Development of a simulation method specialized for flow in a long and narrow region

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Incompressible fluid flows in very long regions

As a familiar example...

Rivers

Blood vessels

Tunnels

Pipes

Straws
Background

Difficult to solve numerically

• MAC (marker and cell) method, fractional-step method
  ➢ By accumulation of numerical error, it is quite difficult to satisfy the equation of continuity precisely.
  ➢ It takes time to converge Poisson's equation of pressure by iterative method.

• Stream function-vorticity method
  ➢ It cannot be calculated in the region of complex shape.
    (Only for 2-dimensional or axisymmetric region)

Example: Calculated by conventional MAC method.

Inflow

Outflow

Pulsating flow

Outflow is smaller than inflow.
## Proposal of a method to solve the flow in a long and thin region in a short time

| Conventional MAC method. (If the calculation time is same as proposed method) |
| Proposed method. |

Even in the conventional MAC method, if the calculation time is sufficient, calculation can be performed accurately.
The long and narrow region is roughly dominated by one-dimensional flow.

Satisfy the equation of continuity from inflow to outflow with one-dimensional flow.
Proposed method

Details of Equations (1)

Original equations

Equation of continuity
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots(1) \]

Incompressible Navier-Stokes equation
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + 1 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \quad \ldots(2) \]
\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + 1 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \quad \ldots(3) \]

Simplification

One-dimensional flow temporally changing in \( x \) direction \( \Rightarrow \) \( v = 0, \ \frac{\partial}{\partial y} = 0 \)
\[ (1) \quad \frac{\partial u}{\partial x} = 0 \quad \ldots(4) \]
\[ (2) \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + 1 \frac{\partial^2 u}{\partial x^2} \]
\[ \rightarrow \quad \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} \quad \ldots(5) \]

Solution of simplified problem

\( u \) does not change in \( x \) direction \( \Rightarrow \) \( u = f(t) \) \( \ldots(6) \)

\( (6) \) is assigned to (5) \( \Rightarrow \) \( f'(t) = -\frac{\partial p}{\partial x} \)
\[ \Rightarrow \quad \text{Integrate at } x \quad \Rightarrow \quad p = -f'x + C \quad \ldots(7) \]
\( C : \text{constant} \)
Details of Equations (2)

Solution of simplified problem

\begin{align*}
(6) \quad u &= f(t) \\
(7) \quad p &= -f'x + C
\end{align*}

Solution of original problem

\[ u = f(t) + \tilde{u}, \quad p = -f'x + c + \tilde{p} \]

Assigned to original equations (1)(2)(3)

\begin{align*}
(1) \quad &\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0 \\
(2) \quad &\frac{\partial \tilde{u}}{\partial t} + (f + \tilde{u}) \frac{\partial \tilde{u}}{\partial x} + v \frac{\partial \tilde{u}}{\partial y} = -\frac{\partial \tilde{p}}{\partial x} + \frac{1}{\text{Re}} \left( \frac{\partial^2 \tilde{u}}{\partial x^2} + \frac{\partial^2 \tilde{u}}{\partial y^2} \right) \\
(3) \quad &\frac{\partial \tilde{v}}{\partial t} + (f + \tilde{u}) \frac{\partial \tilde{v}}{\partial x} + v \frac{\partial \tilde{v}}{\partial y} = -\frac{\partial \tilde{p}}{\partial y} + \frac{1}{\text{Re}} \left( \frac{\partial^2 \tilde{v}}{\partial x^2} + \frac{\partial^2 \tilde{v}}{\partial y^2} \right)
\end{align*}

The form of the equations are almost the same as the original (1)(2)(3)

Equations could be written with \( f(t) \) and \( \tilde{u}, \tilde{p} \) instead of original \( u, p \).

They are solved by conventional MAC method.
Proposed method

**Boundary conditions**

**Wall**

Original problem: \( u = 0 \) (no slip)

\[
\frac{\partial p}{\partial y} = 0
\]

Deviation: \( \tilde{u} = -f(t) \)

\[
\frac{\partial \tilde{p}}{\partial y} = 0
\]

**Pulsating flow**

**Inflow**

Original problem: \( u = 1 + 0.5 \sin \omega t \)

\[
\frac{\partial p}{\partial x} = 0
\]

Deviation: \( \tilde{u} = 0 \)

\[
\frac{\partial \tilde{p}}{\partial x} = 0
\]

**Outflow**

Original problem: \( \frac{\partial u}{\partial x} = 0 \)

\( p = 0 \)

Deviation: \( \frac{\partial \tilde{u}}{\partial x} = 0 \)

\( \tilde{p} = 0 \) (no variation from original pressure)

Solution of simplified problem

\[ f(t) = 1 + 0.5 \sin \omega t \]
Simulation Verification

**Hump and branch region with pulsating inflow**

Proposed method.

Conventional MAC method. (If the calculation time is same as proposed method)

Proposed method.

Conventional MAC method. (If the calculation time is same as proposed method)

Even in the conventional MAC method, if the calculation time is sufficient, calculation can be performed accurately.
Extension of proposed method

**3-dimensional calculation including heat**

It could be used to simulate fire in tunnels, subway, etc.

Proposed method. Upward flow is generated from heat source

Conventional MAC method. (If the calculation time is same as proposed method)

Dimensionless temperature

- Low
- High

Gravity
3-dimensional calculation including heat

Previous example

Inflow: \( u = 1 + 0.5 \sin \omega t \)

Solution of simplified problem
\( f(t) = 1 + 0.5 \sin \omega t \)

• In the case of thermal convection, no inflow in initial stage of calculation.
• One-dimensional flow gradually increases.
• Solution of simplified problem of heat convection may be decided by average value of the velocity around the heat source
(It can accurately express the original solution)
Equations of 3-dimensional calculation including heat

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + (f + \omega) \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + (f + \omega) \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)
\]

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + (f + \omega) \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \frac{Gr}{Re^2} T
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + (f + \omega) \frac{\partial T}{\partial z} = \frac{1}{Re \cdot Pr} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)
\]

\[
w = f(t) + \omega(x, y, z, t), \quad p = -f'(t)z + C + p(x, y, z, t)
\]

Solution of simplified problem

Extension of proposed method
What is a LONG region?

Calculation result by conventional MAC method.

Aspect ratio: 4.0

In the case of this condition, if the aspect ratio is 4.0 or less, it can be calculated without any problem even if by conventional MAC method.
Simulation Verification

Aspect ratio: 25.0

Conventional MAC method.
(If the calculation time is same as proposed method)

Proposed method.

Reynolds number=50, Grashof number=5000, Plandle number=0.71
Reynolds number = 50, Grashof number = 5000, Plandle number = 0.71
The other examples (1)

**Complex shape region**

General coordinate transformation

\[ \xi = \xi(x, y, z) \quad \eta = \eta(x, y, z) \quad \zeta = \zeta(x, y, z) \]

Proposed method.

Conventional MAC method.

(If the calculation time is same as proposed method)
The other examples (1)

Complex shape region

<table>
<thead>
<tr>
<th>Present Method</th>
<th>Standard MAC</th>
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<tbody>
<tr>
<td>Timestep=100</td>
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</tr>
<tr>
<td>Timestep=1000</td>
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</tr>
<tr>
<td>Timestep=5000</td>
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</tbody>
</table>

Flow Rate vs. Time (→upward)

Timestep=100

Timestep=10000

Timestep=50000
The other examples (2)

Region with branch

Branch to 2:1

Actual flow rate will not be 2:1.
(Narrow branch is highly affected by wall friction)

The Solution of simplified problem is set to 2:1 for convenience and corrected with Deviation.
The other examples (2)

Region with branch

Proposed method.

Conventional MAC method.
(If the calculation time is same as proposed method)

Upward flow is generated from heat source

Dimensionless temperature

Low High

Gravity

x step : 100

y

z

Heat source

Heat source

step : 100

😊 😞
The other examples (2)

Region with branch

Timestep=1000

Proposed method.

Conventional MAC method.
(If the calculation time is same as proposed method)
The other examples (2)

**Region with branch**

_Timestep=4000_

- **Conventional MAC method.** (If the calculation time is same as proposed method)
- **Proposed method.**

- **Present Method**

- **Standard MAC**

- **Conventional MAC method.** (If the calculation time is same as proposed method)
The other examples (3)

**Objects in pipe**

Objects calculated by MASK method

Aspect ratio: 25.0
Actual shape

Proposed method. 😊

Conventional MAC method. 😞
(If the calculation time is same as proposed method)
The other examples (3)

Objects in pipe

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</tr>
<tr>
<td>Timestep=4000</td>
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</tr>
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</table>

Flow Rate vs. Time

- **Timestep=100**
- **Timestep=1000**
- **Timestep=4000**
Conclusions

- A method to solve the flow in a long and thin region in a short time is proposed.
- Concept: Solution of original problem = \text{Solution of simplified problem} + \text{Deviation}
- Simulation Verifications are carried out. Better results than conventional MAC method (calculation time is same as proposed method) were obtained.

- Limitations of the proposed method
  - When "solution of simplified problem" is complicated and can not be determined.
  - When not along and thin regions.
Please read this book if you have interest in both circuit & fluid!