Proposal of Obtaining Open Loop Frequency Characteristics of Opamp with Closed Loop Configuration

JianLong Wang*, Yuji Gendai, Anna Kuwana, Haruo Kobayashi (Gunma University)

Abstract: This paper we propose an idea to obtain open loop frequency characteristics of an operational amplifier using corresponding closed loop measurement results. We explain its principle and select two operational amplifiers to verify the proposed method and compare it with the conventional LPF (Low pass filter) and null double injection methods. Our simulations have verified the effectiveness of the proposed method by comparison with the conventional methods.

Keywords: Operational Amplifier, Open Loop, Stability

I. Introduction

Examining the stability of operational amplifier circuits has been a concern since the negative feedback circuit was invented. In control theory, the system is stable if the poles of the closed-loop transfer function are all on the left plane of the complex plane. There are many difficulties when this stability criterion is applied to circuits; this is because knowing the positions of the poles and zeros is difficult due to the questions of equivalent circuit and numerical calculation. As a method for practically dealing with this problem, the frequency characteristic of the transfer function is widely used. From the viewpoint of stability, the input signal V_{in} can be regarded as a disturbance factor of the loop, and the error signal V_r indicates the reverberation when it returns around the loop. The expression of the error signal V_r is given by

$$V_r = \frac{1}{1+KA(s)} V_{\mathrm{in}} \tag{1}$$

Another problem when applying the stability theorem to circuits is that it is difficult to find the stability factor $1 + KA(s)$by simulation. Since the error signal V_r does not exist in practical circuit, Eq. (1) cannot be used. Also V_r can be obtained as a difference between the real signals V_{in} and V_{X}. However, in real circuits the input offset inevitably exists and generates an error.

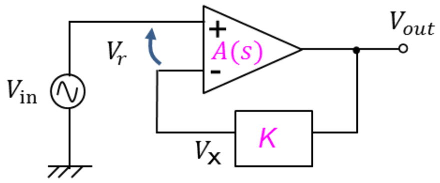


Fig.1 Inverting operational amplifier

We propose an idea to obtain the open loop characteristics $KA(s)$ with corresponding closed loop results and we call this operation as a closed-open conversion method. The reason why the closed-open conversion method has not been used so far is that the numerical error greatly affects the result because the gain of the operational amplifier is large. Considering the feedback control system, and the transfer function of closed-loop is as follows:

$$\frac{V_{out}}{V_{in}} = W(s) = \frac{A(s)}{1+KA(s)} \tag{2}$$

By calculation we can obtain the transfer function of open loop:

$$A(s) = \frac{1}{1/\mathrm{W(s)\text{-}K}} \tag{3}$$

As we know, the gain of the opamp $|A(s)|$ is very large in the low frequency region, so $W(s) \approx 1/\mathrm{K}$ in Eq. (2). Therefore, the resulting $A(s)$ will largely change with a small error in $W(s)$, since $1/W(s)$ is so close to K that the denominator of Eq. (3) becomes very small in magnitude. Simulations yield accurate results even in the low-frequency region, however, this is not true for the measurement results; this leads to the erroneous result for $A(s)$. This is why the closed-open conversion method has not been used. Notice that the gain of the opamp is too large, especially for low frequencies.

However, which has an effect on stability is that the Nyquist diagram is close to the origin point, and at this moment, the gain is much smaller. In a portion on the Nyquist plot places where the gain is small, the numerical accuracy of the closed-open transformation increases, making the proposed closed-open conversion method practical, and it may be used for measurement results. Notice that the low frequency gain is almost independent of stability. As the operational amplifier gain decreases at high frequencies, it approaches the -1 point (Nyquist diagram). Around this point, the closed-loop gain is also small, so that it is easy to obtain the accuracy of mutual conversions.

This time, we select the unity gain buffer connection configuration (feedback factor is $K = 1$) as shown in Fig.2, to introduce our proposed closed-open conversion method. The buffer connection is the easiest to see when looking at the open loop characteristics from the closed loop, and when the gain is 1, the system is most likely to be unstable. Generally, $KA(s)$ is used as the open loop characteristics, and is instead of $A(s)$ in this condition. The closed loop characteristics is as follows:

$$\frac{V_{out}}{V_{in}} = W(s) = \frac{A(s)}{1+KA(s)} \tag{4}$$

By calculation, we obtain the open loop characteristics:

$$A(s)=\frac{W(s)}{1-W(s)} \tag{5}$$

Since the transfer function depends on the load, discussion on the loop stability should be considered as a round transfer function including the load condition. $W(s)$ can be easily obtained by AC analysis. If AC analysis is performed with an actual load on the buffer, it is not necessary to change the load conditions for simulation.

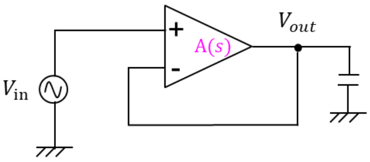


Fig.2 Buffer configuration

II. Bode plot and Nyquist plot

Fig.3 shows the Bode plot which is often used for judging the stability, and the frequency domain which is encircled by the green border. In this area, we can obtain the phase and gain margins to determine the stability.

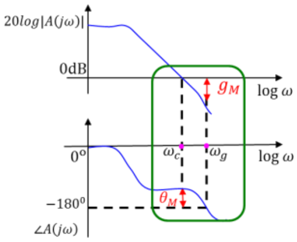


Fig.3 Bode plot and effect area on stability

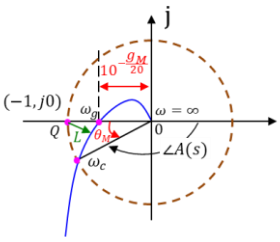


Fig.4 Nyquist plot

Fig.4 shows the Nyquist plot in the high frequency domain corresponding the green border area in Fig.4. Stability is defined by characteristics at around the unit circle (brown broken line), where $|A(s)|$ is small. Nyquist diagram also can show phase and gain margins, and minimum distance to -1 point is a better indicator to determine stability. In order to introduce and verify this theory, we select one operational feedback amplifier whose configuration is as shown in Fig.2 and the transfer function of the operational amplifier is given by

$$A(s)=\frac{10}{(1+s)(1+0.3s)(1+0.06s)} \tag{6}$$

Depict the Nyquist plot of open loop transfer function $KA(s)$ by using Mathematica software at different feedback factor conditions, as shown in Fig.5. Fig.6 shows the Bode plot of stability factor $1/(1+KA(s))$.

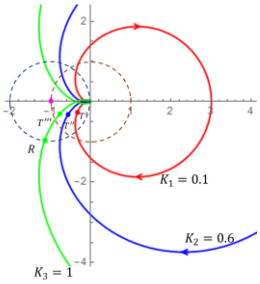


Fig. 5 Nyquist plots with different feedback factor K

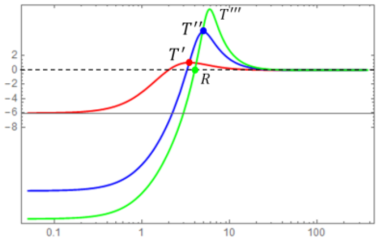


Fig.6 Bode plot of stability factor $1/(1+KA(s))$

We can find out with comparison of Fig.5 and Fig.6, that the closest points T', T'', T''' with the -1 point appear as peak in the Bode plot of the stability factor, and the magnitude is the reciprocal of the closest approach distance. The stability factor peak value is a direct stability index rather than a gain margin or a phase margin.

III. Closed loop characteristics locus in open loop Nyquist plot

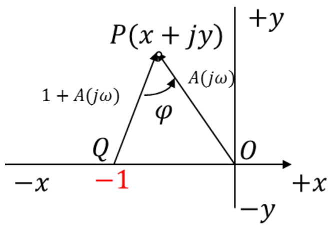


Fig.7 Nyquist plane of open loop transfer function $A(j\omega)$

Fig.7 shows a complex plane where $A(j\omega)$ is represented and point P shows the Nyquist locus of the open loop transfer function. Also the length of vector OP indicates the absolute value of $A(j\omega)$, and the inclination angle indicates the phase angle of $A(j\omega)$. If we choose the point Q at $-1+j0$, the vector QP represents $1+A(j\omega)$. The closed loop transfer function is given by

$$W(j\omega)=\frac{A(j\omega)}{1+1*A(j\omega)}=\frac{\overline{OP}}{\overline{QP}}=Me^{j\varphi} \tag{7}$$

The squared of the length of $W(j\omega)$ is expressed by:

$$M^2=\frac{|\overline{OP}|^2}{|\overline{QP}|^2}=\frac{x^2+y^2}{(x+1)^2+y^2} \tag{8}$$

By rearranging Eq. (8), we can obtain the trajectory equation of M as follows:

$$\{x+\frac{M^2}{M^2-1}\}^2+y^2=(\frac{M}{M^2-1})^2 \tag{9}$$

This is a circumference equation with a center at $(-M^2/(M^2-1)+j0)$ on the real with radius $M/(M^2-1)$. Fig.8 shows a circle group of $M=const.$ with a solid line. The locus of $\varphi=const.$ is an arc passing through the origin O and the point Q. This is clear from the $\angle QPO=\varphi$ relation and the geometrical theorem that the circumference angle is constant. By using different axes on the same complex plane, closed loop and open loop characteristics become a single plot.

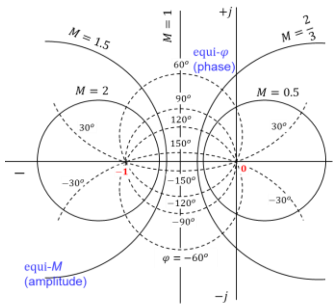
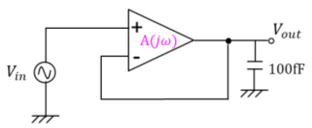


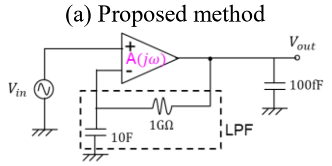
Fig. 8 Closed loop $M\cdot\varphi$ locus in open loop Nyquist plot

IV. Verification and comparison

Conventional low pass filter (LPF) method is often used for checking the open loop characteristics by inserting a LPF with a very low cutoff frequency into the feedback circuit to ensure the DC operating point, where the circuit diagram is shown in Fig.9(b). Regarding to the LPF method, there are two disadvantages: the first is that we need to replace the feedback section with another circuit; this operation is inescapable influence simulation result. The other disadvantage is that we need to measure the transfer characteristics from the positive input due to the loop has been disconnected; the transfer characteristics from the negative input affect the stability.



(a) Proposed method



(b) LPF method

Fig.9 Closed loop system circuits

The internal circuit of the operational amplifier is as shown in Fig.10, and the values of bias voltage V_{bias1} and V_{bias2} are 546.88mV and 366.99mV respectively. At the process of the proposed method, we run the circuit with LTspice and read the output file (text editor) in which the closed loop characteristics are written in a format like {frequency, real part, imaginary part}. Using these data, we can calculate and get open loop characteristics by Eq. (5). We also depict the plot using the data from the LPF method for comparison with the proposed method at one same graph.

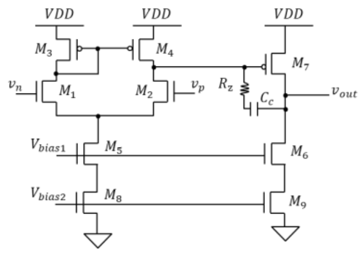


Fig.10 Internal circuit of the opamp

Using the obtained data of the open loop characteristics, we can depict the Bode plot of the open loop transfer function $A(j\omega)$ as the blue line shown in Fig.11. The red line shown in Fig.11 is the simulation result from the LPF method. We also depict the Nyquist plot using the data from two methods as shown in Fig.13, and we can see that two results are consistent.

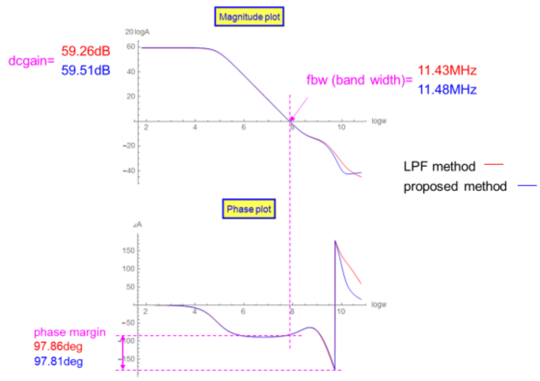


Fig.11 Bode plot of open loop transfer function

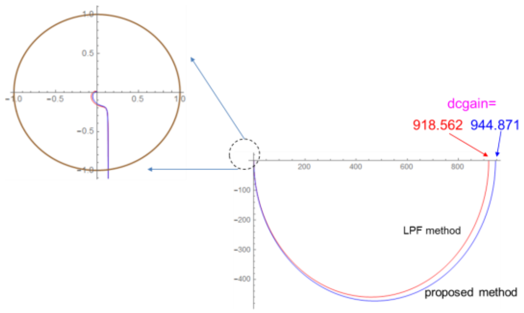


Fig.12 Nyquist plot of open loop transfer function

In the high frequency domain, especially around the unit circle, the simulation results are consistent. In the low frequency region, the difference of DC gain is caused by the difference of operation point.

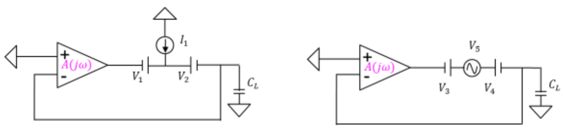
We also have performed simulations by using "Null double injection" method taken from an article by R. D. Middlebrook, and the circuit is shown in Fig.13. The loop gain is equivalent to the following:

$$G_v = \frac{v_f}{v_i}, \quad G_i = \frac{i_f}{i_i} \qquad (10)$$

Here v_f and i_f denote the feedback signals, while v_i and i_i are the input. G_v is the open loop voltage gain, and G_i is the open loop current gain, and they are related through the following equation:

$$G + 1 = (G_v + 1)||(G_i + 1) \quad G = \frac{G_i * G_v - 1}{G_i + G_v + 2} \qquad (11)$$

As shown in Fig.13(a), we inject two batteries and the independent current source I_1 for measuring the open loop gain. We inject two batteries and the independent voltage source V_5 for measuring the open loop gain as shown in Fig.13(b). The circuits need to be analyzed at the same time in order to produce the total gain as the total gain relies on both the open loop current gain and the open loop voltage gain.



(a) Current gain (b) Voltage gain

Fig.13 Null double injection method

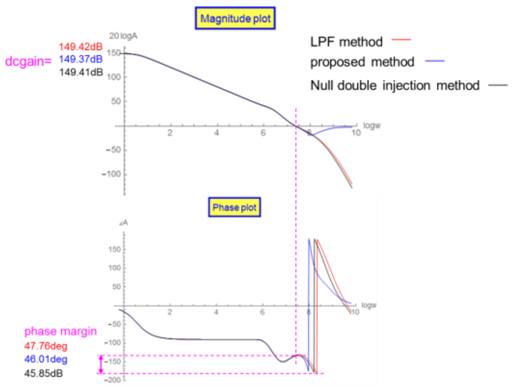


Fig.14 Bode plot of open loop transfer function

We select an LT1128 amplifier, perform its simulations and compare three simulation results. We depict the Bode plot and Nyquist plot using the data from the proposed method, as well as traditional methods including LPF method and null double injection method for comparison at one same graph as shown in Fig.14 and Fig.15.

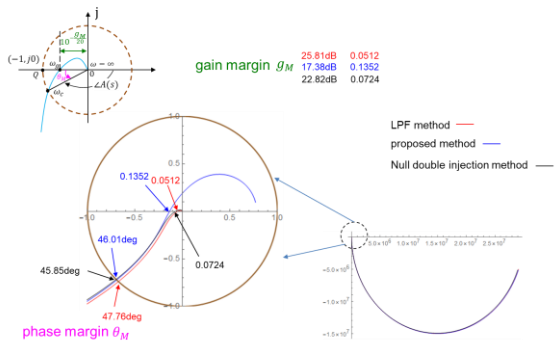


Fig.15 Nyquist plot of the open loop transfer function

By comparison, we can find out that the proposed method can be used for obtaining the open loop characteristics from the closed loop measurement. In the low frequency region, the simulation results are consistent. But in the high frequency region, especially around the unit circle, the simulation results are not consistent. The LPF method need to open up the loop, and that the DC bias point of the circuit will be altered. Since the circuit is linearized around the DC bias point in AC analysis, this will influence the simulation results. Considering the proposed and null double injection methods which can both make measurement without opening up the loop, the proposed approach is simpler and less time-consuming.

V Discussion and Conclusion

In this paper, we have tried the closed-open conversion method to obtain the open loop characteristics (opamp stability etc.) from closed loop operation results. The effectiveness of this method was demonstrated by a practical example. Since the traditional LPF method need to open up the loop, this will influence the simulation results. The null double injection method also does not need opening up the loop although, but compared with the proposed method, the later one is simpler and less time-consuming. The proposed method can be accurate because the open loop gain would not be very high around the frequency where phase and gain margins are evaluated.

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