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Mathematical Analysis and Design of 4-Stage Passive RC Network in RF Front-End System

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Outline

1. Research Background
   - Applications of RC Poly-phase Network
   - Analog Complex Signal Processing Concepts
2. Analysis of 4-Stage Passive RC Network
   - Frequency Responses
   - Image Rejection Ratio
3. Proposed Model of 4-Stage Passive RC Network
   - Model of 4-Stage Passive RC Network
   - Simulation Results
4. Conclusions

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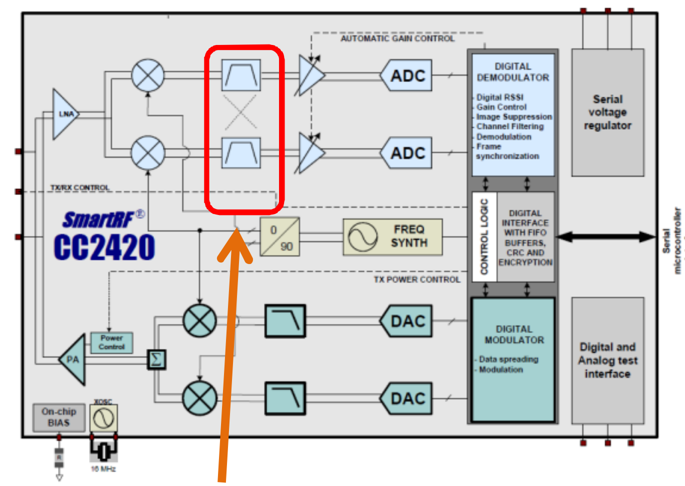
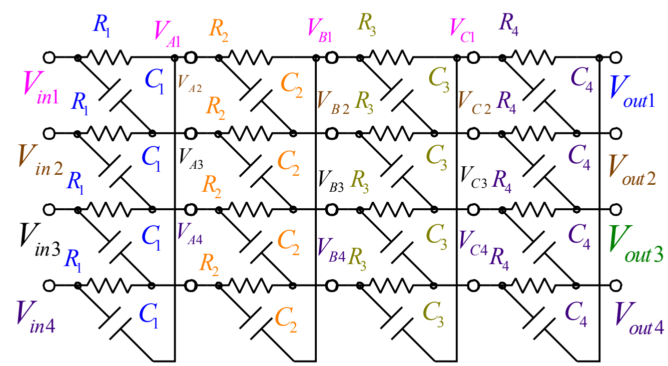
1. Research Background

Typical Applications of RC Poly-phase Network





RC Poly-phase Filter

Applications: Wi-Fi, WiMax, UWB, GSM, WCDMA, LTE , 4G, Cordless Phones, RFID, ZigBee, Bluetooth, TV Set Top Box, Sensing, Radar…

1. Research Background

Research Objective & Design Achievements

Objective

Design of Image Rejection Filter for Blue-tooth Receiver:

- ○ Low image noise
- ○ Simple Model of RC Network

Approach

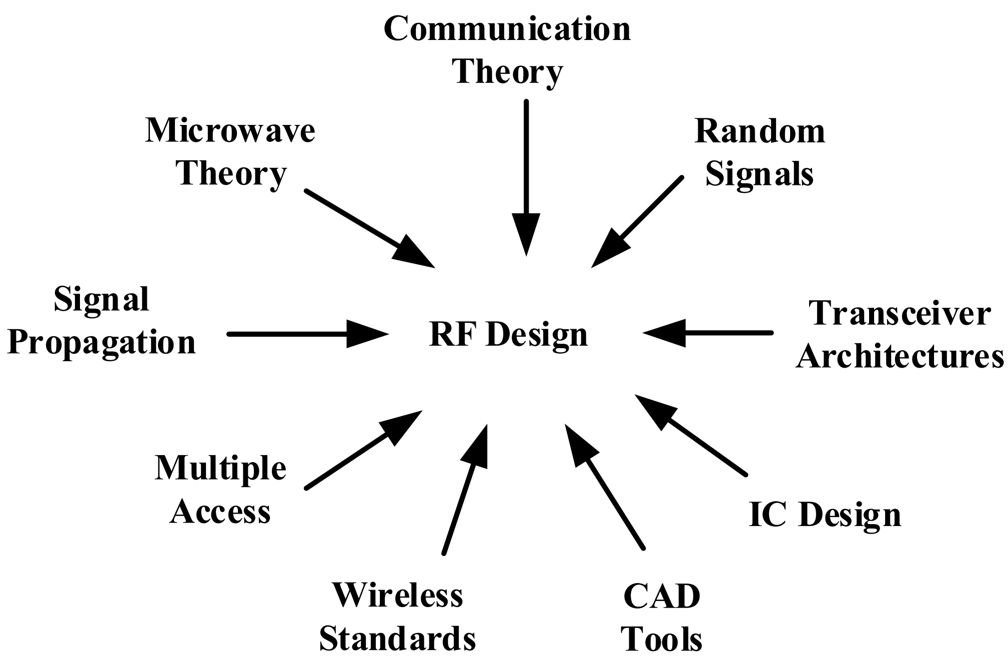
○ Derivation of Transfer Function of 4-Stage Passive RC Network Based on Superposition Principle

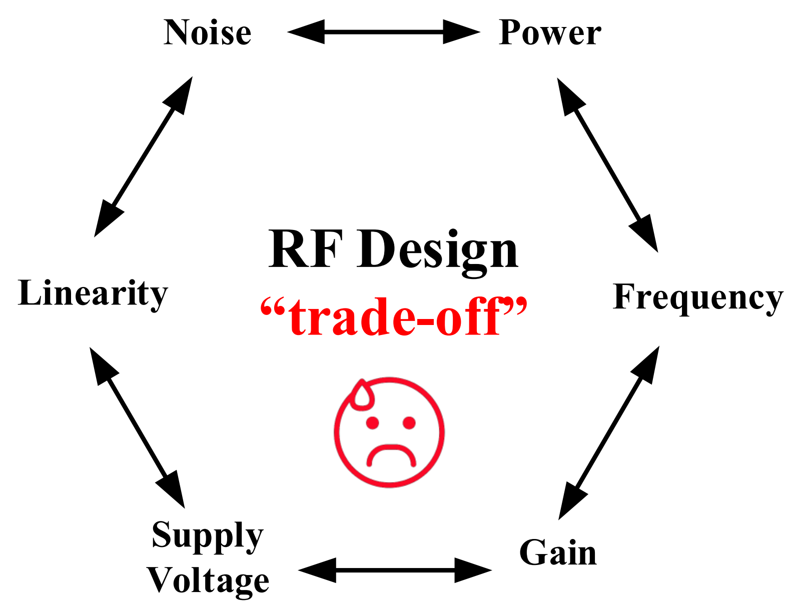
Achievements

- ○ Mathematical Analysis of 4-Stage Passive RC Network
- ○ Image Rejection Ratio: -33dB

1. Research Background

RFIC Design & Trade-offs

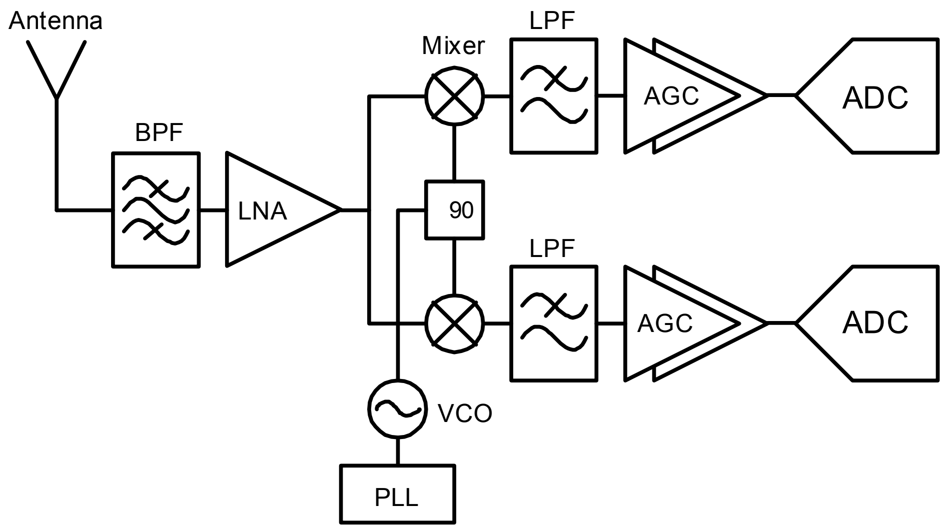




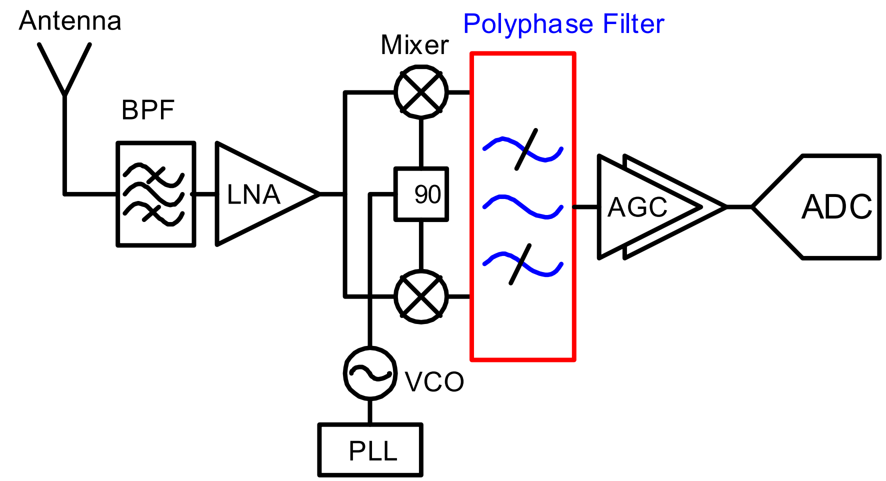
Theoretical Concepts → **Practical Designs**

1. Research Background

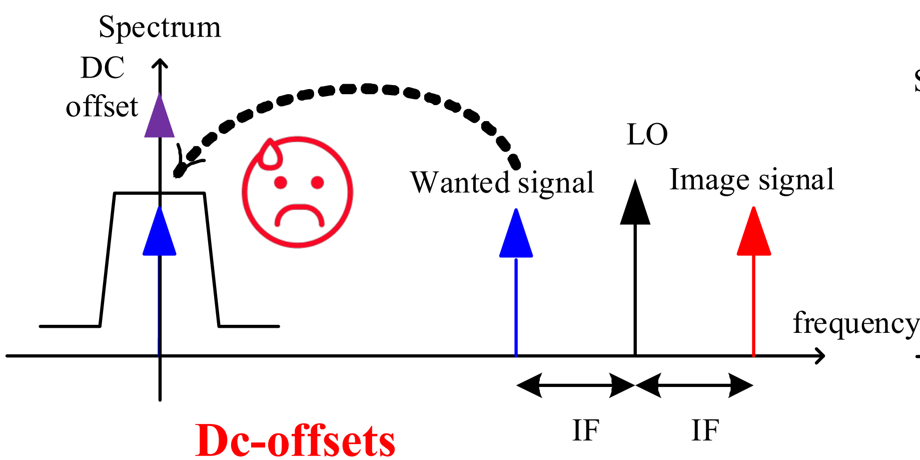
Modern Receiver Architectures

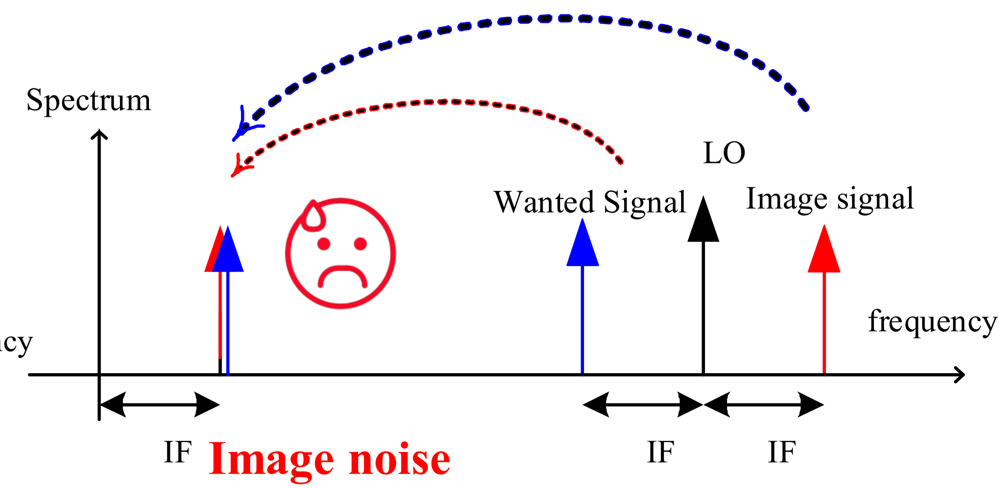


Direct-conversion Receiver



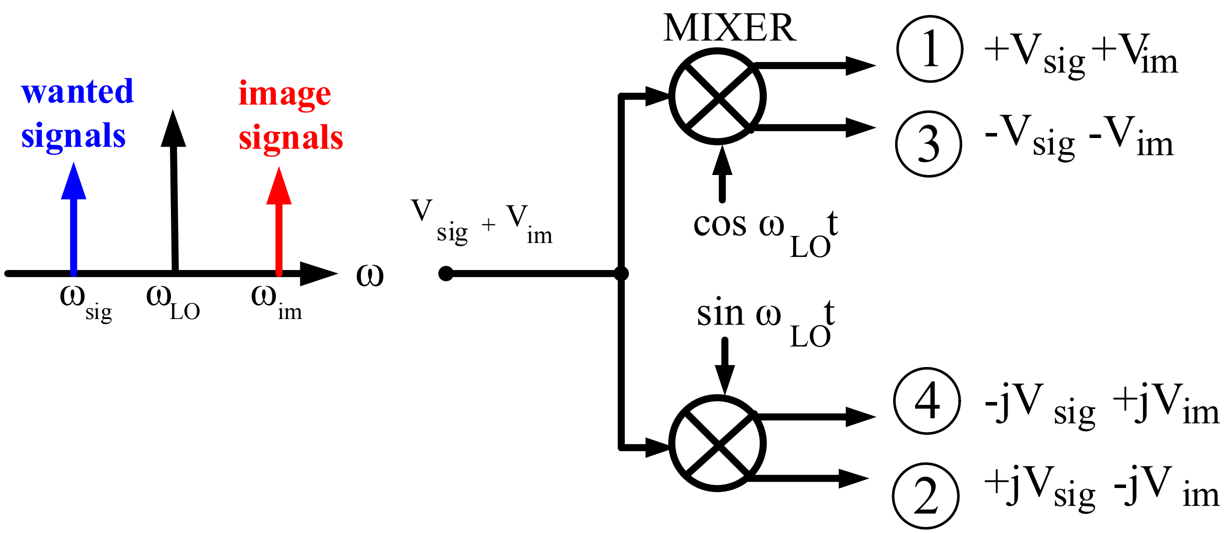
Low-IF Receiver

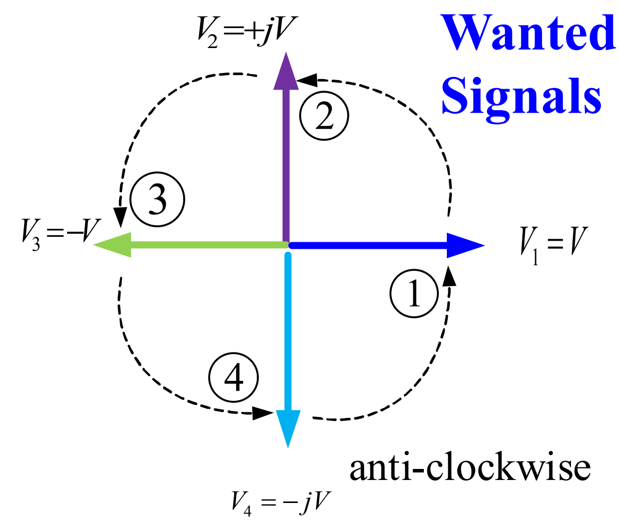


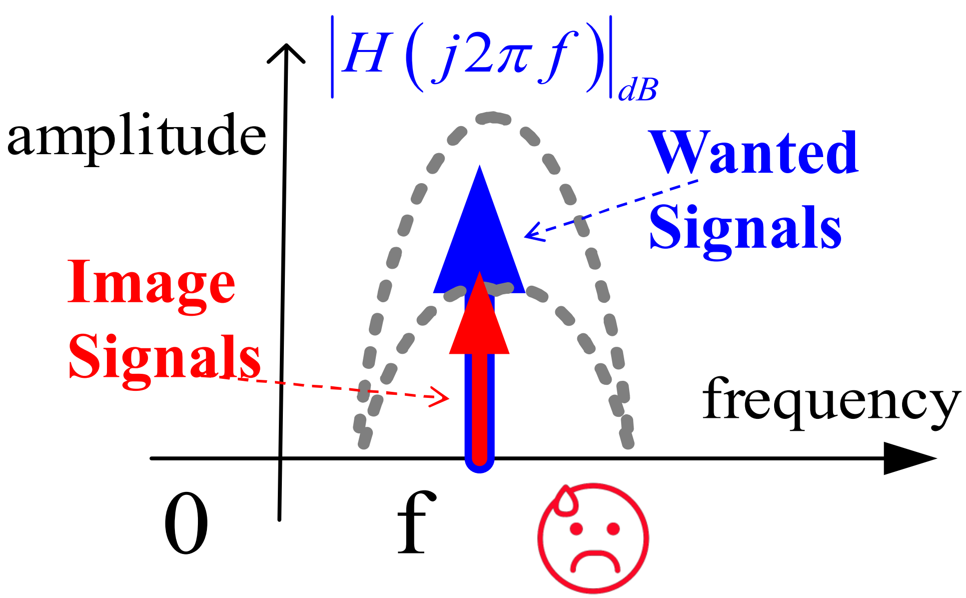


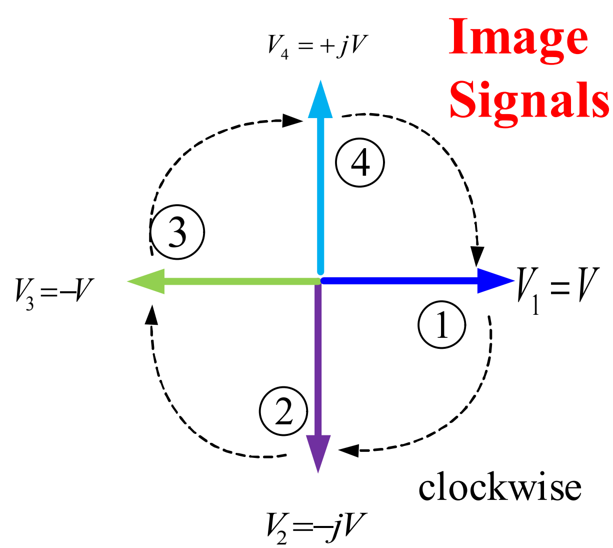
1. Research Background

Frequency Plane of Low-IF Signals







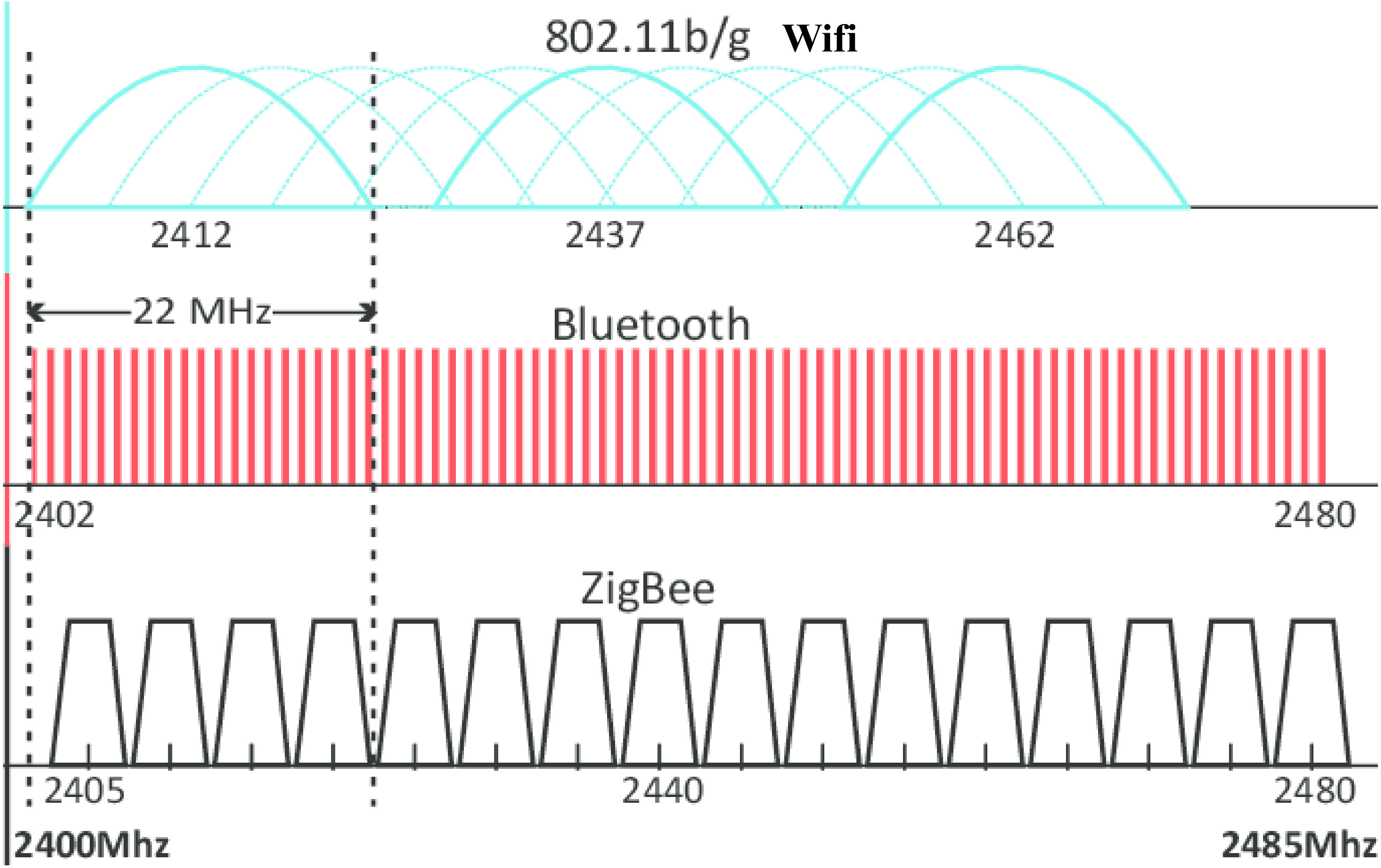


1. Research Background
Wireless Communication Specifications

	Low Energy Bluetooth	Low Power (Wifi) (IEEE 802.11)	ZigBee (IEEE 802.15.4)
Frequency Range	2.402-2.482 GHz	2.40-2.50 GHz	2.402-2.482GHz
Discrete Channels	3	3	16
Max Channel Bandwidth	1~8 MHz	22 MHz	5MHz
Modulation	GFSK	QAM64	QPSK
Nominal Data-Rate	1 Mbps	1 Mbps	250 Kbps
Estimated Max Potential Data-Rate	1 Mbps	54 Mbps	500 Kbps
Nominal Range (0 dBm)	10 m	25 m	75m
Average Power for ten 256-byte massages per day	50uW	570 uW	414uW

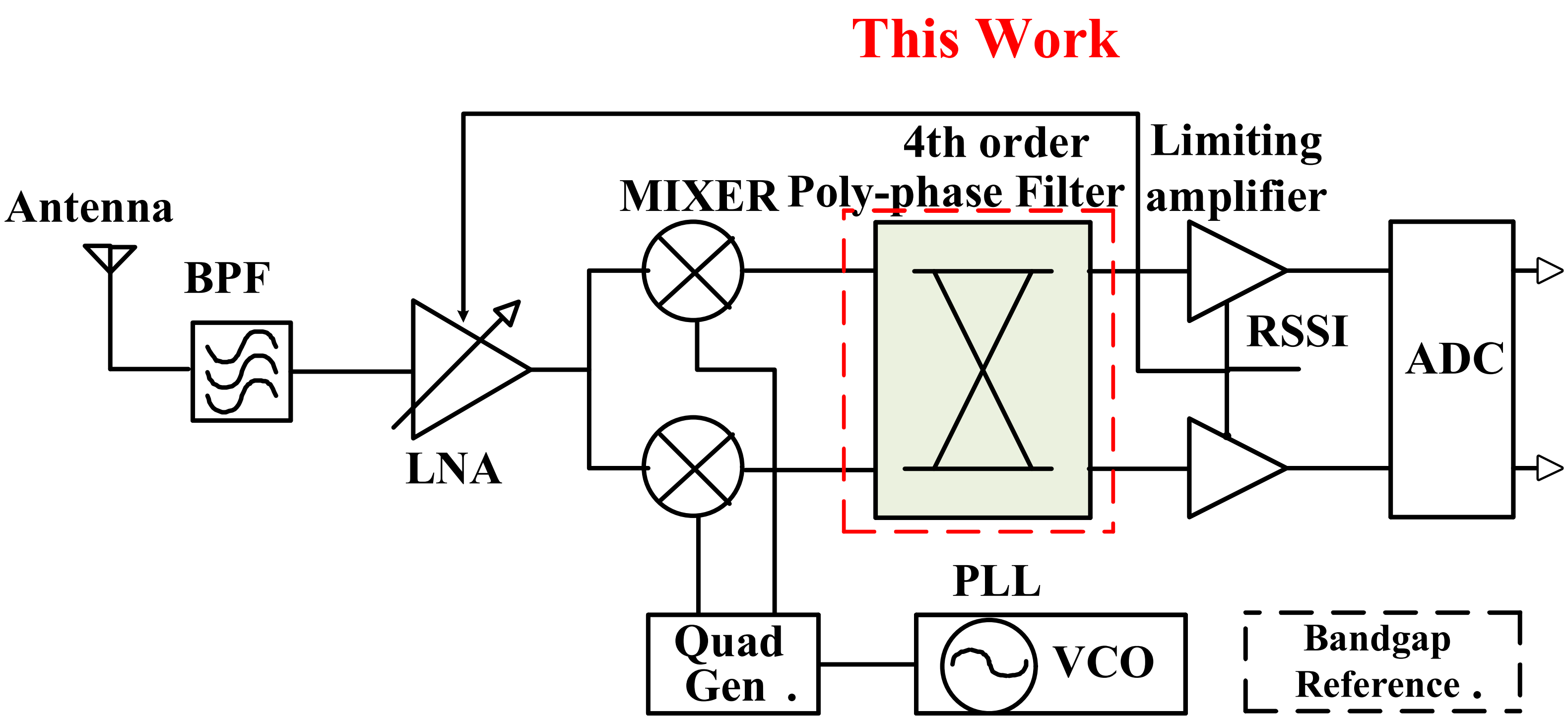
1. Research Background

Signal Bandwidth & Channel Bandwidth



1. Research Background

Low-IF Receiver System Architecture



Merits

- Low-cost
- Small-size
- High-integration

Demerits

- Image Noises
- Power Loss
- Noise Figure

1. Research Background

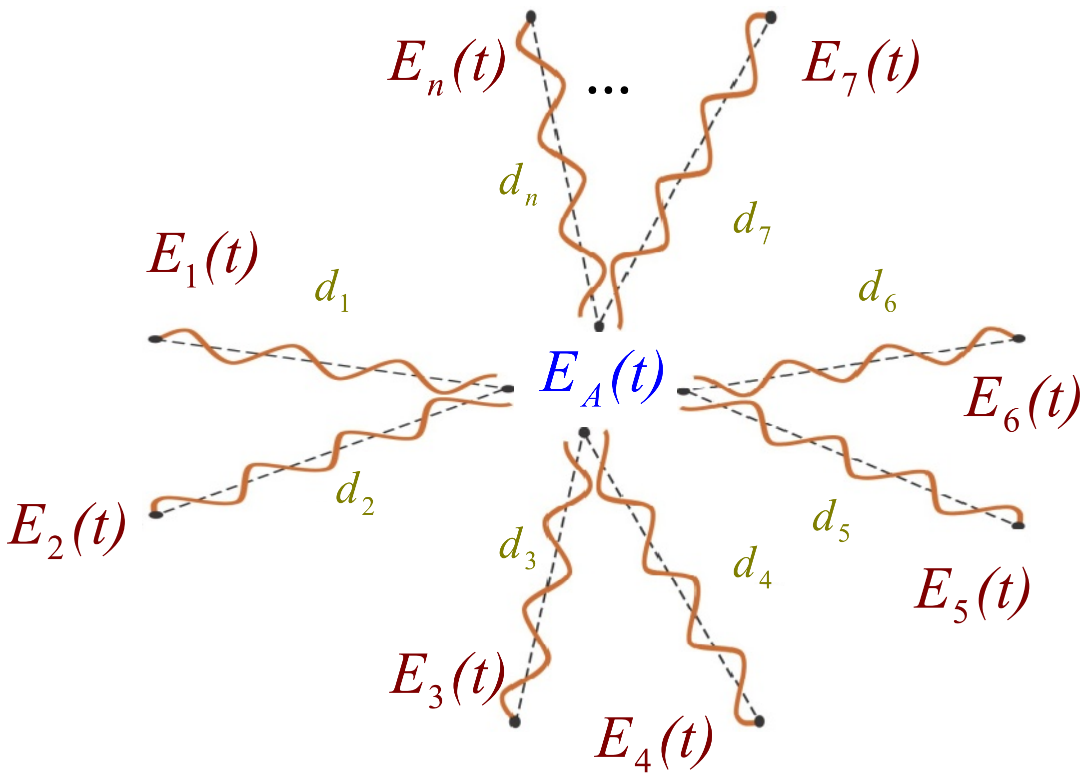
Design Targets of Low-IF Receiver System

Parameter	LNA	Mixer	PPF	RSSI
Input RF frequency	2.4GHz	2.4GHz	5MHz	5MHz
LO frequency		2.405GHz		
Input RF Power	-85 dBm	-68dBm	-60dBm	-60dBm
LO Power		0 dBm		
NF	2dB	10 dB	10dB	10dB
Gain	17 dB	8dB	0dB	60dB
P1dB (input)	>-30dBm	> -10dBm		
Input referent IP3	>-20dBm	> 0 dBm		
Current Consumption	< 5 mA	<10 mA	<5 mA	<5 mA
Supply Voltage	1.8 V	1.8 V	1.8 V	1.8 V
Image Rejection Ratio IMRR			< -30 dB	

1. Research Background

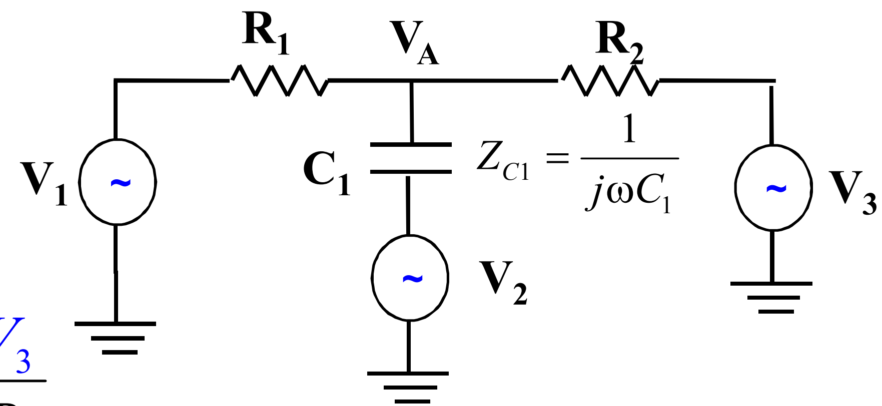
Superposition Principle

$$E_A(t)\sum_{i=1}^{n}\frac{1}{d_i}=\sum_{i=1}^{n}\frac{E_i(t)}{d_i}$$



1. Research Background

Example1 of Superposition Principle



$$V_A\left(\frac{1}{R_1}+\frac{1}{Z_{C1}}+\frac{1}{R_2}\right)=\frac{V_1}{R_1}+\frac{V_2}{Z_{C1}}+\frac{V_3}{R_2}$$

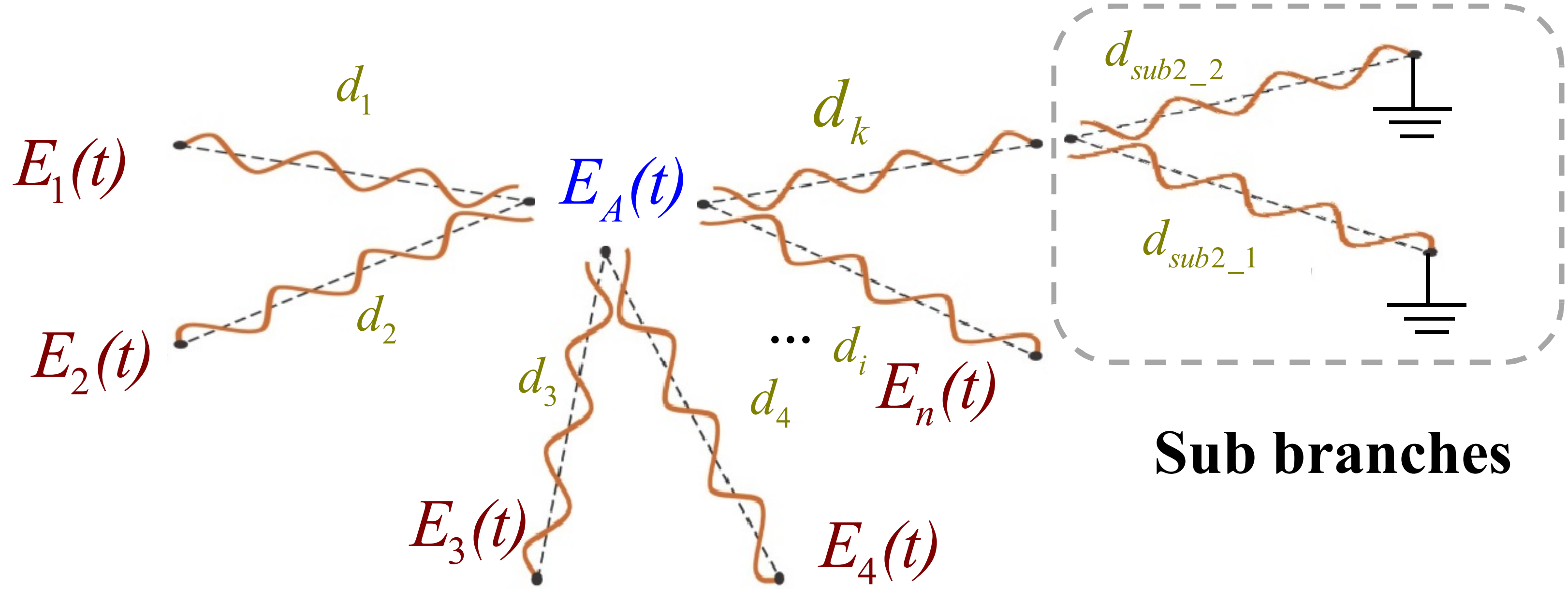
$$V_A\left(\frac{(R_1+Z_{C1})R_2+R_1Z_{C1}}{R_1Z_{C1}R_2}\right)=\frac{V_1}{R_1}+\frac{V_2}{Z_{C1}}+\frac{V_3}{R_2}$$

$$V_A=\frac{V_1Z_{C1}R_2+V_2R_1R_2+V_3R_1Z_{C1}}{(R_1+Z_{C1})R_2+R_1Z_{C1}}$$

$$V_A=\frac{V_1R_2+V_2\,j\omega R_1R_2C_1+V_3R_1}{(j\omega R_1C_1+1)R_2+R_1}$$

1. Research Background

Superposition Principle with Sub Branches

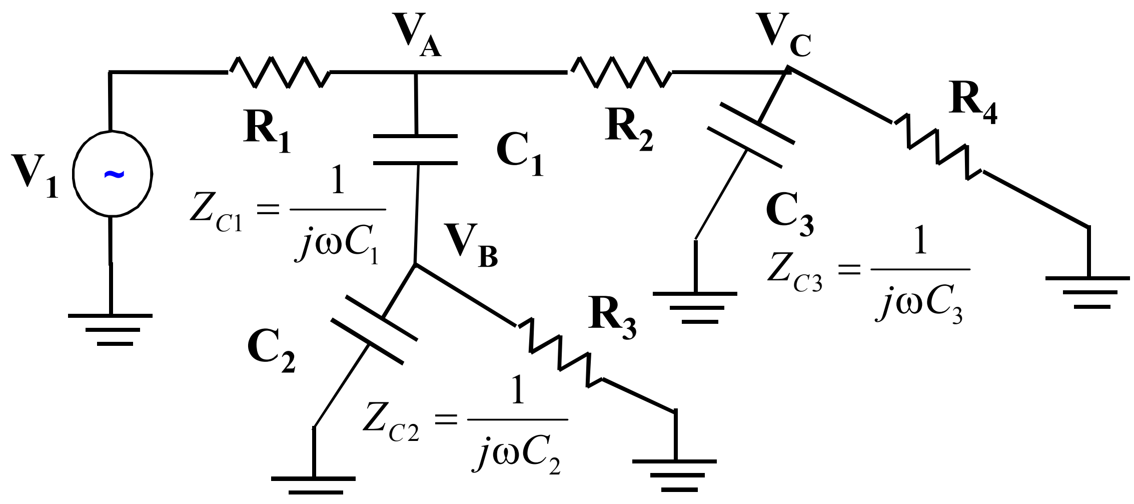


$$E_A(t)\left(\sum_{i=1}^{n}\frac{1}{d_i}+\frac{1}{d_k+\dfrac{1}{\dfrac{1}{d_{k\_sub1}}+\dfrac{1}{d_{k\_sub2}}+\ldots}}\right)=\sum_{i=1}^{n}\frac{E_i(t)}{d_i}$$

1. Research Background

Example2 of Superposition Principle

$$V_B\left(\frac{1}{Z_{C1}}+\frac{1}{Z_{C2}}+\frac{1}{R_3}\right)=\frac{V_A}{Z_{C1}}$$



$$V_C\left(\frac{1}{R_2}+\frac{1}{Z_{C3}}+\frac{1}{R_4}\right)=\frac{V_A}{R_2}$$

$$V_A\left(\frac{1}{R_1}+\frac{1}{Z_{C1}+\dfrac{1}{\dfrac{1}{Z_{C2}}+\dfrac{1}{R_3}}}+\frac{1}{R_2+\dfrac{1}{\dfrac{1}{Z_{C3}}+\dfrac{1}{R_4}}}\right)=\frac{V_1}{R_1}$$

1. Research Background

Energy Propagation

- **Energy Propagation Function:**

$$E(d,t) = E_0 e^{-\alpha d} \cos(\omega t - \beta d + \phi)$$

Wavelength $\lambda = \frac{2\pi}{\beta} [\mathrm{m}]$

- **Lumped circuits: resistors, capacitors, inductors**

  ↖ **neglect time delays (phase)**

- **Distributed circuit elements: transmission lines**

  ↖ **account for time delays (phase change)**

1. Research Background

Some Properties of Hilbert Transforms

Hilbert Transforms Table1:

Name	Function	Positive-Hilbert Transform ($\omega > 0$)	Negative-Hilbert Transform ($\omega < 0$)
Constant	C	0	0
Sine	$\sin(\omega t)$	$-\cos(\omega t)$	$\cos(\omega t)$
Cosine	$\cos(\omega t)$	$\sin(\omega t)$	$-\sin(\omega t)$
Exponential	$e^{j\omega t}$	$-je^{j\omega t}$	$je^{j\omega t}$
	$e^{-j\omega t}$	$je^{-j\omega t}$	$-je^{-j\omega t}$

- **Negative Hilbert transform: -π/2 phase shift (*-j*)**
- **Positive Hilbert transform: +π/2 phase shift (*+j*)**

1. Research Background

Complex Signals

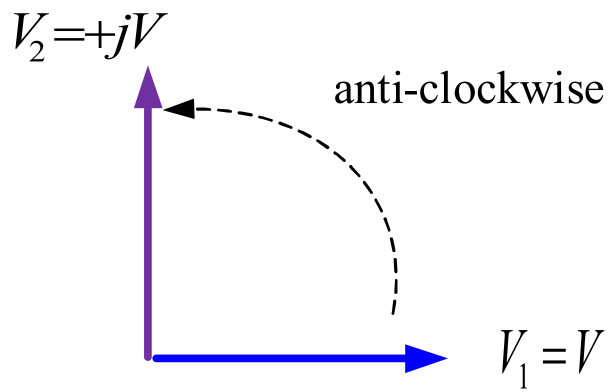
- **Complex signal:** **a set of two (or four) sources with the same frequency and 90-degree separated phase**

Positive Complex Signal

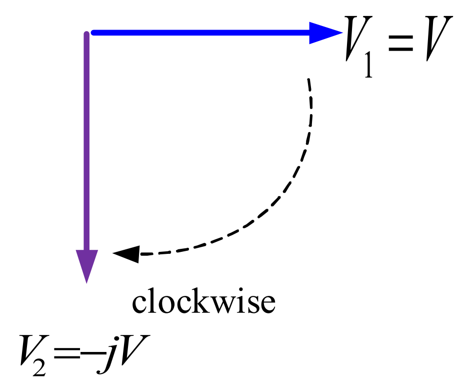
$$S_{Pos\_complex}\{V_1(t);V_2(t)\}=\begin{cases}A\cos(\omega t+\theta)\\+jA\cos(\omega t+\theta)\end{cases}$$

Negative Complex Signal

$$S_{Neg\_complex}\{V_1(t);V_2(t)\}=\begin{cases}A\cos(\omega t+\theta)\\-jA\cos(\omega t+\theta)\end{cases}$$



Positive Hilbert transform



Negative Hilbert transform

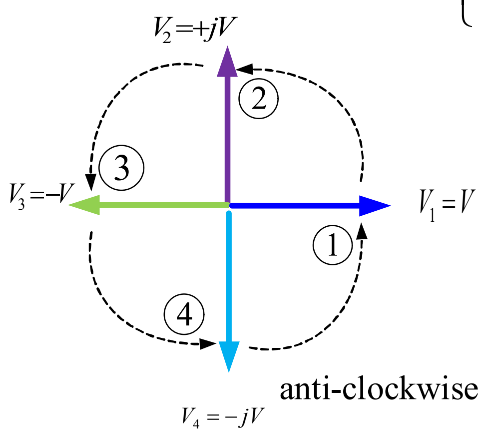
1. Research Background

Poly-phase Signals

- **Poly-phase signal = Complex signal (four sources)**
- **Poly-phase filter = Complex filter**

Positive Poly-phase Signals

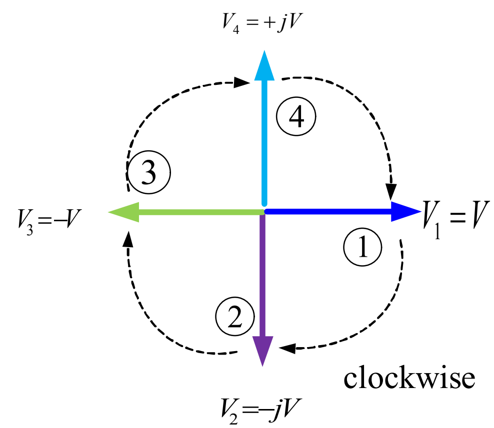
$$S_{Pos\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\}=\begin{cases}A\cos(\omega t+\theta)\\-A\sin(\omega t+\theta)\\-A\cos(\omega t+\theta)\\+A\sin(\omega t+\theta)\end{cases}$$



Positive Hilbert transform

Negative Poly-phase Signals

$$S_{Neg\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\}=\begin{cases}A\cos(\omega t+\theta)\\A\sin(\omega t+\theta)\\-A\cos(\omega t+\theta)\\-A\sin(\omega t+\theta)\end{cases}$$



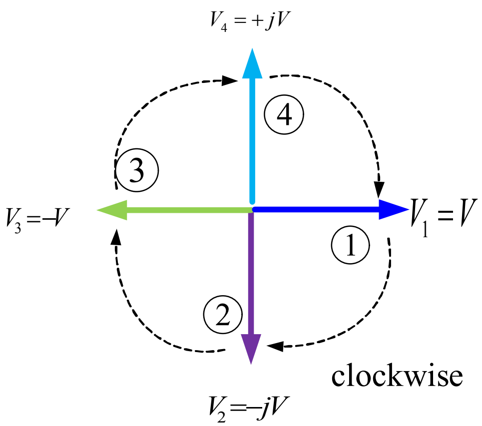
Negative Hilbert transform

1. Research Background

Phase Plane of Poly-phase Signals

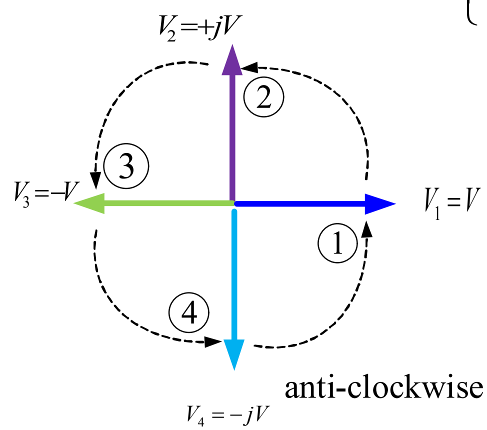
Negative Poly-phase Signals

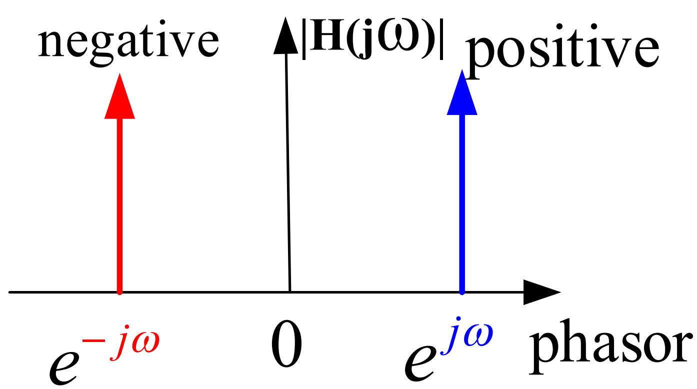
$$S_{Neg\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\}=\begin{cases}A\cos(\omega t+\theta)\\A\sin(\omega t+\theta)\\-A\cos(\omega t+\theta)\\-A\sin(\omega t+\theta)\end{cases}$$



Positive Poly-phase Signals

$$S_{Pos\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\}=\begin{cases}A\cos(\omega t+\theta)\\-A\sin(\omega t+\theta)\\-A\cos(\omega t+\theta)\\+A\sin(\omega t+\theta)\end{cases}$$





1. Research Background

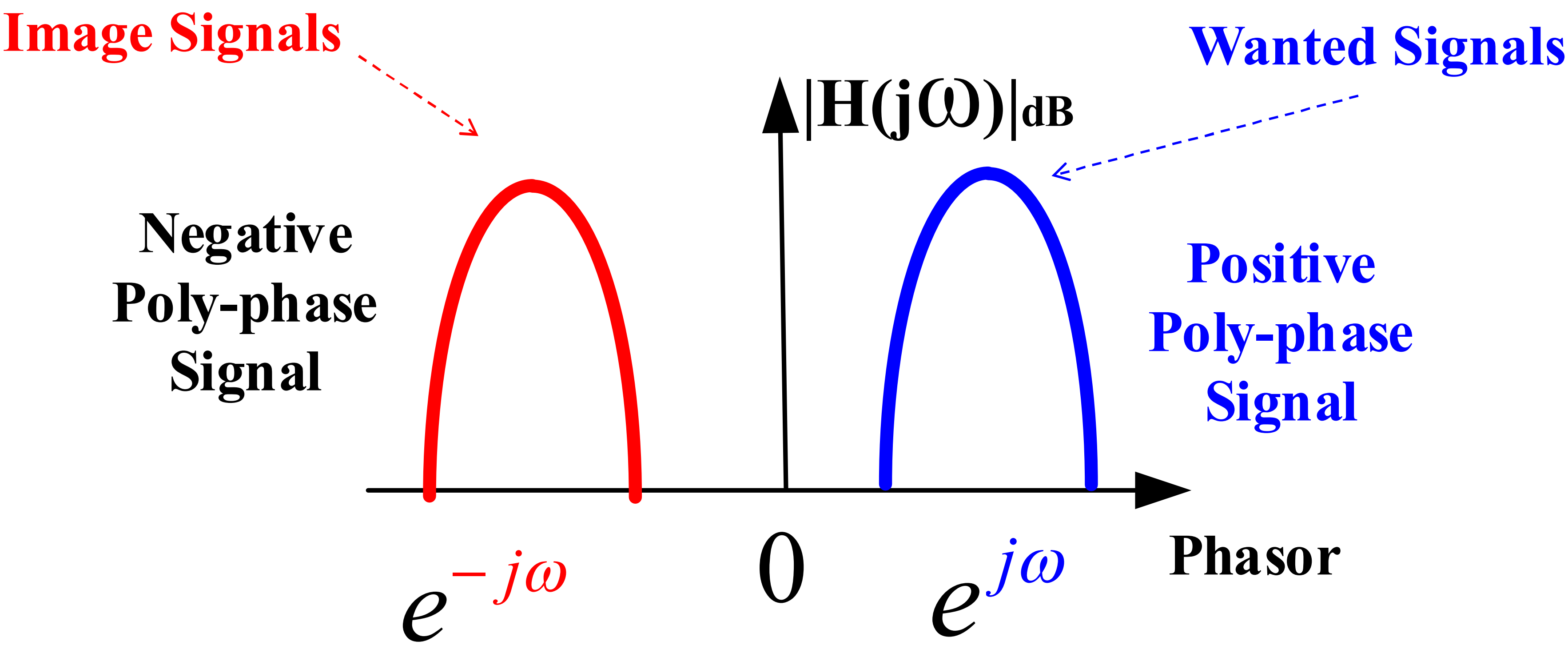
Phase Plane of Received Signals

Negative Poly-phase Signals

$$S_{Neg\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\} = \{1;-j;-1;+j\}V_{neg}(t)$$

Positive Poly-phase Signals

$$S_{Pos\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\} = \{1;j;-1;-j\}V_{pos}(t)$$



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2. **Analysis of 4-Stage Passive RC Network**
   - **Frequency Responses**
   - **Image Rejection Ratio**
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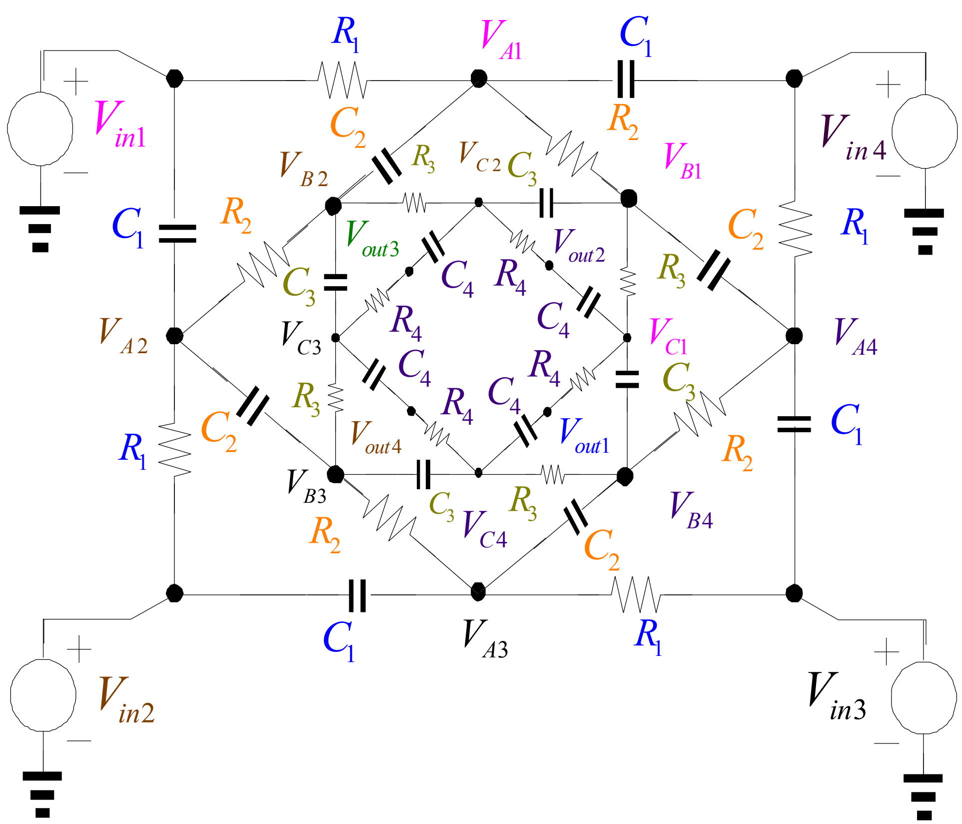
2. Analysis of 4-Stage Passive RC Network

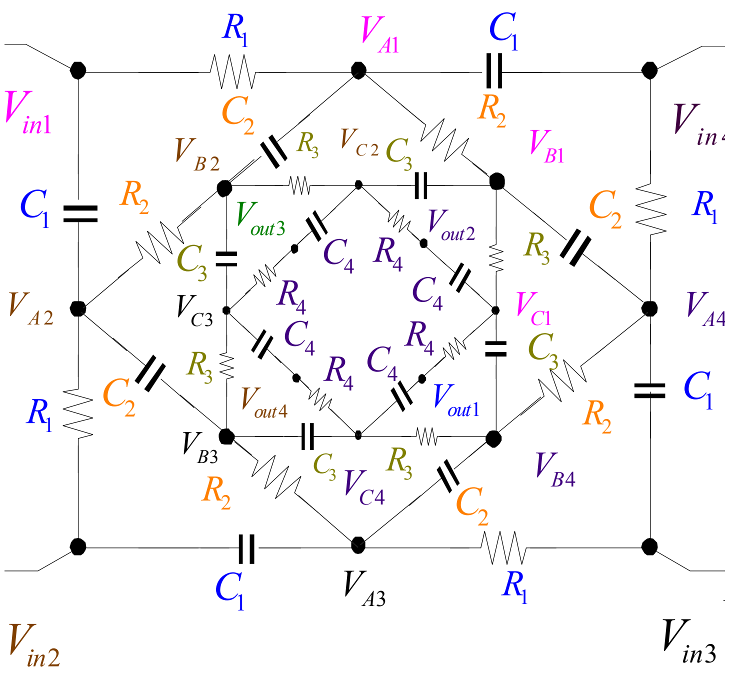
Design of 4-Stage Passive RC Network



Parameter	PPF
Input IF frequency	5MHz
Input IF Power	-60dBm
NF	10dB
Gain	0dB
Current Consumption	<5mA
Supply Voltage	1.8V
Image Rejection Ratio IMRR	< -30 dB

2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages on 1st Loop



$$\begin{cases} V_{A1}\, Y_{total\_A} = \dfrac{V_{in1}}{R_1} + \dfrac{V_{in4}}{Z_{C1}} \\ V_{A2}\, Y_{total\_A} = \dfrac{V_{in2}}{R_1} + \dfrac{V_{in1}}{Z_{C1}} \\ V_{A3}\, Y_{total\_A} = \dfrac{V_{in3}}{R_1} + \dfrac{V_{in2}}{Z_{C1}} \\ V_{A4}\, Y_{total\_A} = \dfrac{V_{in4}}{R_1} + \dfrac{V_{in3}}{Z_{C1}} \end{cases}$$

Here:

$$Y_{total\_A} = \frac{1}{R_1} + \frac{1}{Z_{C1}} + \frac{1}{R_2 + \frac{1}{\frac{1}{Z_{C2}} + \frac{1}{R_3 + \frac{1}{\frac{1}{Z_{C3}} + \frac{2}{Z_{C4}+R_4}}} + \frac{1}{Z_{C3} + \frac{1}{\frac{1}{R_3} + \frac{2}{Z_{C4}+R_4}}}}} + \frac{1}{Z_{C2} + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3 + \frac{1}{\frac{1}{Z_{C3}} + \frac{2}{Z_{C4}+R_4}}} + \frac{1}{Z_{C3} + \frac{1}{\frac{1}{R_3} + \frac{2}{Z_{C4}+R_4}}}}}$$

$$= \frac{\left[2R_1Z_{C1} + (R_1+Z_{C1})(R_2+Z_{C2})\right]\left[2R_3Z_{C3} + (R_3+Z_{C3})(R_4+Z_{C4})\right] + 2(R_3+Z_{C3}+R_4+Z_{C4})\left[(R_1+Z_{C1})R_2Z_{C2} + R_1Z_{C1}(R_2+Z_{C2})\right]}{R_1Z_{C1}\left\{(R_2+Z_{C2})\left[2R_3Z_{C3} + (R_3+Z_{C3})(R_4+Z_{C4})\right] + 2R_2Z_{C2}(R_3+Z_{C3}+R_4+Z_{C4})\right\}}$$

2. Analysis of 4-Stage Passive RC Network

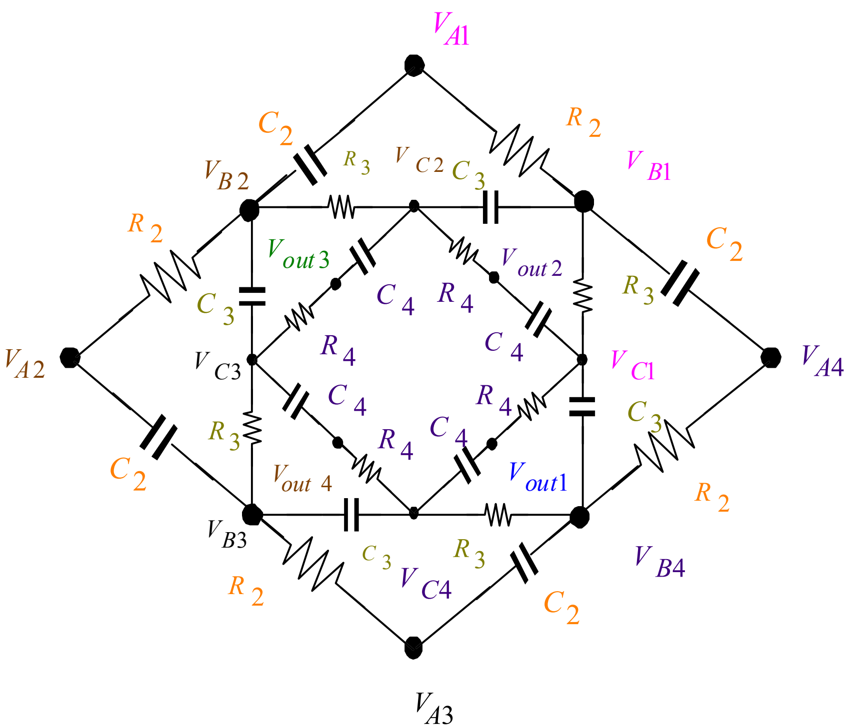
Derivation of Output Voltages on 1st Loop

Voltages on 1st loop

$$
\begin{cases}
V_{A1} = V_{in1}\left(Z_{C1} + \dfrac{V_{in4}}{V_{in1}} R_1\right) \dfrac{(R_2+Z_{C2})\left[2R_3Z_{C3}+(R_3+Z_{C3})(R_4+Z_{C4})\right]+2R_2Z_{C2}(R_3+Z_{C3}+R_4+Z_{C4})}{\left\{\begin{array}{l}\left[2R_1Z_{C1}+(R_1+Z_{C1})(R_2+Z_{C2})\right]\left[2R_3Z_{C3}+(R_3+Z_{C3})(R_4+Z_{C4})\right] \\ +2(R_3+Z_{C3}+R_4+Z_{C4})\left[(R_1+Z_{C1})R_2Z_{C2}+R_1Z_{C1}(R_2+Z_{C2})\right]\end{array}\right\}} \\
V_{A2} = V_{in2}\left(Z_{C1} + \dfrac{V_{in1}}{V_{in2}} R_1\right) \dfrac{(R_2+Z_{C2})\left[2R_3Z_{C3}+(R_3+Z_{C3})(R_4+Z_{C4})\right]+2R_2Z_{C2}(R_3+Z_{C3}+R_4+Z_{C4})}{\left\{\begin{array}{l}\left[2R_1Z_{C1}+(R_1+Z_{C1})(R_2+Z_{C2})\right]\left[2R_3Z_{C3}+(R_3+Z_{C3})(R_4+Z_{C4})\right] \\ +2(R_3+Z_{C3}+R_4+Z_{C4})\left[(R_1+Z_{C1})R_2Z_{C2}+R_1Z_{C1}(R_2+Z_{C2})\right]\end{array}\right\}} \\
V_{A3} = V_{in3}\left(Z_{C1} + \dfrac{V_{in2}}{V_{in3}} R_1\right) \dfrac{(R_2+Z_{C2})\left[2R_3Z_{C3}+(R_3+Z_{C3})(R_4+Z_{C4})\right]+2R_2Z_{C2}(R_3+Z_{C3}+R_4+Z_{C4})}{\left\{\begin{array}{l}\left[2R_1Z_{C1}+(R_1+Z_{C1})(R_2+Z_{C2})\right]\left[2R_3Z_{C3}+(R_3+Z_{C3})(R_4+Z_{C4})\right] \\ +2(R_3+Z_{C3}+R_4+Z_{C4})\left[(R_1+Z_{C1})R_2Z_{C2}+R_1Z_{C1}(R_2+Z_{C2})\right]\end{array}\right\}} \\
V_{A4} = V_{in4}\left(Z_{C1} + \dfrac{V_{in3}}{V_{in4}} R_1\right) \dfrac{(R_2+Z_{C2})\left[2R_3Z_{C3}+(R_3+Z_{C3})(R_4+Z_{C4})\right]+2R_2Z_{C2}(R_3+Z_{C3}+R_4+Z_{C4})}{\left\{\begin{array}{l}\left[2R_1Z_{C1}+(R_1+Z_{C1})(R_2+Z_{C2})\right]\left[2R_3Z_{C3}+(R_3+Z_{C3})(R_4+Z_{C4})\right] \\ +2(R_3+Z_{C3}+R_4+Z_{C4})\left[(R_1+Z_{C1})R_2Z_{C2}+R_1Z_{C1}(R_2+Z_{C2})\right]\end{array}\right\}}
\end{cases}
$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages on 2nd Loop



$$
\begin{cases}
V_{B1}\,Y_{total\_B} = \dfrac{V_{A1}}{R_2} + \dfrac{V_{A4}}{Z_{C2}} \\[2ex]
V_{B2}\,Y_{total\_B} = \dfrac{V_{A2}}{R_2} + \dfrac{V_{A1}}{Z_{C2}} \\[2ex]
V_{B3}\,Y_{total\_B} = \dfrac{V_{A3}}{R_2} + \dfrac{V_{A2}}{Z_{C2}} \\[2ex]
V_{B4}\,Y_{total\_B} = \dfrac{V_{A4}}{R_2} + \dfrac{V_{A3}}{Z_{C2}}
\end{cases}
$$

Here:

$$
Y_{total\_B} = \frac{1}{R_2} + \frac{1}{Z_{C2}} + \frac{1}{R_3 + \dfrac{1}{\dfrac{1}{Z_{C3}} + \dfrac{2}{Z_{C4} + R_4}}} + \frac{1}{Z_{C3} + \dfrac{1}{\dfrac{1}{R_3} + \dfrac{2}{Z_{C4} + R_4}}}
$$

$$
= \frac{(R_2 + Z_{C2})\left[2R_3Z_{C3} + (R_3 + Z_{C3})(R_4 + Z_{C4})\right] + 2R_2Z_{C2}(R_3 + Z_{C3} + R_4 + Z_{C4})}{R_2Z_{C2}\left[2R_3Z_{C3} + (R_3 + Z_{C3})(R_4 + Z_{C4})\right]}
$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages on 2nd Loop

Voltages on 2nd loop

$$\begin{cases}
V_{B1} = \left(V_{A1}Z_{C2} + V_{A4}R_2\right)\dfrac{\left[2R_3Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right]}{\left\{\begin{array}{l}\left(R_2 + Z_{C2}\right)\left[2R_3Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right] \\ +2R_2Z_{C2}\left(R_3 + Z_{C3} + R_4 + Z_{C4}\right)\end{array}\right\}} \\
V_{B2} = \left(V_{A2}Z_{C2} + V_{A1}R_2\right)\dfrac{\left[2R_3Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right]}{\left\{\begin{array}{l}\left(R_2 + Z_{C2}\right)\left[2R_3Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right] \\ +2R_2Z_{C2}\left(R_3 + Z_{C3} + R_4 + Z_{C4}\right)\end{array}\right\}} \\
V_{B3} = \left(V_{A3}Z_{C2} + V_{A2}R_2\right)\dfrac{\left[2R_3Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right]}{\left\{\begin{array}{l}\left(R_2 + Z_{C2}\right)\left[2R_3Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right] \\ +2R_2Z_{C2}\left(R_3 + Z_{C3} + R_4 + Z_{C4}\right)\end{array}\right\}} \\
V_{B4} = \left(V_{A4}Z_{C2} + V_{A3}R_2\right)\dfrac{\left[2R_3Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right]}{\left\{\begin{array}{l}\left(R_2 + Z_{C2}\right)\left[2R_3Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right] \\ +2R_2Z_{C2}\left(R_3 + Z_{C3} + R_4 + Z_{C4}\right)\end{array}\right\}}
\end{cases}$$

2. Analysis of 4-Stage Passive RC Network

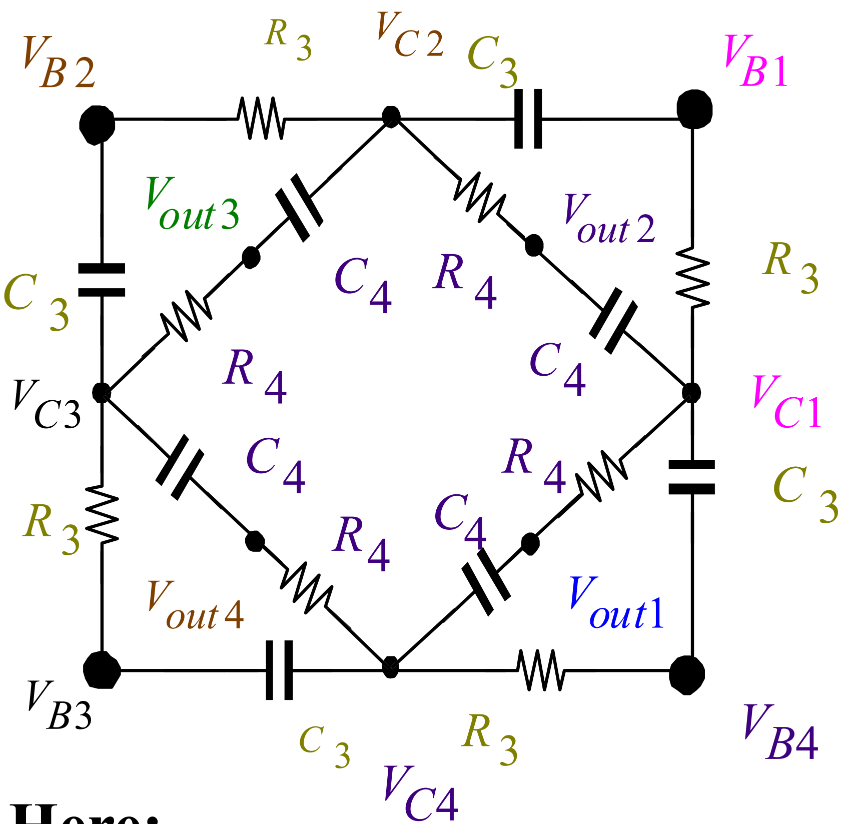
Derivation of Output Voltages on 2nd Loop

Voltages on 2nd loop

$$
\begin{cases}
V_{B1} = V_{in1} \dfrac{\left[\left(Z_{C1} + \dfrac{V_{in4}}{V_{in1}} R_1\right) Z_{C2} + \dfrac{V_{in4}}{V_{in1}}\left(Z_{C1} + \dfrac{V_{in3}}{V_{in4}} R_1\right) R_2\right]\left[2R_3 Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right]}{\left\{\begin{array}{l}\left[2R_1 Z_{C1} + \left(R_1 + Z_{C1}\right)\left(R_2 + Z_{C2}\right)\right]\left[2R_3 Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right] \\ +2\left(R_3 + Z_{C3} + R_4 + Z_{C4}\right)\left[\left(R_1 + Z_{C1}\right) R_2 Z_{C2} + R_1 Z_{C1}\left(R_2 + Z_{C2}\right)\right]\end{array}\right\}} \\
V_{B2} = V_{in2} \dfrac{\left[\left(Z_{C1} + \dfrac{V_{in1}}{V_{in2}} R_1\right) Z_{C2} + \dfrac{V_{in1}}{V_{in2}}\left(Z_{C1} + \dfrac{V_{in4}}{V_{in1}} R_1\right) R_2\right]\left[2R_3 Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right]}{\left\{\begin{array}{l}\left[2R_1 Z_{C1} + \left(R_1 + Z_{C1}\right)\left(R_2 + Z_{C2}\right)\right]\left[2R_3 Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right] \\ +2\left(R_3 + Z_{C3} + R_4 + Z_{C4}\right)\left[\left(R_1 + Z_{C1}\right) R_2 Z_{C2} + R_1 Z_{C1}\left(R_2 + Z_{C2}\right)\right]\end{array}\right\}} \\
V_{B3} = V_{in3} \dfrac{\left[\left(Z_{C1} + \dfrac{V_{in2}}{V_{in3}} R_1\right) Z_{C2} + \dfrac{V_{in2}}{V_{in3}}\left(Z_{C1} + \dfrac{V_{in1}}{V_{in2}} R_1\right) R_2\right]\left[2R_3 Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right]}{\left\{\begin{array}{l}\left[2R_1 Z_{C1} + \left(R_1 + Z_{C1}\right)\left(R_2 + Z_{C2}\right)\right]\left[2R_3 Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right] \\ +2\left(R_3 + Z_{C3} + R_4 + Z_{C4}\right)\left[\left(R_1 + Z_{C1}\right) R_2 Z_{C2} + R_1 Z_{C1}\left(R_2 + Z_{C2}\right)\right]\end{array}\right\}} \\
V_{B4} = V_{in4} \dfrac{\left[\left(Z_{C1} + \dfrac{V_{in3}}{V_{in4}} R_1\right) Z_{C2} + \dfrac{V_{in3}}{V_{in4}}\left(Z_{C1} + \dfrac{V_{in2}}{V_{in3}} R_1\right) R_2\right]\left[2R_3 Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right]}{\left\{\begin{array}{l}\left[2R_1 Z_{C1} + \left(R_1 + Z_{C1}\right)\left(R_2 + Z_{C2}\right)\right]\left[2R_3 Z_{C3} + \left(R_3 + Z_{C3}\right)\left(R_4 + Z_{C4}\right)\right] \\ +2\left(R_3 + Z_{C3} + R_4 + Z_{C4}\right)\left[\left(R_1 + Z_{C1}\right) R_2 Z_{C2} + R_1 Z_{C1}\left(R_2 + Z_{C2}\right)\right]\end{array}\right\}}
\end{cases}
$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages on 3rd Loop



$$
\begin{cases}
V_{C1} Y_{total\_C} = \dfrac{V_{B1}}{R_3} + \dfrac{V_{B4}}{Z_{C3}} \\
V_{C2} Y_{total\_C} = \dfrac{V_{B2}}{R_3} + \dfrac{V_{B1}}{Z_{C3}} \\
V_{C3} Y_{total\_C} = \dfrac{V_{B3}}{R_3} + \dfrac{V_{B2}}{Z_{C3}} \\
V_{C4} Y_{total\_C} = \dfrac{V_{B4}}{R_3} + \dfrac{V_{B3}}{Z_{C3}}
\end{cases}
$$

Here:

$$
Y_{total\_C} = \frac{1}{R_3} + \frac{1}{Z_{C3}} + \frac{2}{Z_{C4} + R_4} = \frac{(Z_{C3} + R_3)(Z_{C4} + R_4) + 2R_3 Z_{C3}}{R_3 Z_{C3} (Z_{C4} + R_4)}
$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages on 3rd Loop

Voltages on 3rd loop

$$
\begin{cases}
V_{C1} = \left(V_{B1}Z_{C3} + V_{B4}R_3\right)\dfrac{\left(Z_{C4} + R_4\right)}{\left(Z_{C3} + R_3\right)\left(Z_{C4} + R_4\right) + 2R_3 Z_{C3}} \\
V_{C2} = \left(V_{B2}Z_{C3} + V_{B1}R_3\right)\dfrac{\left(Z_{C4} + R_4\right)}{\left(Z_{C3} + R_3\right)\left(Z_{C4} + R_4\right) + 2R_3 Z_{C3}} \\
V_{C3} = \left(V_{B3}Z_{C3} + V_{B2}R_3\right)\dfrac{\left(Z_{C4} + R_4\right)}{\left(Z_{C3} + R_3\right)\left(Z_{C4} + R_4\right) + 2R_3 Z_{C3}} \\
V_{C4} = \left(V_{B4}Z_{C3} + V_{B3}R_3\right)\dfrac{\left(Z_{C4} + R_4\right)}{\left(Z_{C3} + R_3\right)\left(Z_{C4} + R_4\right) + 2R_3 Z_{C3}}
\end{cases}
$$

2. Analysis of 4-Stage Passive RC Network

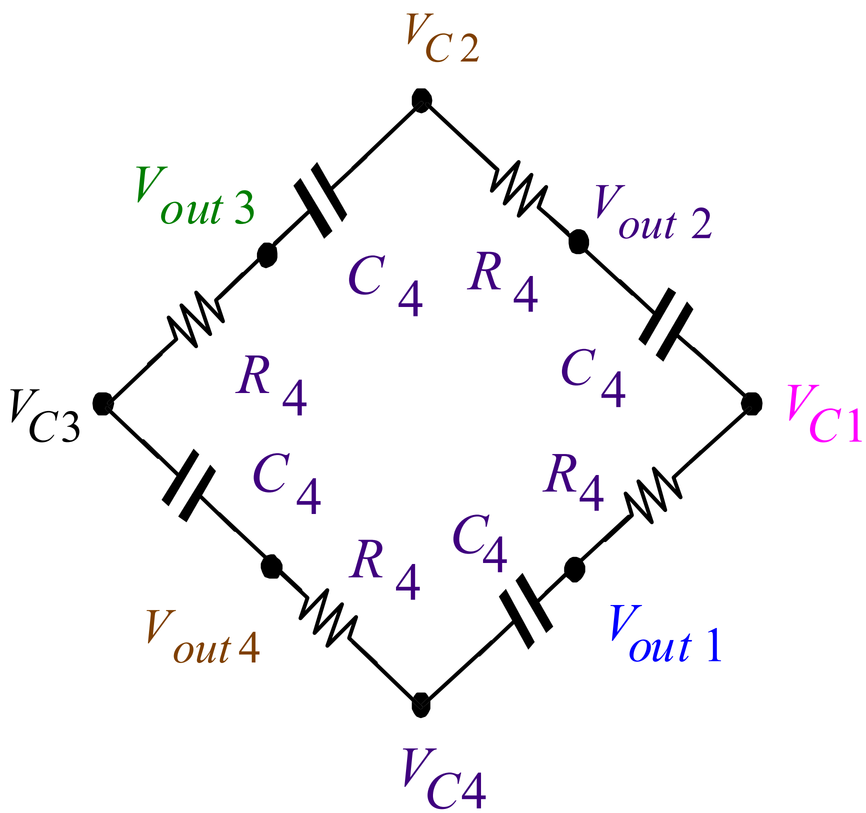
Derivation of Output Voltages on 3rd Loop

Voltages on 3rd loop

$$
\begin{cases}
V_{C1} = V_{in1}\dfrac{\left\{\left[\left(Z_{C1}+\dfrac{V_{in4}}{V_{in1}}R_1\right)Z_{C2}+\dfrac{V_{in4}}{V_{in1}}\left(Z_{C1}+\dfrac{V_{in3}}{V_{in4}}R_1\right)R_2\right]Z_{C3}+\dfrac{V_{in4}}{V_{in1}}\left[\left(Z_{C1}+\dfrac{V_{in3}}{V_{in4}}R_1\right)Z_{C2}+\dfrac{V_{in3}}{V_{in4}}\left(Z_{C1}+\dfrac{V_{in2}}{V_{in3}}R_1\right)R_2\right]R_3\right\}\left(Z_{C4}+R_4\right)}{\left\{\begin{array}{l}\left[2R_1Z_{C1}+\left(R_1+Z_{C1}\right)\left(R_2+Z_{C2}\right)\right]\left[2R_3Z_{C3}+\left(R_3+Z_{C3}\right)\left(R_4+Z_{C4}\right)\right]\\+2\left(R_3+Z_{C3}+R_4+Z_{C4}\right)\left[\left(R_1+Z_{C1}\right)R_2Z_{C2}+R_1Z_{C1}\left(R_2+Z_{C2}\right)\right]\end{array}\right\}} \\[2ex]
V_{C2} = V_{in2}\dfrac{\left\{\left[\left(Z_{C1}+\dfrac{V_{in1}}{V_{in2}}R_1\right)Z_{C2}+\dfrac{V_{in1}}{V_{in2}}\left(Z_{C1}+\dfrac{V_{in4}}{V_{in1}}R_1\right)R_2\right]Z_{C3}+\dfrac{V_{in1}}{V_{in2}}\left[\left(Z_{C1}+\dfrac{V_{in4}}{V_{in1}}R_1\right)Z_{C2}+\dfrac{V_{in4}}{V_{in1}}\left(Z_{C1}+\dfrac{V_{in3}}{V_{in4}}R_1\right)R_2\right]R_3\right\}\left(Z_{C4}+R_4\right)}{\left\{\begin{array}{l}\left[2R_1Z_{C1}+\left(R_1+Z_{C1}\right)\left(R_2+Z_{C2}\right)\right]\left[2R_3Z_{C3}+\left(R_3+Z_{C3}\right)\left(R_4+Z_{C4}\right)\right]\\+2\left(R_3+Z_{C3}+R_4+Z_{C4}\right)\left[\left(R_1+Z_{C1}\right)R_2Z_{C2}+R_1Z_{C1}\left(R_2+Z_{C2}\right)\right]\end{array}\right\}} \\[2ex]
V_{C3} = V_{in3}\dfrac{\left\{\left[\left(Z_{C1}+\dfrac{V_{in2}}{V_{in3}}R_1\right)Z_{C2}+\dfrac{V_{in2}}{V_{in3}}\left(Z_{C1}+\dfrac{V_{in1}}{V_{in2}}R_1\right)R_2\right]Z_{C3}+\dfrac{V_{in2}}{V_{in3}}\left[\left(Z_{C1}+\dfrac{V_{in1}}{V_{in2}}R_1\right)Z_{C2}+\dfrac{V_{in1}}{V_{in2}}\left(Z_{C1}+\dfrac{V_{in4}}{V_{in1}}R_1\right)R_2\right]R_3\right\}\left(Z_{C4}+R_4\right)}{\left\{\begin{array}{l}\left[2R_1Z_{C1}+\left(R_1+Z_{C1}\right)\left(R_2+Z_{C2}\right)\right]\left[2R_3Z_{C3}+\left(R_3+Z_{C3}\right)\left(R_4+Z_{C4}\right)\right]\\+2\left(R_3+Z_{C3}+R_4+Z_{C4}\right)\left[\left(R_1+Z_{C1}\right)R_2Z_{C2}+R_1Z_{C1}\left(R_2+Z_{C2}\right)\right]\end{array}\right\}} \\[2ex]
V_{C4} = V_{in4}\dfrac{\left\{\left[\left(Z_{C1}+\dfrac{V_{in3}}{V_{in4}}R_1\right)Z_{C2}+\dfrac{V_{in3}}{V_{in4}}\left(Z_{C1}+\dfrac{V_{in2}}{V_{in3}}R_1\right)R_2\right]Z_{C3}+\dfrac{V_{in3}}{V_{in4}}\left[\left(Z_{C1}+\dfrac{V_{in2}}{V_{in3}}R_1\right)Z_{C2}+\dfrac{V_{in2}}{V_{in3}}\left(Z_{C1}+\dfrac{V_{in1}}{V_{in2}}R_1\right)R_2\right]R_3\right\}\left(Z_{C4}+R_4\right)}{\left\{\begin{array}{l}\left[2R_1Z_{C1}+\left(R_1+Z_{C1}\right)\left(R_2+Z_{C2}\right)\right]\left[2R_3Z_{C3}+\left(R_3+Z_{C3}\right)\left(R_4+Z_{C4}\right)\right]\\+2\left(R_3+Z_{C3}+R_4+Z_{C4}\right)\left[\left(R_1+Z_{C1}\right)R_2Z_{C2}+R_1Z_{C1}\left(R_2+Z_{C2}\right)\right]\end{array}\right\}}
\end{cases}
$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages on 4th Loop



$$\begin{cases} V_{out1} Y_{total\_D} = \dfrac{V_{C1}}{R_4} + \dfrac{V_{C4}}{Z_{C4}} \\ V_{out2} Y_{total\_D} = \dfrac{V_{C2}}{R_4} + \dfrac{V_{C1}}{Z_{C4}} \\ V_{out3} Y_{total\_D} = \dfrac{V_{C3}}{R_4} + \dfrac{V_{C2}}{Z_{C4}} \\ V_{out4} Y_{total\_D} = \dfrac{V_{C4}}{R_4} + \dfrac{V_{C3}}{Z_{C4}} \end{cases}$$

Here:

$$Y_{total\_D} = \frac{1}{R_4} + \frac{1}{Z_{C4}} = \frac{R_4 + Z_{C4}}{R_4 Z_{C4}}$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages on 4th Loop

Voltages on 4th loop

$$
\begin{cases}
V_{out1} = \left(V_{C1}Z_{C4} + V_{C4}R_4\right)\dfrac{1}{R_4 + Z_{C4}} \\
V_{out2} = \left(V_{C2}Z_{C4} + V_{C1}R_4\right)\dfrac{1}{R_4 + Z_{C4}} \\
V_{out3} = \left(V_{C3}Z_{C4} + V_{C2}R_4\right)\dfrac{1}{R_4 + Z_{C4}} \\
V_{out4} = \left(V_{C4}Z_{C4} + V_{C3}R_4\right)\dfrac{1}{R_4 + Z_{C4}}
\end{cases}
$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages on 4th Loop

Voltages on 4th loop

$$V_{out1}=\frac{V_{in1}\left(Z_{C4}\left(Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]+\left(\frac{V_{in4}}{V_{in1}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]\right\}+\left(\frac{V_{in4}}{V_{in1}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]+\left(\frac{V_{in3}}{V_{in4}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]\right\}\right)+\left(\frac{V_{in4}}{V_{in1}}\right)R_4\left(Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]+\left(\frac{V_{in3}}{V_{in4}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]\right\}+\left(\frac{V_{in3}}{V_{in4}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]+\left(\frac{V_{in2}}{V_{in3}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]\right\}\right)\right)}{\left[2R_1Z_{C1}+(Z_{C2}+R_2)(Z_{C1}+R_1)\right]\left[2R_3Z_{C3}+(Z_{C3}+R_3)(Z_{C4}+R_4)\right]+2(Z_{C3}+R_3+Z_{C4}+R_4)\left[(Z_{C1}+R_1)R_2Z_{C2}+(Z_{C2}+R_2)R_1Z_{C1}\right]}$$

$$V_{out2}=\frac{V_{in2}\left(Z_{C4}\left(Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]+\left(\frac{V_{in1}}{V_{in2}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]\right\}+\left(\frac{V_{in1}}{V_{in2}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]+\left(\frac{V_{in4}}{V_{in1}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]\right\}\right)+\left(\frac{V_{in1}}{V_{in2}}\right)R_4\left(Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]+\left(\frac{V_{in4}}{V_{in1}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]\right\}+\left(\frac{V_{in4}}{V_{in1}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]+\left(\frac{V_{in3}}{V_{in4}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]\right\}\right)\right)}{\left[2R_1Z_{C1}+(Z_{C2}+R_2)(Z_{C1}+R_1)\right]\left[2R_3Z_{C3}+(Z_{C3}+R_3)(Z_{C4}+R_4)\right]+2(Z_{C3}+R_3+Z_{C4}+R_4)\left[(Z_{C1}+R_1)R_2Z_{C2}+(Z_{C2}+R_2)R_1Z_{C1}\right]}$$

$$V_{out3}=\frac{V_{in3}\left(Z_{C4}\left(Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]+\left(\frac{V_{in2}}{V_{in3}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]\right\}+\left(\frac{V_{in2}}{V_{in3}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]+\left(\frac{V_{in1}}{V_{in2}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]\right\}\right)+\left(\frac{V_{in2}}{V_{in3}}\right)R_4\left(Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]+\left(\frac{V_{in1}}{V_{in2}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]\right\}+\left(\frac{V_{in1}}{V_{in2}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]+\left(\frac{V_{in4}}{V_{in1}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]\right\}\right)\right)}{\left[2R_1Z_{C1}+(Z_{C2}+R_2)(Z_{C1}+R_1)\right]\left[2R_3Z_{C3}+(Z_{C3}+R_3)(Z_{C4}+R_4)\right]+2(Z_{C3}+R_3+Z_{C4}+R_4)\left[(Z_{C1}+R_1)R_2Z_{C2}+(Z_{C2}+R_2)R_1Z_{C1}\right]}$$

$$V_{out4}=\frac{V_{in4}\left(Z_{C4}\left(Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in3}}{V_{in4}}\right)\right]+\left(\frac{V_{in3}}{V_{in4}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]\right\}+\left(\frac{V_{in3}}{V_{in4}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]+\left(\frac{V_{in2}}{V_{in3}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]\right\}\right)+\left(\frac{V_{in3}}{V_{in4}}\right)R_4\left(Z_{C3}\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in2}}{V_{in3}}\right)\right]+\left(\frac{V_{in2}}{V_{in3}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]\right\}+\left(\frac{V_{in2}}{V_{in3}}\right)R_3\left\{Z_{C2}\left[Z_{C1}+R_1\left(\frac{V_{in1}}{V_{in2}}\right)\right]+\left(\frac{V_{in1}}{V_{in2}}\right)R_2\left[Z_{C1}+R_1\left(\frac{V_{in4}}{V_{in1}}\right)\right]\right\}\right)\right)}{\left[2R_1Z_{C1}+(Z_{C2}+R_2)(Z_{C1}+R_1)\right]\left[2R_3Z_{C3}+(Z_{C3}+R_3)(Z_{C4}+R_4)\right]+2(Z_{C3}+R_3+Z_{C4}+R_4)\left[(Z_{C1}+R_1)R_2Z_{C2}+(Z_{C2}+R_2)R_1Z_{C1}\right]}$$

2. Analysis of 4-Stage Passive RC Network

Model of Input Wanted Signals

$$S_{Pos\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\}$$

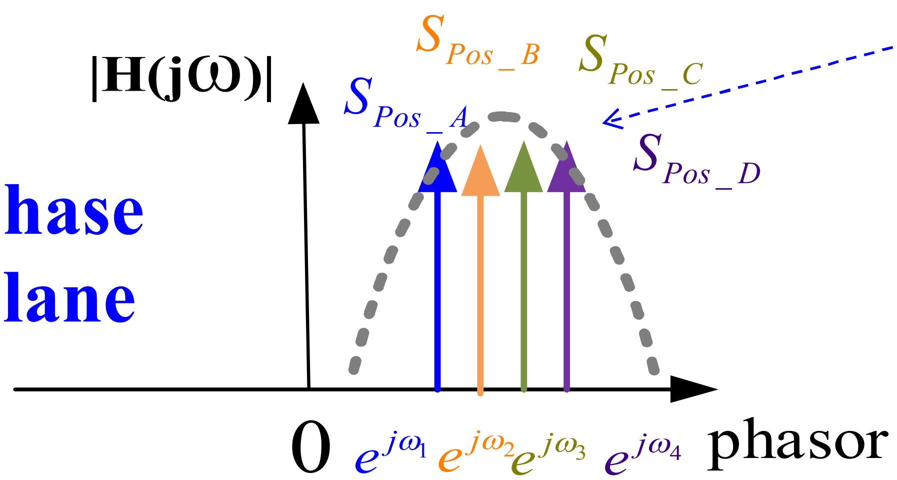
$$=\{1;j;-1;-j\}V_{pos}(t)=\left\{1;e^{j\frac{\pi}{2}};e^{j\pi};e^{j\frac{3\pi}{2}}\right\}\sum_{k=1}^{n}A_k\cos(\omega_k t+\theta_k)$$

Positive poly-phase signals

$$\frac{V_{in1}}{V_{in2}}=\frac{V_{in2}}{V_{in3}}=\frac{V_{in3}}{V_{in4}}=\frac{V_{in4}}{V_{in1}}=-j$$

Wanted Signals

phase plane



2. Analysis of 4-Stage Passive RC Network

Derivation of Output Voltages (Wanted Signal)

Output voltages

$$
\begin{cases}
V_{out1} = V_{in1} \dfrac{(Z_{C4} - jR_4)(Z_{C3} - jR_3)(Z_{C2} - jR_2)(Z_{C1} - jR_1)}{\left\{ \begin{array}{l} \left[2R_1 Z_{C1} + (Z_{C2} + R_2)(Z_{C1} + R_1)\right]\left[2R_3 Z_{C3} + (Z_{C3} + R_3)(Z_{C4} + R_4)\right] \\ +2(Z_{C3} + R_3 + Z_{C4} + R_4)\left[(Z_{C1} + R_1) R_2 Z_{C2} + (Z_{C2} + R_2) R_1 Z_{C1}\right] \end{array} \right\}} \\
V_{out2} = V_{in2} \dfrac{(Z_{C4} - jR_4)(Z_{C3} - jR_3)(Z_{C2} - jR_2)(Z_{C1} - jR_1)}{\left\{ \begin{array}{l} \left[2R_1 Z_{C1} + (Z_{C2} + R_2)(Z_{C1} + R_1)\right]\left[2R_3 Z_{C3} + (Z_{C3} + R_3)(Z_{C4} + R_4)\right] \\ +2(Z_{C3} + R_3 + Z_{C4} + R_4)\left[(Z_{C1} + R_1) R_2 Z_{C2} + (Z_{C2} + R_2) R_1 Z_{C1}\right] \end{array} \right\}} \\
V_{out3} = V_{in3} \dfrac{(Z_{C4} - jR_4)(Z_{C3} - jR_3)(Z_{C2} - jR_2)(Z_{C1} - jR_1)}{\left\{ \begin{array}{l} \left[2R_1 Z_{C1} + (Z_{C2} + R_2)(Z_{C1} + R_1)\right]\left[2R_3 Z_{C3} + (Z_{C3} + R_3)(Z_{C4} + R_4)\right] \\ +2(Z_{C3} + R_3 + Z_{C4} + R_4)\left[(Z_{C1} + R_1) R_2 Z_{C2} + (Z_{C2} + R_2) R_1 Z_{C1}\right] \end{array} \right\}} \\
V_{out4} = V_{in4} \dfrac{(Z_{C4} - jR_4)(Z_{C3} - jR_3)(Z_{C2} - jR_2)(Z_{C1} - jR_1)}{\left\{ \begin{array}{l} \left[2R_1 Z_{C1} + (Z_{C2} + R_2)(Z_{C1} + R_1)\right]\left[2R_3 Z_{C3} + (Z_{C3} + R_3)(Z_{C4} + R_4)\right] \\ +2(Z_{C3} + R_3 + Z_{C4} + R_4)\left[(Z_{C1} + R_1) R_2 Z_{C2} + (Z_{C2} + R_2) R_1 Z_{C1}\right] \end{array} \right\}}
\end{cases}
$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Transfer Function (Wanted Signal)

Transfer Function

$$H=\frac{V_{out1}}{V_{in1}}=\frac{V_{out2}}{V_{in2}}=\frac{V_{out3}}{V_{in3}}=\frac{V_{out4}}{V_{in4}}$$

$$=\frac{\left(Z_{C4}-jR_4\right)\left(Z_{C3}-jR_3\right)\left(Z_{C2}-jR_2\right)\left(Z_{C1}-jR_1\right)}{\left\{\begin{array}{l}\left[2R_1Z_{C1}+\left(Z_{C2}+R_2\right)\left(Z_{C1}+R_1\right)\right]\left[2R_3Z_{C3}+\left(Z_{C3}+R_3\right)\left(Z_{C4}+R_4\right)\right]\\+2\left(Z_{C3}+R_3+Z_{C4}+R_4\right)\left[\left(Z_{C1}+R_1\right)R_2Z_{C2}+\left(Z_{C2}+R_2\right)R_1Z_{C1}\right]\end{array}\right\}}$$

$$Z_{C1}=\frac{1}{j2\pi fC_1};Z_{C2}=\frac{1}{j2\pi fC_2}$$

$$Z_{C3}=\frac{1}{j2\pi fC_3}$$

$$Z_{C4}=\frac{1}{j2\pi fC_4}$$

$$f_1=\frac{1}{2\pi R_1C_1};f_2=\frac{1}{2\pi R_2C_2};f_{21}=\frac{1}{2\pi R_2C_1};$$

$$f_3=\frac{1}{2\pi R_3C_3};f_{31}=\frac{1}{2\pi R_3C_1};f_{32}=\frac{1}{2\pi R_3C_2};$$

$$f_4=\frac{1}{2\pi R_4C_4};f_{41}=\frac{1}{2\pi R_4C_1};f_{42}=\frac{1}{2\pi R_4C_2};f_{43}=\frac{1}{2\pi R_4C_3};$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Transfer Function (Wanted Signal)

Transfer Function

$$
H=\frac{\left\{\begin{array}{l} s^4+js^3\left(f_4+f_3+f_2+f_1\right)-s^2\left[f_4\left(f_3+f_2+f_1\right)+f_3\left(f_2+f_1\right)+f_2f_1\right] \\ -js\left\{f_4\left[f_3\left(f_2+f_1\right)+f_2f_1\right]+f_3f_2f_1\right\}+f_4f_3f_2f_1 \end{array}\right\}}{\left\{\begin{array}{l} s^4+s^3\left(f_4+f_3+f_2+f_1+2\left[\left(f_{43}+f_{42}+f_{41}\right)+\left(f_{32}+f_{31}\right)+f_{21}\right]\right) \\ +s^2\left[\begin{array}{l} f_4\left(f_3+f_2+f_1\right)+f_3\left(f_2+f_1\right)+f_2f_1 \\ +4f_{43}f_{21}+2\left(\begin{array}{l} f_4\left(\left(f_{32}+f_{31}\right)+f_{21}\right)+f_3\left(\left(f_{42}+f_{41}\right)+f_{21}\right) \\ +f_2\left(\left(f_{43}+f_{41}\right)+f_{31}\right)+f_1\left(\left(f_{43}+f_{42}\right)+f_{32}\right) \end{array}\right) \end{array}\right] \\ +s\left\{f_4\left[f_3\left(f_2+f_1\right)+f_2f_1\right]+f_3f_2f_1+2\left(\begin{array}{l} f_4\left(f_3f_{21}+f_2f_{31}+f_1f_{32}\right) \\ +f_3\left(f_2f_{41}+f_1f_{42}\right)+f_2f_1f_{43} \end{array}\right)\right\} \\ +f_4f_3f_2f_1 \end{array}\right\}}
$$

2. Analysis of 4-Stage Passive RC Network

Derivation of Transfer Function (Wanted Signal)

$$a_1 = f_4 + f_3 + f_2 + f_1$$

$$a_2 = f_4\left(f_3 + f_2 + f_1\right) + f_3\left(f_2 + f_1\right) + f_2 f_1$$

$$a_3 = f_4\left[f_3\left(f_2 + f_1\right) + f_2 f_1\right] + f_3 f_2 f_1$$

$$a_4 = f_4 f_3 f_2 f_1$$

$$a_5 = a_1 + 2\left[\left(f_{43} + f_{42} + f_{41}\right) + \left(f_{32} + f_{31}\right) + f_{21}\right]$$

$$a_6 = a_2 + 4 f_{43} f_{21} + 2\begin{pmatrix} f_4\left(\left(f_{32} + f_{31}\right) + f_{21}\right) + f_3\left(\left(f_{42} + f_{41}\right) + f_{21}\right) \\ + f_2\left(\left(f_{43} + f_{41}\right) + f_{31}\right) + f_1\left(\left(f_{43} + f_{42}\right) + f_{32}\right) \end{pmatrix}$$

$$a_7 = a_3 + 2\left(f_4\left(f_3 f_{21} + f_2 f_{31} + f_1 f_{32}\right) + f_3\left(f_2 f_{41} + f_1 f_{42}\right) + f_2 f_1 f_{43}\right)$$

Transfer function

$$H_{Pos}\left(j2\pi f\right) = \frac{s^4 + j a_1 s^3 - a_2 s^2 - j a_3 s + a_4}{s^4 + a_5 s^3 + a_6 s^2 + a_7 s + a_4}$$

2. Analysis of 4-Stage Passive RC Network

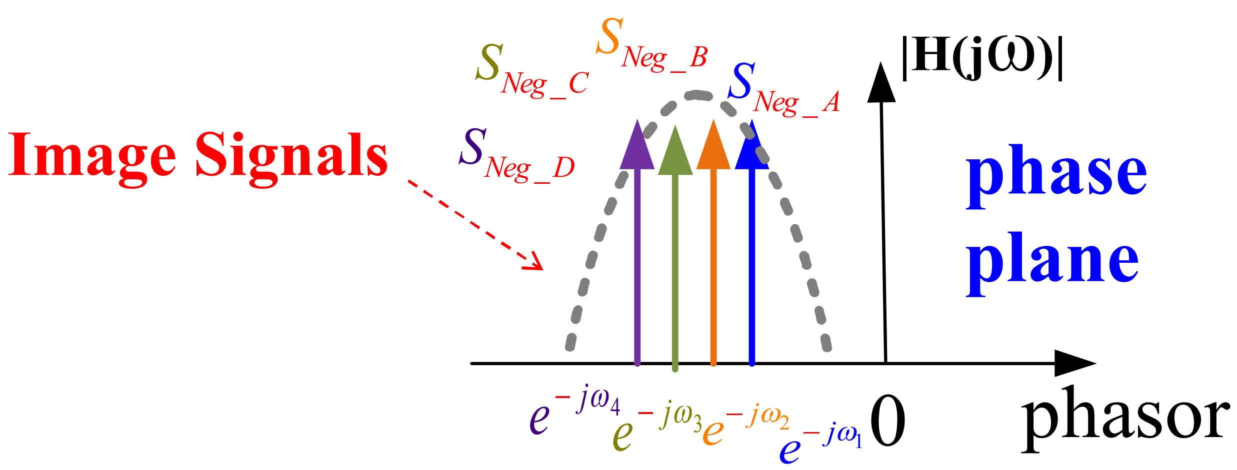
Model of Input Image Signals

$$S_{Neg\_poly}\{V_1(t);V_2(t);V_3(t);V_4(t)\}$$

$$=\{V(t);-jV(t);-V(t);jV(t)\}=\left\{V(t);e^{-j\frac{\pi}{2}}V(t);e^{-j\pi}V(t);e^{-j\frac{3\pi}{2}}V(t)\right\}$$

Negative poly-phase signals

$$\frac{V_{in1}}{V_{in2}}=\frac{V_{in2}}{V_{in3}}=\frac{V_{in3}}{V_{in4}}=\frac{V_{in4}}{V_{in1}}=+j$$



2. Analysis of 4-Stage Passive RC Network

Derivation of Transfer Function (Image Signal)

Transfer Function

$$H = \frac{\left(Z_{C4} + jR_4\right)\left(Z_{C3} + jR_3\right)\left(Z_{C2} + jR_2\right)\left(Z_{C1} + jR_1\right)}{\left\{\begin{array}{l}\left[2R_1 Z_{C1} + \left(Z_{C2} + R_2\right)\left(Z_{C1} + R_1\right)\right]\left[2R_3 Z_{C3} + \left(Z_{C3} + R_3\right)\left(Z_{C4} + R_4\right)\right] \\ +2\left(Z_{C3} + R_3 + Z_{C4} + R_4\right)\left[\left(Z_{C1} + R_1\right)R_2 Z_{C2} + \left(Z_{C2} + R_2\right)R_1 Z_{C1}\right]\end{array}\right\}}$$

$$H = \frac{\left\{s^4 - js^3\left(f_4 + f_3 + f_2 + f_1\right) - s^2\left[f_4\left(f_3 + f_2 + f_1\right) + f_3\left(f_2 + f_1\right) + f_2 f_1\right] + js\left\{f_4\left[f_3\left(f_2 + f_1\right) + f_2 f_1\right] + f_3 f_2 f_1\right\} + f_4 f_3 f_2 f_1\right\}}{\left\{\begin{array}{l} s^4 + s^3\left(f_4 + f_3 + f_2 + f_1 + 2\left[\left(f_{43} + f_{42} + f_{41}\right) + \left(f_{32} + f_{31}\right) + f_{21}\right]\right) + s^2\left[\begin{array}{l} f_4\left(f_3 + f_2 + f_1\right) + f_3\left(f_2 + f_1\right) + f_2 f_1 \\ +4 f_{43} f_{21} + 2\left(\begin{array}{l} f_4\left(\left(f_{32} + f_{31}\right) + f_{21}\right) + f_3\left(\left(f_{42} + f_{41}\right) + f_{21}\right) \\ +f_2\left(\left(f_{43} + f_{41}\right) + f_{31}\right) + f_1\left(\left(f_{43} + f_{42}\right) + f_{32}\right)\end{array}\right)\end{array}\right] \\ +s\left\{f_4\left[f_3\left(f_2 + f_1\right) + f_2 f_1\right] + f_3 f_2 f_1 + 2\left(\begin{array}{l} f_4\left(f_3 f_{21} + f_2 f_{31} + f_1 f_{32}\right) \\ +f_3\left(f_2 f_{41} + f_1 f_{42}\right) + f_2 f_1 f_{43}\end{array}\right)\right\} + f_4 f_3 f_2 f_1 \end{array}\right\}}$$

$$H_{Neg}\left(j2\pi f\right) = \frac{s^4 - ja_1 s^3 - a_2 s^2 + ja_3 s + a_4}{s^4 + a_5 s^3 + a_6 s^2 + a_7 s + a_4}$$

2. Analysis of 4-Stage Passive RC Network

Composed Transfer Function of 4-Stage Passive RC Network

Wanted Signals

$$H_{Pos}\left(j2\pi f\right)=\frac{s^4+ja_1s^3-a_2s^2-ja_3s+a_4}{s^4+a_5s^3+a_6s^2+a_7s+a_4};\forall f>0$$

Image Signals

$$H_{Neg}\left(j2\pi f\right)=\frac{s^4-ja_1s^3-a_2s^2+ja_3s+a_4}{s^4+a_5s^3+a_6s^2+a_7s+a_4};\forall f<0$$

Composed transfer function

$$\left|H\left(f\right)\right|=\frac{\left|f^4+a_1f^3+a_2f^2+a_3f+a_4\right|}{\sqrt{\left(f^4+a_4-a_6f^2\right)^2+\left(a_7f-a_5f^3\right)^2}};\forall f\in R$$

2. Analysis of 4-Stage Passive RC Network

Analysis of Transfer Function (Positive Frequency)

$$\left|H(f)\right| = \frac{f^4 + a_1 f^3 + a_2 f^2 + a_3 f + a_4}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}};\forall f > 0$$

$$\lim_{f \to 0}\left|H(f)\right| = \lim_{f \to 0}\frac{f^4 + a_1 f^3 + a_2 f^2 + a_3 f + a_4}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}} = 1$$

$$\lim_{f \to \infty}\left|H(f)\right| = \lim_{f \to \infty}\frac{f^4 + a_1 f^3 + a_2 f^2 + a_3 f + a_4}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}} = 1$$

$$\min\left(\left|H(f)\right|\right) as\ f = \sqrt[4]{f_4 f_3 f_2 f_1}$$

Applying Cauchy-Schwarz inequality theorem:

$$a,b > 0, a^2 + b^2 \geq 2ab;\ \min\left(a^2 + b^2\right) = 2ab\ as\ "a = b"$$

$$\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2 \geq 2\left|\left(f^4 + a_4 - a_6 f^2\right)\right|\left|\left(a_7 f - a_5 f^3\right)\right|$$

$$\left|\left(f^4 + a_4 - a_6 f^2\right)\right| = \left|\left(a_7 f - a_5 f^3\right)\right|;\Rightarrow \left[\begin{array}{l} f_{\min 1} = \ldots \\ f_{\min 2} = \ldots \end{array}\right. \Rightarrow \left\{\begin{array}{l} \max 1\left(\left|H(f)\right|\right) \\ \max 2\left(\left|H(f)\right|\right) \end{array}\right.$$

2. Analysis of 4-Stage Passive RC Network

Analysis of Transfer Function (Negative Frequency)

$$\left|H(f)\right| = \frac{f^4 - a_1 f^3 + a_2 f^2 - a_3 f + a_4}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}} ; \forall f < 0$$

$$\lim_{f \to 0} \left|H(f)\right| = \lim_{f \to 0} \frac{f^4 - a_1 f^3 + a_2 f^2 - a_3 f + a_4}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}} = 1$$

$$\lim_{f \to -\infty} \left|H(f)\right| = \lim_{f \to -\infty} \frac{f^4 - a_1 f^3 + a_2 f^2 - a_3 f + a_4}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}} = 1$$

$$\min\left(\left|H(f)\right|_{dB}\right) = -\infty; as\ f = -f_1 \vee f = -f_2 \vee f = -f_3 \vee f = -f_4$$

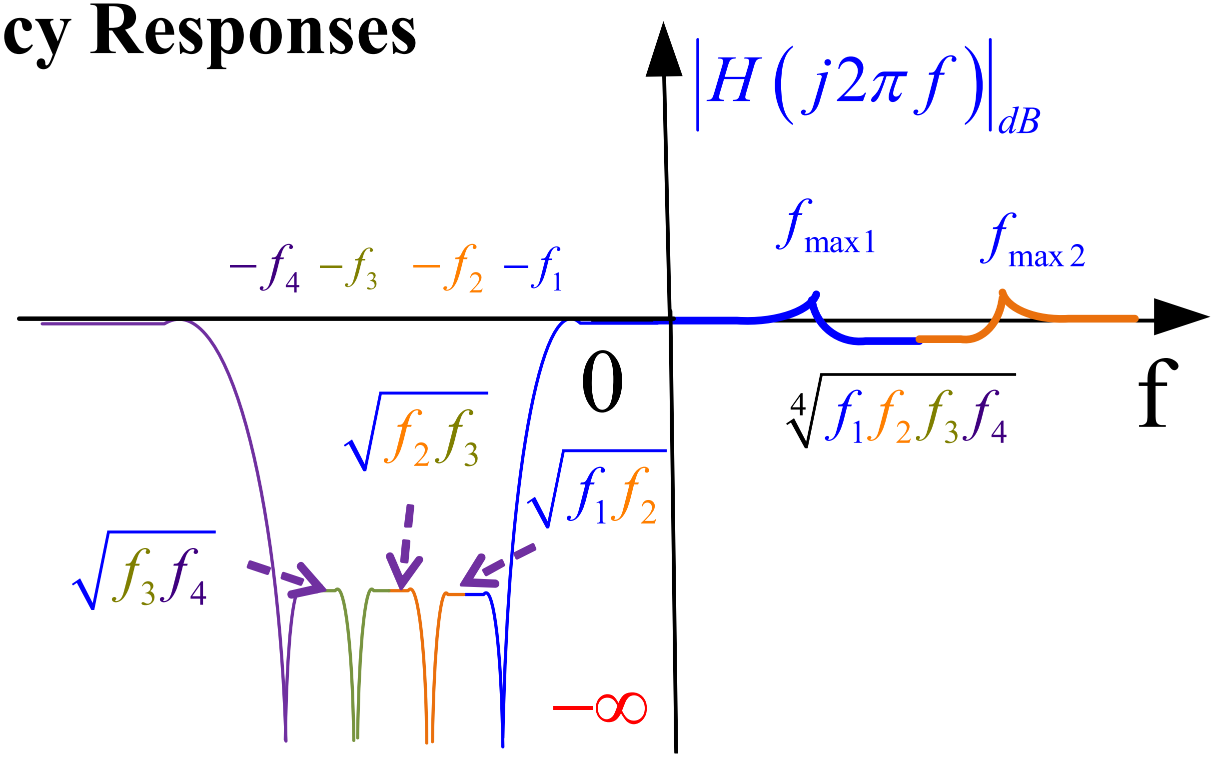
$$\max 1\left(\left|H(f)\right|\right) as\ f = \sqrt{f_1 f_2}; \max 2\left(\left|H(f)\right|\right) as\ f = \sqrt{f_2 f_3}; \max 3\left(\left|H(f)\right|\right) as\ f = \sqrt{f_3 f_4}$$

2. Analysis of 4-Stage Passive RC Network

Frequency Responses

Transfer function $$\left|H(f)\right| = \frac{\left|f^4 + a_1 f^3 + a_2 f^2 + a_3 f + a_4\right|}{\sqrt{\left(f^4 + a_4 - a_6 f^2\right)^2 + \left(a_7 f - a_5 f^3\right)^2}}; \forall f \in R$$

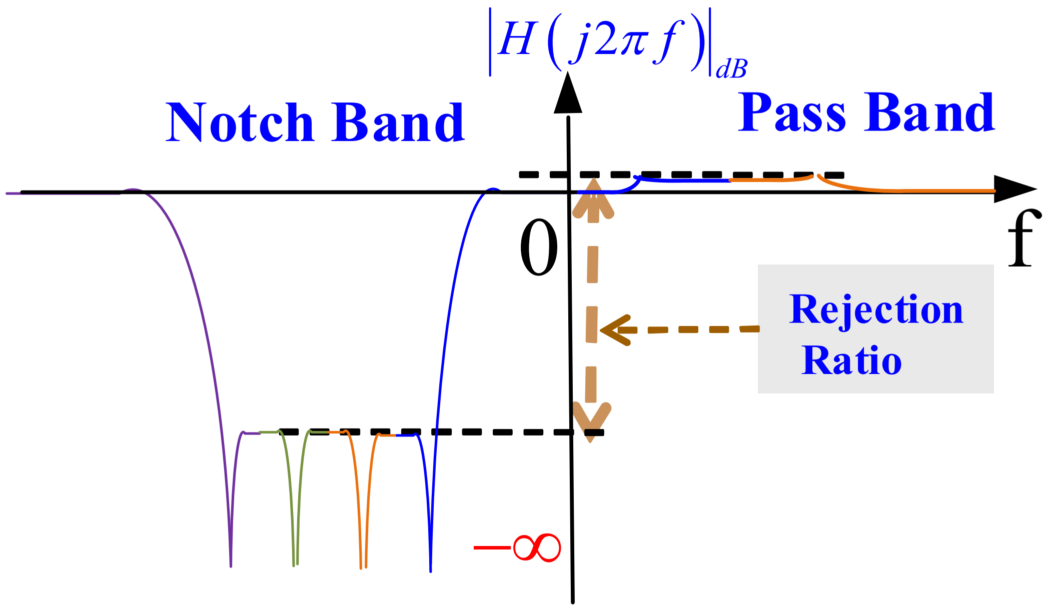
Frequency Responses



2. Analysis of 4-Stage Passive RC Network

Image Rejection Ratio

Image Rejection Ratio = $\dfrac{\text{Pass Band}}{\text{Notch Band}}$



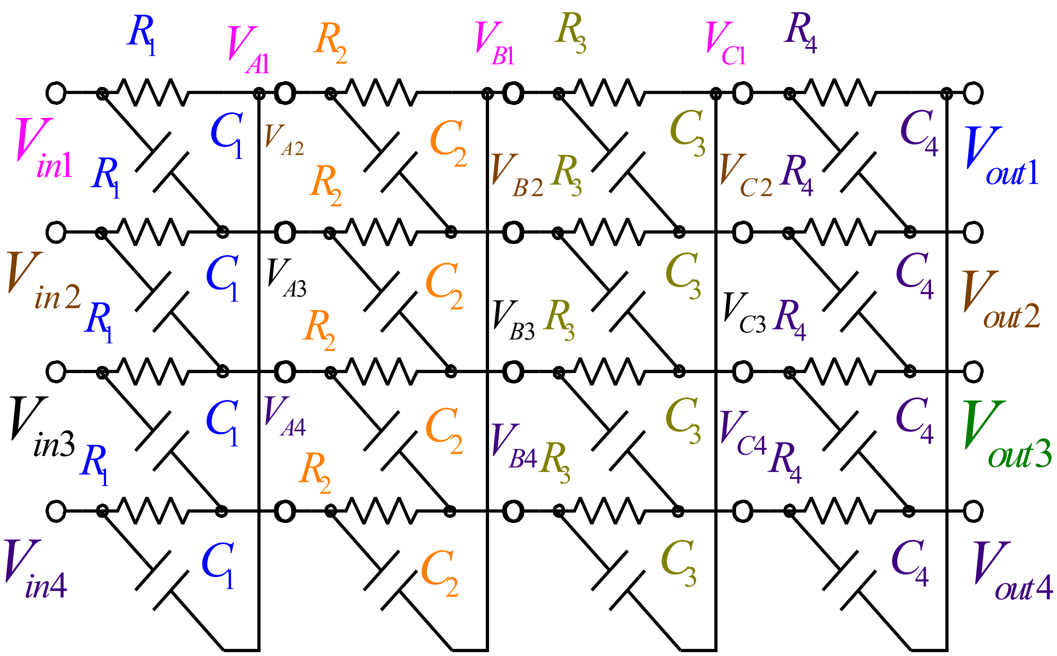
$$IRR = \frac{H_{Pos}(j2\pi f)}{H_{Neg}(j2\pi f)} = \frac{(f_1+f)(f_2+f)(f_3+f)(f_4+f)}{(f_1-f)(f_2-f)(f_3-f)(f_4-f)}$$

Outline

1. Research Background
   - Applications of RC Poly-phase Network
   - Analog Complex Signal Processing Concepts
2. Analysis of 4-Stage Passive RC Network
   - Frequency Responses
   - Image Rejection Ratio
3. **Proposed Model of 4-Stage Passive RC Network**
   - **Model of 4-Stage Passive RC Network**
   - **Simulation Results**
4. Conclusions

3. Proposed Model of 4-Stage Passive RC Network

Model of 4-Stage Passive RC Network



$$R_1 = 1k\Omega; C_1 = 227\,pF;$$
$$R_2 = 1k\Omega; C_2 = 106\,pF;$$
$$R_3 = 1k\Omega; C_3 = 39.8\,pF;$$
$$R_4 = 1k\Omega; C_4 = 15.9\,pF;$$

$$f_1 = 0.7MHz;$$
$$f_2 = 1.5MHz; f_{21} = 0.7MHz;$$
$$f_3 = 4MHz; f_{31} = 0.7MHz; f_{32} = 1.5MHz;$$
$$f_4 = 10MHz; f_{41} = 0.7MHz; f_{42} = 1.5MHz; f_{43} = 4MHz;$$

3. Proposed Model of 4-Stage Passive RC Network

Simplified Model of 4-Stage Passive RC Network

$$a_1 = f_4 + f_3 + f_2 + f_1 = 1.62 * 10^7$$

$$a_2 = f_4\left(f_3 + f_2 + f_1\right) + f_3\left(f_2 + f_1\right) + f_2 f_1 = 7.19 * 10^{13}$$

$$a_3 = f_4\left[f_3\left(f_2 + f_1\right) + f_2 f_1\right] + f_3 f_2 f_1 = 1.03 * 10^{20}$$

$$a_4 = f_4 f_3 f_2 f_1 = 4.2 * 10^{25}$$

$$a_5 = a_1 + 2\left[\left(f_{43} + f_{42} + f_{41}\right) + \left(f_{32} + f_{31}\right) + f_{21}\right] = 3.44 * 10^7$$

$$a_6 = a_2 + 4 f_{43} f_{21} + 2\begin{pmatrix} f_4\left(\left(f_{32} + f_{31}\right) + f_{21}\right) + f_3\left(\left(f_{42} + f_{41}\right) + f_{21}\right) \\ + f_2\left(\left(f_{43} + f_{41}\right) + f_{31}\right) + f_1\left(\left(f_{43} + f_{42}\right) + f_{32}\right) \end{pmatrix} = 1.9 * 10^{14}$$

$$a_7 = a_3 + 2\left(f_4\left(f_3 f_{21} + f_2 f_{31} + f_1 f_{32}\right) + f_3\left(f_2 f_{41} + f_1 f_{42}\right) + f_2 f_1 f_{43}\right) = 2.26 * 10^{20}$$

Transfer function

$$\left|H\left(f\right)\right|_{dB} = \frac{\left|f^4 + 1.62 * 10^7 f^3 + 7.19 * 10^{13} f^2 + 1.03 * 10^{20} f + 4.2 * 10^{25}\right|}{\sqrt{\left(f^4 + 4.2 * 10^{25} - 1.9 * 10^{14} f^2\right)^2 + \left(2.26 * 10^{20} f - 3.44 * 10^7 f^3\right)^2}}$$

3. Proposed Model of 4-Stage Passive RC Network

Analysis of System Model (Positive Frequency)

$$|H| = \frac{f^4 + 1.62*10^7 f^3 + 7.19*10^{13} f^2 + 1.03*10^{20} f + 4.2*10^{25}}{\sqrt{\left(f^4 + 4.2*10^{25} - 1.9*10^{14} f^2\right)^2 + \left(2.26*10^{20} f - 3.44*10^7 f^3\right)^2}}; f > 0$$

$$|H| = \begin{cases} 1; \ f \to 0 \\ 1.16; \ f = 1.55*10^5 \\ 0.938; \ f = 25.5*10^5 \\ 1.18; \ f = 405*10^5 \\ 1; \ f \to \infty \end{cases} \Rightarrow 20\log|H| = \begin{cases} 0dB; \ f \to 0 \\ 1.33dB; \ f = 1.55*10^5 \\ -0.55dB; \ f = 25.5*10^5 \\ 1.42dB; \ f = 405*10^5 \\ 0dB; \ f \to \infty \end{cases}$$

3. Proposed Model of 4-Stage Passive RC Network

Analysis of System Model (Negative Frequency)

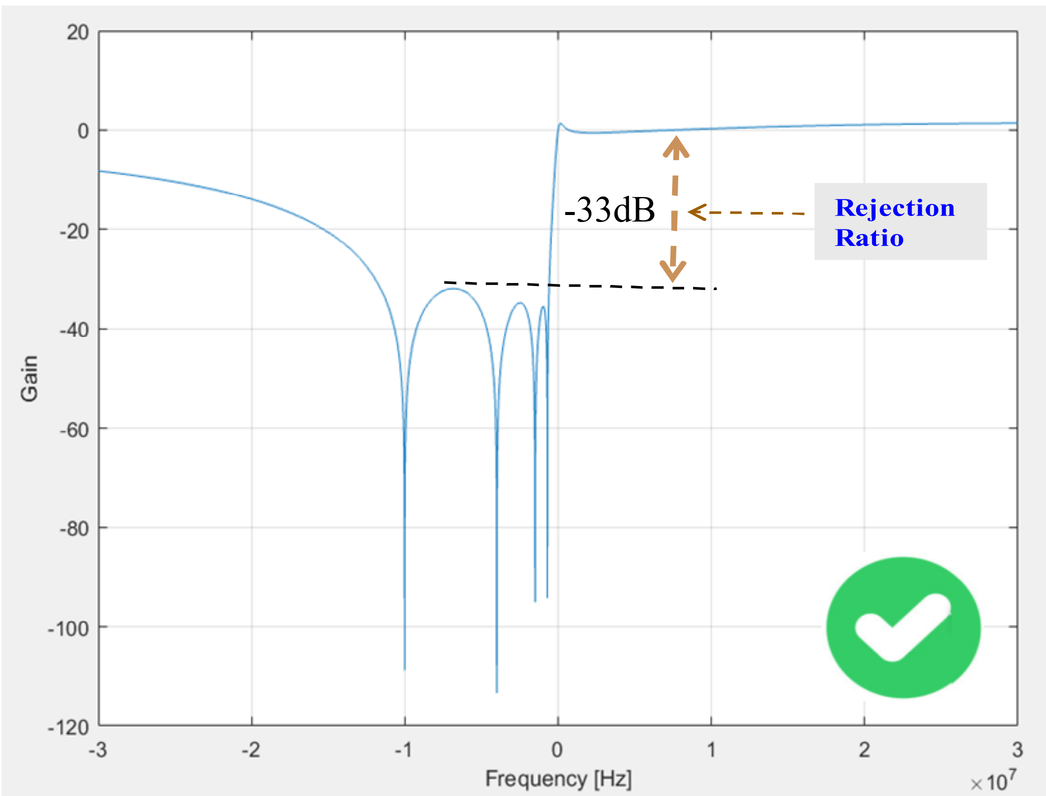
$$|H|_{dB} = \frac{f^4 - 1.62*10^7 f^3 + 7.19*10^{13} f^2 - 1.03*10^{20} f + 4.2*10^{25}}{\sqrt{\left(f^4 + 4.2*10^{25} - 1.9*10^{14} f^2\right)^2 + \left(2.26*10^{20} f - 3.44*10^7 f^3\right)^2}}; f < 0$$

$$|H| = \begin{cases} 1; & f \to 0 \\ 0; & f = -7*10^5 \\ 0.016; & f = -10.2*10^5 \\ 0; & f = -15*10^5 \\ 0.018; & f = -24.5*10^5 \\ 0; & f = -40*10^5 \\ 0.024; & f = -63.2*10^5 \\ 0; & f = -100*10^5 \\ 1; & f \to -\infty \end{cases} \Rightarrow 20\log|H| = \begin{cases} 0dB; & f \to 0 \\ -\infty; & f = -7*10^5 \\ -35.6dB; & f = -10.2*10^5 \\ -\infty; & f = -15*10^5 \\ -34.8dB; & f = -24.5*10^5 \\ -\infty; & f = -40*10^5 \\ -32.1dB; & f = -63.2*10^5 \\ -\infty; & f = -100*10^5 \\ 0dB; & f \to -\infty \end{cases}$$

3. Proposed Model of 4-Stage Passive RC Network

Simulation Results of 4-Stage Passive RC Network

Frequency Responses



Parameter	PPF
Input IF frequency	5MHz
Input IF Power	-60dBm
NF	-
Gain	-0.3dB
Current Consumption	-
Supply Voltage	-
Image Rejection Ratio IMRR	-33dB

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4. Conclusions

This Work:

- **Derivation of transfer function of poly-phase network based on superposition principle**
- **Mathematical analysis and model of 4-stage passive RC poly-phase filter**
- **Image rejection ratio: -33dB**

Future of Work

- **Analysis of IQ mismatches of poly-phase signals**
- **Analysis of Parasitic of RC components**

Thanks for your kind attention!

Questions & Answers

1) **Will the variations of R and C components change the characteristics of this filter?**

   →Yes, they will.

   (The variations of R and C components will cause the IQ mismatches. Therefore, the image rejection ratio will be changed.)

2) **Are the simulation results of RC poly-phase filter in this research best?**

   →No, they aren't.

   (There are some design trade-offs in this research. So, these simulation results are acceptable for the design targets.)