Fast Response, Small Ripple, Low Noise Switching Converter with Digital Charge Time Control and EMI Harmonic Filter

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Outline

1. Research Background
   • Applications of Switching Power Supply
   • Basic Switching Converter Architecture

2. Analysis of Step-down Switching Converter
   • Conventional State-Space Technique
   • Superposition Principle

3. Proposed Design of Buck Converter
   • Ripple Voltage Reduction with Notch Harmonic Filters
   • Simulation Results

4. Conclusions
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4. Conclusions
1. Research Background

Typical Applications of Switching Power Supply

Super Capacitors Battery
Other Power Supply

Switching Converter

Router
PicoPSU

Solar Panel
Panasonic Battery

Camera
Router
Tablet
Laptop
Smartphone
1. Research Background

Research Objective

Objective

Development of switching power supply with
- Fast response & high efficiency
- Low EMI noise
- Small output ripple

Approach

- Analysis of Buck converter system using state-space technique and superposition principle
- EMI reduction using harmonic notch filters
1. Research Background
Design Achievements

Achievements

- Derivation of transfer function of buck converter based on superposition principle
- Overshoot cancelation based on balanced charge-discharge time condition:
  \[ |Z_L| = |Z_C| = 2R \Rightarrow \omega L = \frac{1}{\omega C} = 2R \quad \omega = \frac{1}{\sqrt{LC}} = \frac{1}{2RC} \]
- Ripple reduction from 30mVpp into 0.4mVpp
- Two harmonic notch filters:
  - **-7dB** at the 1\textsuperscript{st} harmonic \[ f_{1\text{st\,harmonic}} = \frac{1}{2\pi \sqrt{L_2 C_2}} = 100k\text{Hz} \]
  - **-2dB** at the 2\textsuperscript{nd} harmonic \[ f_{2\text{nd\,harmonic}} = \frac{1}{2\pi \sqrt{L_3 C_3}} = 300k\text{Hz} \]
1. Research Background

Basic Switching Converter Architecture

**Merits**
- Downsizing
- Light Weight
- High Efficiency

**Demerits**
- Output Ripple
- Switching noise
- Harmonic noise

**High Efficiency Switching**
- Reduce energy consumption
- Extend battery operating time
- Minimize costs of systems
1. Research Background
Trade-offs of Switching Power Supply

<table>
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<tr>
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<th>Linear Regulator</th>
<th>Switching Regulator</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>Inductive</td>
</tr>
<tr>
<td>Efficiency</td>
<td>20-60%</td>
<td>90-95%</td>
</tr>
<tr>
<td>Ripple</td>
<td>Very low</td>
<td>Low</td>
</tr>
<tr>
<td>EMI Noise</td>
<td>Very low</td>
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<td>PCB Area</td>
<td>Very small</td>
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<td>Cost</td>
<td>Lowest</td>
<td>Highest</td>
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</tbody>
</table>
1. Research Background

Required EMI Noises of Switching Converter

Spectrums of PWM pulse < Standard Level
1. Research Background

Superposition Principle

\[ E_A(t) \sum_{i=1}^{n} \frac{1}{d_i} = \sum_{i=1}^{n} \frac{E_i(t)}{d_i} \]
1. Research Background

Example of Superposition Principle

\[ V_A \left( \frac{1}{R_1} + \frac{1}{Z_{C1}} + \frac{1}{Z_{L1}} \right) = \frac{V_1}{R_1} + \frac{V_2}{Z_{C1}} + \frac{V_3}{Z_{L1}} \]

\[ V_A \left( \frac{(R_1 + Z_{C1})Z_{L1} + R_1Z_{C1}}{R_1Z_{C1}Z_{L1}} \right) = \frac{V_1}{R_1} + \frac{V_2}{Z_{C1}} + \frac{V_3}{Z_{L1}} \]

\[ V_A = \frac{V_1Z_{C1}Z_{L1} + V_2R_1Z_{L1} + V_3R_1Z_{C1}}{(R_1 + Z_{C1})Z_{L1} + R_1Z_{C1}} \]

\[ V_A = \frac{V_1j\omega L_1 - V_2\omega^2 R_1 L_1 C_1 + V_3 R_1}{j\omega L_1 - \omega^2 R_1 L_1 C_1 + R_1} \]
1. Research Background

Switching Regulator

Independence of PWM Frequency

\[ \overline{V_{out}} = \frac{T_{ON}}{(T_{ON} + T_{OFF})} \overline{V_{in}} \]

Charge

\[ V_{\text{Charge}}(t_i) = V_{\text{discharge}}(t_{i-1}) \left( 1 - e^{-t/T_{ON}} \right) \]

Discharge

\[ V_{\text{discharge}}(t_i) = V_{\text{charge}}(t_i) e^{-t/T_{OFF}} \]
1. Research Background
Analysis of Switching Control Sources

- **50% Duty Cycle**
  - $V_{DC}$
  - $0.5 \times V_{DC}$
  - Transient time

- **75% Duty Cycle**
  - $V_{DC}$
  - $0.75 \times V_{DC}$
  - Transient time

- **25% Duty Cycle**
  - $V_{DC}$
  - $0.25 \times V_{DC}$
  - Transient time
1. Research Background

Analysis of Square Wave

$f_i = \frac{1}{T}$  

50% Duty Cycle

\[ S_{PWM}(t) = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\sin\left(2\pi \left(\frac{2k-1}{2} f_i\right) t\right)}{2k-1} \]

\[ S_{PWM}(t) = \frac{4}{\pi} \sin(2\pi ft) + \frac{4}{3\pi} \sin(3 \cdot 2\pi ft) \]

+ \frac{4}{5\pi} \sin(5 \cdot 2\pi ft) + ...
1. Research Background

Harmonics of PWM Signals

\[ S_{PWM}(t) = \frac{4}{\pi} \left( \sin(2\pi f_1 t) + \frac{1}{3} \sin(3*2\pi f_1 t) + \frac{1}{5} \sin(5*2\pi f_1 t) + \ldots \right) \]

50% Duty Cycle

75% Duty Cycle

25% Duty Cycle
1. Research Background

Simulations of Harmonics of PWM Signals

Waveforms of PWM signals

Spectrums of PWM signals

\[ f = 200k\text{Hz} \]

\[ f_1 = 200k\text{Hz}, f_2 = 400k\text{Hz}, f_3 = 600k\text{Hz} \]
1. Research Background

Harmonic Notch Filters

\[
V_{\text{out}} = \left( \frac{1}{Z_{\text{in}}} + \frac{1}{Z_{C1} + Z_{L1}} + \frac{1}{Z_{C2} + Z_{L2}} + \ldots + \frac{1}{Z_{Cn} + Z_{Ln}} \right) = \frac{V_{\text{in}}}{Z_{\text{in}}}
\]

\[
V_{\text{out}} = \left( \frac{1}{Z_{\text{in}}} + \frac{j\omega_1 C_1}{1 - \omega_1^2 C_1 L_1} + \frac{j\omega_2 C_2}{1 - \omega_2^2 C_2 L_2} + \ldots + \frac{j\omega_n C_n}{1 - \omega_n^2 C_n L_n} \right) = \frac{V_{\text{in}}}{Z_{\text{in}}}
\]

\[
\Rightarrow V_{\text{out}} = \left[ \frac{1}{Z_{\text{in}}} + \sum_{k=1}^{n} \left( \frac{j\omega_k C_k}{1 - \omega_k^2 C_k L_k} \right) \right] = \frac{V_{\text{in}}}{Z_{\text{in}}}
\]

Superposition principle

Output Voltage

\[
Z_{C1} + Z_{L1} = \frac{1}{j\omega_1 C_1} + j\omega_1 L_1 = \frac{1 - \omega_1^2 C_1 L_1}{j\omega_1 C_1}
\]

\[
\Rightarrow \frac{1}{Z_{C1} + Z_{L1}} = \frac{j\omega_1 C_1}{1 - \omega_1^2 C_1 L_1}
\]
1. Research Background

Transfer Function of Harmonic Notch Filters

\[ V_{out} \left[ \frac{1}{Z_{in}} + \sum_{k=1}^{n} \left( \frac{j \omega_k C_k}{1 - \omega_k^2 C_k L_k} \right) \right] = \frac{V_{in}}{Z_{in}} \]

Transfer Function

\[ H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{Z_{in} \left[ \frac{1}{Z_{in}} + \sum_{k=1}^{n} \left( \frac{j \omega_k C_k}{1 - \omega_k^2 C_k L_k} \right) \right]} \]

\[ H(j\omega) = \frac{1}{1 + Z_{in} \sum_{k=1}^{n} \left( \frac{j \omega_k C_k}{1 - \omega_k^2 C_k L_k} \right)} = \begin{cases} 1 & ; \omega^2 \neq \frac{1}{L_k C_k} ; Z_{in} \approx 0 \\ 0 & ; \omega^2 = \frac{1}{L_k C_k} ; Z_{in} \approx 0 \end{cases} \]

\[ f_k = \omega_k / 2\pi : \text{notch frequency of } L_k C_k \text{ filter} \]
1. Research Background

Frequency Response of Harmonic Notch Filters

\[ |H(j\omega)| = 20 \log(H(j\omega)) = \begin{cases} 
0 \text{dB} & ; \omega^2 \neq \frac{1}{L_k C_k} ; Z_{in} \ll 1 \\
-\infty & ; \omega^2 = \frac{1}{L_k C_k} ; Z_{in} \ll 1 
\end{cases} \]

**Quality factor**

\[ Q_{quality\_factor} = \left| \frac{1}{Z_{L_k}} \right| = \left| \frac{1}{Z_{C_k}} \right| \approx \infty \]

\[ f_k = \frac{\omega_k}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{1}{L_k C_k}} \]

\[ |H(j\omega)|_{dB} \]

1st harmonic \hspace{1cm} 2nd harmonic \hspace{1cm} Nth harmonic

0dB

\[ f_1 = \frac{1}{2\pi \sqrt{L_1 C_1}} \hspace{1cm} f_2 = \frac{1}{2\pi \sqrt{L_2 C_2}} \hspace{1cm} f_n = \frac{1}{2\pi \sqrt{L_n C_n}} \]
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2. Analysis of Step-down Switching Converter
Conventional State-Space Technique

\[ u(m) \to \text{Linear System} \to y(p) \]

\[
\begin{align*}
  u(m) &= \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_i(t) \\ u_m(t) \end{bmatrix} \\
  x(n) &= \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_i(t) \\ x_n(t) \end{bmatrix} \\
  y(p) &= \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_i(t) \\ y_p(t) \end{bmatrix}
\end{align*}
\]

- **Advantages**
  - Modeling, analyzing, and designing a wide range of systems
  - Nonlinear, time-varying, multivariable systems

- **Disadvantages**
  - Not as intuitive as classical method

\[
\begin{align*}
  \frac{dx}{dt} &= A(t)x(t) + B(t)u(t) \\
  y(t) &= C(t)x(t) + D(t)u(t)
\end{align*}
\]
2. Analysis of Step-down Switching Converter

Conventional State-Space Technique

\[ \dot{x}(t) = Ax(t) + Bu(t) \]
\[ y(t) = Cx(t) + Du(t) \]

Laplace transform

\[ sX(s) - x(0) = AX(s) + BU(s) \]
\[ Y(s) = CX(s) + DU(s) \]

assume \( x(0) = 0 \)

\[ X(s) = (sI - A)^{-1}BU(s) \]
\[ Y(s) = [C(sI - A)^{-1}B + D]U(s) \]

\[ \frac{Y(s)}{U(s)} = C[sI - A]^{-1}B + D = \frac{Cadj[sI - A]B + det[sI - A]D}{det[sI - A]} \]
2. Analysis of Step-down Switching Converter

Linear Graph Models of Network

Electronic circuits

Linear graph models
2. Analysis of Step-down Switching Converter

Linear Graph Models of Buck Converter
2. Analysis of Step-down Switching Converter

Conventional State-Space Technique (Switch ON)

\[
\begin{bmatrix}
\ddot{i}_L(t) \\
\ddot{v}_C(t)
\end{bmatrix}
= \begin{bmatrix}
0 & -\frac{1}{L} \\
\frac{1}{C} & -\frac{1}{RC}
\end{bmatrix}
\begin{bmatrix}
i_L(t) \\
v_C(t)
\end{bmatrix}
+ \begin{bmatrix}
\frac{1}{L} \\
0
\end{bmatrix} v_i(t)
\]

\[
y(t) = \begin{bmatrix}
0 & 1
\end{bmatrix}
\begin{bmatrix}
i_L(t) \\
v_C(t)
\end{bmatrix}
+ [0] v_i(t)
\]
2. Analysis of Step-down Switching Converter

Conventional State-Space Technique (Switch ON)

Laplace Transform

\[
\begin{align*}
    sI_L(s) & = 0I_L(s) - \frac{1}{L} V_C(s) + \frac{1}{L} V_i(s) \\
    sV_C(s) & = \frac{I_L(s)}{C} - \frac{V_C(s)}{RC} + 0V_i(s)
\end{align*}
\]

\[
\begin{align*}
    \frac{1}{L} V_C(s) + sC\left(s + \frac{1}{RC}\right)V_C(s) & = \frac{1}{L} V_i(s) \\
    I_L(s) & = C\left(s + \frac{1}{RC}\right)V_C(s)
\end{align*}
\]

\[V_C(s) = \frac{1}{s^2 + \frac{s}{RC} + \frac{1}{LC}} V_i(s)\]

\[
\begin{align*}
    \frac{d^2 v_C}{dt^2} + \frac{1}{RC} \frac{dv_C}{dt} + \frac{1}{LC} v_C &= \frac{1}{LC} v_i \\
    v_C(t) &= V_{out}(t)
\end{align*}
\]
2. Analysis of Step-down Switching Converter

Conventional State-Space Technique (Switch ON)

\[
\frac{d^2 V_{out}(t)}{dt^2} + \frac{1}{RC} \frac{dV_{out}(t)}{dt} + \frac{V_{out}(t)}{LC} = 0
\]

\[
V_{out}(t) = Ae^{st} = A_1e^{s_1t} + A_2e^{s_2t}
\]

\[
\frac{d^2 (Ae^{st})}{dt^2} + \frac{1}{RC} \frac{d(Ae^{st})}{dt} + \frac{(Ae^{st})}{LC} = 0
\]

\[
s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0
\]

\[
s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \quad \vee \quad s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}
\]

\[
\omega_{2RC} = \frac{1}{2RC}; \quad \omega_{LC} = \frac{1}{\sqrt{LC}};
\]

\[
V_{charge}(t) = A_{ch1}e^{-\omega_{2RC}+\sqrt{\left(\omega_{2RC}\right)^2-\omega_{LC}^2}}t + A_{ch2}e^{-\omega_{2RC}-\sqrt{\left(\omega_{2RC}\right)^2-\omega_{LC}^2}}t
\]
2. Analysis of Step-down Switching Converter
Conventional State-Space Technique (Switch OFF)

\[
\begin{bmatrix}
    \dot{i}_L(t) \\
    \dot{v}_C(t)
\end{bmatrix} =
\begin{bmatrix}
    0 & -\frac{1}{L} \\
    -\frac{1}{C} & -\frac{1}{RC}
\end{bmatrix}
\begin{bmatrix}
    i_L(t) \\
    v_C(t)
\end{bmatrix} +
\begin{bmatrix}
    0 \\
    0
\end{bmatrix} v_i(t)
\]

\[
y(t) =
\begin{bmatrix}
    0 & 1
\end{bmatrix}
\begin{bmatrix}
    i_L(t) \\
    v_C(t)
\end{bmatrix} +
\begin{bmatrix}
    0
\end{bmatrix} v_i(t)
\]
2. Analysis of Step-down Switching Converter

Conventional State-Space Technique (Switch OFF)

Laplace Transform

\[
\begin{align*}
sl_L(s) &= 0l_L(s) - \frac{1}{L}v_C(s) \\
sv_C(s) &= \frac{-l_L(s)}{C} - \frac{v_C(s)}{RC} + 0i(s)
\end{align*}
\]

\[
\begin{align*}
\frac{1}{L}v_C(s) - sc\left(s + \frac{1}{RC}\right)v_C(s) &= 0 \\
i_L(s) &= -c\left(s + \frac{1}{RC}\right)v_C(s)
\end{align*}
\]

\[
\left(s^2 - \frac{s}{RC} + \frac{1}{LC}\right)v_C(s) = 0
\]

\[
\frac{d^2v_C}{dt^2} - \frac{1}{RC} \frac{dv_C}{dt} + \frac{1}{LC}v_C = 0 \\
v_C(t) = V_{dis}(t)
\]
2. Analysis of Step-down Switching Converter
Conventional State-Space Technique (Switch OFF)

\[
\frac{d^2 V_{\text{dis}}(t)}{dt^2} - \frac{1}{RC} \frac{dV_{\text{dis}}(t)}{dt} + \frac{V_{\text{dis}}(t)}{LC} = 0
\]

\[
V_{\text{dis}}(t) = A_{\text{dis}} e^{st} = A_3 e^{s_{\text{dis}1} t} + A_3 e^{s_{\text{dis}2} t}
\]

\[
\frac{d^2 \left( A_{\text{dis}} e^{st} \right)}{dt^2} - \frac{1}{RC} \frac{d \left( A_{\text{dis}} e^{st} \right)}{dt} + \left( A_{\text{dis}} e^{st} \right) = 0
\]

\[
s^2 - \frac{1}{RC} s + \frac{1}{LC} = 0
\]

\[
s_{\text{dis}1} = \frac{1}{2RC} + \sqrt{\left( \frac{1}{2RC} \right)^2 - \frac{1}{LC}} \quad \vee \quad s_{\text{dis}2} = \frac{1}{2RC} - \sqrt{\left( \frac{1}{2RC} \right)^2 - \frac{1}{LC}}
\]

\[
\omega_{2RC} = \frac{1}{2RC}; \quad \omega_{LC} = \frac{1}{\sqrt{LC}};
\]

\[
V_{\text{discharge}}(t) = A_{\text{dis}1} e^{\left( \omega_{2RC} + \sqrt{\left( \omega_{2RC} \right)^2 - \omega_{LC}^2} \right) t} + A_{\text{dis}2} e^{\left( \omega_{2RC} + \sqrt{\left( \omega_{2RC} \right)^2 - \omega_{LC}^2} \right) t}
\]
2. Analysis of Step-down Switching Converter

Conventional State-Space Technique

\[
\overline{V_{\text{out}}} = \frac{1}{(T_{\text{ON}} + T_{\text{OFF}})} \left( \int_0^{T_{\text{ON}}} A_{\text{ch}} e^{-(\omega_{2RC} + \sqrt{(\omega_{2RC})^2 - \omega_{LC}^2})t} dt + \int_{T_{\text{ON}}}^{T_{\text{OFF}}} A_{\text{dis}} e^{(\omega_{2RC} - \sqrt{(\omega_{2RC})^2 - \omega_{LC}^2})t} dt \right)
\]

\[
\omega_{2RC} = \omega_{LC} \iff \omega = \frac{1}{\sqrt{LC}} = \frac{1}{2RC}
\]

\[
\omega L = \frac{1}{\omega C} = 2R
\]

\[
\overline{V_{\text{out}}} = \frac{1}{(T_{\text{ON}} + T_{\text{OFF}})} \left( \int_0^{T_{\text{ON}}} A_{\text{ch}} e^{-\omega t} dt + \int_{T_{\text{ON}}}^{T_{\text{OFF}}} A_{\text{dis}} e^{\omega t} dt \right)
\]

\[|Z_L| = |Z_C| = 2R\]

Balanced Charge-Discharge Time Condition
2. Analysis of Step-down Switching Converter

Conventional Switching Buck Converter

<table>
<thead>
<tr>
<th>Input Voltage (Vin)</th>
<th>12V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Voltage (Vo)</td>
<td>5.0V</td>
</tr>
<tr>
<td>Output Current (Io)</td>
<td>1A</td>
</tr>
<tr>
<td>Clock Frequency (Fck)</td>
<td>100kHz</td>
</tr>
</tbody>
</table>

R = 5Ω, L = 318μH, C = 3.18μF

\[ f_{cut \_off} = \frac{1}{2\pi \sqrt{LC}} = 5kHz \]
2. Analysis of Step-down Switching Converter

Analysis Model of Buck Converter

Proposed analysis model

Superposition principle

Output Voltage

Transfer Function

\[ V_o = \frac{1}{Z_L} + \frac{1}{Z_C} + \frac{1}{R} \]

\[ V_o = V_{in} \frac{R Z_C}{R(Z_L + Z_C) + Z_L Z_C} \]

\[ H = \frac{V_o}{V_{in}} = \frac{R Z_C}{R(Z_L + Z_C) + Z_L Z_C} \]

\[ H(j\omega) = \frac{1}{LC} \frac{1}{(j\omega)^2 + j\omega \frac{1}{RC} + \frac{1}{LC}} \]
2. Analysis of Step-down Switching Converter

Balanced Charge-Discharge Time Condition

Transfer Function

\[
H(j\omega) = \frac{1}{LC} \left( \frac{1}{(j\omega)^2 + j\omega \frac{1}{RC} + \frac{1}{LC}} \right)
\]

\[
H(j\omega) = \frac{1}{LC} \left( \frac{1}{(j\omega)^2 + 2j\omega \frac{1}{2RC} + \left(\frac{1}{2RC}\right)^2 + \frac{1}{LC} - \left(\frac{1}{2RC}\right)^2} \right)
\]

Maximum power

\[
\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2 = 0
\]

\[
|Z_L| = |Z_C| = 2R
\]

Balanced Charge-Discharge Time Condition
2. Analysis of Step-down Switching Converter

Transient Response of Buck Converter

Transfer Function

\[ H(j\omega) = \frac{1}{LC} \left( \frac{1}{2RC} \right)^2 + \frac{1}{LC} - \left( \frac{1}{2RC} \right)^2 \]

\[ |Z_L| = |Z_C| < 2R \]

\[ |Z_L| = |Z_C| = 2R \]

\[ |Z_L| = |Z_C| > 2R \]
2. Analysis of Step-down Switching Converter

Max Power Propagation

Transfer Function

\[ H(j\omega) = \frac{1}{LC} \left( j\omega + \frac{1}{2RC} \right)^2 + \frac{1}{LC} - \left( \frac{1}{2RC} \right)^2 \]

Max Power

\[ \frac{1}{LC} - \left( \frac{1}{2RC} \right)^2 = 0 \quad \Rightarrow \quad \omega = \frac{1}{\sqrt{LC}} = \frac{1}{2RC} \]

Rewritten Transfer Function

\[ H(j\omega) = \frac{1}{LC} \left( j\omega + \frac{1}{2RC} \right)^2 \quad \Rightarrow \quad |H(\omega)| = \frac{1}{\left( \frac{1}{2RC} \right)^2 + \omega^2} \]

\[ \omega_{\text{cut-off}} = \frac{1}{\sqrt{LC}} = \frac{1}{2RC} \quad \text{or} \quad f_{\text{cut-off}} = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{4\pi RC} \]

\[ |H(\omega)| = \frac{1}{2} \quad \text{or} \quad |H(\omega)|_{\text{dB}} = 20\log\left(\frac{1}{2}\right) = -3\text{dB} \]
# 2. Analysis of Step-down Switching Converter

## Time Behavior of Buck Converter

### Transfer Function

\[
H(j\omega) = \frac{1}{LC} \left( j\omega + \frac{1}{2RC} \right)^2
\]

Here \( s = j\omega \)

### Laplace Inversion Form

\[
h(t) = \mathcal{L}^{-1} \left\{ \frac{\omega^2}{(s + \omega)^2} \right\} = \omega^2 t e^{-\omega t}
\]

### Output Voltage

\[
\frac{V_{out}(t)}{V_{in}(t)} = h(t) = \omega^2 t e^{-\omega t} \Rightarrow V_{out}(t) = \omega^2 t e^{-\omega t} V_{in}(t)
\]

\[
V_{out}(t) = \left( \frac{1}{2RC} \right)^2 t e^{-\left( \frac{1}{2RC} \right)t} V_{in}(t)
\]

Here \( f_{\text{cut-off}} = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{4\pi RC} \)
2. Analysis of Step-down Switching Converter
Simulation of Balanced Charge-Discharge Time

\[ |Z_L| = |Z_C| = 2R \]

Cutoff frequency

\[ f_{\text{cut-off}} = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{4\pi RC} = 5\text{kHz} \]

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>( L )</td>
<td>318uH</td>
</tr>
<tr>
<td>( Z_L )</td>
<td>( j10 )</td>
</tr>
<tr>
<td>( C )</td>
<td>3.18uF</td>
</tr>
<tr>
<td>( Z_C )</td>
<td>(-j10)</td>
</tr>
<tr>
<td>( R_L )</td>
<td>5</td>
</tr>
<tr>
<td>( f )</td>
<td>5kHz</td>
</tr>
</tbody>
</table>
2. Analysis of Step-down Switching Converter
Waveforms of Balanced Charge-Discharge Time

**Frequency Response**

- \[ |Z_L| = |Z_C| = 2R \]
- \[ f_{\text{cut-off}} = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{4\pi RC} \]

**Step Response**

- Transient state
- Stable state
- Transient state

- Power On
- Power Off

- Without overshoot
- Without undershoot

- Overshoot
- Undershoot

- Cutoff frequency

\[ f_{\text{LC}} = \frac{1}{2\pi \sqrt{LC}} \]

\[ f_{\text{cut-off}} = \frac{1}{4\pi RC} \]

\[ |Z_L| = |Z_C| < 2R \]
2. Analysis of Step-down Switching Converter
Ripple Voltages and EMI Noises

Ripple voltages

Spectrum of ripple voltages

30mV peak-peak

EMI Noises 7dB

1\textsuperscript{st} harmonic \( f = 100\text{KHz} \)

2\textsuperscript{nd} harmonic \( f = 300\text{KHz} \)
1. Research Background
   • Applications of Switching Power Supply
   • Basic Switching Converter Architecture

2. Analysis of Step-down Switching Converter
   • Conventional State-Space Technique
   • Superposition Principle

3. Proposed Design of Buck Converter
   • Ripple Voltage Reduction with Notch Harmonic Filters
   • Simulation Results

4. Conclusions
3. Proposed Design of Buck Converter

Proposed Analysis Model of Buck Converter

Proposed analysis model

\[ V_{out} \left( \frac{1}{Z_L} + \frac{1}{Z_C} + \frac{1}{R} + \frac{1}{Z_{C2} + Z_{L2}} + \frac{1}{Z_{C3} + Z_{L3}} \right) = \frac{V_{in}}{Z_L} \]

\[ V_{out} = V_{in} \frac{Z_C R (Z_{C2} + Z_{L2})(Z_{C3} + Z_{L3})}{\left\{ \left[ R(Z_C + Z_L) + Z_C Z_L \right] (Z_{C2} + Z_{L2}) + RZ_C Z_L \right\} (Z_{C3} + Z_{L3}) + RZ_C Z_L (Z_{C2} + Z_{L2})} \]

Transfer Function

\[ H = \frac{V_o}{V_{in}} = \frac{Z_C R (Z_{C2} + Z_{L2})(Z_{C3} + Z_{L3})}{\left\{ \left[ R(Z_C + Z_L) + Z_C Z_L \right] (Z_{C2} + Z_{L2}) + RZ_C Z_L \right\} (Z_{C3} + Z_{L3}) + RZ_C Z_L (Z_{C2} + Z_{L2})} \]

\[ H(j\omega) = \frac{1}{LC} \left( \frac{1}{L_2 C_2} + (j\omega)^2 \right) \left( \frac{1}{L_3 C_3} + (j\omega)^2 \right) \]

\[ \left\{ \left( j\omega \right)^2 + \frac{1}{RC} \left( j\omega \right) + \frac{1}{LC} \right\} \left( \frac{1}{L_2 C_2} + (j\omega)^2 \right) + \frac{1}{LC} \left( \frac{1}{L_3 C_3} + (j\omega)^2 \right) \]

\[ \left\{ \left( j\omega \right)^2 + \frac{1}{RC} \left( j\omega \right) + \frac{1}{LC} \right\} \left( \frac{1}{L_2 C_2} + (j\omega)^2 \right) + \frac{1}{LC} \left( \frac{1}{L_3 C_3} + (j\omega)^2 \right) \]
3. Proposed Design of Buck Converter

**Simulation of Proposed Buck Converter**

**Transfer Function**

\[
H(j\omega) = \frac{1}{LC} \left( \frac{1}{L_2C_2} + (j\omega)^2 \right) \left( \frac{1}{L_3C_3} + (j\omega)^2 \right)
\]

\[
= \left[ (j\omega)^2 + \frac{1}{RC} (j\omega) + \frac{1}{LC} \right] \left( \frac{1}{L_2C_2} + (j\omega)^2 \right) + \frac{1}{LC} \left( \frac{1}{L_3C_3} + (j\omega)^2 \right) + \frac{1}{LC} \left( \frac{1}{L_2C_2} + (j\omega)^2 \right)
\]

\[
f_{\text{cut-off}} = \frac{1}{2\pi \sqrt{LC}} = 5kHz
\]

\[
f_{1^{\text{st}} \text{harmonic}} = \frac{1}{2\pi \sqrt{L_2C_2}} = 100kHz
\]

\[
f_{2^{\text{nd}} \text{harmonic}} = \frac{1}{2\pi \sqrt{L_3C_3}} = 300kHz
\]
3. Proposed Design of Buck Converter

Frequency Response of Proposed Buck Converter

Resonant frequencies:

\[ f_{1^{\text{st}} \text{ harmonic}} = \frac{1}{2\pi \sqrt{L_2C_2}} = 100kHz \]
\[ f_{2^{\text{nd}} \text{ harmonic}} = \frac{1}{2\pi \sqrt{L_3C_3}} = 300kHz \]
3. Proposed Design of Buck Converter

Analysis of Feedback Voltage Control

Feedback Voltage Control

\[ t = 2RC \]

\[
V_{\text{set}} \left( \frac{1}{Z_f} + \frac{1}{R_f} \right) = \frac{V_{\text{control}}}{Z_f} + \frac{V_{\text{out}}}{R_f}
\]

\[
V_{\text{control}} = V_{\text{set}} + \frac{Z_c f}{R_f} (V_{\text{set}} - V_{\text{out}}) = V_{\text{set}} + \frac{1}{j \omega C_f R_f} (V_{\text{set}} - V_{\text{out}})
\]

Keep small difference
3. Proposed Design of Buck Converter

Proposed Structure of Buck Converter System

\[ R = 5\Omega, \ L = 318\mu H, \ C = 3.18\mu F \]
\[ L2 = 3.18\mu H, \ C2 = 796nF \]
\[ L3 = 531nH, \ C3 = 531nF \]

Input Voltage (Vin) | 12V
---|---
Output Voltage (Vo) | 5.0V
Output Current (Io) | 1A
Clock Frequency (Fck) | 100kHz

Current Step (\(\Delta Io\)) | 1A
---|---
Output Ripple | 0.4mVpp
Over-shoot | 0.1mV
Under-shoot | 0.1mV
3. Proposed Design of Buck Converter

Simulation Waveforms of Proposed System

without Notch harmonics filters

with Notch harmonics filters

Ripple Voltages

Spectrum of ripple voltages

EMI Noises

Spectrum of ripple voltages

without Notch harmonics filters

with Notch harmonics filters

1\textsuperscript{st} harmonic \( f = 100\text{KHz} \)

2\textsuperscript{nd} harmonic \( f = 300\text{KHz} \)

-60dB

-70dB

30mV peak-peak

0.4mV peak-peak

1\textsuperscript{st} harmonic \( f = 100\text{KHz} \)

2\textsuperscript{nd} harmonic \( f = 300\text{KHz} \)
3. Proposed Design of Buck Converter

Transient Response of Proposed System

- Transient state
- Stable state

Power on
Operation time

\[ \tau = 2RC \]

without overshoot

5V

\[ V(\text{vout1}) \]
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4. Conclusions
4. Conclusions

This work:

• Balanced charge-discharge time condition

\[ |Z_L| = |Z_C| = 2R \Rightarrow \omega L = \frac{1}{\omega C} = 2R \quad \omega = \frac{1}{\sqrt{LC}} = \frac{1}{2RC} \]

• Analysis model of Buck converter system based on state-space technique and superposition principle

• EMI and ripple voltage improvement using two harmonic notch filters

→ Ripple reduction from 30mVpp into 0.4mVpp

Future of Work

• Analysis of parasitic of RLC and other components
Thanks for your kind attention!
Questions & Answers

1) Up to now, is the balanced charge-discharge time condition presented?

→ No, it isn’t.

(The proposed condition is used to detect the overshoot voltage of Buck converter.)

2) Why did the author derive the transfer function of Buck converter network based on the superposition principle?

→ Because the frequency responses of Buck converter can be plotted by hand calculation.

(As the Buck converter is analyzed, the proposed method is quicker than the state-space technique.)
3) Are the properties of Buck converter different when the state-space technique and the superposition principle are used to analyze this system?

⇒ No, they aren’t.

(If the transfer function of a network is defined, the properties of this network are expressed by the transient response and the frequency response.)