

Phase Margin Test for Power-Stage of DC-DC Buck Converter

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Abstract. This paper presents our phase margin test for the power-stage of a DC-DC buck converter. The comparison measurement technique is used to measure the self-loop function in a transfer function of the power-stage. The electro-magnetic interference noise caused by voltage ripple is reduced by a passive harmonic notch filter. The practical measurement of the stability test for power-stage of the DC-DC buck converter is performed in both with and without the harmonic notch filter. As a result, the phase margin is improved from 46 degrees to 49 degrees while the voltage ripple is reduced to 5 mVpp from 11 mVpp.

1. Introduction

Nowadays, the requirements for the response time, the overshoot phenomena and the output voltage ripple of these systems are extremely strict in some applications [1]. Therefore, the design of low-voltage high-efficiency DC-DC buck converter becomes challenging and important. In addition, innovative design methods have not only helped to produce a large variety of new integrated circuits but also improved the old ones [2]. The most focus of recent researches is by the necessity on the theory that underlies the design, performance analysis and stability test of the various networks [3]. The fundamental work on the stability test of feedback systems is due to Bode [4]. Employing the Nyquist stability criterion, Bode showed how the notions of phase margin can be exploited to arrive at a simple and useful mean for characterizing the classes of variations in open-loop dynamics which will not destabilize single input feedback systems [5]. The engineering implications of Bode's results are further developed by Hurwitz [6]. Although the Nyquist criterion has been extended to multiloop feedback systems, there has as yet been only limited success in exploiting the multiloop version in the stability analysis of multi-loop feedback systems [7]. The results are interpreted in terms of the classical concept of phase margin, thus strengthening the link between classical and modern feedback synthesis techniques [8]. Furthermore, the transfer function and the self-loop function of a network are very important because they give some useful information about stability and help us optimize the entire performance of a system [9]. From the view point of the complex function, the operating regions of a high-order system are classified as over-damping, critical damping, and under-damping [10]. As a high-order system operates on the under-damping region, the damped oscillation noise causes ringing and makes the network unstable [11]. Ringing or overshoot voltage occurs in both passive and active systems. In other words, it occurs in both feedback and non-feedback systems [12].

To do the stability test of a feedback network, the conventional root locus, Nyquist plot, and Nichols plot techniques all make use of the complex plane [13]. In the root locus method, if a system has poles that are in the right half plane, it will be unstable [14]. Various stabilization conditions were derived using the Nyquist stability criterion [15]. In most cases, Nyquist and Nichols theorems are used in theoretical analysis for feedback systems [16]. To overcome the limitations of the conventional Nyquist stability criterion, the measurement of the phase margin at unity gain of the self-loop function is proposed to evaluate the quality of a power-stage of a DC-DC buck converter.

The motivation for this work on measurement of the self-loop function is coming from the high degree of performance capabilities; therefore, the investigation of the phase margin at unity gain of the self-loop function has been the focal point of extensive research work [17]. In theory, if a network is well defined by a transfer function, we can apply the comparison measurement without disturbing the feedback loop [18].

This paper contains a total of 4 sections. Section 2 presents the mathematical models for the power-stage of the DC-DC buck converter in both cases with and without the harmonic notch filter. Experimental results of measurements of self-loop functions are also described in Section 3. The main points of this work are summarized in Section 4.

2. Mathematical models for power-stage of the DC-DC buck converter

In this section, the simplified models of the power-stage of the DC-DC buck converters are analyzed. The power-stage of the DC-DC converter is an RLC network which is also called 2nd-order low-pass filter as shown in Fig. 1(a). The function of this filter is to convert the switching source into DC voltage [20]. Therefore, we only investigate the transfer function of the power stage [19]. Moreover, linear networks are usually modeled by the complex functions. Complex functions are typically expressed in three forms: magnitude-angular plots (Bode plots), polar charts (Nyquist charts), and magnitude-argument diagrams (Nichols diagrams) [21]. The transfer function and the self-loop function of the power-stage in case of without the harmonic notch filter are given by

$$H(\omega) = \frac{1}{a_0(j\omega)^2 + a_1j\omega + 1}; L(\omega) = a_0(j\omega)^2 + a_1j\omega; \quad (1)$$

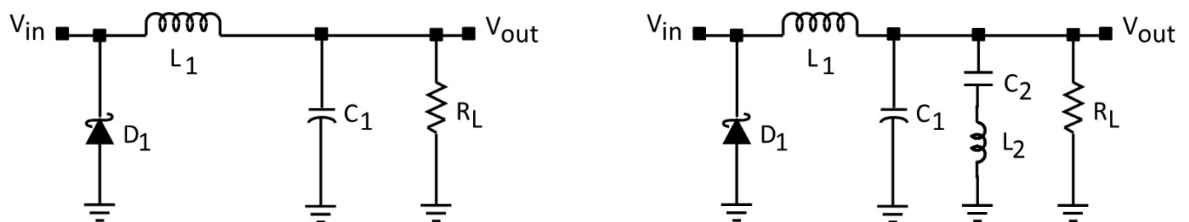
$$a_0 = L_1C_1; a_1 = \frac{L_1}{R_L};$$

The operating regions of a 2nd-order system are over-damping, critical damping, and under-damping [22]. To reduce the ripple of the output voltage, a passive harmonic notch filter is added at the output port. Now, the transfer function and the self-loop function of the power-stage are given by

$$H(\omega) = \frac{b_0(j\omega)^2 + 1}{a_0(j\omega)^4 + a_1(j\omega)^3 + a_2(j\omega)^2 + a_3j\omega + 1}; L(\omega) = a_0(j\omega)^4 + a_1(j\omega)^3 + a_2(j\omega)^2 + a_3j\omega; \quad (2)$$

$$b_0 = L_2C_2; a_0 = L_1C_1L_2C_2; a_1 = \frac{L_1L_2C_2}{R_L}; a_2 = L_1C_1 + L_2C_2 + L_1C_2; a_3 = \frac{L_1}{R_L};$$

The phase margin at unity gain of the self-loop function shows the operating region of a linear system [23]. We can read the phase margin directly from the Nichols plot of the self-loop function [24]. The phase margin test for the implemented circuit is described in section 3.



(a) Simplified model without notch filter; (b) Simplified model with notch filter

Fig.1. Mathematical models of the power-stages of DC-DC buck converters.

3. Phase margin test for power stage of DC-DC buck converter

In this section, the performance of the proposed design of the LC harmonic notch filter for the buck converter has been verified by the practical measurement. Fig. 2 and Fig. 3 show the schematic circuit and the measurement set up for the implemented DC-DC buck converter. The measurement results contained in this paper do not only focus on the transfer function of the power stage, but they also integrate the impact of the voltage ripples, EMI noise and power loss. In the implemented circuit, the chosen components are $L_1 = 530 \mu\text{H}$, $C_1 = 220 \mu\text{F}$, $R_1 = 4 \text{ k}\Omega$, $R_2 = 2 \text{ k}\Omega$, $R_L = 5 \Omega$, $C_2 = 470 \text{ pF}$, $L_2 = 50 \mu\text{H}$. In this work, the network analyzer, spectrum analyzer, signal generator, and oscilloscope, were used to perform the measurements. A laboratory power supply of 12 V was used to do the phase margin test. The operation parameters of this design are described in Table 1.

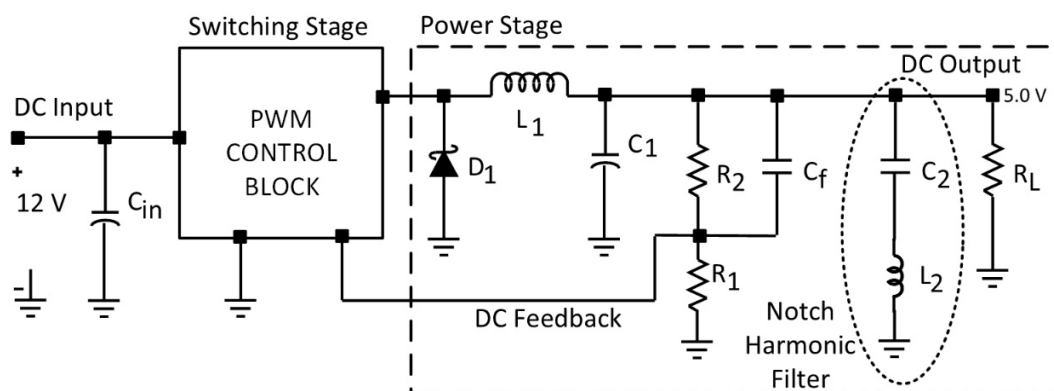


Fig.2. Schematic diagram of the proposed design of the DC-DC converter.

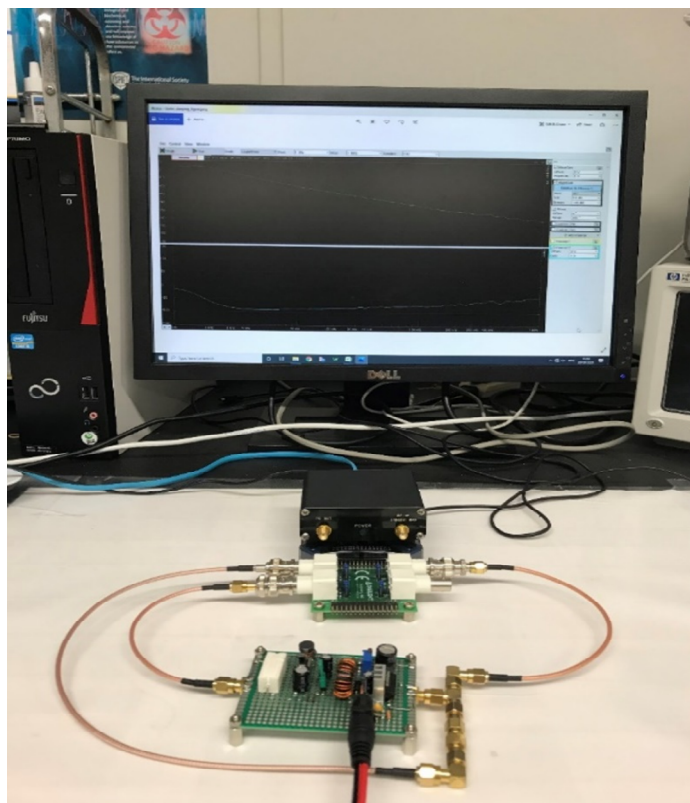
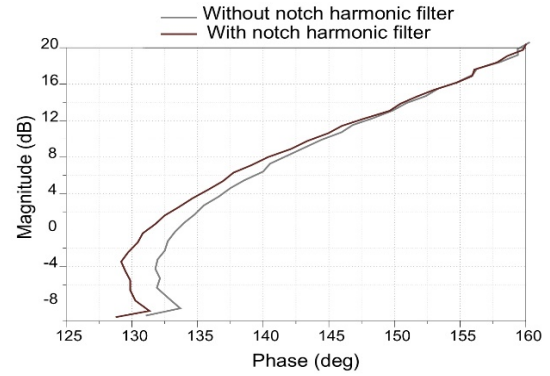
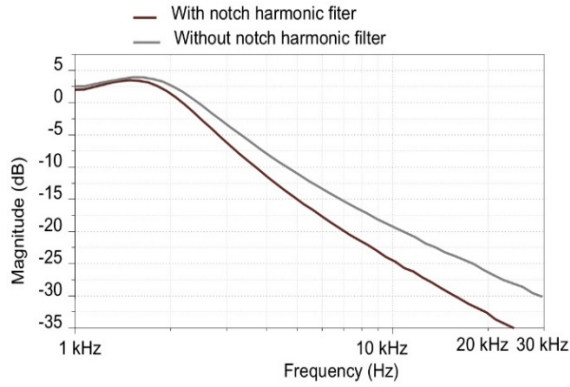


Fig.3. Measurement set up for the implemented DC-DC converter.

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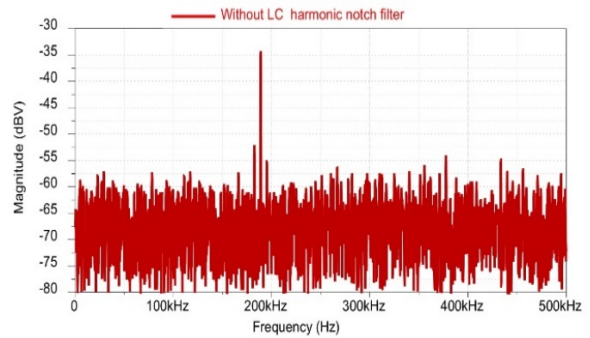
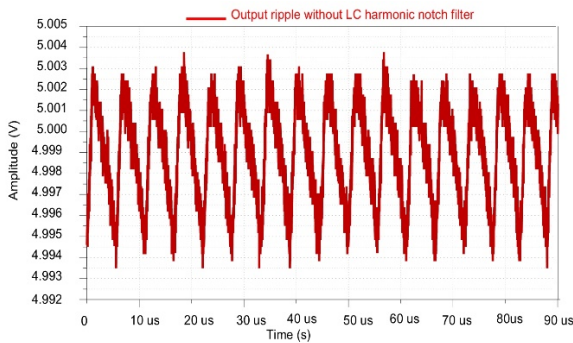
Table 1: Operation parameters of DC-DC buck converter.

Input Voltage (V_{in})	12.0 V
Output Voltage (V_{out})	5.0 V
Output Current (I_o)	1.0 A
Clock Frequency (F_{ck})	180 kHz
Output Ripple	5 mVpp



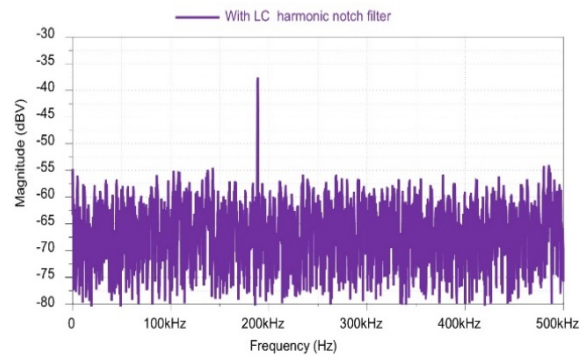
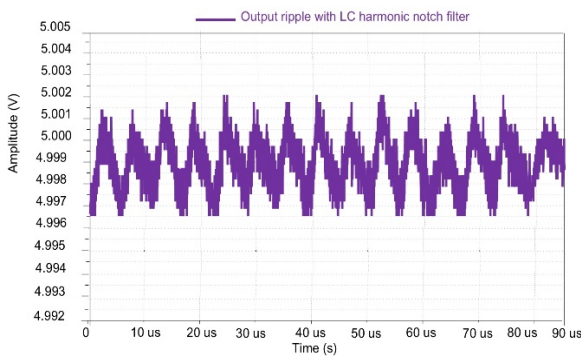
(a) Bode plot of the transfer function; (b) Nichols plot of the self-loop function.

Fig.4. Measurement results of the transfer function and the self-loop function.



(a) Waveform of ripple without notch filter; (b) Spectrum of ripple without notch filter.

Fig.5. Waveforms and spectra of the output ripple without harmonic notch filter.



(a) Waveform of ripple with notch filter; (b) Spectrum of ripple with notch filter.

Fig.6. Waveforms and spectra of the output ripple with harmonic notch filter.

Fig. 4(a) shows the measurement results of the Bode plot of the power-stage. The magnitude of the transfer function of the power stage is reduced by 3 dB after adding the LC harmonic notch filter. The phase margin is improved from 46 degrees (observed at 134 degrees) to 49 degrees (observed at 131 degrees) as shown in Fig. 5(b). As the passive LC harmonic notch filter is added at the output node, the waveform of the output ripple is also kept at low level 5 mVpp at 5 V desired voltage in time domain as shown in Fig. 5(a) and Fig. 6(a). Therefore, the level of the fundamental spectrum is reduced from -35 dBV to -38 dBV at 180 kHz as shown in Fig. 5(b) and Fig. 6(b).

A new phase margin test for the power-stage of a DC-DC buck converter is proposed to overcome the limitation of the conventional Nyquist stability criterion. The comparison measurement can measure the self-loop function of the power-stage [25].

4. Conclusion

This paper describes the approach to do the phase margin test for power-stage of a DC-DC buck converter. The simplified model of the transfer function of the power-stage is a 2nd-order complex function. As the power-stage operates in under-damping region, the overshoot occurs and causes the ringing noise. That makes the system unstable. After adding a passive harmonic notch filter at the output port, the power-stage is a 4th-order complex function and the phase margin is improved from 46 degrees to 49 degrees. The ripple is also reduced from 11 mVpp to 5 mVpp which is compared to the desired output voltage of 5 V. As a result, the spectrum of the voltage ripple is also minimized from -35 dBV to -38 dBV. In this paper not only the results of the mathematical model but also the measurement results of the implemented circuit are provided, including the stability test. The measurement results and the values of theoretical calculation of the self-loop function are almost identical. Therefore, the phase margin test can be used to evaluate the quality of a buck converter. In future work, the parasitic components of the printed circuit board will be investigated.

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