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## DESIGN OF ACTIVE INDUCTOR AND STABILITY TEST FOR PASSIVE RLC LOW-PASS FILTERS

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- 1. Research Background
- Reviews of Complex Functions
- Transfer Function and Its Self-loop Function
- Limitations of Conventional Methods
- 2. Analysis of High-Order Transfer Functions
- Behaviors of High-order Passive Transmission Spaces
- Numerical Examples and Design of Active Inductor
- 3. Experimental Results
- Measurements of Self-loop Functions in RLC networks
- Operating region of Active Serial RLC Low-pass Filter
- 4. Conclusions

## **1. Research Background** Motivation of Study

Large overshoots + ringing + unwanted voltage transients

→ Damped oscillation noise
→ Unstable system



Ringing occurs in
both with and without
feedback systems.

**STABILITY TEST** 

 $\odot$  Ringing affects both input and output signals.

#### **Objectives and Achievements**

## **Objectives**

- Investigation of operating region of high-order systems in both time and frequency domains
- → Over-damping (high delay in rising time)
- $\rightarrow$  Critical damping (max power propagation)
- Under-damping (overshoot and ringing)

#### Achievements

 Design of active inductor and measurement of self-loop function in active serial RLC LPF

## **1. Research Background** Approaching Methods

#### Passive RLC Low-pass Filter Vin = 1 Vin = 1Vi

#### **Balun transformer**



#### **Active RLC Low-pass Filter**



#### Implementation of active RLC LPF



## **1. Research Background** Reviews of Complex Functions

**Complex function with frequency variable** 

$$H(\omega) = \operatorname{Re}(\omega) + j\operatorname{Im}(\omega) = \operatorname{Real}\{H(\omega)\} + j\operatorname{Imag}\{H(\omega)\}$$

#### In complex plane domain

 $H(\omega) = \begin{cases} \operatorname{Re}(\omega) = \operatorname{Real}\{H(\omega)\} \\ \operatorname{Im}(\omega) = \operatorname{Imag}\{H(\omega)\} \\ \operatorname{Fre}(\omega) = \operatorname{angular frequency} \end{cases}$ 

#### In spectrum domain

$$H(\omega) = |H(\omega)|e^{j\theta(\omega)}$$
$$|H(\omega)| = \sqrt{\left[\operatorname{Re}\left\{H(\omega)\right\}\right]^{2} + \left[\operatorname{Im}\left\{H(\omega)\right\}\right]^{2}}$$
$$\theta(\omega) = \arctan\left(\frac{\operatorname{Im}\left\{H(\omega)\right\}}{\operatorname{Re}\left\{H(\omega)\right\}}\right)$$

#### OPOlar chart (Nyquist chart)

Magnitude-frequency, angular-frequency plots (Bode plots)
 Magnitude-angular diagrams (Nicholas diagrams)

**Transfer Function and Its Self-loop Function** 

Linear system  
Input 
$$V_{in}(\omega) \longrightarrow H(\omega) \longrightarrow V_{out}(\omega)$$

 $A(\omega)$  : Open loop function

- $H(\omega)$ : Transfer function
- $L(\omega)$  : Self-loop function

**Transfer function** 

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{A(\omega)}{1 + L(\omega)}$$

 $H(\omega) = \frac{A(\omega)}{0} = \infty$ 

**Unstable system** 

**Constraint for oscillation** 

$$1 + L(\omega) = 0 \quad \Longrightarrow \begin{cases} |L(\omega)| = 1 \\ \angle L(\omega) = -180^{\circ} \end{cases} \quad \Leftrightarrow \qquad \begin{array}{c} \text{PHASE MARGIN} \\ \text{AT UNITY GAIN} \end{array}$$

## **1. Research Background** Signal Flow Graph for Transfer Function

#### **Transfer function**

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{A(\omega)}{1 + L(\omega)}$$

#### **Output voltage**

$$V_{out}(\omega) = A(\omega) \left[ V_{in}(\omega) - \frac{L(\omega)}{A(\omega)} V_{out}(\omega) \right]$$

#### **Negative feedback Network**



Signal flow graph

To meet the specified requirements • High stability • Fast transient response, and

Good steady-state performance.



#### **Proposed Comparison Measurement Technique**



Sequence of steps:

- (i) Measurement of open loop function  $A(\omega)$ ,
- (ii) Measurement of transfer function H(ω), and
- (iii) Derivation of self-loop function.

**Proposed Alternating Current Conservation (1)** 

#### Idea: Alternating current is conserved.

**Incident current = Transmitted current** 



**Proposed Alternating Current Conservation (2)** 



#### **Proposed Alternating Current Conservation (3)**

Alternating current conservation using balun transformer



#### **Self-loop function:**

$$L(\omega) = -\frac{V_{inc}}{V_{trans}}$$

#### Balun transformer (10 mH inductance)



## **1. Research Background** Proposed Widened Superposition Principle



• Multi-source systems, feedback networks (op amps, amplifiers), polyphase filters, complex filters...

#### Limitations of Conventional Methods (1)

[7] Middlebrook, R.D., "Measurement of Loop Gain in Feedback Systems", Int. J. Electronics, vol 38, No. 4, pp. 485-512, 1975.

Measurement of loop gain

- Current injection
- Voltage injection





**Current injection method** 



Voltage injection method

→ Difficult to measure self-loop function in analog circuits

#### Limitations of Conventional Methods (2)

[9] A. S. Sedra and K. C. Smith, "Microelectronic Circuits," 6th ed. Oxford University Press, New York, 2010.

Measurement of loop gain

#### Replica measurement



→ Difficult to measure two real different circuits

#### Limitations of Conventional Methods (3)

- o Conventional Superposition:
- →Solving for every source voltage and current, perhaps several times.
- Conventional measurement of loop gain (Middle Brook's)
- → Applying only in feedback systems (switching DC-DC converters).
- Conventional replica measurement of loop gain
- $\rightarrow$ Using two identical networks (difficult in practical measurement).
- **•Conventional Nyquist's stability condition**
- $\rightarrow$  Using in theoretical analysis for feedback systems (Lab simulation).

 Conventional concepts, analysis and measurement of loop gain are not unique.

## **2.** Analysis of High-Order Transfer Functions **Second-order Parallel RLC Low-pass Filter**



**Apply superposition principle at Vout** 

$$V_{out}\left(\frac{1}{R} + \frac{1}{Z_C} + \frac{1}{Z_L}\right) = \frac{V_{in}}{Z_L};$$

**Transfer function & self-loop function:** 

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + a_0 (j\omega)^2 + a_1 j\omega};$$
$$L(\omega) = a_0 (j\omega)^2 + a_1 j\omega;$$

Where:

$$a_0 = LC; \quad a_1 = \frac{L}{R};$$
  

$$\omega_0 = 1/\sqrt{LC};$$
  

$$|Z_L| = \omega_0 L; \quad |Z_C| = 1/\omega_0 C;$$

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**Operating regions** 

• Over-damping:  $\frac{1}{LC} < \left(\frac{R}{2L}\right)^2 \Leftrightarrow |Z_L| = |Z_C| < R/2$ • Critical damping:  $\frac{1}{LC} = \left(\frac{R}{2L}\right)^2 \Leftrightarrow |Z_L| = |Z_C| = R/2$ • Under-damping:  $\frac{1}{LC} > \left(\frac{R}{2L}\right)^2 \Leftrightarrow |Z_L| = |Z_C| > R/2$ 

### **2. Analysis of High-Order Transfer Functions** Behaviors of Second-order Transfer Function

**Second-order transfer function:**  $H(\omega) = \frac{1}{1 + a_0(j\omega)^2 + a_1j\omega}$ 

Case	Over-damped	Critically damped	Under-damped		
<b>Delta</b> (Δ)	$\frac{1}{a_0} < \left(\frac{a_1}{2a_0}\right)^2 \Longrightarrow \Delta = a_1^2 - 4a_0 > 0$	$\frac{1}{a_0} = \left(\frac{a_1}{2a_0}\right)^2 \Longrightarrow \Delta = a_1^2 - 4a_0 = 0$	$\frac{1}{a_0} > \left(\frac{a_1}{2a_0}\right)^2 \Longrightarrow \Delta = a_1^2 - 4a_0 < 0$		
$\begin{array}{c} \textbf{Module} \\  H(\omega)  \end{array}$	$\frac{\frac{1}{a_0}}{\sqrt{\omega^2 + \left(\frac{a_1}{2a_0} - \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}\right)^2}\sqrt{\omega^2 + \left(\frac{a_1}{2a_0} + \sqrt{\left(\frac{a_1}{2a_0}\right)^2 - \frac{1}{a_0}}\right)^2}}$	$\frac{\frac{1}{a_0}}{\left[\omega^2 + \left(\frac{a_1}{2a_0}\right)^2\right]}$	$\boxed{\frac{\frac{1}{a_{0}}}{\sqrt{\left(\omega - \sqrt{\frac{1}{a_{0}} - \left(\frac{a_{1}}{2a_{0}}\right)^{2}}\right)^{2} + \left(\frac{a_{1}}{2a_{0}}\right)^{2}}\sqrt{\left(\omega + \sqrt{\frac{1}{a_{0}} - \left(\frac{a_{1}}{2a_{0}}\right)^{2}}\right)^{2} + \left(\frac{a_{1}}{2a_{0}}\right)^{2}}}$		
Angular $\theta(\omega)$	$-\arctan\left(\frac{\omega}{\left(\frac{a_1}{2a_0}-\sqrt{\left(\frac{a_1}{2a_0}\right)^2-\frac{1}{a_0}}\right)}-\arctan\left(\frac{\omega}{\left(\frac{a_1}{2a_0}+\sqrt{\left(\frac{a_1}{2a_0}\right)^2-\frac{1}{a_0}}\right)}\right)$	$-2 \arctan\left(\frac{2a_0\omega}{a_1}\right)$	$-\arctan\left(\frac{\omega - \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}}{\frac{a_1}{2a_0}}\right) - \arctan\left(\frac{\omega + \sqrt{\frac{1}{a_0} - \left(\frac{a_1}{2a_0}\right)^2}}{\frac{a_1}{2a_0}}\right)$		
$\omega_{cut} = \frac{a_1}{2a_0}$	$ H(\omega_{cut})  < \frac{2a_0}{a_1}$ $\theta(\omega_{cut}) > -\frac{\pi}{2}$	$ H(\omega_{cut})  = \frac{2a_0}{a_1}  \theta(\omega_{cut}) = -\frac{\pi}{2}$	$ H(\omega_{cut})  > \frac{2a_0}{a_1}$ $\theta(\omega_{cut}) < -\frac{\pi}{2}$		

#### **2. Analysis of High-Order Transfer Functions** Example of Second-order Transfer Function

Magnitude of transfer function

•Under-damping:

$$\frac{\sqrt{3}-1}{2} < \omega < \frac{\sqrt{3}+1}{2} \Rightarrow |H_1(\omega)| > 1 \qquad \left(\omega_1 = \frac{\sqrt{3}-1}{2} < \omega_{cut} = 1\right)$$

 $H_1(\omega) = \frac{1}{(j\omega)^2 + j\omega + 1} \Longrightarrow |H_1(\omega)| = \frac{1}{\sqrt{(\omega - \frac{\sqrt{3}}{2})^2 + \frac{1}{\sqrt{(\omega + \frac{1}{2})^2 + \frac{1}{2}}}}}}}}}}}}}}}}}}}}}}}}}$ 

•Critical damping: H

$$H_{2}(\omega) = \frac{1}{(j\omega)^{2} + 2j\omega + 1} \Longrightarrow |H_{2}(\omega)| = \frac{1}{\omega^{2} + 1} \qquad (\omega_{cut} = 1)$$

•Over-damping:

$$H_{3}(\omega) = \frac{1}{\left(j\omega\right)^{2} + 3j\omega + 1} \Rightarrow \left|H_{3}(\omega)\right| = \frac{1}{\sqrt{\omega^{2} + \left(\frac{3-\sqrt{5}}{2}\right)^{2}}\sqrt{\omega^{2} + \left(\frac{3+\sqrt{5}}{2}\right)^{2}}}$$
$$\frac{3-\sqrt{5}}{2} < \omega < \frac{3+\sqrt{5}}{2} \Rightarrow \left|H_{3}(\omega)\right| < 1 \qquad \left(\omega_{1} = \frac{3-\sqrt{5}}{2} < \omega_{cut} = 1\right)$$

### **2. Analysis of High-Order Transfer Functions** Simulations of Second-order Transfer Function



#### Polar chart of transfer function



#### Magnitude response



## **2. Analysis of High-Order Transfer Functions** Behaviors of Second-order Self-loop Function

**Second-order self-loop function:**  $L(\omega) = j\omega [a_0 j\omega + a_1]$ 

Case	Over-damped		Critically damped		Under-damped	
Delta ( $\Delta$ )	$\Delta = a_1^2 - 4a_0 > 0$		$\Delta = a_1^2 - 4a_0 = 0$		$\Delta = a_1^2 - 4a_0 < 0$	
$ L(\omega) $	$\omega \sqrt{\left(a_0 \omega\right)^2 + a_1^2}$		$\omega \sqrt{\left(a_0 \omega\right)^2 + a_1^2}$		$\omega \sqrt{\left(a_0 \omega\right)^2 + a_1^2}$	
θ(ω)	$\frac{\pi}{2} + \arctan \frac{a_0 \omega}{a_1}$		$\frac{\pi}{2}$ + arctan $\frac{a_0\omega}{a_1}$		$\frac{\pi}{2} + \arctan \frac{a_0 \omega}{a_1}$	
$\omega_{\rm l} = \frac{a_{\rm l}}{2a_{\rm o}}\sqrt{\sqrt{5}-2}$	$ L(\omega_1)  > 1$	$\pi - \theta(\omega_1) > 76.3^{\circ}$	$ L(\omega_1)  = 1$	$\pi - \theta(\omega_1) = 76.3^{\circ}$	$ L(\omega_1)  < 1$	$\pi - \theta(\omega_1) < 76.3^{\circ}$
$\omega_2 = \frac{a_1}{2a_0}$	$ L(\omega_2)  > \sqrt{5}$	$\pi - \theta(\omega_2) > 63.4^{\circ}$	$\left L(\omega_2)\right  = \sqrt{5}$	$\pi - \theta(\omega_2) = 63.4^{\circ}$	$\left L(\omega_2)\right  < \sqrt{5}$	$\pi - \theta(\omega_2) < 63.4^{\circ}$
$\omega_3 = \frac{a_1}{a_0}$	$ L(\omega_3)  > 4\sqrt{2}$	$\pi - \theta(\omega_3) > 45^{\circ}$	$\left L(\omega_3)\right  = 4\sqrt{2}$	$\pi - \theta(\omega_3) = 45^\circ$	$\left L(\omega_3)\right  < 4\sqrt{2}$	$\pi - \theta(\omega_3) < 45^{\circ}$

## **2. Analysis of High-Order Transfer Functions** Behaviors of Second-order Self-loop Function

**Second-order self-loop function:**  $L(\omega) = j\omega[a_0j\omega + a_1]$ 

Unity gain of self-loop function

$$\left|L(\omega)\right| = \omega \sqrt{\left(a_0 \omega\right)^2 + a_1^2} = 1$$

Angular frequency at unity gain

$$\omega_1 = \frac{a_1}{2a_0}\sqrt{\sqrt{5}-2}$$

Phase margin at unity gain of self-loop function

•Under-damping: Phase margin =  $\pi - \theta(\omega_1) < 76.3^\circ$ 

•Critical damping: Phase margin =  $\pi - \theta(\omega_1) = 76.3^\circ$ 

• Over-damping: Phase margin =  $\pi - \theta(\omega_1) > 76.3^\circ$ 

## **2. Analysis of High-Order Transfer Functions** Simulations of Second-order Self-loop Function

•Under-damping: 
$$L_1(\omega) = (j\omega)^2 + j\omega;$$

- •**Critical damping:**  $L_2(\omega) = (j\omega)^2 + 2j\omega;$
- •**Over-damping:**  $L_3(\omega) = (j\omega)^2 + 3j\omega;$

#### Polar chart of self-loop function



#### Magnitude response



## 2. Analysis of High-Order Transfer Functions Summary of Second-order System

#### Magnitude response of transfer function



#### **Transient response**



Magnitude-angular response of self-loop function



#### Over-damping: →Phase margin is 88 degrees.

**Critical damping:** 

→Phase margin is 76.3 degrees.
Under-damping:

 $\rightarrow$  Phase margin is 52 degrees.

## 2. Analysis of High-Order Transfer Functions Mathematical Model of Ideal Op Amp

Ideal op amp



Open-loop function  $A(\omega)$  of op amp

$$A(\omega) = \frac{V_{out}}{V_{in+} - V_{in-}} = \frac{A_0}{1 + \frac{j\omega}{\omega_{bw}}}$$

Gain-bandwidth (GBW), bandwidth fbw

Equivalent model of op amp

 $A(\omega)$ 

 $R_{o}$ 

 $V_{out}$ 

V<sub>in+</sub>

Vin-

Rin



Here, GBW =10 MHz, DC gain Ao = 100000

**Open-loop function and self-loop function** 

$$A(\omega) = \frac{10^5}{1 + j\frac{\omega}{200\pi}}; L(\omega) = j\frac{\omega}{200\pi} = 10^5 \frac{V_{in}}{V_{out}} - 10^5 \frac{V_{in}}$$

## **2. Analysis of High-Order Transfer Functions** Behavior of Open-loop Function of Ideal Op Amp

Open-loop function  $A(\omega)$ 

$$A(\omega) = \frac{10^5}{1 + j\frac{\omega}{200\pi}}$$

#### Nyquist plot of open-loop function





#### **Bode plots of open-loop function**

#### **2. Analysis of High-Order Transfer Functions** Behavior of Self-loop Function of Ideal Op Amp



## 2. Analysis of High-Order Transfer Functions Analysis of Active Inductor



Apply superposition principle at V3

 $V_3\left(\frac{1}{R_2} + \frac{1}{Z_C}\right) = \frac{V_2}{R_2} + \frac{V_4}{Z_C}$  Here,  $V_1 = V_3 = V_5$ 

Approximated value of active inductor

$$Z_{L} = \frac{R_{2}}{R_{1}} \frac{R_{3}}{Z_{C}} Z_{out} = \frac{R_{2}R_{3}}{R_{1}} sCZ_{out}$$

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## **2. Analysis of High-Order Transfer Functions** Simulations of Passive & Active RLC Low-pass Filters



## **3. Proposed Designs and Experimental Results** Implementation of Second-order Serial RLC LPF

Under-shoot occurred at both input and output ports.



#### **Device under test**

#### **3. Proposed Designs and Experimental Results** Measured Transfer Function in Serial RLC LPF



## **3. Proposed Designs and Experimental Results Measured Self-loop Function in Serial RLC LPF**



**Over-damping:**  $\rightarrow$  Phase margin is 80 degrees. Nearly Critical damping:  $\rightarrow$  Phase margin is 75 degrees. **Under-damping:**  $\rightarrow$  Phase margin is 55 degrees.

#### Magnitude response

## **3. Proposed Designs and Experimental Results** Implementation of Active RLC Low-pass Filter

#### **Over-shoot occurred at output port.**



Device under test

#### **3. Proposed Designs and Experimental Results** Measured Transfer Function of Active RLC LPF





## **3. Proposed Designs and Experimental Results** Measured Self-loop Function of Active RLC LPF



Over-damping: →Phase margin is 84 degrees. Nearly Critical damping: →Phase margin is 76 degrees. Under-damping: →Phase margin is 65 degrees.

## 4. Conclusions



#### This work:

- Reviews of complex functions and stability test
- Proposed methods for derivation of transfer function and measurement of self-loop function
- Implementations and measurements of self-loop functions for passive and active second-order RLC lowpass filters
- Theoretically, if phase margin is smaller than 76.3degrees, overshoot occurs in second-order systems.

#### Future of work:

• Stability test for polyphase filters & complex filters

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# Thank you very much!







