2} \]

\[ |H(f)| = \frac{1}{(2\pi)^2 LC} \left( \frac{1}{4\pi RC} \right)^2 + f^2 \Rightarrow |H(f)| = \frac{1}{2} \quad \text{when} \quad f = \frac{1}{4\pi RC} = \frac{1}{2\pi \sqrt{LC}} \]

\[ |Z_L| = |Z_C| = 2R \]

Balanced Charge-Discharge Time Condition
3. Proposed Design of Buck Converter

Conventional Step-down Switching Converter

VCO: Voltage Controlled Oscillator

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Voltage (Vin)</td>
<td>12V</td>
</tr>
<tr>
<td>Output Voltage (Vo)</td>
<td>5.0V</td>
</tr>
<tr>
<td>Output Current (Io)</td>
<td>1.0A</td>
</tr>
<tr>
<td>Clock Frequency (Fck)</td>
<td>200kHz</td>
</tr>
</tbody>
</table>

EMI Reduction using Spread Spectrum of VCO

Circuit with Frequency Modulation
3. Proposed Design of Buck Converter

Simulation of Conventional Buck Converter

\[ f_{\text{cut-off}} = \frac{1}{2\pi \sqrt{LC}} = 5kHz \]

\[ R_{\text{total}} > 5\Omega \]

\[ |Z_L| = |Z_C| = 10\Omega \]

\[ |Z_L| = |Z_C| < 2R \]

Frequency Modulation of VCO Signal
3. Proposed Design of Buck Converter

Waveforms of Conventional Buck Converter

- **Transient state**
- **Stable state**

**Overshoot Voltage**

- **Ripple voltages**
- **Spectrum of ripple voltages**

- **5 us**
- **0.05 mVpp**
3. Proposed Design of Buck Converter
Proposed Design of Step-down Switching Converter

Overshoot improvement with parallel RLC
3. Proposed Design of Buck Converter
Overshoot Improvement with Parallel RLC Network

Proposed model

\[ V_{in} \rightarrow D1 \rightarrow V_{out} \]

\[ Z_{L1} = j\omega L_1 \]

\[ Z_{C1} = \frac{1}{j\omega C_1} \]

\[ R_L \]

\[ \Omega \]

\[ R_2 \]

\[ C_2 \]

\[ L_2 \]

\[ V_{in} \rightarrow V_{out} \]

Superposition principle

\[ V_o = V_{in} \left( \frac{1}{Z_{L1}} + \frac{1}{Z_{C1}} + \frac{1}{R_L} + \frac{1}{R_2 + Z_{L2} + Z_{C2}} \right) = \frac{V_{in}}{Z_{L1}} \]

Output Voltage

\[ V_o = V_{in} \frac{(Z_{L2} + Z_{C2} + R_2)Z_{L1}R_L}{(Z_{L2} + Z_{C2} + R_2)[R_L(Z_{L1} + Z_{C1}) + Z_{L1}Z_{C1}] + Z_{L1}Z_{C1}R_L} \]

Transfer Function

\[ H = \frac{V_o}{V_{in}} = \frac{(Z_{L2} + Z_{C2} + +R_2)Z_{L1}R_L}{(Z_{L2} + Z_{C2} + R_2)[R_L(Z_{L1} + Z_{C1}) + Z_{L1}Z_{C1}] + Z_{L1}Z_{C1}R_L} \]

\[ H(j\omega) = \frac{\frac{1}{L_1C_1} \left( (j\omega)^2 + j\omega \frac{R_2}{L_2} + \frac{1}{L_2C_2} \right)}{\left( (j\omega)^2 + j\omega \frac{R_2}{L_2} + \frac{1}{L_2C_2} \right) \left( (j\omega)^2 + j\omega \frac{1}{R_LC_1} + \frac{1}{L_1C_1} \right) + \frac{(j\omega)^2}{L_2C_1}} \]
3. Proposed Design of Buck Converter
Simulation of Parallel RLC Network

\[ |Z_L| = |Z_C| = 10\Omega < 2R = 40\Omega \]

Cutoff frequency
\[ f_{cut-off} = \frac{1}{2\pi\sqrt{LC}} = 5kHz \]

\[ |Z_L| = |Z_C| = 10\Omega; \quad 2R = 40\Omega \]
3. Proposed Design of Buck Converter

Transient Response of Parallel RLC Network

Transients state

\[ |Z_L| > |Z_C| < 2R \]

Over-shoot

Over-shoot Reduction

Power On

Operation times
3. Proposed Design of Buck Converter

Proposed Design of Step-down Switching Converter

\[ f_{\text{cut-off}} = 5\text{kHz} \]

Overshoot Reduction

\[ f_{\text{par}_{-}\text{RLC}} = 5\text{kHz} \]

Frequency Modulation of VCO Signal
3. Proposed Design of Buck Converter

Overshoot Reduction Waveforms

without parallel RLC network
Overshoot Voltage
>1 Vpp

with parallel RLC network
Overshoot Voltage Reduction
< 0.1 Vpp
3. Proposed Design of Buck Converter

Waveforms of Proposed Buck Converter

- Input Voltage (Vin): 12V
- Output Voltage (Vo): 5.0V
- Output Current (Io): 1A
- Clock Frequency (Fck): 200kHz
- Overshoot: 0.1V
- Ripple Voltage: 0.05mVpp

Ripple voltages

Spectrum of ripple voltages

- Transient state
- Stable state

\[ V_{3MHz} = 10\mu V \]
\[ V_{4MHz} = 3\mu V \]
This work:

- Balanced charge-discharge time condition

\[ |Z_L| = |Z_C| = 2R \Rightarrow \omega L = \frac{1}{\omega C} = 2R \quad \text{and} \quad \omega = \frac{1}{\sqrt{LC}} = \frac{1}{2RC} \]

- Analysis model of Buck converter system based on classical technique and superposition principle

- Overshoot improvement with parallel RLC network
  \( \rightarrow \text{Overshoot reduction from } 1\text{Vpp into } 0.1\text{Vpp} \)

Future of Work

- Analysis of parasitic of RLC and other components
Thanks for your kind attention!

谢谢