電子回路研究会

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群馬大学
GUNMA UNIVERSITY

Measurement of Self-Loop Function and Stability Test for Third-Order Sallen-Key Low-Pass Filters

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Outline

1. Research Background
   - Motivation, objectives and achievements
   - Self-loop function in a transfer function
2. Analysis of High-Order Transfer Functions
   - Operating regions of second-order complex functions
3. Ringing Test for Unity-Gain Amplifiers
   - Behaviors of op amps with feedback networks
4. Ringing Test for High-Order Low-Pass Filters
   - Behaviors of Sallen-Key low-pass filters
5. Conclusions

1. Research Background

Noise in Electronic Systems

Performance of a system

Signal to Noise Ratio:

$$\text{SNR} = \frac{\text{Signal power}}{\text{Noise power}}$$

Performance of a device

Figure of Merit:

$$\text{F} = \frac{\text{Output SNR}}{\text{Input SNR}}$$

Common types of noise:

- Electronic noise
- Thermal noise,
- Intermodulation noise,
- Cross-talk,
- Impulse noise,
- Shot noise, and
- Transit-time noise.

Device noise:

- Flicker noise,
- Thermal noise,
- White noise.

Linear networks

- Overshoot,
- Ringing
- Oscillation noise

1. Research Background

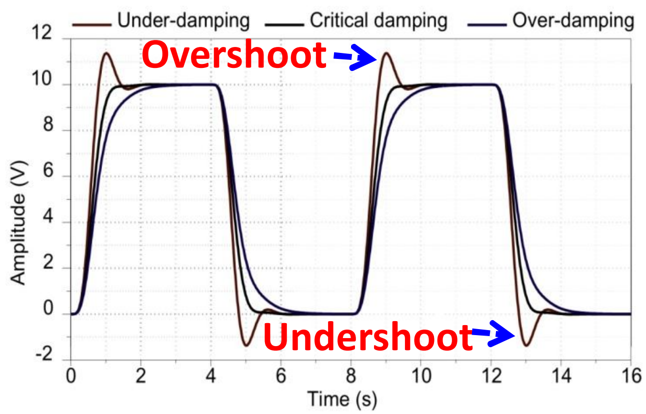
Motivation of Study

Ringing represents a distortion of a signal.

Ringing is overshoot/undershoot voltage or current when it's seen on time domain.

Ringing does the following things:

- **Causes EMI noise,**
- **Increases current flow,**
- **Consumes the power,**
- **Decreases the performance, and**
- **Damages the devices.**



STABILITY TEST

○ **Ringing affects both input and output signals.**

1. Research Background

Objectives and Achievements

Objectives

- ○ **Investigation of operating regions of linear negative feedback networks**

→ **Over-damping (high delay in rising time)**

→ **Critical damping (max power propagation)**

→ **Under-damping (overshoot and ringing)**

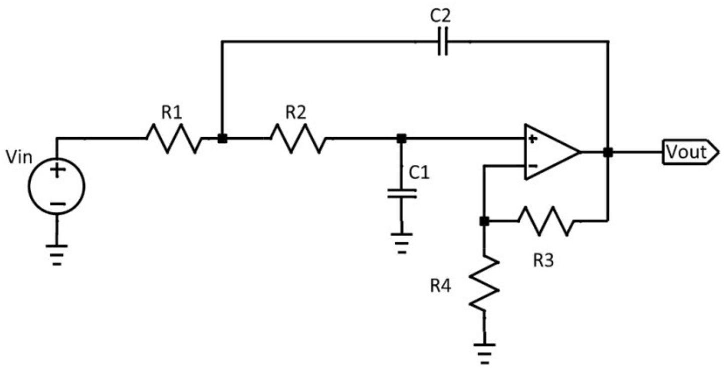
Achievements

- ○ **Measurement of self-loop function and stability test for 3rd-order Sallen-Key low-pass filters.**

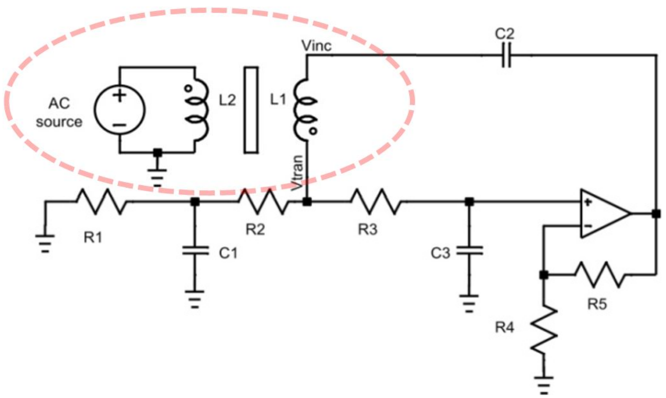
1. Research Background

Approaching Methods

2nd-order Sallen-Key LPF

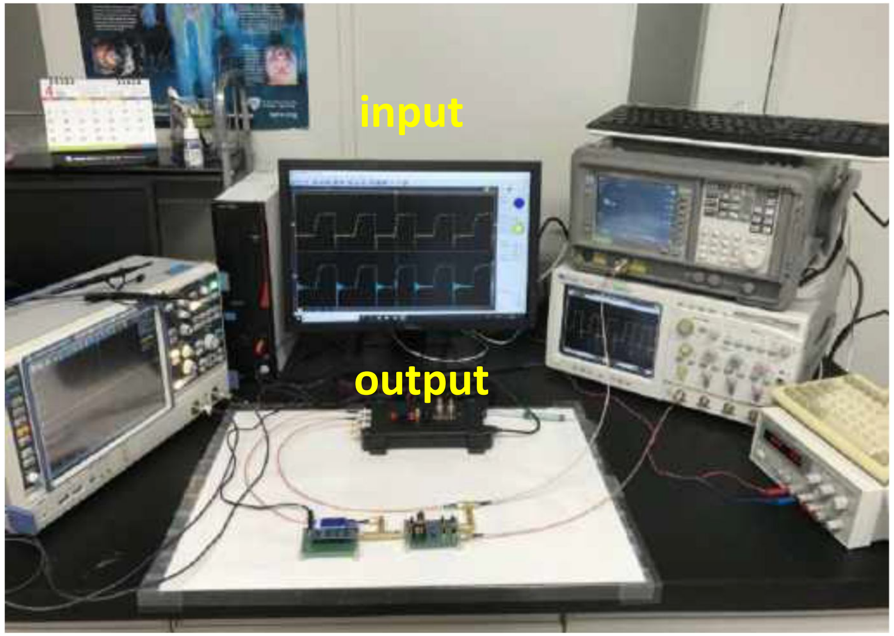


3rd-order Sallen-Key LPF



Balun transformer

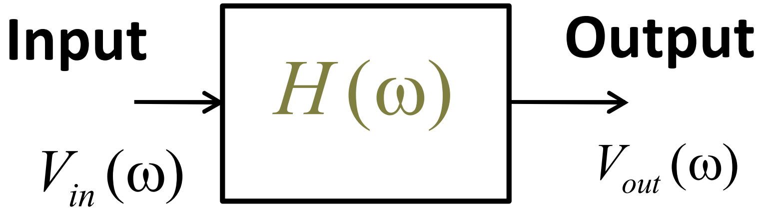
Implemented circuit



1. Research Background

Self-loop Function in A Transfer Function

Linear system



Model of a linear system

$$H(\omega) = \frac{b_0(j\omega)^n + \ldots + b_{n-1}(j\omega) + b_n}{a_0(j\omega)^n + \ldots + a_{n-1}(j\omega) + a_n}$$

Transfer function

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{A(\omega)}{1 + L(\omega)}$$

$A(\omega)$ **: Open loop function**

$H(\omega)$ **: Transfer function**

$L(\omega)$ **: Self-loop function**

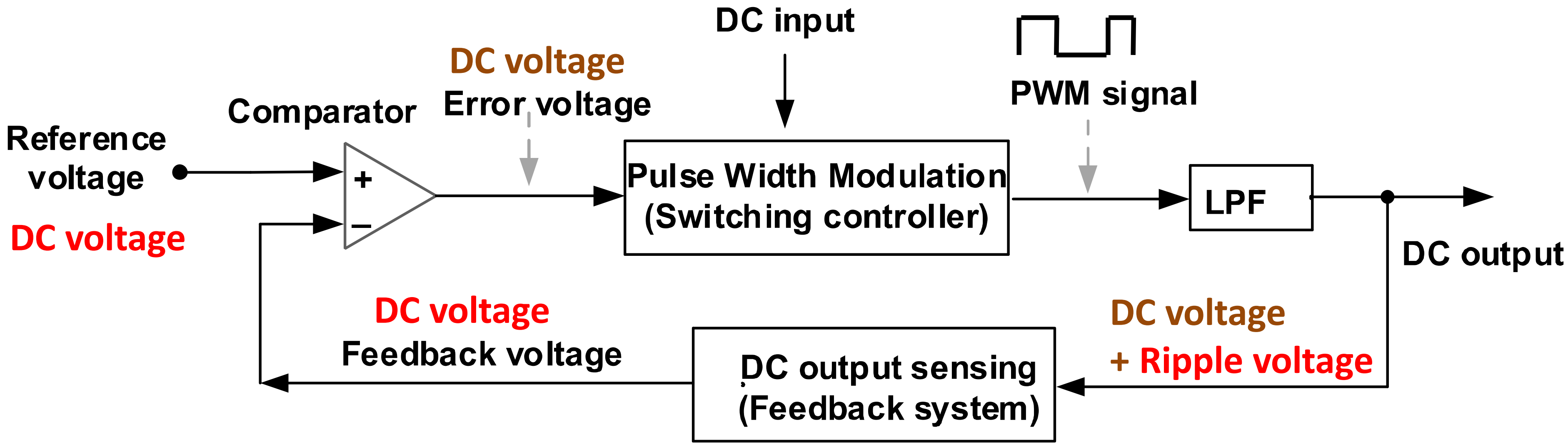
Variable: angular frequency (ω)

- **Polar chart → Nyquist chart**
- **Magnitude-frequency plot** } **Bode plots**
- **Angular-frequency plot** } **Bode plots**
- **Magnitude-angular diagram → Nichols diagram**

1. Research Background

Characteristics of Adaptive Feedback Network

Block diagram of a typical adaptive feedback system



Adaptive feedback is used to control the output source along with the decision source (DC-DC Buck converter).

Transfer function of an adaptive feedback network is significantly different from transfer function of a linear negative feedback network.

→ Loop gain is independent of frequency variable (referent voltage, feedback voltage, and error voltage are DC voltages).

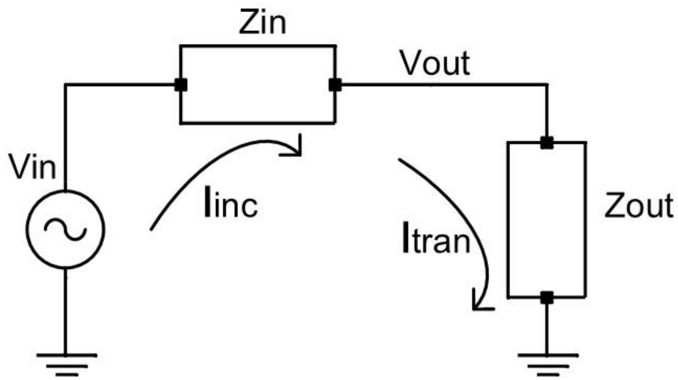
1. Research Background

Alternating Current Conservation

Transfer function

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{1 + \frac{Z_{in}}{Z_{out}}}$$

$$\Rightarrow L(\omega) = \frac{Z_{in}}{Z_{out}};$$

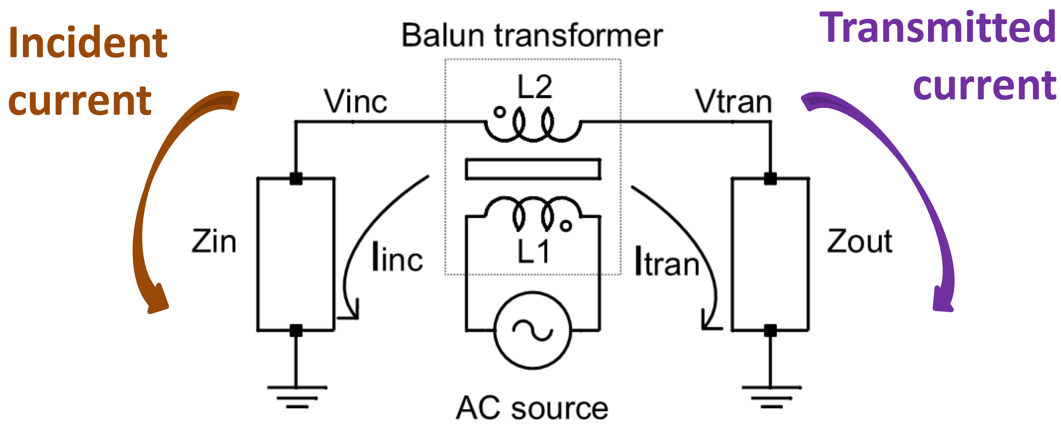


Simplified linear system

Self-loop function

$$\frac{V_{inc}}{Z_{in}} = -\frac{V_{trans}}{Z_{out}} \Rightarrow L(\omega) = -\frac{V_{inc}}{V_{trans}} = \frac{Z_{in}}{Z_{out}}$$

10 mH inductance



Derivation of self-loop function

1. Research Background

Limitations of Conventional Methods

- **Middlebrook's measurement of loop gain**

→ **Applying only in feedback systems (DC-DC converters).**

- **Replica measurement of loop gain**

→ **Using two identical networks (not real measurement).**

- **Nyquist's stability condition**

→ **Theoretical analysis for feedback systems (Lab tool)**

- **Nichols Chart of Loop Gain**

→ **Only used in feedback control theory (Lab tool)**

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2. **Analysis of High-Order Transfer Functions**
- **Operating regions of second-order complex functions**

3. Ringing Test for Unity-Gain Amplifiers
- Behaviors of op amps with feedback networks

4. Ringing Test for High-Order Low-Pass Filters
- Behaviors of Sallen-Key low-pass filters

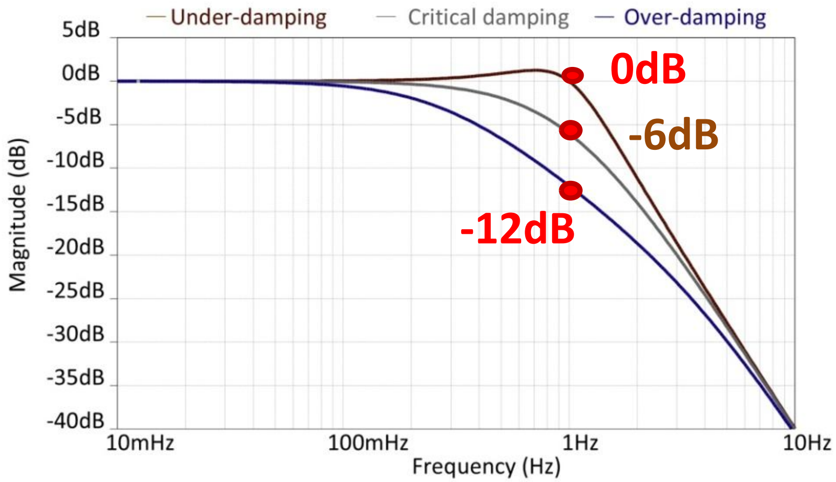
5. Conclusions

2. Analysis of High-Order Transfer Functions

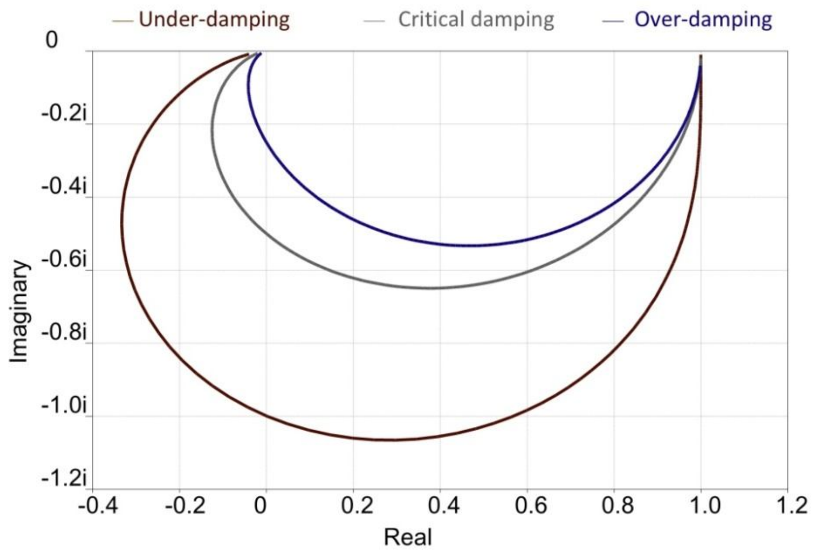
Operating Regions of 2nd-order Transfer Function

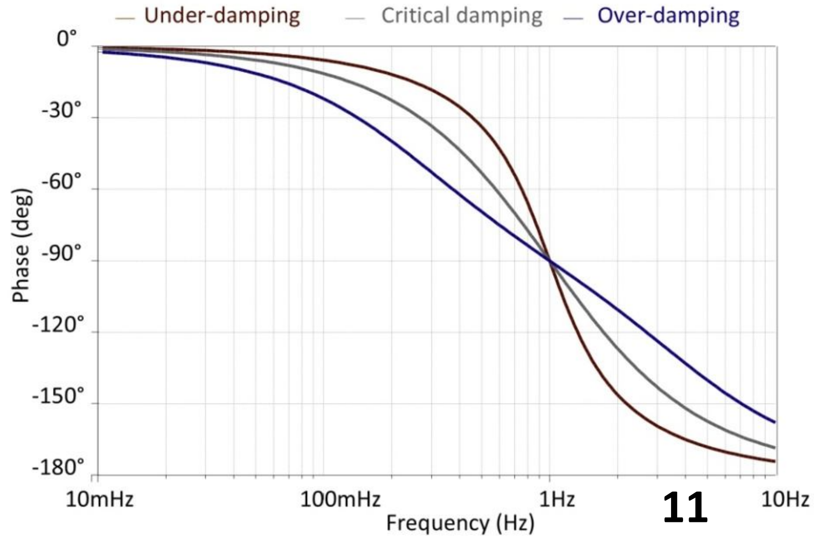
- **Under-damping:** $H_1(\omega) = \dfrac{1}{(j\omega)^2 + j\omega + 1};$
- **Critical damping:** $H_2(\omega) = \dfrac{1}{(j\omega)^2 + 2j\omega + 1};$
- **Over-damping:** $H_3(\omega) = \dfrac{1}{(j\omega)^2 + 3j\omega + 1};$

Bode plot of transfer function



Nyquist chart of transfer function





2. Analysis of High-Order Transfer Functions

Summary of 2nd-order Transfer Function

Second-order transfer function: $H(\omega) = \dfrac{1}{1 + a_0 (j\omega)^2 + a_1 j\omega}$

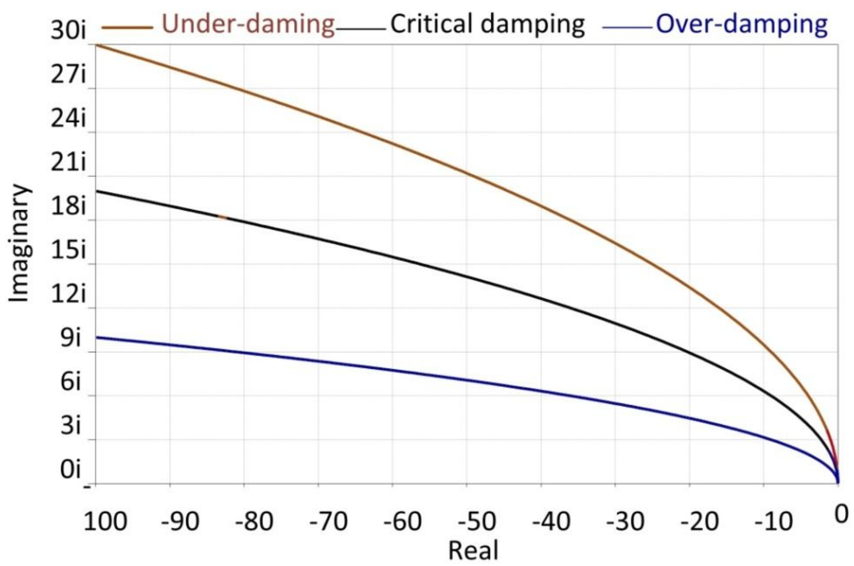
Case	Over-damping	Critical damping	Under-damping
Delta (Δ)	$\frac{1}{a_0} < \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 > 0$	$\frac{1}{a_0} = \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 = 0$	$\frac{1}{a_0} > \left(\frac{a_1}{2a_0}\right)^2 \Rightarrow \Delta = a_1^2 - 4a_0 < 0$
Module $\lvert H(\omega) \rvert$	$\dfrac{\frac{1}{a_0}}{\sqrt{\omega^2 + \left(\frac{a_1}{2a_0} - \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}\right)^2}\sqrt{\omega^2 + \left(\frac{a_1}{2a_0} + \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}\right)^2}}$	$\frac{1}{a_0}\dfrac{1}{\left[\omega^2 + \left(\frac{a_1}{2a_0}\right)^2\right]} = -6dB$	$\dfrac{\frac{1}{a_0}}{\sqrt{\left(\omega - \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}\right)^2 + \left(\frac{a_1}{2a_0}\right)^2}\sqrt{\left(\omega + \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}\right)^2 + \left(\frac{a_1}{2a_0}\right)^2}}$
Angular $\theta(\omega)$	$-\arctan\left(\dfrac{\omega}{\frac{a_1}{2a_0} - \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}}\right) - \arctan\left(\dfrac{\omega}{\frac{a_1}{2a_0} + \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}}\right)$	$-2\arctan\left(\dfrac{2a_0\omega}{a_1}\right)$	$-\arctan\left(\dfrac{\omega - \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}}{\frac{a_1}{2a_0}}\right) - \arctan\left(\dfrac{\omega + \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}}{\frac{a_1}{2a_0}}\right)$
$\omega_{cut} = \dfrac{a_1}{2a_0}$	$\lvert H(\omega_{cut}) \rvert < \dfrac{2a_0}{a_1}$; $\theta(\omega_{cut}) > -\dfrac{\pi}{2}$	$\lvert H(\omega_{cut}) \rvert = \dfrac{2a_0}{a_1}$; $\theta(\omega_{cut}) = -\dfrac{\pi}{2}$	$\lvert H(\omega_{cut}) \rvert > \dfrac{2a_0}{a_1}$; $\theta(\omega_{cut}) < -\dfrac{\pi}{2}$

2. Analysis of High-Order Transfer Functions

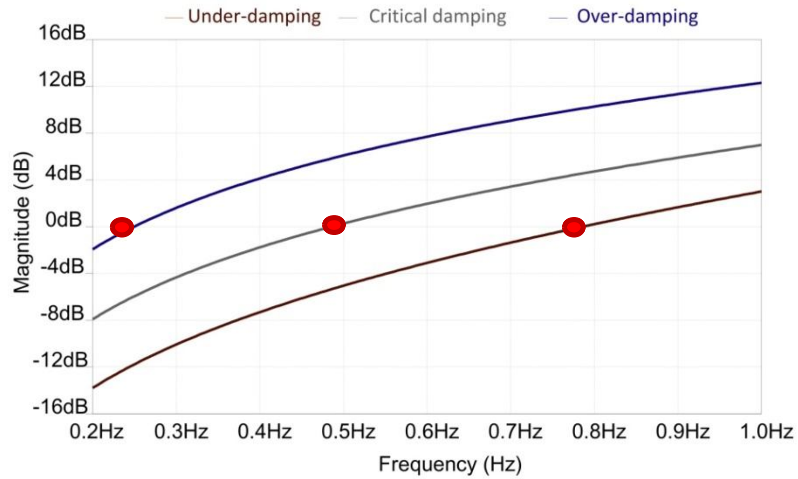
Operating regions of 2nd-order Self-loop Function

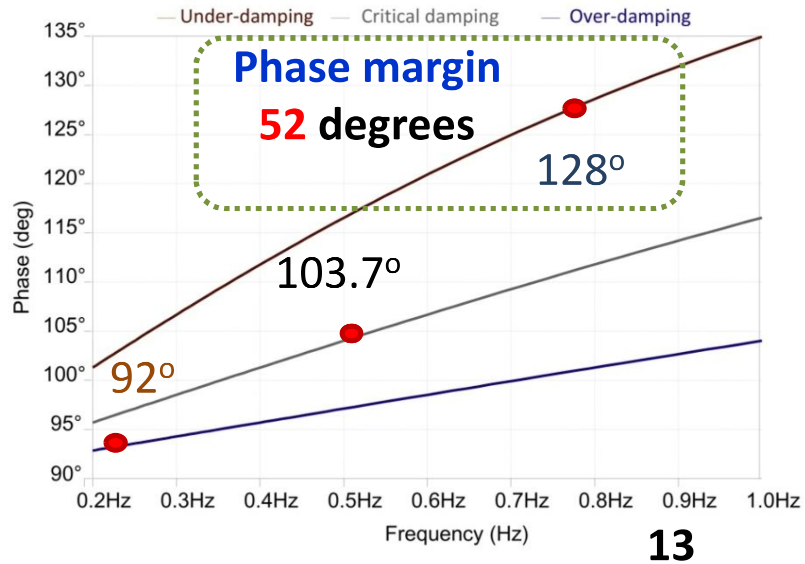
- **Under-damping:** $L_1(\omega) = (j\omega)^2 + j\omega;$
- **Critical damping:** $L_2(\omega) = (j\omega)^2 + 2j\omega;$
- **Over-damping:** $L_3(\omega) = (j\omega)^2 + 3j\omega;$

Nyquist chart of self-loop function



Bode plot of self-loop function





2. Analysis of High-Order Transfer Functions

Summary of 2nd-order Self-loop Function

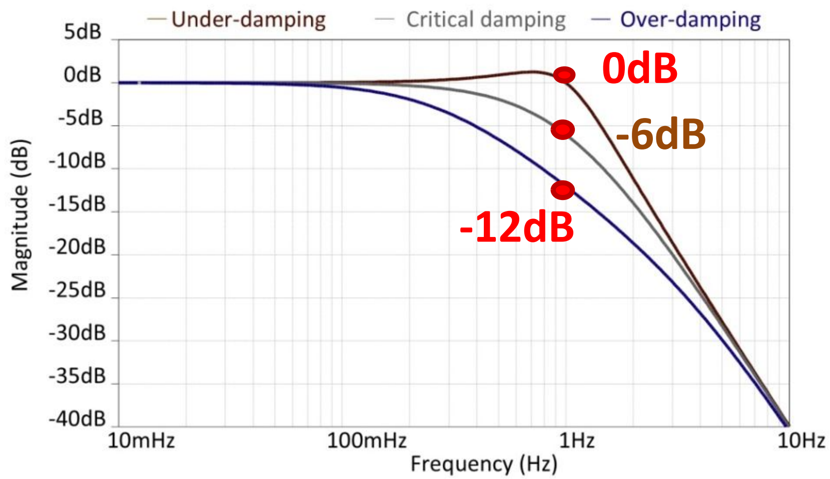
Second-order self-loop function: $L(\omega) = j\omega\left[a_0 j\omega + a_1\right]$

Case	Over-damping		Critical damping		Under-damping	
Delta (Δ)	$\Delta = a_1^2 - 4a_0 > 0$		$\Delta = a_1^2 - 4a_0 = 0$		$\Delta = a_1^2 - 4a_0 < 0$	
$\lvert L(\omega)\rvert$	$\omega\sqrt{(a_0\omega)^2 + a_1^2}$		$\omega\sqrt{(a_0\omega)^2 + a_1^2}$		$\omega\sqrt{(a_0\omega)^2 + a_1^2}$	
$\theta(\omega)$	$\frac{\pi}{2} + \arctan\frac{a_0\omega}{a_1}$		$\frac{\pi}{2} + \arctan\frac{a_0\omega}{a_1}$		$\frac{\pi}{2} + \arctan\frac{a_0\omega}{a_1}$	
$\omega_1 = \frac{a_1}{2a_0}\sqrt{\sqrt{5}-2}$	$\lvert L(\omega_1)\rvert > 1$	$\pi - \theta(\omega_1) > 76.3^o$	$\lvert L(\omega_1)\rvert = 1$	$\pi - \theta(\omega_1) = 76.3^o$	$\lvert L(\omega_1)\rvert < 1$	$\pi - \theta(\omega_1) < 76.3^o$
$\omega_2 = \frac{a_1}{2a_0}$	$\lvert L(\omega_2)\rvert > \sqrt{5}$	$\pi - \theta(\omega_2) > 63.4^o$	$\lvert L(\omega_2)\rvert = \sqrt{5}$	$\pi - \theta(\omega_2) = 63.4^o$	$\lvert L(\omega_2)\rvert < \sqrt{5}$	$\pi - \theta(\omega_2) < 63.4^o$
$\omega_3 = \frac{a_1}{a_0}$	$\lvert L(\omega_3)\rvert > 4\sqrt{2}$	$\pi - \theta(\omega_3) > 45^o$	$\lvert L(\omega_3)\rvert = 4\sqrt{2}$	$\pi - \theta(\omega_3) = 45^o$	$\lvert L(\omega_3)\rvert < 4\sqrt{2}$	$\pi - \theta(\omega_3) < 45^o$

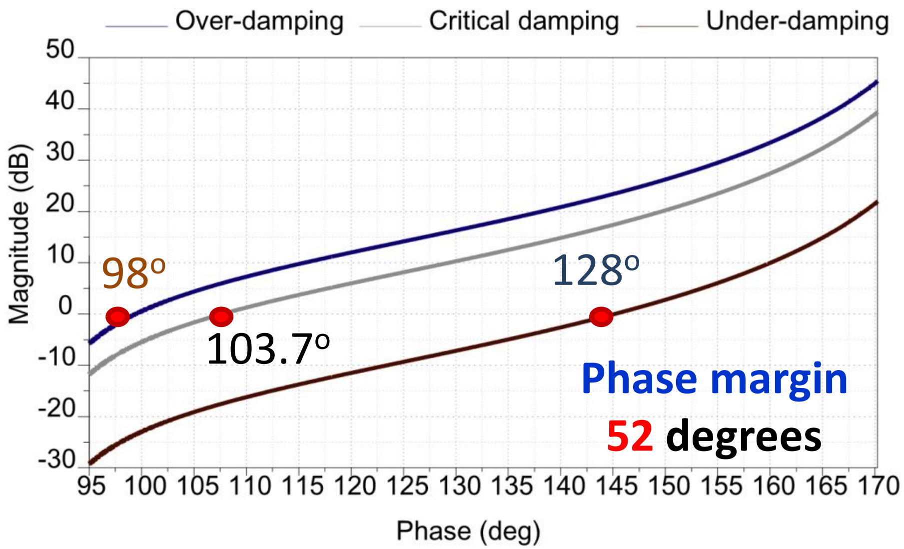
2. Analysis of High-Order Transfer Functions

Operating Regions of 2^{nd}-order System

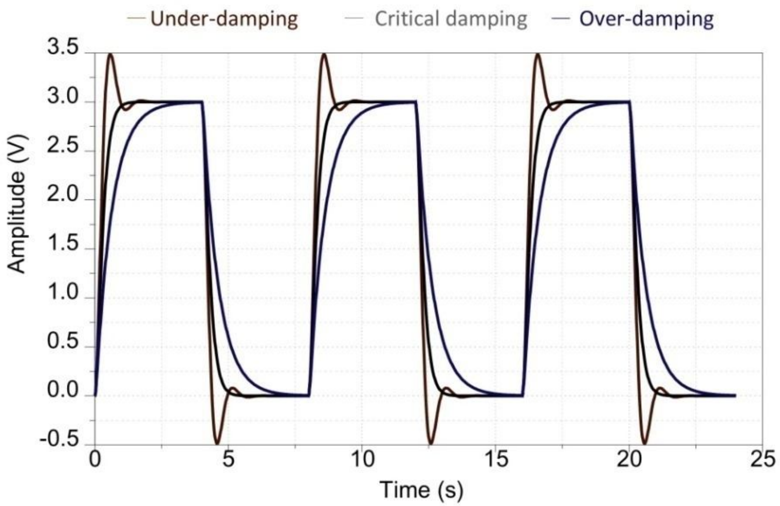
Bode plot of transfer function



Nichols plot of self-loop function



Transient response



Over-damping:

→**Phase margin is 88 degrees.**

Critical damping:

→**Phase margin is 76.3 degrees.**

Under-damping:

→**Phase margin is 52 degrees.**

Outline

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   - **Behaviors of op amps with feedback networks**
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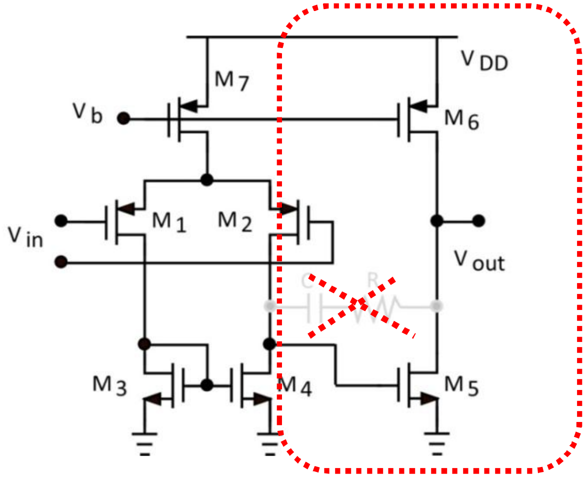
5. Conclusions

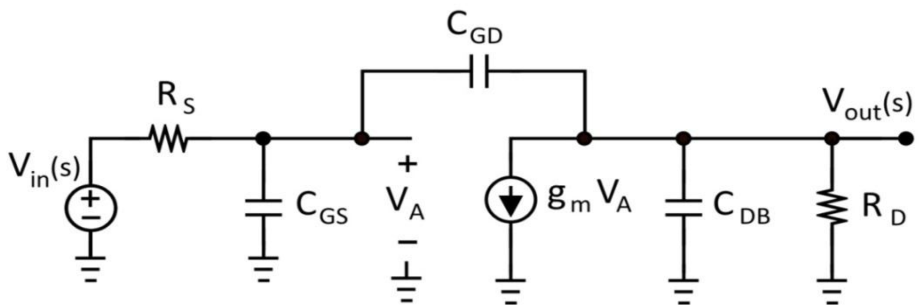
3. Ringing Test for Unity-Gain Amplifiers

Two-stage Op Amp without Miller's Capacitor

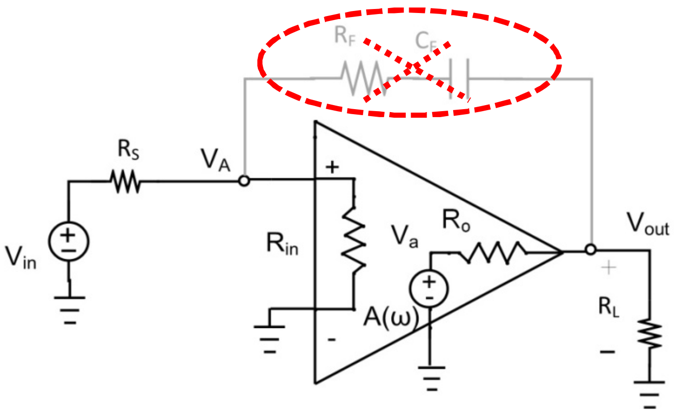
Without frequency compensation



Small signal model



Simplified model



Transfer function H(ω) and self-loop function L(ω)

$$H(\omega) = \frac{b_0 j\omega + b_1}{a_0 (j\omega)^2 + a_1 j\omega + 1};$$

$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega$$

Where,

$$b_0 = R_D R_S \left[(C_{GD} + C_{DB})(C_{GS} + C_{GD}) - C_{GD}^2 \right]$$

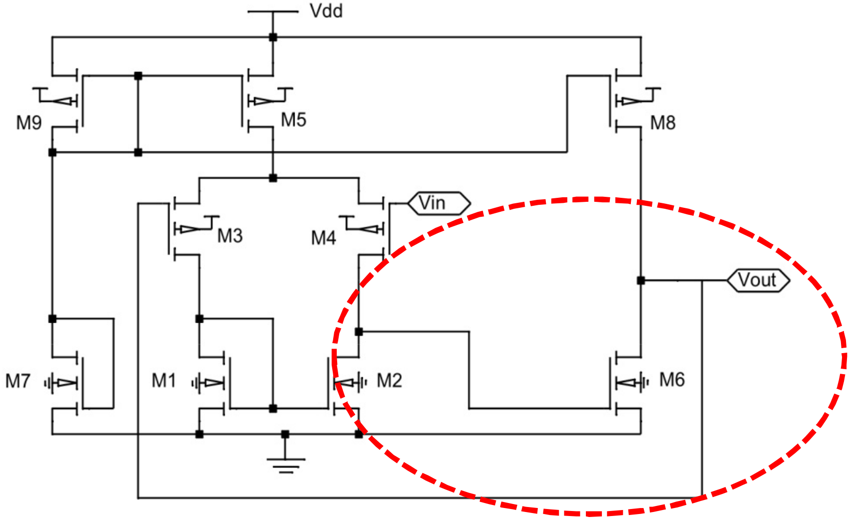
$$b_1 = \left[R_D (C_{GD} + C_{DB}) + R_S (C_{GS} + C_{GD}) + R_D R_S g_m C_{GD} \right]$$

$$a_0 = R_D C_{GD};\ a_1 = -R_D g_m;$$

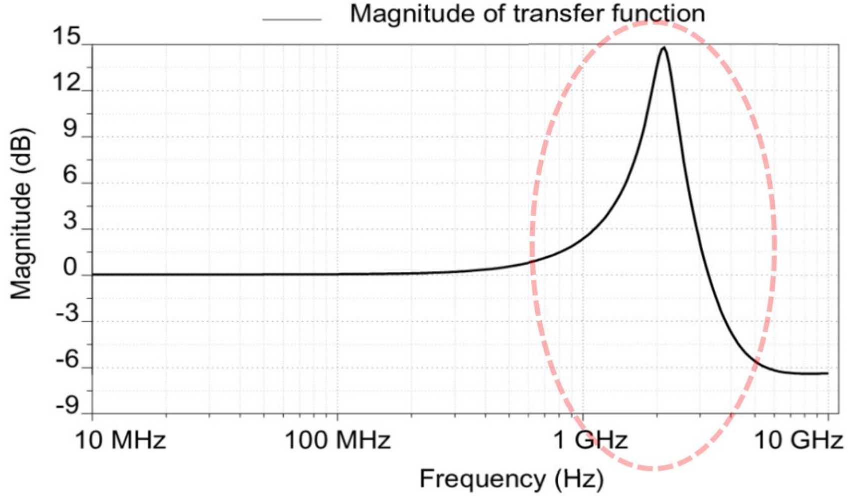
3. Ringing Test for Unity-Gain Amplifiers

Unity-Gain Amplifier without Miller's Capacitor

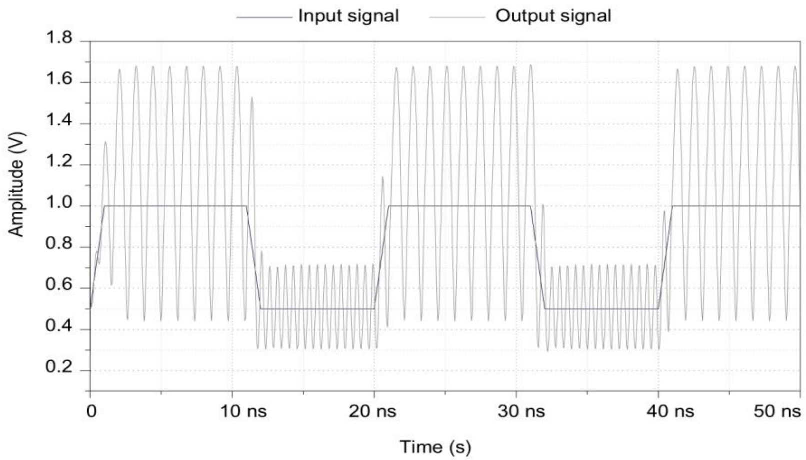
Unity-Gain Amplifier



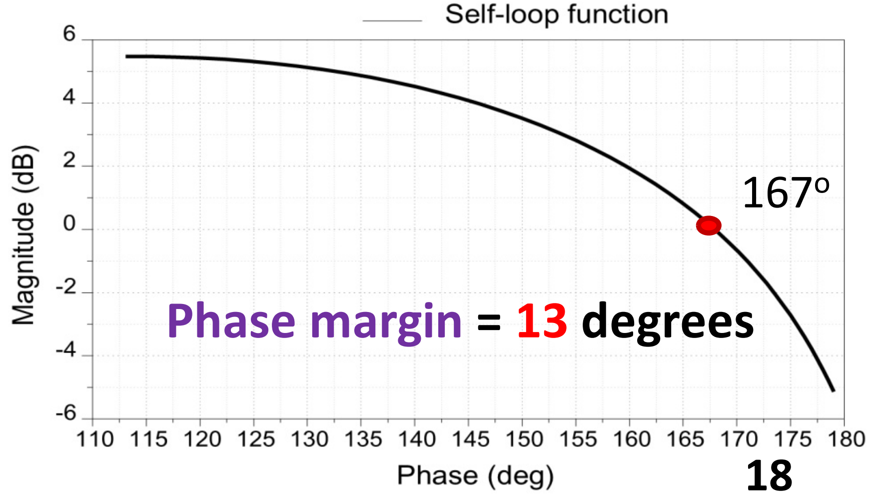
Bode plot of transfer function



Transient response



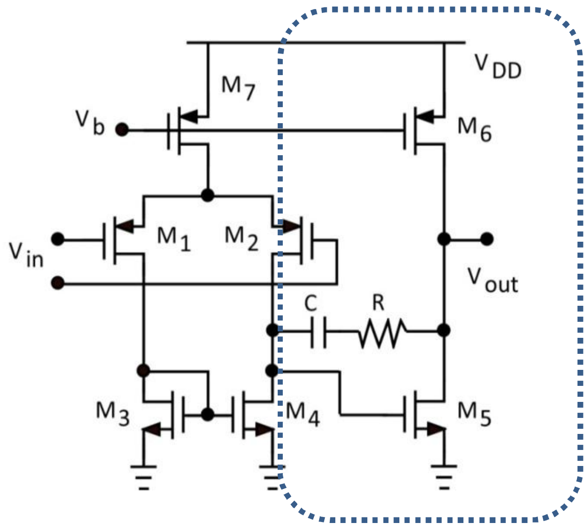
Nichols plot of self-loop function



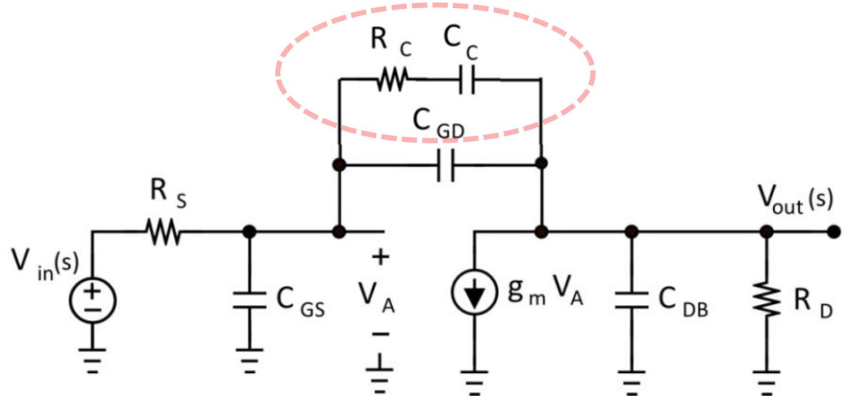
3. Ringing Test for Unity-Gain Amplifiers

Two-stage Op Amp with Frequency Compensation

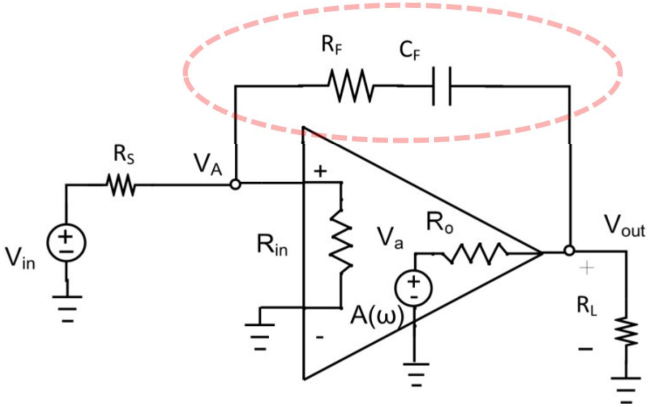
With Miller's capacitor and resistor



Small signal model



Simplified model



Transfer function

$$H(\omega) = \frac{b_0 (j\omega)^3 + b_1 (j\omega)^2 + b_2 j\omega + b_3}{a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega + 1};$$

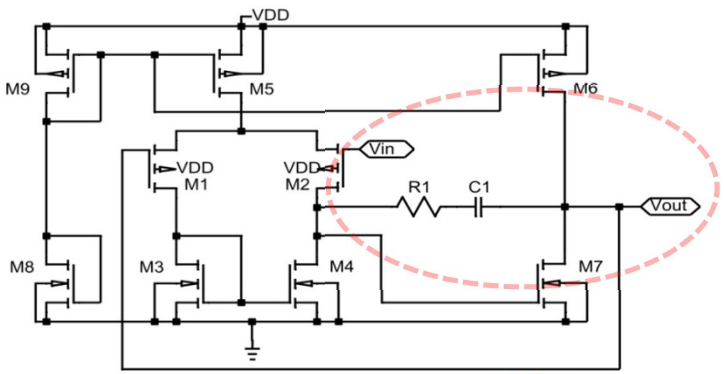
Self-loop function

$$L(\omega) = a_0 (j\omega)^4 + a_1 (j\omega)^3 + a_2 (j\omega)^2 + a_3 j\omega$$

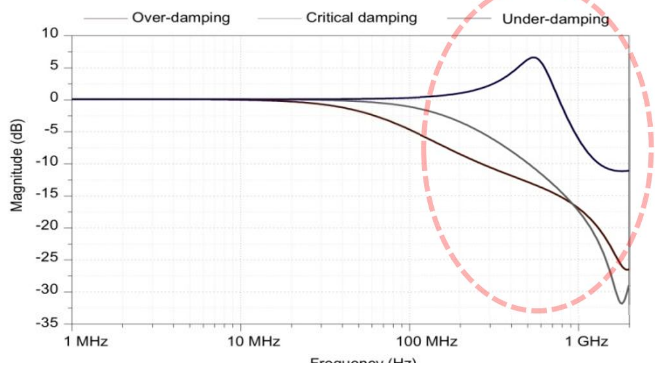
3. Ringing Test for Unity-Gain Amplifiers

Stability Test for Op Amp with Miller's Capacitor

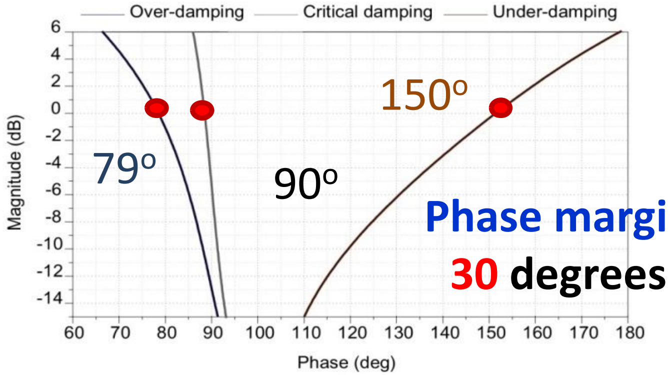
Unity-gain amplifier with Miller's capacitor



Bode plot of transfer function

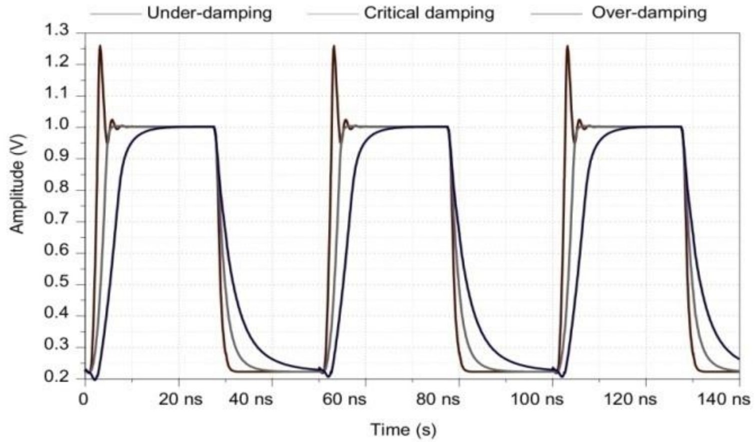


Nichols plot of self-loop function



Phase margin
30 degrees

Transient response



Operating regions

Under-damping:
R1= 2 kΩ, C1 = 1 pF

Critical damping:
R1 = 3.5 kΩ, C1 = 0.2 pF

Over-damping:
R1 = 3.5 kΩ, C1 = 0.8 pF

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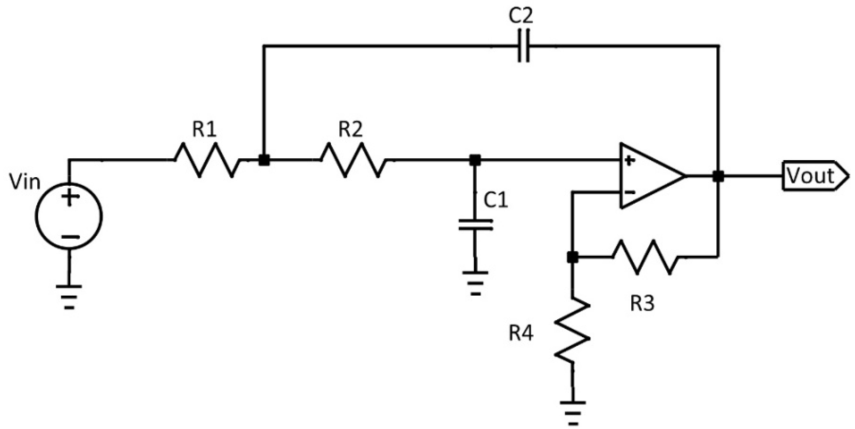
   - **Behaviors of Sallen-Key low-pass filters**
5. Conclusions

4. Ringing Test for High-Order Low-Pass Filters

Review of 2nd-order Sallen-Key Low-pass Filter

2nd-order Sallen-Key LPF



Transfer function

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{b_0}{a_0 (j\omega)^2 + a_1 j\omega + 1};$$

Self-loop function

$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega;$$

Operating regions

•Over-damping: $$\frac{1}{R_1R_2C_1C_2} < \frac{1}{4}\left(R_1C_1 + R_1C_1 - \frac{R_3}{R_4}R_1C_2\right)^2$$

•Critical damping: $$\frac{1}{R_1R_2C_1C_2} = \frac{1}{4}\left(R_1C_1 + R_1C_1 - \frac{R_3}{R_4}R_1C_2\right)^2$$

•Under-damping: $$\frac{1}{R_1R_2C_1C_2} > \frac{1}{4}\left(R_1C_1 + R_1C_1 - \frac{R_3}{R_4}R_1C_2\right)^2$$

Where:

$$\omega_0 = \sqrt{R_1R_2C_1C_2};$$

$$a_0 = R_1R_2C_1C_2;$$

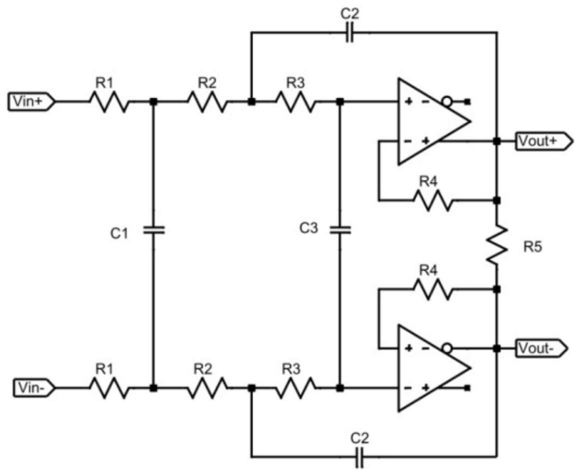
$$a_1 = R_1C_1 + R_1C_1 - \frac{R_3}{R_4}R_1C_2;$$

$$b_0 = 1 + \frac{R_3}{R_4};$$

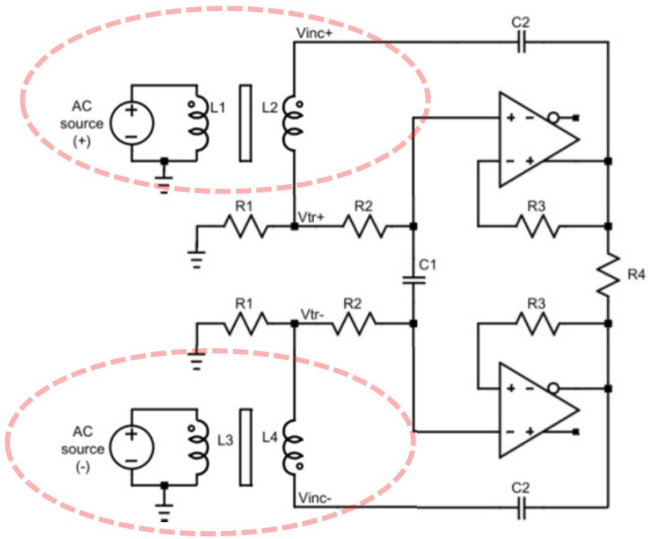
4. Ringing Test for High-Order Low-Pass Filters

Proposed Design of 3rd-Order Sallen-Key LPF

Differential 3rd-order Sallen-Key LPF



Derivation of self-loop function



Transfer function

$$H(\omega) = \frac{b_0}{a_0 (j\omega)^3 + a_1 (j\omega)^2 + a_2 j\omega + 1};$$

Self-loop function

$$L(\omega) = a_0 (j\omega)^3 + a_1 (j\omega)^2 + a_2 j\omega;$$

Where $b_0 = 1 + \frac{R_4}{R_5}; a_0 = R_1 C_1 R_2 C_2 R_3 C_3;$

$$a_1 = R_1 C_1 C_3 (R_2 + R_3) + R_3 C_2 C_3 (R_1 + R_2) - \frac{R_4}{R_5} R_1 C_1 R_2 C_2;$$

$$a_2 = R_1 (C_1 + C_3) + C_3 (R_2 + R_3) - \frac{R_4}{R_5} (R_1 + R_2) C_2;$$

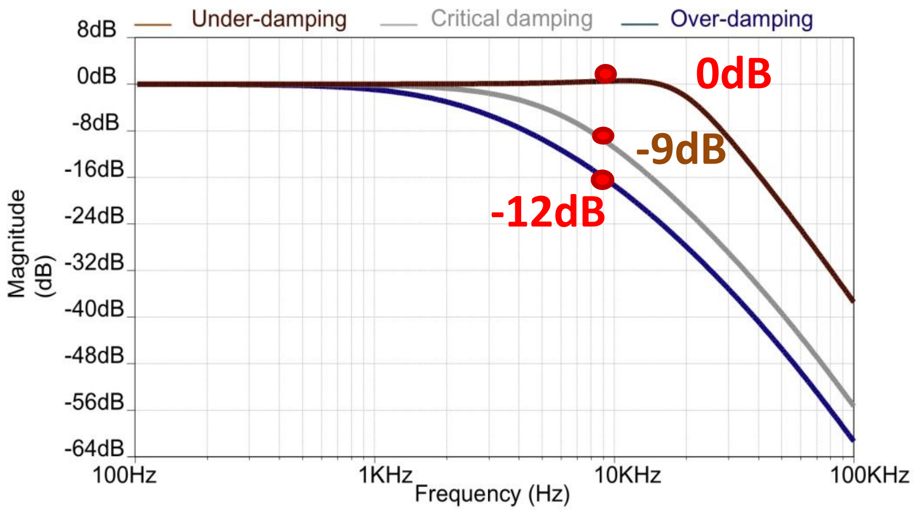
Component parameters

GBW = 10MHz, fo = 10kHz, DC gain (Ao) = 100000, R1 = R2 = R3 = 10 kΩ, R4 = 100 Ω, R5 = 100 kΩ, C1 = 350 pF, C2 = 2 nF.

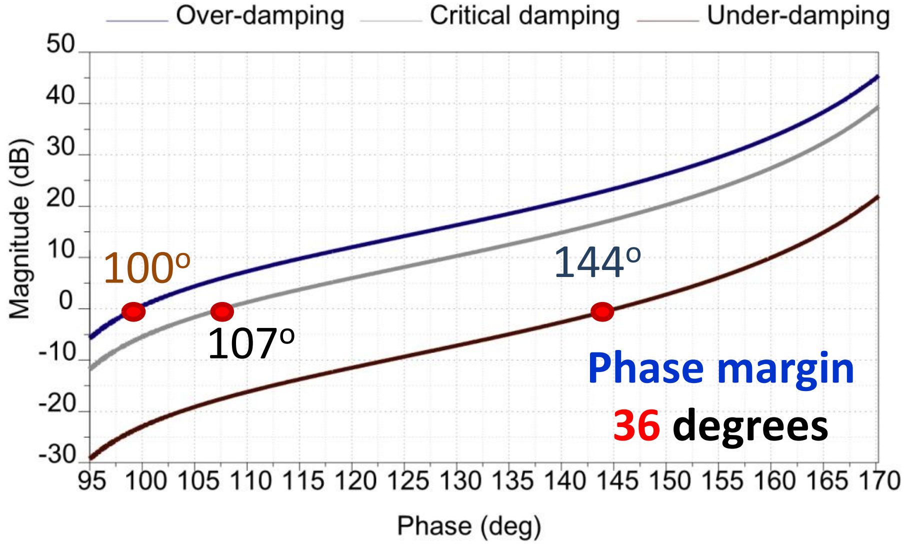
4. Ringing Test for High-Order Low-Pass Filters

Simulation Results of 3rd-Order Sallen-Key LPF

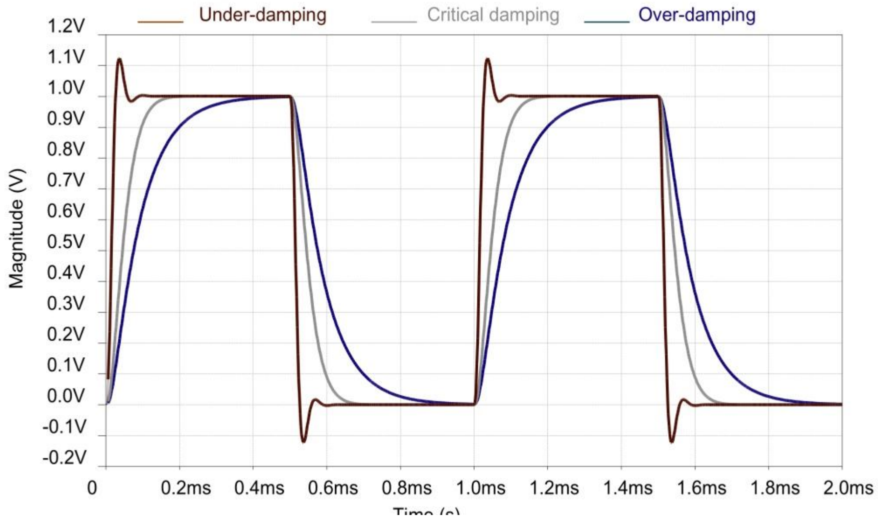
Bode plot of transfer function



Nichols plot of self-loop function



Transient response



Operating regions

Over-damping:

→Phase margin is 80 degrees.

Critical damping:

→Phase margin is 73 degrees.

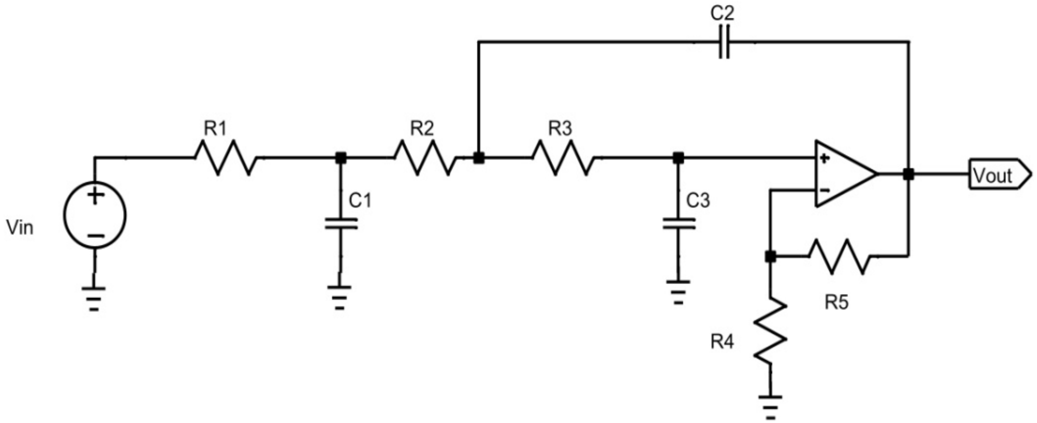
Under-damping:

→Phase margin is 36 degrees.

4. Ringing Test for High-Order Low-Pass Filters

Implemented Circuit of 3rd-Order Sallen-Key LPF

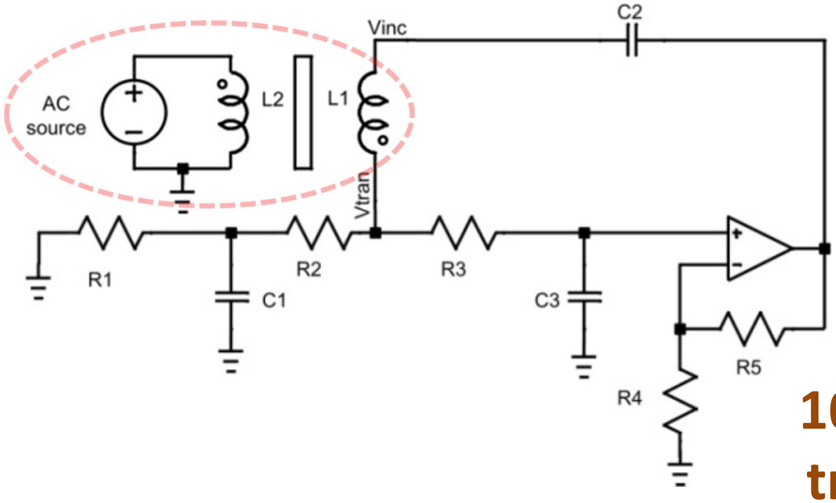
Single ended 3rd -order Sallen-Key LPF



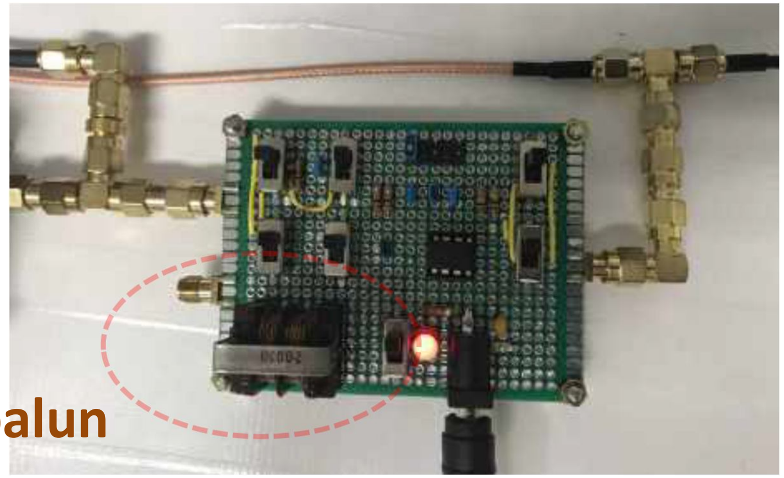
Component parameters

Op Amp: LM358; GBW = 10MHz,
DC gain (Ao) = 100000,
R1 = R2 = R3 = 10 kΩ,
R4 = 100 Ω, R5 = 100 kΩ,
C1 = 350 pF, C2 = 2 nF.

Derivation of self-loop function



Implemented circuit

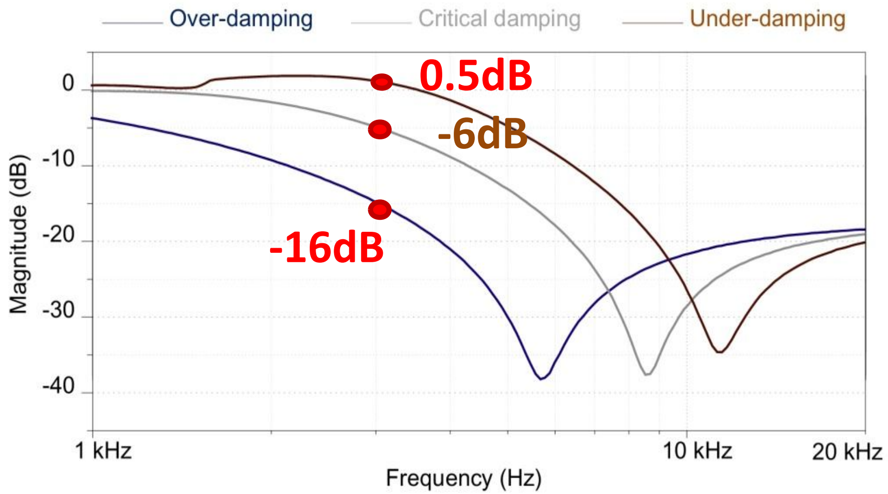


10 mH balun transformer

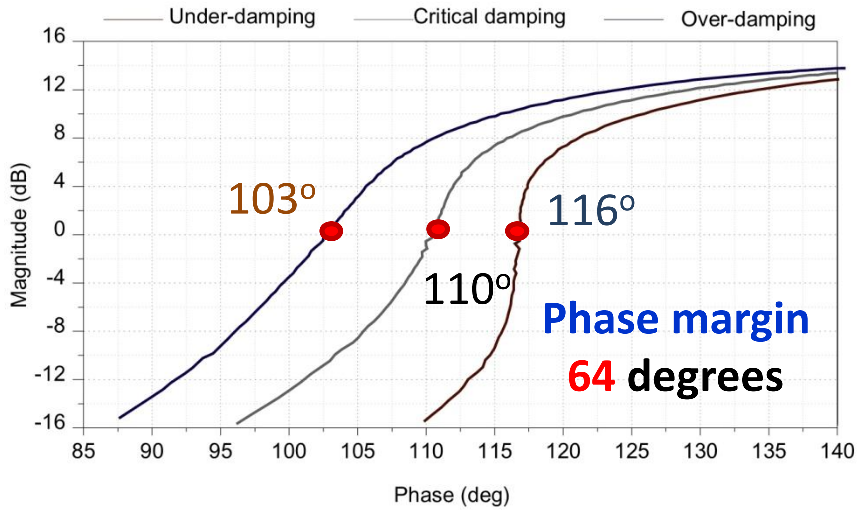
4. Ringing Test for High-Order Low-Pass Filters

Measurement Results of 3rd-Order Sallen-Key LPF

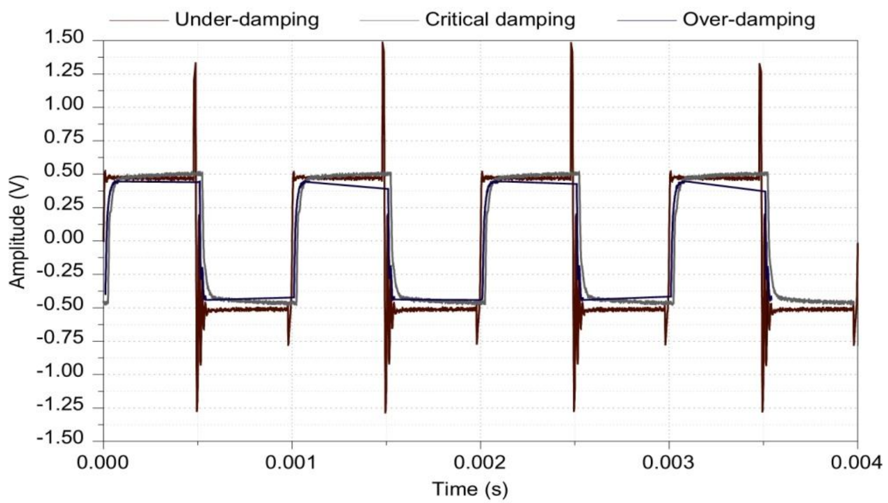
Bode plot of transfer function



Nichols plot of self-loop function



Transient response



Operating regions

Over-damping:

→Phase margin is 77 degrees.

Critical damping:

→Phase margin is 70 degrees.

Under-damping:

→Phase margin is 64 degrees.

Outline

1. Research Background
   - Motivation, objectives and achievements
   - Self-loop function in a transfer function
2. Analysis of High-Order Transfer Functions
   - Operating regions of second-order complex functions
3. Ringing Test for Unity-Gain Amplifiers
   - Behaviors of op amps with feedback networks
4. Ringing Test for High-Order Low-Pass Filters
   - Behaviors of Sallen-Key low-pass filters
5. Conclusions

5. Comparison

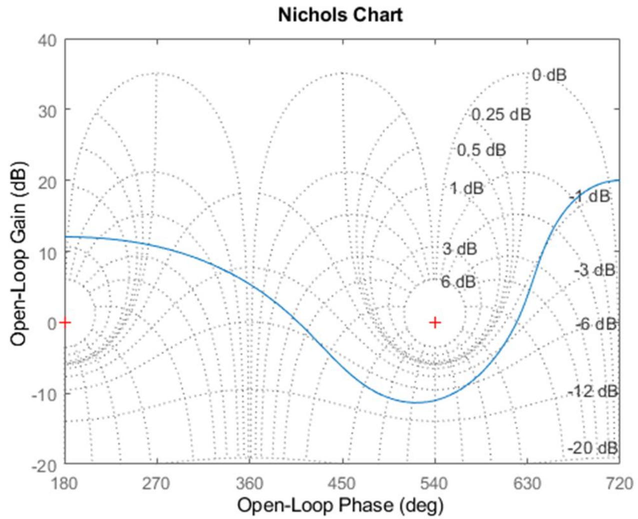
Features	This work	Replica measurement	Middlebrook's method
Main objective	Self-loop function	Loop gain	Loop gain
Transfer function accuracy	Yes	No	No
Ringing Test	Yes	Yes	Yes
Operating region accuracy	Yes	No	No
Phase margin accuracy	Yes	No	No
Passive networks	Yes	No	No

5. Discussions

- **Loop gain is independent of frequency variable.**

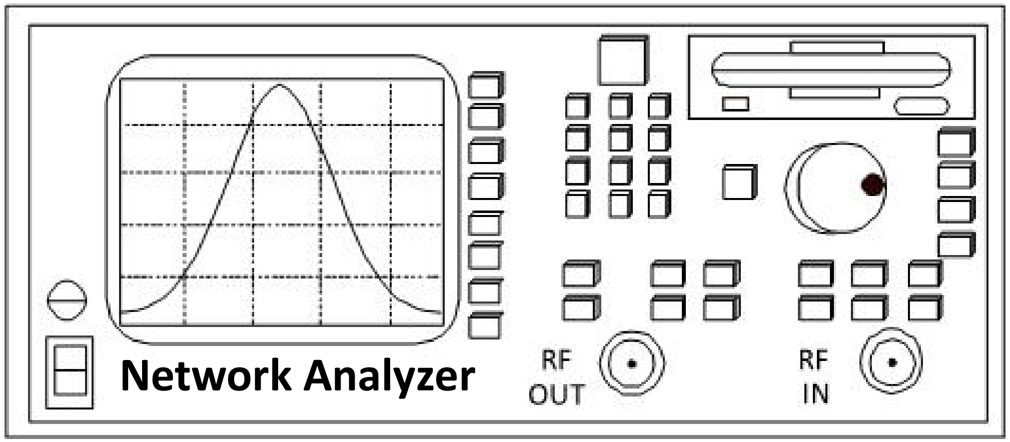
→Loop gain in adaptive feedback network is significantly different from self-loop function in linear negative feedback network.

Nichols chart is only used in MATLAB simulation.



https://www.mathworks.com/help/control/ref/nichols.html

Nichols chart isn't used widely in practical measurements (only used in control theory).



(Technology limitations)

5. Conclusions

This work:

- **Proposal of alternating current conservation for deriving self-loop function in a transfer function**

  → Observation of self-loop function can help us optimize the behavior of a high-order system.

- **Implementations of circuits and measurements of self-loop functions for Sallen-Key low-pass filters**

  →Theoretical concepts of stability test are verified by laboratory simulations and practical experiments.

Future of work:

- **Stability test for parasitic components in transmission lines, printed circuit boards, physical layout layers**

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Thank you very much!

ご清聴ありがとうございした。

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Laboratory