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Phase Margin Test for Power-Stage of DC-DC Buck Converter

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Outline

1. Research Background

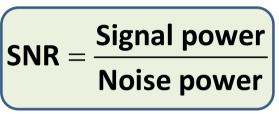
- Motivation, objectives and achievements
- Characteristics of an adaptive feedback network
- 2. Analysis of Power-Stage of DC-DC Converter
- Operating regions of 2nd-order systems
- Phase margin of power-stage of DC-DC converter
- **3. Ripple Reduction for DC-DC Converters**
- LC Harmonic Notch filter
- Nichols chart of power stage of buck converter
- 4. Conclusions

1. Research Background

Noise in Electronic Systems

Performance of a system

Signal to Noise Ratio:



Common types of noise:

- Electronic noise
- Thermal noise,
- Intermodulation noise,
- Cross-talk,
- Impulse noise,
- Shot noise, and
- Transit-time noise.

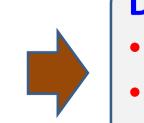
Performance of a device



 $= \frac{\text{Output SNR}}{\text{Input SNR}}$

Device noise:

- Flicker noise,
- Thermal noise,
- White noise.



DC-DC converters

- Overshoot,
- Ringing
- Ripple

1. Research Background

Effects of Ripple and Ringing on Electronic Systems

- Ringing is overshoot/undershoot voltage or current when it's seen on time domain.
- Ripple is wasted power, and has many undesirable effects on analog circuits.
- Ringing does the following things:
- Causes EMI noise,
- Heats components,
- Consumes the power,
- Decreases the performance,
- Damages the devices.

Ripple does the following things:

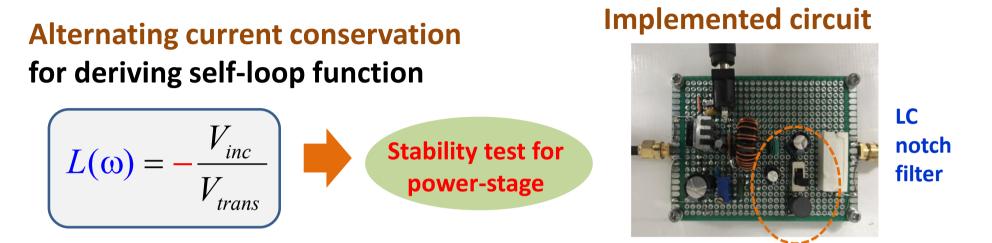
- Increases current flow,
- Increases noise,
- Creates the distortion,
- Causes digital circuits to operate improperly.

1. Research Background Objectives of Study

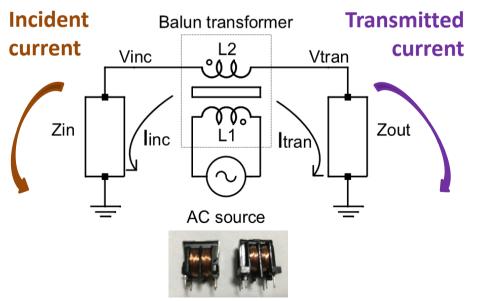
- Investigation of behaviors of power-stage of DC-DC converters
- Ripple reduction for DC-DC buck converter using LC Harmonic Notch filter
- Measurement of self-loop function in powerstage of DC-DC buck converter
- Observation of phase margin at unity gain on the Nichols chart

1. Research Background

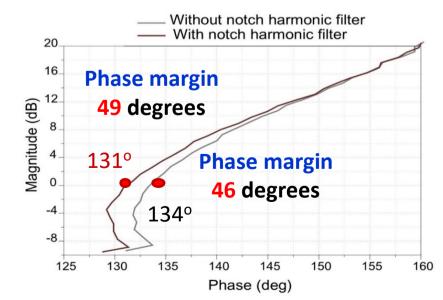
Achievements of Study



Derivation of self-loop function

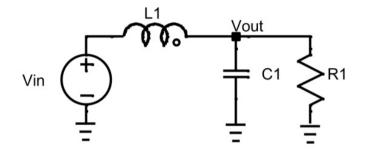


Phase margin of power stage



1. Research Background Approaching Methods

Simplified power-stage

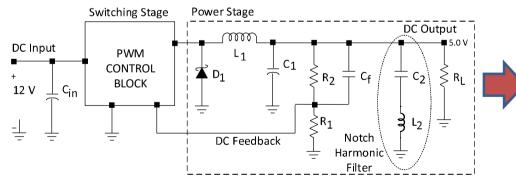


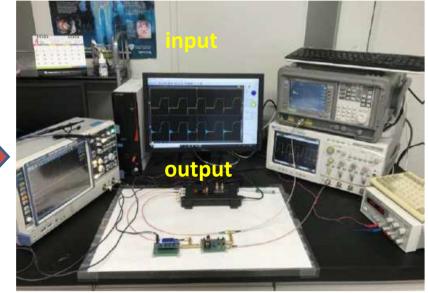




Measurement set up

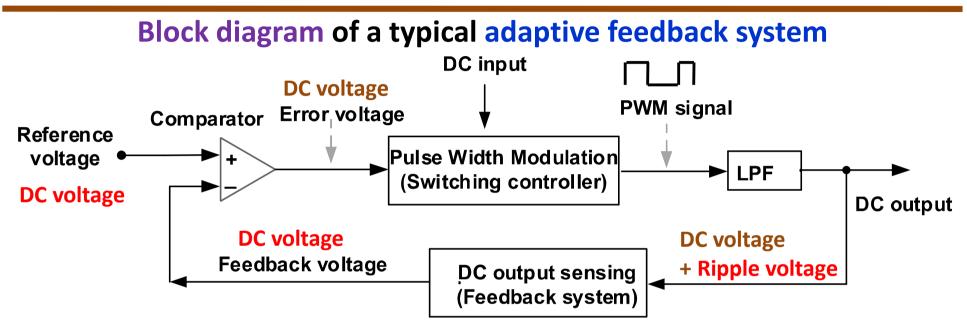
Design of DC-DC buck converter





1. Research Background

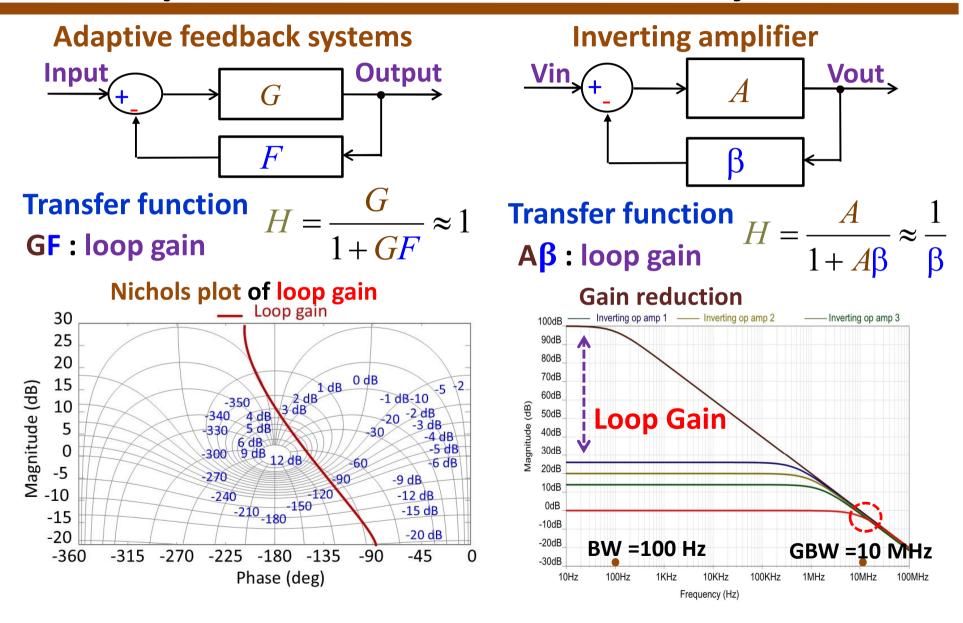
Characteristics of Adaptive Feedback Network



Adaptive feedback is used to control the output source along with the decision source (DC-DC Buck converter).
 Transfer function of an adaptive feedback network is significantly different from transfer function of a linear negative feedback network.

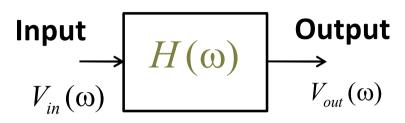
→ Loop gain is independent of frequency variable (referent voltage, feedback voltage, and error voltage are DC voltages).

1. Research Background Loop Gain in Feedback Control Systems



1. Research Background Self-loop Function in A Transfer Function

Linear system



Transfer function

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{A(\omega)}{1 + L(\omega)}$$

○ Polar chart → Nyquist chart
 ○ Magnitude-frequency plot
 ○ Angular-frequency plot
 ○ Magnitude-angular diagram → Nichols diagram

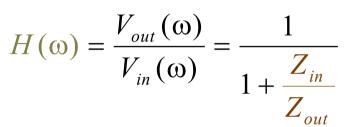
Model of a linear system

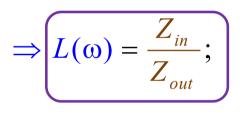
$$H(\boldsymbol{\omega}) = \frac{b_0 (j\omega)^n + \dots + b_{n-1} (j\omega) + b_n}{a_0 (j\omega)^n + \dots + a_{n-1} (j\omega) + a_n}$$

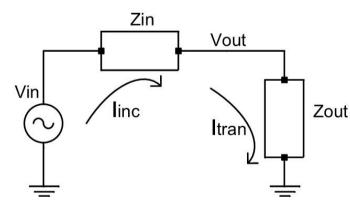
 $A(\omega)$: Numerator function $H(\omega)$: Transfer function $L(\omega)$: Self-loop function Variable: angular frequency (ω)

1. Research Background Alternating Current Conservation

Transfer function

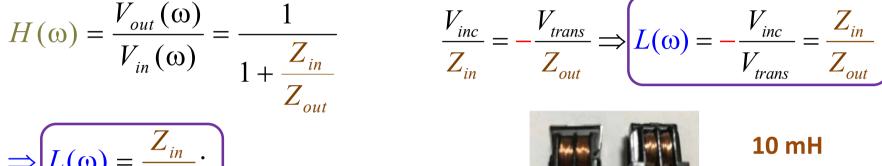




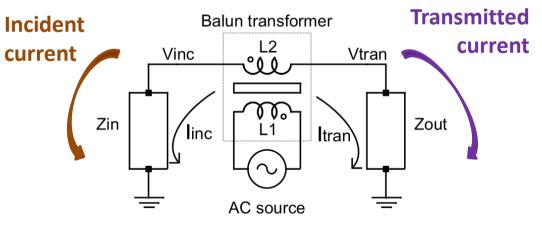


Simplified linear system

Self-loop function



inductance



Derivation of self-loop function

1. Research Background Limitations of Conventional Methods

- Middlebrook's measurement of loop gain
- → Applying only in feedback systems (DC-DC converters).
- **o Replica measurement of loop gain**
- →Using two identical networks (not real measurement).
- Nyquist's stability condition
- \rightarrow Theoretical analysis for feedback systems (Lab tool).
- Nichols chart of loop gain
- \rightarrow Only used in feedback control theory (Lab tool).

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2. Analysis of Power-Stage of DC-DC Converter Behaviors of Second-order Transfer Function

Second-order transfer function: $H(\omega) = \frac{1}{1 + a_0 (j\omega)^2 + a_1 j\omega}$

Case	Over-damping	Critically damping	Under-damping
Delta (Δ)	$\frac{1}{a_0} < \left(\frac{a_1}{2a_0}\right)^2 \Longrightarrow \Delta = a_1^2 - 4a_0 > 0$	$\frac{1}{a_0} = \left(\frac{a_1}{2a_0}\right)^2 \Longrightarrow \Delta = a_1^2 - 4a_0 = 0$	$\frac{1}{a_0} > \left(\frac{a_1}{2a_0}\right)^2 \Longrightarrow \Delta = a_1^2 - 4a_0 < 0$
$\begin{array}{c} \textbf{Module} \\ H(\omega) \end{array}$	$\frac{\frac{1}{a_0}}{\sqrt{\omega^2 + \left(\frac{a_1}{2a_0} - \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}\right)^2}\sqrt{\omega^2 + \left(\frac{a_1}{2a_0} + \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}\right)^2}}$	$\frac{\frac{1}{a_0}}{\left[\omega^2 + \left(\frac{a_1}{2a_0}\right)^2\right]} = \frac{1}{2} = -6dB$	$\frac{\frac{1}{a_0}}{\sqrt{\left(\omega - \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}\right)^2 + \left(\frac{a_1}{2a_0}\right)^2}\sqrt{\left(\omega + \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}\right)^2 + \left(\frac{a_1}{2a_0}\right)^2}}$
$\begin{array}{c} \textbf{Angular} \\ \theta(\omega) \end{array}$	$-\arctan\left(\frac{\omega}{\left(\frac{a_1}{2a_0}-\sqrt{\left(\frac{a_1}{2a_0}\right)^2-\frac{1}{a_0}}\right)}-\arctan\left(\frac{\omega}{\left(\frac{a_1}{2a_0}+\sqrt{\left(\frac{a_1}{2a_0}\right)^2-\frac{1}{a_0}}\right)}\right)$	$-2 \arctan\left(\frac{2a_0\omega}{a_1}\right)$	$-\arctan\left(\frac{\omega - \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}}{\frac{a_1}{2a_0}}\right) - \arctan\left(\frac{\omega + \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}}{\frac{a_1}{2a_0}}\right)$
$\omega_{cut} = \frac{a_1}{2a_0}$	$ H(\omega_{cut}) < \frac{2a_0}{a_1}$ $\theta(\omega_{cut}) > -\frac{\pi}{2}$	$ H(\omega_{cut}) = \frac{2a_0}{a_1} \theta(\omega_{cut}) = -\frac{\pi}{2}$	$ H(\omega_{cut}) > \frac{2a_0}{a_1}$ $\theta(\omega_{cut}) < -\frac{\pi}{2}$

2. Analysis of Power-Stage of DC-DC Converter Behaviors of Second-order Self-loop Function

Second-order self-loop function: $L(\omega) = j\omega [a_0 j\omega + a_1]$

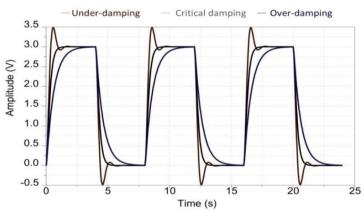
Case	Over-damping		Critical damping		Under-damping	
Delta (Δ)	$\Delta = a_1^2 - 4a_0 > 0$		$\Delta = a_1^2 - 4a_0 = 0$		$\Delta = a_1^2 - 4a_0 < 0$	
$ L(\omega) $	$\omega \sqrt{\left(a_0 \omega\right)^2 + a_1^2}$		$\omega \sqrt{\left(a_0 \omega\right)^2 + a_1^2}$		$\omega \sqrt{\left(a_0 \omega\right)^2 + a_1^2}$	
θ(ω)	$\frac{\pi}{2}$ +	$\frac{\pi}{2} + \arctan \frac{a_0 \omega}{a_1}$ $\frac{\pi}{2} + \arctan \frac{a_0 \omega}{a_1}$		$\operatorname{ctan} \frac{a_0 \omega}{a_1}$	$\frac{\pi}{2} + \arctan \frac{a_0 \omega}{a_1}$	
$\omega_1 = \frac{a_1}{2a_0}\sqrt{\sqrt{5}-2}$	$\left \left L(\omega_1) \right > 1 \right $	$\pi - \theta(\omega_1) > 76.3^{\circ}$	$ L(\omega_1) = 1$	$\pi - \theta(\omega_1) = 76.3^{\circ}$	$ L(\omega_1) < 1$	$\pi - \theta(\omega_1) < 76.3^{\circ}$
$\omega_2 = \frac{a_1}{2a_0}$	$ L(\omega_2) > \sqrt{5}$	$\pi - \theta(\omega_2) > 63.4^{\circ}$	$\left L(\omega_2)\right = \sqrt{5}$	$\pi - \theta(\omega_2) = 63.4^{\circ}$	$\left L(\omega_2)\right < \sqrt{5}$	$\pi - \theta(\omega_2) < 63.4^{\circ}$
$\omega_3 = \frac{a_1}{a_0}$	$\left L(\omega_3)\right > 4\sqrt{2}$	$\pi - \theta(\omega_3) > 45^{\circ}$	$\left L(\omega_3)\right = 4\sqrt{2}$	$\pi - \Theta(\omega_3) = 45^{\circ}$	$\left L(\omega_3)\right < 4\sqrt{2}$	$\pi - \theta(\omega_3) < 45^{\circ}$

2. Analysis of Power-Stage of DC-DC Converter **Operating Regions of 2nd-Order System**

- •Under-damping:
- - $L_2(\omega) = (j\omega)^2 + 2j\omega;$
- - $L_3(\omega) = (j\omega)^2 + 3j\omega;$

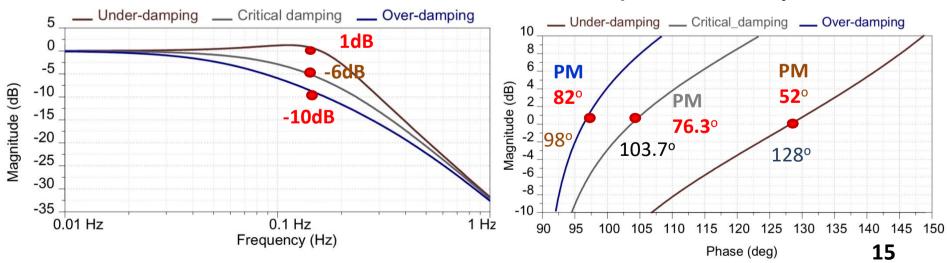
*L*₁(ω) = $(j\omega)^2 + j\omega$; $H_1(\omega) = \frac{1}{(j\omega)^2 + j\omega + 1}$; •Critical damping: $H_2(\omega) = \frac{1}{(j\omega)^2 + 2j\omega + 1}; \bigoplus_{\substack{\geq 2.0\\ = 1.5}}^{\geq 2.5}$ $L_{2}(\omega) = (j\omega) + 2j\omega;$ • Over-damping: $H_{3}(\omega) = \frac{1}{(j\omega)^{2} + 3j\omega + 1};$

Transient response

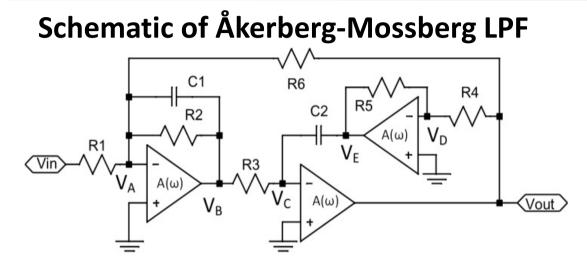


Bode plot of transfer function

Nichols plot of self-loop function



2. Analysis of Power-Stage of DC-DC Converter Implemented Circuit of Åkerberg-Mossberg LPF



Transfer function & self-loop function

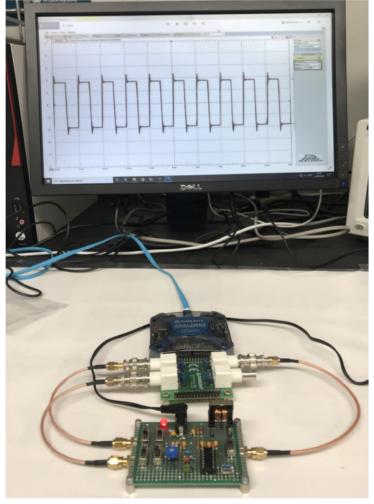
$$H(\omega) = -\frac{b_0}{a_0 (j\omega)^2 + a_1 j\omega + 1};$$

$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega;$$

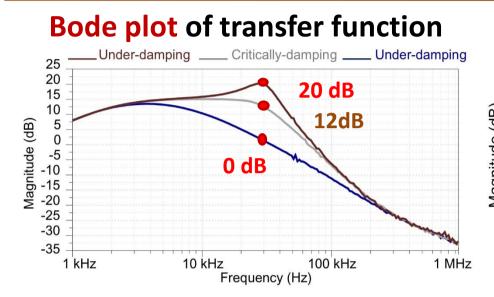
where, $b_0 = \frac{R_6}{R_1};$

$$a_0 = \frac{R_3}{R_4} R_5 R_6 C_1 C_2; a_1 = \frac{R_3 R_5 R_6}{R_4 R_2} C_2;$$

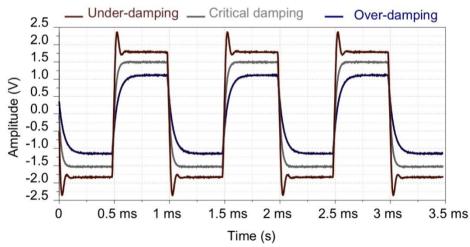
Measurement set up



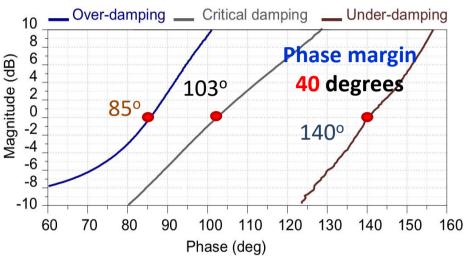
2. Analysis of Power-Stage of DC-DC Converter Measurement Results of Åkerberg-Mossberg LPF



Transient response

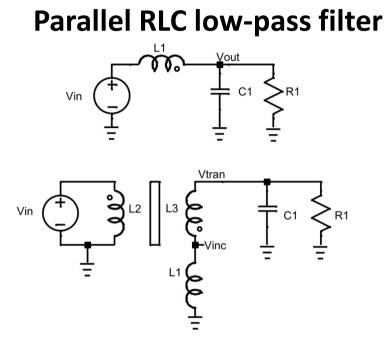


Nichols plot of self-loop function



Over-damping: →Phase margin is 95 degrees. Critical damping: →Phase margin is 77 degrees. Under-damping: →Phase margin is 40 degrees.

2. Analysis of Power-Stage of DC-DC Converter **Behaviors of power-stage of DC-DC Converter**



Transfer function & self-loop function:

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$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + a_0 (j\omega)^2 + a_1 j\omega};$$

$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega;$$

Where: $a_0 = LC; \quad a_1 = \frac{L}{R};$
 $\omega_0 = 1/\sqrt{LC};$
 $|Z_L| = \omega_0 L; \quad |Z_C| = 1/\omega_0 C;$

Operating regions

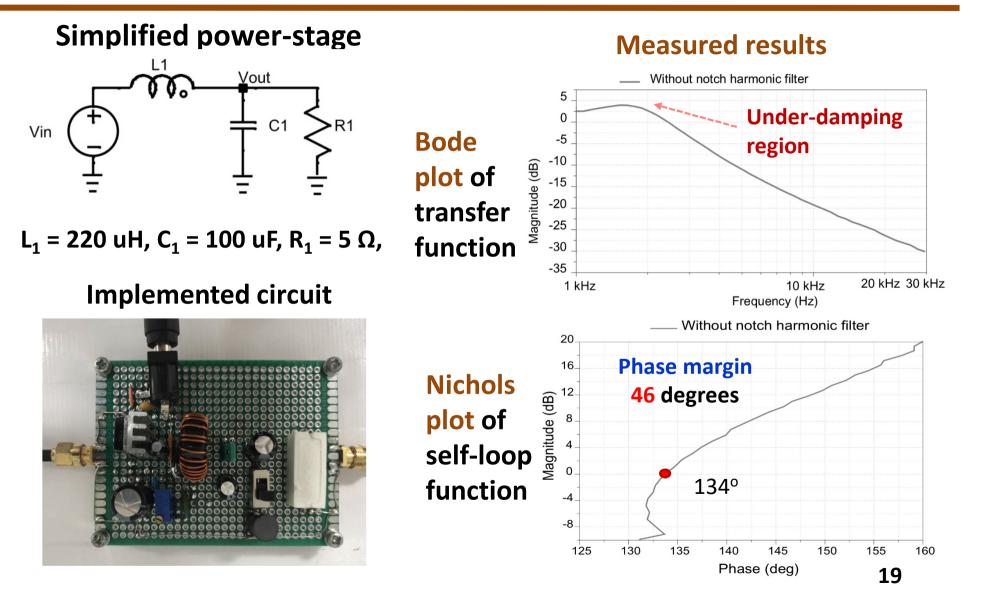
•Over-damping:

•Critical damping:

•Under-damping:

$$\frac{1}{LC} < \left(\frac{R}{2L}\right)^2 \Leftrightarrow |Z_L| = |Z_C| < R/2 \qquad \left| \begin{array}{c} Z_L \\ Z_L \\ Z_L \\ \end{array} \right| = \left| \begin{array}{c} Z_C \\ Z_C \\ \end{array} \right| = 2R \\ Balanced charging \\ discharging time condition \\ \frac{1}{LC} > \left(\frac{R}{2L}\right)^2 \Leftrightarrow |Z_L| = |Z_C| > R/2 \end{array}$$

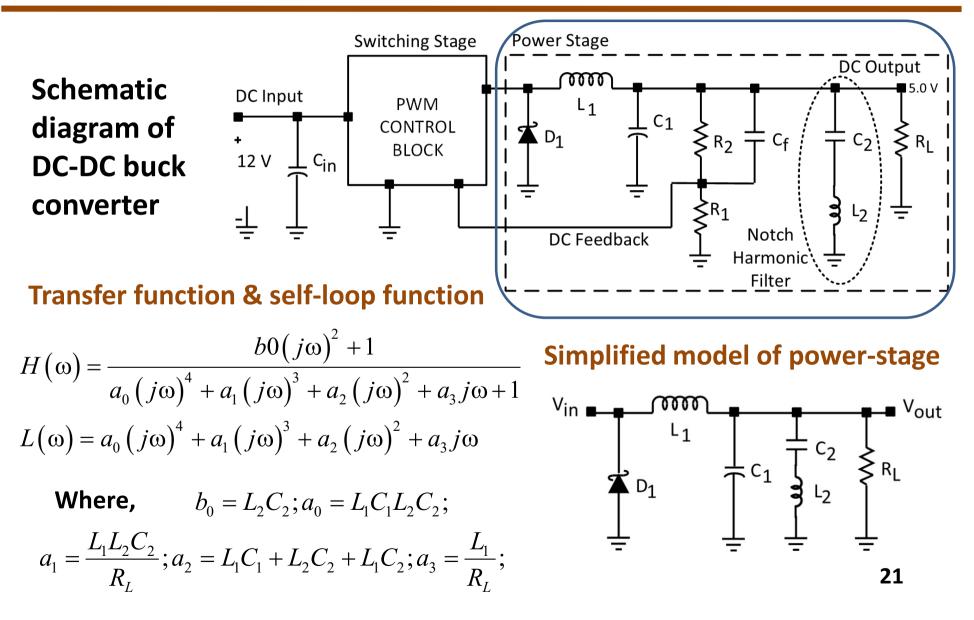
2. Analysis of Power-Stage of DC-DC Converter Phase Margin of Power-Stage of DC-DC Converter



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- LC Harmonic Notch filter
- Nichols chart of power stage of buck converter
- 4. Conclusions

3. Ripple Reduction for DC-DC Converters Ripple Reduction using LC Harmonic Notch Filter

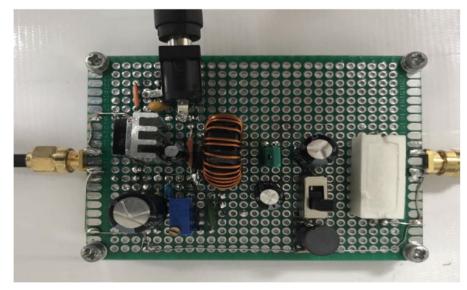


3. Ripple Reduction for DC-DC Converters Implemented Circuit for DC-DC Converter

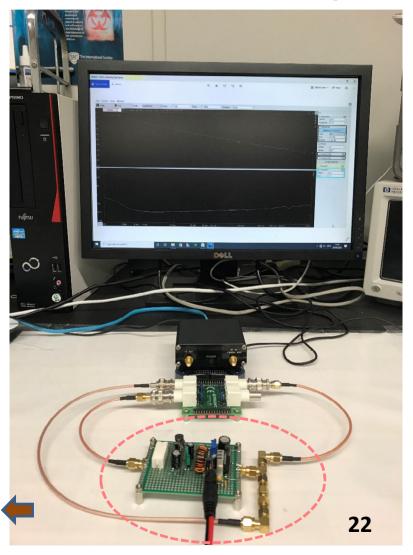
Design parameters

Input voltage (Vin)	12 V		
Output voltage (Vo)	5.0 V		
Output current (Io)	1 A		
Clock frequency (Fck)	180 kHz		
Output ripple	< 10 mVpp		

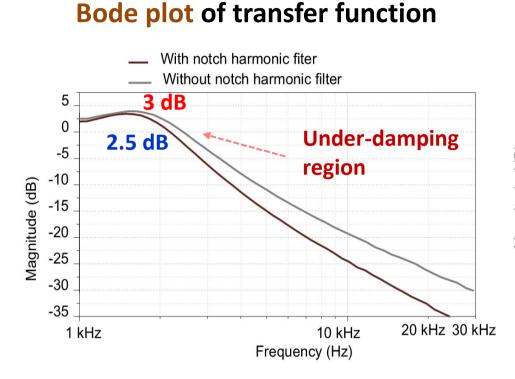
Implemented circuit

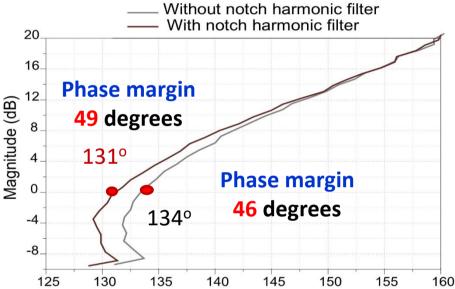


Measurement set up



3. Ripple Reduction for DC-DC Converters Measurement Results of Proposed Design Circuit





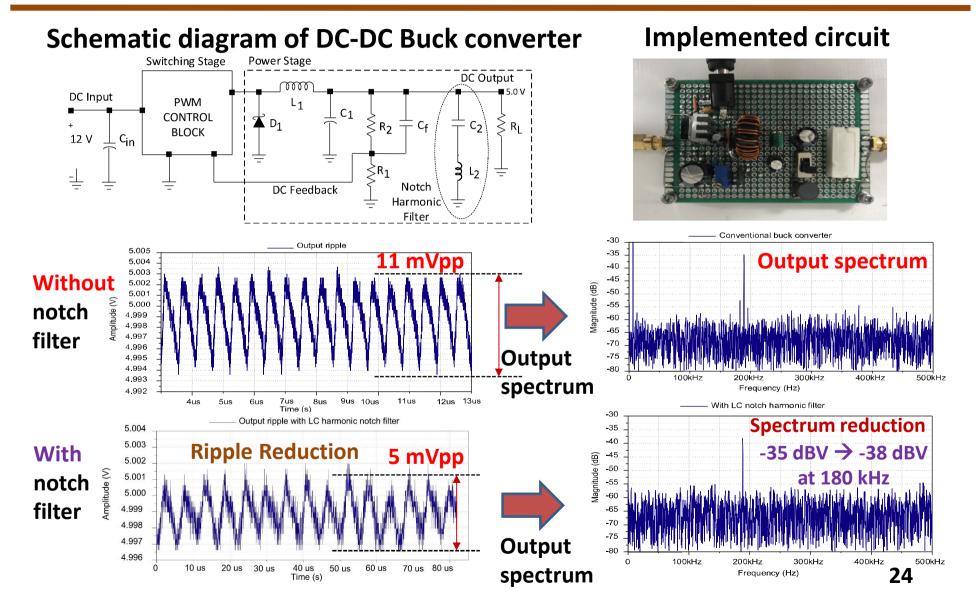
- Reduce the cut-off frequency of the power-stage
- → Reduce the ripple caused by high-order harmonic signals
- Improvement of phase margin of the power-stage

Phase (deg)

→ Reduce the overshoot caused by the passive components 23

Nichols plot of self-loop function

3. Ripple Reduction for DC-DC Converters Ripple Reduction using Harmonic Notch Filter



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- Passive parallel RLC network
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4. Conclusions

4. Discussion and Comparison

Merits of Self-loop Function Derivation Techniques

Features	Comparison measurement	Alternating current conservation	Replica measurement	Middlebrook's method
Main objective	Self-loop function	Self-loop function	Loop gain	Loop gain
Transfer function accuracy	Yes	Yes	Νο	Νο
Breaking feedback loop	Νο	Yes	Yes	Yes
Operating region accuracy	Yes	Yes	Νο	Νο
Phase margin accuracy	Yes	Yes	Νο	Νο
Passive networks	Yes	Yes	Νο	Νο

4. Discussion and Comparison Limitations of Loop Gain on Nichols Charts

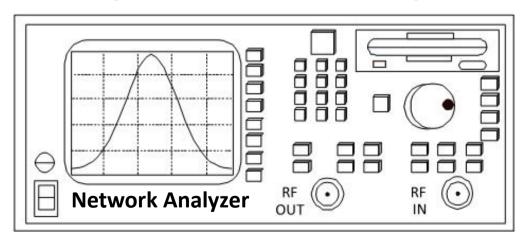
- Loop gain is independent of frequency variable.
- Doop gain in adaptive feedback network is significantly different from self-loop function in linear negative feedback network.

Nichols chart is only used in MATLAB simulation.

Nichols Chart 0 dF 30 0.25 dB 0.5 dB Open-Loop Gain (dB) 0 01 05 1 dB 3 dB -3 dB 6 dB -6 dB -12 dB -10 -20 dB -20 180 270 450 540 630 720 Open-Loop Phase (deg)

https://www.mathworks.com/help/control/ref/nichols.html

Nichols chart isn't used widely in practical measurements (only used in control theory).





4. Conclusions

This work:

- Investigation of behaviors of power-stage in DC-DC converters based on alternating current conservation
- Proposed design of passive LC harmonic notch filters for ripple reduction

→ Phase margin improvement from 46 degrees into 49 degrees

→ Ripple reduction from 11 mVpp into 5 mVpp

Future work:

Stability test for dynamic loads, and parasitic components in printed circuit boards

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Thank you very much! ご清聴ありがとうございした。







