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Analysis and Design of Multi-Tone Signal Generation Algorithms for Reducing Crest Factor

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Outline

- Research Background
- Multi-tone Signal
- Simulation Result for Several Algorithms
- Analysis of Commonality of Four Algorithms
- Conclusion

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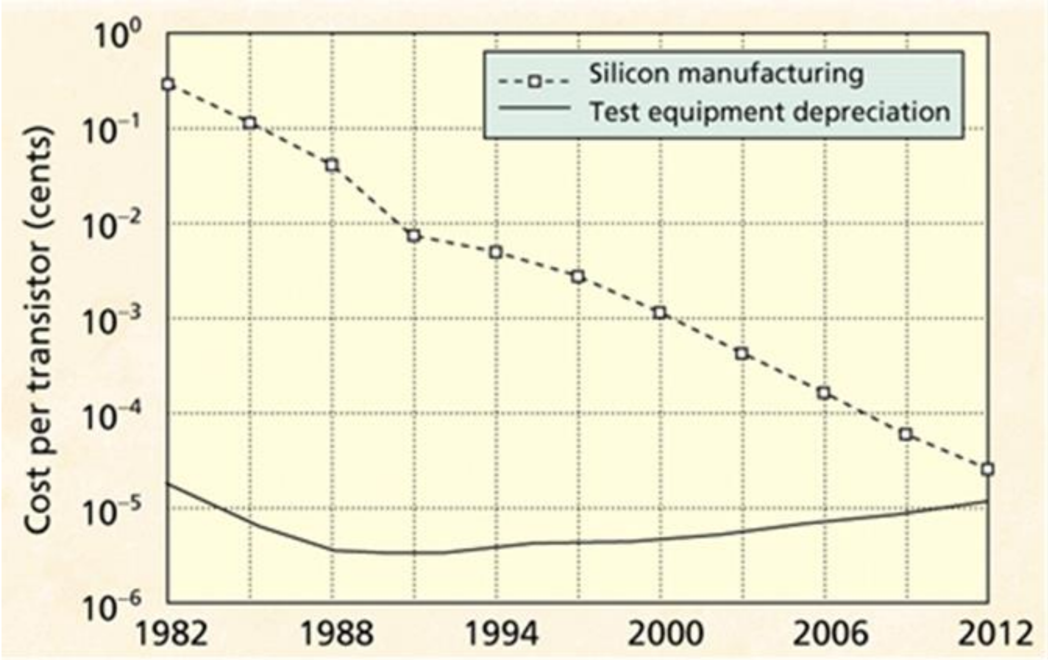
Test Cost Reduction

Decline in silicon manufacturing costs & High integration of LSI

Percentage of test cost : increase

Importance of test cost reduction by shortening test time



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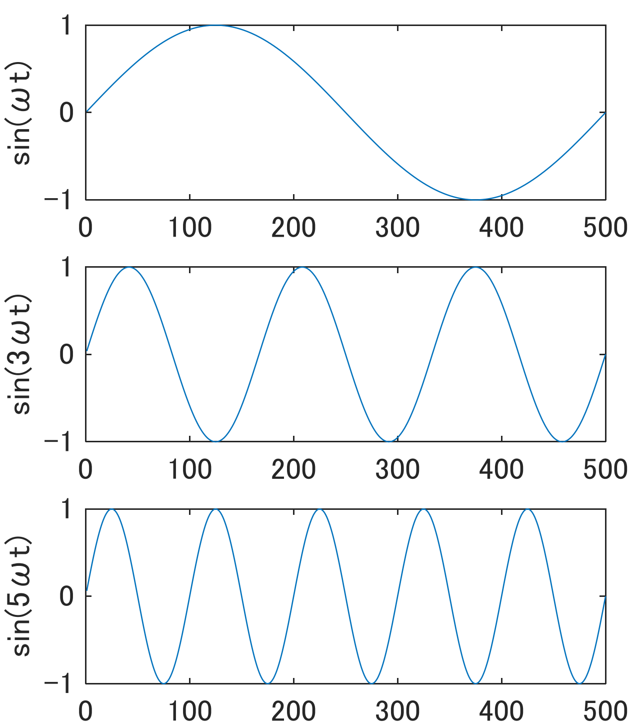
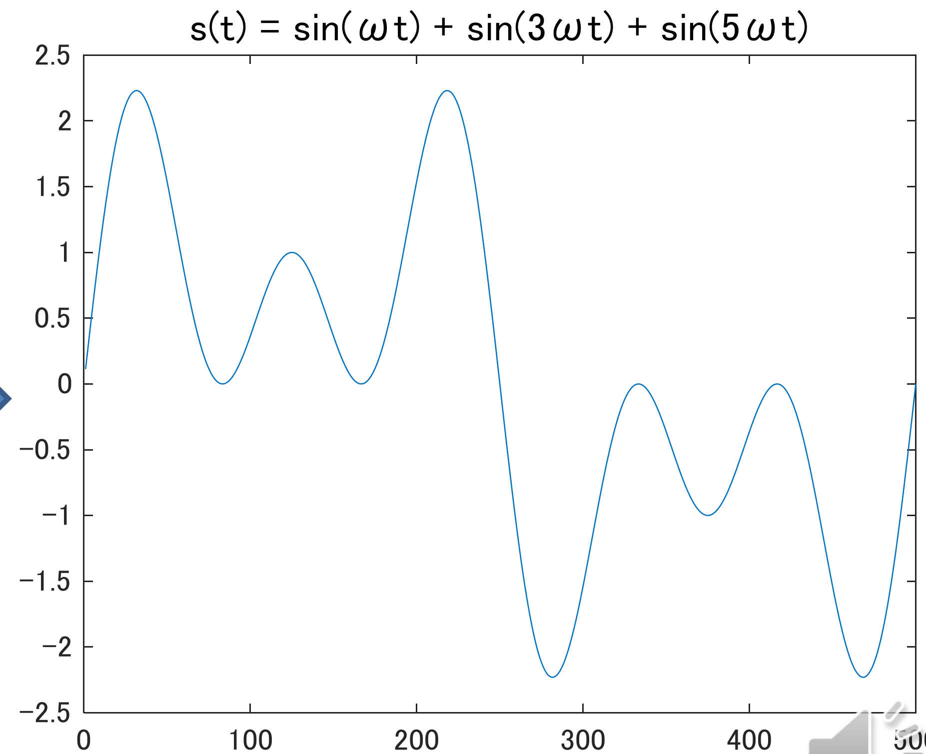
What is Multi-tone Signal?

Sum of multiple tone signals with different frequencies

$$s(t) = \sum_{k=1}^{N} A_k sin(\omega_k t + \theta_k)$$





Frequency Response Measurement

Probe signal

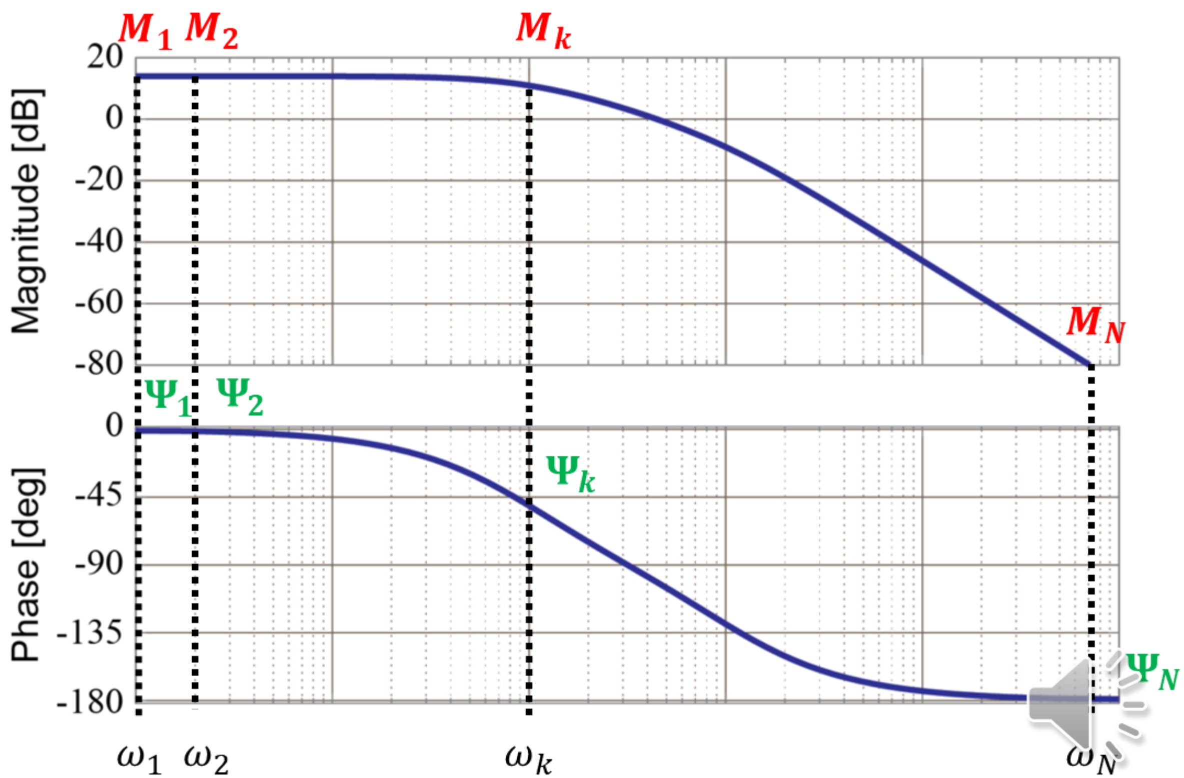
$A_1 sin(\omega_1 t + \theta_1)$
$A_2 sin(\omega_2 t + \theta_2)$
$\vdots$
$A_N sin(\omega_N t + \theta_N)$

Input → Linear system (Filter, etc.) → Output

$B_1 sin(\omega_1 t + \varphi_1)$
$B_2 sin(\omega_2 t + \varphi_2)$
$\vdots$
$B_N sin(\omega_N t + \varphi_N)$

Gain : $M_k = 20\log\frac{B_k}{A_k}$

Phase : $\Psi_k = \varphi_k - \theta_k$



Use of Multi-tone Signals

$A_1 sin(\omega_1 t + \theta_1)$
$A_2 sin(\omega_2 t + \theta_2)$
$\vdots$
$A_N sin(\omega_N t + \theta_N)$

Input → Linear system (Filter, etc.) → Output

$B_1 sin(\omega_1 t + \varphi_1)$
$B_2 sin(\omega_2 t + \varphi_2)$
$\vdots$
$B_N sin(\omega_N t + \varphi_N)$

Single-tone signal

- SNR : Good
- Test time : Long

Multi-tone signal

- Test time : Short
- Amplitude of each tone signal : Small

⇒ SNR : Bad → Test accuracy : Low

Use of Multi-tone Signals

$A_1 sin(\omega_1 t + \theta_1)$

$A_2 sin(\omega_2 t + \theta_2)$

$\vdots$

$A_N sin(\omega_N t + \theta_N)$

Input → **Linear system (Filter, etc.)** → Output

$B_1 sin(\omega_1 t + \varphi_1)$

$B_2 sin(\omega_2 t + \varphi_2)$

$\vdots$

$B_N sin(\omega_N t + \varphi_N)$

Single-tone signal

- SNR : Good
- Test time : Long

Multi-tone signal

- Test time : Short
- Amplitude of each tone signal : Small
  ⇒ SNR : Bad → Test accuracy : Low

Improve by crest factor reduction

What is Crest Factor (CF)?

$$Crest\ Factor[dB] = 20log_{10}\left[\frac{\text{Maximum amplitude}}{\text{effective value}}\right]$$

Comb

Peak

Crest factor (CF) reduction＝Amplitude of each tone signal：Large

Improve SNR for multi-tone signals

Factors for Worsening SNR

When testing a wideband signal device

The number of tones (N) : Increase

Maximum amplitude of multi-tone signal : Increase

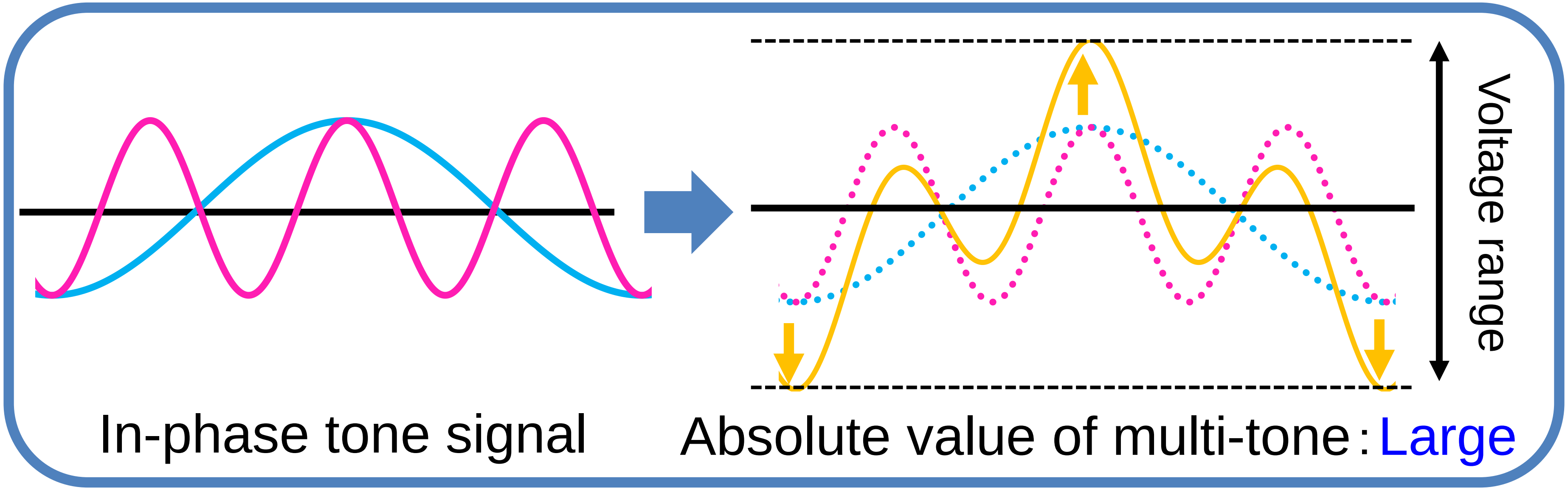
Not designed to handle

Intermodulation distortion (IMD) occurs

In-phase tone signal

For IMD reduction IMD : intermodulation distortion

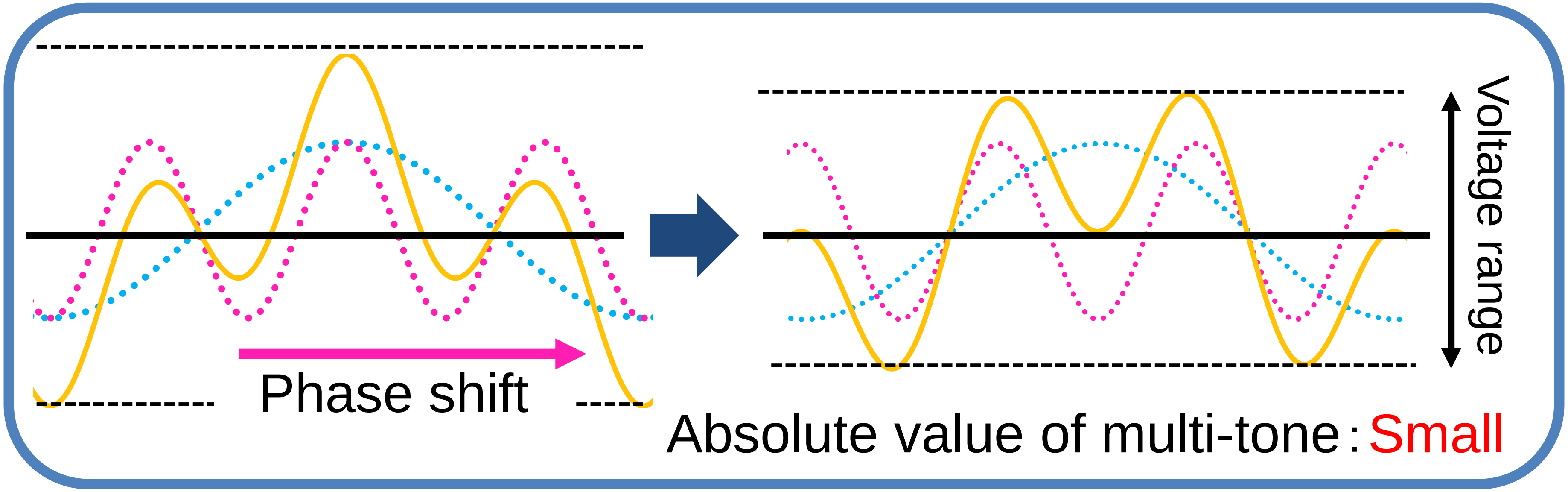
Generates multi-tone signal within a fixed voltage range



Crest factor (CF) : Large

Amplification for each tone : Small ⇒ SNR : Low

Phase Shift of Each Tone



Crest factor (CF)：Reduction

↓

Amplification for each tone：Large

↓

Improved SNR

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In-phase Multi-tone Signal

Basic equation : $s(t) = G\sum_{k=1}^{N} cos(\frac{2\pi f_k t}{T} + \theta_k)$

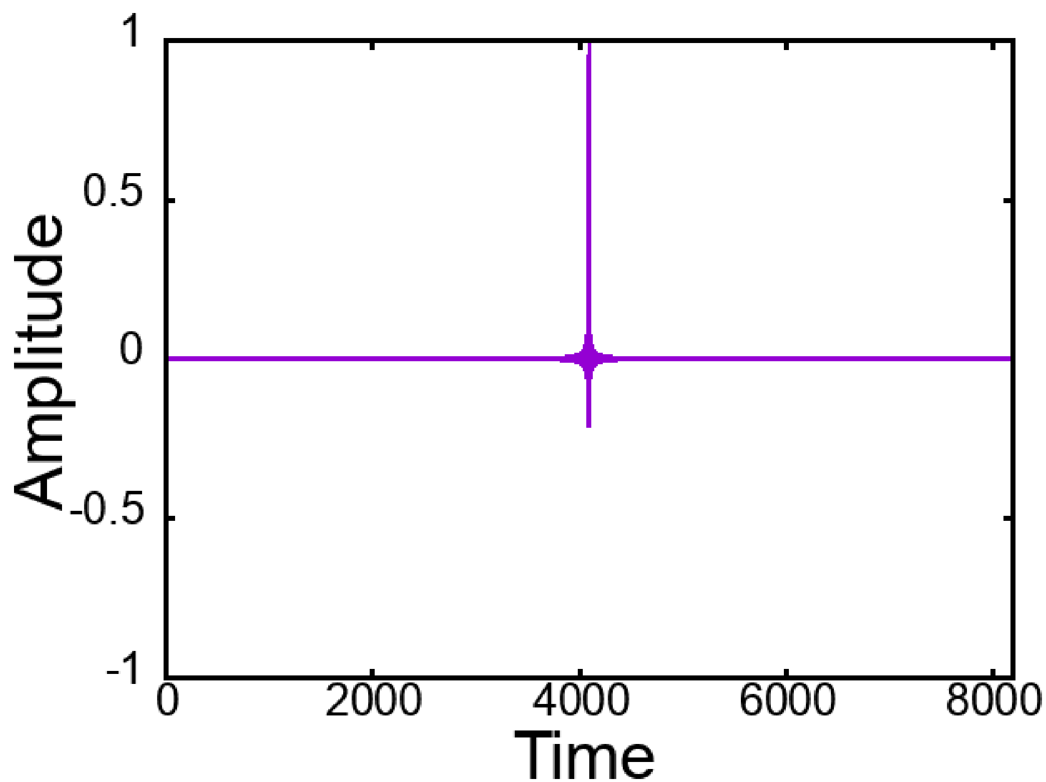
N : number of tones

T : resolution of 1 cycle

θ_k : All 0

$G = 1/A_{max}$: Amplitude of each tone

T = 8192 N = 1024 CF = 33[dB] G = 9.8×10^{-4}

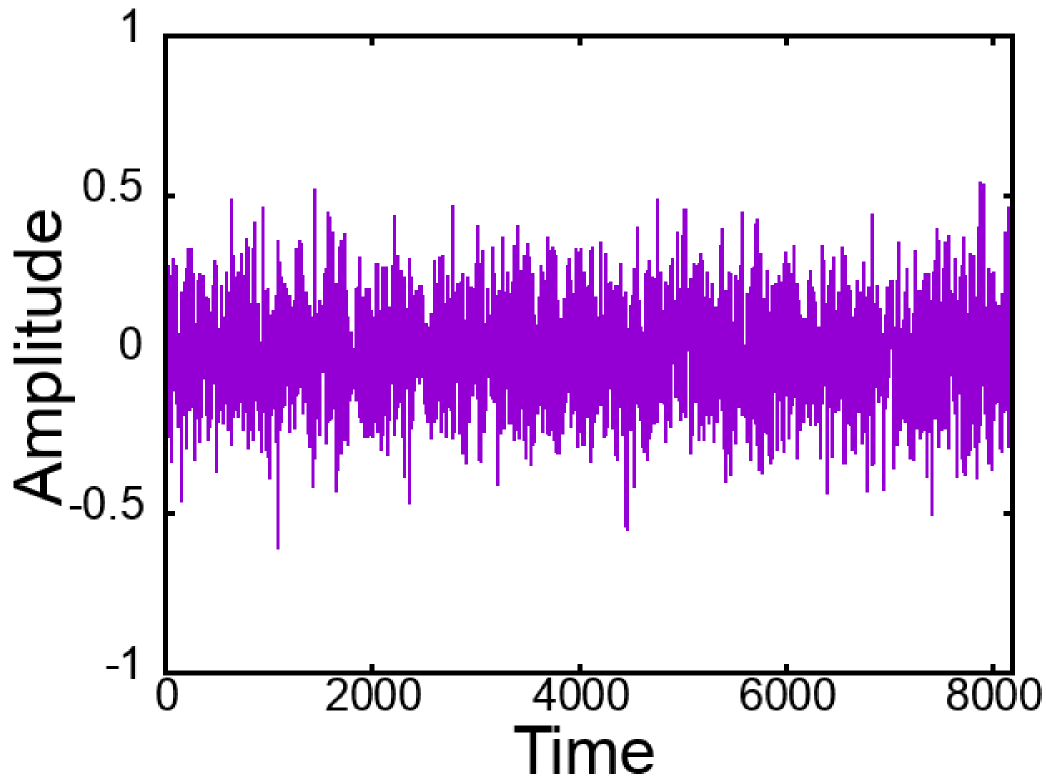


Random Phase Multi-tone Signal Signal

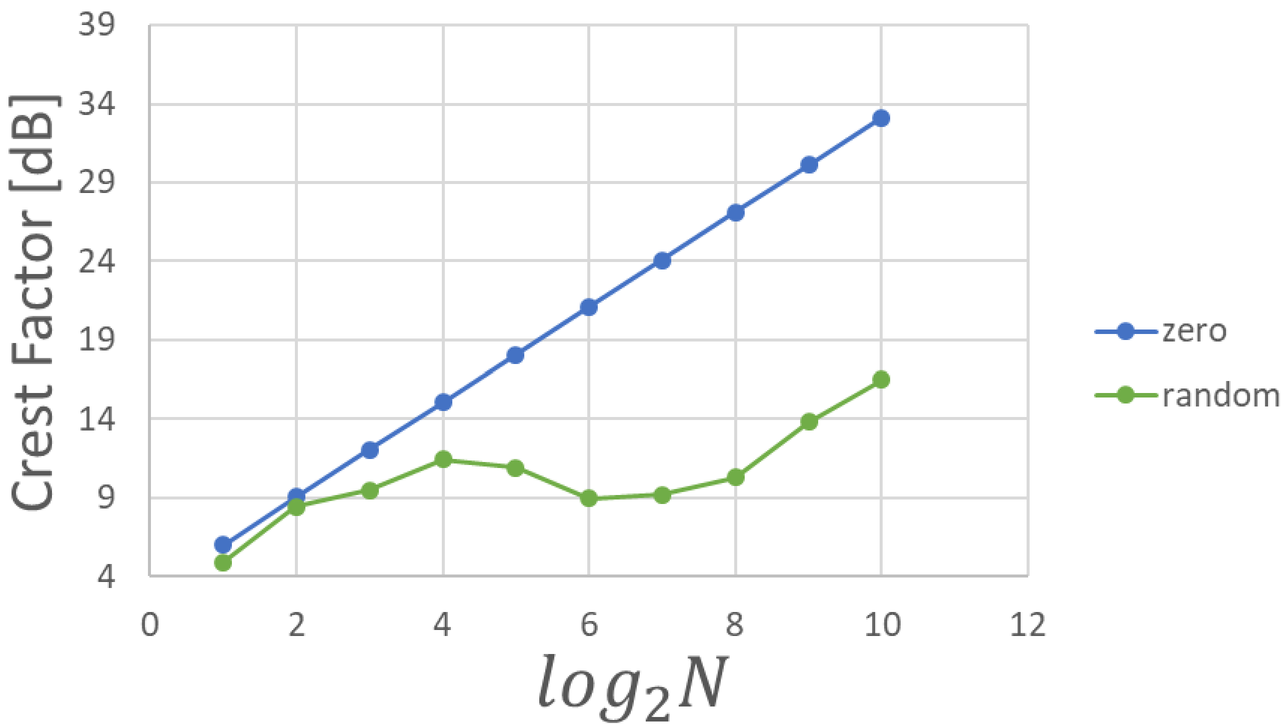
Basic equation : $s(t) = G\sum_{k=1}^{N} cos(\frac{2\pi f_k t}{T} + \theta_k)$

N : number of tones
T : resolution of 1 cycle
θ_k : random number
$G = 1/A_{max}$: Amplitude of each tone

T = 8192 N = 1024 CF = 16[dB] G = 6.6×10^{-3}



Relationship between N and CF



Number of tones N : Large → Crest factor : Large

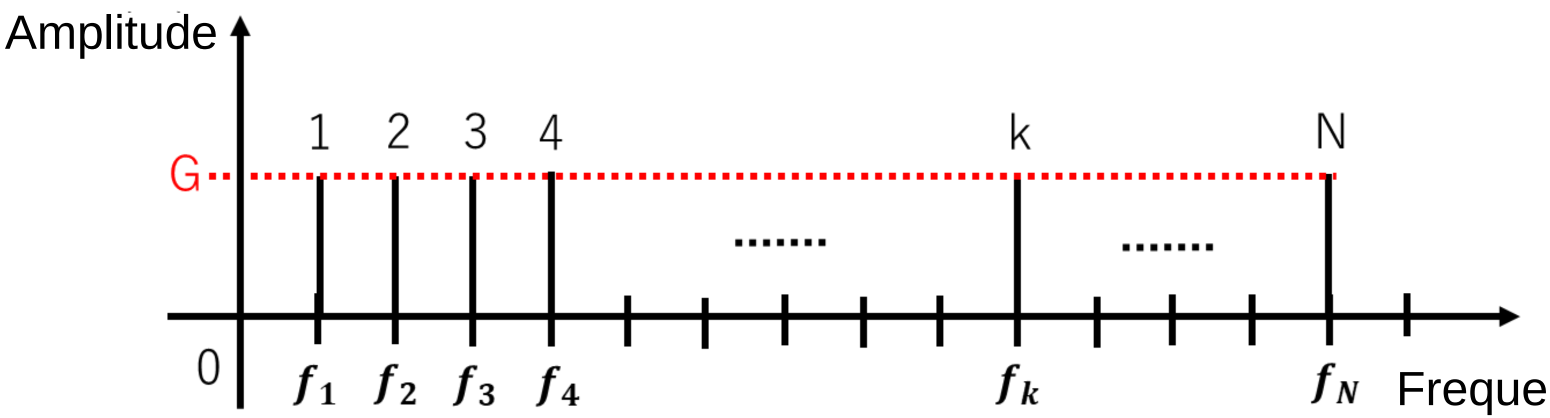
SNR deterioration in wideband test

Crest Factor Reduction Algorithm

Basic equation : $s(t) = G\sum_{k=1}^{N} cos(\frac{2\pi f_k t}{T} + \theta_k)$

N : number of tones
T : resolution of 1 cycle

Newman Phase	$\theta_k = \frac{\pi}{N}(k-1)^2$
Kitayoshi Phase	$\theta_k = \frac{\pi}{N}k(k+1)$
Schroeder Phase	$\theta_k = -\frac{\pi}{N}k(k-1)$
Narahashi Phase	$\theta_k = \frac{\pi}{N-1}(k-1)(k-2)$

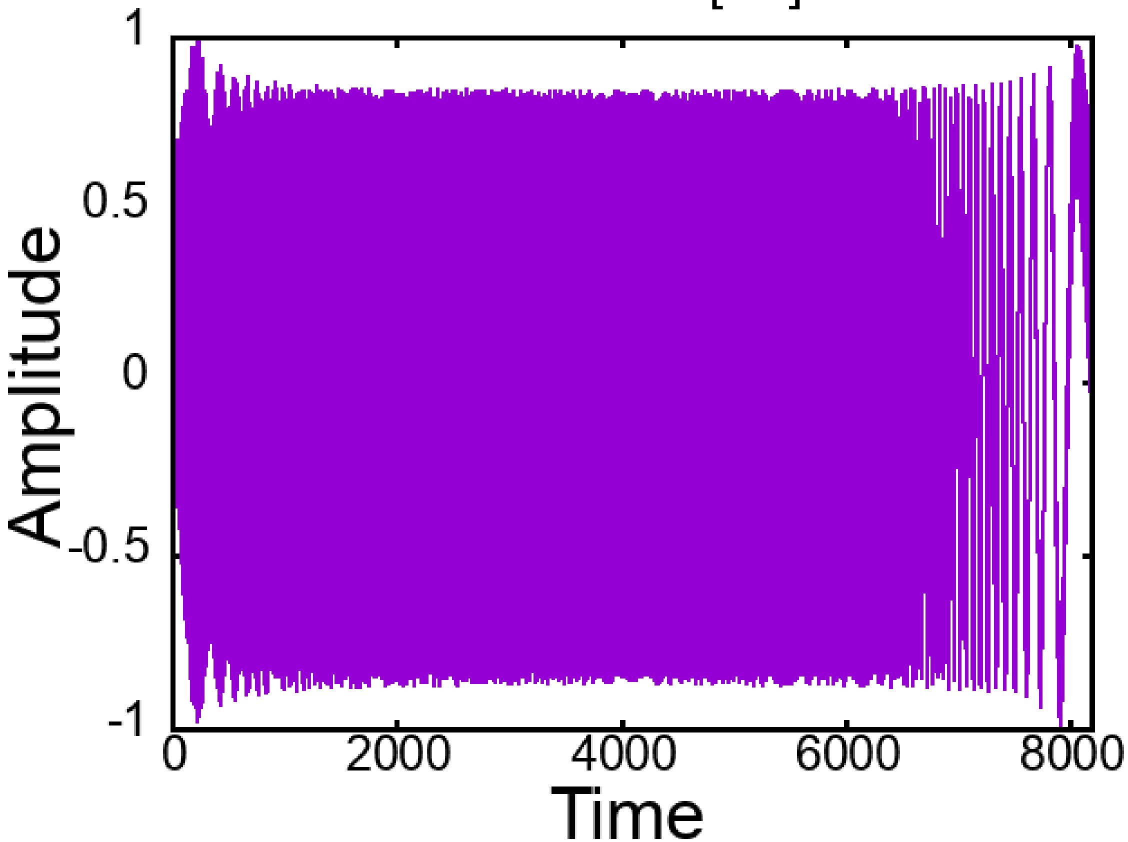


Newman Phase Waveform

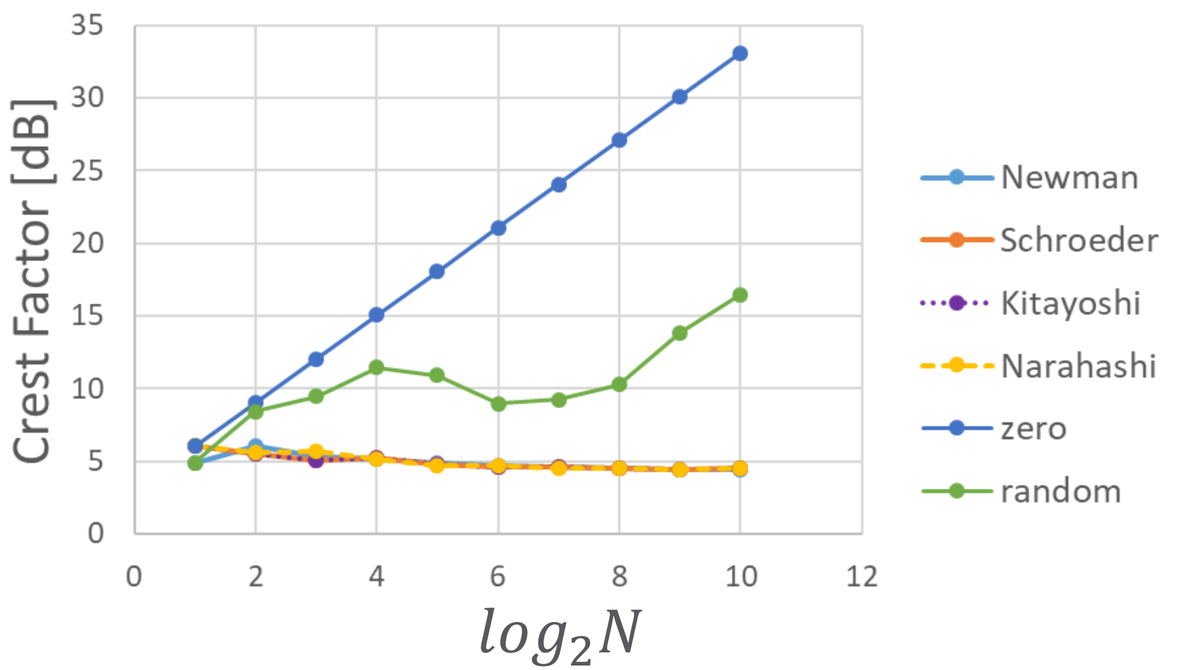
$$s(t) = G\sum_{k=1}^{m} cos(\frac{2\pi f_k t}{T} + \frac{\pi}{N}(k-1)^2)$$

Normalize the amplitude to 1
$G = 1/A_{max}$ = Amplitude of each tone

T = 8192 N = 1024 CF = 4.5[dB] $G = 2.6 \times 10^{-2}$



Relationship between N and CF

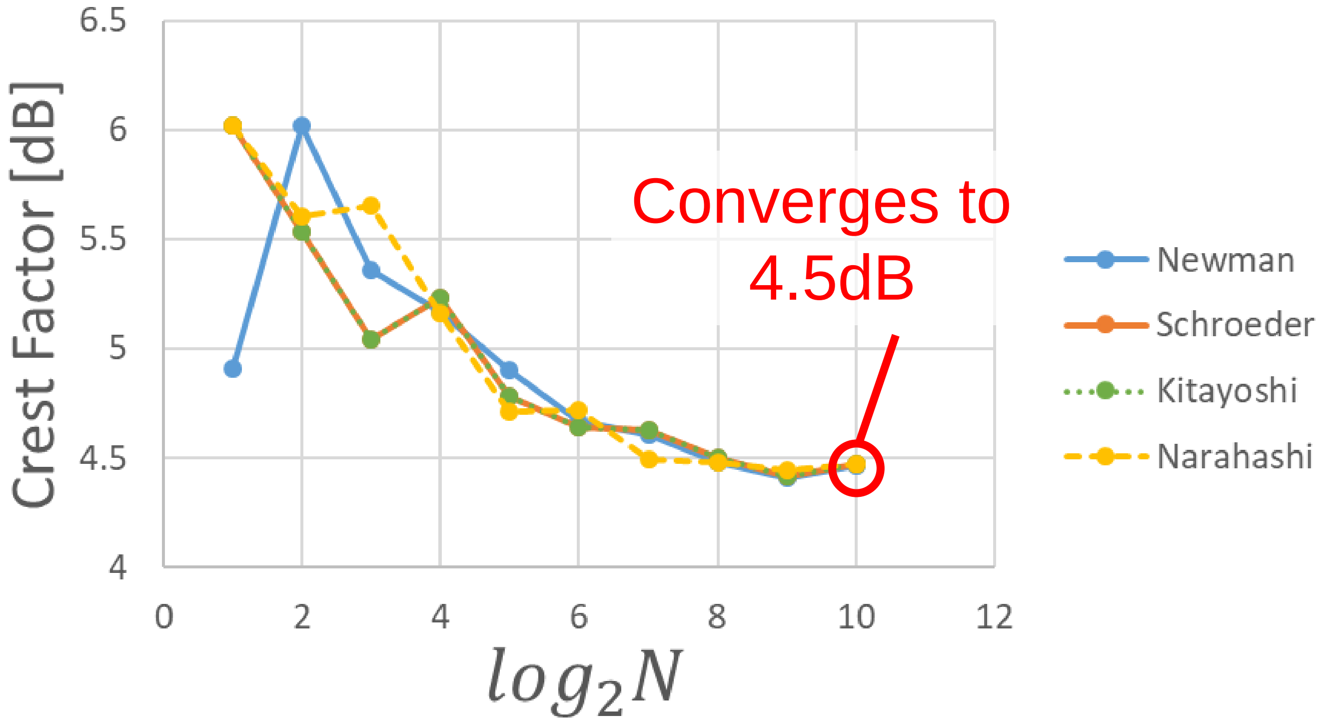


Zero phase・Random phase ： CF increases with N

Four algorithms ： CF reduction

Improve SNR by algorithm

Comparison of Four Algorithms



CF reduction effect is almost equal

Analyze similarity of four algorithms

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Four Algorithms

Newman Phase	$\theta_{\boldsymbol{k}} = \frac{\pi}{N}(k-1)^2$
Kitayoshi Phase	$\theta_{\boldsymbol{k}} = \frac{\pi}{N}k(k+1)$
Schroeder Phase	$\theta_{\boldsymbol{k}} = -\frac{\pi}{N}k(k-1)$
Narahashi Phase	$\theta_{\boldsymbol{k}} = \frac{\pi}{N-1}(k-1)(k-2)$

Derivation of Narahashi Phase

Analysis of commonality of four algorithms

Derivation of Narahashi Phase

Derivation of PAPR (Crest Factor)

$$PAPR = \frac{PEP}{P_{av}} = \frac{max[EP(t)]}{NA^2} = max\left[1 + \frac{2}{N}\sum_{k=1}^{N-1}\sum_{l=k+1}^{N}\cos(2\pi(l-k)\Delta f_0 t + \theta_l - \theta_k)\right]$$

$P_0(t)$: Fluctuation from the average power

$$P_0(t) = \sum_{k=1}^{N-1}\cos(2\pi\cdot 1\cdot\Delta f_0 t + \theta_{k+1} - \theta_k)$$

1st summation term

$$+\sum_{k=1}^{N-2}\cos(2\pi\cdot 2\cdot\Delta f_0 t + \theta_{k+2} - \theta_k) + \cdots + \cos(2\pi\cdot(N-1)\cdot\Delta f_0 t + \theta_N - \theta_1)$$

2nd summation term

Nth summation term

Δf_0 : Frequency spacing between adjacent complex tones

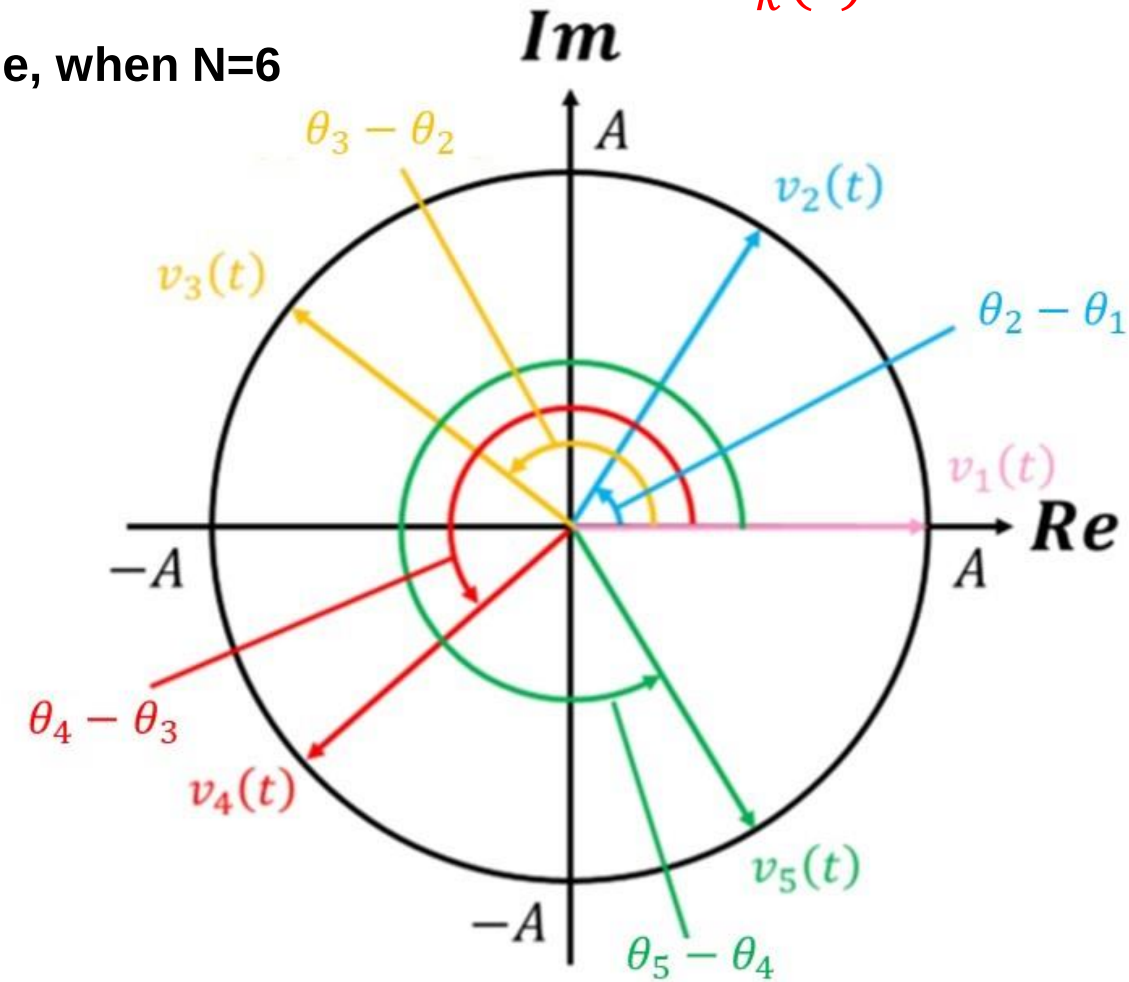
θ_k : Initial phase of each complex tone

Determine θ_k where the 1st summation term becomes zero

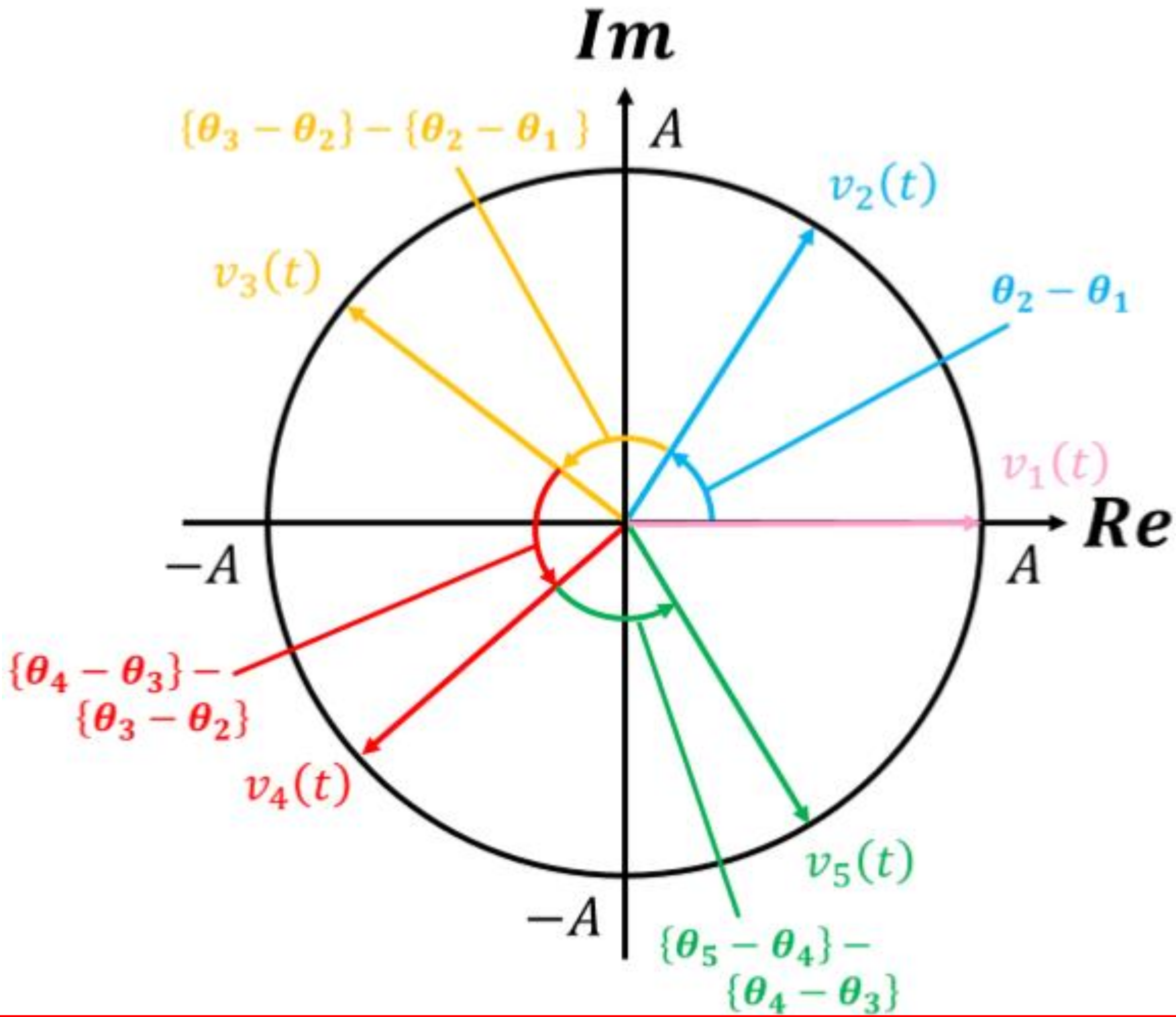
Vector Diagram of 1st Summation Term

$$\text{1st summation term} = \sum_{k=1}^{N-1} \underbrace{\cos(2\pi \cdot 1 \cdot \Delta f_0 t + \theta_{k+1} - \theta_k)}_{v_k(t)}$$

For example, when N=6

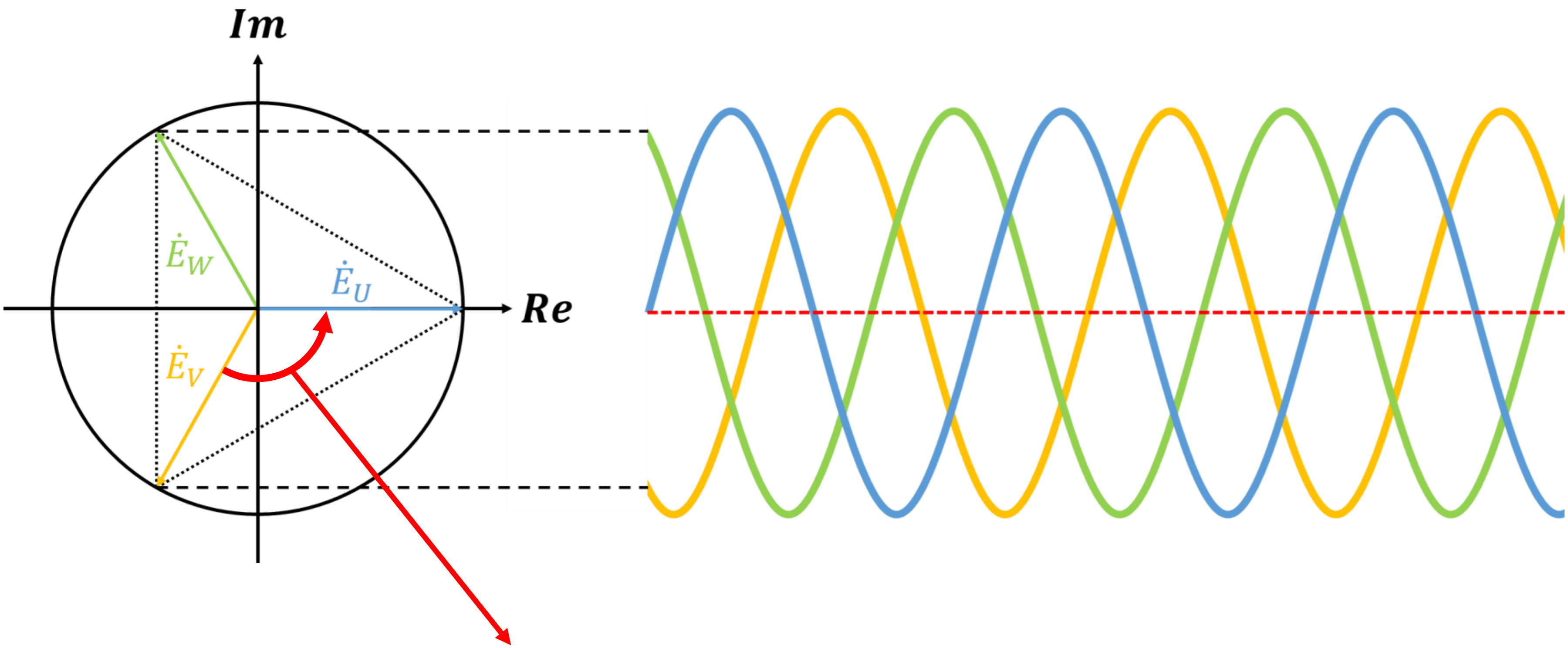


Angle Formed by Each Vector Φ_k



The angle formed by each vector : $\Phi_k = (\theta_{k+1} - \theta_k) - (\theta_k - \theta_{k-1})$

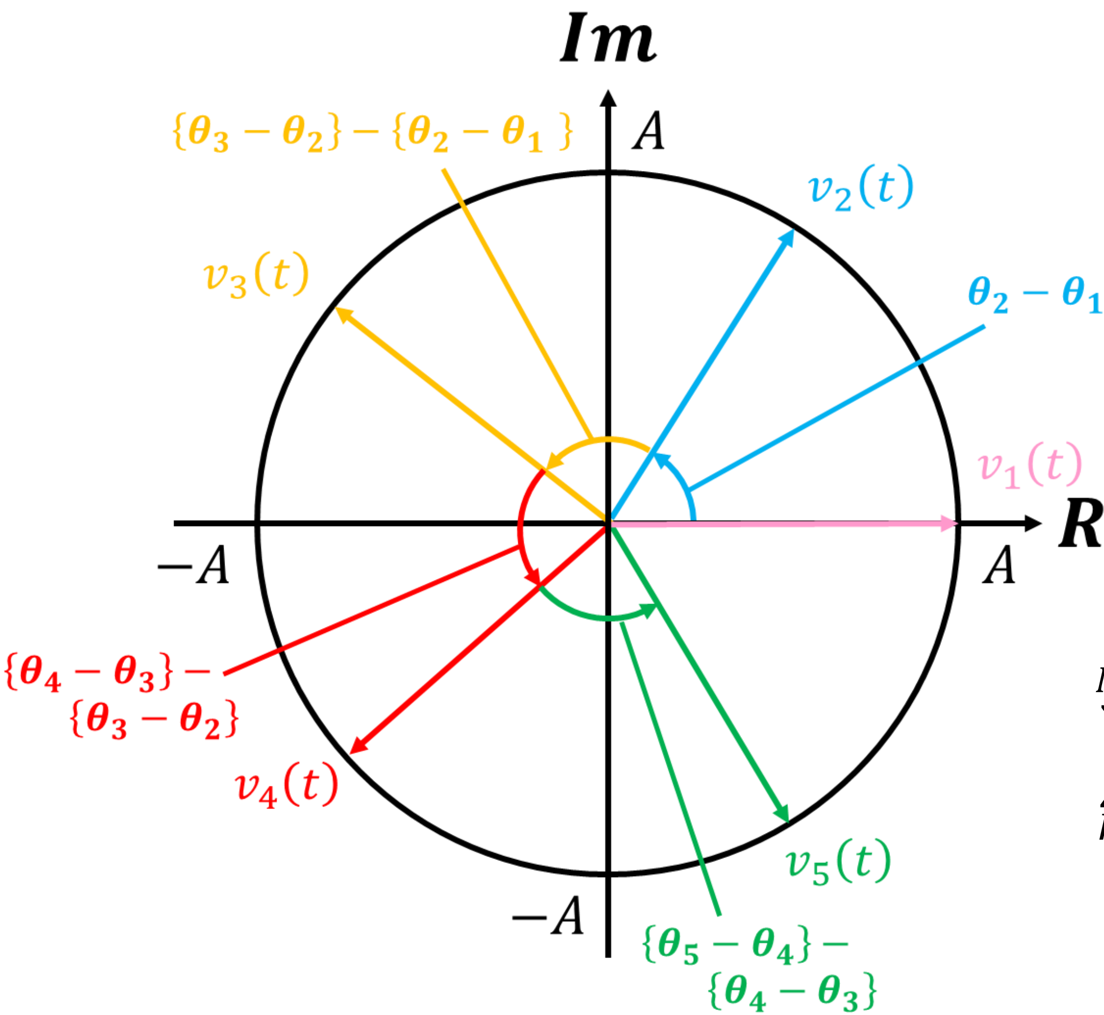
Symmetrical Polyphase AC Circuit



Angles formed by vectors are equal

Sum of the instantaneous values
of symmetrical polyphase AC circuit → always zero

Set 1st Summation Term to Zero



$$\Phi_k = \frac{2\pi}{\text{Number of vectors}} = \frac{2\pi}{N-1}$$

1st summation term : zero

||

$$\underbrace{\sum_{k=1}^{N-1} \cos(2\pi \cdot 1 \cdot \Delta f_0 t + \theta_{k+1} - \theta_k)}_{v_k(t)}$$

Basic equation of Narahashi phase

Narahashi Phase $$\theta_{\boldsymbol{k}} = \frac{\pi}{N-1}(k-1)(k-2)$$

$$\Phi_k = (\theta_{k+1} - \theta_k) - (\theta_k - \theta_{k-1})$$

$$\Phi_k = \frac{2\pi}{\text{Number of vectors}} = \frac{2\pi}{N-1}$$

$$\{\theta_{k+1} - \theta_k\} - \{\theta_k - \theta_{k-1}\} = \frac{2\pi}{N-1}$$

Solving for θ_k

Basic equation of Narahashi phase : $$\theta_k = (k-1)\theta_2 - (k-2)\theta_1 + \frac{(k-1)(k-2)}{N-1}\pi$$

Narahashi Phase

Basic equation of Narahashi phase: $\theta_k = (k-1)\theta_2 - (k-2)\theta_1 + \frac{(k-1)(k-2)}{N-1}\pi$

$\theta_1 = \theta_2 = 0$

$$\theta_k = \frac{(k-1)(k-2)}{N-1}\pi$$

k	1	2	3	4	5	6
θ_k	0	0	$\frac{\pi}{N-1}\cdot 2$	$\frac{\pi}{N-1}\cdot 6$	$\frac{\pi}{N-1}\cdot 12$	$\frac{\pi}{N-1}\cdot 20$

Ψ_k	0	$\frac{\pi}{N-1}\cdot 2$	$\frac{\pi}{N-1}\cdot 4$	$\frac{\pi}{N-1}\cdot 6$	$\frac{\pi}{N-1}\cdot 8$

Φ_k	$\frac{2\pi}{N-1}$	$\frac{2\pi}{N-1}$	$\frac{2\pi}{N-1}$	$\frac{2\pi}{N-1}$

Newman Phase

$$\theta_k = \frac{(k-1)^2}{N}\pi$$

k	1	2	3	4	5	6
θ_k	0	$\frac{\pi}{N}$	$\frac{\pi}{N}\cdot 4$	$\frac{\pi}{N}\cdot 9$	$\frac{\pi}{N}\cdot 16$	$\frac{\pi}{N}\cdot 25$

Ψ_k	$\frac{\pi}{N}$	$\frac{\pi}{N}\cdot 3$	$\frac{\pi}{N}\cdot 5$	$\frac{\pi}{N}\cdot 7$	$\frac{\pi}{N}\cdot 9$

Φ_k	$\frac{2\pi}{N}$	$\frac{2\pi}{N}$	$\frac{2\pi}{N}$	$\frac{2\pi}{N}$

Matches Narahashi phase

What is the difference between the initial phase equations?

Difference between Narahashi and Newman

Basic equation of the Narahashi phase :

$$\theta_k = (k-1)\theta_2 - (k-2)\theta_1 + \frac{(k-1)(k-2)}{N}\pi$$

Solving for k

$$\theta_k = \frac{\pi}{N}k^2 + \left(-\frac{3\pi}{N} + \theta_2 - \theta_1\right)k + \left(\frac{2\pi}{N} - \theta_2 + 2\theta_1\right) \cdots ①$$

Newman Phase :

$$\theta_k = \frac{(k-1)^2}{N}\pi = \frac{\pi}{N}k^2 - 2\cdot\frac{\pi}{N}k + \frac{\pi}{N} \cdots ②$$

Comparing the coefficients between ① and ②

$$\theta_1 = 0\,, \theta_2 = \frac{\pi}{N}$$

Simply changing the setting values of θ_1 and θ_2

Kitayoshi Phase

$$\theta_{\boldsymbol{k}} = \frac{\pi}{N} k(k+1)$$

k	1	2	3	4	5	6
θ_k	$\frac{\pi}{N} \cdot 2$	$\frac{\pi}{N} \cdot 6$	$\frac{\pi}{N} \cdot 12$	$\frac{\pi}{N} \cdot 20$	$\frac{\pi}{N} \cdot 30$	$\frac{\pi}{N} \cdot 42$

Ψ_k	$\frac{\pi}{N} \cdot 4$	$\frac{\pi}{N} \cdot 6$	$\frac{\pi}{N} \cdot 8$	$\frac{\pi}{N} \cdot 10$	$\frac{\pi}{N} \cdot 12$

Φ_k	$\frac{\pi}{N} \cdot 2$	$\frac{\pi}{N} \cdot 2$	$\frac{\pi}{N} \cdot 2$	$\frac{\pi}{N} \cdot 2$

Matches Narahashi phase and Newman phase

Difference between Narahashi and Kitayoshi

Basic equation of the Narahashi phase :

$$\theta_k = (k-1)\theta_2 - (k-2)\theta_1 + \frac{(k-1)(k-2)}{N}\pi$$

Solving for k

$$\theta_k = \frac{\pi}{N}k^2 + \left(-\frac{3\pi}{N} + \theta_2 - \theta_1\right)k + \left(\frac{2\pi}{N} - \theta_2 + 2\theta_1\right) \cdots ①$$

Kitayoshi Phase :

$$\theta_k = \frac{\pi}{N}k(k+1) = \frac{\pi}{N}k^2 + \frac{\pi}{N}k \cdots ②$$

Comparing the coefficients between ① and ②

$$\theta_1 = \frac{2\pi}{N}, \theta_2 = \frac{6\pi}{N}$$

Simply changing the setting values of θ_1 and θ_2

Schroeder Phase

$$\theta_{\boldsymbol{k}} = \frac{\pi}{N} k(k-1)$$

k	1	2	3	4	5	6
θ_k	0	$\frac{\pi}{N} \cdot 2$	$\frac{\pi}{N} \cdot 6$	$\frac{\pi}{N} \cdot 12$	$\frac{\pi}{N} \cdot 20$	$\frac{\pi}{N} \cdot 30$

Ψ_k	$\frac{\pi}{N} \cdot 2$	$\frac{\pi}{N} \cdot 4$	$\frac{\pi}{N} \cdot 6$	$\frac{\pi}{N} \cdot 8$	$\frac{\pi}{N} \cdot 10$

Φ_k	$\frac{\pi}{N} \cdot 2$	$\frac{\pi}{N} \cdot 2$	$\frac{\pi}{N} \cdot 2$	$\frac{\pi}{N} \cdot 2$

Matches Narahashi phase and Newman phase and Kitayoshi phase

Difference between Narahashi and Schroeder

Basic equation of the Narahashi phase:

$$\theta_k = (k-1)\theta_2 - (k-2)\theta_1 + \frac{(k-1)(k-2)}{N}\pi$$

Solving for k

$$\theta_k = \frac{\pi}{N}k^2 + \left(-\frac{3\pi}{N} + \theta_2 - \theta_1\right)k + \left(\frac{2\pi}{N} - \theta_2 + 2\theta_1\right) \cdots ①$$

Schroeder Phase:

$$\theta_k = \frac{\pi}{N}k(k-1) = \frac{\pi}{N}k^2 - \frac{\pi}{N}k \cdots ②$$

Comparing the coefficients between ① and ②

$$\theta_1 = 0, \theta_2 = \frac{2\pi}{N}$$

Simply changing the setting values of θ_1 and θ_2

Unification of Initial Phase Setting Equations

Basic equation of Narahashi phase : $\theta_k = (k-1)\theta_2 - (k-2)\theta_1 + \frac{(k-1)(k-2)}{N}\pi$

$\theta_1 = 0\,, \theta_2 = \frac{\pi}{N}$ → Newman phase : $\theta_k = \frac{(k-1)^2}{N}\pi$

$\theta_1 = \frac{2\pi}{N}, \theta_2 = \frac{6\pi}{N}$ → Kitayoshi phase : $\theta_k = \frac{\pi}{N}k(k+1)$

$\theta_1 = 0\,, \theta_2 = \frac{2\pi}{N}$ → Schroeder phase : $\theta_k = \frac{\pi}{N}k(k-1)$

Equation can be unified by think of 1st and 2nd phases as "initial values" ← Similarity of CF reduction effect

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Conclusion

- In algorithms for reducing CF, we can unify the four algorithms by analyzing the second derivative of θ_k.
- Proper multi-tone signal generation algorithms can reduce CF.

Four Algorithms References

Newman Phase

D. J. Newman, "An L1 Extremal Problem for Polynomials," Proc. Amer. Math. Soc., no.16, pp. 1287-1290 (Dec. 1965).

Kitayoshi Phase

H. Kitayoshi, S. Sumida, K. Shirakawa, S. Takeshita, "DSP Synthesized Signal Source for Analog Testing Stimulus and New Test Method", IEEE International Test Conference, (Jan. 1985).

Schroeder Phase

M. R. Schroeder, "Synthesis of Low-Peak-Factor Signals and Binary Sequence with Low Autocorrelation," IEEE Trans. Information Theory, vol. 16, pp. 85-89 (Jan. 1970).

Narahashi Phase

S. Narahashi, T. Nojima, "Initial Phase Setting Method to Reduce Peak-to-Average Power Ratio (PAPR) of Multi-tone Signal," IEICE Transactions, vol. J78-B-II, no.11, pp.663-670 (Nov. 1995).

Q&A

Q. Would you please explain more details about the improvement of SNR in your research?
How to evaluate the improvement of SNR in the practical measurements?
Do you plain to extend the research in future work?

A. Reduce the crest factor.