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Fast Response, Small Ripple, Low Noise Switching Converter with Digital Charge Time Control and EMI Harmonic Filter

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Outline

1. Research Background
    - Applications of Switching Power Supply
    - Basic Switching Converter Architecture
2. Analysis of Step-down Switching Converter
    - Conventional State-Space Technique
    - Superposition Principle
3. Proposed Design of Buck Converter
    - Ripple Voltage Reduction with Notch Harmonic Filters
    - Simulation Results
4. Conclusions

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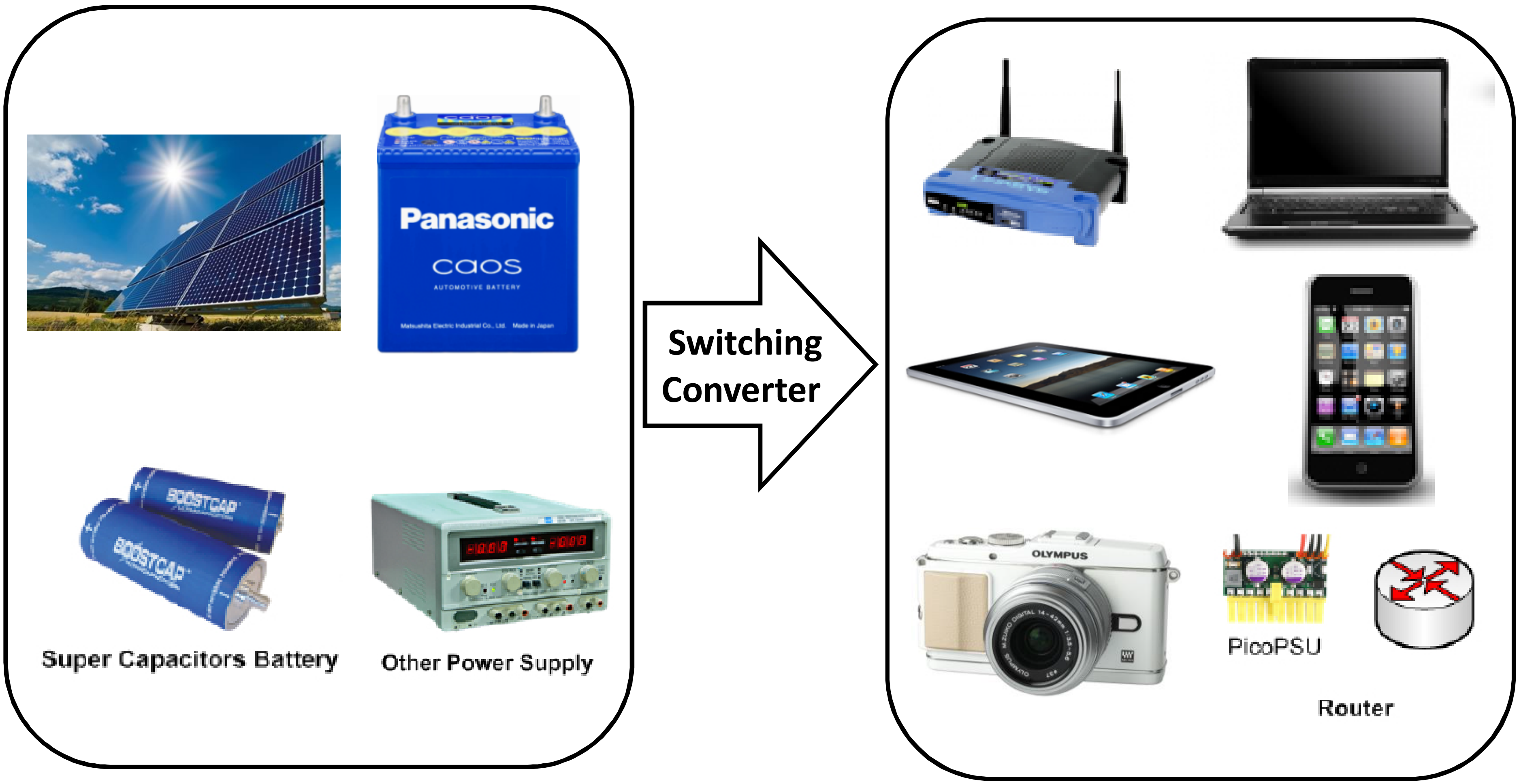
    - Ripple Voltage Reduction with Notch Harmonic Filters
    - Simulation Results
4. Conclusions

1. Research Background

Typical Applications of Switching Power Supply



1. Research Background

Research Objective

Objective

Development of switching power supply with

- Fast response & high efficiency
- Low EMI noise
- Small output ripple

Approach

- Analysis of Buck converter system using state-space technique and superposition principle
- EMI reduction using harmonic notch filters

1. Research Background

Design Achievements

Achievements

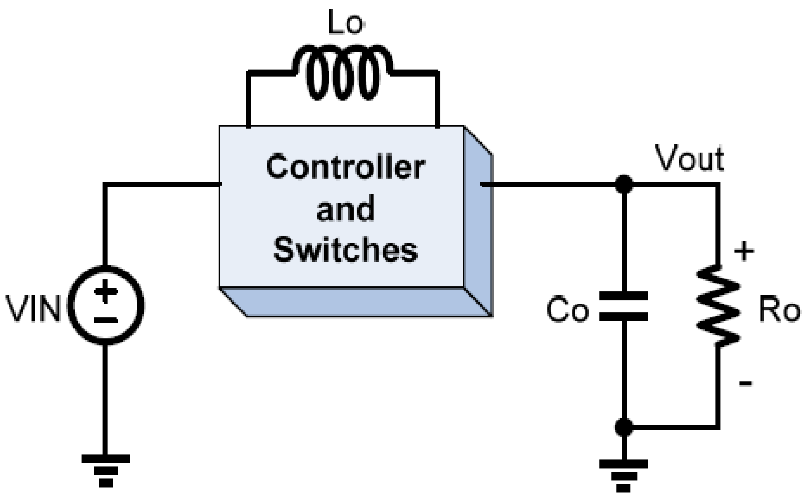
- Derivation of transfer function of buck converter based on **superposition principle**
- **Overshoot cancelation** based on **balanced charge-discharge time condition:**

$$|Z_L| = |Z_C| = 2R \Rightarrow \omega L = \frac{1}{\omega C} = 2R \qquad \omega = \frac{1}{\sqrt{LC}} = \frac{1}{2RC}$$

- **Ripple reduction** from **30mVpp** into **0.4mVpp**
- **Two harmonic notch filters:**
  - **-7dB** at the 1^{st} harmonic $\quad f_{1^{st}\ harmonic} = \frac{1}{2\pi\sqrt{L_2 C_2}} = 100kHz$
  - **-2dB** at the 2^{nd} harmonic $\quad f_{2^{nd}\ harmonic} = \frac{1}{2\pi\sqrt{L_3 C_3}} = 300kHz$

1. Research Background

Basic Switching Converter Architecture



Basic Switching Converter

Merits
- Downsizing
- Light Weight
- High Efficiency

High Efficiency Switching

➔ Reduce energy consumption

➔ Extend battery operating time

➔ Minimize costs of systems

Demerits
- Output Ripple
- Switching noise
- Harmonic noise

1. Research Background

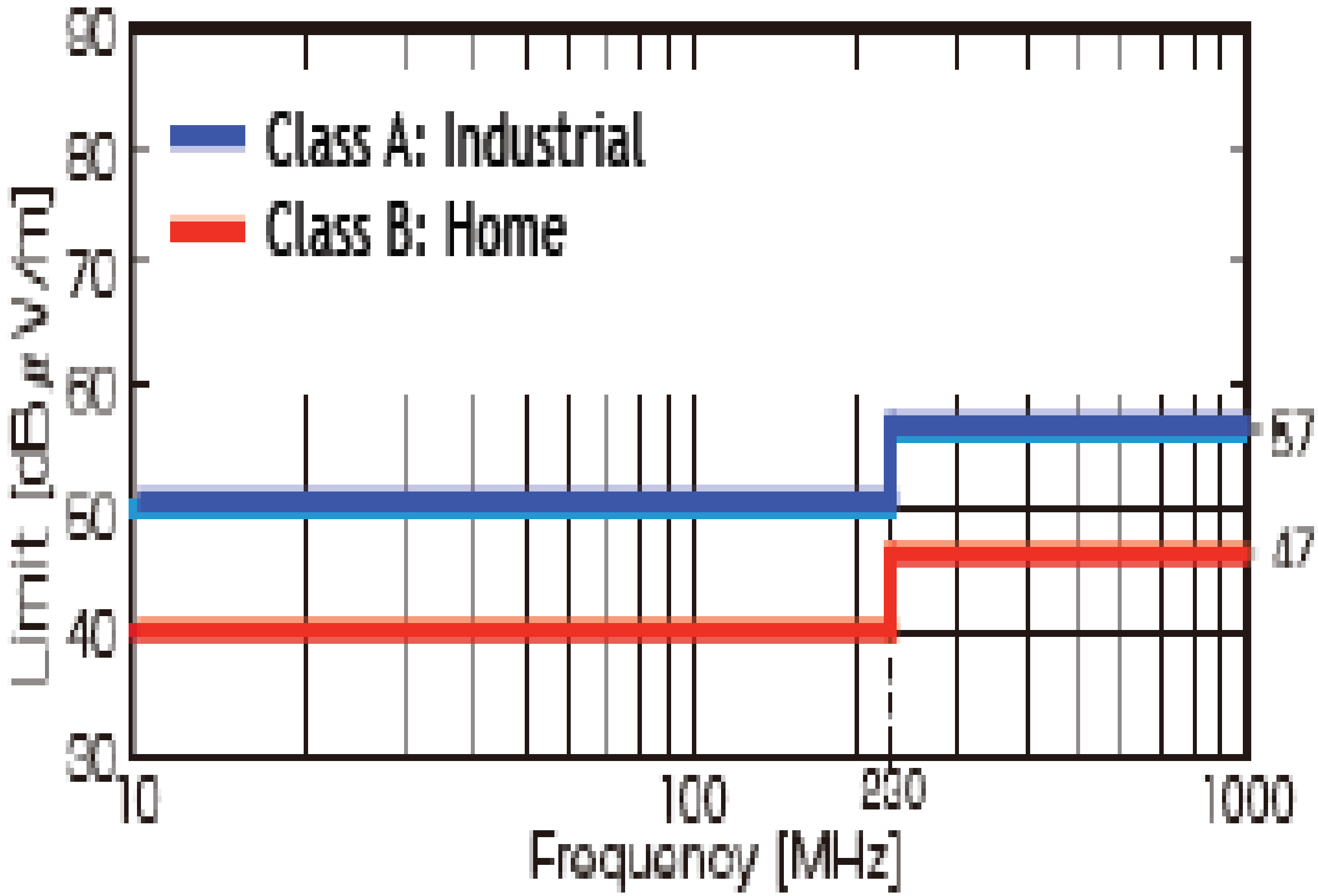
Trade-offs of Switching Power Supply

	Linear Regulator	Switching Regulator	
		Inductive	Charge Pump
Efficiency	20-60%	90-95%	75-90%
Ripple	Very low	Low	Moderate
EMI Noise	Very low	Moderate	Low
PCB Area	Very small	Largest	Medium
Cost	Lowest	Highest	Medium

1. Research Background

Required EMI Noises of Switching Converter

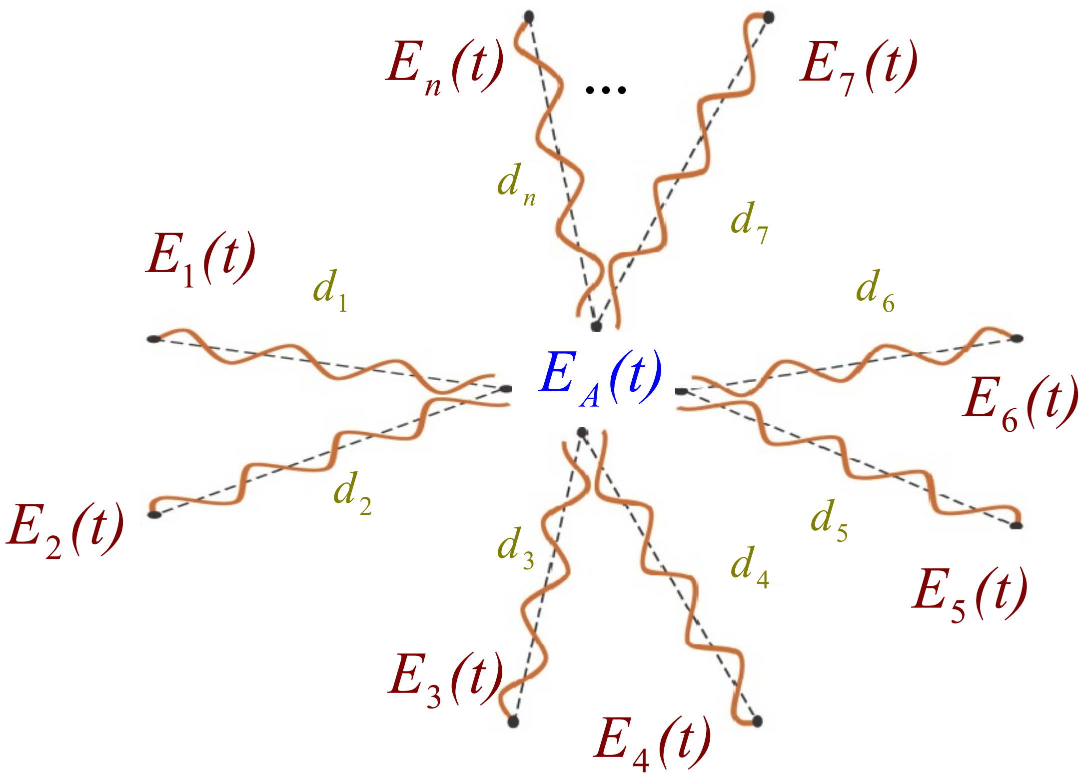
Spectrums of PWM pulse < Standard Level



1. Research Background

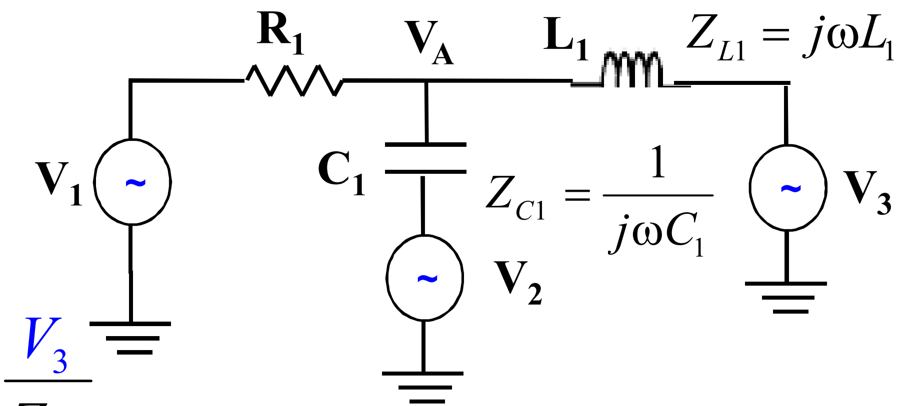
Superposition Principle

$$E_A(t)\sum_{i=1}^{n}\frac{1}{d_i}=\sum_{i=1}^{n}\frac{E_i(t)}{d_i}$$



1. Research Background

Example of Superposition Principle



$$V_A\left(\frac{1}{R_1}+\frac{1}{Z_{C1}}+\frac{1}{Z_{L1}}\right)=\frac{V_1}{R_1}+\frac{V_2}{Z_{C1}}+\frac{V_3}{Z_{L1}}$$

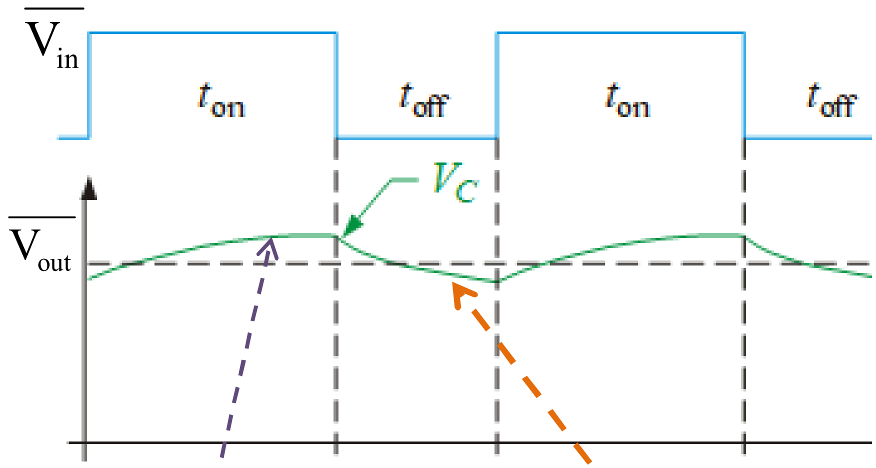
$$V_A\left(\frac{(R_1+Z_{C1})Z_{L1}+R_1Z_{C1}}{R_1Z_{C1}Z_{L1}}\right)=\frac{V_1}{R_1}+\frac{V_2}{Z_{C1}}+\frac{V_3}{Z_{L1}}$$

$$V_A=\frac{V_1Z_{C1}Z_{L1}+V_2R_1Z_{L1}+V_3R_1Z_{C1}}{(R_1+Z_{C1})Z_{L1}+R_1Z_{C1}}$$

$$V_A=\frac{V_1 j\omega L_1 - V_2\omega^2R_1L_1C_1+V_3R_1}{j\omega L_1-\omega^2R_1L_1C_1+R_1}$$

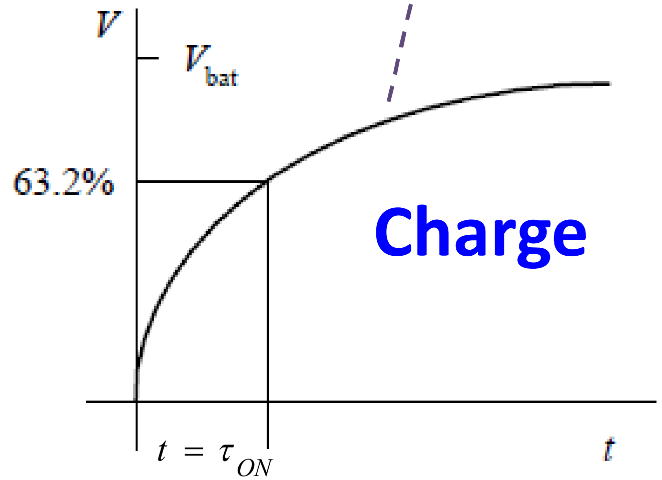
1. Research Background

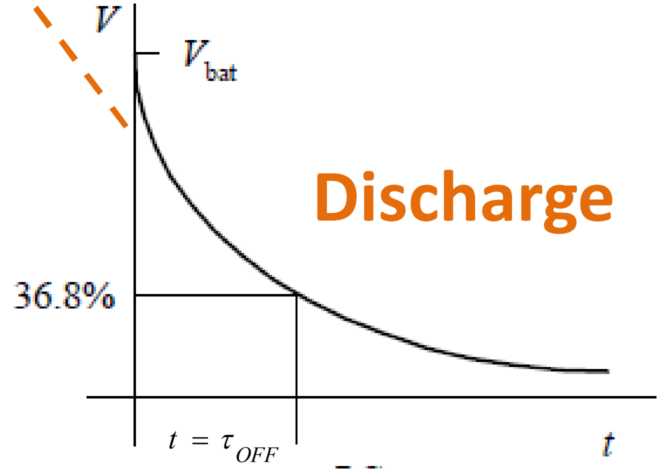
Switching Regulator



Independence of PWM Frequency

$$\overline{V_{out}} = \frac{T_{ON}}{\left(T_{ON} + T_{OFF}\right)} \overline{V_{in}}$$



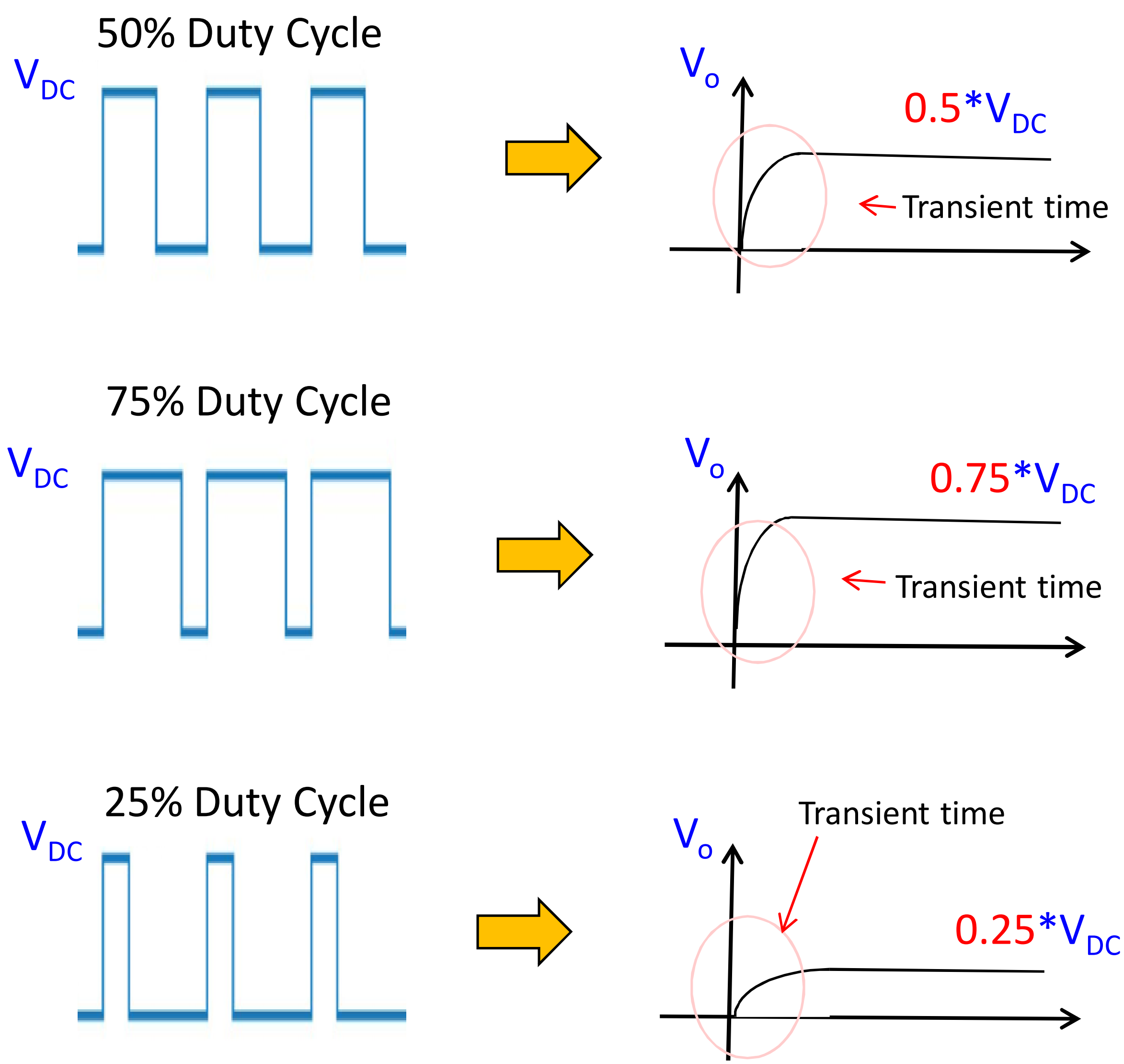


$$V_{Charge}(t_i) = \overline{V_{discharge}(t_{i-1})}\left(1 - e^{\frac{-t}{\tau_{ON}}}\right)$$

$$V_{discharge}(t_i) = \overline{V_{charge}(t_i)} e^{\frac{-t}{\tau_{OFF}}}$$

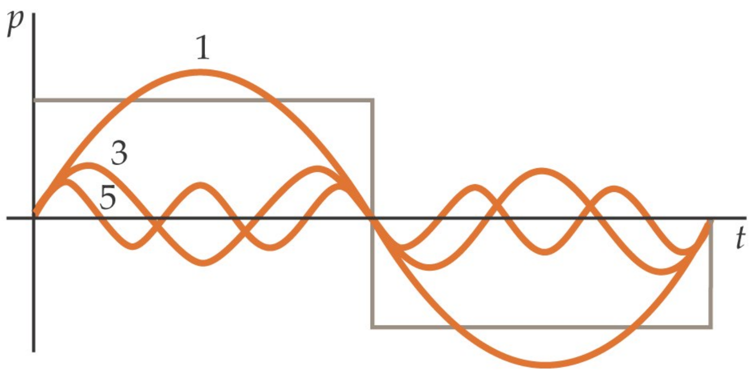
1. Research Background

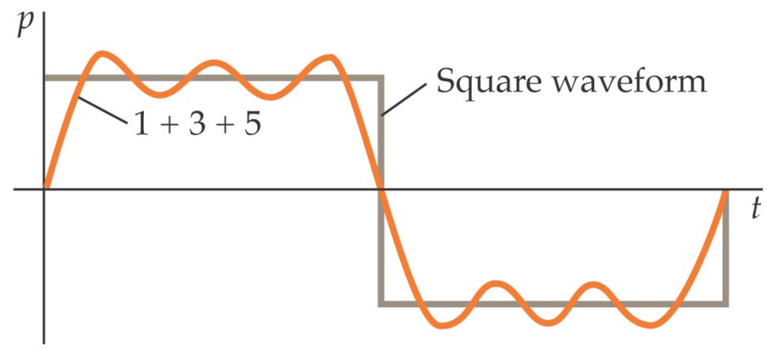
Analysis of Switching Control Sources



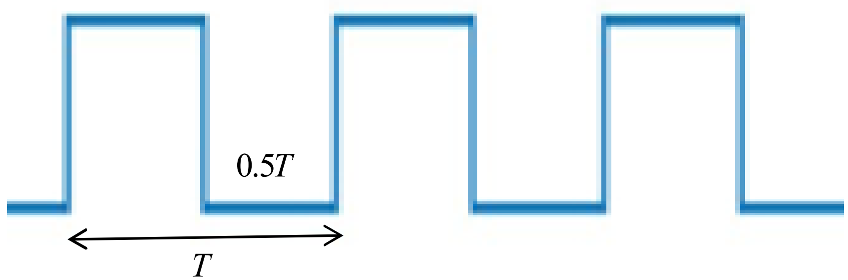
1. Research Background

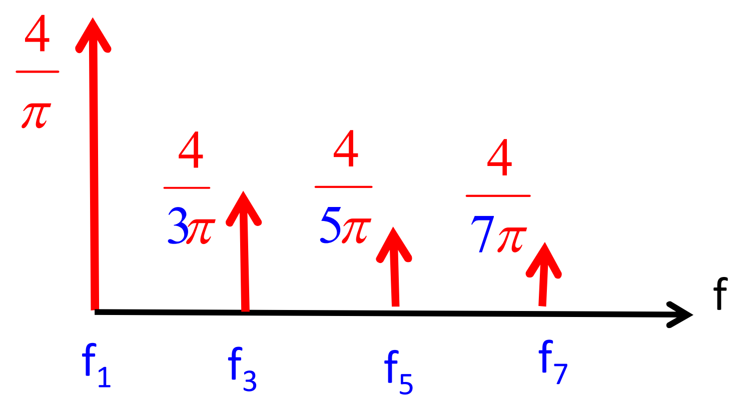
Analysis of Square Wave





$f_1 = 1/T$ 50% Duty Cycle





$$S_{PWM}(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin\left(2\pi(2k-1)(f_1)t\right)}{2k-1}$$

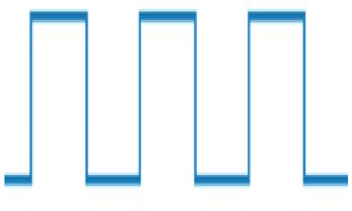
$$S_{PWM}(t) = \frac{4}{\pi}\sin(2\pi ft) + \frac{4}{3\pi}\sin(3*2\pi ft) + \frac{4}{5\pi}\sin(5*2\pi ft) + \ldots$$

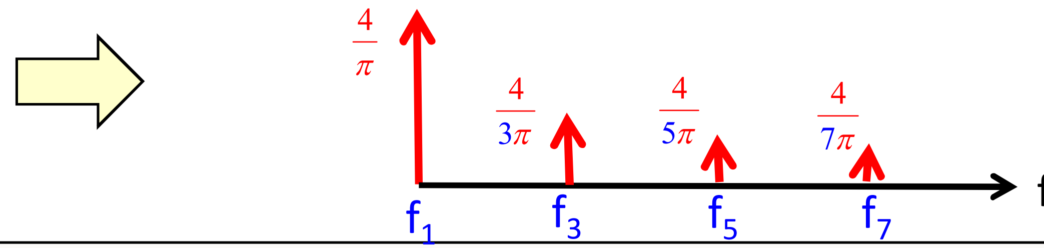
1. Research Background

Harmonics of PWM Signals

50% Duty Cycle

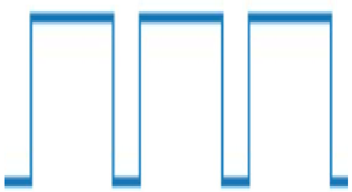
$$S_{PWM}(t)=\frac{4}{\pi}\left(\sin(2\pi ft)+\frac{1}{3}\sin(3*2\pi ft)+\frac{1}{5}\sin(5*2\pi ft)+...\right)$$

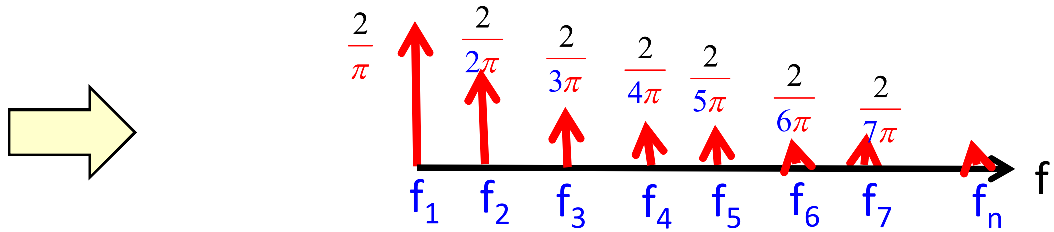




75% Duty Cycle

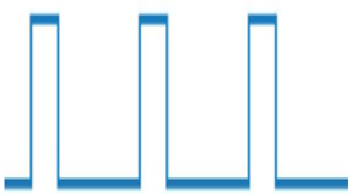
$$S_{PWM}(t)=\frac{4}{\pi}\left(\sin(2\pi ft)+\frac{1}{2}\sin(2*2\pi ft)+\frac{1}{3}\sin(3*2\pi ft)+\frac{1}{4}\sin(4*2\pi ft)+\frac{1}{5}\sin(5*2\pi ft)+...\right)$$

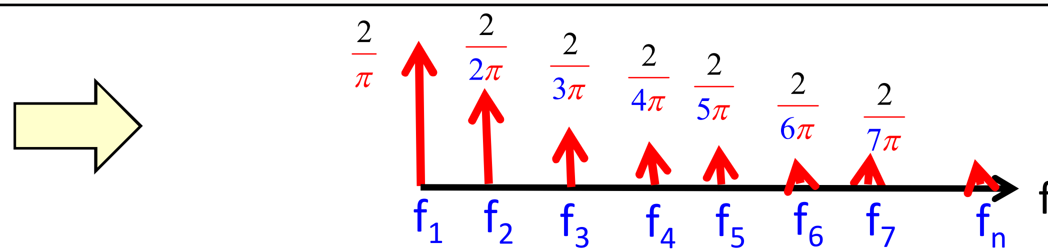




$$S_{PWM}(t)=\frac{4}{\pi}\left(\sin(2\pi ft)-\frac{1}{2}\sin(2*2\pi ft)+\frac{1}{3}\sin(3*2\pi ft)-\frac{1}{4}\sin(4*2\pi ft)+\frac{1}{5}\sin(5*2\pi ft)+...\right)$$

25% Duty Cycle

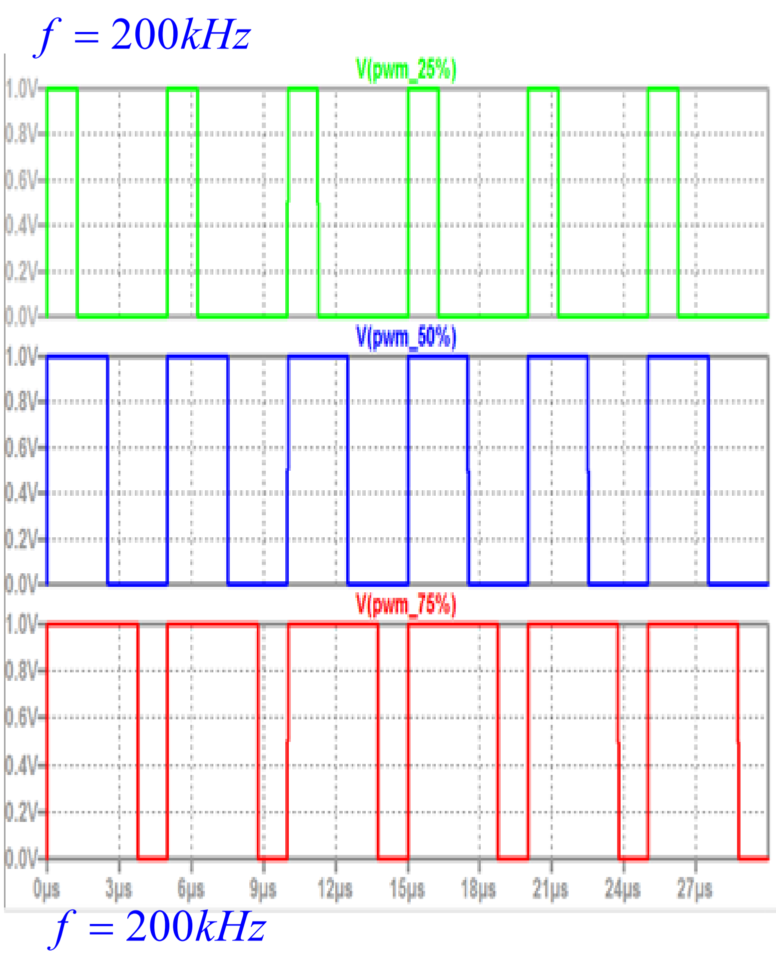




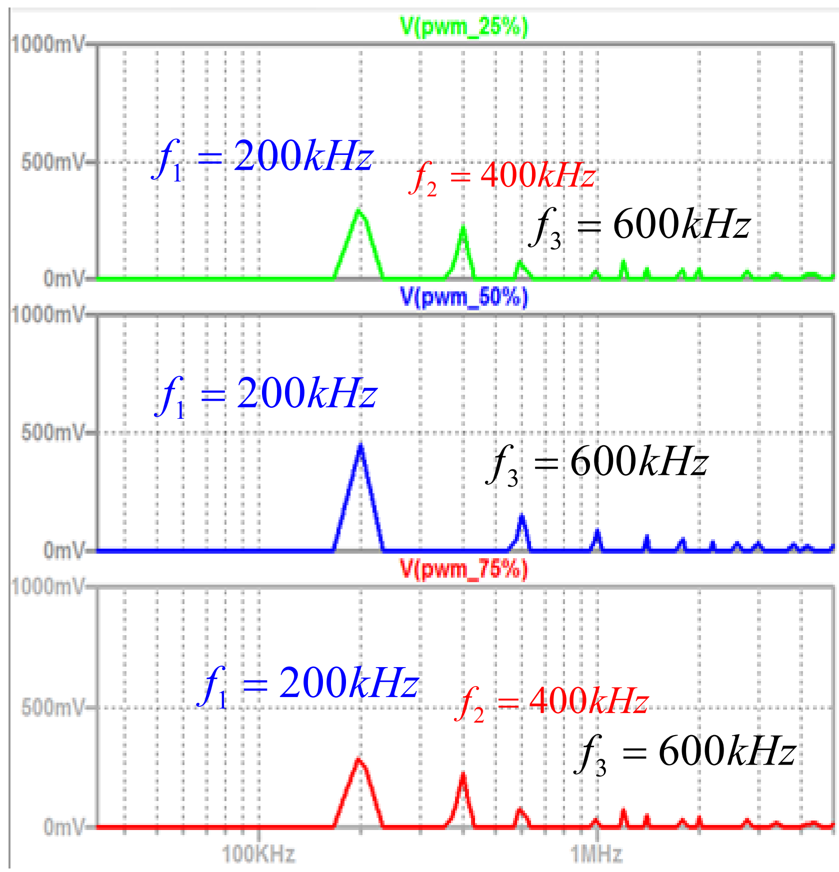
1. Research Background

Simulations of Harmonics of PWM Signals

Waveforms of PWM signals

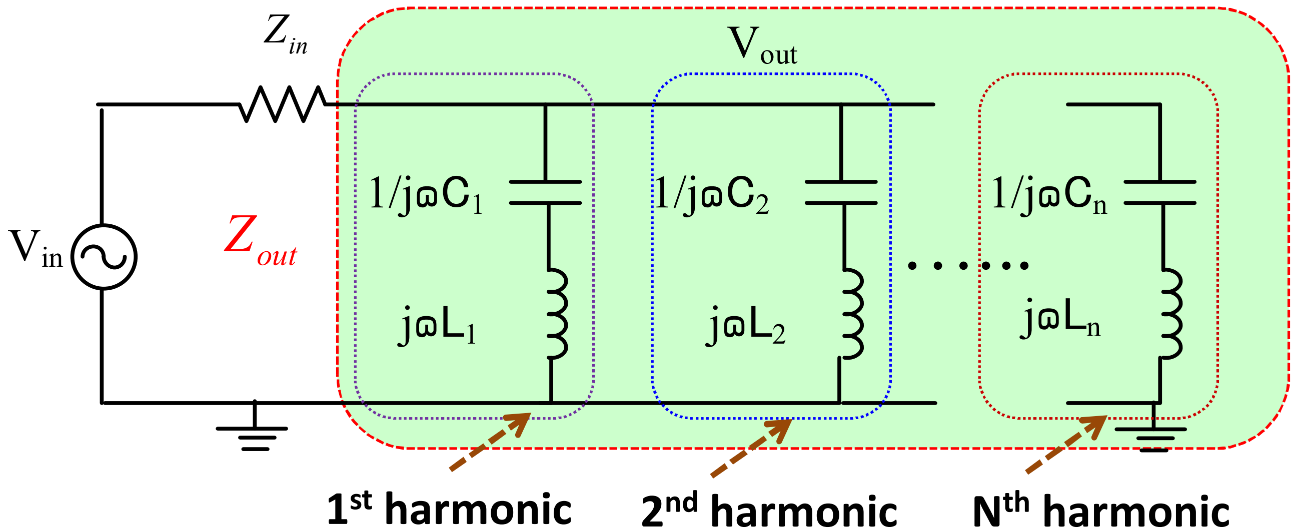


Spectrums of PWM signals



1. Research Background

Harmonic Notch Filters



$$Z_{C1} + Z_{L1} = \frac{1}{j\omega_1 C_1} + j\omega_1 L_1$$

$$= \frac{1 - \omega_1^2 C_1 L_1}{j\omega_1 C_1}$$

$$\Rightarrow \frac{1}{Z_{C1} + Z_{L1}} = \frac{j\omega_1 C_1}{1 - \omega_1^2 C_1 L_1}$$

Superposition principle

$$V_{out}\left(\frac{1}{Z_{in}} + \frac{1}{Z_{C1} + Z_{L1}} + \frac{1}{Z_{C2} + Z_{L2}} + \ \ldots + \frac{1}{Z_{Cn} + Z_{Ln}}\right) = \frac{V_{in}}{Z_{in}}$$

Output Voltage

$$V_{out}\left(\frac{1}{Z_{in}} + \frac{j\omega_1 C_1}{1 - \omega_1^2 C_1 L_1} + \frac{j\omega_2 C_2}{1 - \omega_2^2 C_2 L_2} + \ldots + \frac{j\omega_n C_n}{1 - \omega_n^2 C_n L_n}\right) = \frac{V_{in}}{Z_{in}}$$

$$\Rightarrow V_{out}\left[\frac{1}{Z_{in}} + \sum_{k=1}^{n}\left(\frac{j\omega_k C_k}{1 - \omega_k^2 C_k L_k}\right)\right] = \frac{V_{in}}{Z_{in}}$$

1. Research Background

Transfer Function of Harmonic Notch Filters

$$V_{out}\left[\frac{1}{Z_{in}}+\sum_{k=1}^{n}\left(\frac{j\omega_k C_k}{1-\omega_k^2 C_k L_k}\right)\right]=\frac{V_{in}}{Z_{in}}$$

Transfer Function

$$H(j\omega)=\frac{V_{out}}{V_{in}}=\frac{1}{Z_{in}\left[\frac{1}{Z_{in}}+\sum_{k=1}^{n}\left(\frac{j\omega_k C_k}{1-\omega_k^2 C_k L_k}\right)\right]}$$

$$H(j\omega)=\frac{1}{1+Z_{in}\sum_{k=1}^{n}\left(\frac{j\omega_k C_k}{1-\omega_k^2 C_k L_k}\right)}=\begin{cases}1 & ;\omega^2\neq\frac{1}{L_k C_k};Z_{in}\cong 0\\ 0 & ;\omega^2=\frac{1}{L_k C_k};Z_{in}\cong 0\end{cases}$$

$f_k = \omega_k / 2\pi$: notch frequency of $L_k C_k$ filter

1. Research Background

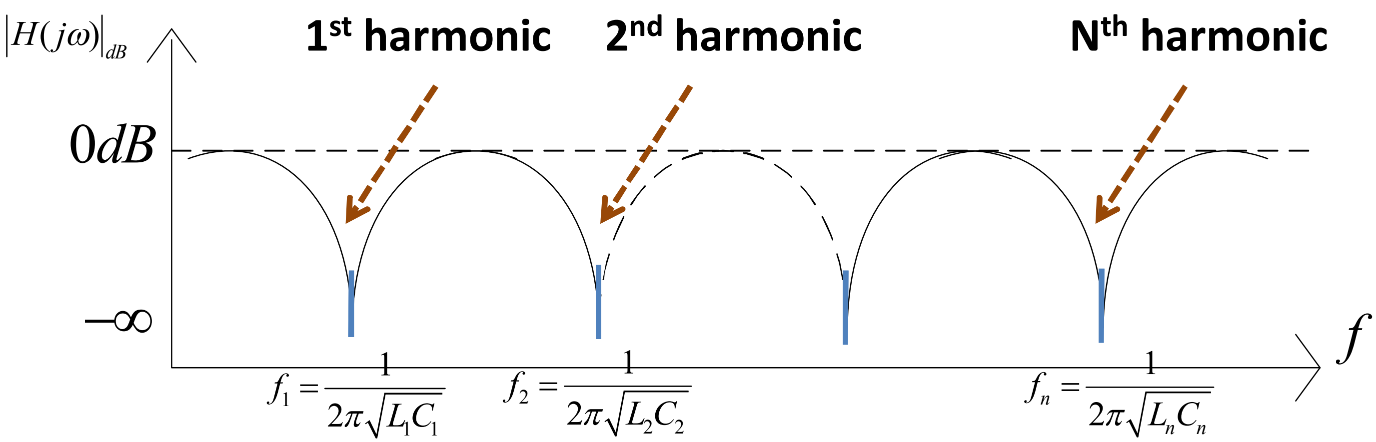
Frequency Response of Harmonic Notch Filters

$$|H(j\omega)| = 20\log\left(H(j\omega)\right) = \begin{cases} 0dB & ;\omega^2 \neq \dfrac{1}{L_k C_k}; Z_{in} << 1 \\ -\infty & ;\omega^2 = \dfrac{1}{L_k C_k}; Z_{in} << 1 \end{cases}$$

Quality factor

$$Q_{quality\_factor} = \left\|\frac{1}{Z_{L_k}}\right\| = \left\|\frac{1}{Z_{C_k}}\right\| \cong \infty$$

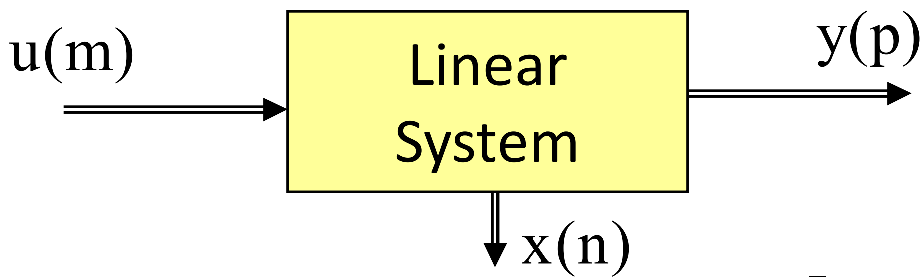
$$f_k = \frac{\omega_k}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{L_k C_k}}$$



Outline

2. Analysis of Step-down Switching Converter

Conventional State-Space Technique



$$u(m) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_i(t) \\ u_m(t) \end{bmatrix} \qquad x(n) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_i(t) \\ x_n(t) \end{bmatrix} \qquad y(p) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_i(t) \\ y_p(t) \end{bmatrix}$$

Input vector   State vector   Output vector

$$\boxed{\begin{aligned} \dot{x} &= \frac{dx}{dt} = A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) + D(t)u(t) \end{aligned}}$$

- **Advantages**
  - **Modeling, analyzing, and designing a wide range of systems**
  - **Nonlinear, time-varying, multivariable systems**
- **Disadvantages**
  - **Not as intuitive as classical method**

2. Analysis of Step-down Switching Converter

Conventional State-Space Technique

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

Laplace transform

$$sX(s) - x(0) = AX(s) + BU(s)$$

$$Y(s) = CX(s) + DU(s)$$

$$assume \quad x(0) = 0$$

$$X(s) = (sI - A)^{-1} BU(s)$$

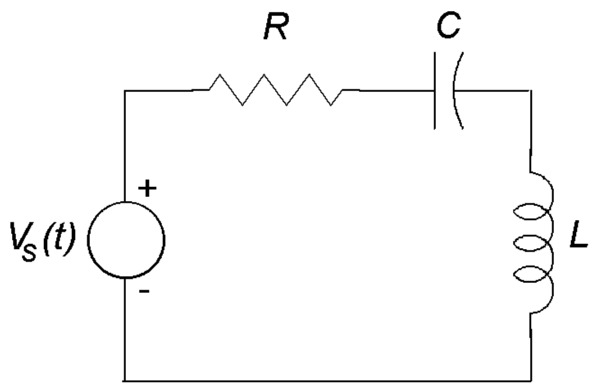
$$Y(s) = [C(sI - A)^{-1} B + D]U(s)$$

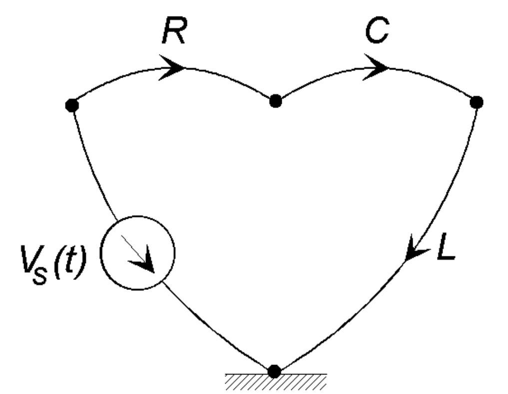
$$\frac{Y(s)}{U(s)} = C[sI - A]^{-1} B + D = \frac{C adj[sI - A]B + \det[sI - A]D}{\det[sI - A]}$$

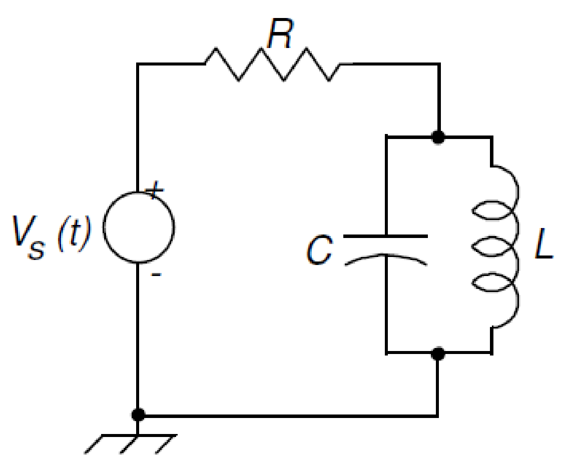
2. Analysis of Step-down Switching Converter

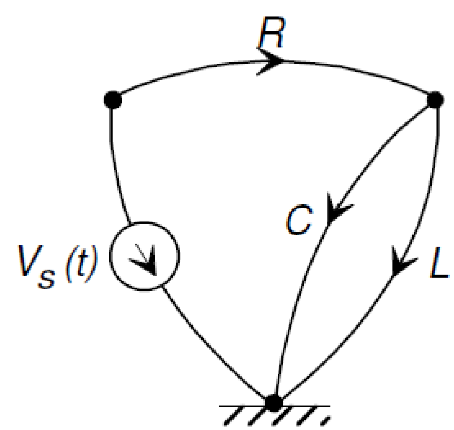
Linear Graph Models of Network

Electronic circuits | **Linear graph models**



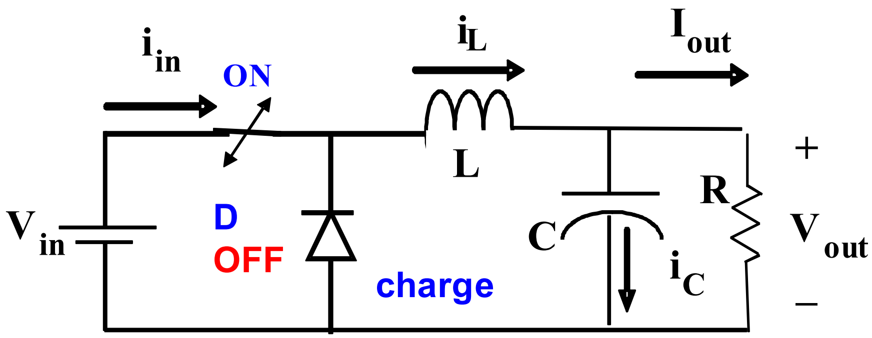




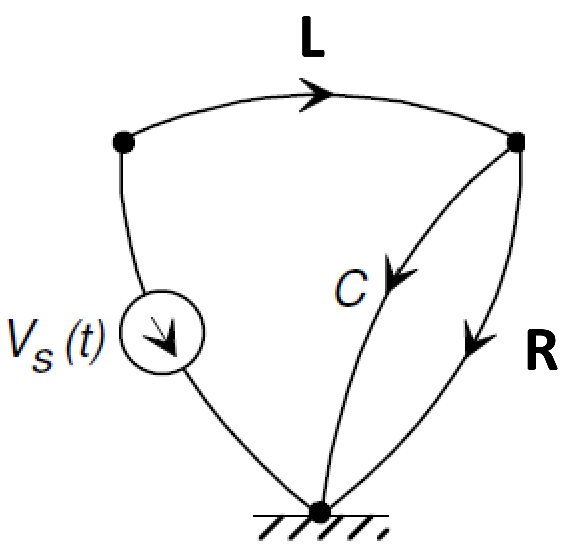


2. Analysis of Step-down Switching Converter

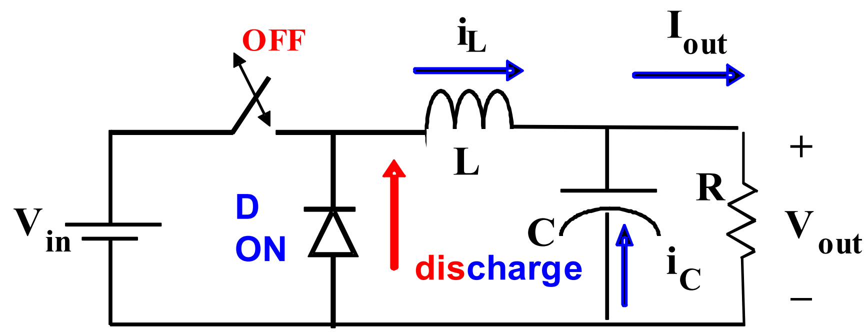
Linear Graph Models of Buck Converter



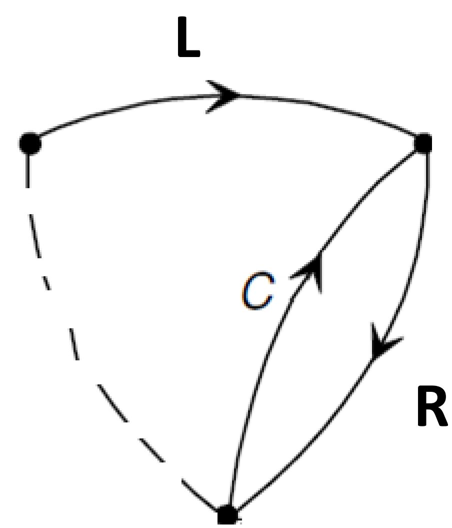
Switch ON



Linear graph



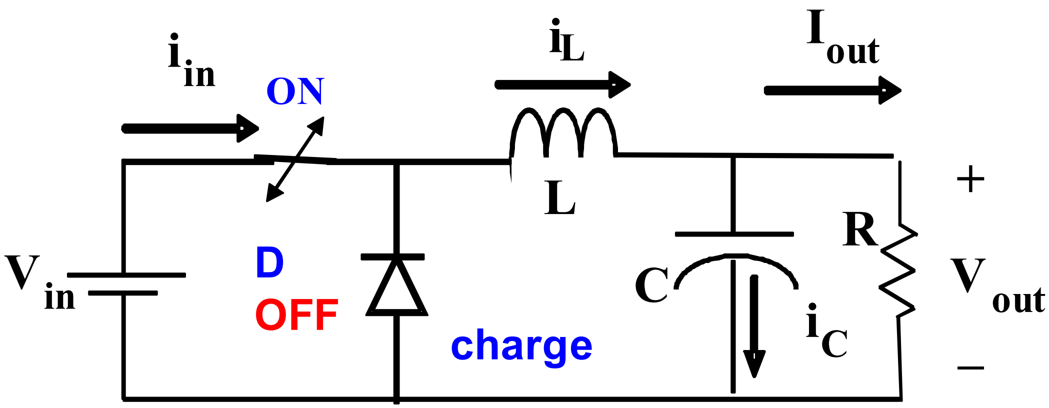
Switch OFF

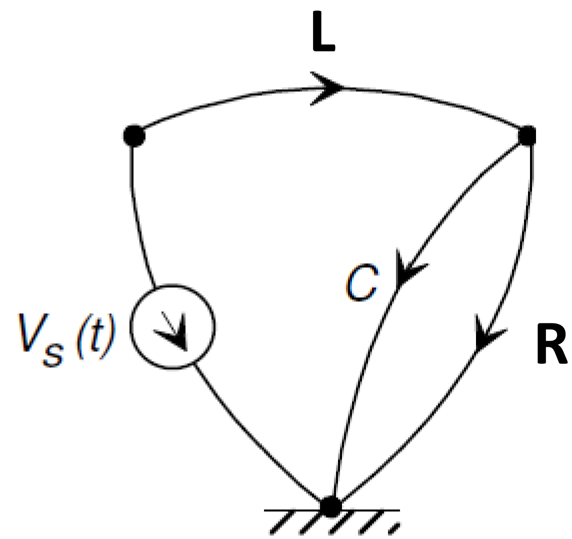


Linear graph

2. Analysis of Step-down Switching Converter

Conventional State-Space Technique (Switch ON)





Linear graph

$$\begin{bmatrix} \dot{i}_L(t) \\ \dot{v}_C(t) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_i(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + [0] v_i(t)$$

2. Analysis of Step-down Switching Converter

Conventional State-Space Technique (Switch ON)

Laplace Transform

$$\begin{cases} sI_L(s) & = & 0I_L(s) - \frac{1}{L}V_C(s) + \frac{1}{L}V_i(s) \\ sV_C(s) & = & \frac{I_L(s)}{C} - \frac{V_C(s)}{RC} + 0V_i(s) \end{cases}$$

$$\Rightarrow \begin{cases} \frac{1}{L}V_C(s) + sC\left(s + \frac{1}{RC}\right)V_C(s) & = & \frac{1}{L}V_i(s) \\ I_L(s) & = & C\left(s + \frac{1}{RC}\right)V_C(s) \end{cases}$$

$$\Rightarrow V_C(s) = \frac{\frac{1}{LC}}{\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right)}V_i(s)$$

$$\Rightarrow \boxed{\frac{d^2 v_C}{dt^2} + \frac{1}{RC}\frac{d(v_C)}{dt} + \frac{1}{LC}v_C = \frac{1}{LC}v_i} \qquad v_C(t) = V_{out}(t)$$

2. Analysis of Step-down Switching Converter

Conventional State-Space Technique (Switch ON)

$$\frac{d^2V_{out}(t)}{dt^2}+\frac{1}{RC}\frac{dV_{out}(t)}{dt}+\frac{V_{out}(t)}{LC}=0$$

$$V_{out}(t)=Ae^{st}=A_1e^{s_1t}+A_2e^{s_2t}$$

$$\frac{d^2\left(Ae^{st}\right)}{dt^2}+\frac{1}{RC}\frac{d\left(Ae^{st}\right)}{dt}+\frac{\left(Ae^{st}\right)}{LC}=0$$

$$s^2+\frac{1}{RC}s+\frac{1}{LC}=0$$

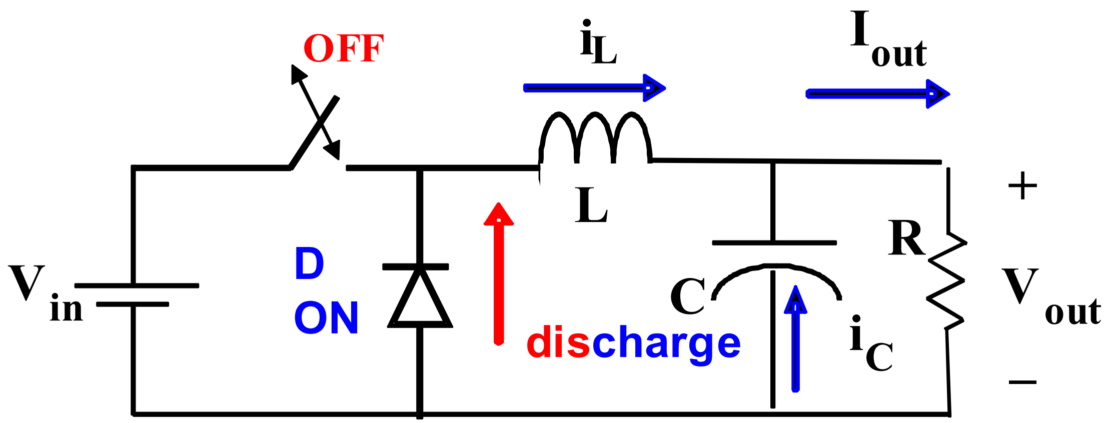
$$s_1=-\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2-\frac{1}{LC}} \quad \vee \quad s_2=-\frac{1}{2RC}-\sqrt{\left(\frac{1}{2RC}\right)^2-\frac{1}{LC}}$$

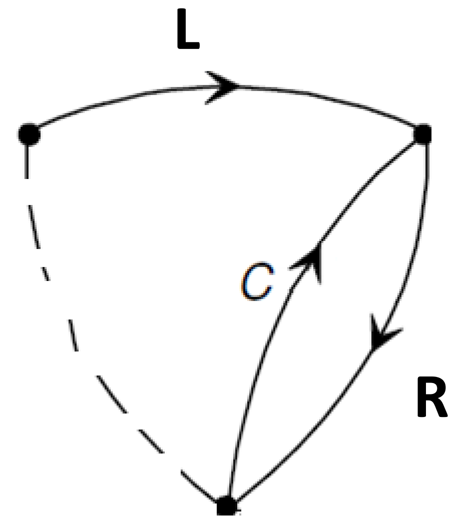
$$\omega_{2RC}=\frac{1}{2RC};\omega_{LC}=\frac{1}{\sqrt{LC}};$$

$$V_{charge}(t)=A_{ch1}e^{\left(-\omega_{2RC}+\sqrt{(\omega_{2RC})^2-\omega_{LC}^2}\right)t}+A_{ch2}e^{\left(-\omega_{2RC}-\sqrt{(\omega_{2RC})^2-\omega_{LC}^2}\right)t}$$

2. Analysis of Step-down Switching Converter

Conventional State-Space Technique (Switch OFF)





Linear graph

$$\begin{bmatrix} \dot{i}_L(t) \\ \dot{v}_C(t) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_i(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + [0] v_i(t)$$

2. Analysis of Step-down Switching Converter

Conventional State-Space Technique (Switch OFF)

Laplace Transform

$$\begin{cases} sI_L(s) & = & 0I_L(s) - \frac{1}{L}V_C(s) \\ sV_C(s) & = & -\frac{I_L(s)}{C} - \frac{V_C(s)}{RC} + 0V_i(s) \end{cases}$$

$$\Rightarrow \begin{cases} \frac{1}{L}V_C(s) - sC\left(s + \frac{1}{RC}\right)V_C(s) & = & 0 \\ I_L(s) & = & -C\left(s + \frac{1}{RC}\right)V_C(s) \end{cases}$$

$$\Rightarrow \left(s^2 - \frac{s}{RC} + \frac{1}{LC}\right)V_C(s) = 0$$

$$\Rightarrow \boxed{\frac{d^2 v_C}{dt^2} - \frac{1}{RC}\frac{d(v_C)}{dt} + \frac{1}{LC}v_C = 0}$$

$$v_C(t) = V_{dis}(t)$$

2. Analysis of Step-down Switching Converter

Conventional State-Space Technique (Switch OFF)

$$\frac{d^2V_{dis}(t)}{dt^2}-\frac{1}{RC}\frac{dV_{dis}(t)}{dt}+\frac{V_{dis}(t)}{LC}=0$$

$$V_{dis}(t)=A_{dis}e^{st}=A_3e^{s_{dis1}t}+A_3e^{s_{dis2}t}$$

$$\frac{d^2\left(A_{dis}e^{st}\right)}{dt^2}-\frac{1}{RC}\frac{d\left(A_{dis}e^{st}\right)}{dt}+\frac{\left(A_{dis}e^{st}\right)}{LC}=0$$

$$s^2-\frac{1}{RC}s+\frac{1}{LC}=0$$

$$s_{dis1}=\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2-\frac{1}{LC}} \quad \vee \quad s_{dis2}=\frac{1}{2RC}-\sqrt{\left(\frac{1}{2RC}\right)^2-\frac{1}{LC}}$$

$$\omega_{2RC}=\frac{1}{2RC};\omega_{LC}=\frac{1}{\sqrt{LC}};$$

$$V_{discharge}(t)=A_{dis1}e^{\left(\omega_{2RC}+\sqrt{\left(\omega_{2RC}\right)^2-\omega_{LC}^2}\right)t}+A_{dis2}e^{\left(\omega_{2RC}+\sqrt{\left(\omega_{2RC}\right)^2-\omega_{LC}^2}\right)t}$$

2. Analysis of Step-down Switching Converter

Conventional State-Space Technique

$$\overline{V_{out}} = \frac{1}{(T_{ON}+T_{OFF})}\left(\begin{aligned} &\int_0^{T_{ON}} \left(A_{ch1} e^{\left(-\omega_{2RC}+\sqrt{(\omega_{2RC})^2-\omega_{LC}^2}\right)t} + A_{ch2} e^{\left(-\omega_{2RC}-\sqrt{(\omega_{2RC})^2-\omega_{LC}^2}\right)t} \right) dt \\ &+ \int_{T_{ON}}^{T_{OFF}} \left\{ A_{dis1} e^{\left(\omega_{2RC}+\sqrt{(\omega_{2RC})^2-\omega_{LC}^2}\right)t} + A_{dis2} e^{\left(\omega_{2RC}-\sqrt{(\omega_{2RC})^2-\omega_{LC}^2}\right)t} \right\} dt \end{aligned} \right)$$

$$\omega_{2RC} = \omega_{LC} \Leftrightarrow \omega = \frac{1}{\sqrt{LC}} = \frac{1}{2RC}$$

$$\omega L = \frac{1}{\omega C} = 2R$$

$$|Z_L| = |Z_C| = 2R$$

Balanced Charge-Discharge Time Condition

$$\overline{V_{out}} = \frac{1}{(T_{ON}+T_{OFF})}\left(\int_0^{T_{ON}} A_{ch} e^{-\omega t} dt + \int_{T_{ON}}^{T_{OFF}} A_{dis} e^{\omega t} dt \right)$$

2. Analysis of Step-down Switching Converter

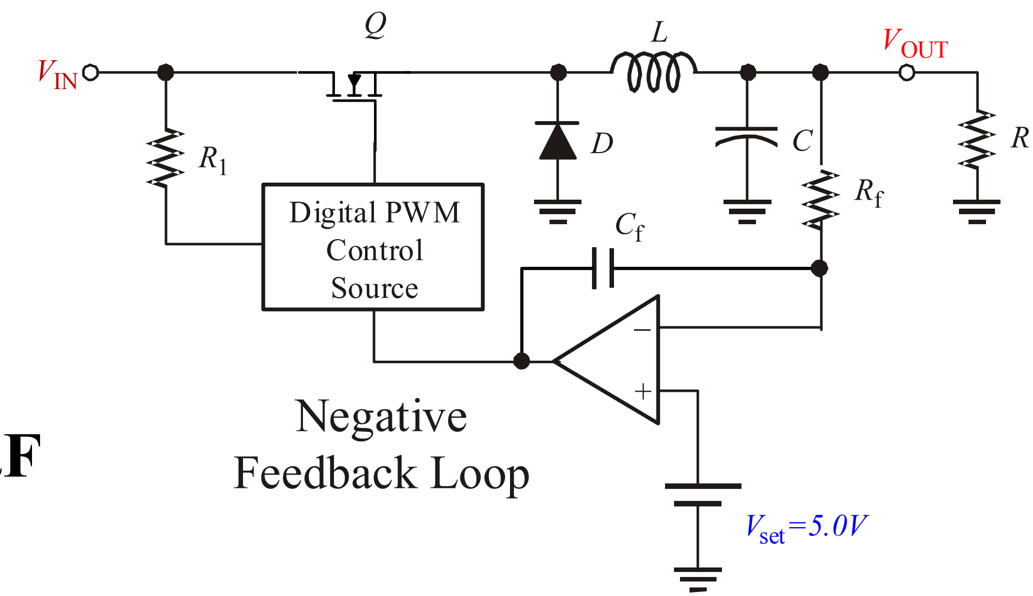
Conventional Switching Buck Converter

Input Voltage (Vin)	12V
Output Voltage (Vo)	5.0V
Output Current (Io)	1A
Clock Frequency (Fck)	100kHz

R = 5Ω, L = 318µH, C = 3.18µF

$$f_{cut\_off} = \frac{1}{2\pi\sqrt{LC}} = 5kHz$$

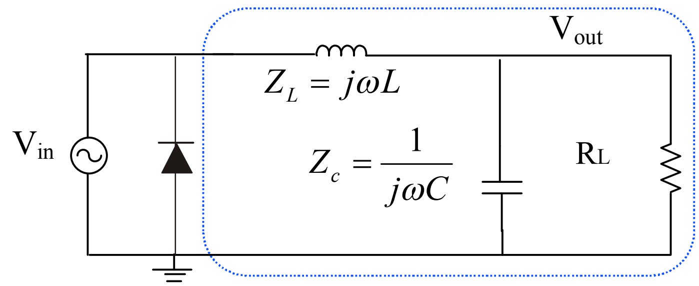
Switching Buck Converter



2. Analysis of Step-down Switching Converter

Analysis Model of Buck Converter

Proposed analysis model



Superposition principle

$$V_o\left(\frac{1}{Z_L}+\frac{1}{Z_C}+\frac{1}{R}\right)=\frac{V_{in}}{Z_L}$$

Output Voltage

$$V_o = V_{in}\frac{RZ_C}{R\left(Z_L+Z_C\right)+Z_LZ_C}$$

Transfer Function

$$H=\frac{V_o}{V_{in}}=\frac{RZ_C}{R\left(Z_L+Z_C\right)+Z_LZ_C}$$

$$H\left(j\omega\right)=\frac{\frac{1}{LC}}{\left(j\omega\right)^2+j\omega\frac{1}{RC}+\frac{1}{LC}}$$

2. Analysis of Step-down Switching Converter

Balanced Charge-Discharge Time Condition

Transfer Function

$$H(j\omega)=\frac{\frac{1}{LC}}{(j\omega)^2+j\omega\frac{1}{RC}+\frac{1}{LC}} \Rightarrow H(j\omega)=\frac{\frac{1}{LC}}{(j\omega)^2+2j\omega\frac{1}{2RC}+\left(\frac{1}{2RC}\right)^2+\frac{1}{LC}-\left(\frac{1}{2RC}\right)^2}$$

$$H(j\omega)=\frac{\frac{1}{LC}}{\left(j\omega+\frac{1}{2RC}\right)^2+\frac{1}{LC}-\left(\frac{1}{2RC}\right)^2}$$

Maximum power

$$\Rightarrow \boxed{\frac{1}{LC}-\left(\frac{1}{2RC}\right)^2=0}$$

$$\Rightarrow \omega L=\frac{1}{\omega C}=2R$$

$$|Z_L|=|Z_C|=2R$$

Balanced Charge-Discharge Time Condition

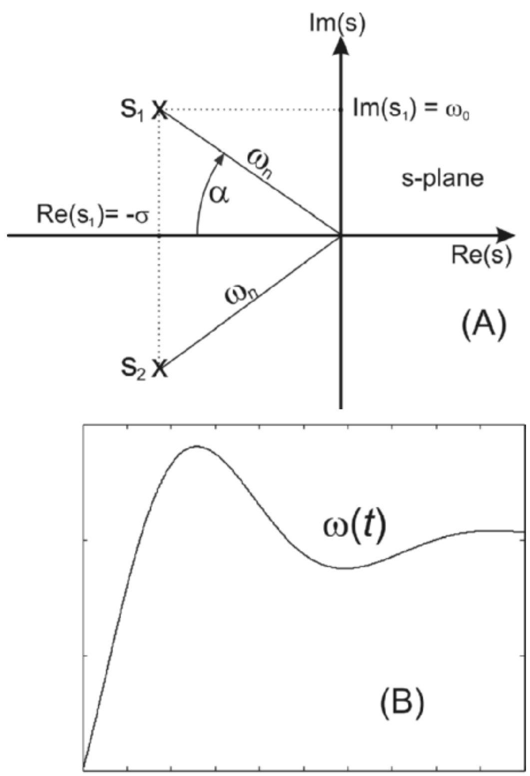
2. Analysis of Step-down Switching Converter

Transient Response of Buck Converter

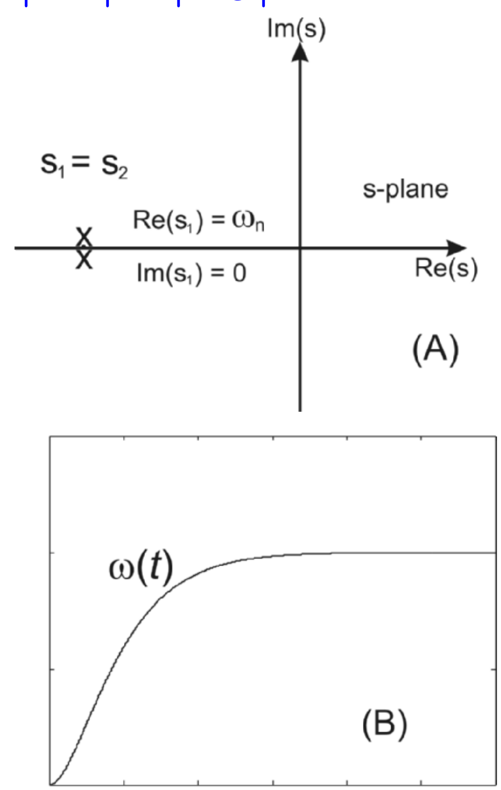
Transfer Function

$$H(j\omega)=\frac{\frac{1}{LC}}{\left(j\omega+\frac{1}{2RC}\right)^2+\frac{1}{LC}-\left(\frac{1}{2RC}\right)^2}$$

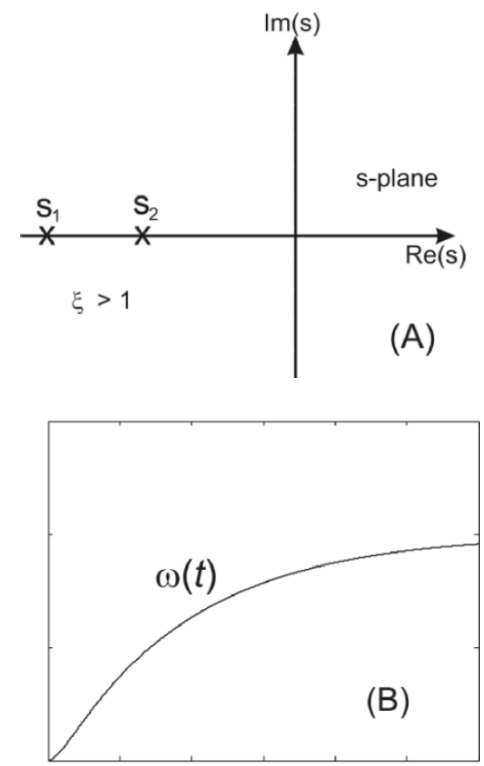
$$|Z_L|=|Z_C|<2R$$



$$|Z_L|=|Z_C|=2R$$



$$|Z_L|=|Z_C|>2R$$



2. Analysis of Step-down Switching Converter

Max Power Propagation

Transfer Function

$$H(j\omega) = \frac{\frac{1}{LC}}{\left(j\omega + \frac{1}{2RC}\right)^2 + \frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$$

Max Power

$$\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2 = 0 \quad \Rightarrow \quad \omega = \frac{1}{\sqrt{LC}} = \frac{1}{2RC}$$

Rewritten Transfer Function

$$H(j\omega) = \frac{\frac{1}{LC}}{\left(j\omega + \frac{1}{2RC}\right)^2} \quad \Rightarrow \quad |H(\omega)| = \frac{\frac{1}{LC}}{\left(\frac{1}{2RC}\right)^2 + \omega^2}$$

$$\omega_{cut\_off} = \frac{1}{\sqrt{LC}} = \frac{1}{2RC} \qquad or \qquad f_{cut\_off} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{4\pi RC}$$

$$|H(\omega)| = \frac{1}{2} \qquad or \qquad |H(\omega)|_{dB} = 20\log\left(\frac{1}{2}\right) = -3dB$$

2. Analysis of Step-down Switching Converter

Time Behavior of Buck Converter

Transfer Function

$$H(j\omega) = \frac{\frac{1}{LC}}{\left(j\omega + \frac{1}{2RC}\right)^2}$$

Here $s = j\omega$

$$\Rightarrow \quad H(s) = \frac{\frac{1}{LC}}{\left(j\omega + \frac{1}{2RC}\right)^2} = \frac{\omega^2}{(s+\omega)^2}$$

Laplace Inversion Form

$$h(t) = \mathcal{L}^{-1}\left\{\frac{\omega^2}{(s+\omega)^2}\right\} = \omega^2 t e^{-\omega t}$$

Output Voltage

$$\frac{V_{out}(t)}{V_{in}(t)} = h(t) = \omega^2 t e^{-\omega t} \Rightarrow V_{out}(t) = \omega^2 t e^{-\omega t} V_{in}(t)$$

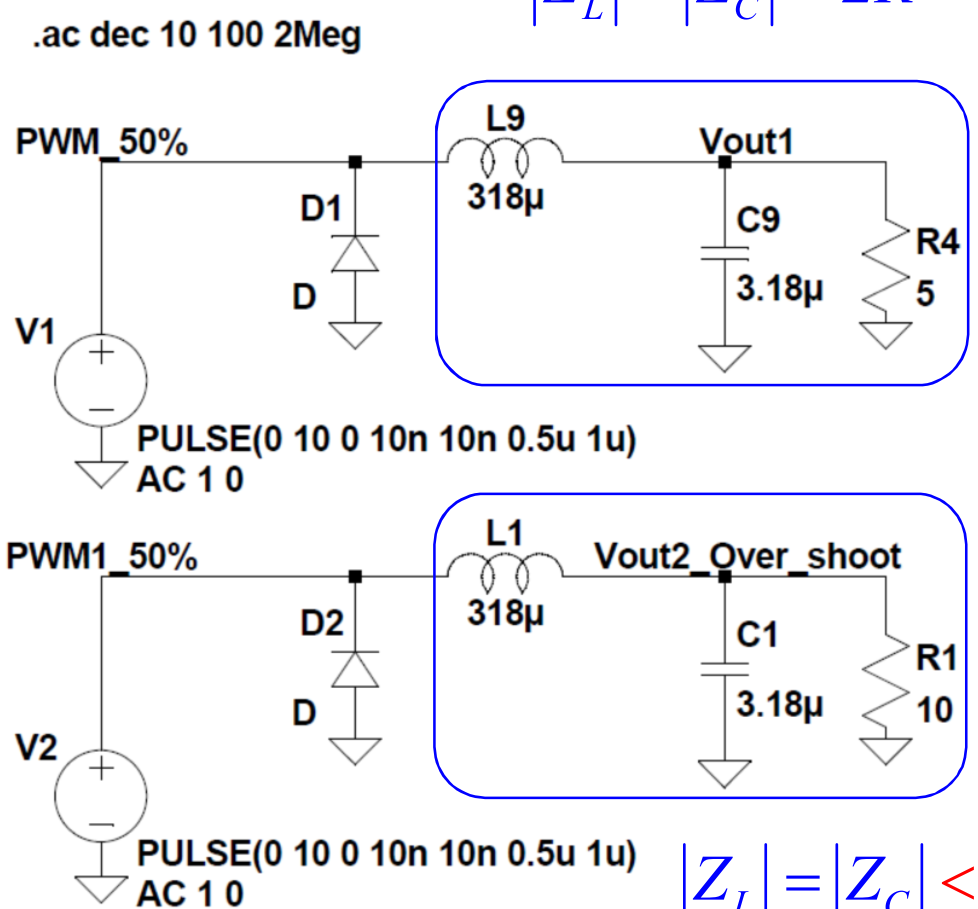
$$V_{out}(t) = \left(\frac{1}{2RC}\right)^2 t e^{-\left(\frac{1}{2RC}\right)t} V_{in}(t)$$

Here $f_{cut\_off} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{4\pi RC}$

2. Analysis of Step-down Switching Converter

Simulation of Balanced Charge-Discharge Time

$$|Z_L| = |Z_C| = 2R$$



$$|Z_L| = |Z_C| < 2R$$

Cutoff frequency

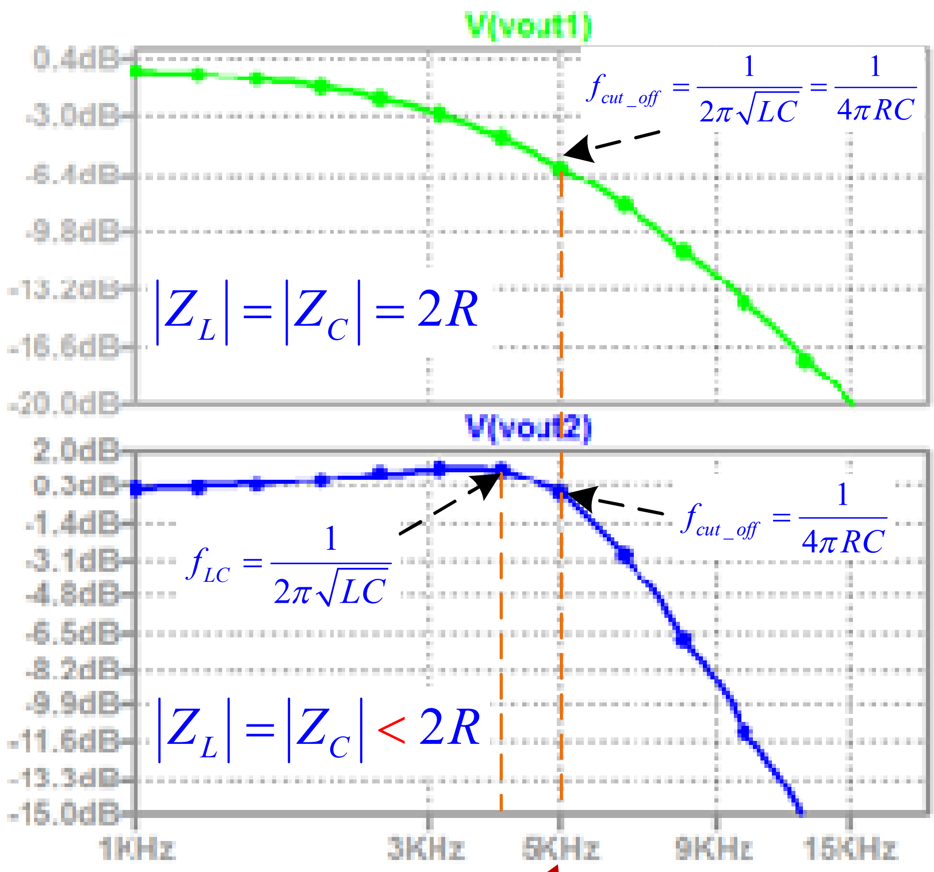
$$f_{cut\_off} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{4\pi RC} = 5kHz$$

L	318uH
Z_L	j10
C	3.18uF
Z_C	-j10
R_L	5
f	5kHz

2. Analysis of Step-down Switching Converter

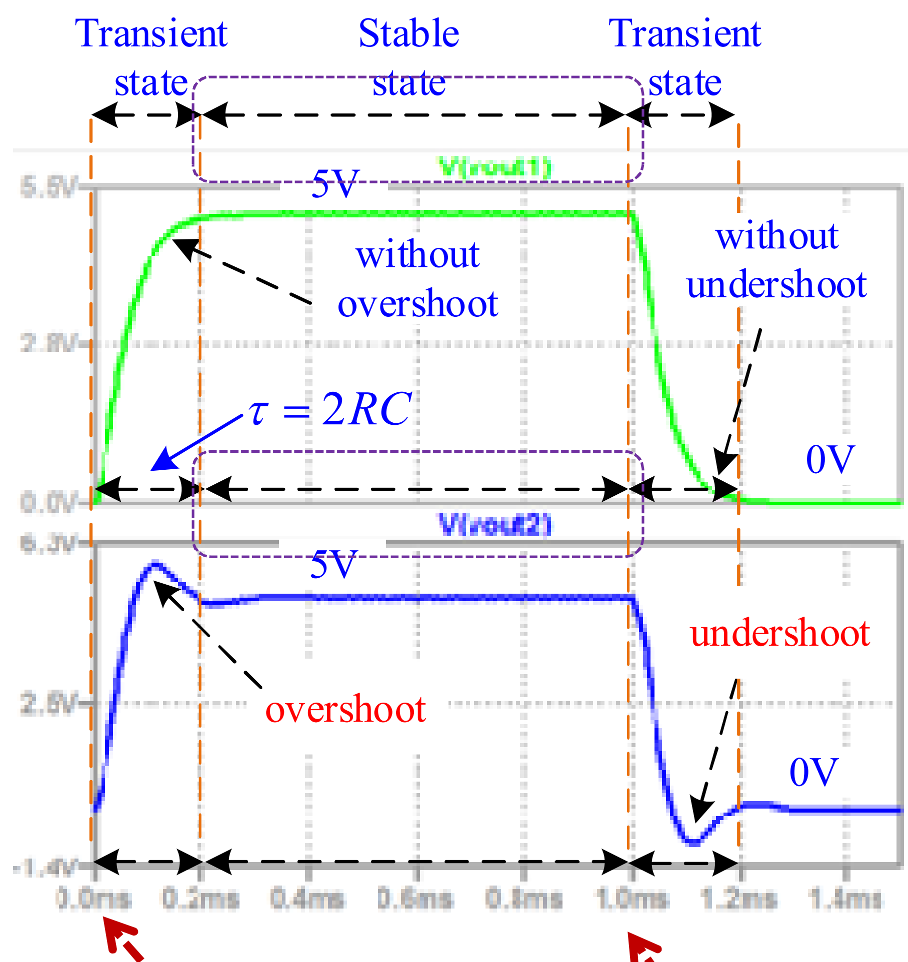
Waveforms of Balanced Charge-Discharge Time

Frequency Response



Cutoff frequency

Step Response

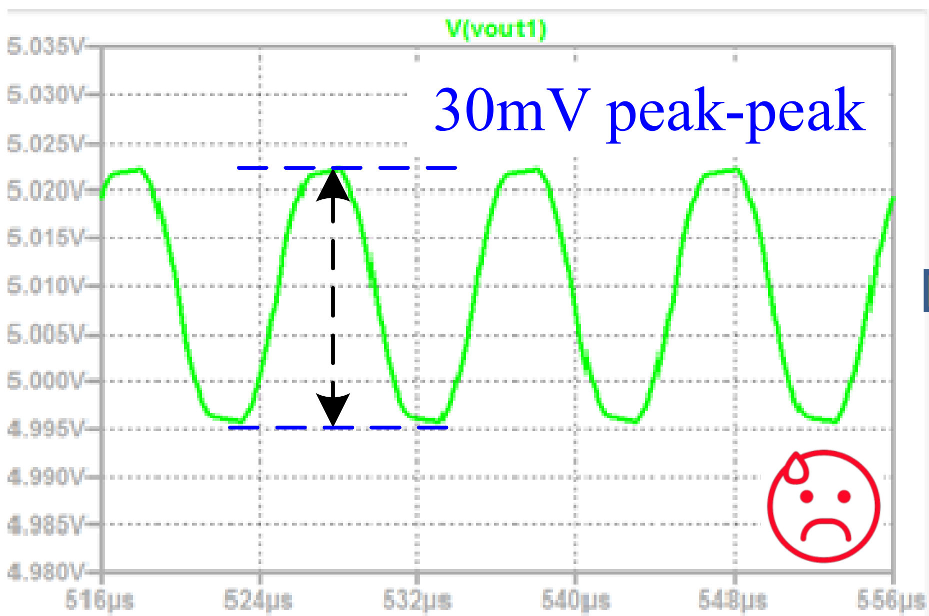


Power On **Power Off**

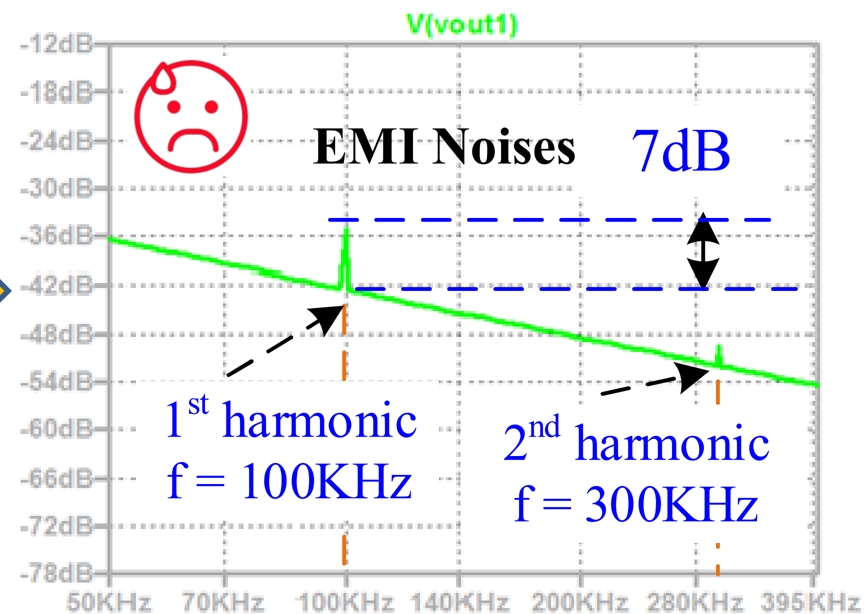
2. Analysis of Step-down Switching Converter

Ripple Voltages and EMI Noises

Ripple voltages



Spectrum of ripple voltages



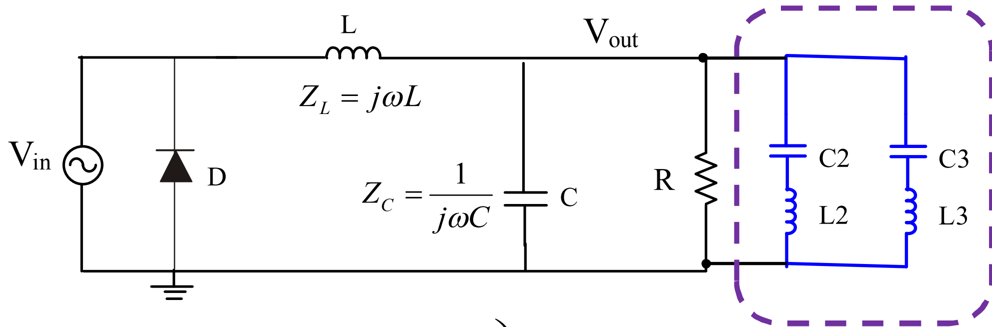
Outline

1. Research Background
   - Applications of Switching Power Supply
   - Basic Switching Converter Architecture
2. Analysis of Step-down Switching Converter
   - Conventional State-Space Technique
   - Superposition Principle
3. **Proposed Design of Buck Converter**
   - **Ripple Voltage Reduction with Notch Harmonic Filters**
   - **Simulation Results**
4. Conclusions

3. Proposed Design of Buck Converter

Proposed Analysis Model of Buck Converter

Proposed analysis model



Notch harmonics filters

$$V_{out}\left(\frac{1}{Z_L}+\frac{1}{Z_C}+\frac{1}{R}+\frac{1}{Z_{C2}+Z_{L2}}+\frac{1}{Z_{C3}+Z_{L3}}\right)=\frac{V_{in}}{Z_L}$$

$$V_{out}=V_{in}\frac{Z_C R\left(Z_{C2}+Z_{L2}\right)\left(Z_{C3}+Z_{L3}\right)}{\left\{\left[R\left(Z_C+Z_L\right)+Z_C Z_L\right]\left(Z_{C2}+Z_{L2}\right)+RZ_C Z_L\right\}\left(Z_{C3}+Z_{L3}\right)+RZ_C Z_L\left(Z_{C2}+Z_{L2}\right)}$$

Transfer Function

$$H=\frac{V_o}{V_{in}}=\frac{Z_C R\left(Z_{C2}+Z_{L2}\right)\left(Z_{C3}+Z_{L3}\right)}{\left\{\left[R\left(Z_C+Z_L\right)+Z_C Z_L\right]\left(Z_{C2}+Z_{L2}\right)+RZ_C Z_L\right\}\left(Z_{C3}+Z_{L3}\right)+RZ_C Z_L\left(Z_{C2}+Z_{L2}\right)}$$

$$H\left(j\omega\right)=\frac{\frac{1}{LC}\left(\frac{1}{L_2C_2}+\left(j\omega\right)^2\right)\left(\frac{1}{L_3C_3}+\left(j\omega\right)^2\right)}{\left\{\left[\left(j\omega\right)^2+\frac{1}{RC}\left(j\omega\right)+\frac{1}{LC}\right]\left(\frac{1}{L_2C_2}+\left(j\omega\right)^2\right)+\frac{1}{LC}\frac{\left(j\omega\right)}{L_2}\right\}\left(\frac{1}{L_3C_3}+\left(j\omega\right)^2\right)+\frac{1}{LC}\frac{\left(j\omega\right)}{L_3}\left(\frac{1}{L_2C_2}+\left(j\omega\right)^2\right)}$$

3. Proposed Design of Buck Converter

Simulation of Proposed Buck Converter

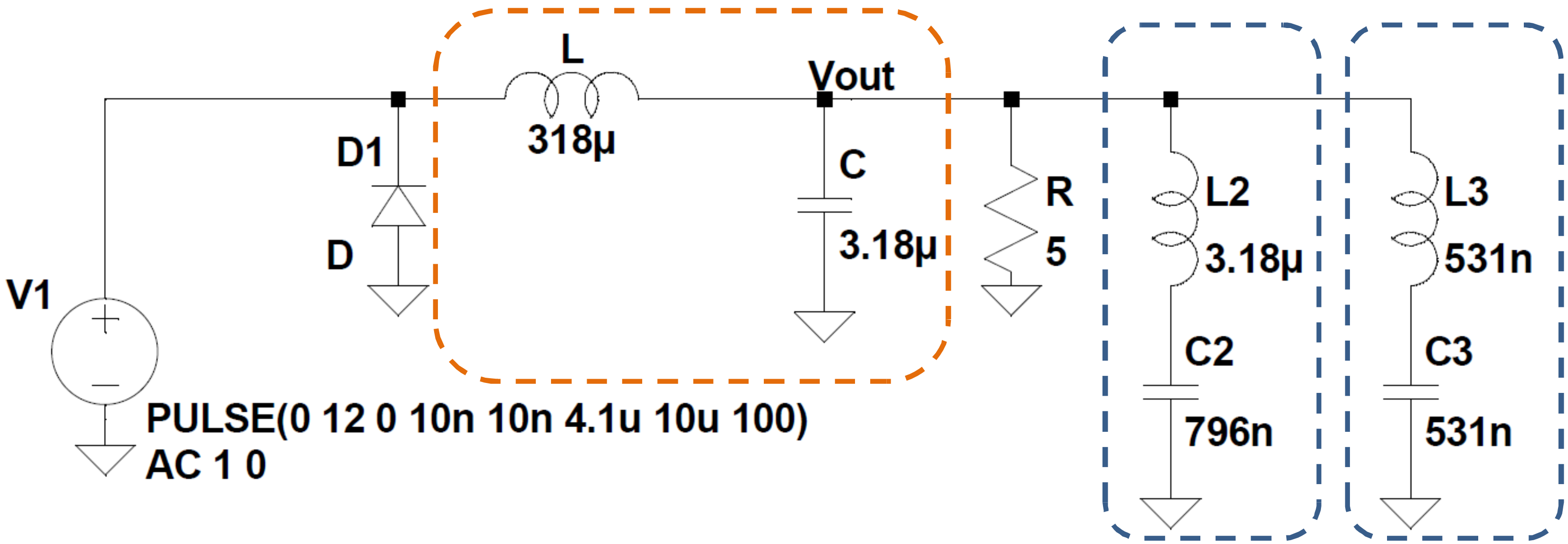
Transfer Function

$$H(j\omega)=\frac{\frac{1}{LC}\left(\frac{1}{L_2C_2}+(j\omega)^2\right)\left(\frac{1}{L_3C_3}+(j\omega)^2\right)}{\left\{\left[(j\omega)^2+\frac{1}{RC}(j\omega)+\frac{1}{LC}\right]\left(\frac{1}{L_2C_2}+(j\omega)^2\right)+\frac{1}{LC}\frac{(j\omega)}{L_2}\right\}\left(\frac{1}{L_3C_3}+(j\omega)^2\right)+\frac{1}{LC}\frac{(j\omega)}{L_3}\left(\frac{1}{L_2C_2}+(j\omega)^2\right)}$$

.tran 1500u

$$f_{cut\_off}=\frac{1}{2\pi\sqrt{LC}}=5kHz$$

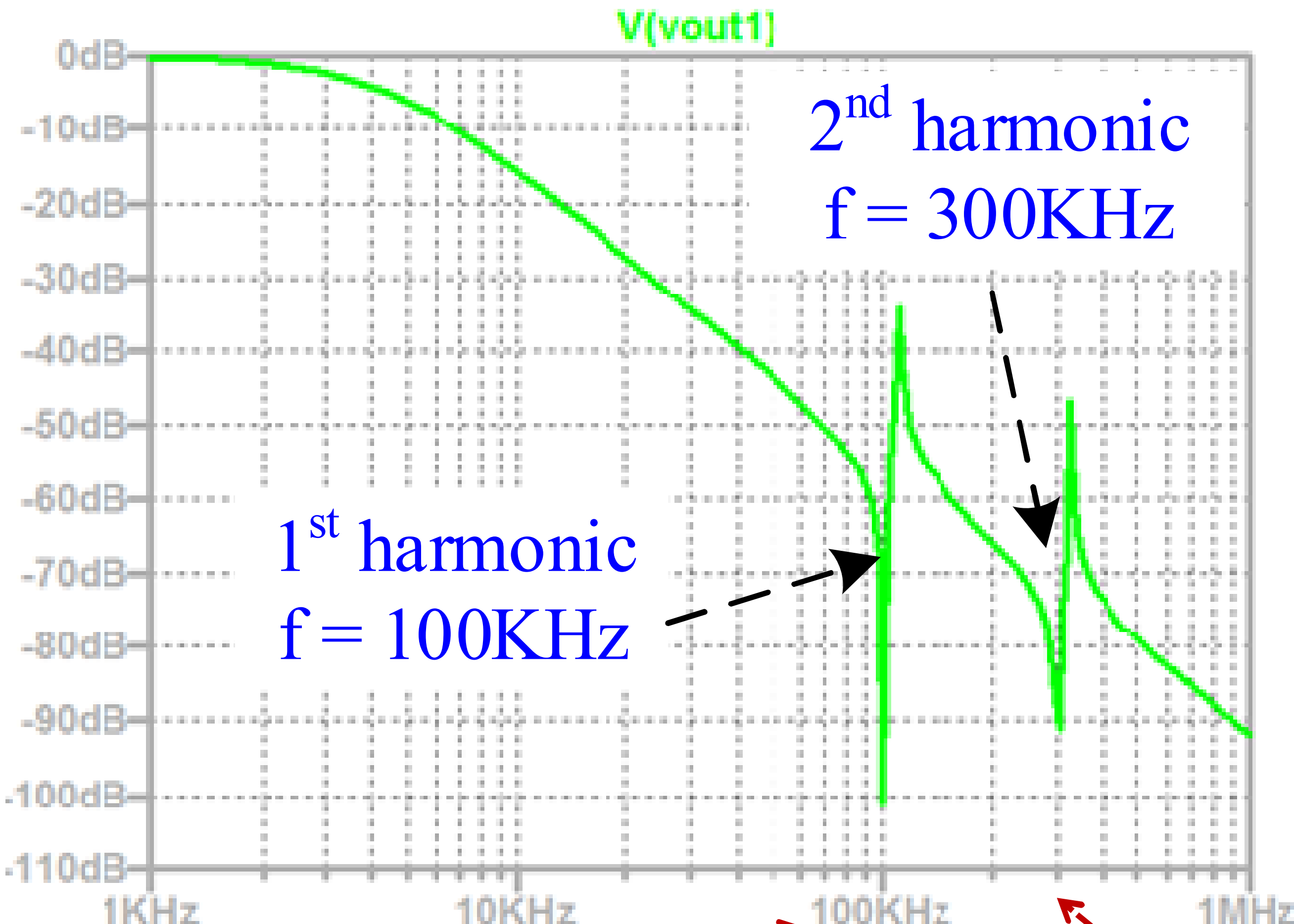
$$f_{1^{st}\ harmonic}=\frac{1}{2\pi\sqrt{L_2C_2}}=100kHz$$



$$f_{2^{nd}\ harmonic}=\frac{1}{2\pi\sqrt{L_3C_3}}=300kHz$$

3. Proposed Design of Buck Converter

Frequency Response of Proposed Buck Converter

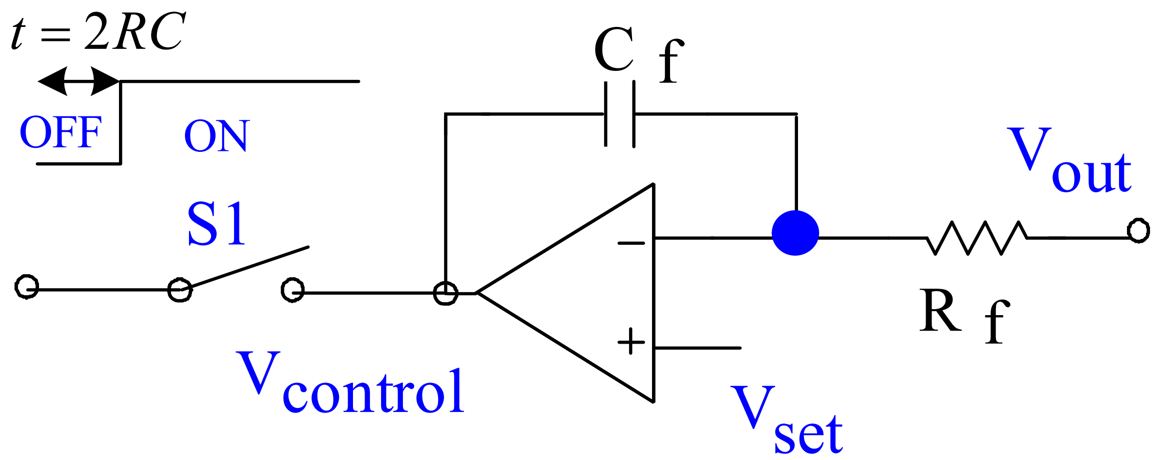


Resonant frequencies: $f_{1^{st}\ harmonic} = \dfrac{1}{2\pi\sqrt{L_2C_2}} = 100kHz$ $f_{2^{nd}\ harmonic} = \dfrac{1}{2\pi\sqrt{L_3C_3}} = 300kHz$

3. Proposed Design of Buck Converter

Analysis of Feedback Voltage Control

Feedback Voltage Control



$$V_{set}\left(\frac{1}{Z_f}+\frac{1}{R_f}\right)=\frac{V_{control}}{Z_f}+\frac{V_{out}}{R_f}$$

Keep small difference

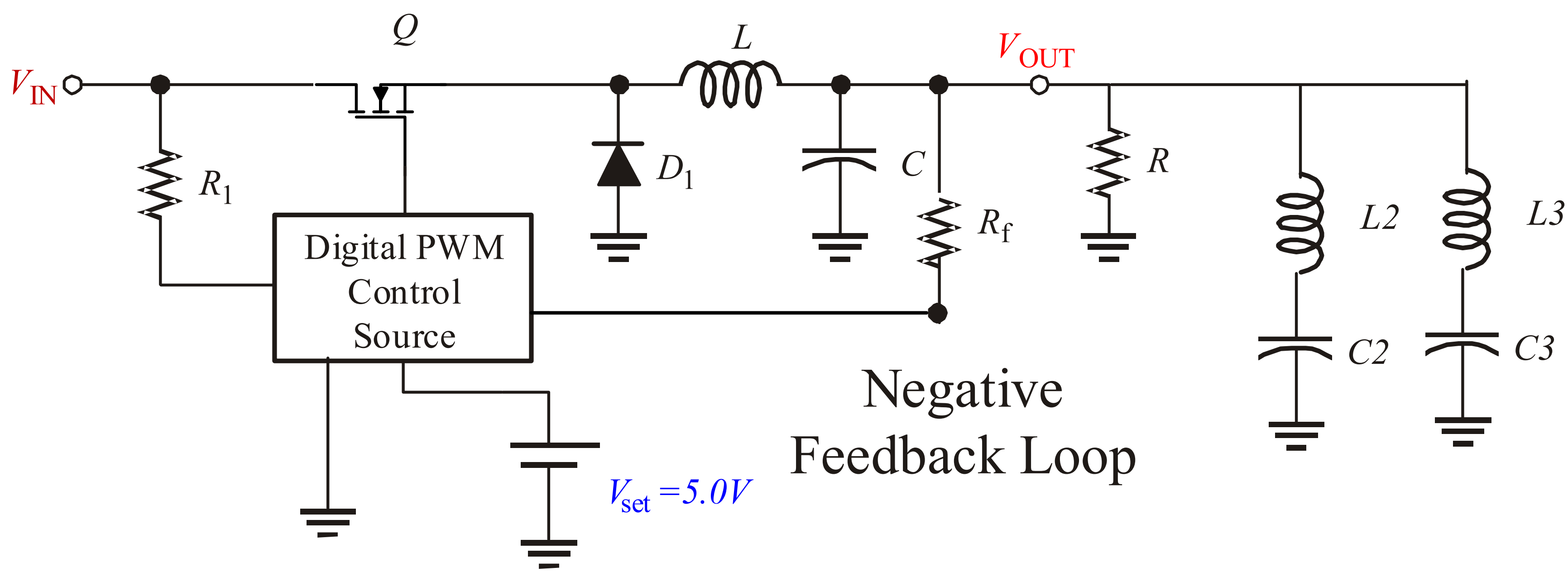
$$V_{control}=V_{set}+\frac{Z_{Cf}}{R_f}\left(V_{set}-V_{out}\right)=V_{set}+\boxed{\frac{1}{j\omega C_f R_f}\left(V_{set}-V_{out}\right)}$$

3. Proposed Design of Buck Converter

Proposed Structure of Buck Converter System

$R = 5\Omega$, $L = 318\mu H$, $C = 3.18\mu F$

$L2 = 3.18\mu H$, $C2 = 796nF$
$L3 = 531nH$, $C3 = 531nF$



Input Voltage (Vin)	**12V**
Output Voltage (Vo)	**5.0V**
Output Current (Io)	**1A**
Clock Frequency (Fck)	**100kHz**

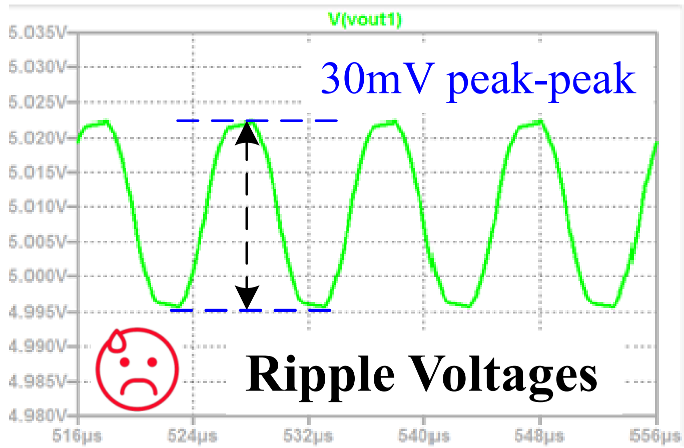
Current Step (⊿Io)	**1A**
Output Ripple	**0.4mVpp**
Over-shoot	**0.1mV**
Under-shoot	**0.1mV**

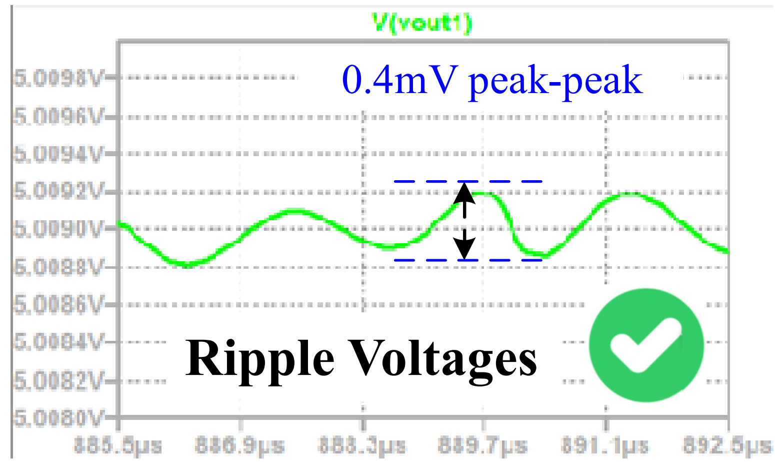
3. Proposed Design of Buck Converter

Simulation Waveforms of Proposed System

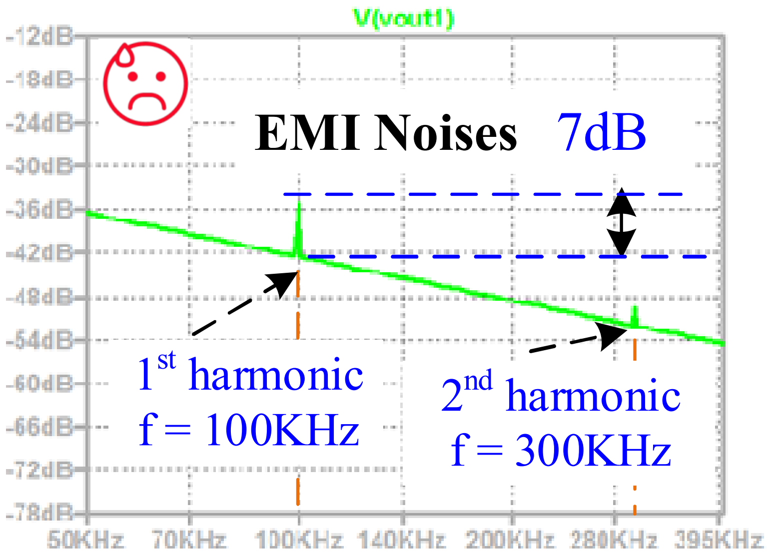
without Notch harmonics filters

with Notch harmonics filters

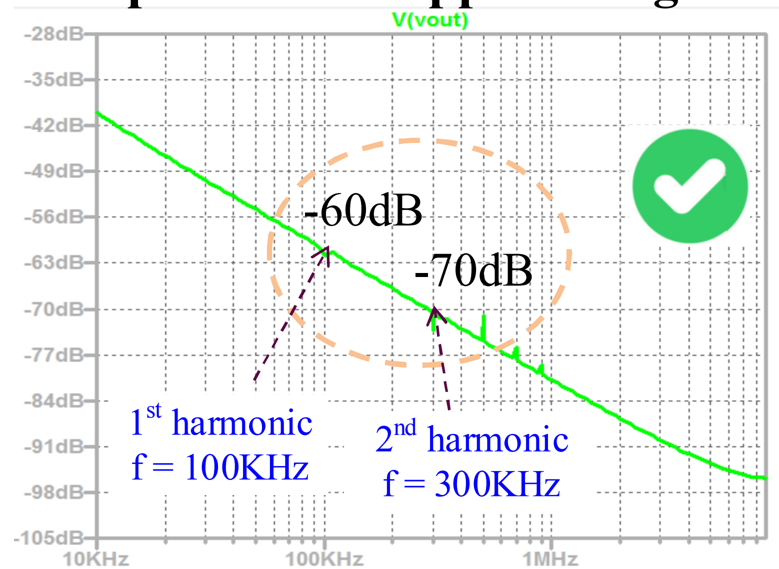




Spectrum of ripple voltages

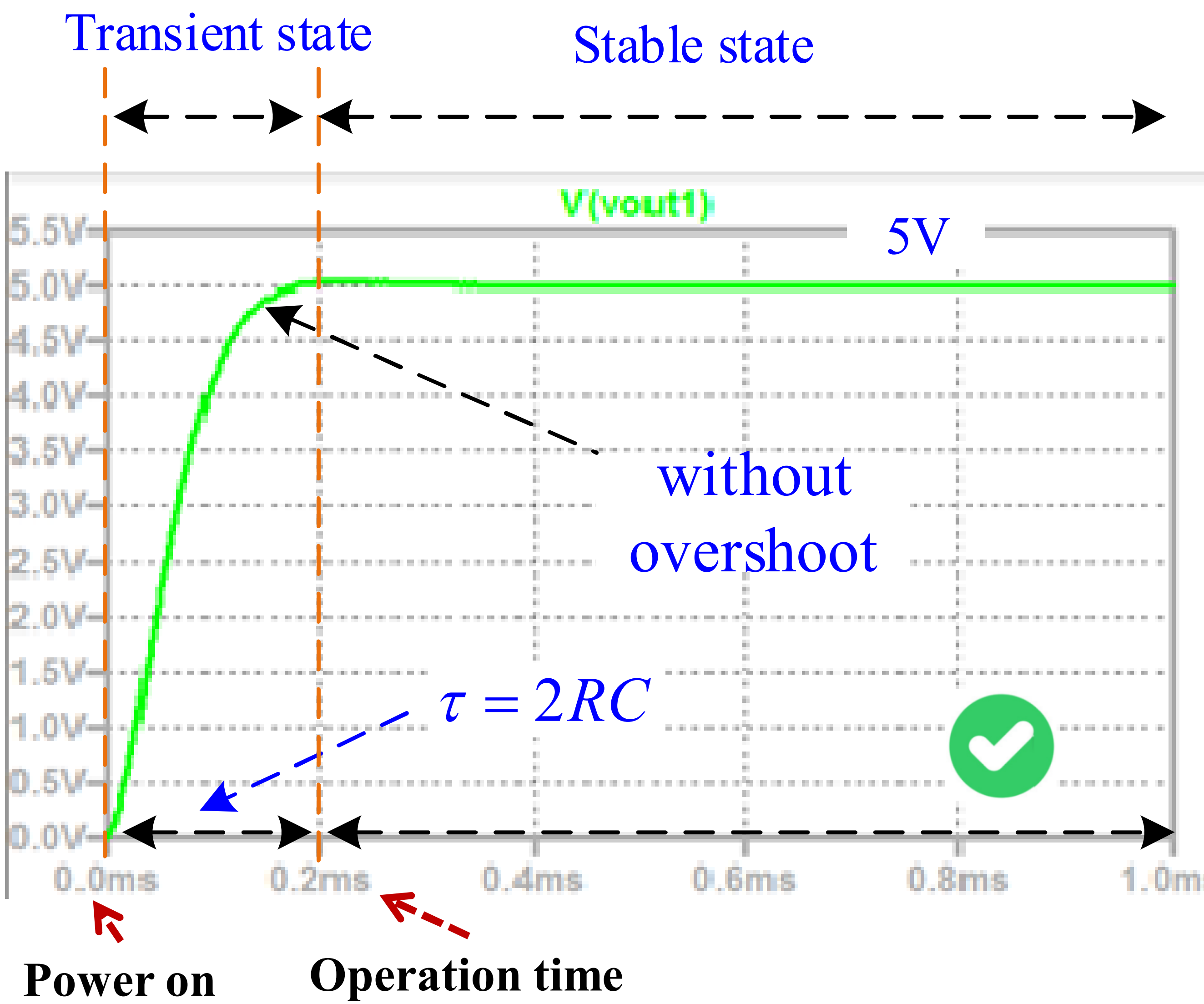


Spectrum of ripple voltages



3. Proposed Design of Buck Converter

Transient Response of Proposed System



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4. **Conclusions**

4. Conclusions

This work:

- **Balanced charge-discharge time condition**

$$\left|Z_L\right| = \left|Z_C\right| = 2R \Rightarrow \omega L = \frac{1}{\omega C} = 2R \qquad \omega = \frac{1}{\sqrt{LC}} = \frac{1}{2RC}$$

- **Analysis model of Buck converter system based on state-space technique and superposition principle**
- **EMI and ripple voltage improvement using two harmonic notch filters**

  → Ripple reduction from 30mVpp into 0.4mVpp

Future of Work

- **Analysis of parasitic of RLC and other components**

Thanks for your kind attention!

Questions & Answers

1) **Up to now, is the balanced charge-discharge time condition presented?**

   → **No, it isn't.**

   (The proposed condition is used to detect the overshoot voltage of Buck converter.)

2) **Why did the author derive the transfer function of Buck converter network based on the superposition principle?**

   → **Because the frequency responses of Buck converter can be plotted by hand calculation.**

   (As the Buck converter is analyzed, the proposed method is quicker than the state-space technique.)

Questions & Answers

3) **Are the properties of Buck converter different when the state-space technique and the superposition principle are used to analyze this system?**

   → **No, they aren't.**

   (If the transfer function of a network is defined, the properties of this network are expressed by the transient response and the frequency response.)