Measurement of Self-Loop Function and Stability Test for Third-Order Sallen-Key Low-Pass Filters

MinhTri Tran, Anna Kuwana, Haruo Kobayashi

Division of Electronics and Informatics, Faculty of Science and Technology, Gunma University, Japan
Outline

1. Research Background
   • Motivation, objectives and achievements
   • Self-loop function in a transfer function
2. Analysis of High-Order Transfer Functions
   • Operating regions of second-order complex functions
3. Ringing Test for Unity-Gain Amplifiers
   • Behaviors of op amps with feedback networks
4. Ringing Test for High-Order Low-Pass Filters
   • Behaviors of Sallen-Key low-pass filters
5. Conclusions
1. Research Background
Noise in Electronic Systems

Performance of a system

Signal to Noise Ratio:

\[ SNR = \frac{\text{Signal power}}{\text{Noise power}} \]

Performance of a device

Figure of Merit:

\[ F = \frac{\text{Output SNR}}{\text{Input SNR}} \]

Common types of noise:
• Electronic noise
• Thermal noise,
• Intermodulation noise,
• Cross-talk,
• Impulse noise,
• Shot noise, and
• Transit-time noise.

Device noise:
• Flicker noise,
• Thermal noise,
• White noise.

Linear networks
• Overshoot,
• Ringing
• Oscillation noise
1. Research Background

Motivation of Study

Ringing represents a distortion of a signal. Ringing is overshoot/undershoot voltage or current when it’s seen on time domain.

Ringing does the following things:

• Causes EMI noise,
• Increases current flow,
• Consumes the power,
• Decreases the performance, and
• Damages the devices.

○ Ringing affects both input and output signals.
1. Research Background

Objectives and Achievements

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<td>- Investigation of operating regions of linear negative feedback networks</td>
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<tr>
<td>- Over-damping (high delay in rising time)</td>
</tr>
<tr>
<td>- Critical damping (max power propagation)</td>
</tr>
<tr>
<td>- Under-damping (overshoot and ringing)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Achievements</th>
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<tr>
<td>- Measurement of self-loop function and stability test for 3\textsuperscript{rd}-order Sallen-Key low-pass filters.</td>
</tr>
</tbody>
</table>
1. Research Background

Approaching Methods

2\textsuperscript{nd}-order Sallen-Key LPF

3\textsuperscript{rd}-order Sallen-Key LPF

Balun transformer

Implemented circuit
1. Research Background

**Self-loop Function in A Transfer Function**

**Linear system**

\[ H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{A(\omega)}{1 + L(\omega)} \]

**Transfer function**

\[ H(\omega) = \frac{b_0(j\omega)^n + \ldots + b_{n-1}(j\omega) + b_n}{a_0(j\omega)^n + \ldots + a_{n-1}(j\omega) + a_n} \]

- **Model of a linear system**
- **A(\omega)**: Open loop function
- **H(\omega)**: Transfer function
- **L(\omega)**: Self-loop function
- Variable: angular frequency \((\omega)\)

- Polar chart \(\rightarrow\) Nyquist chart
- Magnitude-frequency plot
- Angular-frequency plot
- Magnitude-angular diagram \(\rightarrow\) Nichols diagram

**Bode plots**
1. Research Background
Characteristics of Adaptive Feedback Network

Block diagram of a typical adaptive feedback system

Adaptive feedback is used to control the output source along with the decision source (DC-DC Buck converter). Transfer function of an adaptive feedback network is significantly different from transfer function of a linear negative feedback network. → Loop gain is independent of frequency variable (referent voltage, feedback voltage, and error voltage are DC voltages).
1. Research Background

Alternating Current Conservation

Transfer function

\[ H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{1 + \frac{Z_{in}}{Z_{out}}} \]

\[ \Rightarrow L(\omega) = \frac{Z_{in}}{Z_{out}} \]

Self-loop function

\[ \frac{V_{inc}}{Z_{in}} = -\frac{V_{trans}}{Z_{out}} \Rightarrow L(\omega) = -\frac{V_{inc}}{V_{trans}} = \frac{Z_{in}}{Z_{out}} \]

Simplified linear system

10 mH inductance

Incident current

Transmitted current

Derivation of self-loop function
1. Research Background

Limitations of Conventional Methods

- **Middlebrook’s measurement of loop gain**
  - Applying only in feedback systems (*DC-DC converters*).

- **Replica measurement of loop gain**
  - Using two identical networks (*not real measurement*).

- **Nyquist’s stability condition**
  - Theoretical analysis for feedback systems (*Lab tool*)

- **Nichols Chart of Loop Gain**
  - Only used in feedback control theory (*Lab tool*)
Outline

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2. Analysis of High-Order Transfer Functions
   • Operating regions of second-order complex functions

3. Ringing Test for Unity-Gain Amplifiers
   • Behaviors of op amps with feedback networks

4. Ringing Test for High-Order Low-Pass Filters
   • Behaviors of Sallen-Key low-pass filters

5. Conclusions
2. Analysis of High-Order Transfer Functions

Operating Regions of 2\textsuperscript{nd}-order Transfer Function

- **Under-damping:** \( H_1(\omega) = \frac{1}{(j\omega)^2 + j\omega + 1}; \)
- **Critical damping:** \( H_2(\omega) = \frac{1}{(j\omega)^2 + 2j\omega + 1}; \)
- **Over-damping:** \( H_3(\omega) = \frac{1}{(j\omega)^2 + 3j\omega + 1}; \)

**Nyquist chart of transfer function**

**Bode plot of transfer function**

- Under-damping
- Critical damping
- Over-damping

- 0dB
- -6dB
- -12dB

- 10mHz 100mHz 1Hz 10Hz
- Frequency (Hz)

- 0°
- -30°
- -120°

- 10mHz 100mHz 1Hz 10Hz
- Frequency (Hz)
## 2. Analysis of High-Order Transfer Functions

### Summary of 2nd-order Transfer Function

**Second-order transfer function:**

\[
H(\omega) = \frac{1}{1 + a_0(j\omega)^2 + a_1j\omega}
\]

<table>
<thead>
<tr>
<th>Case</th>
<th>Over-damping</th>
<th>Critical damping</th>
<th>Under-damping</th>
</tr>
</thead>
</table>
| Delta (\(\Delta\)) | \[
\frac{1}{a_0} < \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 > 0
\]
| Module | \[
\sqrt{\omega^2 + \left(\frac{a_1}{2a_0}\right)^2} = -6dB
\]
| Angular \(\theta(\omega)\) | \[-\arctan\left(\frac{\omega}{\sqrt{\frac{a_1}{2a_0}} - 1}\right)\] | \[-2 \arctan\left(\frac{2a_0\omega}{a_1}\right)\] | \[-\arctan\left(\frac{\omega}{\sqrt{\frac{a_1}{2a_0}}} - 1\right)\] |
| \(\omega_{\text{cut}} = \frac{a_1}{2a_0}\) | \[|H(\omega_{\text{cut}})| < \frac{2a_0}{a_1}\] | \[\theta(\omega_{\text{cut}}) > -\frac{\pi}{2}\] | \[|H(\omega_{\text{cut}})| > \frac{2a_0}{a_1}\] |
2. Analysis of High-Order Transfer Functions

Operating regions of 2\textsuperscript{nd}-order Self-loop Function

- **Under-damping**: \( L_1(\omega) = (j\omega)^2 + j\omega; \)
- **Critical damping**: \( L_2(\omega) = (j\omega)^2 + 2j\omega; \)
- **Over-damping**: \( L_3(\omega) = (j\omega)^2 + 3j\omega; \)

**Bode plot of self-loop function**

**Nyquist chart of self-loop function**

- **Phase margin**: 52 degrees
- **Bode plot**:
  - **Magnitude (dB)**
  - **Frequency (Hz)**
  - **Phase margin**: 128°
  - **Phase (deg)**
  - **Frequency (Hz)**

**Graphical representations**
# 2. Analysis of High-Order Transfer Functions

## Summary of 2\textsuperscript{nd}-order Self-loop Function

**Second-order self-loop function:** \( L(\omega) = j\omega [a_0 j\omega + a_1] \)

<table>
<thead>
<tr>
<th>Case</th>
<th>Over-damping</th>
<th>Critical damping</th>
<th>Under-damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta ((\Delta))</td>
<td>(\Delta = a_1^2 - 4a_0 &gt; 0)</td>
<td>(\Delta = a_1^2 - 4a_0 = 0)</td>
<td>(\Delta = a_1^2 - 4a_0 &lt; 0)</td>
</tr>
<tr>
<td>(</td>
<td>L(\omega)</td>
<td>)</td>
<td>(\omega\sqrt{(a_0\omega)^2 + a_1^2})</td>
</tr>
<tr>
<td>(\theta(\omega))</td>
<td>(\frac{\pi}{2} + \arctan \frac{a_0\omega}{a_1})</td>
<td>(\frac{\pi}{2} + \arctan \frac{a_0\omega}{a_1})</td>
<td>(\frac{\pi}{2} + \arctan \frac{a_0\omega}{a_1})</td>
</tr>
<tr>
<td>(\omega_1 = \frac{a_1}{2a_0}\sqrt{5 - 2})</td>
<td>(</td>
<td>L(\omega_1)</td>
<td>&gt; 1)</td>
</tr>
<tr>
<td>(\omega_2 = \frac{a_1}{2a_0})</td>
<td>(</td>
<td>L(\omega_2)</td>
<td>&gt; \sqrt{5})</td>
</tr>
<tr>
<td>(\omega_3 = \frac{a_1}{a_0})</td>
<td>(</td>
<td>L(\omega_3)</td>
<td>&gt; 4\sqrt{2})</td>
</tr>
</tbody>
</table>
2. Analysis of High-Order Transfer Functions

Operating Regions of 2\textsuperscript{nd}-order System

**Bode plot of transfer function**

- Under-damping
- Critical damping
- Over-damping

**Nichols plot of self-loop function**

- Over-damping
- Critical damping
- Under-damping

**Transient response**

- Under-damping
- Critical damping
- Over-damping

**Over-damping:**

\[ \rightarrow \text{Phase margin is 88 degrees.} \]

**Critical damping:**

\[ \rightarrow \text{Phase margin is 76.3 degrees.} \]

**Under-damping:**

\[ \rightarrow \text{Phase margin is 52 degrees.} \]
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5. Conclusions
3. Ringing Test for Unity-Gain Amplifiers
Two-stage Op Amp without Miller’s Capacitor

Without frequency compensation

Small signal model

Transfer function $H(\omega)$ and self-loop function $L(\omega)$

$$H(\omega) = \frac{b_0 j\omega + b_1}{a_0 (j\omega)^2 + a_1 j\omega + 1};$$

$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega$$

Where,

$$b_0 = R_D R_S \left[ (C_{GD} + C_{DB})(C_{GS} + C_{GD}) - C_{GD}^2 \right]$$

$$b_1 = \left[ R_D (C_{GD} + C_{DB}) + R_S (C_{GS} + C_{GD}) + R_D R_S g_m C_{GD} \right]$$

$$a_0 = R_D C_{GD}; \quad a_1 = -R_D g_m;$$
3. Ringing Test for Unity-Gain Amplifiers

**Unity-Gain Amplifier without Miller’s Capacitor**

- **Bode plot of transfer function**
  - Magnitude of transfer function
  - Frequency (Hz)
  - 10 MHz to 10 GHz

- **Nichols plot of self-loop function**
  - Self-loop function
  - Magnitude (dB)
  - Phase (deg)
  - Phase margin = 13 degrees

- **Transient response**
3. Ringing Test for Unity-Gain Amplifiers

Two-stage Op Amp with Frequency Compensation

With Miller’s capacitor and resistor

Small signal model

Simplified model

Transfer function

\[ H(\omega) = \frac{b_0 (j\omega)^3 + b_1 (j\omega)^2 + b_2 j\omega + b_3}{a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega + 1}; \]

Self-loop function

\[ L(\omega) = a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega \]
3. **Ringing Test for Unity-Gain Amplifiers**

**Stability Test for Op Amp with Miller’s Capacitor**

Unity-gain amplifier with Miller’s capacitor

**Bode plot of transfer function**

**Nichols plot of self-loop function**

**Transient response**

**Operating regions**

**Under-damping:**
R1 = 2 kΩ, C1 = 1 pF

**Critical damping:**
R1 = 3.5 kΩ, C1 = 0.2 pF

**Over-damping:**
R1 = 3.5 kΩ, C1 = 0.8 pF
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4. Ringing Test for High-Order Low-Pass Filters

Review of 2\textsuperscript{nd}-order Sallen-Key Low-pass Filter

\textbf{2\textsuperscript{nd}-order Sallen-Key LPF}

![Diagram of 2\textsuperscript{nd}-order Sallen-Key Low-pass Filter]

\textbf{Transfer function}

\[ H(\omega) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{b_0}{a_0 (j\omega)^2 + a_1 j\omega + 1}; \]

\textbf{Self-loop function}

\[ L(\omega) = a_0 (j\omega)^2 + a_1 j\omega; \]

\textbf{Operating regions}

- **Over-damping:**
  \[ \frac{1}{R_1 R_2 C_1 C_2} < \frac{1}{4} \left( R_1 C_1 + R_1 C_1 - \frac{R_3}{R_4} R_1 C_2 \right)^2 \]

- **Critical damping:**
  \[ \frac{1}{R_1 R_2 C_1 C_2} = \frac{1}{4} \left( R_1 C_1 + R_1 C_1 - \frac{R_3}{R_4} R_1 C_2 \right)^2 \]

- **Under-damping:**
  \[ \frac{1}{R_1 R_2 C_1 C_2} > \frac{1}{4} \left( R_1 C_1 + R_1 C_1 - \frac{R_3}{R_4} R_1 C_2 \right)^2 \]

\textbf{Where:}

- \( \omega_0 = \sqrt{R_1 R_2 C_1 C_2} \)
- \( a_0 = R_1 R_2 C_1 C_2 \)
- \( a_1 = R_1 C_1 + R_1 C_1 - \frac{R_3}{R_4} R_1 C_2 \)
- \( b_0 = 1 + \frac{R_3}{R_4} \)
4. Ringing Test for High-Order Low-Pass Filters

Proposed Design of 3rd-Order Sallen-Key LPF

Differential 3rd-order Sallen-Key LPF

Transfer function

\[ H(\omega) = \frac{b_0}{a_0 (j\omega)^3 + a_1 (j\omega)^2 + a_2 j\omega + 1}; \]

Self-loop function

\[ L(\omega) = a_0 (j\omega)^3 + a_1 (j\omega)^2 + a_2 j\omega; \]

Where \( b_0 = 1 + \frac{R_4}{R_5}; a_0 = R_1C_1R_2C_2R_3C_3; \)

\[ a_1 = R_1C_1C_3(R_2 + R_3) + R_3C_2C_3(R_1 + R_2) - \frac{R_4}{R_5}R_1C_1R_2C_2; \]

\[ a_2 = R_1(C_1 + C_3) + C_3(R_2 + R_3) - \frac{R_4}{R_5}(R_1 + R_2)C_2; \]

Component parameters

GBW = 10MHz, \( f_o = 10k\text{Hz}, \) DC gain (Ao) = 100000, R1 = R2 = R3 = 10 kΩ, R4 = 100 Ω, R5 = 100 kΩ, C1 = 350 pF, C2 = 2 nF.
4. Ringing Test for High-Order Low-Pass Filters
Simulation Results of 3rd-Order Sallen-Key LPF

**Bode plot of transfer function**

**Nichols plot of self-loop function**

**Transient response**

**Operating regions**

**Over-damping:**
- Phase margin is 80 degrees.

**Critical damping:**
- Phase margin is 73 degrees.

**Under-damping:**
- Phase margin is 36 degrees.
4. Ringing Test for High-Order Low-Pass Filters

Implemented Circuit of 3rd-Order Sallen-Key LPF

**Single ended 3rd-order Sallen-Key LPF**

\[ \text{Component parameters} \]

Op Amp: LM358; GBW = 10MHz, DC gain (Ao) = 100000, R1 = R2 = R3 = 10 kΩ, R4 = 100 Ω, R5 = 100 kΩ, C1 = 350 pF, C2 = 2 nF.

**Implemented circuit**

10 mH balun transformer
4. Ringing Test for High-Order Low-Pass Filters

Measurement Results of 3\textsuperscript{rd}-Order Sallen-Key LPF

**Over-damping:**
- Phase margin is 77 degrees.

**Critical damping:**
- Phase margin is 70 degrees.

**Under-damping:**
- Phase margin is 64 degrees.
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## 5. Comparison

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<th>Replica measurement</th>
<th>Middlebrook’s method</th>
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<td><strong>Main objective</strong></td>
<td>Self-loop function</td>
<td>Loop gain</td>
<td>Loop gain</td>
</tr>
<tr>
<td><strong>Transfer function accuracy</strong></td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Ringing Test</strong></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Operating region accuracy</strong></td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Phase margin accuracy</strong></td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Passive networks</strong></td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>
5. Discussions

- Loop gain is independent of frequency variable.

→ Loop gain in adaptive feedback network is significantly different from self-loop function in linear negative feedback network.

Nichols chart is only used in MATLAB simulation.

Nichols chart isn’t used widely in practical measurements (only used in control theory).

https://www.mathworks.com/help/control/ref/nichols.html

(Technology limitations)
5. Conclusions

This work:

• Proposal of alternating current conservation for deriving self-loop function in a transfer function
  → Observation of self-loop function can help us optimize the behavior of a high-order system.

• Implementations of circuits and measurements of self-loop functions for Sallen-Key low-pass filters
  → Theoretical concepts of stability test are verified by laboratory simulations and practical experiments.

Future of work:

• Stability test for parasitic components in transmission lines, printed circuit boards, physical layout layers
References


Thank you very much!
ご清聴ありがとうございます。