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IPS04

Development of a simulation method specialized for flow in a long and narrow region

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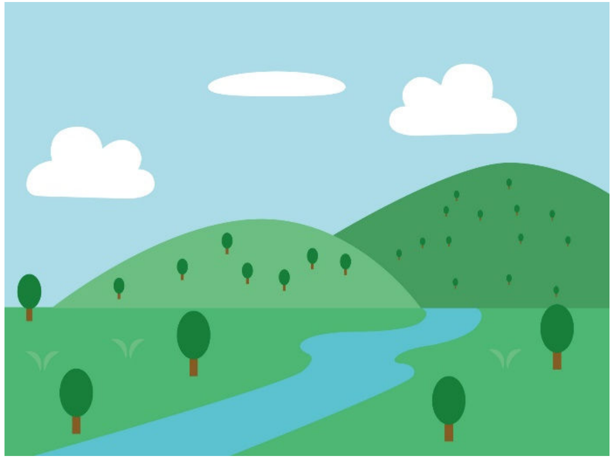
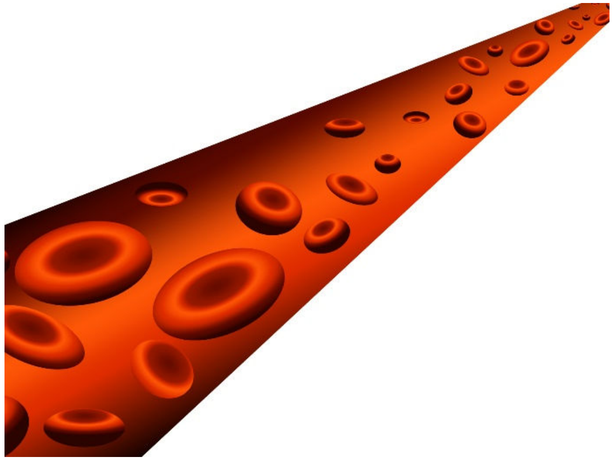
Incompressible fluid flows in very long regions

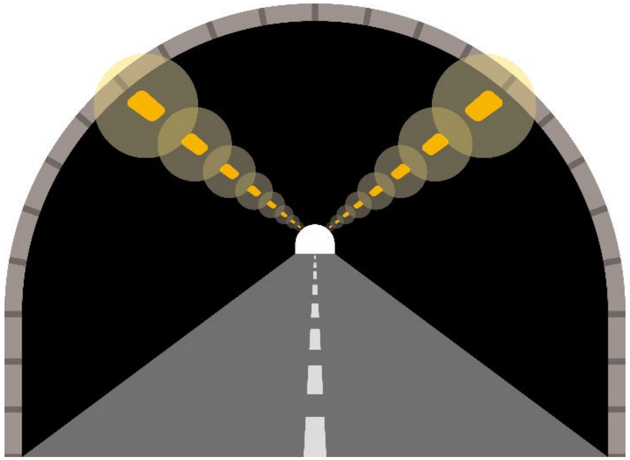
As a familiar example...



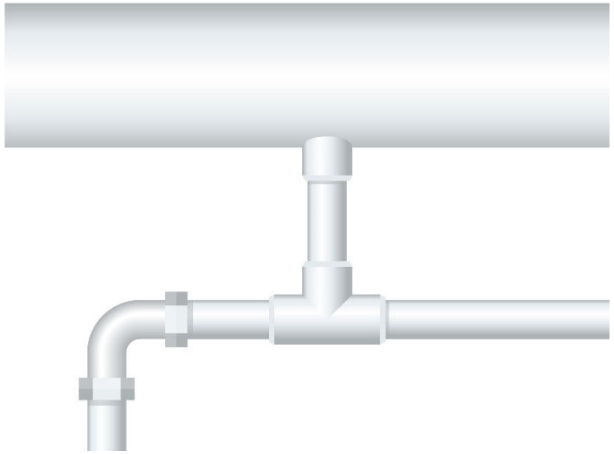
Rivers



Blood vessels



Tunnels



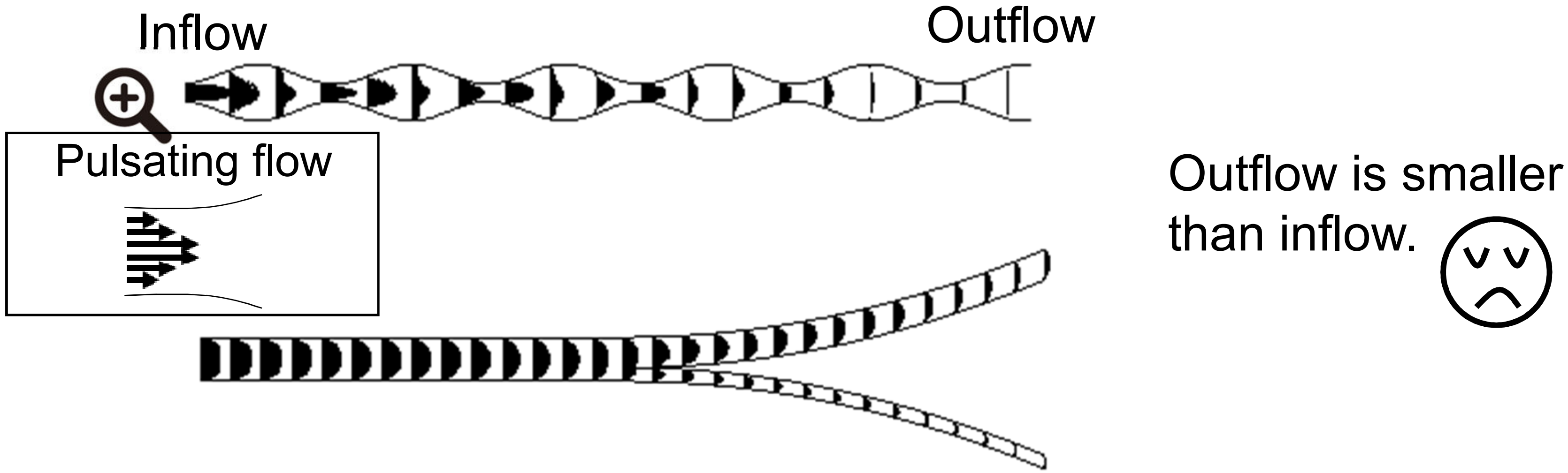
Pipes

Straws

Difficult to solve numerically

- MAC (marker and cell) method, fractional-step method
  - ➢ By accumulation of numerical error,
    it is quite difficult satisfy the equation of continuity precisely.
  - ➢ It takes time to converge Poisson's equation of pressure
    by iterative method.
- Stream function-vorticity method
  - ➢ It can not be calculated in the region of complex shape.
    (Only for 2-dimensional or axisymmetric region)

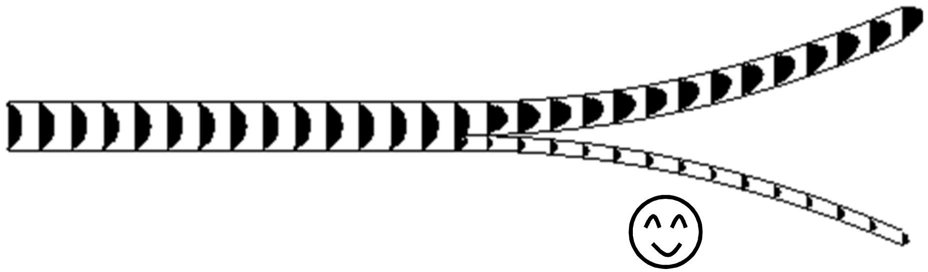
Example: Calculated by conventional MAC method.



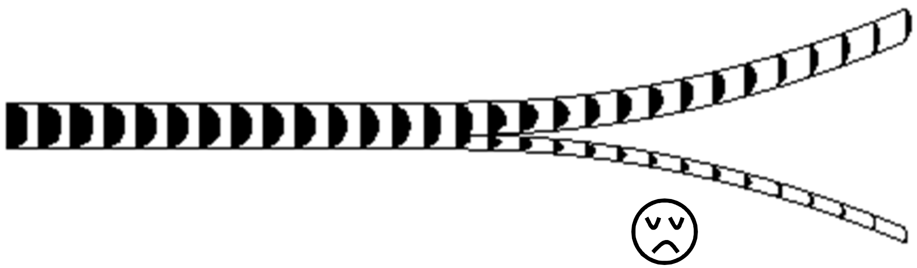
Proposal of a method to solve the flow in a long and thin region in a short time

Proposed method.

Conventional MAC method.
(If the calculation time is
same as proposed method)



Proposed method.



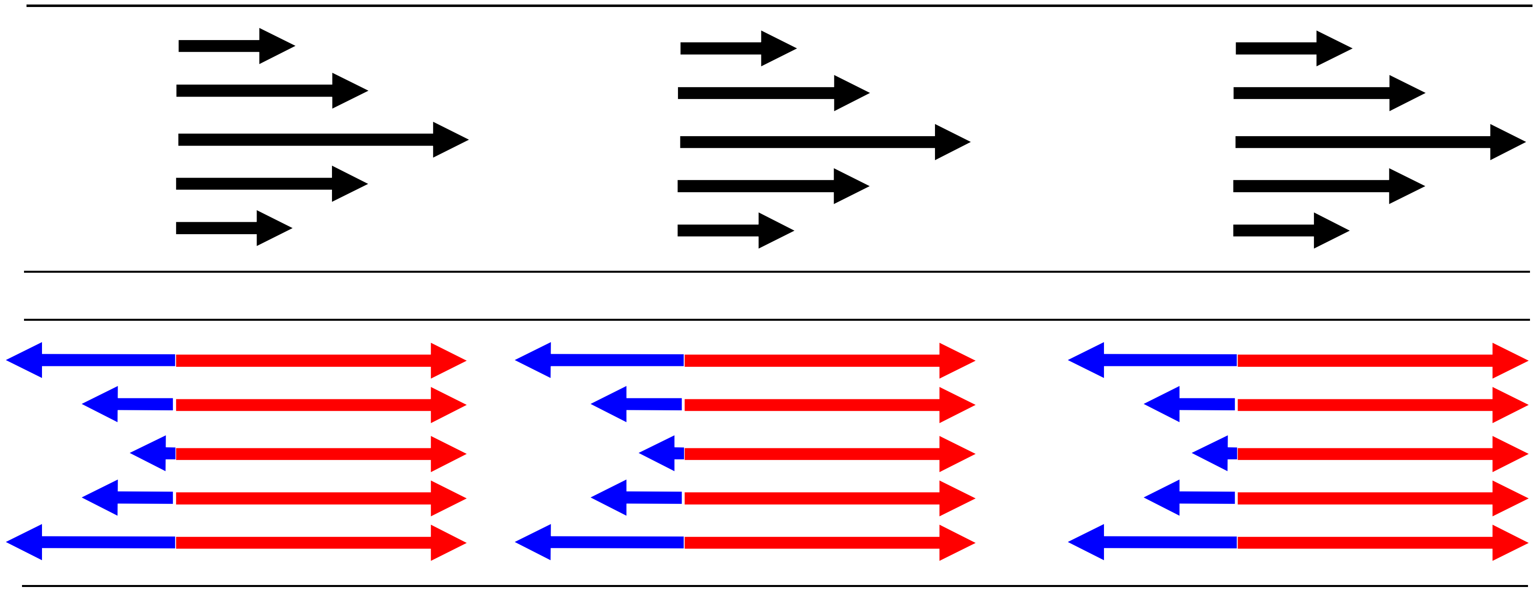
Conventional MAC method.
(If the calculation time is
same as proposed method)

Even in the conventional MAC method, if the calculation time is sufficient, calculation can be performed accurately.

Concept

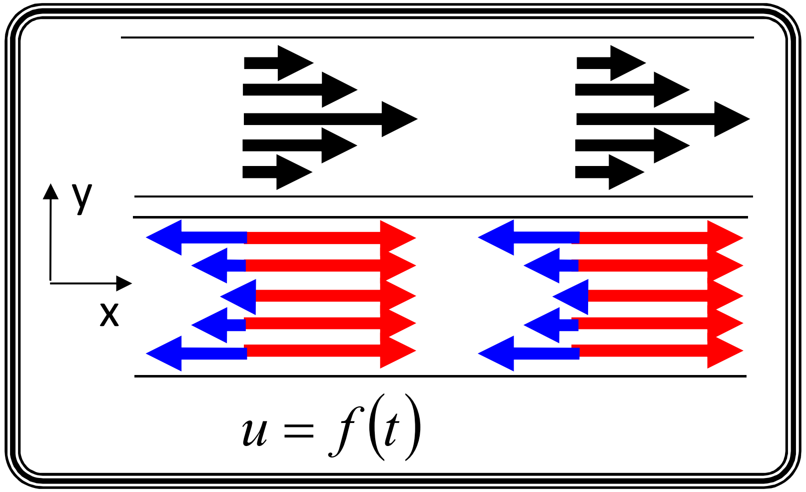
Solution of original problem

＝Solution of simplified problem ＋ Deviation



- The long and narrow region is roughly dominated by one-dimensional flow
- Satisfy the equation of continuity from inflow to outflow with one-dimensional flow

Details of Equations (1)



Original equations

Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad ...(1)$$

Incompressible Navier-Stokes equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad ...(2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad ...(3)$$

Simplification

One-dimensional flow temporally changing in x direction ➡ $v = 0, \frac{\partial}{\partial y} = 0$

(1) ➡ $\frac{\partial u}{\partial x} = 0 \quad ...(4)$

(2) ➡ $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial x^2}$

➡ $\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} \quad ...(5)$

Solution of simplified problem

(4) ➡ u does not change in x direction ➡ $u = f(t) \quad ...(6)$

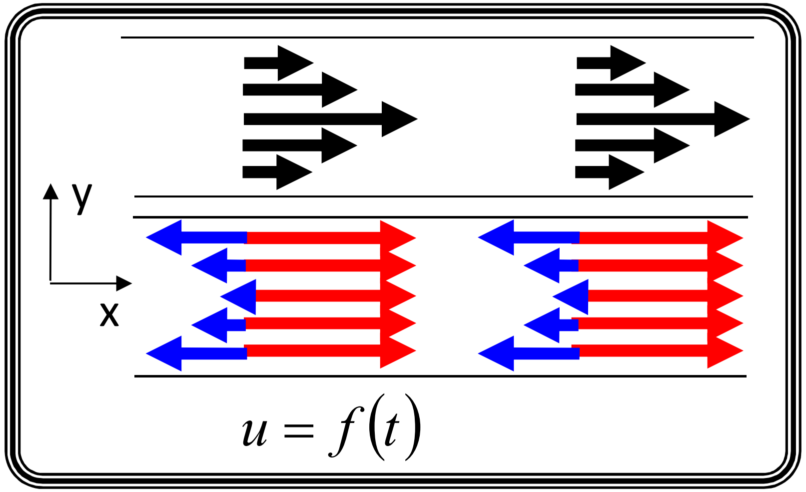
(6) is assigned to (5) ➡ $f'(t) = -\frac{\partial p}{\partial x}$ ➡ Integrate at x ➡ $p = -f'x + C \quad ...(7)$

C : constant

Details of Equations (2)

Solution of simplified problem

(6) $u = f(t)$ (7) $p = -f'x + C$



Solution of original problem
＝Solution of simplified problem＋Deviation

$$u = \underline{f(t)} + \underline{\tilde{u}}, \quad p = \underline{-f'x + c} + \underline{\tilde{p}}$$

⬇

Assigned to original equations (1)(2)(3)

(1) ➡ $$\frac{\partial \tilde{u}}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(2) ➡ $$\frac{\partial \tilde{u}}{\partial t} + (f + \tilde{u})\frac{\partial \tilde{u}}{\partial x} + v\frac{\partial \tilde{u}}{\partial y} = -\frac{\partial \tilde{p}}{\partial x} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \tilde{u}}{\partial x^2} + \frac{\partial^2 \tilde{u}}{\partial y^2}\right)$$

(3) ➡ $$\frac{\partial v}{\partial t} + (f + \tilde{u})\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial \tilde{p}}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

➡ The form of the equations are almost the same as the original (1)(2)(3)

Equations could be written with $f(t)$ and $\tilde{u}, \tilde{p}$ instead of original u, p.
They are solved by conventional MAC method.

Boundary conditions

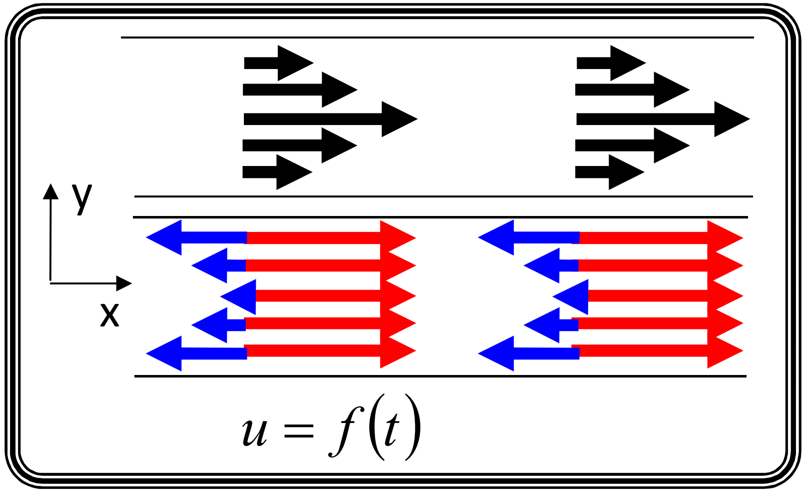
Wall

Original problem: $u = 0$ (no slip)

$$\frac{\partial p}{\partial y} = 0$$

Deviation: $\tilde{u} = -f(t)$

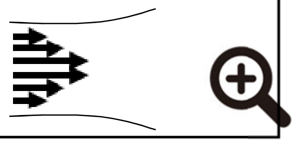
$$\frac{\partial \tilde{p}}{\partial y} = 0$$



Solution of simplified problem

$f(t) = 1 + 0.5 \sin \omega t$

Pulsating flow



Inflow

Original problem: $u = 1 + 0.5 \sin \omega t$

$$\frac{\partial p}{\partial x} = 0$$

Deviation: $\tilde{u} = 0$

$$\frac{\partial \tilde{p}}{\partial x} = 0$$

Outflow

Original problem: $\frac{\partial u}{\partial x} = 0$

$$p = 0$$

Deviation: $\frac{\partial \tilde{u}}{\partial x} = 0$

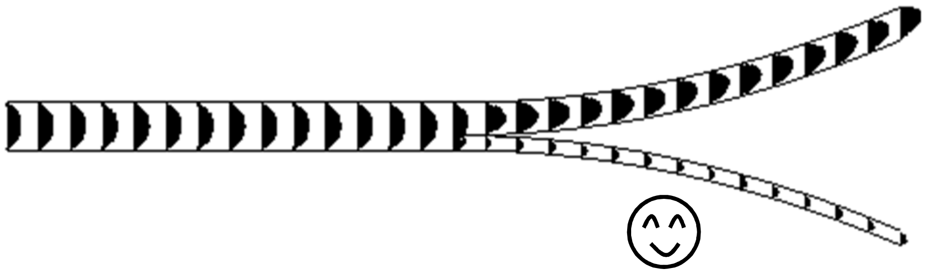
$$\tilde{p} = 0$$

(no variation from original pressure)

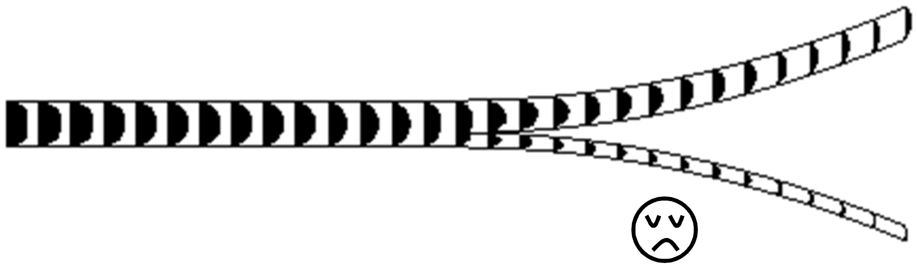
Hump and branch region with pulsating inflow

Proposed method.

Conventional MAC method.
(If the calculation time is
same as proposed method)



Proposed method.



Conventional MAC method.
(If the calculation time is
same as proposed method)

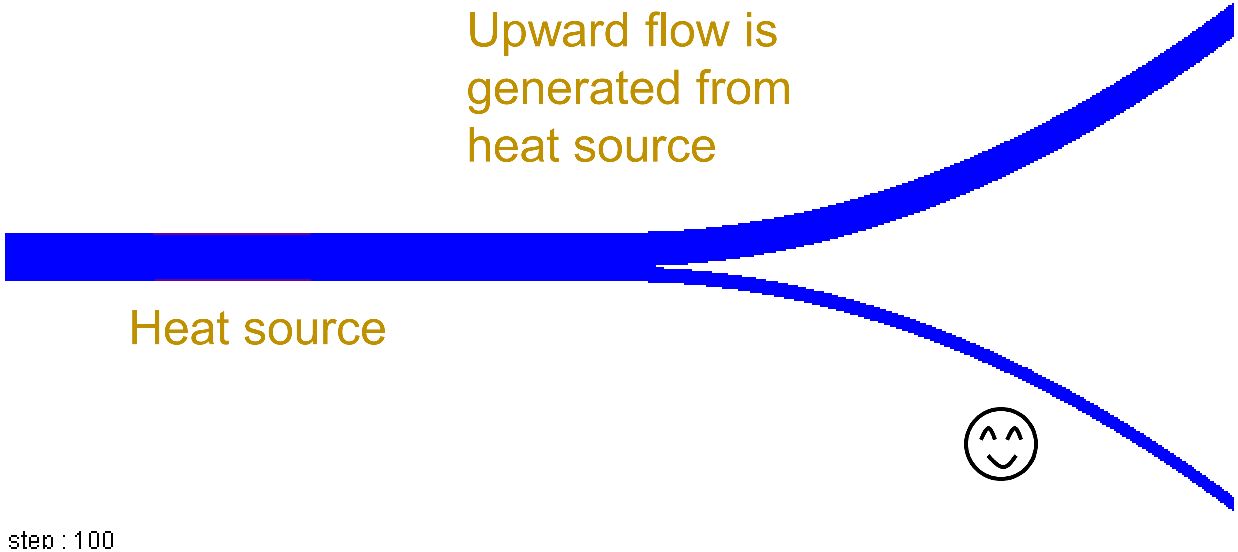
Even in the conventional MAC method, if the calculation time is sufficient, calculation can be performed accurately.

Extension of proposed method

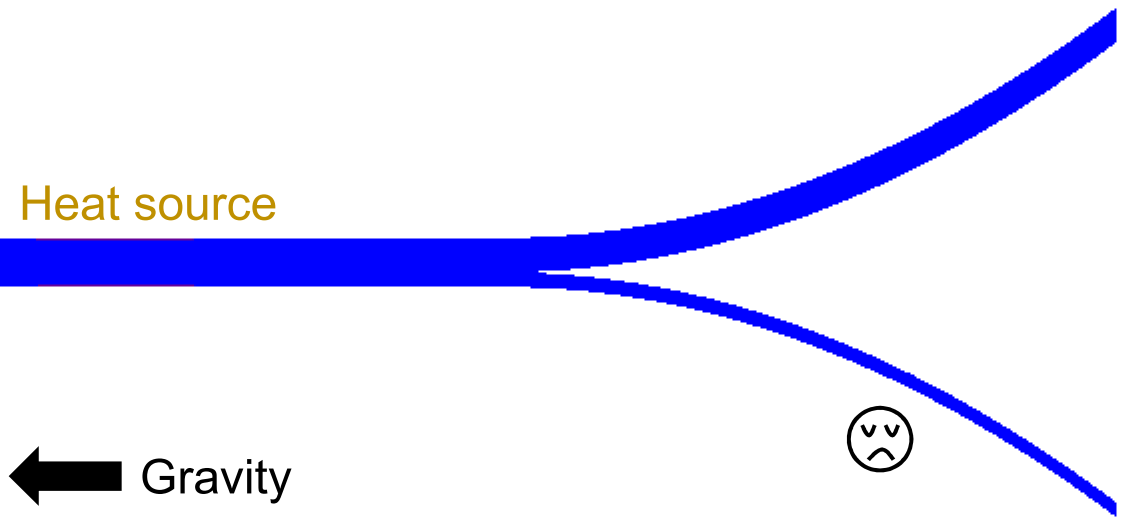
3-dimensional calculation including heat

It could be used to simulate fire in tunnels, subway, etc.

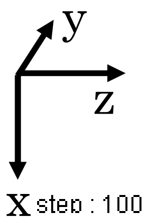
Proposed method.



Conventional MAC method.
(If the calculation time is same as proposed method)



Dimensionless temperature



Gravity

3-dimensional calculation including heat

Previous example

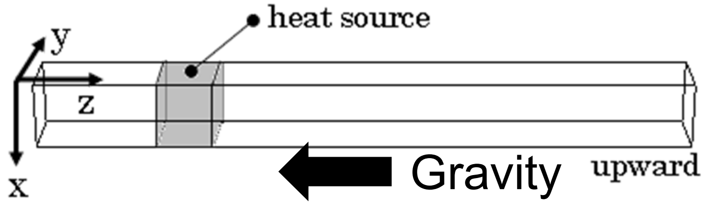
Inflow: $u = 1 + 0.5\sin\omega t$

Solution of simplified problem

$f(t) = 1 + 0.5\sin\omega t$

- In the case of thermal convection, no inflow in initial stage of calculation.
- One-dimensional flow gradually increases.
- Solution of simplified problem of heat convection may be decided by average value of the velocity around the heat source (It can accurately express the original solution)

Equations of 3-dimensional calculation including heat



$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \tilde{w}}{\partial z}=0$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+(f+\tilde{w})\frac{\partial u}{\partial z}=-\frac{\partial \tilde{p}}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+(f+\tilde{w})\frac{\partial v}{\partial z}=-\frac{\partial \tilde{p}}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{\partial \tilde{w}}{\partial t}+u\frac{\partial \tilde{w}}{\partial x}+v\frac{\partial \tilde{w}}{\partial y}+(f+\tilde{w})\frac{\partial \tilde{w}}{\partial z}=-\frac{\partial \tilde{p}}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \tilde{w}}{\partial x^2}+\frac{\partial^2 \tilde{w}}{\partial y^2}+\frac{\partial^2 \tilde{w}}{\partial z^2}\right)+\frac{Gr}{\mathrm{Re}^2}T$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+(f+\tilde{w})\frac{\partial T}{\partial z}=\frac{1}{\mathrm{Re}\cdot\mathrm{Pr}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)$$

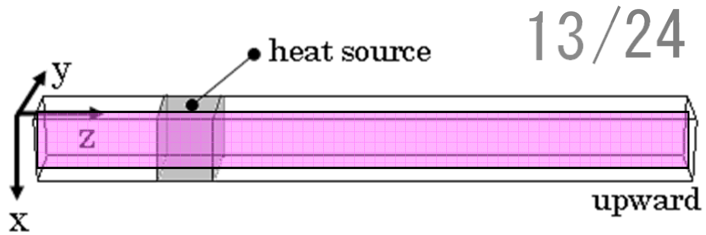
Buoyancy term

$$w=f(t)+\tilde{w}(x,y,z,t),\quad p=-f'(t)z+C+\tilde{p}(x,y,z,t)$$

Solution of simplified problem

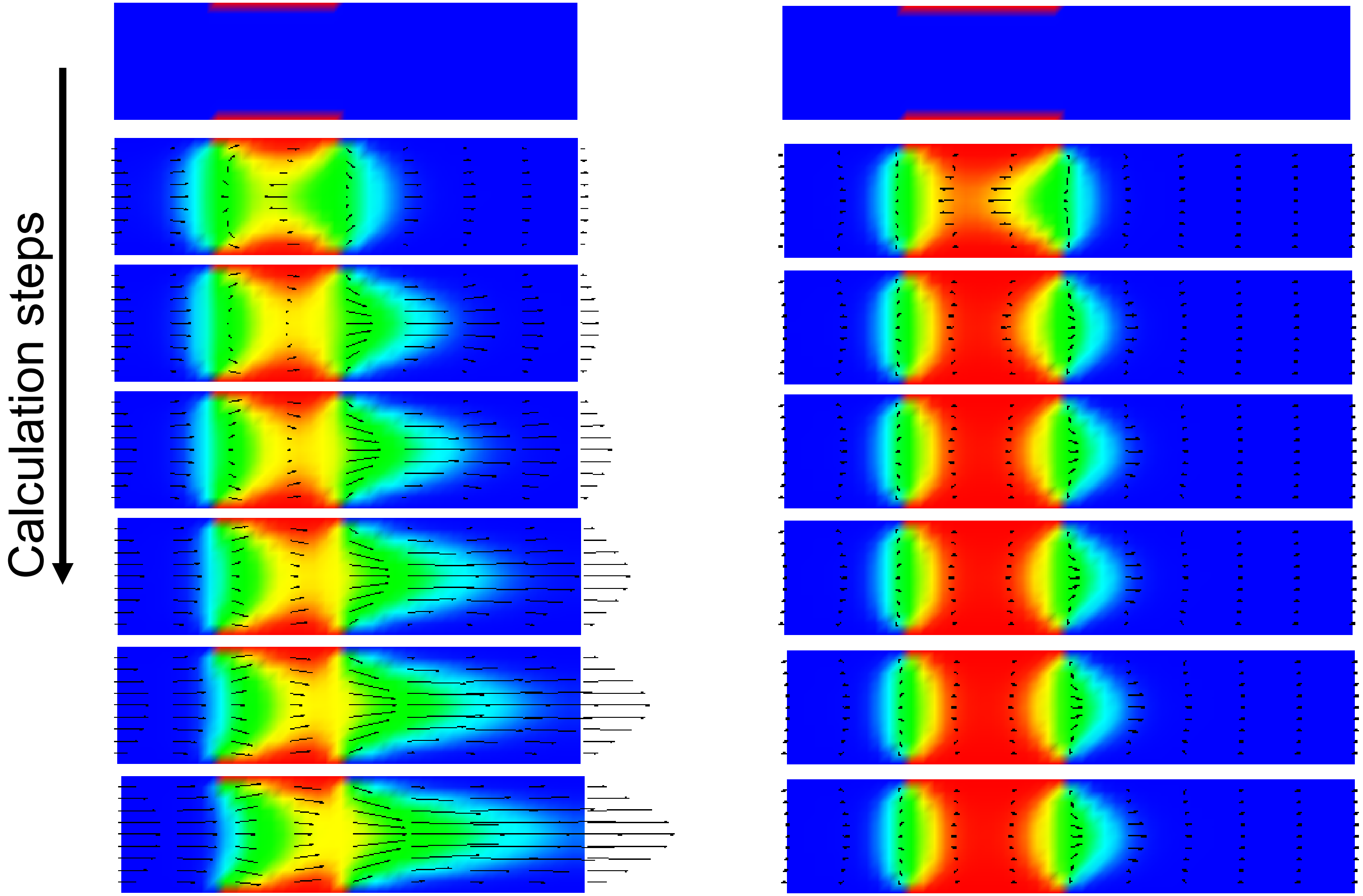
Deviation

What is a LONG region?



Calculation result by conventional MAC method.

Aspect ratio: 4.0 | Aspect ratio: 5.0

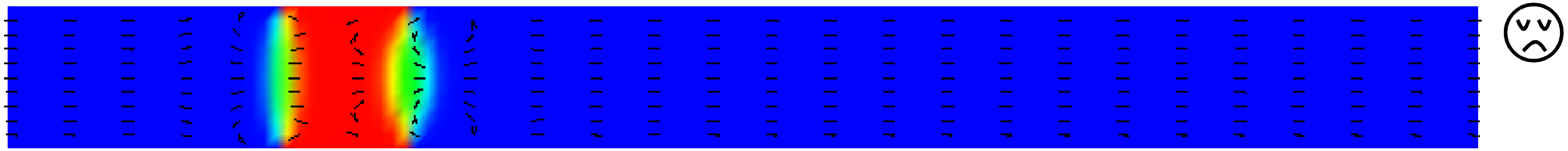


In the case of this condition, if the aspect ratio is 4.0 or less,
it can be calculated without any problem even if by conventional MAC method.

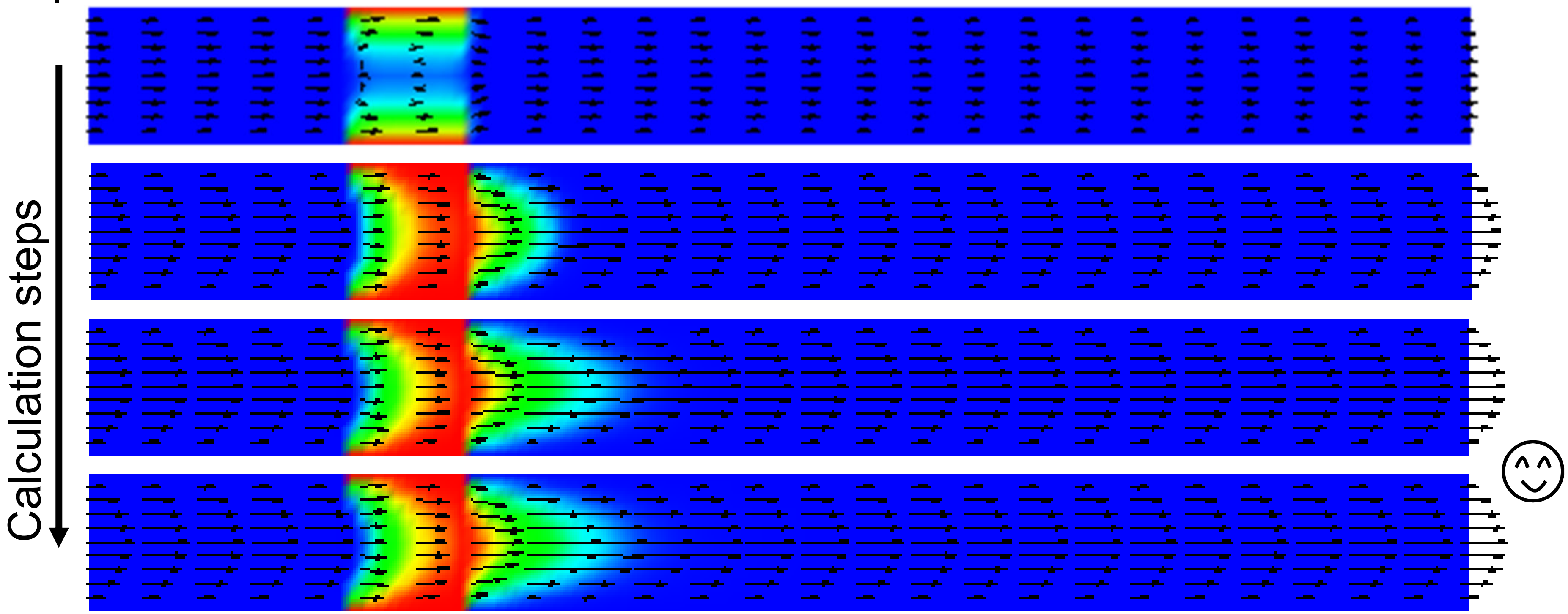
Aspect ratio: 25.0

Actual shape(Difficult to see)

Conventional MAC method.
(If the calculation time is same as proposed method)



Proposed method.



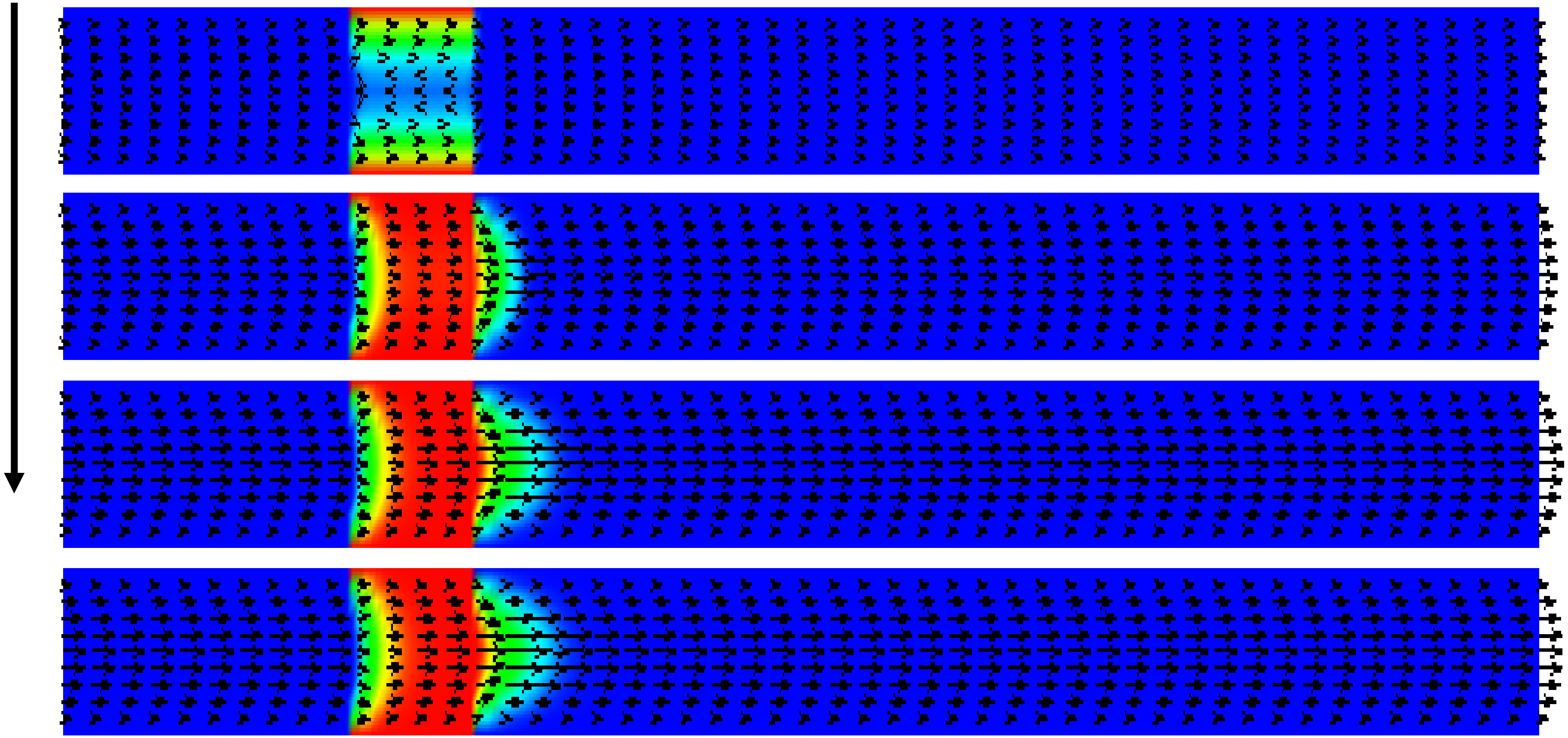
Reynolds number=50, Grashof number=5000, Plandle number=0.71

Aspect ratio: 50.0

Actual shape(Difficult to see)

Proposed method.

Calculation steps



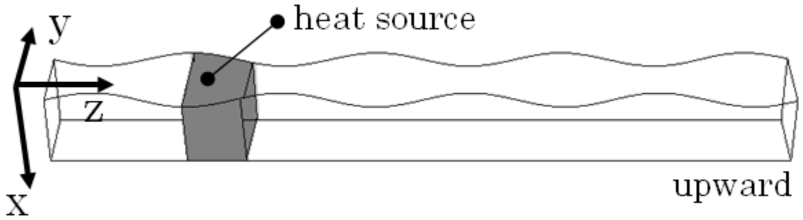
Reynolds number=50, Grashof number=5000, Plandle number=0.71

The other examples (1)

Complex shape region

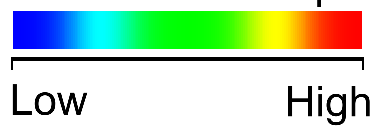
General coordinate transformation

$$\xi = \xi(x,y,z) \quad \eta = \eta(x,y,z) \quad \zeta = \zeta(x,y,z)$$

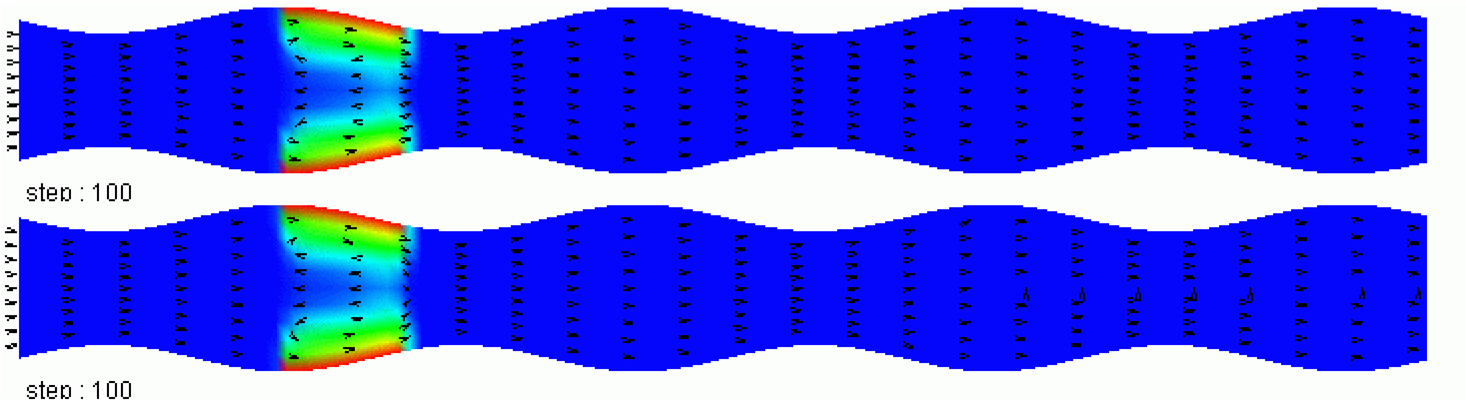


Aspect ratio: 25.0
Actual shape

Dimensionless temperature



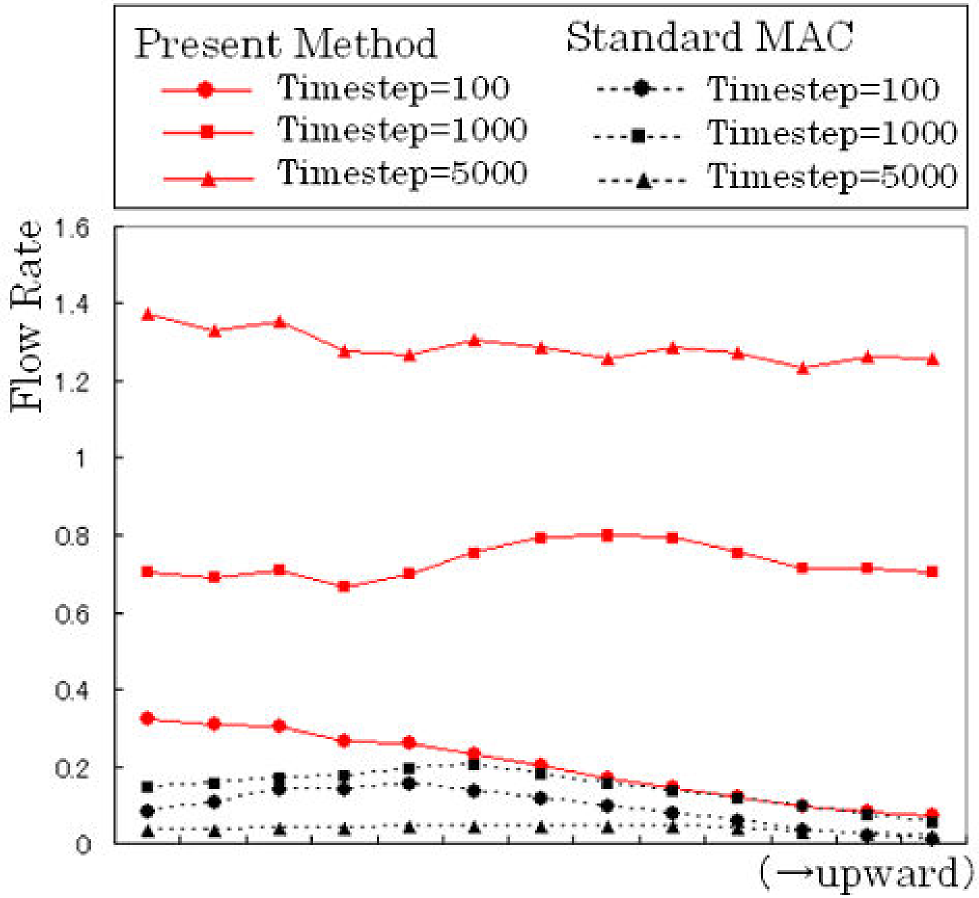
Proposed method.



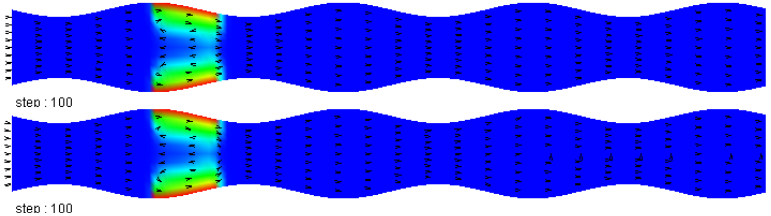
Conventional MAC method.
(If the calculation time is same as proposed method)

The other examples (1)

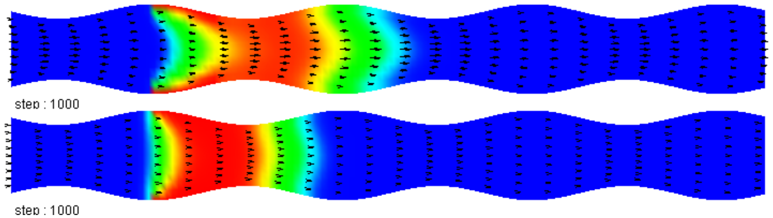
Complex shape region



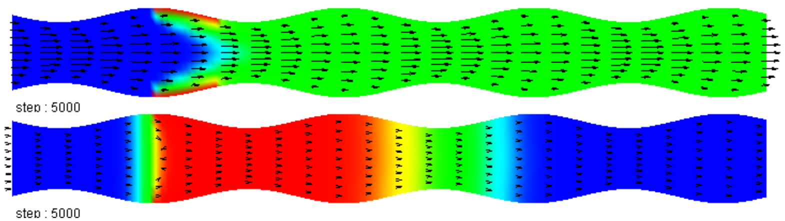
Timestep=100



Timestep=10000

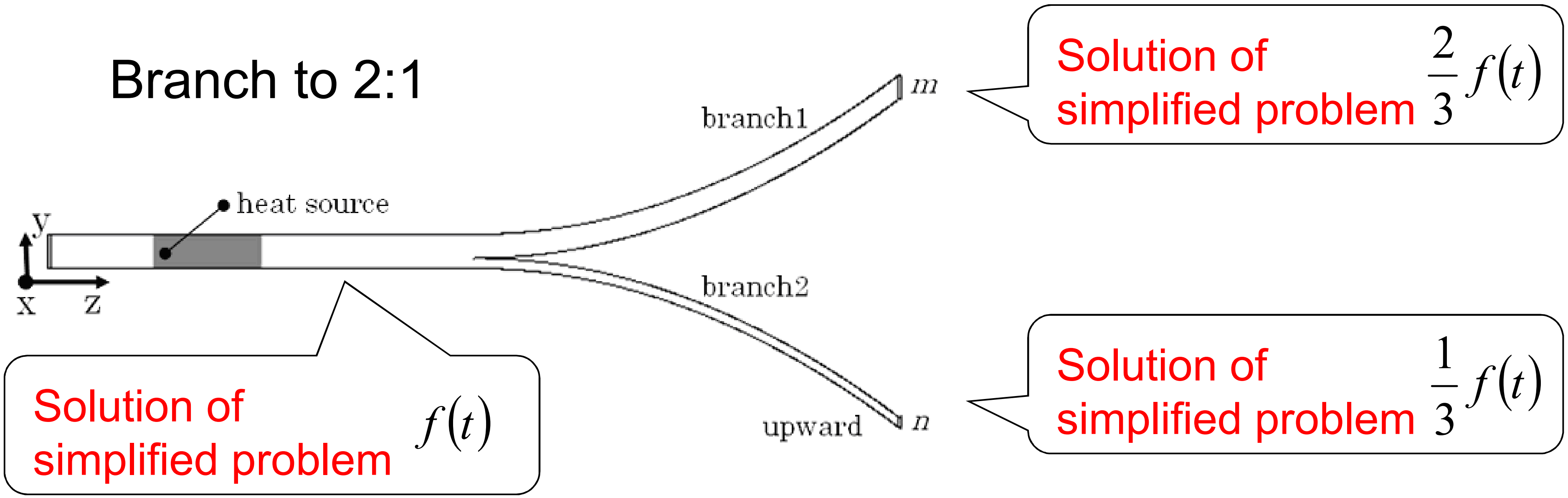


Timestep=50000



Region with branch

Branch to 2:1

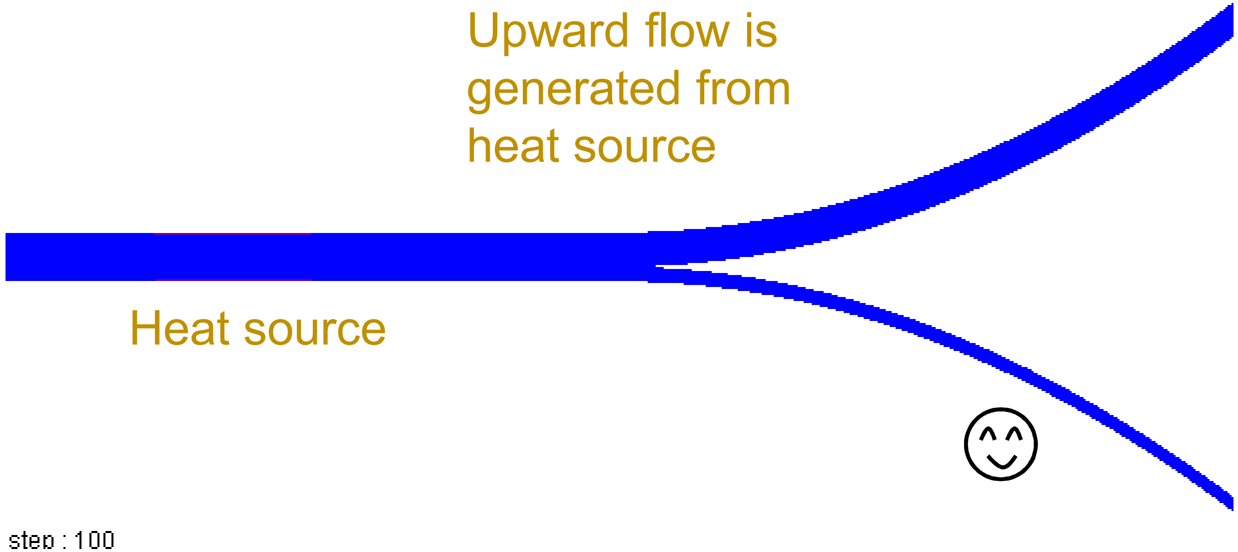


Actual flow rate will not be 2:1.
(Narrow branch is highly affected by wall friction)

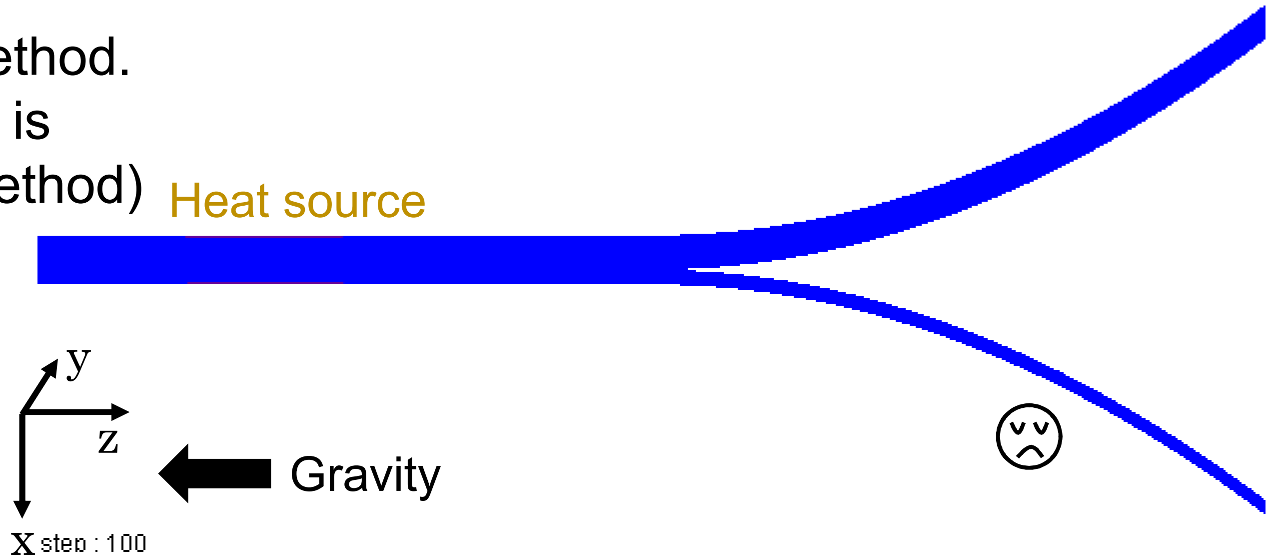
The Solution of simplified problem is set to 2:1 for convenience and corrected with Deviation.

Region with branch

Proposed method.



Conventional MAC method.
(If the calculation time is same as proposed method)



Dimensionless temperature

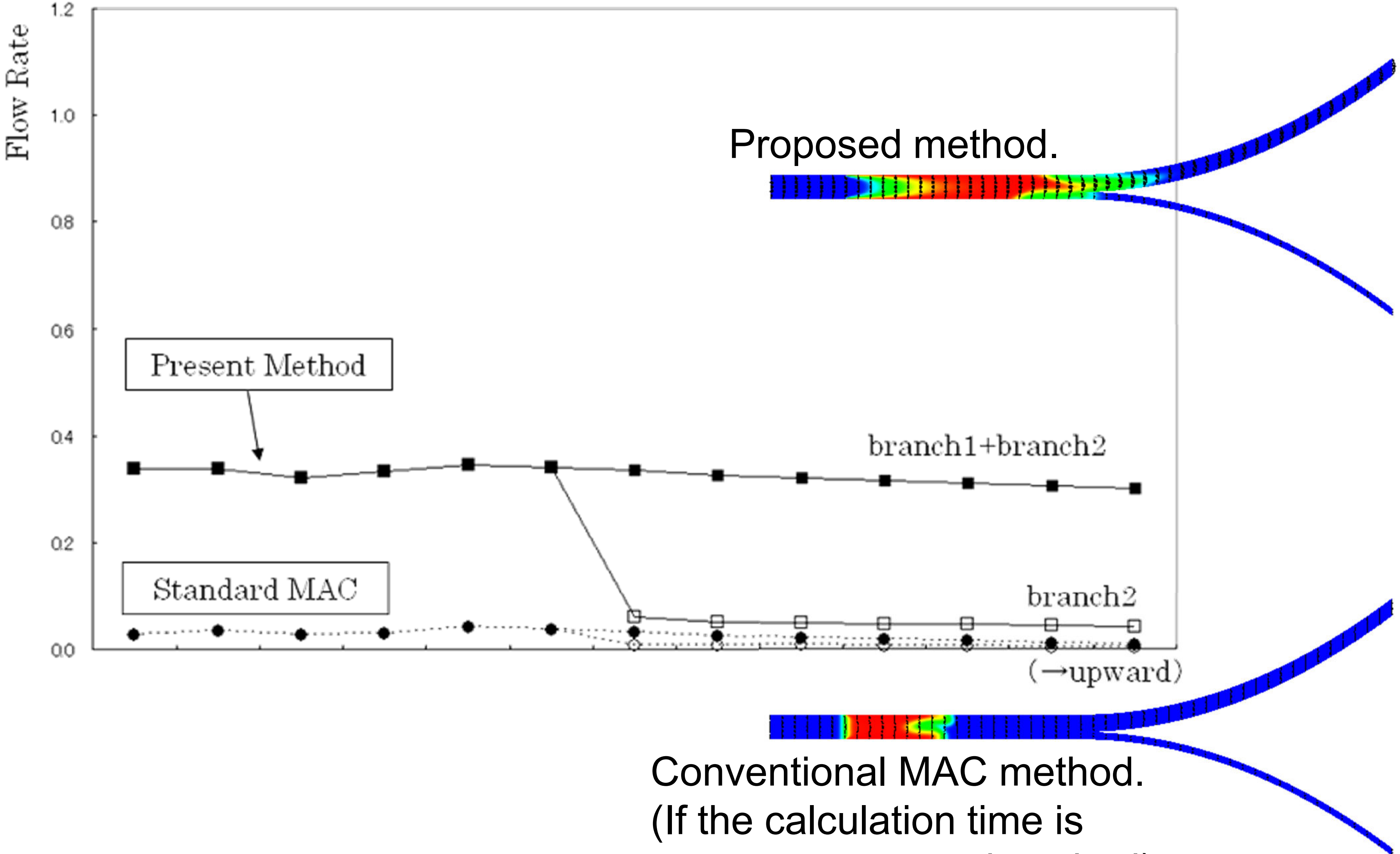
Low High

Gravity

The other examples (2)

Region with branch

Timestep=1000



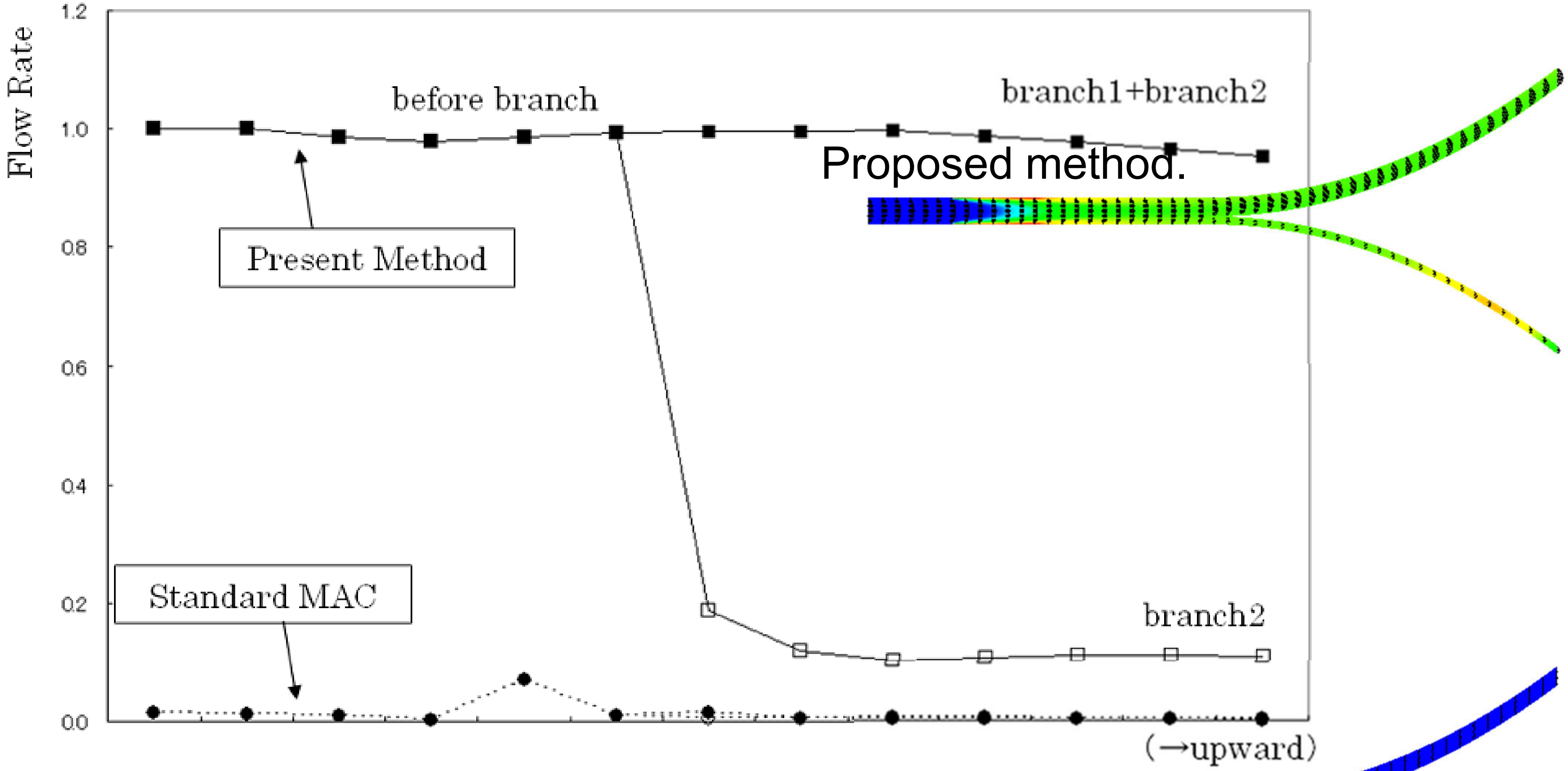
Proposed method.

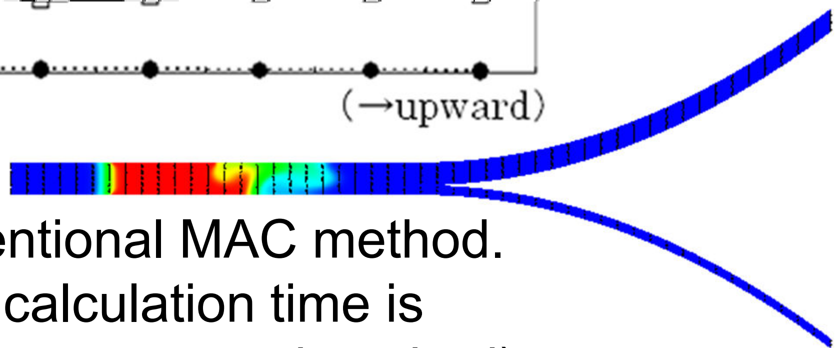
Conventional MAC method.
(If the calculation time is
same as proposed method)

The other examples (2)

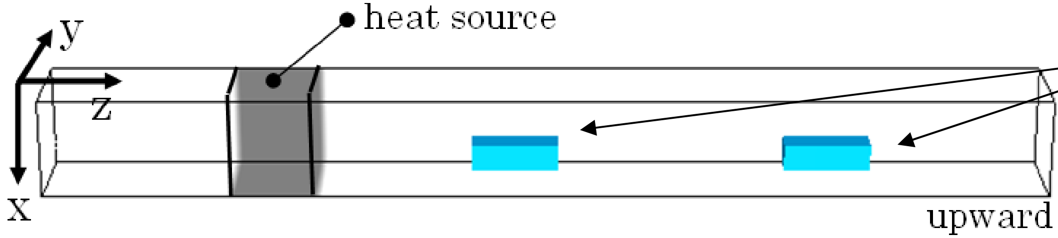
Region with branch

Timestep=4000





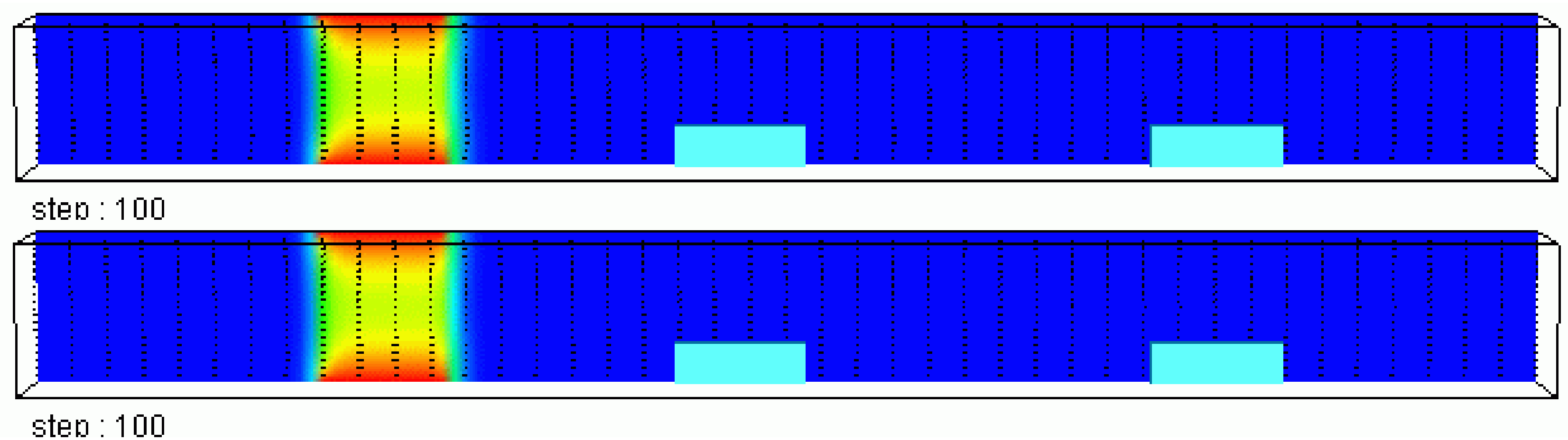
Objects in pipe



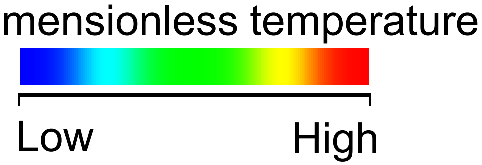
Objects calculated by MASK method

Aspect ratio: 25.0
Actual shape

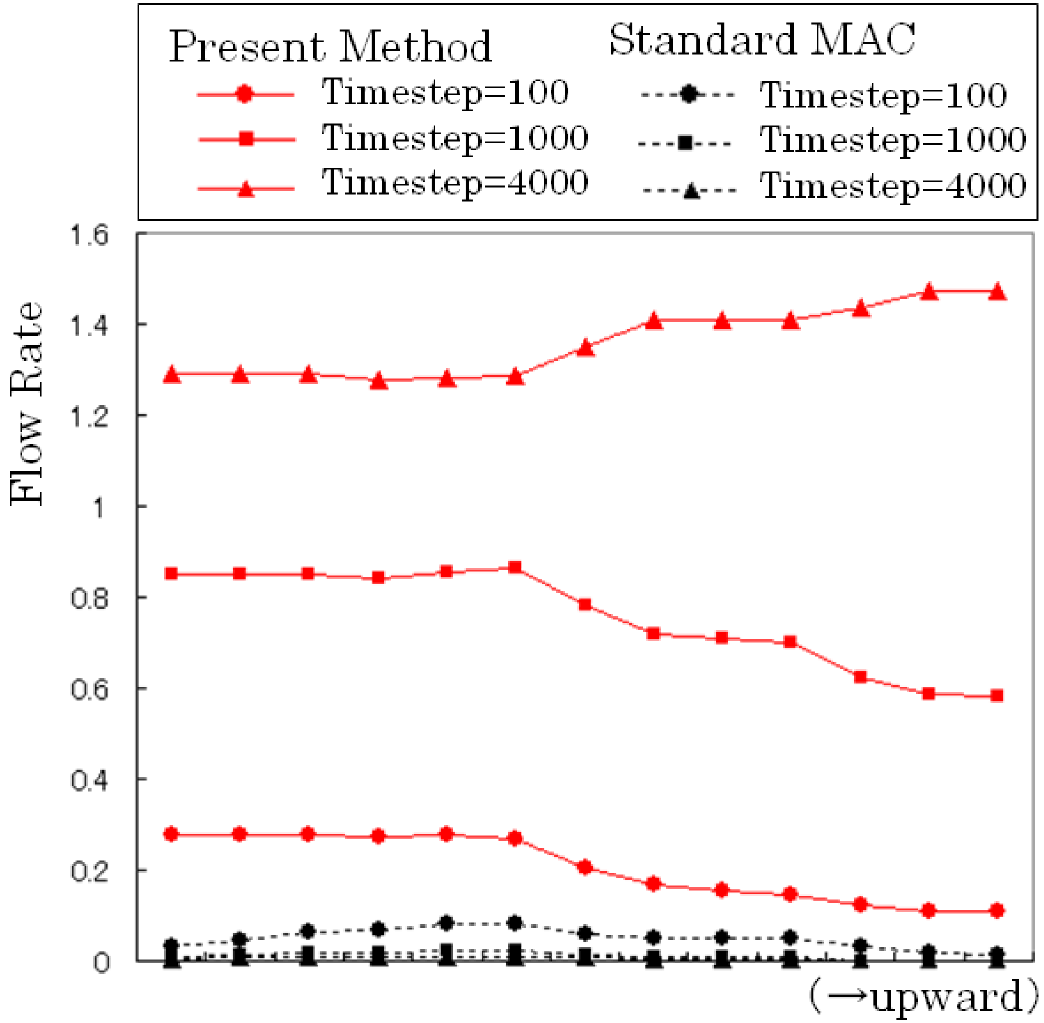
Proposed method.



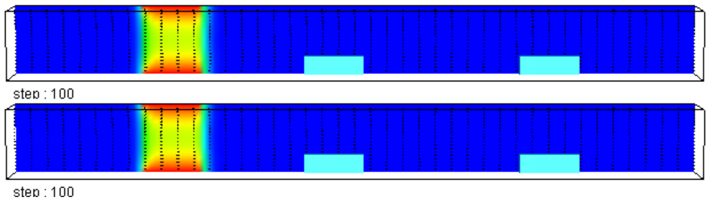
Conventional MAC method.
(If the calculation time is
same as proposed method)



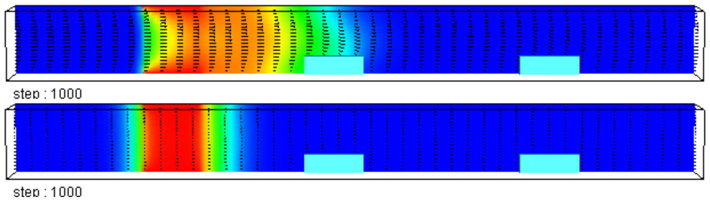
Objects in pipe



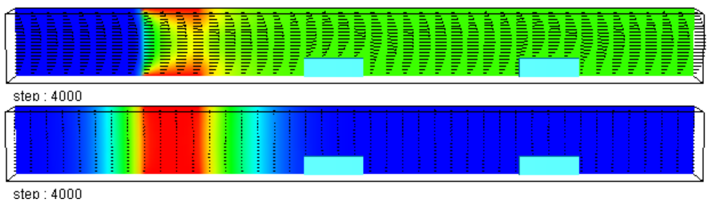
Timestep=100



Timestep=1000



Timestep=4000

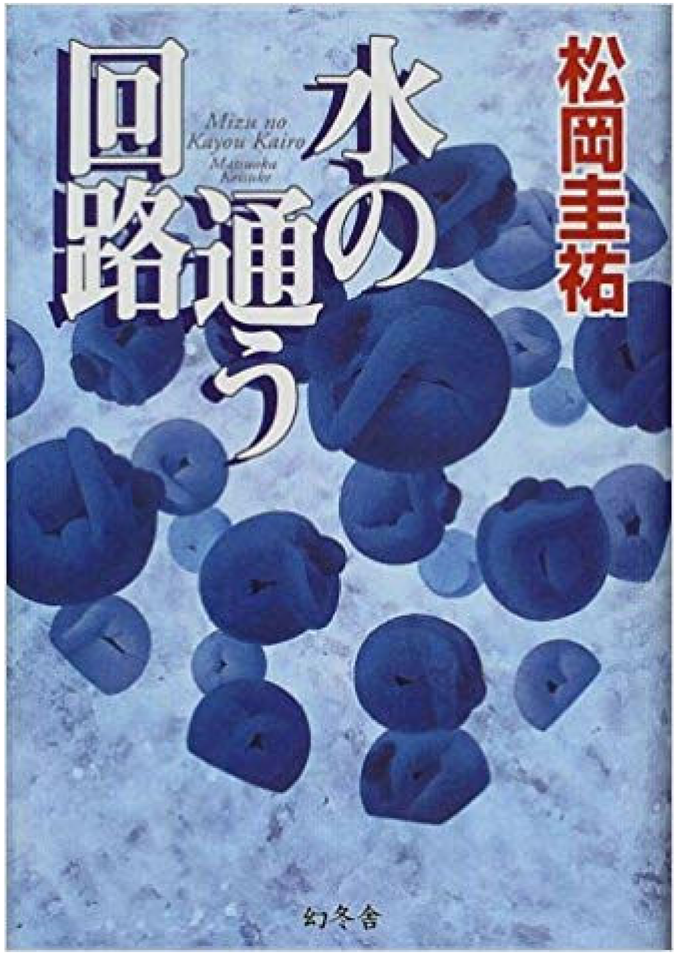


Conclusions

- A method to solve the flow in a long and thin region in a short time is proposed.
- Concept: Solution of original problem
  ＝Solution of simplified problem ＋ Deviation
- Simulation Verifications are carried out.
  Better results than conventional MAC method (calculation time is same as proposed method) were obtained.

- Limitations of the proposed method
  - When "solution of simplified problem" is complicated and can not be determined.
  - When not along and thin regions.

Circuit philosophy



Meaningful words

Turbid water
in the water passing circuit.

Please read this book if you have interest in both circuit & fluid!