Advanced Data Converter Session

22 Oct. 2020
14:00-14:15

Digital-to-Analog Converter Architectures Based on Polygonal and Prime Numbers

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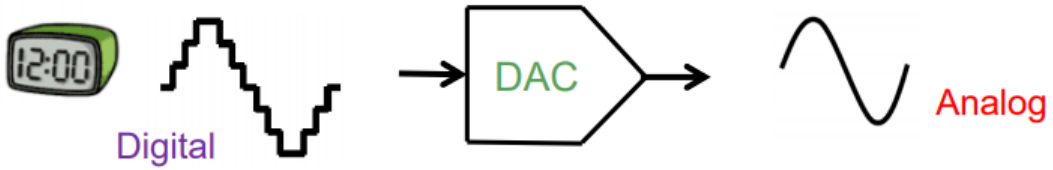
Shuhei Yamamoto, Anna Kuwana, Haruo Kobayashi

Oyama National College : Kazuyoshi Kubo

Research Objective

- **Interesting properties of integers**

- **Possibility of new configurations of DAC**



Outline

- Research Background
- Triangular Number DAC
- Polygonal number DAC
- Prime number DAC
- Summary

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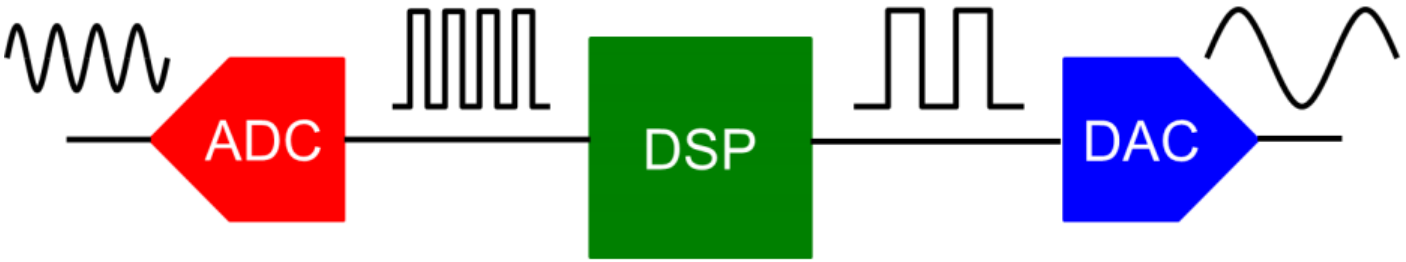
Importance of ADC / DAC

- Rapid development of digital electronics technology
- A natural signal is analog



DACs are Everywhere !

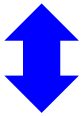
Communication equipment

Electronic measuring instrument

Audio systems

Integer Theory and Electronic Circuit Design

Many interesting properties of Integers



Currently
No Link

Electronic circuit designs

Our research here makes their links !

Carolus Fridericus Gauss
(1777-1855)

Integer theory is
the queen of Mathematics

Outline

- Research Background
- **Triangular Number DAC**
- Polygonal Number DAC
- Prime Number DAC
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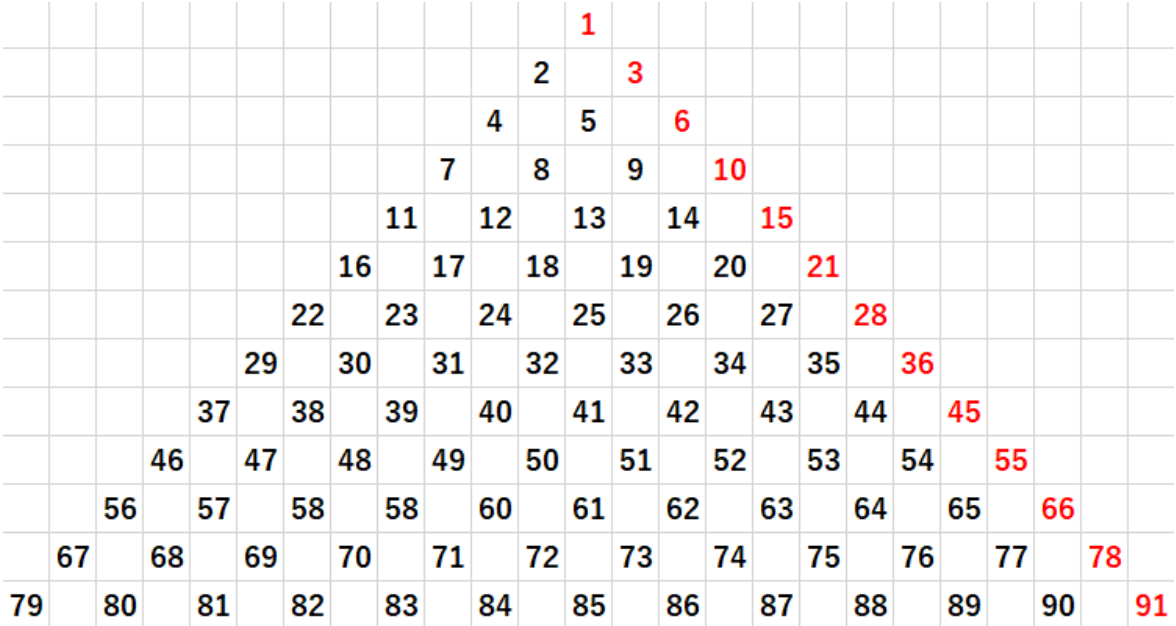
What is Triangular Number ?

Triangular Number: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, ..n(n+1)/2



Theory of Trigonometric Numbers

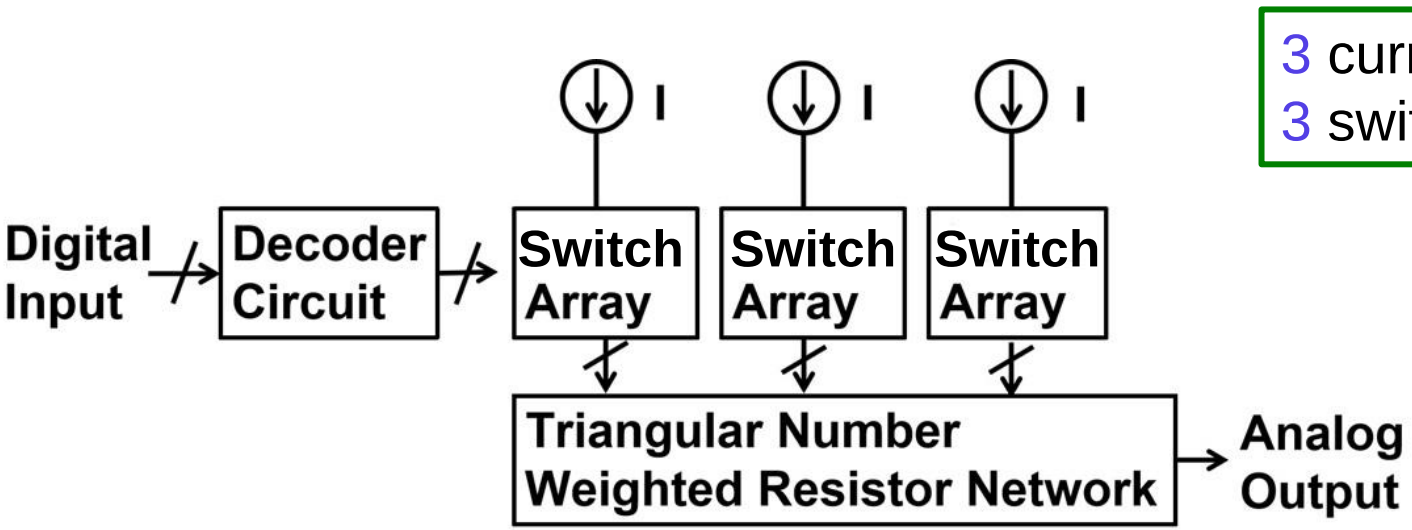
Any natural number

expressed by

Sum of 3 triangular numbers

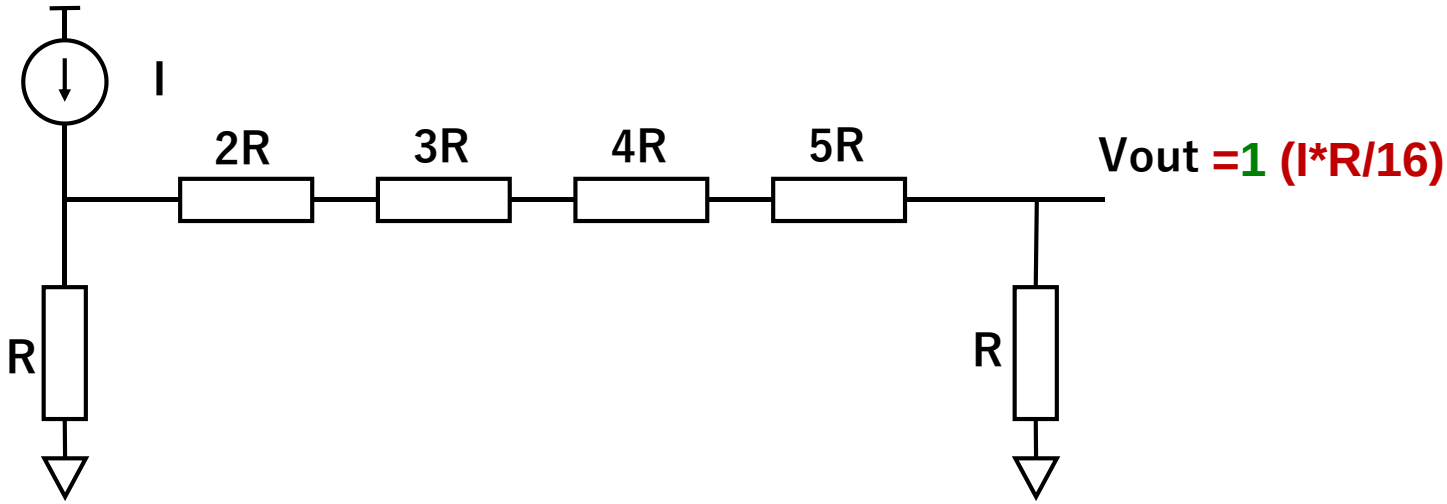
1:	1
2:	1+1
3:	3
4:	1+3
5:	1+1+3
6:	6
7:	1+6
8:	1+1+6
9:	3+6
10:	10
11:	1+10
12:	1+1+10
13:	3+10
14:	1+3+10
15:	15
16:	1+15
17:	1+1+15
18:	3+15
19:	1+3+15
20:	10+10
21:	21
22:	1+21
23:	1+1+21
24:	3+21
25:	10+15
26:	1+10+15
27:	1+10+21
28:	28
29:	1+28
30:	1+1+28

Proposed Triangular Number DAC



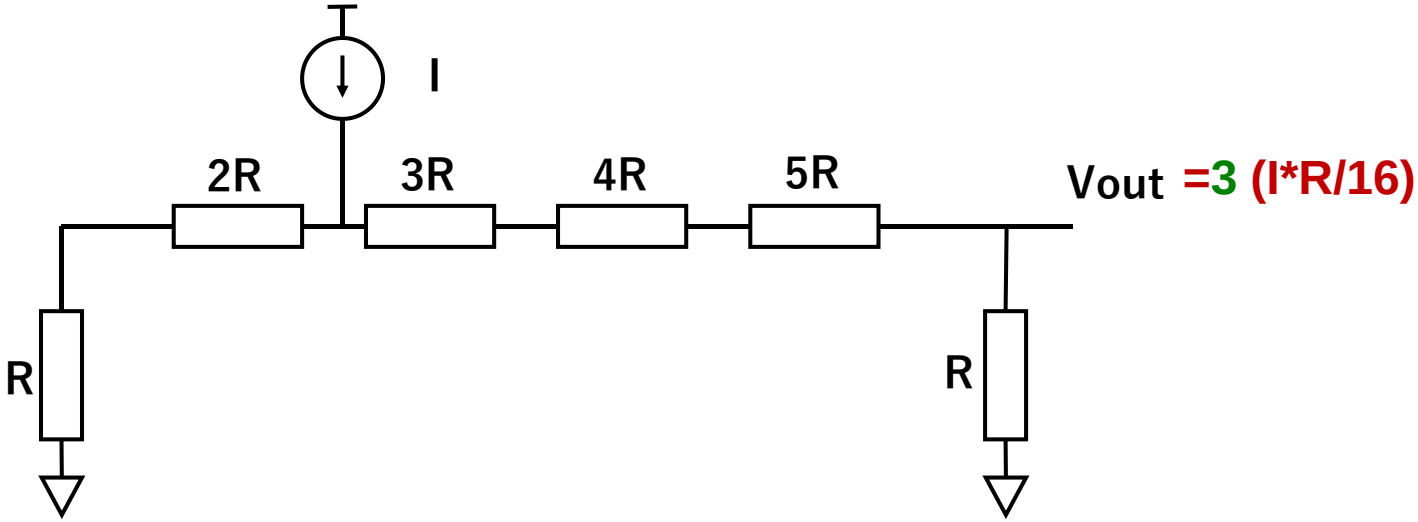
Triangular Number Weighted Voltage (1)

Triangular Number： 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, ..n(n+1)/2



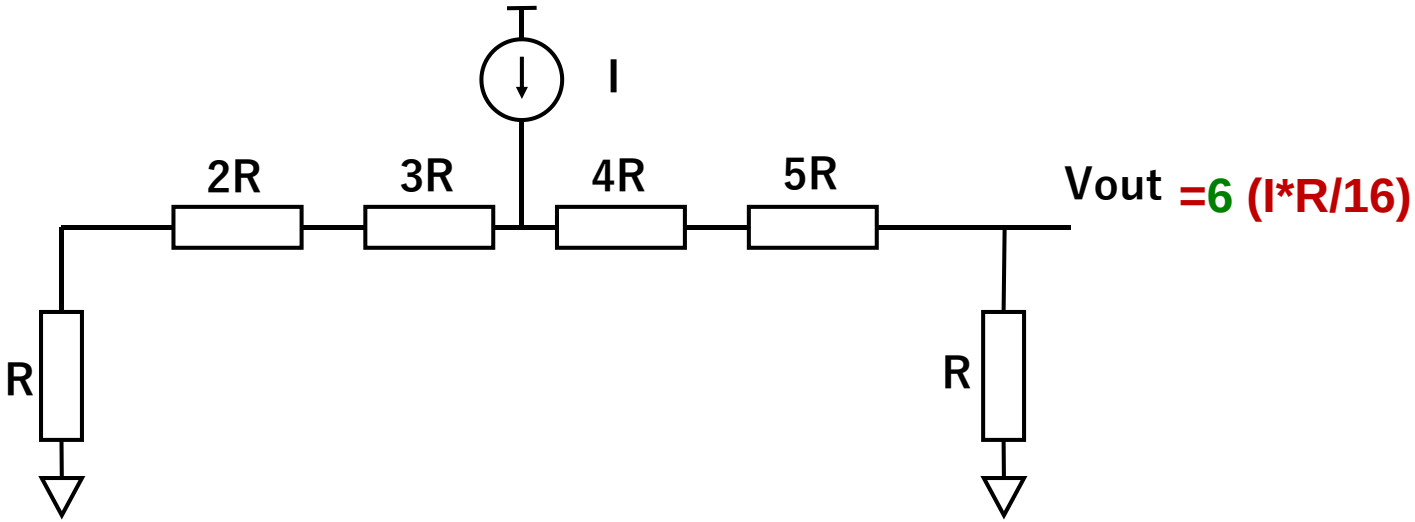
Triangular Number Weighted Voltage (2)

Triangular Number: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, ..n(n+1)/2



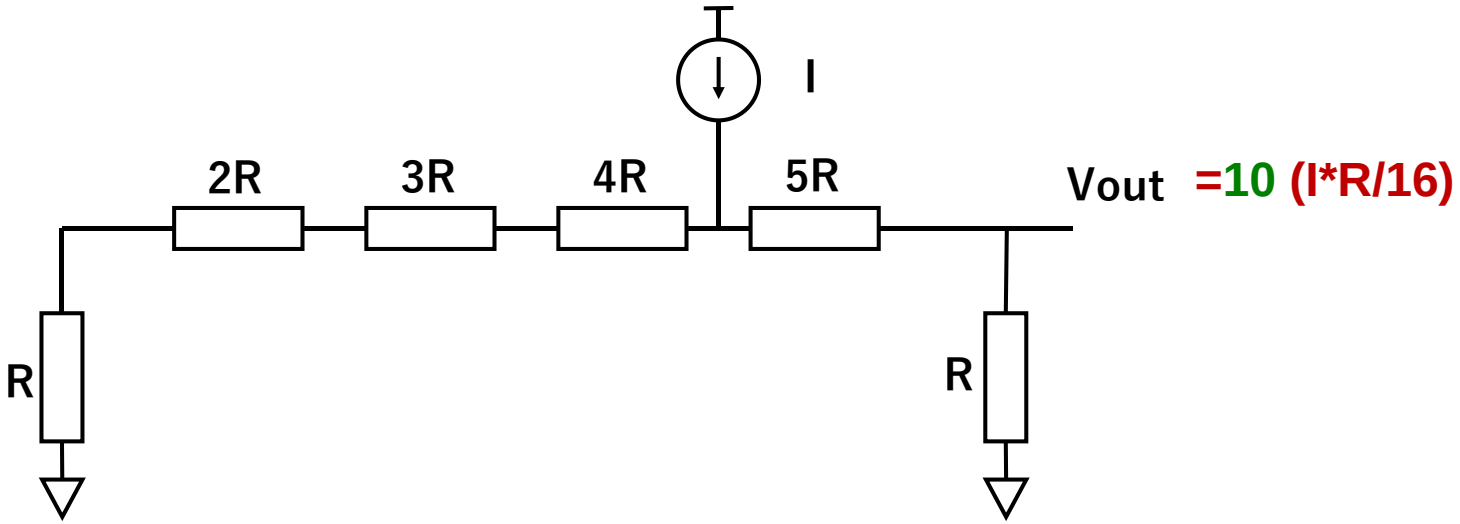
Triangular Number Weighted Voltage (3)

Triangular Number: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, ..n(n+1)/2



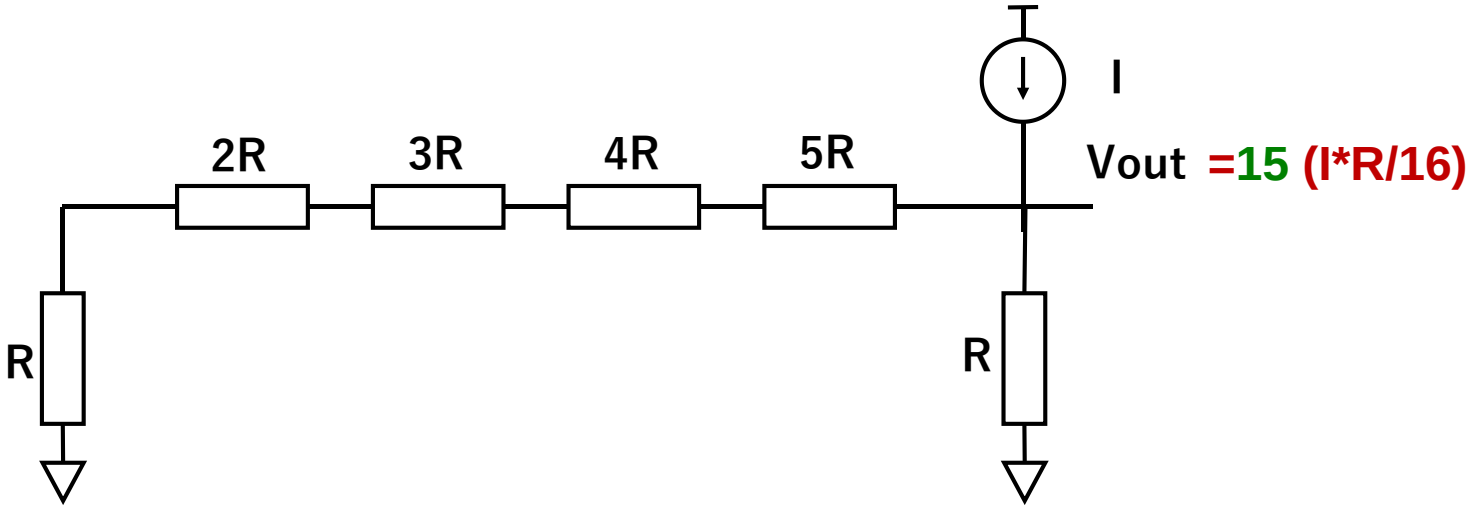
Triangular Number Weighted Voltage (4)

Triangular Number: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, ..n(n+1)/2



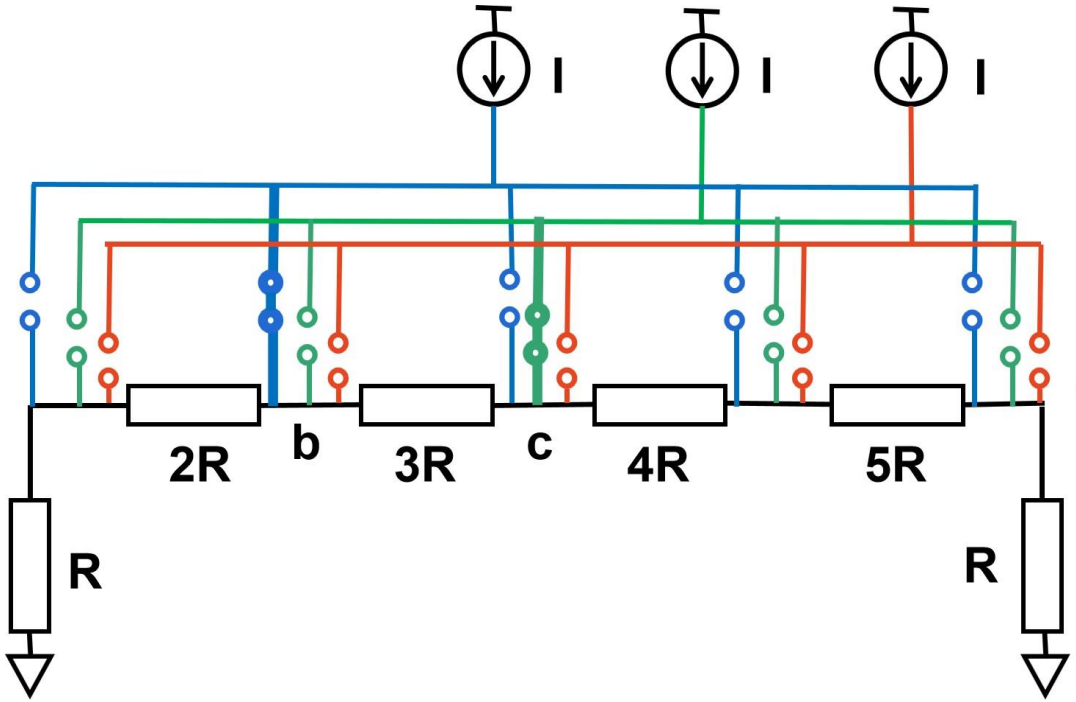
Triangular Number Weighted Voltage (5)

Triangular Number： 1, 3, 6, 10, **15,** 21, 28, 36, 45, 55, 66, 78, 91, ..n(n+1)/2



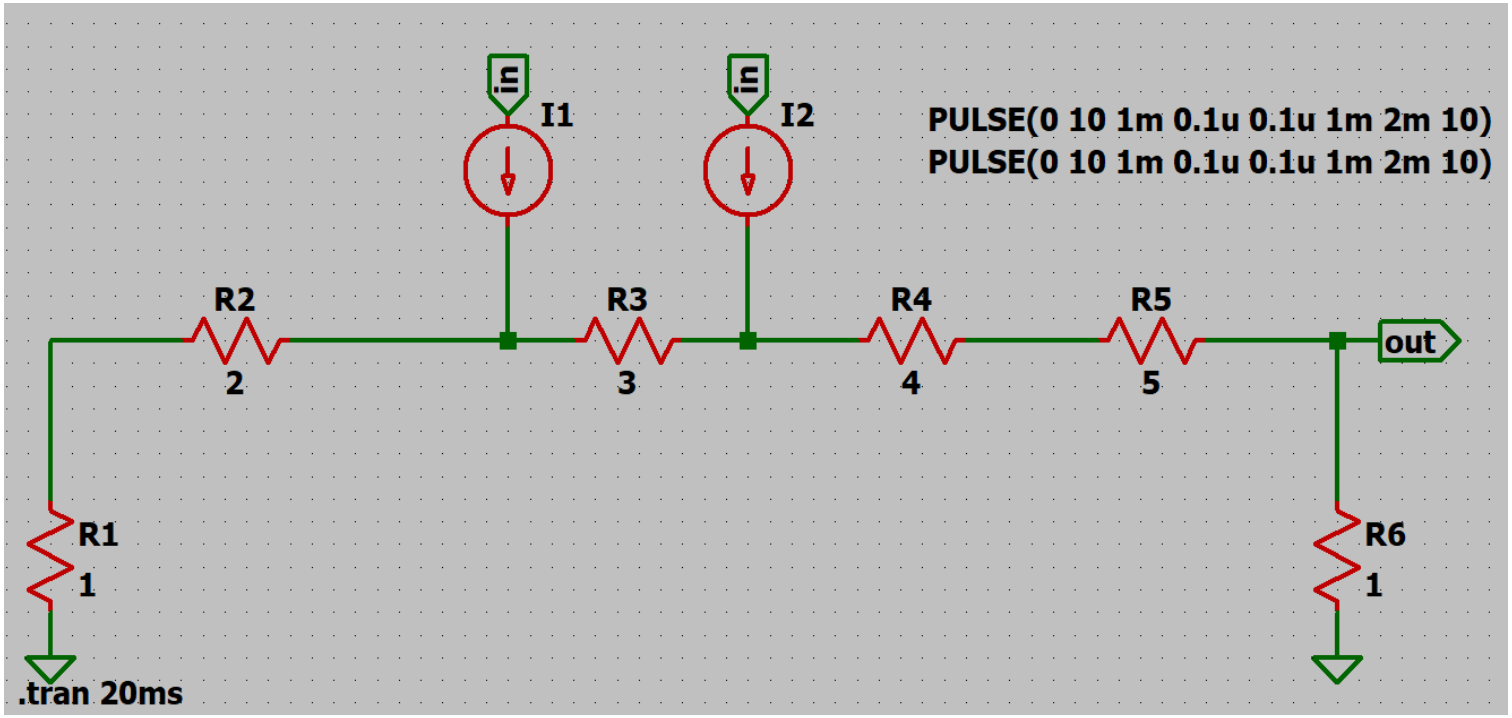
When Digital Input is 9: Calculation

Triangular Number: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, ..n(n+1)/2

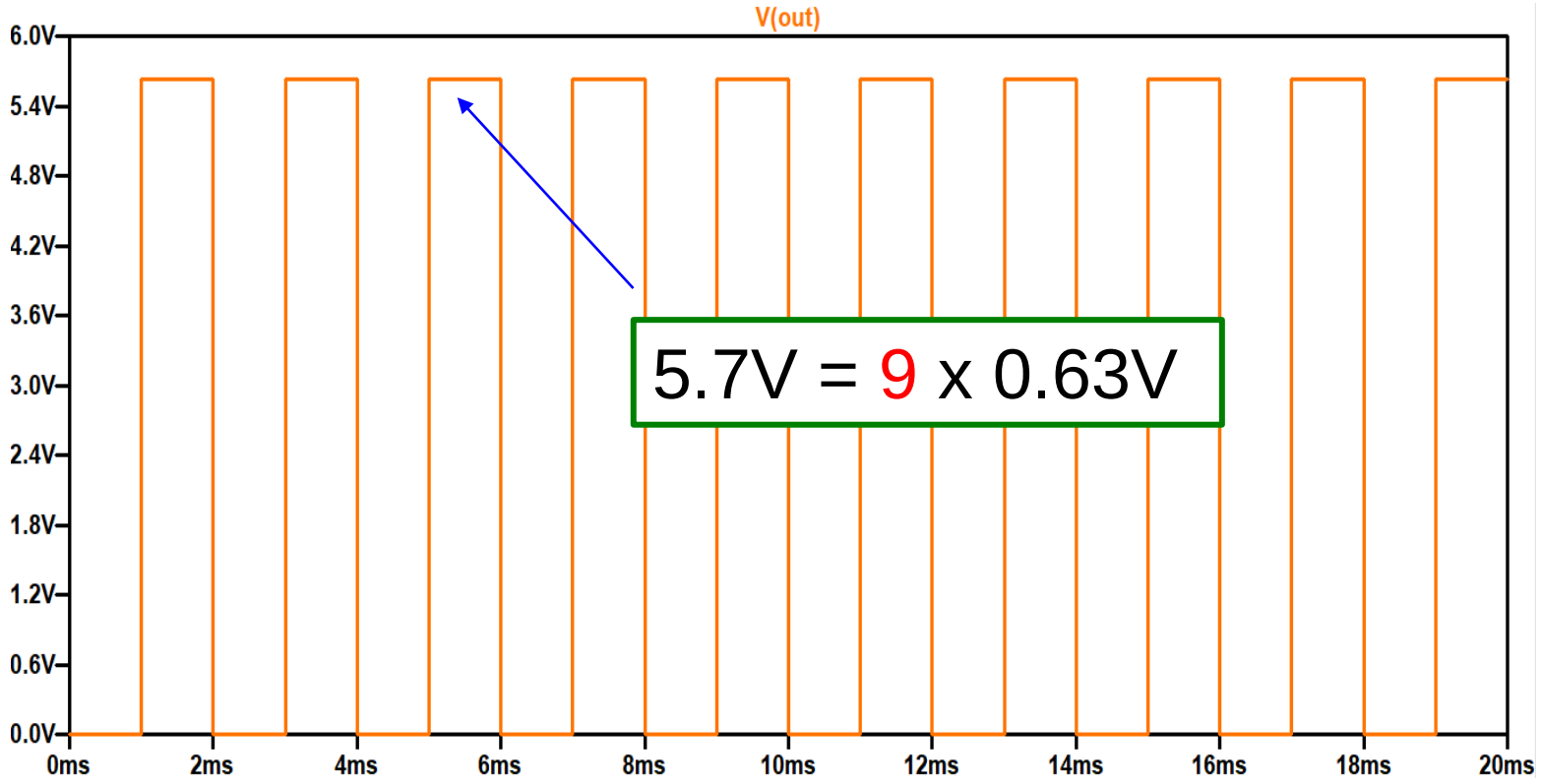


Vout = (3 + 6) Vo = 9 (1/16)RI

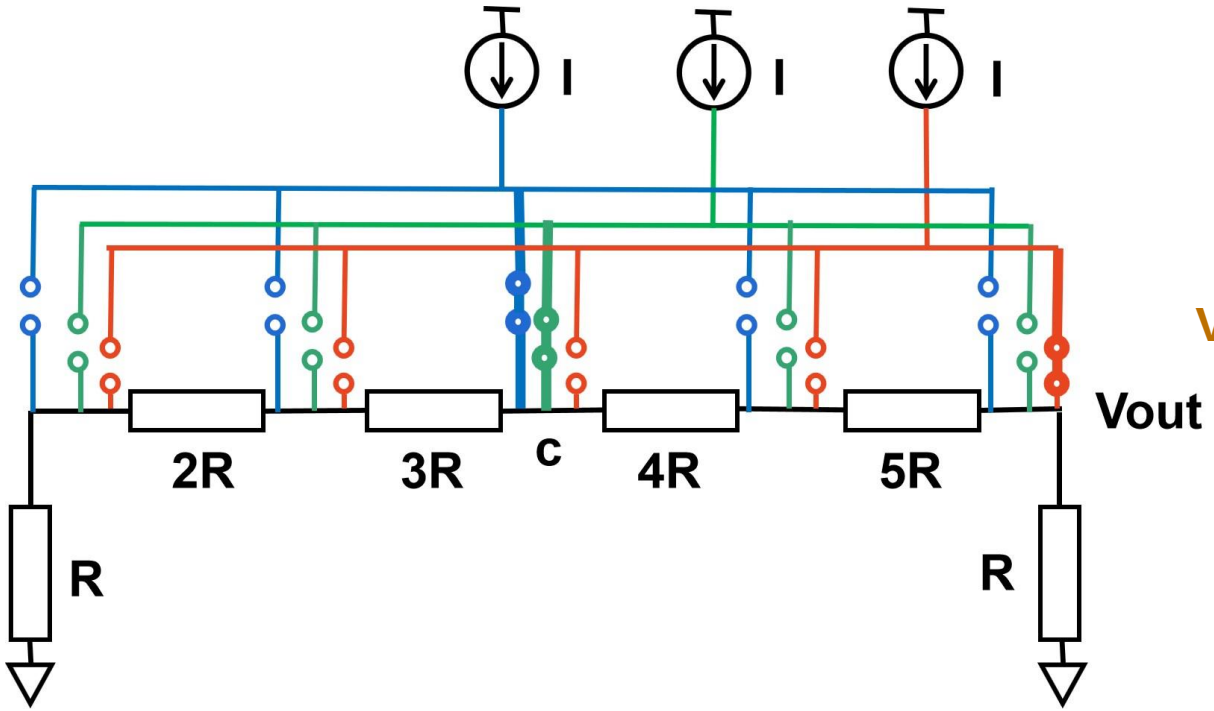
When Digital Input is 9: Simulation



When Digital Input is 9: Simulation Result

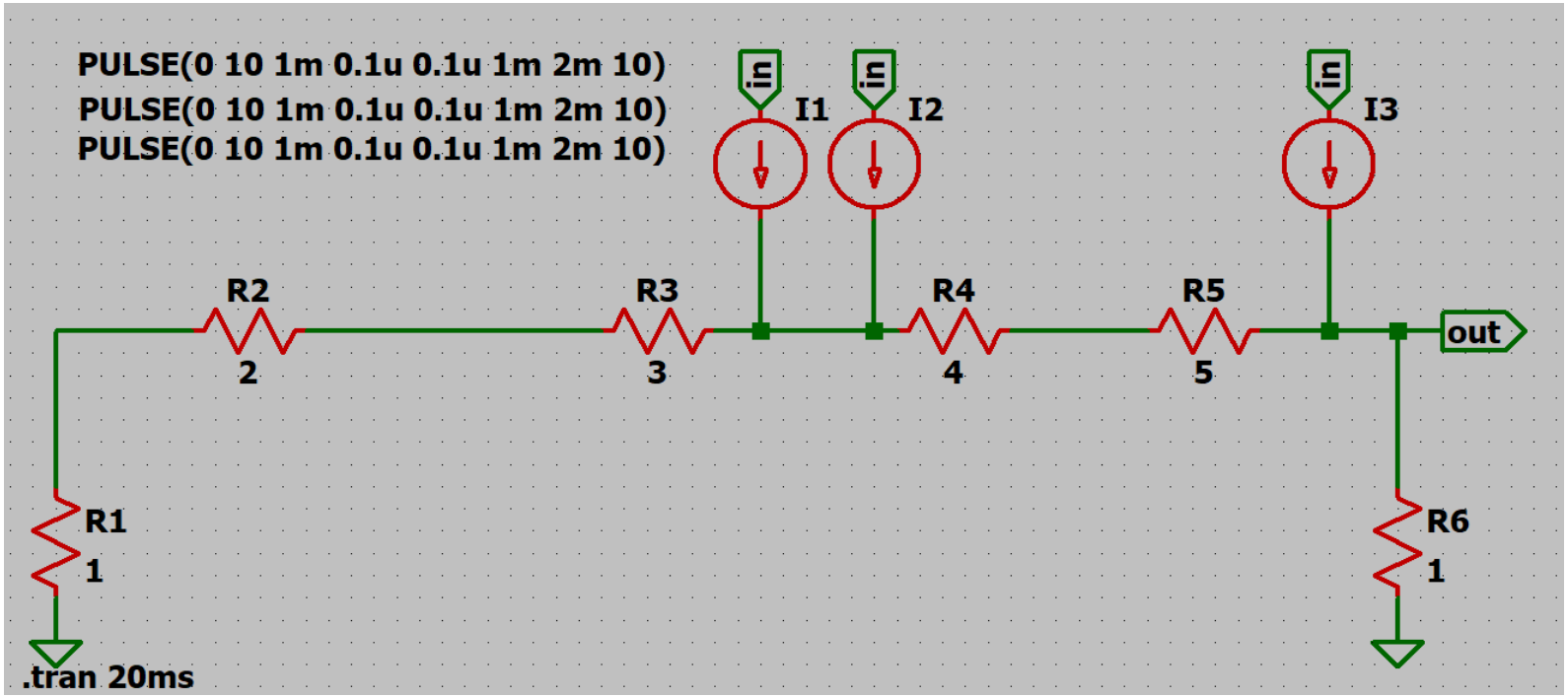


When Digital Input is 27: Calculation

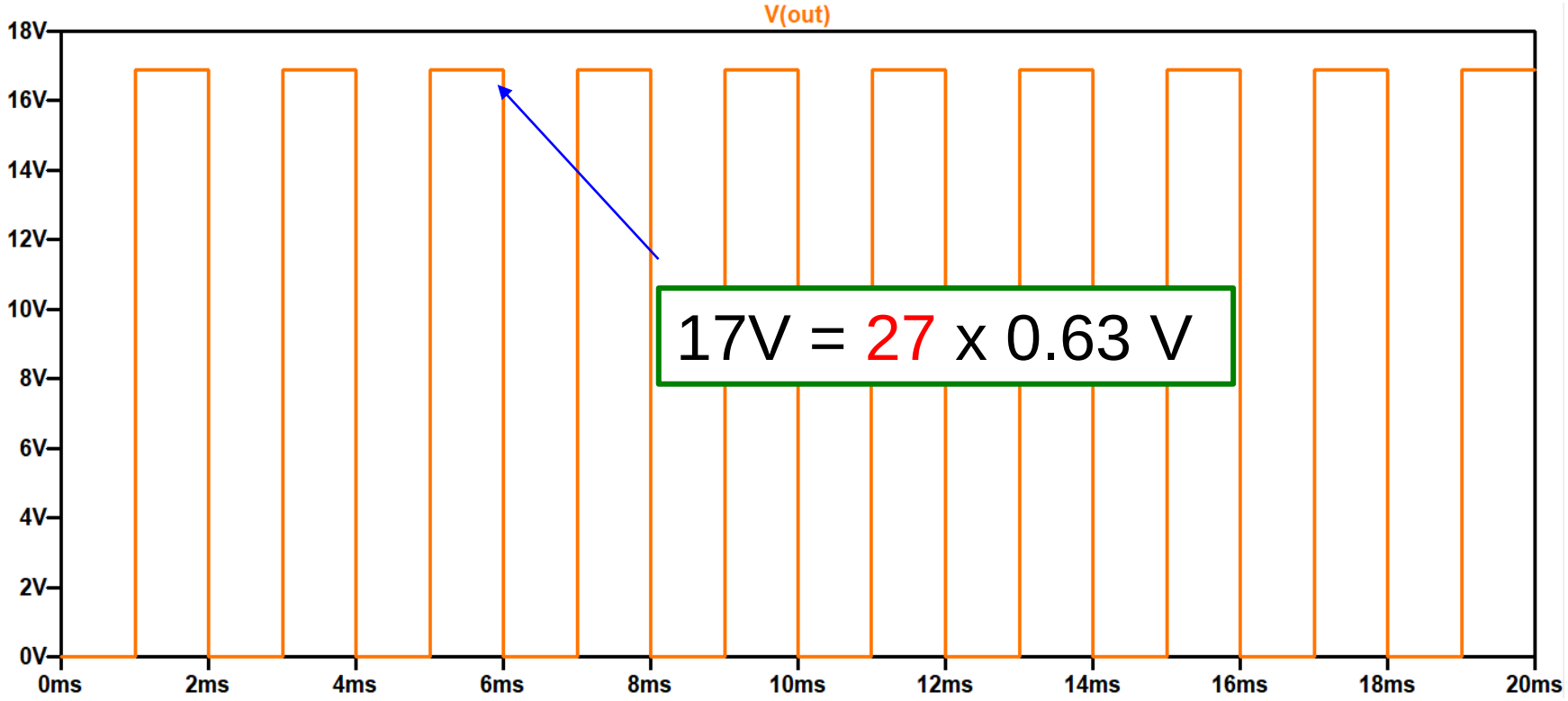


Vout = (6+6+15)Vo=27Vo.

When Digital Input is 27: Simulation



When Digital Input is 27: Simulation Result



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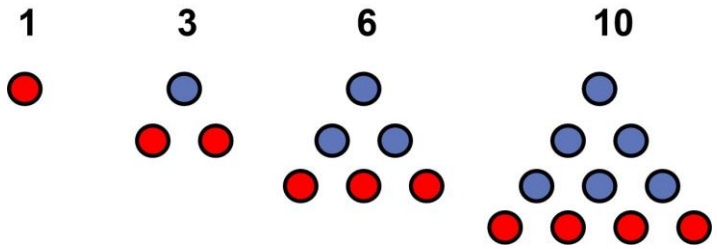
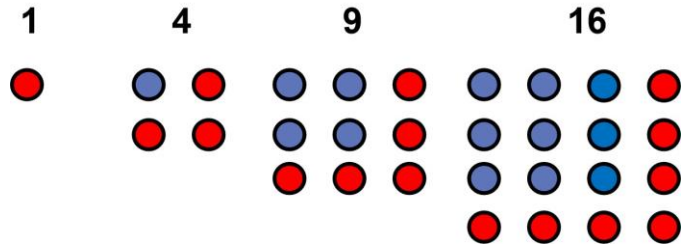
Polygonal Number

Polygonal number

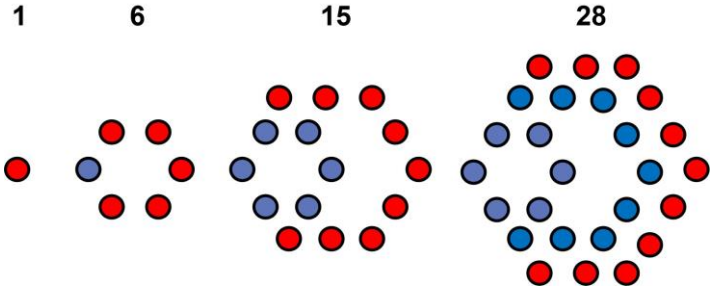
Represented as dots or pebbles arranged in the shape of a regular polygon.



(a)Triangular numbers.



(b) Square numbers.



(c) Hexagonal numbers.

Polygonal Number Theorem

Any natural number

expressed by

Sum of N N-angular numbers

k-th of N-angular number, m(N, k) can be expressed by

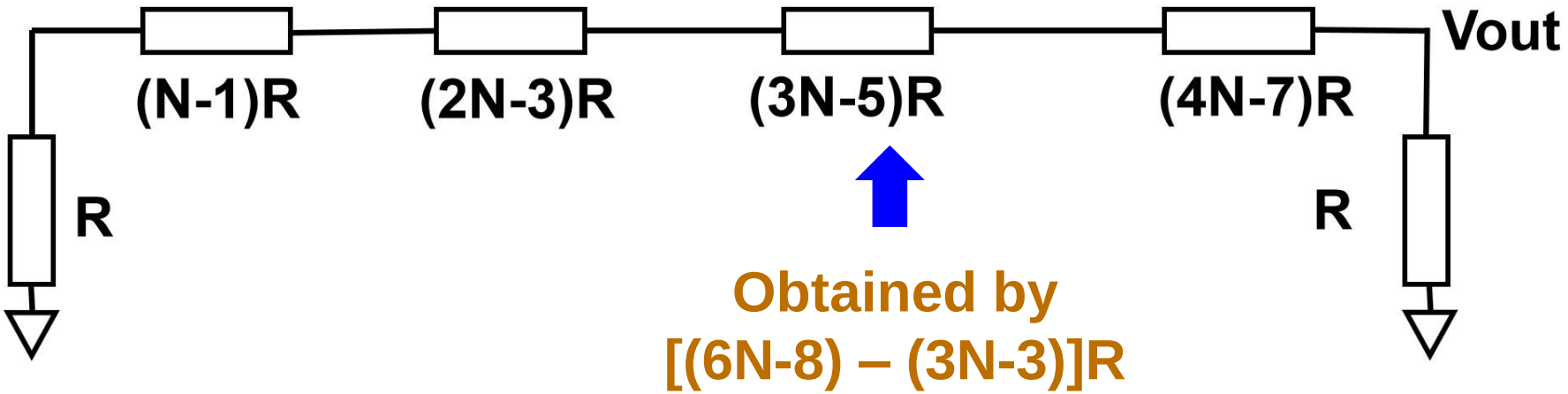
$$m(N, k) = (1/2)\, k\, [(N-2)k - (N-4)]$$

Then N-angular numbers are given by

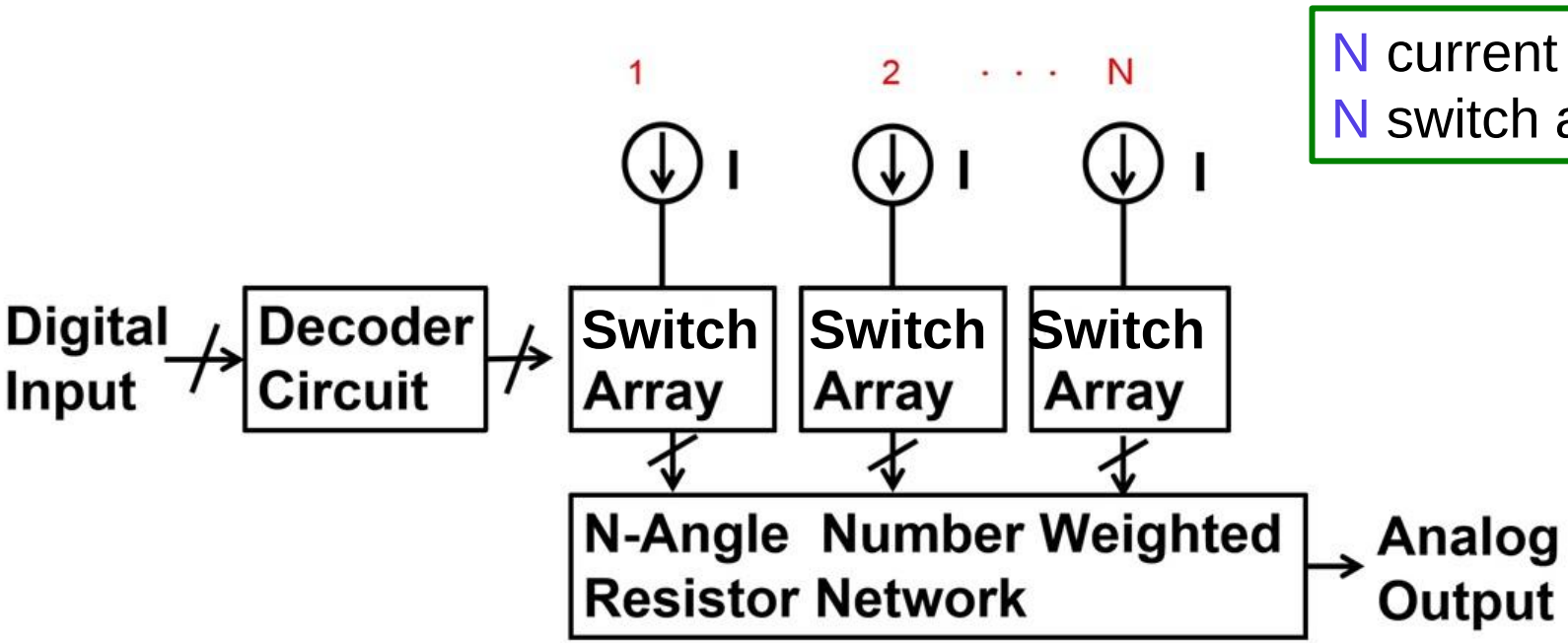
1, N, 3N-3, 6N-8, 10N-15, …

for k=1, 2, 3, 4, 5, …

N-angle Number Weighted Resistor Network

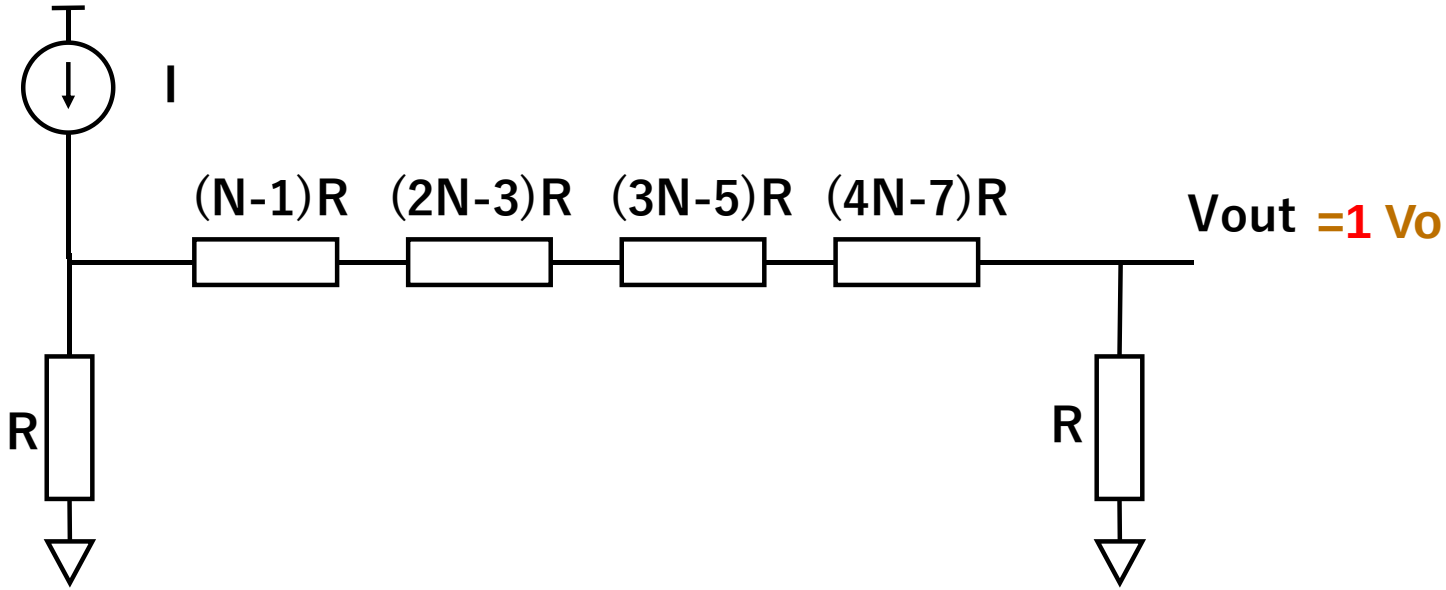


DAC Configuration based on N-angular Number



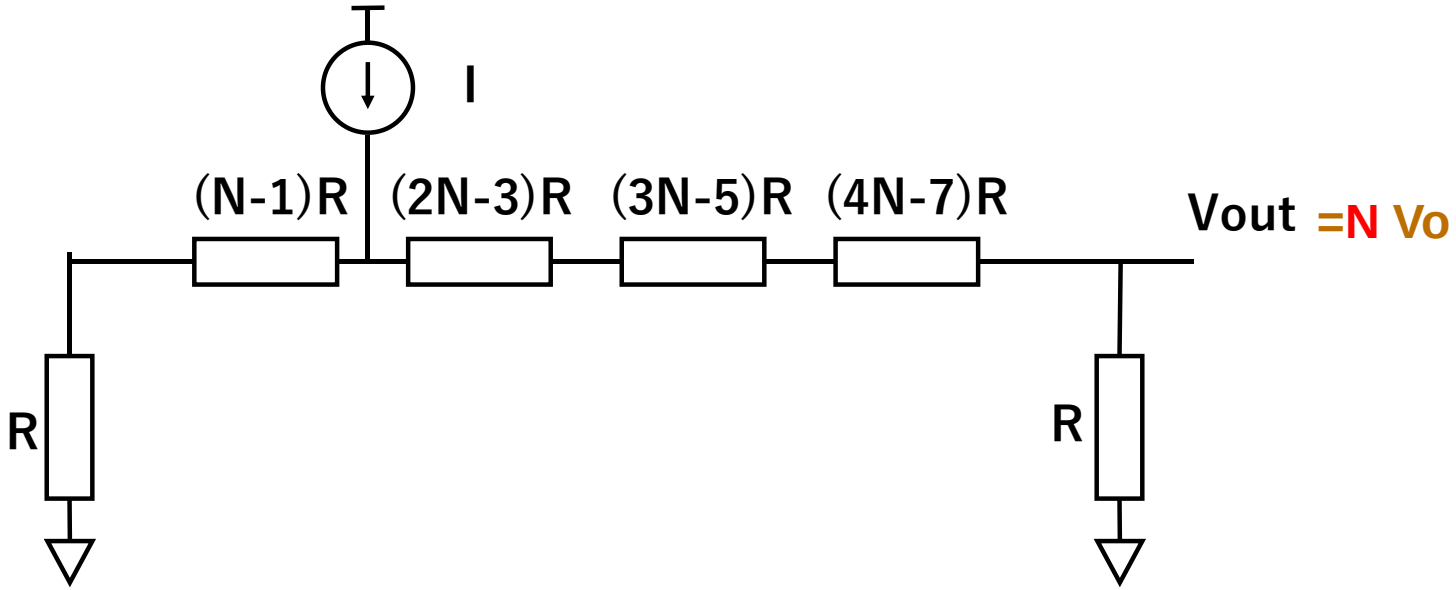
N-angular Number Weighted Voltage (1)

N-angular Number: 1, N, 3N-3, 6N-8, 10N-15, …



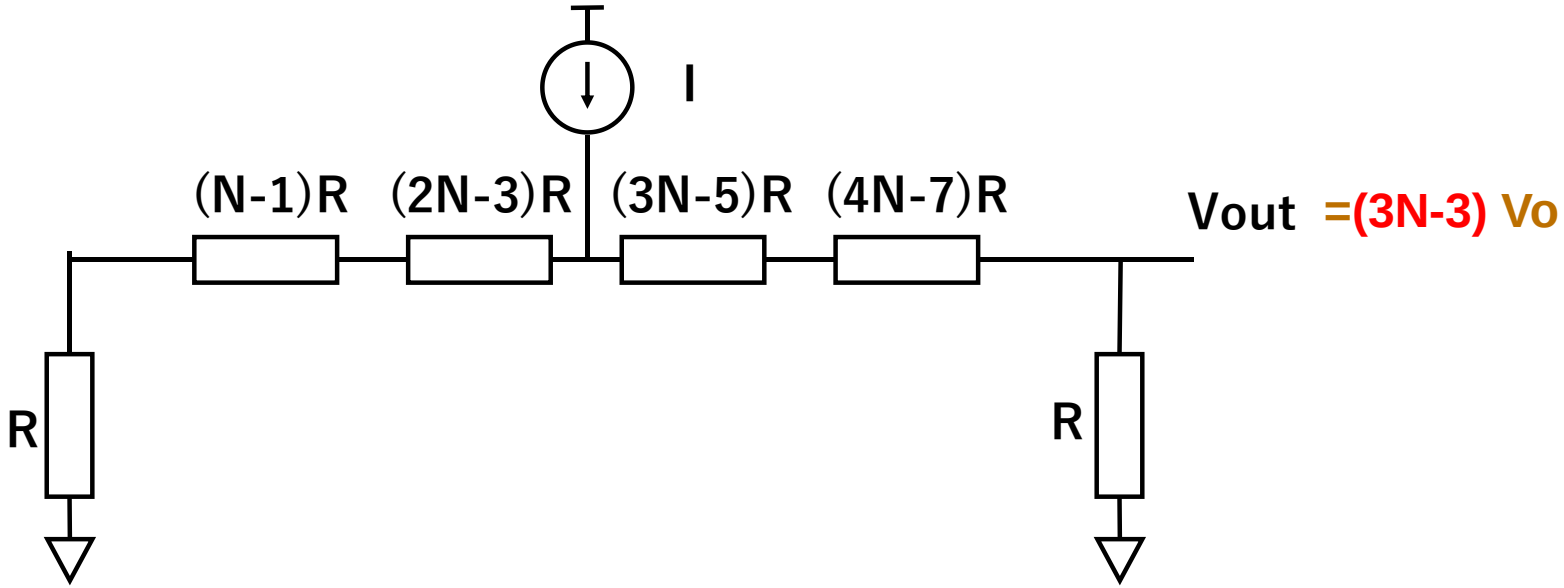
N-angular Number Weighted Voltage (2)

N-angular Number: 1, N, 3N-3, 6N-8, 10N-15, …



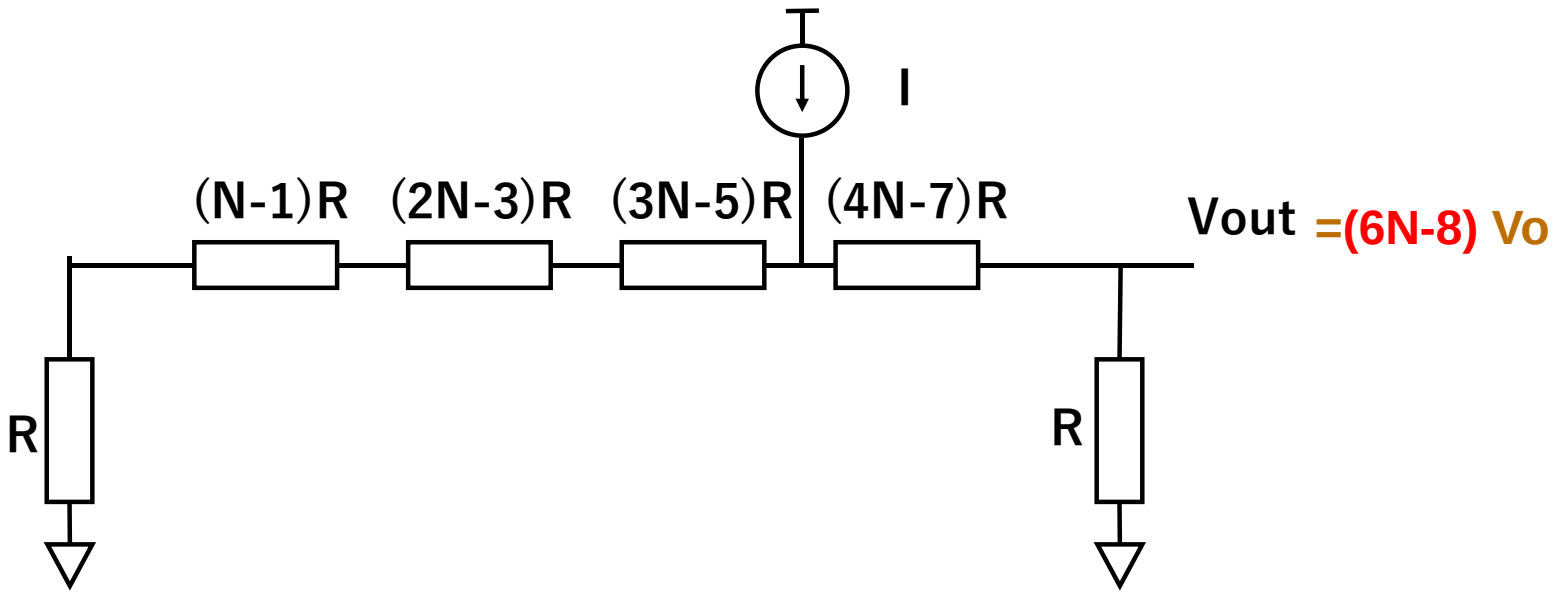
N-angular Number Weighted Voltage (3)

N-angular Number: 1, N, 3N-3, 6N-8, 10N-15, …



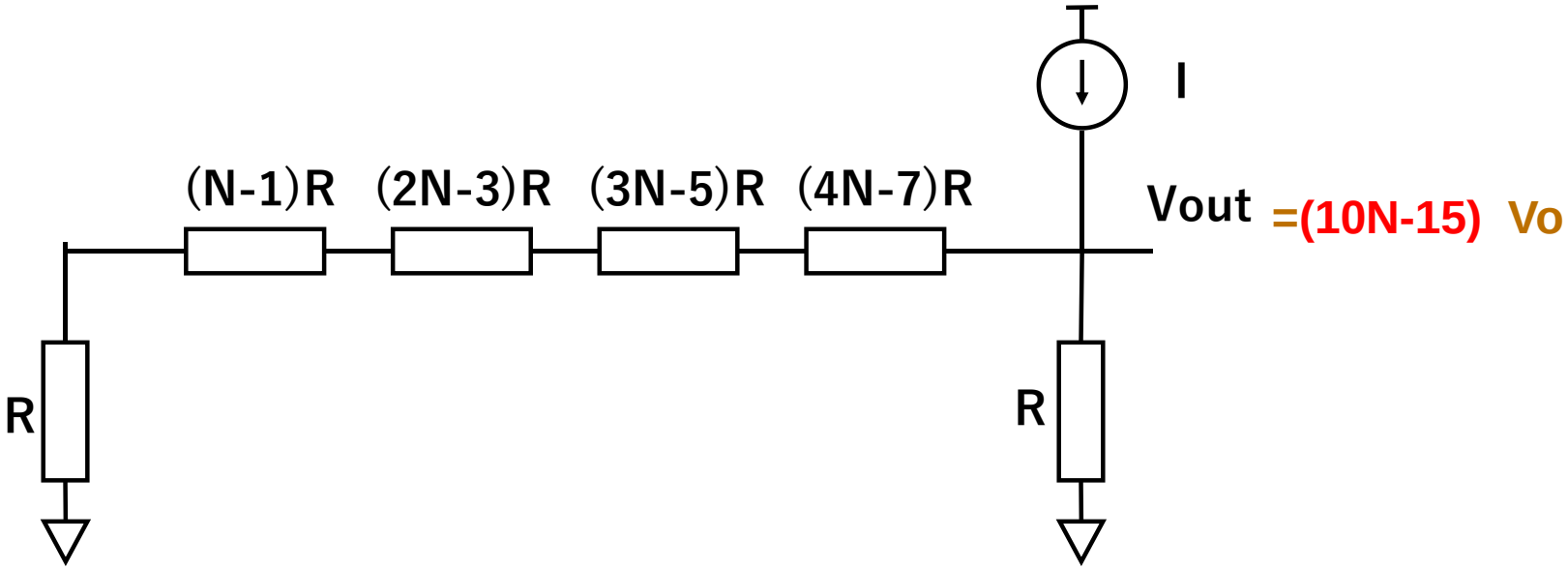
N-angular Number Weighted Voltage (4)

N-angular Number: 1, N, 3N-3, 6N-8, 10N-15, …



N-angular Number Weighted Voltage (5)

N-angular Number: 1, N, 3N-3, 6N-8, 10N-15, …



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Goldbach's Conjecture

Goldbach's Conjecture:

All even numbers can be represented by sum of two prime numbers.

+	2	3	5	7	11	13	17	19
2	4	5	7	9	13	15	19	21
3	5	6	8	10	14	16	20	22
5	7	8	10	12	16	18	22	24
7	9	10	12	14	18	20	24	26
11	13	14	16	18	22	24	28	30
13	15	16	18	20	24	26	30	32
17	19	20	22	24	28	30	34	36
19	21	22	24	26	30	32	36	38

Prime Numbers

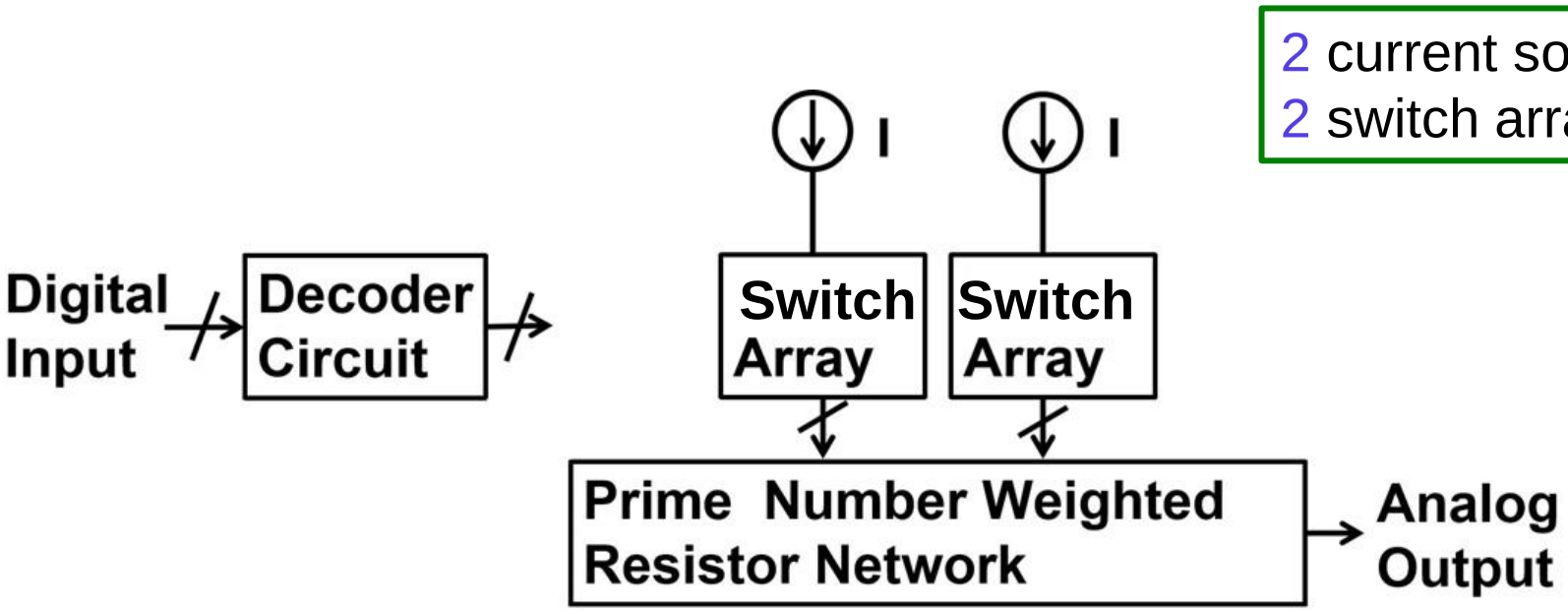
Prime numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ……

All even numbers are represented by two prime numbers

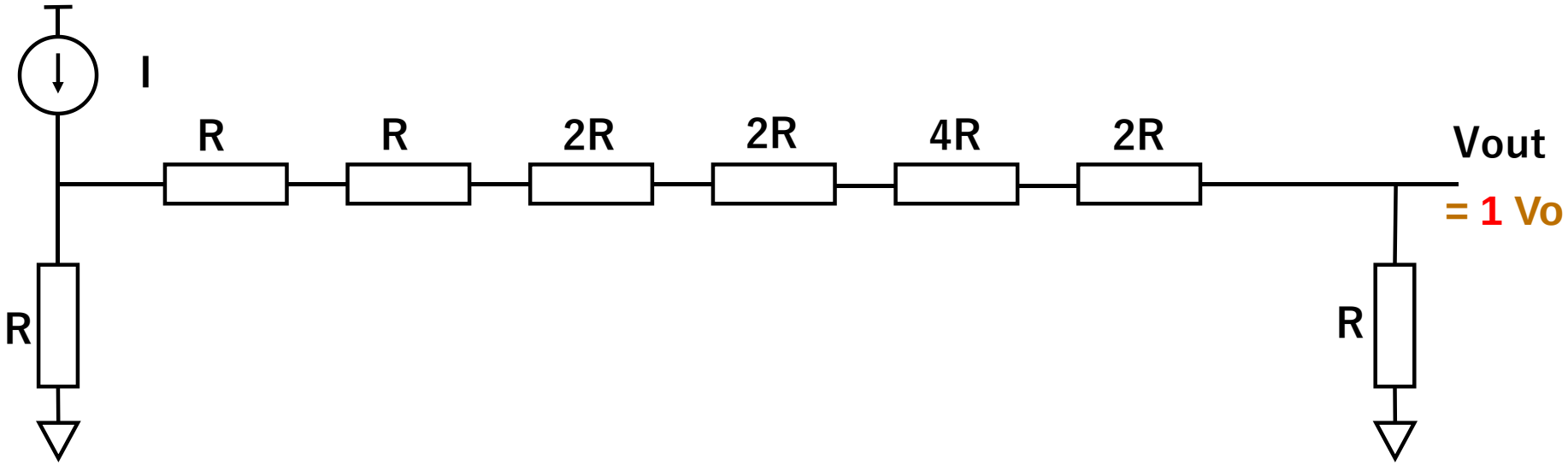
2:	2	32:	13+19
4:	2+2	34:	17+17
6:	3+3	36:	17+19
8:	3+5	38:	19+19
10:	3+7	40:	17+23
12:	5+7	42:	19+23
14:	7+7	44:	13+31
16:	5+11	46:	23+23
18:	7+11	48:	19+29
20:	7+13	50:	19+31
22:	11+11	52:	23+29
24:	11+13	54:	23+31
26:	13+13	56:	19+37
28:	11+17	58:	29+29
30:	13+17	60:	29+31

Proposed Prime Number DAC



Proposed Prime Number DAC Operation (1)

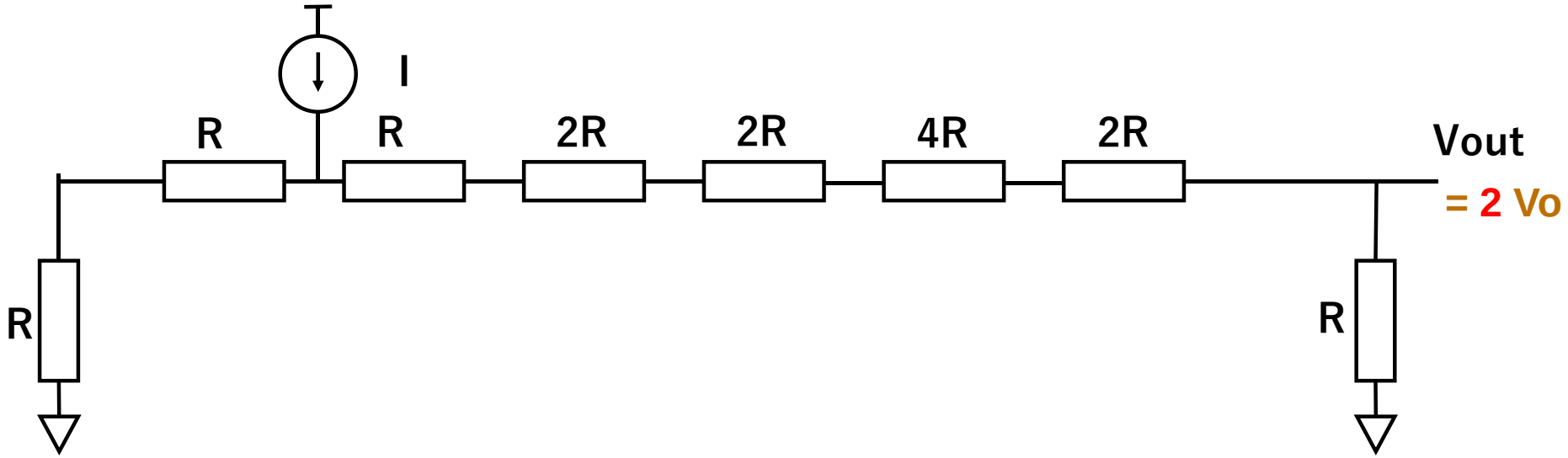
Prime numbers: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ……



Here Vo=(1/14)RI

Proposed Prime Number DAC Operation (2)

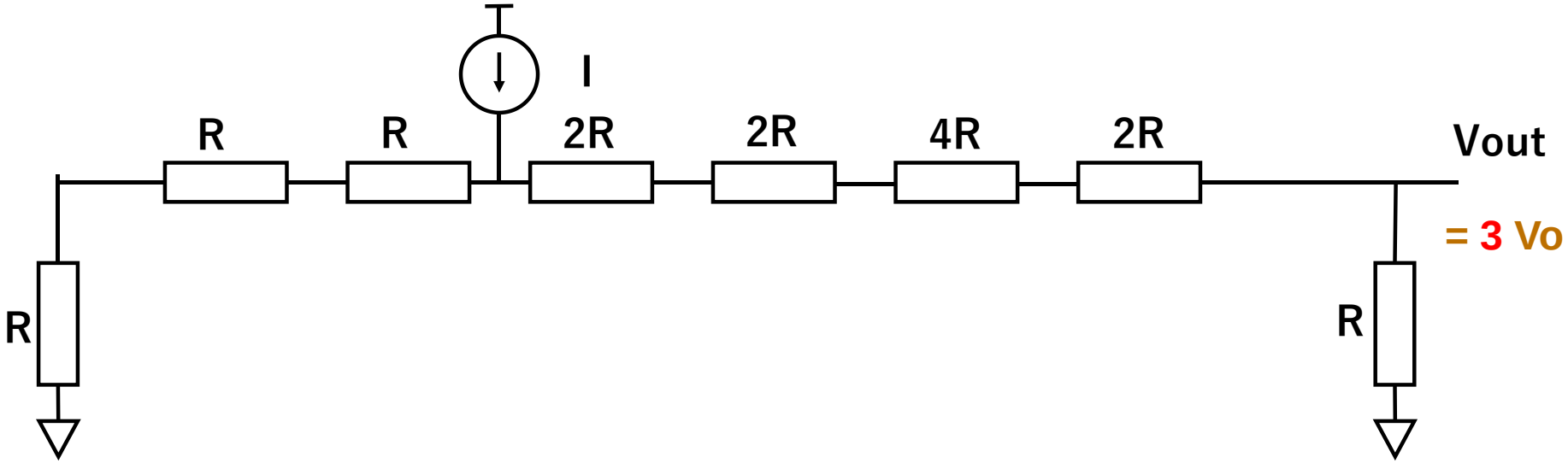
Prime numbers: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ……



Here Vo=(1/14)RI

Proposed Prime Number DAC Operation (3)

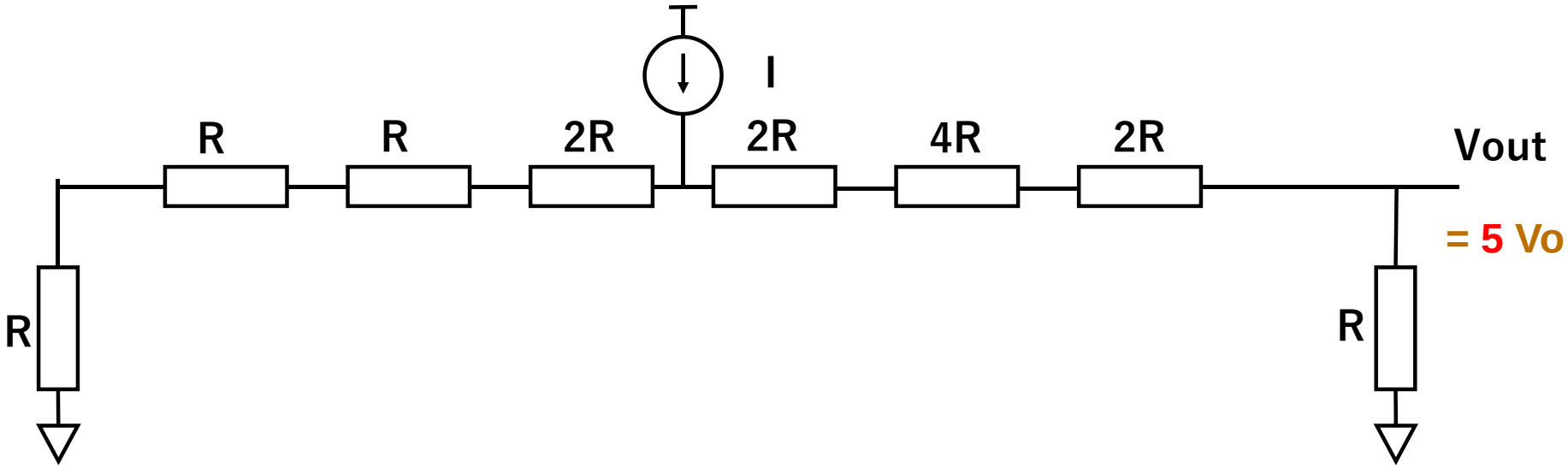
Prime numbers: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ……



Here Vo=(1/14)RI

Proposed Prime Number DAC Operation (4)

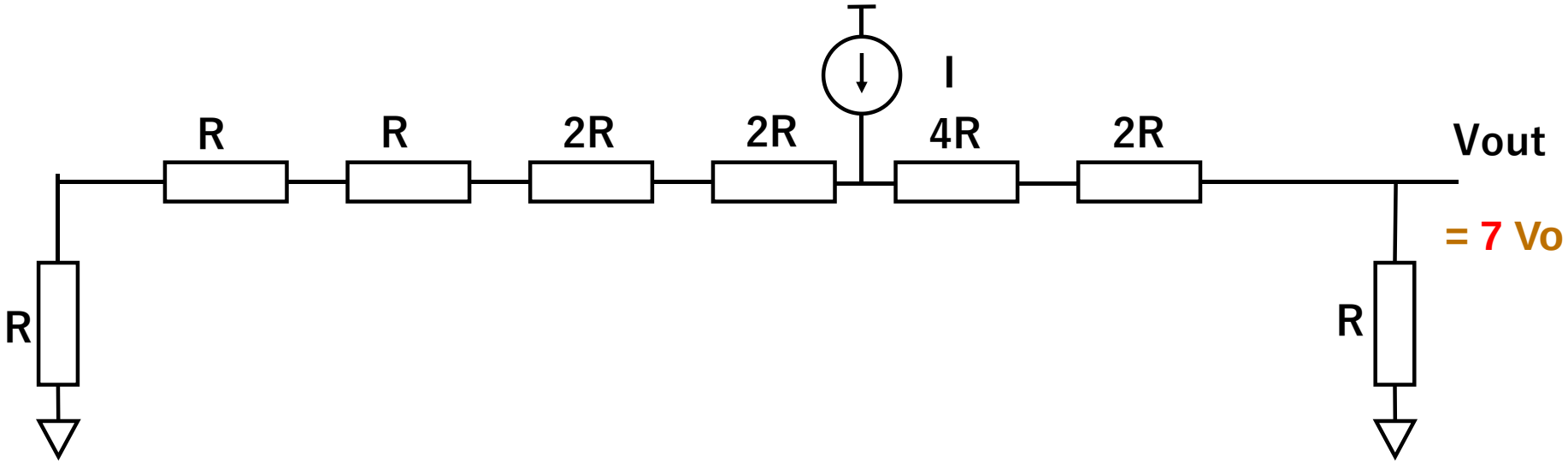
Prime numbers: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ……



Here Vo=(1/14)RI

Proposed Prime Number DAC Operation (5)

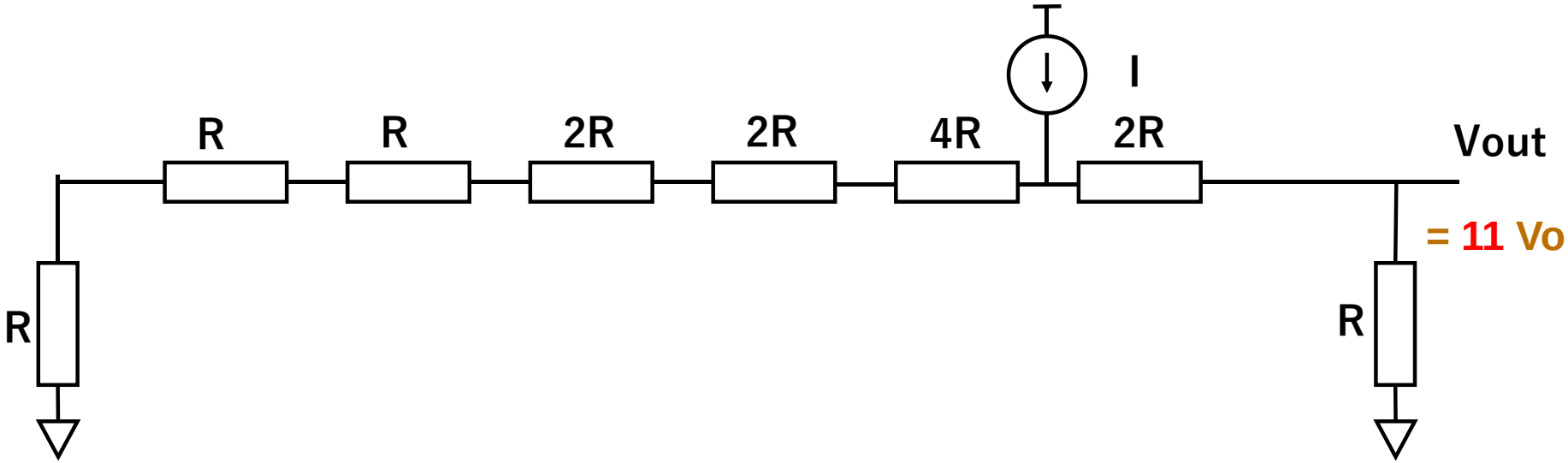
Prime numbers: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ……



Here Vo=(1/14)RI

Proposed Prime Number DAC Operation (6)

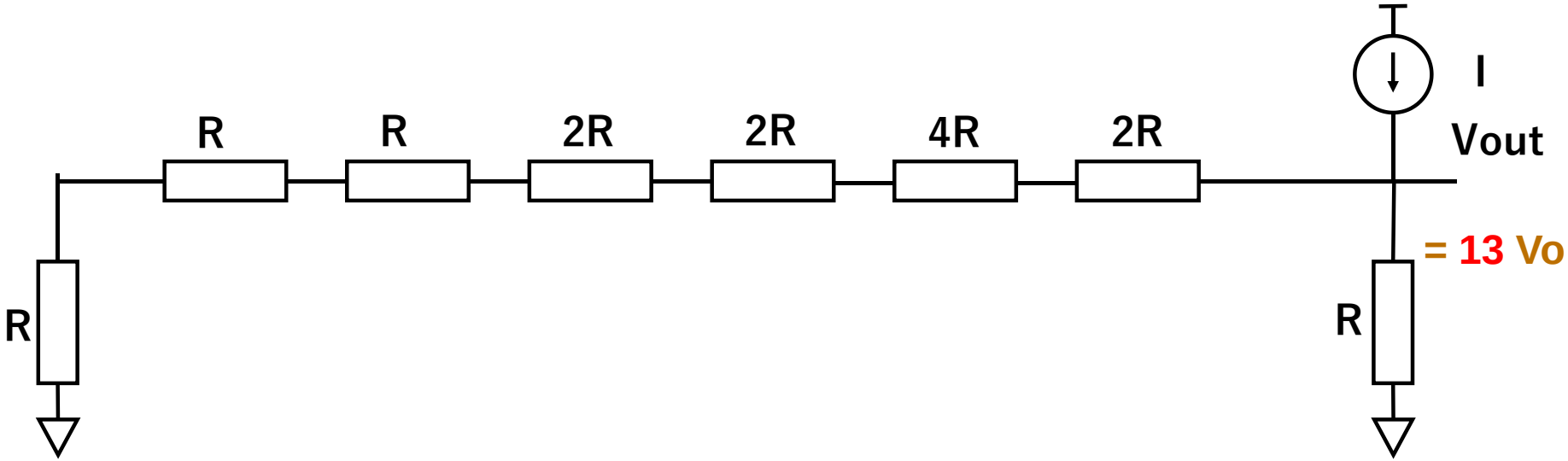
Prime numbers: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ……



Here Vo=(1/14)RI

Proposed Prime Number DAC Operation (7)

Prime numbers: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ……



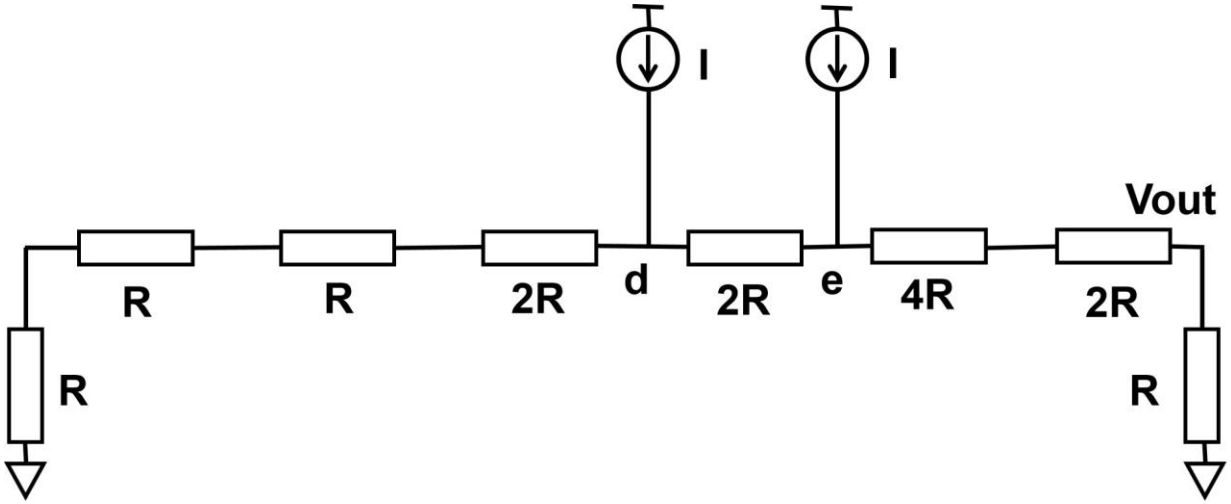
Here Vo=(1/14)RI

Digital Input with Addition of 2 Prime Numbers

Digital Input				Digital Input			
1	←	2:	2	16	←	32:	13+19
2	←	4:	2+2	17	←	34:	17+17
3	←	6:	3+3	18	←	36:	17+19
4	←	8:	3+5	19	←	38:	19+19
5	←	10:	3+7	20	←	40:	17+23
6	←	12:	5+7	21	←	42:	19+23
7	←	14:	7+7	22	←	44:	13+31
8	←	16:	5+11	23	←	46:	23+23
9	←	18:	7+11	24	←	48:	19+29
10	←	20:	7+13	25	←	50:	19+31
11	←	22:	11+11	26	←	52:	23+29
12	←	24:	11+13	27	←	54:	23+31
13	←	26:	13+13	28	←	56:	19+37
14	←	28:	11+17	29	←	58:	29+29
15	←	30:	13+17	30	←	60:	29+31

Prime Number DAC Operation for digital input = 6

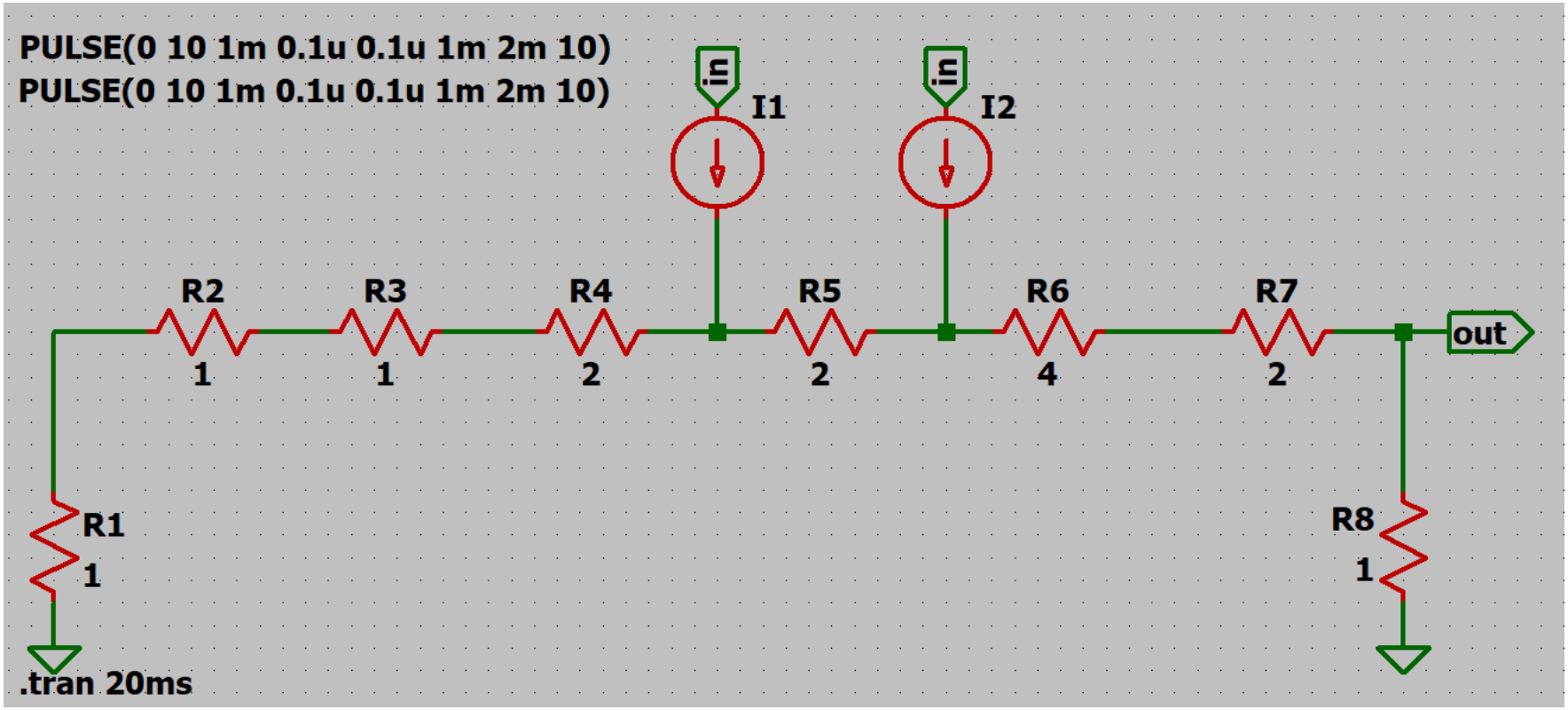
Prime numbers: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ……



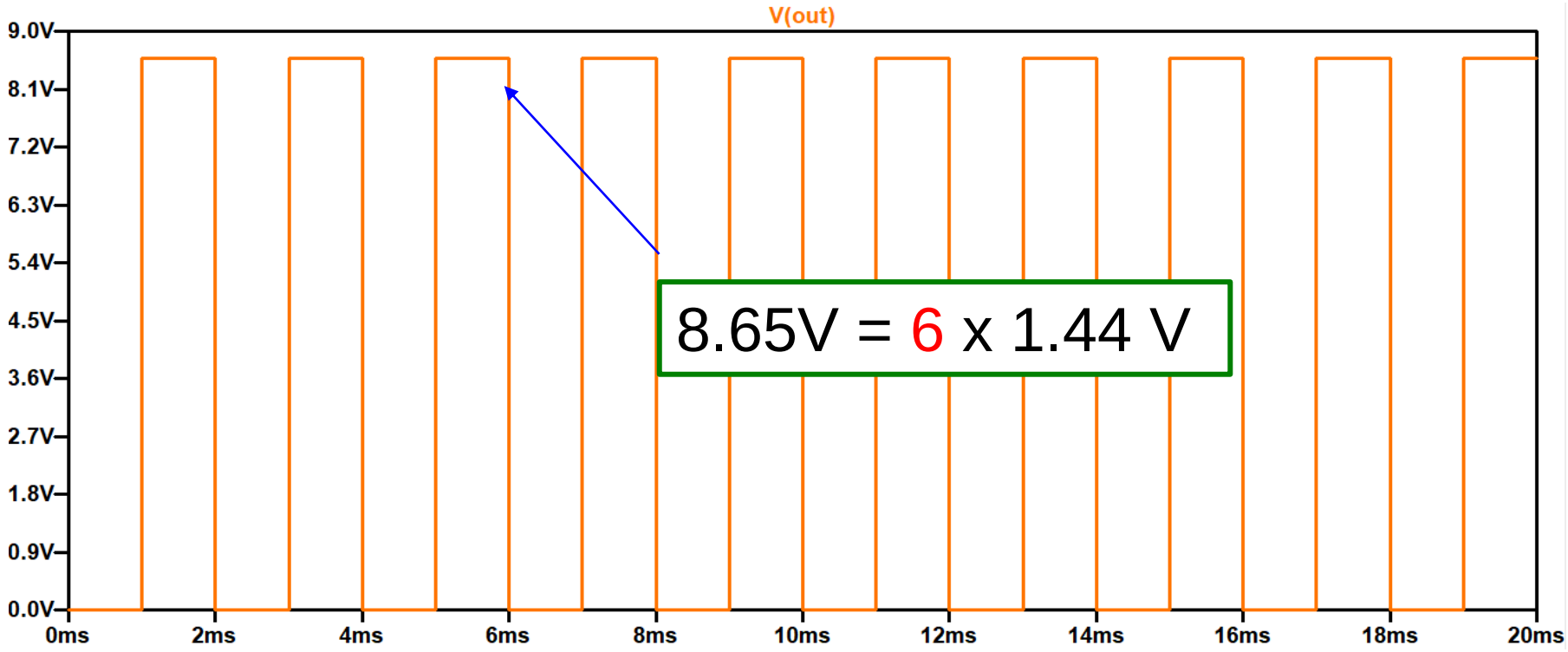
= (5+7) Vo
= 6 (2 Vo)

Here Vo=(1/14)RI

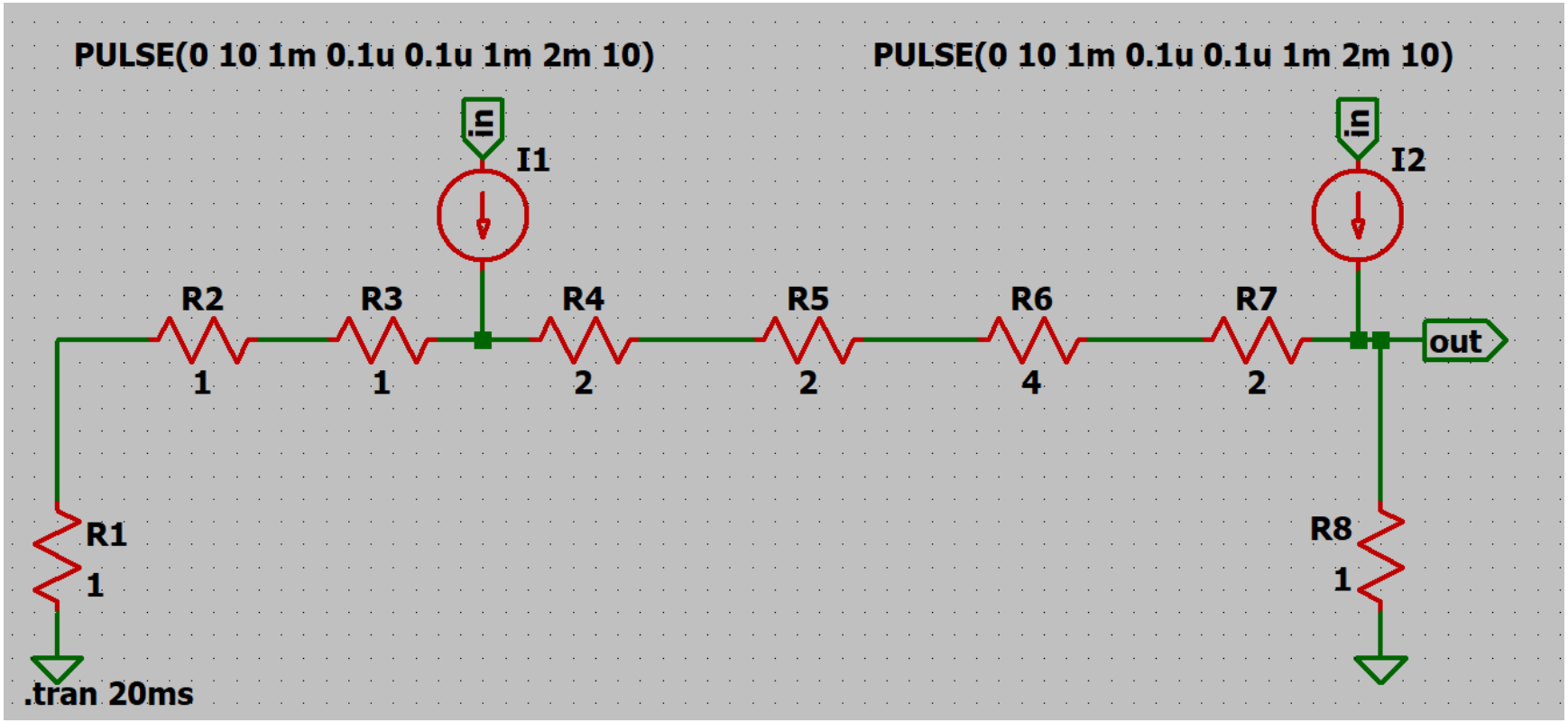
Prime number DAC operation for digital input = 6



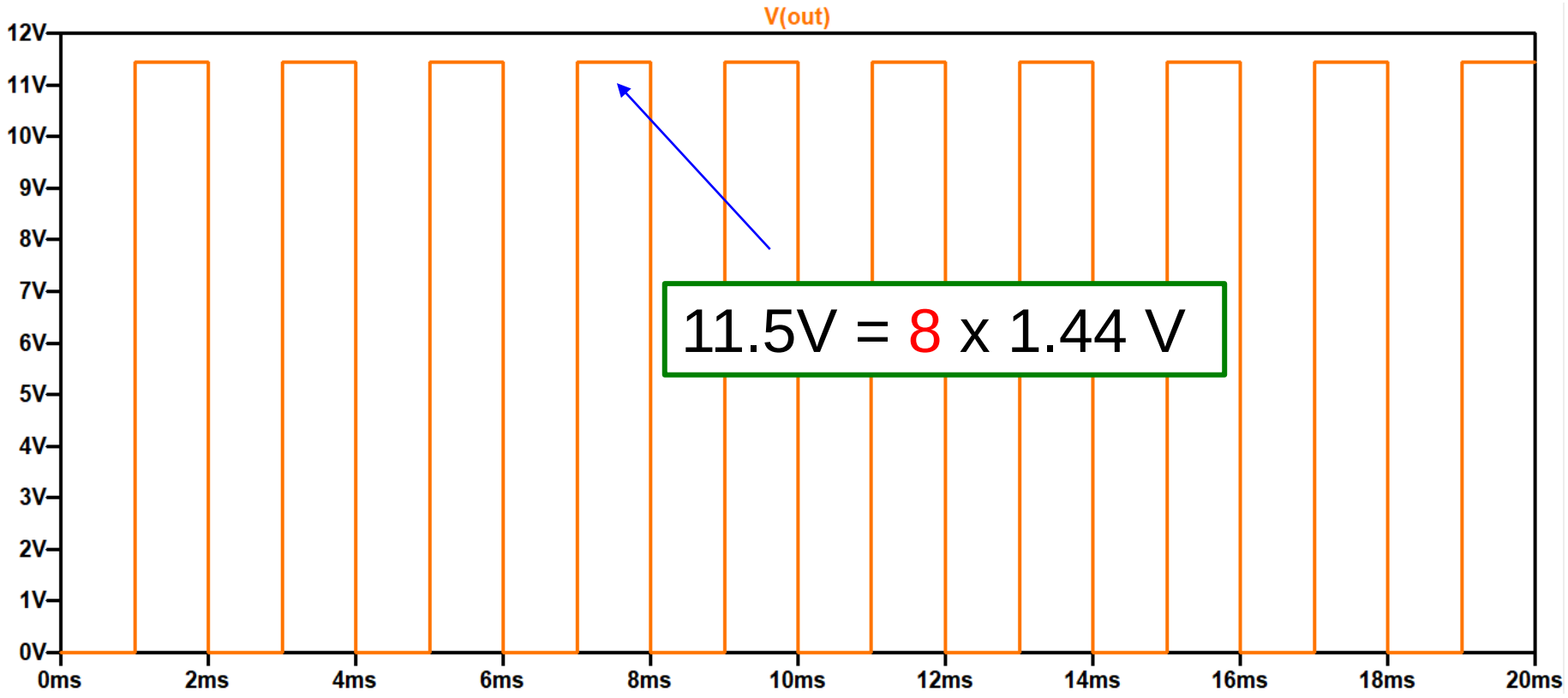
Prime number DAC operation for digital input = 6



In case that digital input is 8



In case that digital input is 8



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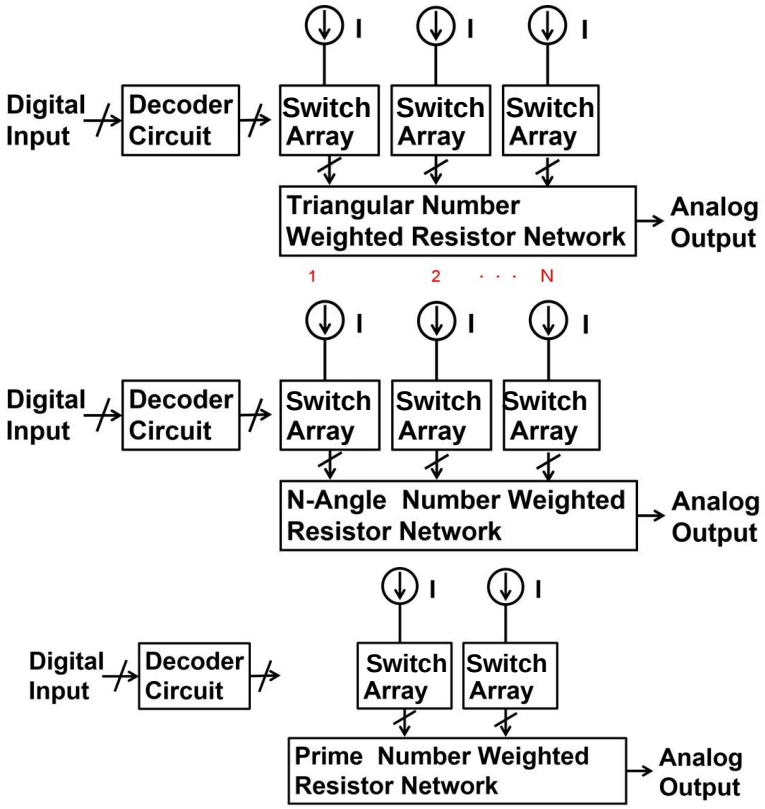
Summary



Triangular number DAC

3 current sources
3 switch arrays

Polygonal number DAC

N current sources
N switch arrays

Prime number DAC

2 current sources
2 switch arrays

Conclusion

- **Completely new DAC architectures based on integer theory**
- **Discussions on their pros and cons are left for the future work.**

Acknowledgements

The authors thank Mr. Minh Tri Tran

for having the simulation together.